Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/1.1.2.2-c-x-^m-a+b-x^2-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1071]. This is test number [19].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (1071)	% 0.00 (0)
Mathematica	% 100.00 (1071)	% 0.00 (0)
Maple	% 70.49 (755)	% 29.51 (316)
Maxima	% 59.01 (632)	% 40.99 (439)
Fricas	% 62.93 (674)	% 37.07 (397)
Sympy	% 94.58 (1013)	% 5.42 (58)
Giac	% 57.52 (616)	% 42.48 (455)
Mupad	% 64.89 (695)	% 35.11 (376)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

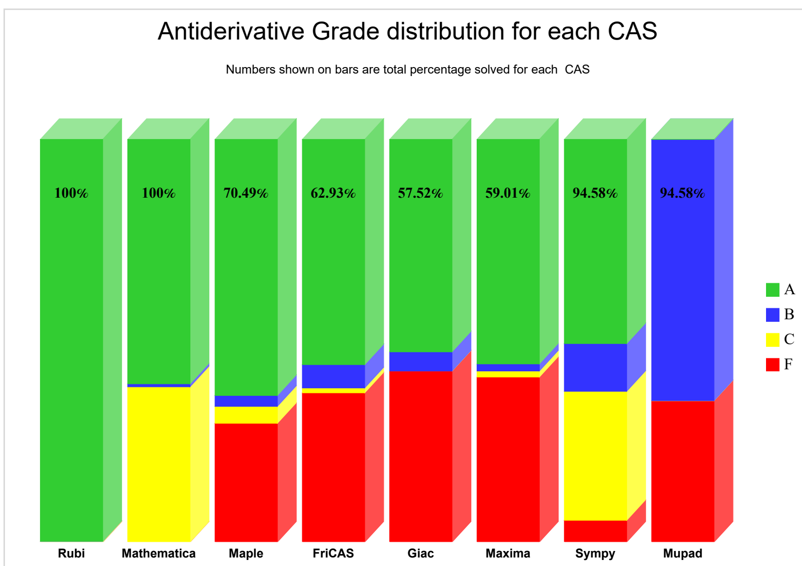
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

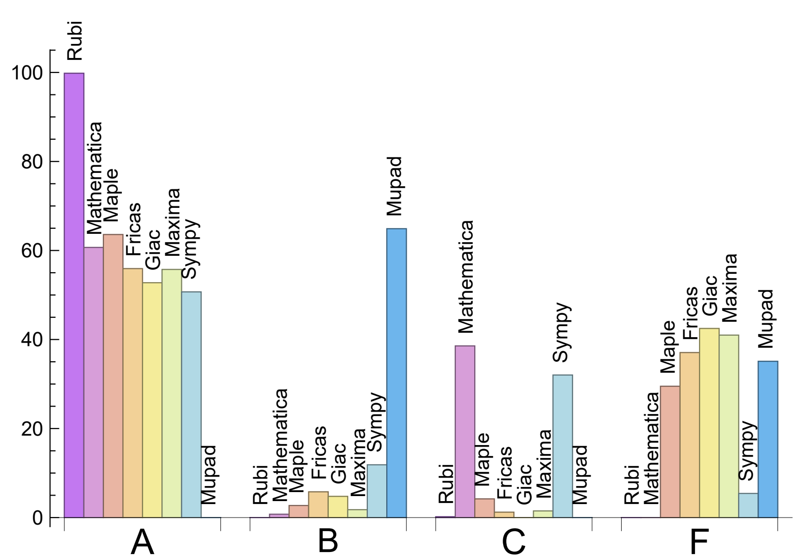
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.81	0.00	0.19	0.00
Mathematica	60.69	0.75	38.56	0.00
Maple	63.59	2.71	4.20	29.51
Maxima	55.74	1.77	1.49	40.99
Fricas	55.93	5.79	1.21	37.07
Sympy	50.70	11.86	32.03	5.42
Giac	52.75	4.76	0.00	42.48
Mupad	0.00	64.89	0.00	35.11

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	316	100.00 %	0.00 %	0.00 %
Maxima	439	98.86 %	0.00 %	1.14 %
Fricas	397	94.21 %	5.79 %	0.00 %
Sympy	58	0.00 %	98.28 %	1.72 %
Giac	455	100.00 %	0.00 %	0.00 %
Mupad	376	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

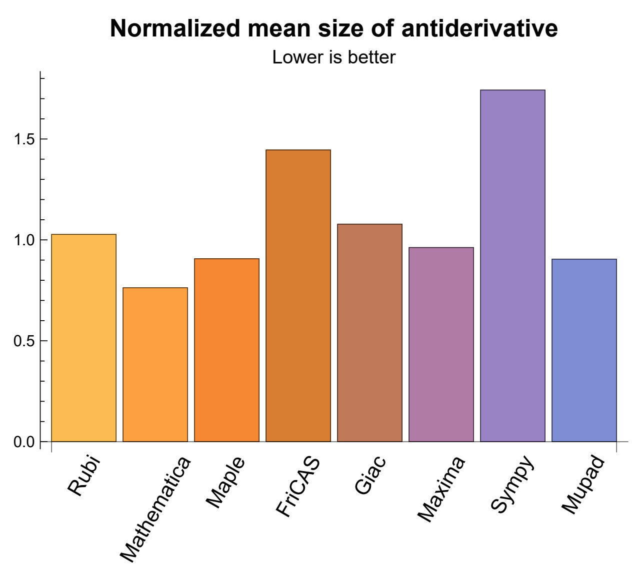
1.3 Performance

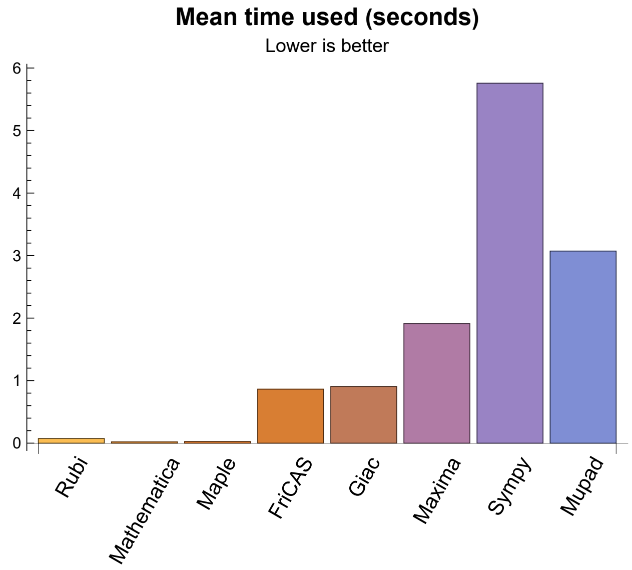
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.07	113.00	1.03	78.00	1.00
Mathematica	0.02	55.70	0.76	51.00	0.75
Maple	0.03	69.55	0.91	49.00	0.84
Maxima	1.91	69.75	0.96	55.00	0.88
Fricas	0.86	108.34	1.45	67.00	1.11
Sympy	5.76	129.84	1.74	46.00	0.94
Giac	0.91	74.24	1.08	57.00	0.88
Mupad	3.07	60.01	0.90	41.00	0.83

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
```

```
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))
```

```
except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

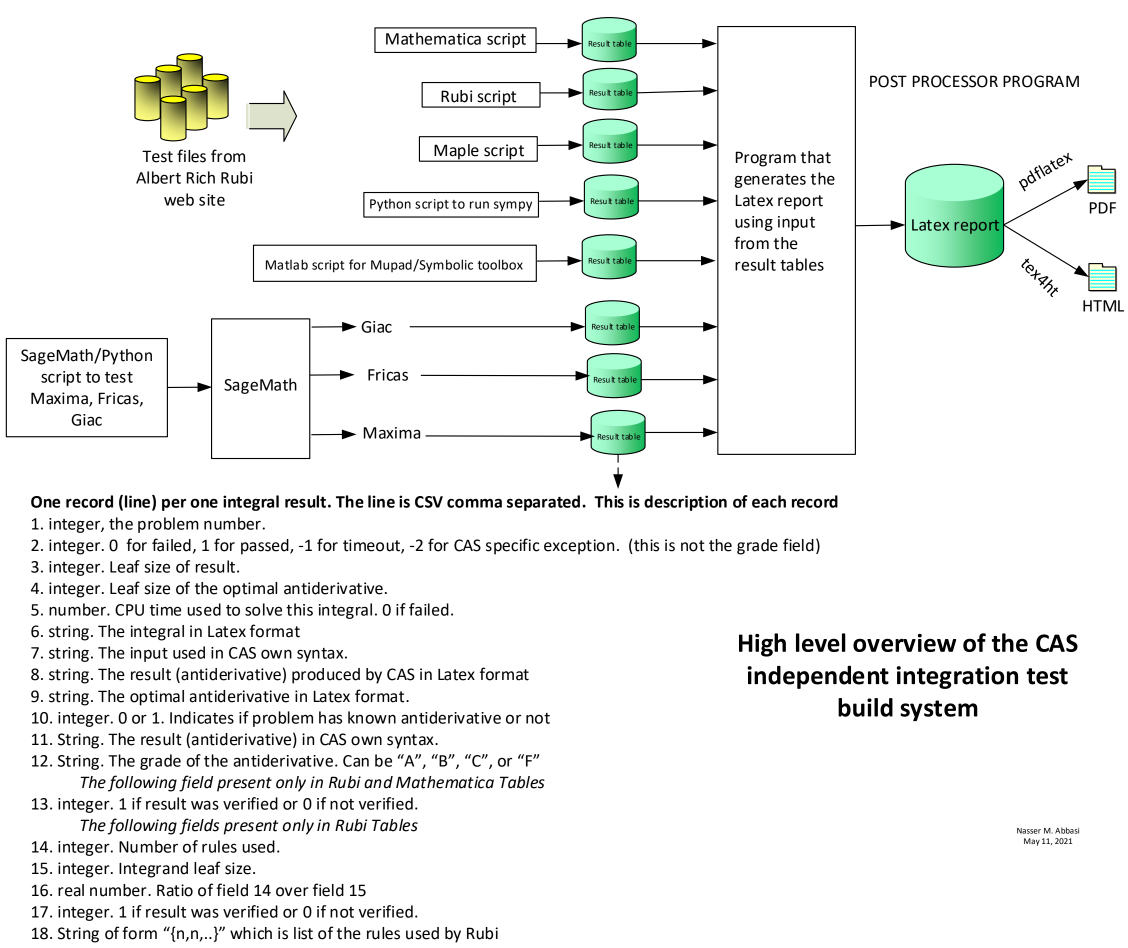
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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B grade: { }

C grade: { 662, 664 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 296, 298, 300, 304, 306, 308, 312, 313, 315, 317, 321, 323, 325, 329, 331, 333, 337, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 378, 379, 380, 383, 384, 385, 386, 387, 388, 389, 390, 391, 394, 397, 398, 399, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 422, 425, 426, 427, 428, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 496, 497, 498, 499, 500, 501, 503, 505, 506, 507, 508, 509, 510, 511, 512, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 529, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 554, 555, 556, 557, 558, 560, 561, 562, 563, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 665, 666, 667, 668, 669, 677, 678, 679, 680, 681, 689, 690, 691, 692, 693, 701, 702, 703, 704, 705, 706, 714, 715, 716, 717, 718, 726, 727, 728, 729, 742, 743, 744, 745, 751, 752, 753, 759, 760, 761, 768, 769, 770, 775, 776, 777, 778, 784, 785, 786, 884, 885, 886, 887, 928, 929, 930, 931, 940, 941, 942, 943, 944, 945, 946, 947, 948, 955, 956, 957, 958, 959, 970, 971, 972, 973, 974, 980, 981, 982, 983, 984, 987, 988, 989, 990, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071 }

B grade: { 38, 65, 90, 101, 102, 196, 197, 559 }

C grade: { 293, 294, 295, 297, 299, 301, 302, 303, 305, 307, 309, 310, 311, 314, 316, 318, 319, 320, 322, 324, 326, 327, 328, 330, 332, 334, 335, 336, 338, 359, 360, 374, 376, 377, 381, 382, 392, 393, 395, 396, 400, 401, 402, 418, 419, 420, 421, 423, 424, 429, 430, 431, 432, 433, 451, 462, 473, 484, 495, 502, 504, 513, 515, 528, 530, 542, 553, 564, 575, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 661, 662, 663, 664, 670, 671, 672, 673, 674, 675, 676, 682, 683, 684, 685, 686, 687, 688, 694, 695, 696, 697, 698, 699, 700, 707, 708, 709, 710, 711, 712, 713, 719, 720, 721, 722, 723, 724, 725, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 746, 747, 748, 749, 750, 754, 755, 756, 757, 758, 762, 763, 764, 765, 766, 767, 771, 772, 773, 774, 779, 780, 781, 782, 783, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 932, 933, 934, 935, 936, 937, 938, 939, 949, 950, 951, 952, 953, 954, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 975, 976, 977, 978, 979, 985, 986, 991, 992, 993, 994, 995, 996, 997, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1061, 1063 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 343, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 608, 610, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 638, 642, 644, 646, 648, 650, 661, 663, 665, 666, 667, 668, 677, 678, 679, 680, 689, 690, 691, 692, 701, 702, 703, 704, 705, 714, 715, 716, 717, 726, 727, 728, 729, 742, 743, 744, 745, 759, 760, 761, 775, 776, 777, 778, 928, 929, 930, 931, 940, 941, 942, 943, 946, 947, 948, 957, 958, 959, 972, 973, 974, 982, 983, 984, 987, 988, 989, 990, 1039, 1040, 1041, 1042, 1061, 1063, 1065 }

B grade: { 33, 38, 58, 65, 90, 91, 101, 102, 196, 197, 198, 337, 339, 340, 341, 342, 422, 433, 607, 609, 611, 637, 639, 640, 641, 643, 645, 647, 649 }

C grade: { 352, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 887, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 915 }

F grade: { 344, 345, 346, 347, 348, 349, 350, 351, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 662, 664, 669, 670, 671, 672, 673, 674, 675, 676, 681, 682, 683, 684, 685, 686, 687, 688, 693, 694, 695, 696, 697, 698, 699, 700, 706, 707, 708, 709, 710, 711, 712, 713, 718, 719, 720, 721, 722, 723, 724, 725, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 884, 885, 886, 888, 889, 890, 912, 913, 914, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 932, 933, 934, 935, 936, 937, 938, 939, 944, 945, 949, 950, 951, 952, 953, 954, 955, 956, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 975, 976, 977, 978, 979, 980, 981, 985, 986, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1066, 1067, 1068, 1069, 1070, 1071 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 200, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 478, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 518, 520, 522, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 567, 569, 572, 574, 576, 577, 578, 580, 581, 582, 584, 585, 586, 588, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 677, 678, 679, 680, 681, 682, 683, 689, 690, 691, 692, 693, 694, 695, 701, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 726, 727, 728, 729, 730, 731, 732, 742, 745, 775, 778, 1039, 1040, 1041, 1042, 1061, 1063, 1065 }

B grade: { 38, 65, 90, 101, 102, 174, 196, 197, 198, 199, 201, 202, 203, 312, 337, 517, 519, 521, 523 }

C grade: { 475, 477, 479, 480, 481, 482, 484, 566, 568, 570, 571, 573, 575, 579, 583, 587 }

F grade: { 257, 258, 261, 262, 263, 344, 345, 346, 347, 348, 349, 350, 351, 352, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 696, 697, 698, 699, 700, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 733, 734, 735, 736, 737, 738, 739, 740, 741, 743, 744, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 776, 777, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1066, 1067, 1068, 1069, 1070, 1071 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 200, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 342, 343, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 478, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514, 515, 516, 517, 518, 519, 520, 522, 523, 525, 527, 529, 530, 531, 532, 533, 534, 535, 536, 539, 540, 541, 542, 543, 544, 545, 546, 547, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 567, 569, 572, 574, 578, 579, 581, 582, 583, 584, 585, 586, 587, 588, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 677, 678, 679, 680, 681, 682, 683, 689, 690, 691, 693, 694, 695, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 720, 726, 727, 728, 729, 730, 732, 742, 743, 744, 745, 759, 760, 761, 775, 776, 777, 778, 928, 929, 930, 931, 940, 941, 942, 943, 946, 947, 948, 955, 956, 957, 958, 959, 972, 973, 974, 982, 983, 984, 987, 988, 989, 990, 1039, 1040, 1041, 1042, 1061, 1063, 1065 }

B grade: { 33, 38, 58, 65, 90, 91, 101, 102, 174, 196, 197, 198, 199, 201, 202, 203, 204, 205, 206, 207, 208, 221, 248, 250, 252, 263, 312, 335, 337, 338, 339, 340, 341, 372, 383, 390, 403, 415, 416, 434, 435, 510, 511, 521, 524, 526, 528, 537, 538, 548, 576, 577, 580, 692, 701, 718, 719, 731, 944, 945, 985, 986 }

C grade: { 475, 477, 479, 480, 481, 482, 484, 566, 568, 570, 571, 573, 575 }

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2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 134, 136, 138, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 200, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 229, 231, 232, 233, 234, 235, 236, 237, 238, 242, 243, 244, 245, 247, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 312, 313, 314, 315, 316, 317, 318, 319, 320, 338, 339, 340, 341, 342, 343, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 441, 442, 443, 444, 446, 447, 448, 449, 451, 452, 453, 454, 455, 457, 458, 459, 460, 462, 463, 464, 465, 466, 468, 470, 471, 473, 474, 476, 479, 481, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 503, 504, 507, 509, 511, 518, 520, 522, 524, 526, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 576, 577, 578, 580, 581, 582, 584, 585, 586, 588, 608, 610, 612, 638, 640, 642, 644, 646, 648, 668, 672, 673, 674, 675, 676, 679, 680, 684, 685, 686, 687, 688, 689, 690, 691, 692, 696, 697, 698, 699, 700, 705, 709, 710, 711, 712, 713, 717, 721, 722, 723, 724, 725, 728, 729, 733, 734, 735, 736, 737, 775, 776, 946, 947, 957, 958, 972, 973, 982, 983, 987, 988, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042 }

B grade: { 17, 33, 38, 57, 58, 65, 89, 90, 91, 101, 102, 131, 133, 135, 137, 139, 141, 156, 158, 174, 186, 196, 197, 198, 199, 201, 202, 203, 204, 228, 230, 239, 240, 241, 246, 248, 257, 258, 261, 262, 263, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 365, 367, 368, 383, 384, 385, 386, 403, 404, 405, 406, 407, 408, 434, 435, 436, 437, 438, 439, 440, 445, 450, 456, 461, 467, 472, 478, 502, 505, 506, 508, 510, 512, 513, 514, 515, 516, 517, 519, 521, 523, 525, 527, 528, 529, 530, 531, 649, 665, 666, 667, 677, 678, 701, 702, 703, 704, 714, 715, 716, 726, 727, 742, 928, 929, 940, 941, 974, 989 }

C grade: { 344, 345, 346, 347, 348, 349, 350, 351, 352, 469, 475, 477, 480, 482, 483, 484, 571, 572, 573, 574, 575, 579, 583, 587, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 609, 611, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 639, 641, 643, 645, 647, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 669, 670, 671, 681, 682, 683, 693, 694, 695, 706, 707, 708, 718, 719, 720, 730, 731, 732, 739, 740, 741, 746, 747, 748, 749, 751, 752, 753, 755, 756, 757, 758, 763, 764, 765, 768, 769, 770, 773, 774, 779, 780, 781, 782, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 925, 926, 927, 932, 933, 934, 935, 937, 938, 939, 944, 945, 949, 950, 951, 952, 953, 954, 955, 956, 960, 961, 962, 963, 964, 965, 966, 967, 968, 970, 971, 975, 976, 977, 978, 980, 981, 984, 985, 986, 992, 993, 994, 995, 996, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1053, 1054, 1058, 1059, 1068 }

F grade: { 296, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 738, 743, 744, 745, 750, 754, 759, 760, 761, 762, 766, 767, 771, 772, 777, 778, 783, 924, 930, 931, 936, 942, 943, 948, 959, 969, 979, 990, 991, 997, 1050, 1051, 1052, 1055, 1056, 1057, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1069, 1070, 1071 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 343, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 441, 442, 443, 444, 445, 446, 447, 448, 449, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 473, 474, 476, 478, 480, 482, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 567, 569, 571, 573, 575, 576, 578, 579, 580, 582, 583, 584, 585, 586, 587, 588, 665, 666, 667, 668, 669, 670, 671, 677, 678, 679, 680, 681, 682, 683, 689, 690, 691, 692, 693, 694, 695, 701, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 726, 727, 728, 729, 730, 731, 732, 1042 }

B grade: { 38, 65, 90, 101, 102, 196, 197, 312, 337, 339, 340, 341, 342, 365, 366, 367, 368, 382, 383, 384, 385, 386, 402, 403, 404, 405, 406, 407, 408, 432, 433, 434, 435, 436, 437, 438, 439, 440, 450, 461, 472, 537, 538, 549, 550, 552, 577, 581, 1039, 1040, 1041 }

C grade: { }

F grade: { 344, 345, 346, 347, 348, 349, 350, 351, 352, 475, 477, 479, 481, 483, 566, 568, 570, 572, 574, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 696, 697, 698, 699, 700, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 353, 354, 355, 356, 357, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 380, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 399, 400, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 428, 429, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 465, 467, 468, 469, 470, 471, 472, 473, 474, 476, 478, 479, 480, 481, 482, 483, 484, 485, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 504, 505, 507, 509, 510, 511, 512, 513, 514, 515, 516, 518, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 661, 663, 665, 666, 667, 668, 669, 670, 671, 674, 675, 677, 678, 679, 680, 681, 682, 683, 686, 687, 689, 690, 691, 692, 693, 694, 695, 698, 699, 701, 702, 703, 704, 705, 706, 707, 708, 711, 712, 714, 715, 716, 717, 718, 719, 720, 723, 724, 726, 727, 728, 729, 730, 731, 732, 735, 736, 789, 790, 795, 796, 801, 802, 807, 808, 811, 812, 813, 814, 818, 819, 825, 826, 832, 833, 839, 840, 846, 847, 853, 854, 857, 858, 859, 860, 861, 862, 866, 867, 873, 874, 880, 881, 887, 888, 894, 895, 901, 902, 908, 909, 915, 916, 928, 929, 930, 931, 940, 941, 942, 943, 946, 947, 948, 957, 958, 959, 972, 973, 974, 982, 983, 984, 987, 988, 989, 990, 1013, 1014, 1021, 1022, 1028, 1029, 1035, 1036, 1039, 1040, 1041, 1042, 1048, 1049, 1061, 1063, 1065 }

C grade: { }

F grade: { 344, 345, 346, 347, 348, 349, 350, 351, 352, 361, 362, 378, 379, 382, 397, 398, 401, 402, 425, 426, 427, 430, 431, 432, 433, 464, 466, 475, 477, 486, 497, 506, 508, 517, 519, 555, 566, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 662, 664, 672, 673, 676, 684, 685, 688, 696, 697, 700, 709, 710, 713, 721, 722, 725, 733, 734, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 791, 792, 793, 794, 797, 798, 799, 800, 803, 804, 805, 806, 809, 810, 815, 816, 817, 820, 821, 822, 823, 824, 827, 828, 829, 830, 831, 834, 835, 836, 837, 838, 841, 842, 843, 844, 845, 848, 849, 850, 851, 852, 855, 856, 863, 864, 865, 868, 869, 870, 871, 872, 875, 876, 877, 878, 879, 882, 883, 884, 885, 886, 889, 890, 891, 892, 893, 896, 897, 898, 899, 900, 903, 904, 905, 906, 907, 910, 911, 912, 913, 914, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 932, 933, 934, 935, 936, 937, 938, 939, 944, 945, 949, 950, 951, 952, 953, 954, 955, 956, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 975, 976, 977, 978, 979, 980, 981, 985, 986, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1015, 1016, 1017, 1018, 1019, 1020, 1023, 1024, 1025, 1026, 1027, 1030, 1031, 1032, 1033, 1034, 1037, 1038, 1043, 1044, 1045, 1046, 1047, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1066, 1067, 1068, 1069, 1070, 1071 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.000	1.315	0.537	0.060	1.094	0.029

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.000	1.353	0.706	0.059	0.915	0.021

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.000	1.344	0.739	0.058	1.114	0.020

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.82	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.000	1.336	0.707	0.059	1.026	0.021

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.002	0.000	0.000	1.344	0.816	0.056	0.943	0.016

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	11	10	14	11
normalized size	1	1.00	1.00	0.92	1.08	0.85	0.77	1.08	0.85
time (sec)	N/A	0.004	0.001	0.004	1.345	0.805	0.083	1.024	0.021

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	13	5	10	10
normalized size	1	1.00	1.00	1.10	1.00	1.30	0.50	1.00	1.00
time (sec)	N/A	0.004	0.001	0.004	1.333	0.562	0.079	0.984	0.023

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	17	10	20	11
normalized size	1	1.00	1.00	0.92	1.08	1.31	0.77	1.54	0.85
time (sec)	N/A	0.004	0.003	0.005	1.363	0.845	0.107	1.063	4.925

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.005	0.002	0.006	1.287	0.633	0.115	1.107	0.026

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.005	0.002	0.004	1.333	0.823	0.119	0.992	0.024

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15
normalized size	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.005	0.002	0.006	1.303	0.907	0.133	1.085	0.027

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15
normalized size	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.005	0.002	0.006	1.418	0.832	0.135	1.034	0.026

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.018	0.001	0.001	1.341	0.612	0.065	1.071	0.037

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.011	0.001	0.001	1.312	0.690	0.065	1.292	0.031

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.018	0.001	0.002	1.354	0.620	0.064	1.044	0.031

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.010	0.001	0.001	1.347	0.738	0.066	1.129	0.034

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	14	24	24	14	24
normalized size	1	1.00	1.00	1.56	0.88	1.50	1.50	0.88	1.50
time (sec)	N/A	0.002	0.002	0.000	1.349	0.553	0.065	1.052	0.031

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
normalized size	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.007	0.001	0.001	1.311	0.495	0.064	1.052	0.029

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	24	21	20	24	21
normalized size	1	1.00	1.00	0.96	1.04	0.91	0.87	1.04	0.91
time (sec)	N/A	0.013	0.001	0.003	1.393	0.559	0.098	1.067	0.033

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	22
normalized size	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92
time (sec)	N/A	0.010	0.001	0.004	1.356	0.855	0.097	1.072	0.032

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	24	27	24	32	23
normalized size	1	1.00	1.00	0.89	0.89	1.00	0.89	1.19	0.85
time (sec)	N/A	0.014	0.001	0.007	1.387	0.813	0.131	1.139	4.937

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	22	26	22	22	24
normalized size	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.04
time (sec)	N/A	0.009	0.001	0.006	1.319	0.862	0.139	0.877	0.026

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	26	28	24	34	24
normalized size	1	1.00	1.00	0.96	1.08	1.17	1.00	1.42	1.00
time (sec)	N/A	0.013	0.001	0.007	1.277	0.798	0.172	1.145	0.043

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	26	26	27	26	25
normalized size	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.89
time (sec)	N/A	0.010	0.001	0.004	1.304	0.791	0.175	1.050	0.034

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	25	24	24	26	24	26
normalized size	1	1.00	1.58	1.32	1.26	1.26	1.37	1.26	1.37
time (sec)	N/A	0.003	0.001	0.006	1.376	0.847	0.187	1.107	0.034

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
normalized size	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.010	0.001	0.004	1.381	0.794	0.197	1.060	0.036

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
normalized size	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.014	0.001	0.006	1.379	0.702	0.209	1.086	0.035

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
normalized size	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.010	0.001	0.004	1.337	0.832	0.220	1.048	0.035

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.028	0.002	0.000	1.362	0.816	0.069	1.167	0.042

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.027	0.002	0.001	1.315	0.479	0.070	1.180	0.040

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.025	0.002	0.001	1.351	0.583	0.068	0.967	0.044

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	43	36	35	35	37	35	35
normalized size	1	1.00	1.26	1.06	1.03	1.03	1.09	1.03	1.03
time (sec)	N/A	0.032	0.002	0.002	1.372	0.711	0.069	0.951	0.043

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	36	14	35	37	14	35
normalized size	1	1.00	1.00	2.25	0.88	2.19	2.31	0.88	2.19
time (sec)	N/A	0.002	0.002	0.000	1.335	0.800	0.068	0.995	0.057

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	36	33	37	36	33
normalized size	1	1.00	1.00	0.87	0.92	0.85	0.95	0.92	0.85
time (sec)	N/A	0.018	0.004	0.002	1.307	0.759	0.109	0.961	0.036

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	36	38	37	46	34
normalized size	1	1.00	1.00	0.88	0.90	0.95	0.92	1.15	0.85
time (sec)	N/A	0.021	0.007	0.008	1.379	0.730	0.140	1.078	0.036

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	37	39	37	46	37
normalized size	1	1.00	1.00	0.88	0.92	0.98	0.92	1.15	0.92
time (sec)	N/A	0.019	0.005	0.007	1.358	0.792	0.190	1.144	4.902

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	39	39	37	47	36
normalized size	1	1.00	1.00	0.87	1.00	1.00	0.95	1.21	0.92
time (sec)	N/A	0.021	0.004	0.007	1.286	0.786	0.241	1.054	0.045

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	36	35	35	37	35	37
normalized size	1	1.00	2.26	1.89	1.84	1.84	1.95	1.84	1.95
time (sec)	N/A	0.003	0.007	0.004	1.320	0.915	0.266	1.042	0.027

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	43	36	37	37	39	37	37
normalized size	1	1.00	1.08	0.90	0.92	0.92	0.98	0.92	0.92
time (sec)	N/A	0.019	0.004	0.005	1.368	0.777	0.292	1.017	0.057

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.019	0.004	0.005	1.286	0.796	0.319	0.835	0.047

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.019	0.007	0.006	1.322	0.860	0.346	1.041	0.031

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.014	0.002	0.001	1.346	0.743	0.068	0.933	0.041

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.013	0.002	0.000	1.375	0.726	0.068	1.026	0.040

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.013	0.002	0.000	1.322	0.755	0.067	1.027	0.041

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	31
normalized size	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89
time (sec)	N/A	0.011	0.001	0.000	1.360	0.722	0.065	0.722	0.038

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	36	29	32	32
normalized size	1	1.00	1.00	0.97	0.94	1.06	0.85	0.94	0.94
time (sec)	N/A	0.013	0.004	0.003	1.383	0.830	0.105	1.042	0.041

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	36	36	34	36
normalized size	1	1.00	1.00	0.92	0.92	0.97	0.97	0.92	0.97
time (sec)	N/A	0.013	0.004	0.006	1.336	0.794	0.148	1.087	4.802

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	33	37	34	33	34
normalized size	1	1.00	1.00	0.97	0.97	1.09	1.00	0.97	1.00
time (sec)	N/A	0.013	0.005	0.005	1.343	0.803	0.197	1.078	0.029

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	37	37	39	37	35
normalized size	1	1.00	1.00	0.92	0.95	0.95	1.00	0.95	0.90
time (sec)	N/A	0.013	0.004	0.005	1.247	0.824	0.256	1.086	0.027

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.013	0.004	0.006	1.487	0.822	0.273	1.196	0.032

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.015	0.006	0.005	1.424	0.739	0.303	1.027	0.029

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	57	57
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.94	0.83	0.83
time (sec)	N/A	0.049	0.003	0.001	1.432	0.741	0.078	0.873	0.026

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	57	57
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.94	0.83	0.83
time (sec)	N/A	0.044	0.002	0.000	1.359	0.715	0.078	1.011	0.023

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83
time (sec)	N/A	0.043	0.002	0.001	1.375	0.610	0.080	1.157	0.024

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	58	57	57	65	57	57
normalized size	1	1.00	0.96	0.81	0.79	0.79	0.90	0.79	0.79
time (sec)	N/A	0.091	0.002	0.000	1.369	0.739	0.093	1.021	0.024

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	66	57	56	56	63	56	56
normalized size	1	1.00	1.25	1.08	1.06	1.06	1.19	1.06	1.06
time (sec)	N/A	0.065	0.002	0.002	1.269	0.682	0.077	1.056	0.024

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	66	57	56	56	63	56	56
normalized size	1	1.00	1.94	1.68	1.65	1.65	1.85	1.65	1.65
time (sec)	N/A	0.036	0.002	0.002	1.363	0.725	0.077	1.185	0.025

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	58	14	57	65	14	57
normalized size	1	1.00	1.00	3.62	0.88	3.56	4.06	0.88	3.56
time (sec)	N/A	0.002	0.002	0.002	1.399	0.767	0.077	1.057	0.023

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	58	55	65	58	55
normalized size	1	1.00	1.00	0.86	0.89	0.85	1.00	0.89	0.85
time (sec)	N/A	0.034	0.004	0.003	1.373	0.587	0.136	0.923	0.028

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	58	61	63	68	56
normalized size	1	1.00	1.00	0.89	0.91	0.95	0.98	1.06	0.88
time (sec)	N/A	0.037	0.005	0.005	1.369	0.793	0.169	1.056	0.030

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	59	61	63	70	59
normalized size	1	1.00	1.00	0.89	0.92	0.95	0.98	1.09	0.92
time (sec)	N/A	0.036	0.007	0.005	1.329	0.908	0.223	0.881	0.031

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	61	61	65	72	59
normalized size	1	1.00	1.00	0.89	0.95	0.95	1.02	1.12	0.92
time (sec)	N/A	0.034	0.005	0.006	1.398	0.819	0.283	1.189	0.040

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	61	61	63	70	59
normalized size	1	1.00	1.00	0.89	0.95	0.95	0.98	1.09	0.92
time (sec)	N/A	0.033	0.005	0.007	1.410	0.870	0.362	1.049	0.042

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	61	61	61	69	58
normalized size	1	1.00	1.00	0.86	0.94	0.94	0.94	1.06	0.89
time (sec)	N/A	0.031	0.005	0.007	1.351	0.852	0.440	1.060	4.772

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	69	58	57	57	61	57	59
normalized size	1	1.00	3.63	3.05	3.00	3.00	3.21	3.00	3.11
time (sec)	N/A	0.004	0.004	0.007	1.370	0.685	0.474	1.069	4.751

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	67	58	59	59	63	59	58
normalized size	1	1.00	1.68	1.45	1.48	1.48	1.58	1.48	1.45
time (sec)	N/A	0.018	0.006	0.004	1.344	0.816	0.506	1.064	4.736

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	67	58	59	59	63	59	58
normalized size	1	1.00	1.08	0.94	0.95	0.95	1.02	0.95	0.94
time (sec)	N/A	0.028	0.004	0.007	1.376	0.849	0.550	0.979	0.041

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.032	0.004	0.006	1.356	0.838	0.591	0.945	0.040

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.031	0.004	0.005	1.346	0.700	0.628	1.018	0.041

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83
time (sec)	N/A	0.025	0.002	0.001	1.320	0.745	0.077	1.045	0.024

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	57	57
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.94	0.83	0.83
time (sec)	N/A	0.024	0.002	0.002	1.310	0.642	0.077	0.983	0.024

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83
time (sec)	N/A	0.022	0.002	0.001	1.311	0.740	0.076	1.008	0.025

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	56	56	63	56	56
normalized size	1	1.00	1.00	0.86	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.022	0.002	0.002	1.382	0.811	0.076	1.094	0.024

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	55	54	54	61	54	54
normalized size	1	1.00	1.00	0.89	0.87	0.87	0.98	0.87	0.87
time (sec)	N/A	0.019	0.001	0.001	1.380	0.678	0.072	1.144	0.022

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	55	59	58	55	55
normalized size	1	1.00	1.00	0.92	0.90	0.97	0.95	0.90	0.90
time (sec)	N/A	0.021	0.004	0.005	1.388	0.831	0.132	1.126	0.026

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	55	59	60	55	57
normalized size	1	1.00	1.00	0.92	0.92	0.98	1.00	0.92	0.95
time (sec)	N/A	0.022	0.004	0.005	1.364	0.798	0.174	1.133	0.025

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	58	59	63	58	58
normalized size	1	1.00	1.00	0.89	0.92	0.94	1.00	0.92	0.92
time (sec)	N/A	0.022	0.004	0.005	1.378	0.866	0.227	0.938	0.047

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	58	59	61	58	59
normalized size	1	1.00	1.00	0.92	0.95	0.97	1.00	0.95	0.97
time (sec)	N/A	0.022	0.004	0.007	1.435	0.793	0.294	1.118	4.789

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	57	59	60	57	57
normalized size	1	1.00	1.00	0.92	0.95	0.98	1.00	0.95	0.95
time (sec)	N/A	0.022	0.005	0.008	1.443	0.704	0.362	1.035	0.039

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	58	59	59	63	59	58
normalized size	1	1.00	1.00	0.89	0.91	0.91	0.97	0.91	0.89
time (sec)	N/A	0.022	0.004	0.006	1.362	0.763	0.431	1.039	0.038

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	59	59	63	59	58
normalized size	1	1.00	1.00	0.87	0.88	0.88	0.94	0.88	0.87
time (sec)	N/A	0.023	0.004	0.006	1.359	0.729	0.469	1.091	0.038

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.023	0.004	0.006	1.368	0.695	0.505	1.080	4.747

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.022	0.004	0.004	1.324	0.607	0.536	1.099	0.040

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.023	0.006	0.007	1.394	0.533	0.567	1.129	0.040

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	108	91	90	90	105	90	90
normalized size	1	1.00	0.84	0.71	0.70	0.70	0.81	0.70	0.70
time (sec)	N/A	0.210	0.003	0.002	1.296	0.619	0.088	1.055	0.103

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	108	91	90	90	107	90	90
normalized size	1	1.00	0.98	0.83	0.82	0.82	0.97	0.82	0.82
time (sec)	N/A	0.171	0.003	0.001	1.345	0.603	0.088	1.005	4.566

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	106	91	90	90	104	90	90
normalized size	1	1.00	1.16	1.00	0.99	0.99	1.14	0.99	0.99
time (sec)	N/A	0.142	0.003	0.001	1.283	0.654	0.087	0.996	4.593

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	106	91	90	90	105	90	90
normalized size	1	1.00	1.47	1.26	1.25	1.25	1.46	1.25	1.25
time (sec)	N/A	0.116	0.003	0.001	1.289	0.761	0.087	1.079	0.090

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	103	90	89	89	102	89	89
normalized size	1	1.00	1.94	1.70	1.68	1.68	1.92	1.68	1.68
time (sec)	N/A	0.084	0.003	0.001	1.354	0.836	0.087	1.127	0.092

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	106	91	90	90	105	90	90
normalized size	1	1.00	3.12	2.68	2.65	2.65	3.09	2.65	2.65
time (sec)	N/A	0.046	0.003	0.000	1.433	0.766	0.085	1.183	0.092

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	91	14	90	99	14	14
normalized size	1	1.00	1.00	5.69	0.88	5.62	6.19	0.88	0.88
time (sec)	N/A	0.002	0.002	0.001	1.276	0.580	0.084	1.124	4.612

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	91	88	102	91	88
normalized size	1	1.00	1.00	0.89	0.91	0.88	1.02	0.91	0.88
time (sec)	N/A	0.059	0.004	0.003	1.299	0.863	0.183	0.900	4.618

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	90	91	94	100	101	89
normalized size	1	1.00	1.00	0.91	0.92	0.95	1.01	1.02	0.90
time (sec)	N/A	0.060	0.005	0.007	1.378	1.152	0.217	1.059	0.059

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	90	92	94	104	103	92
normalized size	1	1.00	1.00	0.89	0.91	0.93	1.03	1.02	0.91
time (sec)	N/A	0.061	0.005	0.007	1.325	0.789	0.271	1.121	0.055

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	89	91	94	97	102	91
normalized size	1	1.00	1.00	0.95	0.97	1.00	1.03	1.09	0.97
time (sec)	N/A	0.057	0.005	0.007	1.345	0.871	0.334	1.051	0.053

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	90	94	94	100	105	92
normalized size	1	1.00	1.00	0.93	0.97	0.97	1.03	1.08	0.95
time (sec)	N/A	0.057	0.005	0.008	1.386	0.880	0.416	0.997	0.052

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	90	94	94	99	105	91
normalized size	1	1.00	1.00	0.95	0.99	0.99	1.04	1.11	0.96
time (sec)	N/A	0.058	0.005	0.007	1.330	0.968	0.507	1.128	5.110

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	90	94	94	99	105	92
normalized size	1	1.00	1.00	0.89	0.93	0.93	0.98	1.04	0.91
time (sec)	N/A	0.054	0.005	0.008	1.380	0.748	0.618	1.215	0.061

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	90	94	94	99	103	94
normalized size	1	1.00	1.00	0.91	0.95	0.95	1.00	1.04	0.95
time (sec)	N/A	0.051	0.005	0.009	1.348	0.735	0.733	1.076	5.148

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	94	94	97	102	91
normalized size	1	1.00	1.00	0.89	0.94	0.94	0.97	1.02	0.91
time (sec)	N/A	0.054	0.005	0.009	1.360	1.250	0.833	1.056	5.089

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	100	91	90	90	97	90	92
normalized size	1	1.00	5.26	4.79	4.74	4.74	5.11	4.74	4.84
time (sec)	N/A	0.003	0.005	0.006	1.324	0.794	0.896	1.068	0.076

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	106	91	92	92	99	92	92
normalized size	1	1.00	2.65	2.28	2.30	2.30	2.48	2.30	2.30
time (sec)	N/A	0.017	0.004	0.007	1.347	0.836	0.937	1.002	0.077

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	104	91	92	92	99	92	91
normalized size	1	1.00	1.68	1.47	1.48	1.48	1.60	1.48	1.47
time (sec)	N/A	0.029	0.004	0.007	1.363	0.752	0.992	1.061	0.079

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	106	91	92	92	99	92	92
normalized size	1	1.00	1.26	1.08	1.10	1.10	1.18	1.10	1.10
time (sec)	N/A	0.041	0.004	0.006	1.394	0.879	1.078	1.202	4.967

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	92	92	99	92	92
normalized size	1	1.00	1.00	0.86	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.055	0.005	0.007	1.304	0.892	1.141	1.057	0.080

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	92	92	99	92	92
normalized size	1	1.00	1.00	0.84	0.85	0.85	0.92	0.85	0.85
time (sec)	N/A	0.054	0.004	0.006	1.415	0.878	1.205	1.051	4.893

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	92	92	99	92	92
normalized size	1	1.00	1.00	0.84	0.85	0.85	0.92	0.85	0.85
time (sec)	N/A	0.052	0.004	0.006	1.314	0.766	1.281	1.219	4.862

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	92	92	99	92	91
normalized size	1	1.00	1.00	0.86	0.87	0.87	0.93	0.87	0.86
time (sec)	N/A	0.052	0.004	0.007	1.337	0.602	1.336	1.053	0.084

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	90	90	107	90	90
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83
time (sec)	N/A	0.048	0.003	0.003	1.362	0.652	0.088	1.096	4.945

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	90	90	107	90	90
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83
time (sec)	N/A	0.039	0.003	0.002	1.324	0.791	0.087	1.140	0.097

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	90	90	107	90	90
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83
time (sec)	N/A	0.039	0.003	0.001	1.335	0.800	0.086	1.097	0.096

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	90	90	105	90	90
normalized size	1	1.00	1.00	0.86	0.85	0.85	0.99	0.85	0.85
time (sec)	N/A	0.040	0.003	0.002	1.372	0.593	0.085	0.912	4.970

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	88	87	87	102	87	87
normalized size	1	1.00	1.00	0.87	0.86	0.86	1.01	0.86	0.86
time (sec)	N/A	0.037	0.001	0.000	1.304	0.725	0.083	1.178	0.053

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	88	92	99	88	88
normalized size	1	1.00	1.00	0.89	0.88	0.92	0.99	0.88	0.88
time (sec)	N/A	0.039	0.017	0.004	1.301	0.927	0.179	1.071	0.057

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	89	89	92	100	89	91
normalized size	1	1.00	1.00	0.91	0.91	0.94	1.02	0.91	0.93
time (sec)	N/A	0.038	0.008	0.006	1.382	0.534	0.223	1.099	0.053

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	91	92	102	91	91
normalized size	1	1.00	1.00	0.89	0.91	0.92	1.02	0.91	0.91
time (sec)	N/A	0.039	0.012	0.006	1.213	0.619	0.275	0.974	4.959

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	89	91	92	102	91	91
normalized size	1	1.00	1.00	0.87	0.89	0.90	1.00	0.89	0.89
time (sec)	N/A	0.037	0.006	0.005	1.356	0.646	0.333	0.934	4.798

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	89	91	92	100	91	91
normalized size	1	1.00	1.00	0.87	0.89	0.90	0.98	0.89	0.89
time (sec)	N/A	0.039	0.012	0.007	1.360	0.896	0.417	0.939	0.047

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	91	92	99	91	91
normalized size	1	1.00	1.00	0.89	0.91	0.92	0.99	0.91	0.91
time (sec)	N/A	0.040	0.014	0.007	1.396	0.591	0.499	0.925	4.578

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	89	91	92	97	91	92
normalized size	1	1.00	1.00	0.91	0.93	0.94	0.99	0.93	0.94
time (sec)	N/A	0.038	0.013	0.007	1.331	0.839	0.571	1.105	0.074

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	88	90	92	95	90	90
normalized size	1	1.00	1.00	0.89	0.91	0.93	0.96	0.91	0.91
time (sec)	N/A	0.039	0.006	0.008	1.407	0.696	0.694	0.875	4.525

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	91	92	92	99	92	91
normalized size	1	1.00	1.00	0.88	0.88	0.88	0.95	0.88	0.88
time (sec)	N/A	0.039	0.011	0.006	1.360	0.818	0.788	0.817	0.074

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	92	92	99	92	92
normalized size	1	1.00	1.00	0.86	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.040	0.014	0.006	1.360	0.847	0.836	1.137	0.076

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	68	68	67	68	69	67
normalized size	1	1.00	1.00	0.86	0.86	0.85	0.86	0.87	0.85
time (sec)	N/A	0.056	0.006	0.005	1.358	0.858	0.175	0.988	0.064

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	71	72	170	119	77	65
normalized size	1	1.00	1.00	0.88	0.89	2.10	1.47	0.95	0.80
time (sec)	N/A	0.035	0.035	0.005	2.921	1.008	0.200	1.078	0.056

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	57	56	56	58	56
normalized size	1	1.00	1.00	0.86	0.86	0.85	0.85	0.88	0.85
time (sec)	N/A	0.044	0.006	0.005	1.347	0.621	0.166	1.075	0.083

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	60	60	148	107	65	54
normalized size	1	1.00	1.00	0.88	0.88	2.18	1.57	0.96	0.79
time (sec)	N/A	0.030	0.026	0.004	2.936	0.984	0.191	1.099	0.052

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	46	46	45	44	47	45
normalized size	1	1.00	1.00	0.87	0.87	0.85	0.83	0.89	0.85
time (sec)	N/A	0.035	0.005	0.003	1.363	0.699	0.157	1.048	4.727

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	50	126	95	55	43
normalized size	1	1.00	1.00	0.89	0.91	2.29	1.73	1.00	0.78
time (sec)	N/A	0.024	0.023	0.003	2.839	1.180	0.180	0.995	0.070

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	34	33	32	35	33
normalized size	1	1.00	1.00	0.88	0.85	0.82	0.80	0.88	0.82
time (sec)	N/A	0.027	0.005	0.002	1.307	1.153	0.150	1.121	4.646

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	37	99	80	40	32
normalized size	1	1.00	1.00	0.90	0.88	2.36	1.90	0.95	0.76
time (sec)	N/A	0.020	0.019	0.004	2.803	1.053	0.168	1.012	0.069

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	22	20	24	22
normalized size	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.018	0.004	0.003	1.323	1.073	0.135	1.091	0.037

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	82	56	26	23
normalized size	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.74
time (sec)	N/A	0.012	0.008	0.002	2.972	0.773	0.146	1.119	0.034

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.003	0.002	0.001	1.338	0.680	0.108	1.068	4.632

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
normalized size	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.005	0.004	0.003	2.903	1.027	0.134	0.974	4.695

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	23	18	15	24	18
normalized size	1	1.00	1.00	0.95	1.05	0.82	0.68	1.09	0.82
time (sec)	N/A	0.011	0.005	0.005	1.332	0.868	0.200	1.093	0.081

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	29	82	65	29	26
normalized size	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.76
time (sec)	N/A	0.012	0.012	0.006	2.891	0.787	0.178	1.126	4.620

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	33	33	31	43	31
normalized size	1	1.00	1.00	0.91	0.94	0.94	0.89	1.23	0.89
time (sec)	N/A	0.022	0.007	0.006	1.371	0.912	0.263	1.148	0.074

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	40	106	87	40	37
normalized size	1	1.00	1.00	0.91	0.93	2.47	2.02	0.93	0.86
time (sec)	N/A	0.017	0.020	0.007	2.884	1.030	0.218	1.119	4.672

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	47	45	42	57	46
normalized size	1	1.00	1.00	0.90	0.96	0.92	0.86	1.16	0.94
time (sec)	N/A	0.028	0.007	0.007	1.359	0.652	0.304	1.192	0.078

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	52	52	132	100	52	48
normalized size	1	1.00	1.00	0.90	0.90	2.28	1.72	0.90	0.83
time (sec)	N/A	0.025	0.023	0.008	2.886	0.804	0.257	0.617	0.063

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	58	58	56	70	58
normalized size	1	1.00	1.00	0.89	0.92	0.92	0.89	1.11	0.92
time (sec)	N/A	0.035	0.007	0.008	1.358	0.878	0.347	0.628	4.645

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	61	62	154	112	62	59
normalized size	1	1.00	1.00	0.88	0.90	2.23	1.62	0.90	0.86
time (sec)	N/A	0.035	0.025	0.007	2.925	0.946	0.296	0.637	0.065

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	66	69	69	68	81	68
normalized size	1	1.00	1.00	0.88	0.92	0.92	0.91	1.08	0.91
time (sec)	N/A	0.041	0.007	0.007	1.331	0.889	0.386	0.626	4.667

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	83	85	88	104	88	103	90
normalized size	1	1.00	0.88	0.90	0.94	1.11	0.94	1.10	0.96
time (sec)	N/A	0.076	0.032	0.011	1.335	0.611	0.313	0.606	0.085

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	90	93	234	151	95	88
normalized size	1	1.00	0.89	0.86	0.89	2.23	1.44	0.90	0.84
time (sec)	N/A	0.046	0.057	0.008	2.916	0.708	0.340	0.645	0.071

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	74	77	93	80	92	79
normalized size	1	1.00	0.87	0.89	0.93	1.12	0.96	1.11	0.95
time (sec)	N/A	0.066	0.025	0.010	1.360	0.917	0.295	0.626	4.485

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	78	82	212	134	84	77
normalized size	1	1.00	0.89	0.85	0.89	2.30	1.46	0.91	0.84
time (sec)	N/A	0.037	0.057	0.010	3.009	0.996	0.325	0.642	4.555

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	63	65	81	66	80	68
normalized size	1	1.00	0.86	0.90	0.93	1.16	0.94	1.14	0.97
time (sec)	N/A	0.054	0.025	0.011	1.346	0.569	0.274	0.654	0.069

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	68	71	190	124	73	66
normalized size	1	1.00	0.90	0.86	0.90	2.41	1.57	0.92	0.84
time (sec)	N/A	0.032	0.053	0.009	2.967	0.953	0.308	0.631	4.586

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	52	54	70	53	67	57
normalized size	1	1.00	0.86	0.91	0.95	1.23	0.93	1.18	1.00
time (sec)	N/A	0.042	0.020	0.010	1.373	0.750	0.260	0.619	0.077

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	57	59	164	107	61	56
normalized size	1	1.00	0.91	0.86	0.89	2.48	1.62	0.92	0.85
time (sec)	N/A	0.027	0.043	0.009	2.972	0.950	0.287	0.639	0.089

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	41	43	56	39	49	45
normalized size	1	1.00	0.86	0.93	0.98	1.27	0.89	1.11	1.02
time (sec)	N/A	0.032	0.023	0.010	1.385	0.801	0.233	0.618	0.046

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	43	45	136	83	42	43
normalized size	1	1.00	0.93	0.78	0.82	2.47	1.51	0.76	0.78
time (sec)	N/A	0.017	0.034	0.010	2.944	0.827	0.255	0.629	4.591

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	32	35	29	48	29
normalized size	1	1.00	0.82	0.91	0.97	1.06	0.88	1.45	0.88
time (sec)	N/A	0.024	0.008	0.009	1.318	0.765	0.193	0.639	0.046

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	36	120	78	35	33
normalized size	1	1.00	1.00	0.80	0.80	2.67	1.73	0.78	0.73
time (sec)	N/A	0.012	0.021	0.007	2.964	0.648	0.209	0.638	4.756

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	14
normalized size	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	0.88
time (sec)	N/A	0.003	0.002	0.000	1.298	0.657	0.158	0.642	0.031

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	33
normalized size	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73
time (sec)	N/A	0.010	0.023	0.005	2.962	0.890	0.211	0.653	4.740

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	37	47	34	47	34
normalized size	1	1.00	0.87	0.92	0.97	1.24	0.89	1.24	0.89
time (sec)	N/A	0.027	0.015	0.010	1.391	0.963	0.295	0.632	4.701

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	46	49	136	92	47	44
normalized size	1	1.00	0.95	0.81	0.86	2.39	1.61	0.82	0.77
time (sec)	N/A	0.017	0.037	0.010	2.968	0.982	0.290	0.646	0.073

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	46	52	73	51	51	51
normalized size	1	1.00	0.84	0.94	1.06	1.49	1.04	1.04	1.04
time (sec)	N/A	0.036	0.040	0.011	1.366	0.714	0.363	0.603	0.078

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	59	64	172	114	59	58
normalized size	1	1.00	0.99	0.87	0.94	2.53	1.68	0.87	0.85
time (sec)	N/A	0.026	0.043	0.011	3.014	0.891	0.347	0.614	4.729

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	61	70	90	68	86	67
normalized size	1	1.00	0.86	0.92	1.06	1.36	1.03	1.30	1.02
time (sec)	N/A	0.044	0.052	0.014	1.320	0.873	0.441	0.633	4.804

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	70	75	198	126	70	70
normalized size	1	1.00	0.99	0.86	0.93	2.44	1.56	0.86	0.86
time (sec)	N/A	0.033	0.044	0.012	2.914	0.913	0.397	0.647	4.850

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	68	73	79	99	78	99	78
normalized size	1	1.00	0.85	0.91	0.99	1.24	0.98	1.24	0.98
time (sec)	N/A	0.054	0.057	0.014	1.320	0.607	0.467	0.639	0.119

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	91	81	86	220	138	81	80
normalized size	1	1.00	0.97	0.86	0.91	2.34	1.47	0.86	0.85
time (sec)	N/A	0.043	0.056	0.014	2.974	1.010	0.446	0.620	4.560

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	79	84	92	112	94	110	89
normalized size	1	1.00	0.85	0.90	0.99	1.20	1.01	1.18	0.96
time (sec)	N/A	0.065	0.076	0.013	1.305	0.864	0.523	0.628	4.731

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	97	101	111	137	119	114	111
normalized size	1	1.00	0.85	0.89	0.97	1.20	1.04	1.00	0.97
time (sec)	N/A	0.103	0.030	0.011	1.363	0.802	0.482	0.628	4.725

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	85	91	99	125	104	102	100
normalized size	1	1.00	0.85	0.91	0.99	1.25	1.04	1.02	1.00
time (sec)	N/A	0.082	0.028	0.012	1.293	0.821	0.466	0.615	0.077

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	75	80	89	115	92	92	90
normalized size	1	1.00	0.86	0.92	1.02	1.32	1.06	1.06	1.03
time (sec)	N/A	0.072	0.025	0.010	1.344	0.778	0.436	0.625	4.493

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	69	77	103	78	80	78
normalized size	1	1.00	0.85	0.93	1.04	1.39	1.05	1.08	1.05
time (sec)	N/A	0.058	0.021	0.011	1.340	0.787	0.416	0.609	0.083

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	48	58	66	91	68	62	68
normalized size	1	1.00	0.74	0.89	1.02	1.40	1.05	0.95	1.05
time (sec)	N/A	0.046	0.057	0.010	1.279	1.293	0.380	0.661	4.749

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	46	55	69	53	42	52
normalized size	1	1.00	0.80	0.94	1.12	1.41	1.08	0.86	1.06
time (sec)	N/A	0.038	0.018	0.008	1.299	0.869	0.319	0.623	0.059

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	36	36	36	22	37
normalized size	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.004	0.007	0.007	1.346	0.889	0.265	0.618	0.032

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	26	27	14	28
normalized size	1	1.00	1.00	0.94	0.88	1.62	1.69	0.88	1.75
time (sec)	N/A	0.003	0.002	0.002	1.326	0.899	0.235	0.627	4.623

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	49	60	90	56	59	56
normalized size	1	1.00	0.80	0.91	1.11	1.67	1.04	1.09	1.04
time (sec)	N/A	0.038	0.038	0.012	1.325	0.913	0.407	0.624	4.675

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	62	77	119	80	82	75
normalized size	1	1.00	0.88	0.93	1.15	1.78	1.19	1.22	1.12
time (sec)	N/A	0.047	0.059	0.013	1.339	0.879	0.508	0.619	0.080

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	74	79	92	134	90	80	88
normalized size	1	1.00	0.86	0.92	1.07	1.56	1.05	0.93	1.02
time (sec)	N/A	0.060	0.051	0.015	1.424	0.944	0.546	0.646	4.672

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	85	90	103	145	104	110	101
normalized size	1	1.00	0.89	0.95	1.08	1.53	1.09	1.16	1.06
time (sec)	N/A	0.068	0.071	0.013	1.362	0.888	0.594	0.654	4.692

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	96	101	114	156	116	119	111
normalized size	1	1.00	0.86	0.90	1.02	1.39	1.04	1.06	0.99
time (sec)	N/A	0.080	0.061	0.014	1.337	0.946	0.653	0.634	4.855

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	99	99	105	278	162	96	99
normalized size	1	1.00	0.89	0.89	0.95	2.50	1.46	0.86	0.89
time (sec)	N/A	0.048	0.062	0.010	2.954	0.858	0.489	0.613	0.076

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	88	88	93	256	144	84	87
normalized size	1	1.00	0.90	0.90	0.95	2.61	1.47	0.86	0.89
time (sec)	N/A	0.041	0.049	0.010	2.858	0.929	0.465	0.641	0.070

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	77	82	230	133	73	77
normalized size	1	1.00	0.91	0.91	0.96	2.71	1.56	0.86	0.91
time (sec)	N/A	0.035	0.046	0.011	2.970	0.846	0.437	0.630	4.725

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	63	68	202	107	54	64
normalized size	1	1.00	0.89	0.85	0.92	2.73	1.45	0.73	0.86
time (sec)	N/A	0.025	0.049	0.010	2.900	0.803	0.398	0.636	4.746

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	47	59	188	110	45	56
normalized size	1	1.00	0.86	0.73	0.92	2.94	1.72	0.70	0.88
time (sec)	N/A	0.020	0.041	0.008	2.996	0.708	0.332	0.598	4.771

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	49	62	190	110	50	55
normalized size	1	1.00	0.89	0.75	0.95	2.92	1.69	0.77	0.85
time (sec)	N/A	0.018	0.029	0.008	3.039	0.713	0.306	0.615	4.738

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	51	58	188	105	45	55
normalized size	1	1.00	0.89	0.82	0.94	3.03	1.69	0.73	0.89
time (sec)	N/A	0.016	0.038	0.005	3.026	0.930	0.319	0.633	4.656

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	66	71	202	116	57	66
normalized size	1	1.00	0.89	0.87	0.93	2.66	1.53	0.75	0.87
time (sec)	N/A	0.026	0.040	0.012	2.981	0.941	0.418	0.637	4.673

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	79	79	86	238	138	71	80
normalized size	1	1.00	0.91	0.91	0.99	2.74	1.59	0.82	0.92
time (sec)	N/A	0.034	0.050	0.015	2.956	1.011	0.476	0.640	4.672

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	90	89	97	264	150	80	92
normalized size	1	1.00	0.90	0.89	0.97	2.64	1.50	0.80	0.92
time (sec)	N/A	0.042	0.054	0.014	2.911	0.887	0.529	0.638	5.021

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	101	101	108	286	162	93	102
normalized size	1	1.00	0.89	0.89	0.96	2.53	1.43	0.82	0.90
time (sec)	N/A	0.056	0.056	0.014	3.024	0.950	0.579	0.607	4.985

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	169	199	242	346	260	168	242
normalized size	1	1.00	0.78	0.92	1.12	1.60	1.20	0.78	1.12
time (sec)	N/A	0.255	0.048	0.018	1.615	0.697	2.151	0.635	5.295

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	158	188	231	335	245	157	230
normalized size	1	1.00	0.77	0.92	1.13	1.63	1.20	0.77	1.12
time (sec)	N/A	0.214	0.030	0.018	1.578	0.974	2.060	0.634	0.401

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	145	177	220	322	233	139	220
normalized size	1	1.00	0.77	0.94	1.17	1.71	1.24	0.74	1.17
time (sec)	N/A	0.189	0.039	0.019	1.595	0.771	2.025	0.634	0.425

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	116	166	209	300	219	119	207
normalized size	1	1.00	0.65	0.93	1.17	1.68	1.22	0.66	1.16
time (sec)	N/A	0.169	0.033	0.012	1.525	0.966	1.835	0.615	5.362

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	101	150	190	190	202	99	192
normalized size	1	1.00	5.32	7.89	10.00	10.00	10.63	5.21	10.11
time (sec)	N/A	0.003	0.021	0.010	1.496	0.887	1.699	0.640	0.115

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	90	133	179	179	190	88	181
normalized size	1	1.00	2.31	3.41	4.59	4.59	4.87	2.26	4.64
time (sec)	N/A	0.017	0.017	0.009	1.468	0.890	1.419	0.627	4.886

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	79	116	168	168	178	77	170
normalized size	1	1.00	1.36	2.00	2.90	2.90	3.07	1.33	2.93
time (sec)	N/A	0.027	0.018	0.010	1.457	0.857	1.327	0.626	0.105

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	68	99	157	157	167	66	159
normalized size	1	1.00	0.88	1.29	2.04	2.04	2.17	0.86	2.06
time (sec)	N/A	0.039	0.017	0.009	1.429	0.558	1.182	0.633	0.097

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	57	82	146	146	155	55	148
normalized size	1	1.00	0.63	0.90	1.60	1.60	1.70	0.60	1.63
time (sec)	N/A	0.068	0.016	0.010	1.445	1.002	1.079	0.635	4.819

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	65	135	135	143	44	136
normalized size	1	1.00	0.64	0.90	1.88	1.88	1.99	0.61	1.89
time (sec)	N/A	0.053	0.011	0.007	1.446	0.800	1.007	0.644	0.099

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	48	124	124	131	33	125
normalized size	1	1.00	0.66	0.91	2.34	2.34	2.47	0.62	2.36
time (sec)	N/A	0.039	0.014	0.009	1.506	0.867	0.945	0.639	4.828

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	31	113	113	119	22	114
normalized size	1	1.00	0.71	0.91	3.32	3.32	3.50	0.65	3.35
time (sec)	N/A	0.025	0.008	0.008	1.392	0.790	0.926	0.623	0.108

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	103	110	14	14
normalized size	1	1.00	1.00	0.94	0.88	6.44	6.88	0.88	0.88
time (sec)	N/A	0.003	0.003	0.001	1.297	0.901	0.893	0.587	0.126

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	120	147	214	398	223	136	210
normalized size	1	1.00	0.72	0.89	1.29	2.40	1.34	0.82	1.27
time (sec)	N/A	0.128	0.104	0.018	1.648	0.913	1.308	0.632	5.464

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	136	167	231	427	245	159	229
normalized size	1	1.00	0.74	0.91	1.26	2.32	1.33	0.86	1.24
time (sec)	N/A	0.186	0.130	0.020	1.670	0.954	1.464	0.592	0.522

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	151	198	246	442	260	174	243
normalized size	1	1.00	0.70	0.91	1.13	2.04	1.20	0.80	1.12
time (sec)	N/A	0.220	0.103	0.022	1.708	0.800	1.549	0.607	5.775

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	162	209	257	453	270	187	255
normalized size	1	1.00	0.72	0.92	1.14	2.00	1.19	0.83	1.13
time (sec)	N/A	0.233	0.127	0.022	1.707	1.105	1.604	0.644	1.088

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	166	228	248	718	314	162	241
normalized size	1	1.00	0.72	0.99	1.07	3.11	1.36	0.70	1.04
time (sec)	N/A	0.167	0.094	0.022	3.391	0.686	2.040	0.594	4.898

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	155	217	236	692	299	150	231
normalized size	1	1.00	0.71	1.00	1.08	3.17	1.37	0.69	1.06
time (sec)	N/A	0.141	0.080	0.023	3.247	0.837	1.919	0.646	0.402

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	144	203	222	664	274	131	218
normalized size	1	1.00	0.70	0.98	1.07	3.21	1.32	0.63	1.05
time (sec)	N/A	0.124	0.076	0.024	3.173	1.051	1.816	0.635	0.444

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	134	124	213	650	277	122	210
normalized size	1	1.00	0.68	0.63	1.08	3.30	1.41	0.62	1.07
time (sec)	N/A	0.113	0.078	0.018	3.165	0.878	1.550	0.617	4.964

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	138	124	219	654	289	128	207
normalized size	1	1.00	0.70	0.63	1.11	3.30	1.46	0.65	1.05
time (sec)	N/A	0.122	0.073	0.017	3.156	0.955	1.456	0.631	4.757

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	138	122	221	654	291	128	205
normalized size	1	1.00	0.69	0.61	1.11	3.29	1.46	0.64	1.03
time (sec)	N/A	0.123	0.060	0.018	3.155	0.757	1.353	0.636	4.753

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	138	122	221	654	291	128	205
normalized size	1	1.00	0.69	0.61	1.10	3.27	1.46	0.64	1.02
time (sec)	N/A	0.120	0.073	0.019	3.090	0.602	1.274	0.634	0.146

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	138	122	221	654	291	128	205
normalized size	1	1.00	0.69	0.61	1.10	3.25	1.45	0.64	1.02
time (sec)	N/A	0.119	0.072	0.016	3.075	1.156	1.190	0.631	4.770

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	138	122	221	654	291	128	204
normalized size	1	1.00	0.68	0.60	1.09	3.24	1.44	0.63	1.01
time (sec)	N/A	0.116	0.062	0.015	3.078	0.860	1.154	0.602	4.736

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	138	122	221	654	291	128	204
normalized size	1	1.00	0.68	0.60	1.09	3.22	1.43	0.63	1.00
time (sec)	N/A	0.111	0.066	0.015	3.141	0.863	1.084	0.609	4.794

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	138	122	221	654	291	128	204
normalized size	1	1.00	0.68	0.60	1.08	3.21	1.43	0.63	1.00
time (sec)	N/A	0.108	0.064	0.016	3.187	0.930	1.036	0.632	4.738

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	138	124	219	654	286	128	206
normalized size	1	1.00	0.67	0.60	1.07	3.19	1.40	0.62	1.00
time (sec)	N/A	0.106	0.058	0.016	3.095	0.841	1.027	0.617	0.167

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	131	156	212	650	272	122	209
normalized size	1	1.00	0.72	0.86	1.17	3.59	1.50	0.67	1.15
time (sec)	N/A	0.100	0.105	0.006	3.110	0.906	1.036	0.601	4.743

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	147	206	225	664	282	134	220
normalized size	1	1.00	0.70	0.99	1.08	3.18	1.35	0.64	1.05
time (sec)	N/A	0.131	0.099	0.023	3.131	1.104	1.348	0.634	5.089

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	157	219	240	700	304	148	234
normalized size	1	1.00	0.71	1.00	1.09	3.18	1.38	0.67	1.06
time (sec)	N/A	0.141	0.088	0.024	3.297	0.968	1.454	0.632	5.106

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	169	230	251	726	316	159	246
normalized size	1	1.00	0.73	0.99	1.08	3.12	1.36	0.68	1.06
time (sec)	N/A	0.158	0.093	0.026	3.351	1.012	1.500	0.598	5.892

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	25	23	22	26	23
normalized size	1	1.00	1.00	0.93	0.89	0.82	0.79	0.93	0.82
time (sec)	N/A	0.022	0.006	0.003	1.356	0.750	0.141	0.620	0.049

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	42	80	49	29	23
normalized size	1	1.00	1.00	0.87	1.35	2.58	1.58	0.94	0.74
time (sec)	N/A	0.012	0.010	0.004	2.969	1.059	0.152	0.628	4.567

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	15	15	12	16	15
normalized size	1	1.00	1.00	1.00	0.94	0.94	0.75	1.00	0.94
time (sec)	N/A	0.003	0.003	0.002	1.293	0.763	0.115	0.611	0.033

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	31	68	46	18	16
normalized size	1	1.00	1.00	0.67	1.29	2.83	1.92	0.75	0.67
time (sec)	N/A	0.007	0.004	0.003	2.996	0.712	0.140	0.588	0.177

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	25	20	15	26	21
normalized size	1	1.00	1.00	1.00	1.09	0.87	0.65	1.13	0.91
time (sec)	N/A	0.012	0.006	0.006	1.324	1.219	0.214	0.578	4.537

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	44	82	58	31	25
normalized size	1	1.00	1.00	0.88	1.33	2.48	1.76	0.94	0.76
time (sec)	N/A	0.012	0.012	0.005	2.888	0.838	0.186	0.622	4.610

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	33	35	33	31	43	31
normalized size	1	1.00	1.00	0.94	1.00	0.94	0.89	1.23	0.89
time (sec)	N/A	0.024	0.008	0.006	1.388	0.825	0.273	0.583	0.073

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	29	34	35	42	29	53	32
normalized size	1	1.00	0.83	0.97	1.00	1.20	0.83	1.51	0.91
time (sec)	N/A	0.027	0.013	0.008	1.348	0.817	0.196	0.630	0.044

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	38	52	127	71	39	34
normalized size	1	1.00	1.02	0.83	1.13	2.76	1.54	0.85	0.74
time (sec)	N/A	0.014	0.030	0.007	2.997	0.832	0.214	0.608	4.684

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	16	15	16	15
normalized size	1	1.00	1.00	1.00	0.94	0.94	0.88	0.94	0.88
time (sec)	N/A	0.003	0.002	0.000	1.377	1.105	0.156	0.605	0.021

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	38	52	126	71	39	34
normalized size	1	1.00	1.02	0.83	1.13	2.74	1.54	0.85	0.74
time (sec)	N/A	0.012	0.017	0.004	2.958	0.857	0.220	0.582	4.514

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	35	39	41	53	34	51	36
normalized size	1	1.00	0.88	0.98	1.02	1.32	0.85	1.28	0.90
time (sec)	N/A	0.028	0.020	0.011	1.365	0.962	0.299	0.626	0.055

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	47	65	140	83	50	45
normalized size	1	1.00	0.97	0.81	1.12	2.41	1.43	0.86	0.78
time (sec)	N/A	0.018	0.036	0.010	2.977	0.873	0.298	0.585	4.628

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	44	51	57	80	49	56	55
normalized size	1	1.00	0.85	0.98	1.10	1.54	0.94	1.08	1.06
time (sec)	N/A	0.038	0.033	0.013	1.310	0.841	0.367	0.600	4.591

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	25	35	38	38	36	26	37
normalized size	1	1.00	1.25	1.75	1.90	1.90	1.80	1.30	1.85
time (sec)	N/A	0.004	0.009	0.006	1.315	0.962	0.275	0.623	0.038

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	52	76	188	105	53	54
normalized size	1	1.00	0.84	0.78	1.13	2.81	1.57	0.79	0.81
time (sec)	N/A	0.020	0.031	0.009	2.894	0.894	0.315	0.634	4.616

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	26	26	16	26
normalized size	1	1.00	1.00	1.00	0.94	1.53	1.53	0.94	1.53
time (sec)	N/A	0.003	0.002	0.000	1.341	0.977	0.241	0.613	0.026

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	61	73	188	99	49	55
normalized size	1	1.00	0.88	0.95	1.14	2.94	1.55	0.77	0.86
time (sec)	N/A	0.018	0.041	0.005	2.993	0.896	0.327	0.619	4.600

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	45	55	62	92	56	63	57
normalized size	1	1.00	0.79	0.96	1.09	1.61	0.98	1.11	1.00
time (sec)	N/A	0.038	0.037	0.012	1.366	0.731	0.419	0.625	0.065

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	56	86	202	107	61	66
normalized size	1	1.00	0.88	0.72	1.10	2.59	1.37	0.78	0.85
time (sec)	N/A	0.026	0.047	0.012	2.906	0.872	0.427	0.630	4.605

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	68	79	121	78	84	76
normalized size	1	1.00	0.87	0.99	1.14	1.75	1.13	1.22	1.10
time (sec)	N/A	0.051	0.056	0.013	1.306	1.150	0.515	0.631	4.610

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	25	35	60	60	60	39	59
normalized size	1	1.00	0.69	0.97	1.67	1.67	1.67	1.08	1.64
time (sec)	N/A	0.030	0.010	0.007	1.361	0.992	0.435	0.592	4.595

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	81	72	124	324	160	77	96
normalized size	1	1.00	0.74	0.66	1.14	2.97	1.47	0.71	0.88
time (sec)	N/A	0.037	0.051	0.010	2.982	0.965	0.498	0.626	4.759

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	48	49	16	48
normalized size	1	1.00	1.00	1.00	0.94	2.82	2.88	0.94	2.82
time (sec)	N/A	0.003	0.004	0.000	1.308	0.809	0.404	0.623	4.690

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	79	107	117	320	146	71	99
normalized size	1	1.00	0.79	1.07	1.17	3.20	1.46	0.71	0.99
time (sec)	N/A	0.034	0.044	0.004	2.901	0.918	0.518	0.621	4.795

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	67	87	106	180	104	85	101
normalized size	1	1.00	0.74	0.96	1.16	1.98	1.14	0.93	1.11
time (sec)	N/A	0.064	0.035	0.013	1.394	0.840	0.640	0.606	5.238

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	92	78	130	334	155	83	110
normalized size	1	1.00	0.78	0.66	1.10	2.83	1.31	0.70	0.93
time (sec)	N/A	0.049	0.055	0.016	2.904	0.817	0.657	0.631	5.175

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	83	102	123	209	126	106	120
normalized size	1	1.00	0.78	0.96	1.16	1.97	1.19	1.00	1.13
time (sec)	N/A	0.087	0.068	0.016	1.353	0.719	0.763	0.643	5.351

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	17	13	12	18	14
normalized size	1	1.00	1.00	0.93	1.13	0.87	0.80	1.20	0.93
time (sec)	N/A	0.009	0.004	0.004	1.345	1.298	0.135	0.623	5.108

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	17	15	12	18	16
normalized size	1	1.00	1.00	0.89	0.94	0.83	0.67	1.00	0.89
time (sec)	N/A	0.010	0.005	0.004	1.366	0.963	0.137	0.620	0.056

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	24	28	22	32	22
normalized size	1	1.00	1.00	0.88	0.92	1.08	0.85	1.23	0.85
time (sec)	N/A	0.017	0.006	0.007	1.342	0.917	0.205	0.626	0.059

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	24	28	22	32	22
normalized size	1	1.00	1.00	0.85	0.89	1.04	0.81	1.19	0.81
time (sec)	N/A	0.018	0.004	0.005	1.338	0.687	0.210	0.593	0.059

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	20	0	82	83	23	23
normalized size	1	1.00	0.93	0.67	0.00	2.73	2.77	0.77	0.77
time (sec)	N/A	0.027	0.011	0.006	0.000	0.830	0.166	0.581	0.167

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	44	33	0	102	87	33	29
normalized size	1	1.00	1.19	0.89	0.00	2.76	2.35	0.89	0.78
time (sec)	N/A	0.033	0.016	0.006	0.000	0.941	0.227	0.586	0.293

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	25	28	40	22	36	24
normalized size	1	1.00	0.89	0.89	1.00	1.43	0.79	1.29	0.86
time (sec)	N/A	0.020	0.013	0.011	1.349	0.722	0.204	0.629	0.035

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	25	28	40	22	36	26
normalized size	1	1.00	0.87	0.83	0.93	1.33	0.73	1.20	0.87
time (sec)	N/A	0.019	0.014	0.010	1.355	0.561	0.206	0.635	0.036

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	0	106	60	37	38
normalized size	1	1.00	1.06	1.00	0.00	3.12	1.76	1.09	1.12
time (sec)	N/A	0.025	0.016	0.007	0.000	0.867	0.249	0.630	5.138

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	0	105	66	36	38
normalized size	1	1.00	1.06	1.00	0.00	3.09	1.94	1.06	1.12
time (sec)	N/A	0.011	0.011	0.003	0.000	0.827	0.232	0.617	4.638

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	38	0	182	104	58	46
normalized size	1	1.00	1.00	0.76	0.00	3.64	2.08	1.16	0.92
time (sec)	N/A	0.053	0.022	0.007	0.000	0.826	0.319	0.639	4.876

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	18	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.004	0.005	0.003	1.325	0.716	5.450	0.628	4.505

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	18	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.004	0.005	0.003	1.293	0.685	2.406	0.603	0.026

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	18	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.004	0.005	0.001	1.333	1.288	0.952	0.631	0.025

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	16	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.76	0.90	0.62	0.71
time (sec)	N/A	0.004	0.005	0.003	1.315	0.722	1.303	0.612	0.025

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	13	14	17	13	14
normalized size	1	1.00	1.00	0.79	0.68	0.74	0.89	0.68	0.74
time (sec)	N/A	0.004	0.005	0.004	1.296	0.626	0.227	0.628	0.026

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	13	14	17	13	15
normalized size	1	1.00	1.00	0.84	0.68	0.74	0.89	0.68	0.79
time (sec)	N/A	0.004	0.005	0.004	1.353	0.806	0.384	0.593	0.026

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	15	17	13	15
normalized size	1	1.00	1.00	0.74	0.68	0.79	0.89	0.68	0.79
time (sec)	N/A	0.004	0.006	0.003	1.321	0.952	0.560	0.597	0.027

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	19	13	15
normalized size	1	1.00	1.00	0.74	0.68	0.68	1.00	0.68	0.79
time (sec)	N/A	0.004	0.006	0.003	1.312	0.903	1.239	0.586	0.027

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	29	34	24	25
normalized size	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.69
time (sec)	N/A	0.009	0.009	0.004	1.360	0.836	10.605	0.621	0.045

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	29	34	24	26
normalized size	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.72
time (sec)	N/A	0.008	0.008	0.005	1.262	1.024	5.506	0.625	0.037

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	29	34	24	26
normalized size	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.72
time (sec)	N/A	0.009	0.008	0.006	1.362	0.948	2.546	0.629	0.038

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	27	34	24	26
normalized size	1	1.00	0.83	0.75	0.67	0.75	0.94	0.67	0.72
time (sec)	N/A	0.008	0.008	0.005	1.338	0.822	1.705	0.582	0.037

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	27	24	26	32	24	26
normalized size	1	1.00	0.88	0.79	0.71	0.76	0.94	0.71	0.76
time (sec)	N/A	0.009	0.008	0.005	1.303	1.029	0.748	0.628	0.035

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	27	24	26	32	24	26
normalized size	1	1.00	0.88	0.79	0.71	0.76	0.94	0.71	0.76
time (sec)	N/A	0.008	0.009	0.005	1.333	0.829	0.942	0.632	0.038

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	27	24	26	32	24	26
normalized size	1	1.00	0.88	0.79	0.71	0.76	0.94	0.71	0.76
time (sec)	N/A	0.008	0.009	0.005	1.352	0.678	1.111	0.636	0.037

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	27	25	26	32	25	26
normalized size	1	1.00	0.88	0.79	0.74	0.76	0.94	0.74	0.76
time (sec)	N/A	0.008	0.009	0.004	1.308	0.649	1.710	0.601	0.032

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	35	40	49	35	35
normalized size	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.013	0.012	0.006	1.340	0.645	19.691	0.608	0.044

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	35	40	49	35	35
normalized size	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.012	0.010	0.004	1.336	0.666	10.623	0.626	0.044

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	35	40	49	35	35
normalized size	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.013	0.010	0.005	1.366	0.851	5.734	0.626	0.042

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	35	38	49	35	35
normalized size	1	1.00	0.80	0.75	0.69	0.75	0.96	0.69	0.69
time (sec)	N/A	0.013	0.010	0.006	1.338	0.924	2.175	0.580	0.042

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	38	35	37	48	35	35
normalized size	1	1.00	0.84	0.78	0.71	0.76	0.98	0.71	0.71
time (sec)	N/A	0.012	0.010	0.007	1.335	0.848	2.101	0.586	0.044

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	38	35	37	46	35	35
normalized size	1	1.00	0.87	0.81	0.74	0.79	0.98	0.74	0.74
time (sec)	N/A	0.012	0.011	0.006	1.386	0.869	2.312	0.578	0.044

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	38	35	37	48	35	35
normalized size	1	1.00	0.84	0.78	0.71	0.76	0.98	0.71	0.71
time (sec)	N/A	0.012	0.012	0.006	1.349	0.936	2.802	0.579	0.045

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	38	36	37	46	36	37
normalized size	1	1.00	0.87	0.81	0.77	0.79	0.98	0.77	0.79
time (sec)	N/A	0.012	0.011	0.004	1.347	0.752	3.855	0.610	0.042

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	203	152	194	170	192	196	67
normalized size	1	1.00	0.94	0.71	0.90	0.79	0.89	0.91	0.31
time (sec)	N/A	0.198	0.068	0.011	3.075	0.983	50.315	0.623	4.486

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	78	143	186	165	180	178	54
normalized size	1	1.00	0.38	0.70	0.91	0.81	0.88	0.87	0.26
time (sec)	N/A	0.152	0.027	0.005	3.035	0.963	15.293	0.648	0.087

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	189	140	185	124	172	178	55
normalized size	1	1.00	0.94	0.69	0.92	0.61	0.85	0.88	0.27
time (sec)	N/A	0.163	0.039	0.007	2.900	0.959	5.892	0.648	0.094

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	54	132	172	126	165	182	38
normalized size	1	1.00	0.28	0.69	0.90	0.66	0.86	0.95	0.20
time (sec)	N/A	0.144	0.025	0.007	2.978	0.665	3.167	0.601	0.071

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	146	132	172	126	160	182	37
normalized size	1	1.00	0.76	0.69	0.90	0.66	0.83	0.95	0.19
time (sec)	N/A	0.146	0.034	0.004	3.042	0.956	5.942	0.631	0.084

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	27	140	186	142	170	190	54
normalized size	1	1.00	0.13	0.69	0.92	0.70	0.84	0.94	0.27
time (sec)	N/A	0.167	0.006	0.009	3.042	0.849	12.160	0.624	4.501

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	29	143	187	167	178	178	53
normalized size	1	1.00	0.14	0.70	0.92	0.82	0.87	0.87	0.26
time (sec)	N/A	0.161	0.007	0.009	3.131	0.676	30.395	0.643	0.090

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	29	152	198	193	190	200	66
normalized size	1	1.00	0.13	0.71	0.92	0.90	0.88	0.93	0.31
time (sec)	N/A	0.175	0.006	0.011	3.044	0.889	109.164	0.621	4.481

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	221	158	206	192	0	196	80
normalized size	1	1.00	0.96	0.69	0.90	0.83	0.00	0.85	0.35
time (sec)	N/A	0.166	0.103	0.013	3.025	1.132	0.000	0.653	4.515

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	43	149	195	185	595	199	64
normalized size	1	1.00	0.20	0.68	0.89	0.85	2.73	0.91	0.29
time (sec)	N/A	0.148	0.013	0.012	2.994	0.998	153.056	0.617	0.081

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	198	158	195	187	440	199	64
normalized size	1	1.00	0.91	0.72	0.89	0.86	2.02	0.91	0.29
time (sec)	N/A	0.148	0.101	0.010	2.884	0.948	78.161	0.647	4.651

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	29	158	194	182	578	199	64
normalized size	1	1.00	0.13	0.72	0.89	0.83	2.65	0.91	0.29
time (sec)	N/A	0.147	0.006	0.008	3.022	0.694	50.142	0.616	0.086

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	199	149	194	179	434	199	64
normalized size	1	1.00	0.91	0.68	0.89	0.82	1.99	0.91	0.29
time (sec)	N/A	0.151	0.102	0.009	3.051	0.913	75.795	0.651	0.090

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	27	158	208	208	700	210	77
normalized size	1	1.00	0.12	0.69	0.90	0.90	3.04	0.91	0.33
time (sec)	N/A	0.188	0.006	0.016	2.983	0.945	150.184	0.618	0.083

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	29	161	209	228	0	196	77
normalized size	1	1.00	0.13	0.70	0.91	0.99	0.00	0.85	0.33
time (sec)	N/A	0.177	0.007	0.015	3.063	1.066	0.000	0.642	4.683

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	29	172	221	251	0	220	87
normalized size	1	1.00	0.12	0.71	0.91	1.03	0.00	0.91	0.36
time (sec)	N/A	0.191	0.007	0.017	3.035	0.494	0.000	0.658	4.690

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	242	170	218	254	0	209	87
normalized size	1	1.00	1.01	0.71	0.91	1.06	0.00	0.87	0.36
time (sec)	N/A	0.177	0.113	0.014	3.091	1.224	0.000	0.654	4.688

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	45	169	222	260	0	212	85
normalized size	1	1.00	0.19	0.70	0.92	1.07	0.00	0.88	0.35
time (sec)	N/A	0.170	0.017	0.016	2.971	0.964	0.000	0.659	0.083

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	223	169	221	257	0	211	85
normalized size	1	1.00	0.92	0.70	0.91	1.06	0.00	0.87	0.35
time (sec)	N/A	0.164	0.109	0.016	2.894	0.949	0.000	0.658	4.669

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	29	175	217	250	0	209	86
normalized size	1	1.00	0.12	0.73	0.91	1.05	0.00	0.87	0.36
time (sec)	N/A	0.164	0.006	0.010	2.995	1.049	0.000	0.647	0.083

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	220	166	217	241	0	209	86
normalized size	1	1.00	0.92	0.69	0.91	1.01	0.00	0.87	0.36
time (sec)	N/A	0.172	0.083	0.010	3.023	0.914	0.000	0.651	4.667

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	27	178	230	263	0	220	99
normalized size	1	1.00	0.11	0.71	0.92	1.05	0.00	0.88	0.39
time (sec)	N/A	0.191	0.006	0.020	3.002	0.898	0.000	0.652	0.102

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	29	181	231	283	0	208	99
normalized size	1	1.00	0.12	0.72	0.92	1.13	0.00	0.83	0.39
time (sec)	N/A	0.185	0.006	0.020	2.967	1.020	0.000	0.653	4.657

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	29	192	243	306	0	232	109
normalized size	1	1.00	0.11	0.73	0.92	1.16	0.00	0.88	0.41
time (sec)	N/A	0.212	0.007	0.022	3.050	0.758	0.000	0.626	4.648

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	48	66	86	117	122	194	33
normalized size	1	1.00	0.83	1.14	1.48	2.02	2.10	3.34	0.57
time (sec)	N/A	0.034	0.016	0.007	2.967	0.942	3.102	0.634	0.083

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	72	84	117	105	84	47
normalized size	1	1.00	1.00	0.67	0.78	1.08	0.97	0.78	0.44
time (sec)	N/A	0.061	0.025	0.008	3.046	0.959	4.146	0.635	0.079

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	24	67	79	110	99	79	42
normalized size	1	1.00	0.24	0.66	0.78	1.09	0.98	0.78	0.42
time (sec)	N/A	0.055	0.007	0.007	2.967	1.082	1.749	0.631	0.044

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	67	79	110	97	79	42
normalized size	1	1.00	1.00	0.68	0.80	1.11	0.98	0.80	0.42
time (sec)	N/A	0.055	0.016	0.006	2.959	0.937	0.843	0.603	0.041

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	22	62	74	107	90	74	37
normalized size	1	1.00	0.24	0.67	0.80	1.16	0.98	0.80	0.40
time (sec)	N/A	0.055	0.005	0.006	2.870	0.849	0.547	0.631	0.037

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	76	62	74	107	90	74	37
normalized size	1	1.00	0.83	0.67	0.80	1.16	0.98	0.80	0.40
time (sec)	N/A	0.053	0.013	0.005	2.893	0.657	0.734	0.594	0.027

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	20	67	79	120	97	79	42
normalized size	1	1.00	0.20	0.68	0.80	1.21	0.98	0.80	0.42
time (sec)	N/A	0.056	0.005	0.007	2.988	0.853	1.351	0.634	0.037

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	22	67	79	129	99	79	42
normalized size	1	1.00	0.22	0.66	0.78	1.28	0.98	0.78	0.42
time (sec)	N/A	0.054	0.006	0.008	2.932	1.234	2.435	0.628	0.039

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	22	72	86	136	105	86	48
normalized size	1	1.00	0.20	0.67	0.80	1.26	0.97	0.80	0.44
time (sec)	N/A	0.057	0.005	0.010	3.048	0.640	5.465	0.626	0.040

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	121	79	91	148	277	91	55
normalized size	1	1.00	0.99	0.65	0.75	1.21	2.27	0.75	0.45
time (sec)	N/A	0.061	0.050	0.011	2.997	0.584	8.345	0.622	4.617

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	30	74	86	141	264	86	51
normalized size	1	1.00	0.27	0.65	0.76	1.25	2.34	0.76	0.45
time (sec)	N/A	0.059	0.011	0.011	2.996	0.941	5.308	0.602	4.623

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	106	74	86	140	257	86	51
normalized size	1	1.00	0.94	0.65	0.76	1.24	2.27	0.76	0.45
time (sec)	N/A	0.059	0.045	0.010	2.959	1.096	3.387	0.629	4.677

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	22	74	86	140	257	86	50
normalized size	1	1.00	0.19	0.65	0.76	1.24	2.27	0.76	0.44
time (sec)	N/A	0.060	0.005	0.006	2.936	1.181	2.166	0.631	0.035

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	107	74	86	141	264	86	50
normalized size	1	1.00	0.95	0.65	0.76	1.25	2.34	0.76	0.44
time (sec)	N/A	0.059	0.045	0.006	2.838	1.134	2.989	0.604	4.663

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	20	79	92	148	366	92	55
normalized size	1	1.00	0.16	0.65	0.75	1.21	3.00	0.75	0.45
time (sec)	N/A	0.065	0.005	0.014	3.037	0.903	4.563	0.635	4.677

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	22	79	92	158	366	91	55
normalized size	1	1.00	0.18	0.65	0.75	1.30	3.00	0.75	0.45
time (sec)	N/A	0.061	0.005	0.012	2.944	0.904	8.330	0.634	0.077

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	22	84	97	163	384	98	59
normalized size	1	1.00	0.17	0.64	0.74	1.24	2.93	0.75	0.45
time (sec)	N/A	0.065	0.007	0.016	3.042	1.034	18.732	0.639	0.070

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	135	82	99	173	481	94	62
normalized size	1	1.00	1.05	0.64	0.77	1.34	3.73	0.73	0.48
time (sec)	N/A	0.068	0.046	0.012	3.014	0.927	23.354	0.685	4.733

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	32	82	99	175	481	94	62
normalized size	1	1.00	0.25	0.64	0.77	1.36	3.73	0.73	0.48
time (sec)	N/A	0.070	0.012	0.010	2.911	0.660	15.486	0.647	0.066

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	121	82	97	171	481	92	62
normalized size	1	1.00	0.94	0.64	0.75	1.33	3.73	0.71	0.48
time (sec)	N/A	0.068	0.031	0.012	2.969	0.655	9.810	0.640	4.687

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	22	86	99	175	481	94	61
normalized size	1	1.00	0.17	0.67	0.77	1.36	3.73	0.73	0.47
time (sec)	N/A	0.069	0.005	0.008	3.020	0.684	6.379	0.589	0.037

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	121	86	99	173	481	94	61
normalized size	1	1.00	0.94	0.67	0.77	1.34	3.73	0.73	0.47
time (sec)	N/A	0.068	0.033	0.007	2.896	0.791	8.459	0.610	4.750

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	20	87	102	178	653	99	65
normalized size	1	1.00	0.14	0.63	0.74	1.29	4.73	0.72	0.47
time (sec)	N/A	0.070	0.005	0.013	2.976	0.994	12.635	0.628	4.718

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	22	87	102	188	653	99	65
normalized size	1	1.00	0.16	0.63	0.74	1.36	4.73	0.72	0.47
time (sec)	N/A	0.071	0.005	0.015	2.853	0.941	22.417	0.625	4.721

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	22	92	107	193	678	106	69
normalized size	1	1.00	0.15	0.63	0.73	1.31	4.61	0.72	0.47
time (sec)	N/A	0.075	0.006	0.014	2.951	0.721	51.156	0.638	0.075

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	24	23	23	26	24	11
normalized size	1	1.00	1.00	1.60	1.53	1.53	1.73	1.60	0.73
time (sec)	N/A	0.007	0.004	0.009	2.969	0.940	0.291	0.624	0.032

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	100	22	69	68	108	94	68	57
normalized size	1	1.37	0.30	0.95	0.93	1.48	1.29	0.93	0.78
time (sec)	N/A	0.257	0.005	0.050	2.971	0.918	1.781	0.630	4.767

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	432	97	367	1999	540	389
normalized size	1	1.00	0.91	4.45	1.00	3.78	20.61	5.57	4.01
time (sec)	N/A	0.043	0.055	0.008	1.330	0.936	4.497	0.690	5.113

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	72	291	79	251	1221	365	272
normalized size	1	1.00	0.91	3.68	1.00	3.18	15.46	4.62	3.44
time (sec)	N/A	0.032	0.035	0.006	1.339	0.810	2.847	0.610	4.988

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	178	61	157	683	224	167
normalized size	1	1.00	0.92	2.92	1.00	2.57	11.20	3.67	2.74
time (sec)	N/A	0.022	0.031	0.007	1.340	0.862	1.665	0.658	4.793

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	93	43	85	306	117	93
normalized size	1	1.00	0.93	2.16	1.00	1.98	7.12	2.72	2.16
time (sec)	N/A	0.015	0.023	0.006	1.315	1.061	0.943	0.654	4.751

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	35	25	33	94	43	34
normalized size	1	1.00	1.00	1.40	1.00	1.32	3.76	1.72	1.36
time (sec)	N/A	0.007	0.014	0.003	1.297	0.939	0.412	0.689	4.801

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	88	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	2.26	0.00	-0.03
time (sec)	N/A	0.008	0.008	0.257	0.000	1.005	1.247	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	374	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	9.59	0.00	-0.03
time (sec)	N/A	0.008	0.007	0.260	0.000	0.970	6.223	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	1556	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	39.90	0.00	-0.03
time (sec)	N/A	0.008	0.008	0.256	0.000	0.781	20.437	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	45	0	0	0	92	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	2.09	0.00	-0.02
time (sec)	N/A	0.012	0.014	0.261	0.000	0.733	2.949	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	95	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	2.16	0.00	-0.02
time (sec)	N/A	0.010	0.008	0.255	0.000	0.877	1.250	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	39	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	1.03	0.00	-0.03
time (sec)	N/A	0.009	0.008	0.257	0.000	0.945	7.151	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	0	0	0	102	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	2.17	0.00	-0.02
time (sec)	N/A	0.012	0.012	0.254	0.000	1.041	11.565	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	0	0	0	92	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	2.04	0.00	-0.02
time (sec)	N/A	0.012	0.012	0.260	0.000	1.023	21.424	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	92	0	0	343	0	-1
normalized size	1	1.00	1.06	2.56	0.00	0.00	9.53	0.00	-0.03
time (sec)	N/A	0.008	0.009	0.281	0.000	1.009	6.163	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	73	57	110	57	55
normalized size	1	1.00	0.62	0.59	0.91	0.71	1.38	0.71	0.69
time (sec)	N/A	0.045	0.026	0.007	1.356	0.865	1.418	0.605	4.587

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	53	46	87	43	44
normalized size	1	1.00	0.66	0.61	0.90	0.78	1.47	0.73	0.75
time (sec)	N/A	0.033	0.019	0.006	1.323	0.553	0.704	0.592	4.659

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	33	34	63	29	33
normalized size	1	1.00	0.74	0.66	0.87	0.89	1.66	0.76	0.87
time (sec)	N/A	0.022	0.014	0.008	1.337	0.610	0.318	0.595	4.650

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	39	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	2.17	0.78	0.78
time (sec)	N/A	0.003	0.002	0.001	1.334	0.734	0.168	0.604	4.627

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	39	27	77	56	33	29
normalized size	1	1.00	1.00	1.05	0.73	2.08	1.51	0.89	0.78
time (sec)	N/A	0.024	0.008	0.003	1.363	0.992	1.409	0.592	4.658

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	59	63	51	106	42	46	35
normalized size	1	1.00	1.26	1.34	1.09	2.26	0.89	0.98	0.74
time (sec)	N/A	0.025	0.033	0.005	1.369	0.494	1.926	0.611	4.795

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	39	85	73	131	92	72	54
normalized size	1	1.00	0.55	1.20	1.03	1.85	1.30	1.01	0.76
time (sec)	N/A	0.039	0.009	0.006	1.324	1.016	3.735	0.596	4.910

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	39	105	93	157	117	92	74
normalized size	1	1.00	0.41	1.11	0.98	1.65	1.23	0.97	0.78
time (sec)	N/A	0.052	0.008	0.009	1.340	0.968	5.907	0.677	4.942

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	77	77	69	146	117	64	-1
normalized size	1	1.00	0.82	0.82	0.73	1.55	1.24	0.68	-0.01
time (sec)	N/A	0.033	0.034	0.008	1.305	1.112	5.773	0.691	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	64	57	49	119	92	50	-1
normalized size	1	1.00	0.91	0.81	0.70	1.70	1.31	0.71	-0.01
time (sec)	N/A	0.021	0.024	0.005	1.359	0.911	3.527	0.715	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	49	36	28	94	41	37	35
normalized size	1	1.00	1.07	0.78	0.61	2.04	0.89	0.80	0.76
time (sec)	N/A	0.010	0.014	0.003	1.328	0.902	1.831	0.661	4.673

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	63	54	28	88	56	57	56
normalized size	1	1.00	1.50	1.29	0.67	2.10	1.33	1.36	1.33
time (sec)	N/A	0.012	0.072	0.004	1.373	0.751	1.417	0.652	4.816

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	42	59	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	2.00	2.81	0.81
time (sec)	N/A	0.004	0.005	0.003	1.315	0.806	0.706	0.681	4.567

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	36	38	68	112	37
normalized size	1	1.00	0.70	0.64	0.82	0.86	1.55	2.55	0.84
time (sec)	N/A	0.011	0.009	0.004	1.283	0.833	0.912	0.684	4.767

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	56	49	359	138	73
normalized size	1	1.00	0.62	0.57	0.82	0.72	5.28	2.03	1.07
time (sec)	N/A	0.018	0.010	0.006	1.315	0.981	1.298	0.653	4.707

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	76	60	575	166	93
normalized size	1	1.00	0.58	0.54	0.83	0.65	6.25	1.80	1.01
time (sec)	N/A	0.029	0.012	0.006	1.337	0.865	1.742	0.697	4.946

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	73	68	133	57	64
normalized size	1	1.00	0.62	0.59	0.91	0.85	1.66	0.71	0.80
time (sec)	N/A	0.048	0.026	0.006	1.328	0.990	3.813	0.682	4.604

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	53	57	109	43	53
normalized size	1	1.00	0.66	0.61	0.90	0.97	1.85	0.73	0.90
time (sec)	N/A	0.035	0.019	0.005	1.317	0.848	2.239	0.642	4.659

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	33	45	85	29	42
normalized size	1	1.00	0.74	0.66	0.87	1.18	2.24	0.76	1.11
time (sec)	N/A	0.024	0.014	0.005	1.328	0.967	1.133	0.645	4.637

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	32	61	14	14
normalized size	1	1.00	1.00	0.83	0.78	1.78	3.39	0.78	0.78
time (sec)	N/A	0.004	0.003	0.003	1.325	0.940	0.592	0.672	4.567

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	52	40	100	78	48	42
normalized size	1	1.00	0.93	0.96	0.74	1.85	1.44	0.89	0.78
time (sec)	N/A	0.036	0.021	0.006	1.278	0.850	1.924	0.631	4.696

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	37	75	63	119	88	63	47
normalized size	1	1.00	0.59	1.19	1.00	1.89	1.40	1.00	0.75
time (sec)	N/A	0.037	0.009	0.006	1.342	1.012	2.333	0.677	4.816

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	76	102	90	136	71	70	52
normalized size	1	1.00	1.12	1.50	1.32	2.00	1.04	1.03	0.76
time (sec)	N/A	0.040	0.037	0.006	1.370	1.277	2.984	0.647	4.905

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	39	122	110	157	119	92	72
normalized size	1	1.00	0.42	1.33	1.20	1.71	1.29	1.00	0.78
time (sec)	N/A	0.054	0.010	0.010	1.333	0.915	5.331	0.644	4.941

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	39	142	130	179	148	109	89
normalized size	1	1.00	0.34	1.22	1.12	1.54	1.28	0.94	0.77
time (sec)	N/A	0.069	0.010	0.016	1.384	0.642	8.264	0.650	5.212

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	94	95	87	168	148	76	-1
normalized size	1	1.00	0.82	0.83	0.76	1.46	1.29	0.66	-0.01
time (sec)	N/A	0.042	0.136	0.006	1.356	0.905	8.195	0.662	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	83	75	67	145	119	63	-1
normalized size	1	1.00	0.91	0.82	0.74	1.59	1.31	0.69	-0.01
time (sec)	N/A	0.031	0.118	0.005	1.334	0.962	5.206	0.640	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	51	43	124	70	49	37
normalized size	1	1.00	1.00	0.78	0.66	1.91	1.08	0.75	0.57
time (sec)	N/A	0.015	0.089	0.003	1.314	1.060	2.905	0.628	4.608

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	50	69	43	112	88	73	40
normalized size	1	1.00	0.79	1.10	0.68	1.78	1.40	1.16	0.63
time (sec)	N/A	0.018	0.008	0.004	1.344	0.834	2.369	0.670	5.153

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	52	92	66	112	78	114	-1
normalized size	1	1.00	0.85	1.51	1.08	1.84	1.28	1.87	-0.02
time (sec)	N/A	0.018	0.008	0.004	1.317	0.912	2.026	0.709	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	35	68	86	17
normalized size	1	1.00	1.00	0.86	0.81	1.67	3.24	4.10	0.81
time (sec)	N/A	0.005	0.006	0.004	1.336	0.863	0.911	0.642	5.295

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	36	49	94	166	71
normalized size	1	1.00	0.70	0.64	0.82	1.11	2.14	3.77	1.61
time (sec)	N/A	0.012	0.010	0.003	1.386	1.318	1.179	0.687	5.655

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	56	60	420	192	91
normalized size	1	1.00	0.62	0.57	0.82	0.88	6.18	2.82	1.34
time (sec)	N/A	0.021	0.011	0.004	1.375	0.887	1.613	0.638	5.710

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	76	71	648	220	111
normalized size	1	1.00	0.58	0.54	0.83	0.77	7.04	2.39	1.21
time (sec)	N/A	0.029	0.013	0.006	1.424	0.829	2.124	0.686	5.781

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	73	79	158	57	75
normalized size	1	1.00	0.62	0.59	0.91	0.99	1.98	0.71	0.94
time (sec)	N/A	0.047	0.028	0.006	1.326	0.902	9.131	0.660	4.863

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	53	68	133	43	64
normalized size	1	1.00	0.66	0.61	0.90	1.15	2.25	0.73	1.08
time (sec)	N/A	0.034	0.021	0.005	1.357	0.739	6.275	0.922	4.752

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	33	56	109	29	53
normalized size	1	1.00	0.74	0.66	0.87	1.47	2.87	0.76	1.39
time (sec)	N/A	0.024	0.017	0.004	1.370	0.606	3.832	0.992	4.744

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	43	85	14	14
normalized size	1	1.00	1.00	0.83	0.78	2.39	4.72	0.78	0.78
time (sec)	N/A	0.003	0.004	0.003	1.341	0.463	2.100	1.199	4.595

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	62	66	54	126	105	62	59
normalized size	1	1.00	0.86	0.92	0.75	1.75	1.46	0.86	0.82
time (sec)	N/A	0.043	0.027	0.005	1.326	0.959	3.556	1.187	4.682

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	37	88	76	142	112	82	66
normalized size	1	1.00	0.46	1.10	0.95	1.78	1.40	1.02	0.82
time (sec)	N/A	0.045	0.010	0.006	1.379	0.975	3.223	1.130	4.948

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	39	116	104	145	117	88	71
normalized size	1	1.00	0.45	1.35	1.21	1.69	1.36	1.02	0.83
time (sec)	N/A	0.048	0.010	0.006	1.349	0.897	3.727	1.164	5.007

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	87	139	127	158	99	87	72
normalized size	1	1.00	0.98	1.56	1.43	1.78	1.11	0.98	0.81
time (sec)	N/A	0.053	0.042	0.010	1.469	0.965	4.528	1.218	5.201

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	39	159	147	179	150	109	89
normalized size	1	1.00	0.35	1.41	1.30	1.58	1.33	0.96	0.79
time (sec)	N/A	0.067	0.011	0.018	1.381	0.855	7.576	1.113	5.430

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	39	179	167	201	175	126	106
normalized size	1	1.00	0.28	1.31	1.22	1.47	1.28	0.92	0.77
time (sec)	N/A	0.083	0.011	0.036	1.439	0.704	11.535	1.160	5.678

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	105	113	105	190	175	91	-1
normalized size	1	1.00	0.77	0.83	0.77	1.40	1.29	0.67	-0.01
time (sec)	N/A	0.054	0.154	0.007	1.264	0.822	11.108	1.090	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	94	93	85	167	150	77	-1
normalized size	1	1.00	0.84	0.83	0.76	1.49	1.34	0.69	-0.01
time (sec)	N/A	0.042	0.137	0.008	1.342	0.854	7.223	1.080	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	76	66	58	146	97	63	37
normalized size	1	1.00	0.90	0.79	0.69	1.74	1.15	0.75	0.44
time (sec)	N/A	0.022	0.110	0.003	1.322	0.895	4.214	1.226	4.460

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	52	85	59	140	117	87	40
normalized size	1	1.00	0.63	1.02	0.71	1.69	1.41	1.05	0.48
time (sec)	N/A	0.025	0.008	0.005	1.323	0.623	3.631	1.113	5.050

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	54	110	84	141	112	132	-1
normalized size	1	1.00	0.63	1.28	0.98	1.64	1.30	1.53	-0.01
time (sec)	N/A	0.027	0.008	0.006	1.321	0.925	3.165	1.125	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	54	130	104	140	105	168	-1
normalized size	1	1.00	0.66	1.59	1.27	1.71	1.28	2.05	-0.01
time (sec)	N/A	0.028	0.010	0.010	1.401	1.036	3.662	1.139	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	46	95	113	71
normalized size	1	1.00	1.00	0.86	0.81	2.19	4.52	5.38	3.38
time (sec)	N/A	0.005	0.008	0.003	1.409	0.987	1.260	0.993	5.073

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	36	60	121	220	91
normalized size	1	1.00	0.70	0.64	0.82	1.36	2.75	5.00	2.07
time (sec)	N/A	0.011	0.011	0.006	1.528	0.682	1.591	1.142	5.369

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	56	71	481	246	111
normalized size	1	1.00	0.62	0.57	0.82	1.04	7.07	3.62	1.63
time (sec)	N/A	0.019	0.012	0.006	1.414	0.944	2.160	1.156	5.618

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	76	82	721	274	131
normalized size	1	1.00	0.58	0.54	0.83	0.89	7.84	2.98	1.42
time (sec)	N/A	0.028	0.015	0.007	1.390	1.085	2.793	1.183	5.981

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	64	61	96	93	1012	300	151
normalized size	1	1.00	0.55	0.53	0.83	0.80	8.72	2.59	1.30
time (sec)	N/A	0.042	0.016	0.007	1.447	1.105	3.488	1.230	6.365

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	75	72	116	104	1346	328	171
normalized size	1	1.00	0.54	0.51	0.83	0.74	9.61	2.34	1.22
time (sec)	N/A	0.056	0.019	0.008	1.536	1.406	4.423	1.101	6.869

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	94	91	153	145	301	113	141
normalized size	1	1.00	0.58	0.57	0.95	0.90	1.87	0.70	0.88
time (sec)	N/A	0.100	0.058	0.008	1.470	1.030	103.129	1.068	4.837

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	83	80	133	134	277	99	130
normalized size	1	1.00	0.59	0.57	0.95	0.96	1.98	0.71	0.93
time (sec)	N/A	0.080	0.044	0.008	1.453	0.905	79.370	1.096	4.836

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	72	69	113	123	253	85	119
normalized size	1	1.00	0.59	0.57	0.93	1.01	2.07	0.70	0.98
time (sec)	N/A	0.071	0.041	0.006	1.482	0.775	62.850	1.010	4.809

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	61	58	93	112	230	71	108
normalized size	1	1.00	0.60	0.57	0.92	1.11	2.28	0.70	1.07
time (sec)	N/A	0.058	0.035	0.006	1.429	0.886	48.285	1.077	4.741

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	73	101	204	57	97
normalized size	1	1.00	0.62	0.59	0.91	1.26	2.55	0.71	1.21
time (sec)	N/A	0.046	0.030	0.006	1.419	1.220	37.198	1.146	4.735

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	53	90	180	43	86
normalized size	1	1.00	0.66	0.61	0.90	1.53	3.05	0.73	1.46
time (sec)	N/A	0.035	0.023	0.005	1.388	0.867	27.022	0.960	4.600

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	33	78	156	29	29
normalized size	1	1.00	0.74	0.66	0.87	2.05	4.11	0.76	0.76
time (sec)	N/A	0.024	0.019	0.005	1.294	0.822	18.496	1.058	4.739

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	65	133	14	14
normalized size	1	1.00	1.00	0.83	0.78	3.61	7.39	0.78	0.78
time (sec)	N/A	0.003	0.006	0.003	1.321	0.712	12.960	1.188	4.813

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	84	94	82	170	160	90	87
normalized size	1	1.00	0.78	0.87	0.76	1.57	1.48	0.83	0.81
time (sec)	N/A	0.070	0.040	0.005	1.402	0.967	10.477	1.168	5.309

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	37	118	106	188	167	116	95
normalized size	1	1.00	0.31	1.00	0.90	1.59	1.42	0.98	0.81
time (sec)	N/A	0.071	0.012	0.007	1.432	1.067	9.267	1.144	5.445

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	39	148	136	192	175	124	132
normalized size	1	1.00	0.31	1.17	1.08	1.52	1.39	0.98	1.05
time (sec)	N/A	0.076	0.013	0.007	1.456	0.895	8.727	1.145	5.651

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	39	168	156	190	175	124	149
normalized size	1	1.00	0.31	1.33	1.24	1.51	1.39	0.98	1.18
time (sec)	N/A	0.076	0.013	0.012	1.471	1.058	7.118	1.096	5.992

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	39	190	178	189	173	122	105
normalized size	1	1.00	0.30	1.48	1.39	1.48	1.35	0.95	0.82
time (sec)	N/A	0.079	0.012	0.023	1.452	0.911	7.741	1.127	6.197

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	109	213	201	202	153	121	106
normalized size	1	1.00	0.83	1.63	1.53	1.54	1.17	0.92	0.81
time (sec)	N/A	0.083	0.050	0.051	1.480	1.055	9.314	1.087	6.621

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	39	233	221	223	204	143	123
normalized size	1	1.00	0.25	1.50	1.43	1.44	1.32	0.92	0.79
time (sec)	N/A	0.101	0.013	0.128	1.507	1.077	14.979	1.102	6.678

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	39	253	241	245	231	160	140
normalized size	1	1.00	0.22	1.41	1.35	1.37	1.29	0.89	0.78
time (sec)	N/A	0.122	0.013	0.335	1.505	1.190	21.819	1.092	7.225

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	138	169	161	255	258	133	-1
normalized size	1	1.00	0.68	0.84	0.80	1.26	1.28	0.66	-0.00
time (sec)	N/A	0.114	0.219	0.013	1.461	1.242	30.595	1.209	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	127	149	141	234	231	119	-1
normalized size	1	1.00	0.71	0.84	0.79	1.31	1.30	0.67	-0.01
time (sec)	N/A	0.086	0.193	0.009	1.400	1.063	20.001	1.166	0.000

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	116	129	121	211	204	105	-1
normalized size	1	1.00	0.75	0.84	0.79	1.37	1.32	0.68	-0.01
time (sec)	N/A	0.070	0.182	0.007	1.375	1.056	13.568	1.244	0.000

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	98	96	88	190	151	91	37
normalized size	1	1.00	0.80	0.79	0.72	1.56	1.24	0.75	0.30
time (sec)	N/A	0.042	0.140	0.002	1.323	0.838	8.348	1.123	4.620

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	52	117	91	184	173	115	40
normalized size	1	1.00	0.42	0.95	0.74	1.50	1.41	0.93	0.33
time (sec)	N/A	0.044	0.009	0.005	1.458	0.924	7.465	1.217	6.031

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	54	146	120	189	175	160	-1
normalized size	1	1.00	0.42	1.14	0.94	1.48	1.37	1.25	-0.01
time (sec)	N/A	0.049	0.010	0.008	1.425	1.197	6.843	1.146	0.000

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	54	166	140	191	175	200	-1
normalized size	1	1.00	0.42	1.29	1.09	1.48	1.36	1.55	-0.01
time (sec)	N/A	0.050	0.011	0.010	1.448	1.044	7.823	1.079	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	54	186	160	187	167	240	-1
normalized size	1	1.00	0.43	1.48	1.27	1.48	1.33	1.90	-0.01
time (sec)	N/A	0.051	0.011	0.018	1.474	1.172	8.788	1.201	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	54	206	180	184	160	276	-1
normalized size	1	1.00	0.44	1.66	1.45	1.48	1.29	2.23	-0.01
time (sec)	N/A	0.052	0.011	0.034	1.509	1.038	9.863	1.119	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	68	150	167	111
normalized size	1	1.00	1.00	0.86	0.81	3.24	7.14	7.95	5.29
time (sec)	N/A	0.005	0.008	0.003	1.495	0.984	2.450	1.062	6.277

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	36	82	175	328	131
normalized size	1	1.00	0.70	0.64	0.82	1.86	3.98	7.45	2.98
time (sec)	N/A	0.011	0.014	0.006	1.566	0.941	2.978	1.092	6.849

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	56	93	604	354	151
normalized size	1	1.00	0.62	0.57	0.82	1.37	8.88	5.21	2.22
time (sec)	N/A	0.021	0.015	0.005	1.513	1.153	3.928	1.224	7.416

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	76	104	867	382	171
normalized size	1	1.00	0.58	0.54	0.83	1.13	9.42	4.15	1.86
time (sec)	N/A	0.030	0.017	0.006	1.551	1.300	4.969	1.295	8.173

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	64	61	96	115	1182	408	191
normalized size	1	1.00	0.55	0.53	0.83	0.99	10.19	3.52	1.65
time (sec)	N/A	0.045	0.018	0.006	1.565	1.349	6.262	1.170	8.840

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	75	72	116	126	1540	436	211
normalized size	1	1.00	0.54	0.51	0.83	0.90	11.00	3.11	1.51
time (sec)	N/A	0.056	0.021	0.007	1.532	1.813	7.799	1.246	9.629

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	86	83	136	137	1950	462	231
normalized size	1	1.00	0.52	0.51	0.83	0.84	11.89	2.82	1.41
time (sec)	N/A	0.072	0.022	0.008	1.589	1.736	9.066	1.233	10.465

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	24	40	28	61	34	25
normalized size	1	1.00	0.59	0.52	0.87	0.61	1.33	0.74	0.54
time (sec)	N/A	0.020	0.011	0.003	2.975	0.868	1.972	1.031	4.547

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	39	46	45	42	75	43	30
normalized size	1	1.00	0.62	0.73	0.71	0.67	1.19	0.68	0.48
time (sec)	N/A	0.016	0.013	0.006	2.935	0.974	4.564	1.172	0.028

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	26	23	44	23	20
normalized size	1	1.00	0.71	0.61	0.84	0.74	1.42	0.74	0.65
time (sec)	N/A	0.015	0.007	0.002	2.964	0.828	0.629	1.014	0.021

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	36	32	31	37	54	36	23
normalized size	1	1.00	0.80	0.71	0.69	0.82	1.20	0.80	0.51
time (sec)	N/A	0.010	0.015	0.004	2.967	0.954	2.707	1.045	0.034

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	27	11	16
normalized size	1	1.00	1.00	0.80	0.73	0.73	1.80	0.73	1.07
time (sec)	N/A	0.002	0.002	0.003	1.315	0.943	0.200	1.042	0.019

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	19	29	22	29	16
normalized size	1	1.00	1.00	0.74	0.70	1.07	0.81	1.07	0.59
time (sec)	N/A	0.004	0.006	0.003	2.980	0.948	0.214	0.992	0.032

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	19	44	39	38	22
normalized size	1	1.00	1.00	0.83	0.63	1.47	1.30	1.27	0.73
time (sec)	N/A	0.016	0.005	0.005	2.939	1.001	1.245	1.029	0.028

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	34	21	35	19	40	19
normalized size	1	1.00	1.00	1.36	0.84	1.40	0.76	1.60	0.76
time (sec)	N/A	0.005	0.006	0.005	2.938	0.732	0.249	1.148	0.028

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	41	35	57	24	43	25
normalized size	1	1.00	0.95	1.05	0.90	1.46	0.62	1.10	0.64
time (sec)	N/A	0.016	0.018	0.006	2.810	0.998	1.670	1.206	0.030

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	20	34	42	27
normalized size	1	1.00	1.00	0.83	0.78	1.11	1.89	2.33	1.50
time (sec)	N/A	0.003	0.003	0.003	2.937	0.761	0.928	1.007	0.033

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	32	55	49	64	63	55	45
normalized size	1	1.00	0.56	0.96	0.86	1.12	1.11	0.96	0.79
time (sec)	N/A	0.022	0.005	0.006	3.012	1.098	3.197	1.052	0.031

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	34	40	28	61	52	28
normalized size	1	1.00	0.59	0.74	0.87	0.61	1.33	1.13	0.61
time (sec)	N/A	0.021	0.013	0.004	2.912	0.955	2.008	1.017	4.532

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	39	46	45	45	167	33	32
normalized size	1	1.00	0.62	0.73	0.71	0.71	2.65	0.52	0.51
time (sec)	N/A	0.016	0.013	0.006	3.000	0.900	4.655	1.077	4.563

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	26	23	44	32	23
normalized size	1	1.00	0.71	0.94	0.84	0.74	1.42	1.03	0.74
time (sec)	N/A	0.015	0.009	0.005	2.968	0.743	0.671	1.140	0.022

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	36	32	31	40	124	26	27
normalized size	1	1.00	0.80	0.71	0.69	0.89	2.76	0.58	0.60
time (sec)	N/A	0.010	0.016	0.006	2.952	0.954	2.684	1.088	0.035

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	11	18	27	11	18
normalized size	1	1.00	1.00	1.47	0.73	1.20	1.80	0.73	1.20
time (sec)	N/A	0.002	0.002	0.003	1.277	0.651	0.203	1.081	0.020

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	19	32	22	19	18
normalized size	1	1.00	1.00	0.74	0.70	1.19	0.81	0.70	0.67
time (sec)	N/A	0.003	0.006	0.003	2.943	1.467	0.215	1.000	0.024

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	28	76	40	32
normalized size	1	1.00	1.00	0.83	1.17	0.93	2.53	1.33	1.07
time (sec)	N/A	0.015	0.004	0.006	2.865	0.852	1.320	1.176	4.552

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	34	21	36	20	39	21
normalized size	1	1.00	1.00	1.36	0.84	1.44	0.80	1.56	0.84
time (sec)	N/A	0.005	0.007	0.005	2.923	0.743	0.245	1.144	0.027

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	41	51	38	97	45	35
normalized size	1	1.00	0.95	1.05	1.31	0.97	2.49	1.15	0.90
time (sec)	N/A	0.016	0.027	0.006	2.993	0.917	1.733	1.072	4.655

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	14	21	76	73	31
normalized size	1	1.00	1.00	1.39	0.78	1.17	4.22	4.06	1.72
time (sec)	N/A	0.003	0.003	0.003	2.897	0.905	0.966	1.050	0.033

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	32	55	65	45	139	57	49
normalized size	1	1.00	0.56	0.96	1.14	0.79	2.44	1.00	0.86
time (sec)	N/A	0.022	0.006	0.005	3.018	0.864	3.275	1.055	0.030

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	34	40	28	61	34	28
normalized size	1	1.00	0.59	0.74	0.87	0.61	1.33	0.74	0.61
time (sec)	N/A	0.020	0.010	0.006	2.941	0.922	1.918	0.967	5.314

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	49	61	57	42	167	44	-1
normalized size	1	1.00	0.68	0.85	0.79	0.58	2.32	0.61	-0.01
time (sec)	N/A	0.019	0.012	0.007	2.916	0.926	4.563	1.186	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	26	23	44	23	23
normalized size	1	1.00	0.71	0.94	0.84	0.74	1.42	0.74	0.74
time (sec)	N/A	0.014	0.007	0.005	2.911	0.884	0.656	1.065	5.511

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	47	43	37	124	37	-1
normalized size	1	1.00	0.85	0.87	0.80	0.69	2.30	0.69	-0.02
time (sec)	N/A	0.013	0.014	0.006	2.894	0.851	2.799	1.156	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	11	11	27	11	11
normalized size	1	1.00	1.00	1.47	0.73	0.73	1.80	0.73	0.73
time (sec)	N/A	0.002	0.002	0.004	1.303	0.837	0.202	1.033	0.060

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	35	31	29	22	30	29
normalized size	1	1.00	1.03	0.97	0.86	0.81	0.61	0.83	0.81
time (sec)	N/A	0.006	0.006	0.003	2.974	0.739	0.215	1.112	5.032

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	19	28	82	24	24
normalized size	1	1.00	1.00	0.83	0.63	0.93	2.73	0.80	0.80
time (sec)	N/A	0.017	0.005	0.005	3.009	0.924	1.371	1.013	5.338

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	48	48	33	35	19	44	39
normalized size	1	1.00	1.41	1.41	0.97	1.03	0.56	1.29	1.15
time (sec)	N/A	0.008	0.010	0.003	2.930	0.634	0.249	1.231	5.548

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	55	41	35	38	97	29	29
normalized size	1	1.00	1.41	1.05	0.90	0.97	2.49	0.74	0.74
time (sec)	N/A	0.016	0.008	0.004	2.959	1.178	1.738	1.071	5.330

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	14	20	76	42	31
normalized size	1	1.00	1.00	1.39	0.78	1.11	4.22	2.33	1.72
time (sec)	N/A	0.003	0.003	0.003	3.014	0.898	0.975	1.096	5.098

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	32	55	49	45	139	41	43
normalized size	1	1.00	0.56	0.96	0.86	0.79	2.44	0.72	0.75
time (sec)	N/A	0.021	0.005	0.005	2.955	0.668	3.360	0.981	5.190

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	24	40	28	68	37	27
normalized size	1	1.00	0.59	0.52	0.87	0.61	1.48	0.80	0.59
time (sec)	N/A	0.022	0.014	0.004	2.948	0.486	2.081	1.060	5.127

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	48	55	45	72	83	0	-1
normalized size	1	1.00	0.67	0.76	0.62	1.00	1.15	0.00	-0.01
time (sec)	N/A	0.021	0.016	0.008	2.939	0.627	4.672	0.000	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	26	23	49	25	22
normalized size	1	1.00	0.71	0.61	0.84	0.74	1.58	0.81	0.71
time (sec)	N/A	0.014	0.009	0.004	3.032	0.906	0.686	0.978	5.145

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	41	31	67	61	0	-1
normalized size	1	1.00	0.80	0.76	0.57	1.24	1.13	0.00	-0.02
time (sec)	N/A	0.014	0.016	0.005	2.982	0.794	2.759	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	18	31	12	11
normalized size	1	1.00	1.00	0.80	0.73	1.20	2.07	0.80	0.73
time (sec)	N/A	0.002	0.002	0.002	1.308	0.987	0.218	1.083	0.070

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	29	19	59	34	0	28
normalized size	1	1.00	1.00	0.81	0.53	1.64	0.94	0.00	0.78
time (sec)	N/A	0.006	0.008	0.003	2.992	0.883	0.385	0.000	5.027

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	52	44	24	24
normalized size	1	1.00	1.00	0.83	1.17	1.73	1.47	0.80	0.80
time (sec)	N/A	0.015	0.004	0.006	2.929	1.097	1.264	1.171	4.732

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	49	43	21	64	32	0	41
normalized size	1	1.00	1.44	1.26	0.62	1.88	0.94	0.00	1.21
time (sec)	N/A	0.008	0.007	0.005	2.915	0.912	0.417	0.000	4.775

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	55	41	51	65	27	29	29
normalized size	1	1.00	1.41	1.05	1.31	1.67	0.69	0.74	0.74
time (sec)	N/A	0.016	0.014	0.004	2.897	0.889	1.701	1.212	4.792

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	37	0	31
normalized size	1	1.00	1.00	0.83	0.78	0.78	2.06	0.00	1.72
time (sec)	N/A	0.003	0.003	0.004	2.892	0.893	1.000	0.000	4.740

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	C	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	32	55	65	72	68	43	43
normalized size	1	1.00	0.56	0.96	1.14	1.26	1.19	0.75	0.75
time (sec)	N/A	0.021	0.005	0.007	2.985	0.932	3.367	1.112	4.695

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	39	36	53	35	68	46	36
normalized size	1	1.00	0.70	0.64	0.95	0.62	1.21	0.82	0.64
time (sec)	N/A	0.031	0.019	0.006	1.322	0.869	0.827	1.145	4.674

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	59	51	124	95	54	-1
normalized size	1	1.00	0.85	0.81	0.70	1.70	1.30	0.74	-0.01
time (sec)	N/A	0.020	0.026	0.006	1.351	0.734	4.030	1.037	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	25	33	23	44	30	24
normalized size	1	1.00	0.75	0.69	0.92	0.64	1.22	0.83	0.67
time (sec)	N/A	0.022	0.013	0.005	1.345	0.843	0.488	1.092	4.729

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	39	31	93	42	40	56
normalized size	1	1.00	1.00	0.80	0.63	1.90	0.86	0.82	1.14
time (sec)	N/A	0.012	0.018	0.005	1.262	0.942	2.253	1.182	4.818

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	20	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	1.33	0.87	0.87
time (sec)	N/A	0.003	0.002	0.002	1.397	0.956	0.382	0.998	4.584

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	23	20
normalized size	1	1.00	1.00	0.84	0.52	2.36	0.68	0.92	0.80
time (sec)	N/A	0.006	0.004	0.003	1.356	0.643	1.023	1.242	0.123

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	17	60	19	22	19
normalized size	1	1.00	1.00	1.16	0.68	2.40	0.76	0.88	0.76
time (sec)	N/A	0.017	0.005	0.005	1.380	1.037	1.061	0.993	4.873

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	19	30	17
normalized size	1	1.00	1.00	0.95	0.89	0.89	1.00	1.58	0.89
time (sec)	N/A	0.005	0.004	0.005	1.354	1.253	0.686	1.156	4.600

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	61	48	36	105	42	51	38
normalized size	1	1.00	1.22	0.96	0.72	2.10	0.84	1.02	0.76
time (sec)	N/A	0.026	0.053	0.005	1.347	0.914	2.251	1.086	4.753

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	26	36	27	46	55	25
normalized size	1	1.00	0.66	0.59	0.82	0.61	1.05	1.25	0.57
time (sec)	N/A	0.010	0.006	0.003	1.265	0.858	0.903	1.147	4.622

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	37	68	56	135	97	75	57
normalized size	1	1.00	0.50	0.92	0.76	1.82	1.31	1.01	0.77
time (sec)	N/A	0.038	0.007	0.005	1.290	1.004	4.249	1.113	4.732

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	36	53	46	68	52	41
normalized size	1	1.00	0.69	0.65	0.96	0.84	1.24	0.95	0.75
time (sec)	N/A	0.033	0.017	0.007	1.313	0.932	0.942	0.985	4.721

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	71	57	49	159	71	51	-1
normalized size	1	1.00	1.04	0.84	0.72	2.34	1.04	0.75	-0.01
time (sec)	N/A	0.023	0.034	0.008	1.325	1.003	3.257	1.150	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	23	32	34	41	32	22
normalized size	1	1.00	0.75	0.72	1.00	1.06	1.28	1.00	0.69
time (sec)	N/A	0.022	0.011	0.004	1.377	0.921	0.575	1.062	4.698

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	59	37	29	130	37	39	36
normalized size	1	1.00	1.37	0.86	0.67	3.02	0.86	0.91	0.84
time (sec)	N/A	0.013	0.052	0.006	1.391	1.244	1.706	1.230	0.091

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	24	24	14	14
normalized size	1	1.00	1.00	0.94	0.88	1.50	1.50	0.88	0.88
time (sec)	N/A	0.003	0.003	0.003	1.324	0.700	0.540	1.046	0.040

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	23	17	14	14
normalized size	1	1.00	1.00	0.94	0.88	1.44	1.06	0.88	0.88
time (sec)	N/A	0.002	0.004	0.003	1.332	0.949	0.629	1.117	0.029

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	43	31	126	184	39	33
normalized size	1	1.00	0.80	1.05	0.76	3.07	4.49	0.95	0.80
time (sec)	N/A	0.027	0.006	0.005	1.360	0.923	1.731	1.117	4.776

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	26	34	35	46	50	35
normalized size	1	1.00	0.71	0.68	0.89	0.92	1.21	1.32	0.92
time (sec)	N/A	0.008	0.006	0.005	1.294	1.081	0.856	1.150	4.635

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	68	35	63	51	171	73	72	53
normalized size	1	0.99	0.51	0.91	0.74	2.48	1.06	1.04	0.77
time (sec)	N/A	0.039	0.007	0.006	1.311	0.807	3.386	1.119	4.937

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	40	37	54	50	233	106	38
normalized size	1	1.00	0.61	0.56	0.82	0.76	3.53	1.61	0.58
time (sec)	N/A	0.016	0.008	0.007	1.313	0.929	1.233	1.160	5.125

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	90	75	89	227	367	65	-1
normalized size	1	1.00	0.99	0.82	0.98	2.49	4.03	0.71	-0.01
time (sec)	N/A	0.029	0.138	0.008	1.415	0.649	5.122	1.209	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	39	36	52	58	138	44	38
normalized size	1	1.00	0.72	0.67	0.96	1.07	2.56	0.81	0.70
time (sec)	N/A	0.031	0.018	0.005	1.351	0.622	1.092	1.021	5.201

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	80	54	65	199	303	51	-1
normalized size	1	1.00	1.25	0.84	1.02	3.11	4.73	0.80	-0.02
time (sec)	N/A	0.020	0.117	0.008	1.372	0.860	2.992	1.051	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	33	47	92	24	24
normalized size	1	1.00	0.78	0.69	0.92	1.31	2.56	0.67	0.67
time (sec)	N/A	0.022	0.012	0.005	1.265	0.875	1.073	1.100	5.174

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	34	37	44	17	17
normalized size	1	1.00	1.00	0.86	1.62	1.76	2.10	0.81	0.81
time (sec)	N/A	0.005	0.005	0.006	1.309	1.003	0.756	1.076	5.128

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	35	46	14	14
normalized size	1	1.00	1.00	0.83	0.78	1.94	2.56	0.78	0.78
time (sec)	N/A	0.003	0.004	0.003	1.332	0.827	1.002	1.018	4.944

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	31	47	95	27	28
normalized size	1	1.00	0.74	0.67	0.79	1.21	2.44	0.69	0.72
time (sec)	N/A	0.005	0.008	0.003	1.342	0.918	0.828	1.038	4.885

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	36	57	45	197	740	50	47
normalized size	1	1.00	0.61	0.97	0.76	3.34	12.54	0.85	0.80
time (sec)	N/A	0.035	0.007	0.005	1.338	0.866	2.889	1.145	5.199

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	42	39	50	59	165	64	42
normalized size	1	1.00	0.70	0.65	0.83	0.98	2.75	1.07	0.70
time (sec)	N/A	0.014	0.009	0.004	1.343	0.833	1.287	1.203	5.181

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	92	37	78	66	241	864	73	73
normalized size	1	1.05	0.42	0.89	0.75	2.74	9.82	0.83	0.83
time (sec)	N/A	0.050	0.008	0.006	1.354	1.014	5.169	1.075	5.253

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	53	48	72	72	354	121	78
normalized size	1	1.00	0.62	0.56	0.84	0.84	4.12	1.41	0.91
time (sec)	N/A	0.022	0.011	0.006	1.398	0.957	1.731	1.261	5.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	114	111	285	359	3181	91	-1
normalized size	1	1.00	0.87	0.85	2.18	2.74	24.28	0.69	-0.01
time (sec)	N/A	0.054	0.195	0.033	1.686	1.010	13.422	1.191	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	61	58	92	102	454	72	80
normalized size	1	1.00	0.65	0.62	0.98	1.09	4.83	0.77	0.85
time (sec)	N/A	0.053	0.031	0.007	1.638	0.902	6.389	1.126	4.945

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	101	88	255	331	2980	78	-1
normalized size	1	1.00	0.95	0.83	2.41	3.12	28.11	0.74	-0.01
time (sec)	N/A	0.042	0.132	0.015	1.673	0.962	9.036	1.200	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	50	47	73	91	364	55	63
normalized size	1	1.00	0.67	0.63	0.97	1.21	4.85	0.73	0.84
time (sec)	N/A	0.045	0.025	0.008	1.364	0.862	6.081	0.980	4.880

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	103	59	95	17	68
normalized size	1	1.00	1.00	0.86	4.90	2.81	4.52	0.81	3.24
time (sec)	N/A	0.005	0.006	0.005	1.334	0.621	1.472	1.223	4.764

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	53	80	272	41	41
normalized size	1	1.00	0.66	0.61	0.90	1.36	4.61	0.69	0.69
time (sec)	N/A	0.033	0.019	0.007	1.407	0.880	5.911	0.946	4.805

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	85	71	199	29	68
normalized size	1	1.00	0.70	0.64	1.93	1.61	4.52	0.66	1.55
time (sec)	N/A	0.011	0.012	0.005	1.380	0.591	1.599	0.986	4.876

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	33	69	180	24	24
normalized size	1	1.00	0.74	0.66	0.87	1.82	4.74	0.63	0.63
time (sec)	N/A	0.023	0.014	0.006	1.301	0.850	5.861	0.826	4.824

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	70	82	517	43	70
normalized size	1	1.00	0.62	0.57	1.03	1.21	7.60	0.63	1.03
time (sec)	N/A	0.021	0.013	0.007	1.388	0.809	1.870	1.081	4.761

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	57	90	14	14
normalized size	1	1.00	1.00	0.83	0.78	3.17	5.00	0.78	0.78
time (sec)	N/A	0.004	0.005	0.004	1.332	0.875	5.728	0.970	4.597

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	51	48	61	91	1265	55	61
normalized size	1	1.00	0.66	0.62	0.79	1.18	16.43	0.71	0.79
time (sec)	N/A	0.016	0.013	0.004	1.367	0.863	2.192	1.117	4.602

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	36	85	73	329	5250	81	75
normalized size	1	1.00	0.38	0.89	0.77	3.46	55.26	0.85	0.79
time (sec)	N/A	0.059	0.006	0.008	1.356	0.824	7.904	1.084	4.853

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	64	61	82	103	400	90	76
normalized size	1	1.00	0.64	0.61	0.82	1.03	4.00	0.90	0.76
time (sec)	N/A	0.026	0.013	0.007	1.433	0.828	2.928	1.122	4.715

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	132	37	108	96	373	5540	104	113
normalized size	1	1.05	0.29	0.86	0.76	2.96	43.97	0.83	0.90
time (sec)	N/A	0.078	0.009	0.007	1.390	1.112	12.428	1.130	4.959

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	75	72	108	116	668	147	97
normalized size	1	1.00	0.57	0.55	0.82	0.88	5.06	1.11	0.73
time (sec)	N/A	0.041	0.014	0.008	1.422	1.056	3.837	1.159	4.803

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	24	40	23	44	34	21
normalized size	1	1.00	0.59	0.52	0.87	0.50	0.96	0.74	0.46
time (sec)	N/A	0.019	0.009	0.003	2.886	0.795	1.287	1.127	0.025

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	34	34	33	37	39	36	25
normalized size	1	1.00	0.76	0.76	0.73	0.82	0.87	0.80	0.56
time (sec)	N/A	0.009	0.012	0.006	2.906	0.745	0.731	1.088	0.031

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	26	18	27	23	15
normalized size	1	1.00	0.71	0.61	0.84	0.58	0.87	0.74	0.48
time (sec)	N/A	0.014	0.006	0.003	2.914	0.759	0.378	1.133	0.018

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	19	29	22	29	17
normalized size	1	1.00	1.00	0.74	0.70	1.07	0.81	1.07	0.63
time (sec)	N/A	0.006	0.005	0.006	2.923	0.765	0.226	1.072	0.033

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	10	11	9
normalized size	1	1.00	1.00	0.80	0.73	0.73	0.67	0.73	0.60
time (sec)	N/A	0.002	0.001	0.003	1.318	0.611	0.151	1.118	0.015

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	6	16	7	16	6
normalized size	1	1.00	1.00	0.70	0.60	1.60	0.70	1.60	0.60
time (sec)	N/A	0.001	0.004	0.002	2.964	0.815	0.153	1.168	0.027

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	9	35	8	29	12
normalized size	1	1.00	1.00	0.75	0.45	1.75	0.40	1.45	0.60
time (sec)	N/A	0.010	0.003	0.003	2.961	0.932	1.018	1.064	0.037

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	18	15	23	12
normalized size	1	1.00	1.00	0.83	0.78	1.00	0.83	1.28	0.67
time (sec)	N/A	0.003	0.003	0.003	2.979	0.783	0.759	1.117	0.022

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	30	24	57	44	43	25
normalized size	1	1.00	0.95	0.77	0.62	1.46	1.13	1.10	0.64
time (sec)	N/A	0.017	0.011	0.005	2.939	0.806	2.101	1.105	0.028

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	27	22	29	28	32	42	19
normalized size	1	1.00	0.73	0.59	0.78	0.76	0.86	1.14	0.51
time (sec)	N/A	0.007	0.005	0.004	2.863	0.760	1.294	1.220	0.021

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	32	44	38	64	63	55	33
normalized size	1	1.00	0.56	0.77	0.67	1.12	1.11	0.96	0.58
time (sec)	N/A	0.023	0.005	0.005	2.949	0.825	3.890	1.180	0.029

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	34	40	23	46	43	23
normalized size	1	1.00	0.59	0.74	0.87	0.50	1.00	0.93	0.50
time (sec)	N/A	0.019	0.009	0.004	2.951	1.010	1.288	1.241	0.041

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	34	34	33	40	39	26	27
normalized size	1	1.00	0.76	0.76	0.73	0.89	0.87	0.58	0.60
time (sec)	N/A	0.010	0.012	0.007	2.930	0.939	0.731	1.138	0.030

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	26	18	29	23	18
normalized size	1	1.00	0.71	0.94	0.84	0.58	0.94	0.74	0.58
time (sec)	N/A	0.014	0.006	0.006	2.899	0.937	0.383	0.979	0.017

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	19	32	22	19	19
normalized size	1	1.00	1.00	0.74	0.70	1.19	0.81	0.70	0.70
time (sec)	N/A	0.005	0.006	0.006	2.879	0.870	0.226	1.178	0.022

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	11	11	12	11	11
normalized size	1	1.00	1.00	1.47	0.73	0.73	0.80	0.73	0.73
time (sec)	N/A	0.002	0.002	0.003	1.334	0.982	0.150	1.128	4.556

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	6	19	7	6	6
normalized size	1	1.00	1.00	0.70	0.60	1.90	0.70	0.60	0.60
time (sec)	N/A	0.001	0.004	0.003	2.978	0.942	0.154	1.121	0.008

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	25	18	26	31	20
normalized size	1	1.00	1.00	0.75	1.25	0.90	1.30	1.55	1.00
time (sec)	N/A	0.010	0.003	0.004	2.989	0.570	1.064	0.938	0.116

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	14	14	41	33	14
normalized size	1	1.00	1.00	1.39	0.78	0.78	2.28	1.83	0.78
time (sec)	N/A	0.003	0.003	0.003	2.902	0.763	0.795	1.115	0.022

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	30	40	38	99	45	35
normalized size	1	1.00	0.95	0.77	1.03	0.97	2.54	1.15	0.90
time (sec)	N/A	0.016	0.011	0.006	2.932	0.911	2.172	1.206	0.026

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	27	32	29	21	80	73	22
normalized size	1	1.00	0.73	0.86	0.78	0.57	2.16	1.97	0.59
time (sec)	N/A	0.007	0.004	0.004	2.972	0.925	1.329	1.132	0.021

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	32	44	54	45	136	57	49
normalized size	1	1.00	0.56	0.77	0.95	0.79	2.39	1.00	0.86
time (sec)	N/A	0.024	0.005	0.005	2.926	0.860	3.963	0.977	4.505

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	34	40	23	44	34	22
normalized size	1	1.00	0.59	0.74	0.87	0.50	0.96	0.74	0.48
time (sec)	N/A	0.019	0.009	0.005	2.932	0.958	1.252	1.122	4.740

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	49	45	37	39	37	-1
normalized size	1	1.00	0.80	0.91	0.83	0.69	0.72	0.69	-0.02
time (sec)	N/A	0.013	0.013	0.006	2.809	0.940	0.718	1.236	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	26	18	27	23	18
normalized size	1	1.00	0.71	0.94	0.84	0.58	0.87	0.74	0.58
time (sec)	N/A	0.013	0.006	0.004	2.920	0.909	0.366	0.966	4.861

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	35	31	29	22	30	29
normalized size	1	1.00	1.00	0.97	0.86	0.81	0.61	0.83	0.81
time (sec)	N/A	0.008	0.006	0.006	2.908	0.904	0.224	1.183	0.101

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	11	11	10	11	11
normalized size	1	1.00	1.00	1.47	0.73	0.73	0.67	0.73	0.73
time (sec)	N/A	0.002	0.002	0.003	1.272	0.761	0.152	1.071	0.145

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	22	18	16	7	17	16
normalized size	1	1.00	2.26	1.16	0.95	0.84	0.37	0.89	0.84
time (sec)	N/A	0.003	0.003	0.002	2.958	1.028	0.156	0.985	4.746

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	9	18	26	14	20
normalized size	1	1.00	1.00	0.75	0.45	0.90	1.30	0.70	1.00
time (sec)	N/A	0.010	0.003	0.003	2.909	0.546	1.071	1.062	0.122

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	14	18	37	23	14
normalized size	1	1.00	1.00	1.39	0.78	1.00	2.06	1.28	0.78
time (sec)	N/A	0.003	0.002	0.004	2.842	0.848	0.805	1.062	4.747

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	54	30	24	38	99	29	29
normalized size	1	1.00	1.38	0.77	0.62	0.97	2.54	0.74	0.74
time (sec)	N/A	0.016	0.021	0.006	2.930	0.924	2.133	1.099	4.881

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	32	29	28	76	42	31
normalized size	1	1.00	0.68	0.86	0.78	0.76	2.05	1.14	0.84
time (sec)	N/A	0.007	0.004	0.004	2.961	0.839	1.349	1.132	4.874

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	32	44	38	45	136	41	57
normalized size	1	1.00	0.56	0.77	0.67	0.79	2.39	0.72	1.00
time (sec)	N/A	0.023	0.005	0.007	2.921	0.755	3.821	1.043	5.319

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	24	40	23	49	36	23
normalized size	1	1.00	0.59	0.52	0.87	0.50	1.07	0.78	0.50
time (sec)	N/A	0.019	0.009	0.005	2.912	0.687	1.213	0.995	5.295

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	43	33	67	53	0	-1
normalized size	1	1.00	0.80	0.80	0.61	1.24	0.98	0.00	-0.02
time (sec)	N/A	0.013	0.014	0.005	2.946	1.230	0.888	0.000	0.000

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	26	18	31	24	18
normalized size	1	1.00	0.71	0.61	0.84	0.58	1.00	0.77	0.58
time (sec)	N/A	0.014	0.008	0.004	2.963	0.905	0.375	1.084	5.067

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	29	19	59	36	0	31
normalized size	1	1.00	1.00	0.81	0.53	1.64	1.00	0.00	0.86
time (sec)	N/A	0.007	0.006	0.006	2.927	0.879	0.392	0.000	0.124

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	14	11	11
normalized size	1	1.00	1.00	0.80	0.73	0.73	0.93	0.73	0.73
time (sec)	N/A	0.002	0.002	0.003	1.324	0.687	0.164	1.135	4.986

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	6	47	17	0	15
normalized size	1	1.00	1.00	0.84	0.32	2.47	0.89	0.00	0.79
time (sec)	N/A	0.003	0.003	0.003	2.979	0.788	0.320	0.000	0.107

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	25	43	8	14	14
normalized size	1	1.00	1.00	0.75	1.25	2.15	0.40	0.70	0.70
time (sec)	N/A	0.010	0.003	0.004	2.945	0.758	1.019	1.132	5.272

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	15	0	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	0.83	0.00	0.78
time (sec)	N/A	0.003	0.002	0.003	2.951	0.873	0.765	0.000	5.038

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	54	30	40	65	46	29	29
normalized size	1	1.00	1.38	0.77	1.03	1.67	1.18	0.74	0.74
time (sec)	N/A	0.016	0.022	0.005	2.924	0.872	2.062	1.074	5.179

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	22	29	21	36	0	31
normalized size	1	1.00	0.68	0.59	0.78	0.57	0.97	0.00	0.84
time (sec)	N/A	0.007	0.006	0.004	2.963	0.573	1.286	0.000	5.082

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	C	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	32	44	54	72	65	43	60
normalized size	1	1.00	0.56	0.77	0.95	1.26	1.14	0.75	1.05
time (sec)	N/A	0.023	0.005	0.006	2.888	0.988	3.876	1.107	5.070

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	21	11	65	14	22	11
normalized size	1	1.00	1.00	1.24	0.65	3.82	0.82	1.29	0.65
time (sec)	N/A	0.002	0.007	0.003	1.304	0.982	0.948	1.144	0.036

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	21	11	58	39	27	15
normalized size	1	1.00	1.00	1.24	0.65	3.41	2.29	1.59	0.88
time (sec)	N/A	0.003	0.008	0.004	2.826	0.960	1.014	1.110	0.036

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	24	57	39	23	20
normalized size	1	1.00	1.00	0.84	0.96	2.28	1.56	0.92	0.80
time (sec)	N/A	0.006	0.007	0.003	1.329	0.880	1.026	1.138	0.082

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	12	68	17	28	25
normalized size	1	1.00	1.00	0.81	0.46	2.62	0.65	1.08	0.96
time (sec)	N/A	0.005	0.007	0.003	1.382	0.711	0.968	1.203	0.092

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	21	13	59	17	22	20
normalized size	1	1.00	1.00	1.11	0.68	3.11	0.89	1.16	1.05
time (sec)	N/A	0.006	0.008	0.005	1.329	0.915	0.981	1.119	5.119

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	21	13	62	46	28	25
normalized size	1	1.00	1.00	1.11	0.68	3.26	2.42	1.47	1.32
time (sec)	N/A	0.003	0.008	0.007	2.858	0.955	1.040	1.054	0.101

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	26	74	46	25	22
normalized size	1	1.00	1.00	0.85	0.96	2.74	1.70	0.93	0.81
time (sec)	N/A	0.006	0.007	0.005	1.387	0.637	1.051	1.142	0.129

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	14	74	20	30	27
normalized size	1	1.00	1.00	0.82	0.50	2.64	0.71	1.07	0.96
time (sec)	N/A	0.005	0.006	0.005	1.347	0.991	0.997	1.159	5.095

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	23	20
normalized size	1	1.00	1.00	0.84	0.52	2.36	0.68	0.92	0.80
time (sec)	N/A	0.005	0.006	0.001	1.355	0.958	1.015	1.084	0.002

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	13	72	46	28	25
normalized size	1	1.00	1.00	0.81	0.50	2.77	1.77	1.08	0.96
time (sec)	N/A	0.005	0.006	0.005	2.956	0.775	1.074	1.146	0.114

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	26	63	46	25	22
normalized size	1	1.00	1.00	0.85	0.96	2.33	1.70	0.93	0.81
time (sec)	N/A	0.006	0.006	0.003	1.320	0.880	1.091	1.053	0.125

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	14	74	20	30	27
normalized size	1	1.00	1.00	0.82	0.50	2.64	0.71	1.07	0.96
time (sec)	N/A	0.005	0.007	0.005	1.322	0.902	1.023	1.274	5.125

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	6	23	19	9	14
normalized size	1	1.00	1.00	0.94	0.38	1.44	1.19	0.56	0.88
time (sec)	N/A	0.002	0.004	0.004	2.891	0.834	1.021	1.074	4.738

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	103	152	0	0	46	0	-1
normalized size	1	1.00	0.56	0.83	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.133	0.065	0.058	0.000	0.783	27.189	0.000	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	85	221	0	0	46	0	-1
normalized size	1	1.00	0.28	0.73	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.230	0.048	0.028	0.000	1.029	9.379	0.000	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	85	138	0	0	46	0	-1
normalized size	1	1.00	0.56	0.90	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.090	0.044	0.027	0.000	0.872	2.765	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	56	205	0	0	46	0	-1
normalized size	1	1.00	0.21	0.76	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.194	0.014	0.029	0.000	1.074	1.072	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	54	119	0	0	46	0	-1
normalized size	1	1.00	0.43	0.94	0.00	0.00	0.37	0.00	-0.01
time (sec)	N/A	0.074	0.013	0.026	0.000	0.953	0.987	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	54	194	0	0	49	0	-1
normalized size	1	1.00	0.21	0.74	0.00	0.00	0.19	0.00	-0.00
time (sec)	N/A	0.194	0.013	0.037	0.000	0.868	1.353	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	56	120	0	0	49	0	-1
normalized size	1	1.00	0.44	0.95	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.076	0.014	0.031	0.000	0.855	2.858	0.000	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	56	219	0	0	53	0	-1
normalized size	1	1.00	0.18	0.72	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.222	0.015	0.034	0.000	0.896	9.755	0.000	0.000

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	102	163	0	0	46	0	-1
normalized size	1	1.00	0.48	0.77	0.00	0.00	0.22	0.00	-0.00
time (sec)	N/A	0.130	0.078	0.031	0.000	0.917	50.256	0.000	0.000

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	89	232	0	0	46	0	-1
normalized size	1	1.00	0.27	0.71	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.256	0.063	0.029	0.000	0.899	18.194	0.000	0.000

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	89	150	0	0	46	0	-1
normalized size	1	1.00	0.49	0.83	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.105	0.055	0.012	0.000	0.704	6.123	0.000	0.000

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	57	218	0	0	46	0	-1
normalized size	1	1.00	0.19	0.73	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.224	0.013	0.013	0.000	0.966	3.398	0.000	0.000

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	55	134	0	0	46	0	-1
normalized size	1	1.00	0.36	0.88	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.090	0.012	0.013	0.000	0.604	2.626	0.000	0.000

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	55	208	0	0	49	0	-1
normalized size	1	1.00	0.19	0.70	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.230	0.014	0.017	0.000	0.996	2.484	0.000	0.000

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	57	125	0	0	49	0	-1
normalized size	1	1.00	0.38	0.82	0.00	0.00	0.32	0.00	-0.01
time (sec)	N/A	0.093	0.014	0.014	0.000	0.761	4.503	0.000	0.000

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	57	216	0	0	53	0	-1
normalized size	1	1.00	0.19	0.73	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.227	0.013	0.018	0.000	0.973	10.931	0.000	0.000

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	57	135	0	0	53	0	-1
normalized size	1	1.00	0.38	0.89	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.092	0.015	0.038	0.000	0.640	33.123	0.000	0.000

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	57	234	0	0	53	0	-1
normalized size	1	1.00	0.17	0.71	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.270	0.014	0.039	0.000	0.891	83.792	0.000	0.000

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	74	237	0	0	53	0	-1
normalized size	1	1.00	0.58	1.85	0.00	0.00	0.41	0.00	-0.01
time (sec)	N/A	0.064	0.031	0.063	0.000	0.784	10.494	0.000	0.000

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	74	133	0	0	53	0	-1
normalized size	1	1.00	0.63	1.14	0.00	0.00	0.45	0.00	-0.01
time (sec)	N/A	0.077	0.024	0.030	0.000	0.637	2.728	0.000	0.000

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	51	229	0	0	53	0	-1
normalized size	1	1.00	0.52	2.31	0.00	0.00	0.54	0.00	-0.01
time (sec)	N/A	0.043	0.015	0.029	0.000	1.102	1.046	0.000	0.000

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	51	124	0	0	53	0	-1
normalized size	1	1.00	0.54	1.32	0.00	0.00	0.56	0.00	-0.01
time (sec)	N/A	0.047	0.013	0.031	0.000	0.746	0.929	0.000	0.000

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	51	225	0	0	56	0	-1
normalized size	1	1.00	0.52	2.30	0.00	0.00	0.57	0.00	-0.01
time (sec)	N/A	0.042	0.013	0.040	0.000	0.845	1.293	0.000	0.000

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	51	129	0	0	49	0	-1
normalized size	1	1.00	0.53	1.34	0.00	0.00	0.51	0.00	-0.01
time (sec)	N/A	0.048	0.013	0.034	0.000	1.189	2.855	0.000	0.000

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	87	141	0	0	44	0	-1
normalized size	1	1.00	0.56	0.90	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.090	0.030	0.030	0.000	1.121	19.569	0.000	0.000

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	69	210	0	0	44	0	-1
normalized size	1	1.00	0.25	0.77	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.186	0.026	0.029	0.000	0.983	6.834	0.000	0.000

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	69	125	0	0	44	0	-1
normalized size	1	1.00	0.54	0.98	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.071	0.024	0.013	0.000	0.934	1.928	0.000	0.000

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	56	132	0	0	44	0	-1
normalized size	1	1.00	0.24	0.56	0.00	0.00	0.19	0.00	-0.00
time (sec)	N/A	0.166	0.013	0.014	0.000	0.687	0.930	0.000	0.000

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	54	104	0	0	44	0	-1
normalized size	1	1.00	0.56	1.07	0.00	0.00	0.45	0.00	-0.01
time (sec)	N/A	0.058	0.014	0.016	0.000	0.981	1.089	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	54	196	0	0	48	0	-1
normalized size	1	1.00	0.20	0.73	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.188	0.013	0.017	0.000	0.775	1.637	0.000	0.000

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	56	123	0	0	48	0	-1
normalized size	1	1.00	0.43	0.95	0.00	0.00	0.37	0.00	-0.01
time (sec)	N/A	0.073	0.013	0.016	0.000	1.050	3.894	0.000	0.000

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	56	219	0	0	51	0	-1
normalized size	1	1.00	0.18	0.72	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.220	0.013	0.018	0.000	1.055	12.686	0.000	0.000

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	74	128	0	0	44	0	-1
normalized size	1	1.00	0.48	0.84	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.087	0.031	0.042	0.000	1.013	22.732	0.000	0.000

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	60	197	0	0	44	0	-1
normalized size	1	1.00	0.23	0.74	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.189	0.028	0.036	0.000	0.996	7.301	0.000	0.000

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	59	115	0	0	44	0	-1
normalized size	1	1.00	0.47	0.92	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.071	0.025	0.014	0.000	0.936	2.046	0.000	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	59	197	0	0	44	0	-1
normalized size	1	1.00	0.22	0.74	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.187	0.013	0.013	0.000	0.938	1.284	0.000	0.000

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	59	114	0	0	44	0	-1
normalized size	1	1.00	0.47	0.90	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.070	0.021	0.020	0.000	0.840	1.800	0.000	0.000

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	57	197	0	0	48	0	-1
normalized size	1	1.00	0.19	0.67	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.219	0.013	0.021	0.000	1.030	3.162	0.000	0.000

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	59	124	0	0	48	0	-1
normalized size	1	1.00	0.38	0.81	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.090	0.013	0.020	0.000	0.835	7.963	0.000	0.000

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	59	219	0	0	51	0	-1
normalized size	1	1.00	0.18	0.66	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.254	0.013	0.021	0.000	1.036	24.979	0.000	0.000

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	80	219	0	0	44	0	-1
normalized size	1	1.00	0.52	1.41	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.089	0.056	0.042	0.000	0.957	22.873	0.000	0.000

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	74	385	0	0	44	0	-1
normalized size	1	1.00	0.24	1.27	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.216	0.040	0.041	0.000	0.770	7.328	0.000	0.000

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	79	218	0	0	44	0	-1
normalized size	1	1.00	0.51	1.40	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.088	0.044	0.016	0.000	1.039	3.832	0.000	0.000

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	59	382	0	0	44	0	-1
normalized size	1	1.00	0.20	1.26	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.224	0.013	0.016	0.000	0.789	2.824	0.000	0.000

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	79	216	0	0	44	0	-1
normalized size	1	1.00	0.50	1.38	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.088	0.039	0.023	0.000	0.941	4.284	0.000	0.000

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	57	384	0	0	48	0	-1
normalized size	1	1.00	0.17	1.15	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.255	0.013	0.024	0.000	1.010	7.659	0.000	0.000

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	59	227	0	0	48	0	-1
normalized size	1	1.00	0.32	1.23	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.110	0.015	0.022	0.000	1.012	13.967	0.000	0.000

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	59	410	0	0	51	0	-1
normalized size	1	1.00	0.16	1.13	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.295	0.013	0.024	0.000	0.777	41.151	0.000	0.000

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	61	235	0	0	51	0	-1
normalized size	1	1.00	0.57	2.20	0.00	0.00	0.48	0.00	-0.01
time (sec)	N/A	0.043	0.033	0.039	0.000	0.924	6.445	0.000	0.000

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	61	131	0	0	51	0	-1
normalized size	1	1.00	0.69	1.49	0.00	0.00	0.58	0.00	-0.01
time (sec)	N/A	0.047	0.022	0.033	0.000	0.967	1.870	0.000	0.000

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	53	165	0	0	51	0	-1
normalized size	1	1.00	0.79	2.46	0.00	0.00	0.76	0.00	-0.01
time (sec)	N/A	0.030	0.016	0.017	0.000	1.180	0.859	0.000	0.000

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	117	0	0	51	0	-1
normalized size	1	1.00	0.89	1.86	0.00	0.00	0.81	0.00	-0.02
time (sec)	N/A	0.034	0.017	0.020	0.000	0.841	1.036	0.000	0.000

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	51	228	0	0	54	0	-1
normalized size	1	1.00	0.48	2.13	0.00	0.00	0.50	0.00	-0.01
time (sec)	N/A	0.043	0.017	0.023	0.000	0.901	1.584	0.000	0.000

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	53	132	0	0	54	0	-1
normalized size	1	1.00	0.54	1.35	0.00	0.00	0.55	0.00	-0.01
time (sec)	N/A	0.046	0.015	0.027	0.000	0.940	3.771	0.000	0.000

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	59	230	0	0	51	0	-1
normalized size	1	1.00	0.54	2.09	0.00	0.00	0.46	0.00	-0.01
time (sec)	N/A	0.042	0.025	0.047	0.000	0.794	7.335	0.000	0.000

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	59	126	0	0	51	0	-1
normalized size	1	1.00	0.63	1.34	0.00	0.00	0.54	0.00	-0.01
time (sec)	N/A	0.048	0.021	0.040	0.000	0.687	2.103	0.000	0.000

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	58	227	0	0	51	0	-1
normalized size	1	1.00	0.57	2.25	0.00	0.00	0.50	0.00	-0.01
time (sec)	N/A	0.039	0.017	0.022	0.000	0.907	1.274	0.000	0.000

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	59	122	0	0	51	0	-1
normalized size	1	1.00	0.61	1.27	0.00	0.00	0.53	0.00	-0.01
time (sec)	N/A	0.046	0.025	0.025	0.000	1.083	1.791	0.000	0.000

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	58	228	0	0	54	0	-1
normalized size	1	1.00	0.41	1.63	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.056	0.021	0.026	0.000	1.214	3.163	0.000	0.000

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	58	133	0	0	54	0	-1
normalized size	1	1.00	0.44	1.01	0.00	0.00	0.41	0.00	-0.01
time (sec)	N/A	0.063	0.019	0.026	0.000	0.954	8.112	0.000	0.000

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	66	0	0	36	0	-1
normalized size	1	1.00	1.14	3.14	0.00	0.00	1.71	0.00	-0.05
time (sec)	N/A	0.009	0.007	0.057	0.000	0.719	0.796	0.000	0.000

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	23	73	0	0	32	0	-1
normalized size	1	1.00	0.34	1.09	0.00	0.00	0.48	0.00	-0.01
time (sec)	N/A	0.038	0.006	0.051	0.000	0.646	0.753	0.000	0.000

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	64	66	0	0	0	54	0	-1
normalized size	1	1.28	1.32	0.00	0.00	0.00	1.08	0.00	-0.02
time (sec)	N/A	0.019	0.022	0.269	0.000	0.940	2.848	0.000	0.000

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	54	0	-1
normalized size	1	1.26	1.30	0.00	0.00	0.00	1.08	0.00	-0.02
time (sec)	N/A	0.018	0.014	0.266	0.000	0.822	1.173	0.000	0.000

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	53	0	-1
normalized size	1	1.26	1.30	0.00	0.00	0.00	1.06	0.00	-0.02
time (sec)	N/A	0.018	0.016	0.260	0.000	0.958	1.012	0.000	0.000

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	66	68	0	0	0	53	0	-1
normalized size	1	1.38	1.42	0.00	0.00	0.00	1.10	0.00	-0.02
time (sec)	N/A	0.020	0.018	0.263	0.000	0.945	1.394	0.000	0.000

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	66	68	0	0	0	53	0	-1
normalized size	1	1.32	1.36	0.00	0.00	0.00	1.06	0.00	-0.02
time (sec)	N/A	0.020	0.018	0.266	0.000	0.816	2.938	0.000	0.000

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	54	0	-1
normalized size	1	1.26	1.30	0.00	0.00	0.00	1.08	0.00	-0.02
time (sec)	N/A	0.020	0.018	0.261	0.000	0.886	2.643	0.000	0.000

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	48	0	-1
normalized size	1	1.26	1.30	0.00	0.00	0.00	0.96	0.00	-0.02
time (sec)	N/A	0.020	0.018	0.264	0.000	0.889	1.868	0.000	0.000

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	65	0	0	0	53	0	-1
normalized size	1	1.26	1.30	0.00	0.00	0.00	1.06	0.00	-0.02
time (sec)	N/A	0.017	0.014	0.000	0.000	0.912	1.025	0.000	0.000

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	57	57	0	0	0	41	0	-1
normalized size	1	1.24	1.24	0.00	0.00	0.00	0.89	0.00	-0.02
time (sec)	N/A	0.019	0.011	0.268	0.000	0.940	3.056	0.000	0.000

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	66	65	0	0	0	53	0	-1
normalized size	1	1.29	1.27	0.00	0.00	0.00	1.04	0.00	-0.02
time (sec)	N/A	0.020	0.019	0.273	0.000	1.004	8.061	0.000	0.000

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	104	16	16	16	202	0	24
normalized size	1	1.00	6.12	0.94	0.94	0.94	11.88	0.00	1.41
time (sec)	N/A	0.011	0.108	0.013	1.881	0.903	10.061	0.000	5.193

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	127	104	0	16	18	105	0	-1
normalized size	1	7.47	6.12	0.00	0.94	1.06	6.18	0.00	-0.06
time (sec)	N/A	0.066	0.069	0.278	1.890	0.714	5.288	0.000	0.000

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	103	14	13	16	97	0	13
normalized size	1	1.00	6.87	0.93	0.87	1.07	6.47	0.00	0.87
time (sec)	N/A	0.012	0.109	0.007	1.906	0.678	52.957	0.000	5.431

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	123	103	0	13	26	94	0	-1
normalized size	1	8.20	6.87	0.00	0.87	1.73	6.27	0.00	-0.07
time (sec)	N/A	0.065	0.065	0.327	1.915	0.953	5.586	0.000	0.000

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	57	1795	57	55
normalized size	1	1.00	0.62	0.59	0.80	0.71	22.44	0.71	0.69
time (sec)	N/A	0.049	0.025	0.006	1.374	0.491	2.848	0.922	5.254

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	46	700	43	44
normalized size	1	1.00	0.66	0.61	0.80	0.78	11.86	0.73	0.75
time (sec)	N/A	0.035	0.018	0.006	1.342	0.736	1.877	1.028	4.803

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	34	223	29	33
normalized size	1	1.00	0.74	0.66	0.79	0.89	5.87	0.76	0.87
time (sec)	N/A	0.023	0.013	0.007	1.318	0.780	1.216	1.204	4.720

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	42	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	2.33	0.78	0.78
time (sec)	N/A	0.003	0.003	0.005	1.370	0.889	0.195	1.119	4.657

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	126	0	97	102	46	98	115
normalized size	1	1.00	1.25	0.00	0.96	1.01	0.46	0.97	1.14
time (sec)	N/A	0.079	0.056	0.291	3.035	0.932	1.066	2.406	4.735

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	37	0	103	155	42	115	125
normalized size	1	1.00	0.35	0.00	0.96	1.45	0.39	1.07	1.17
time (sec)	N/A	0.071	0.009	0.316	2.912	0.782	1.195	2.482	4.880

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	39	0	155	199	42	140	217
normalized size	1	1.00	0.29	0.00	1.15	1.47	0.31	1.04	1.61
time (sec)	N/A	0.094	0.008	0.282	2.914	0.687	1.387	2.369	5.109

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	94	0	0	0	29	0	-1
normalized size	1	1.00	0.30	0.00	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.283	0.053	0.293	0.000	1.026	0.933	0.000	0.000

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	62	0	0	0	29	0	-1
normalized size	1	1.00	0.21	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.161	0.049	0.281	0.000	0.943	0.841	0.000	0.000

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	46	0	0	0	26	0	37
normalized size	1	1.00	0.17	0.00	0.00	0.00	0.10	0.00	0.14
time (sec)	N/A	0.124	0.005	0.299	0.000	0.696	0.784	0.000	4.592

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	49	0	0	0	29	0	40
normalized size	1	1.00	0.19	0.00	0.00	0.00	0.11	0.00	0.15
time (sec)	N/A	0.125	0.009	0.283	0.000	1.009	0.827	0.000	4.777

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	51	0	0	0	34	0	-1
normalized size	1	1.00	0.18	0.00	0.00	0.00	0.12	0.00	-0.00
time (sec)	N/A	0.158	0.010	0.296	0.000	0.837	0.946	0.000	0.000

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	57	1795	57	55
normalized size	1	1.00	0.62	0.59	0.80	0.71	22.44	0.71	0.69
time (sec)	N/A	0.047	0.026	0.006	1.303	0.885	2.941	0.982	4.629

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	46	700	43	44
normalized size	1	1.00	0.66	0.61	0.80	0.78	11.86	0.73	0.75
time (sec)	N/A	0.036	0.018	0.006	1.354	0.807	1.996	0.966	4.663

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	35	66	29	33
normalized size	1	1.00	0.74	0.66	0.79	0.92	1.74	0.76	0.87
time (sec)	N/A	0.023	0.014	0.006	1.351	0.891	0.751	1.084	4.715

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	42	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	2.33	0.78	0.78
time (sec)	N/A	0.004	0.004	0.003	1.327	0.826	0.374	1.056	4.583

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	93	0	97	122	46	98	125
normalized size	1	1.00	0.92	0.00	0.96	1.21	0.46	0.97	1.24
time (sec)	N/A	0.066	0.037	0.284	2.971	0.776	1.116	2.358	4.666

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	37	0	103	290	42	116	136
normalized size	1	1.00	0.36	0.00	0.99	2.79	0.40	1.12	1.31
time (sec)	N/A	0.066	0.008	0.287	2.971	1.006	1.219	2.366	4.889

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	39	0	155	380	42	141	212
normalized size	1	1.00	0.29	0.00	1.15	2.81	0.31	1.04	1.57
time (sec)	N/A	0.088	0.009	0.278	3.019	0.957	1.412	2.551	5.161

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	94	0	0	0	29	0	-1
normalized size	1	1.00	0.16	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.462	0.053	0.288	0.000	0.978	0.996	0.000	0.000

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	62	0	0	0	29	0	-1
normalized size	1	1.00	0.11	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.369	0.052	0.283	0.000	0.926	0.906	0.000	0.000

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	46	0	0	0	26	0	37
normalized size	1	1.00	0.08	0.00	0.00	0.00	0.05	0.00	0.07
time (sec)	N/A	0.307	0.006	0.302	0.000	0.780	0.812	0.000	5.167

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	538	538	49	0	0	0	29	0	40
normalized size	1	1.00	0.09	0.00	0.00	0.00	0.05	0.00	0.07
time (sec)	N/A	0.303	0.011	0.279	0.000	0.822	0.888	0.000	5.467

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	575	51	0	0	0	34	0	-1
normalized size	1	1.00	0.09	0.00	0.00	0.00	0.06	0.00	-0.00
time (sec)	N/A	0.373	0.011	0.334	0.000	0.655	0.984	0.000	0.000

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	68	136	57	64
normalized size	1	1.00	0.62	0.59	0.80	0.85	1.70	0.71	0.80
time (sec)	N/A	0.049	0.027	0.005	1.384	0.537	6.578	0.648	5.188

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	57	112	43	53
normalized size	1	1.00	0.66	0.61	0.80	0.97	1.90	0.73	0.90
time (sec)	N/A	0.038	0.019	0.007	1.335	0.906	3.979	0.646	5.100

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	45	88	29	42
normalized size	1	1.00	0.74	0.66	0.79	1.18	2.32	0.76	1.11
time (sec)	N/A	0.024	0.015	0.006	1.324	0.928	2.541	0.694	5.054

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	32	65	14	14
normalized size	1	1.00	1.00	0.83	0.78	1.78	3.61	0.78	0.78
time (sec)	N/A	0.004	0.005	0.002	1.333	0.654	1.369	0.626	4.990

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	144	0	109	111	49	110	133
normalized size	1	1.00	1.23	0.00	0.93	0.95	0.42	0.94	1.14
time (sec)	N/A	0.082	0.045	0.297	3.057	0.980	1.303	1.464	5.001

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	37	0	116	129	46	131	141
normalized size	1	1.00	0.32	0.00	1.00	1.11	0.40	1.13	1.22
time (sec)	N/A	0.084	0.009	0.295	2.901	0.981	1.414	1.423	5.416

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	39	0	152	174	42	139	191
normalized size	1	1.00	0.30	0.00	1.15	1.32	0.32	1.05	1.45
time (sec)	N/A	0.087	0.010	0.303	3.085	0.867	1.536	1.155	5.491

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	79	0	0	0	29	0	-1
normalized size	1	1.00	0.24	0.00	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.228	0.078	0.293	0.000	1.137	1.271	0.000	0.000

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	67	0	0	0	29	0	-1
normalized size	1	1.00	0.22	0.00	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.185	0.063	0.300	0.000	0.694	1.138	0.000	0.000

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	47	0	0	0	26	0	37
normalized size	1	1.00	0.16	0.00	0.00	0.00	0.09	0.00	0.13
time (sec)	N/A	0.166	0.007	0.306	0.000	1.003	0.990	0.000	5.164

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	50	0	0	0	29	0	40
normalized size	1	1.00	0.18	0.00	0.00	0.00	0.10	0.00	0.14
time (sec)	N/A	0.154	0.010	0.289	0.000	0.787	1.093	0.000	5.613

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	52	0	0	0	34	0	-1
normalized size	1	1.00	0.18	0.00	0.00	0.00	0.12	0.00	-0.00
time (sec)	N/A	0.150	0.010	0.279	0.000	0.712	1.087	0.000	0.000

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	16	9	24	56	9	25
normalized size	1	1.00	1.00	1.23	0.69	1.85	4.31	0.69	1.92
time (sec)	N/A	0.002	0.005	0.003	1.319	0.957	2.908	0.561	5.131

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	46	1690	61	48
normalized size	1	1.00	0.62	0.59	0.80	0.58	21.12	0.76	0.60
time (sec)	N/A	0.046	0.029	0.006	1.355	0.839	2.771	0.585	5.316

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	35	631	47	36
normalized size	1	1.00	0.66	0.61	0.80	0.59	10.69	0.80	0.61
time (sec)	N/A	0.034	0.021	0.006	1.297	0.913	1.784	0.573	5.207

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	24	178	30	24
normalized size	1	1.00	0.74	0.66	0.79	0.63	4.68	0.79	0.63
time (sec)	N/A	0.023	0.013	0.005	1.400	0.825	1.142	0.573	5.006

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	24	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	1.33	0.78	0.78
time (sec)	N/A	0.003	0.003	0.003	1.296	0.919	0.396	0.572	4.671

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	70	0	86	235	41	87	106
normalized size	1	1.00	0.81	0.00	1.00	2.73	0.48	1.01	1.23
time (sec)	N/A	0.051	0.030	0.274	2.975	0.827	1.015	1.079	4.828

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	37	0	118	344	41	119	138
normalized size	1	1.00	0.34	0.00	1.07	3.13	0.37	1.08	1.25
time (sec)	N/A	0.065	0.007	0.277	2.946	0.913	1.189	1.137	5.011

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	39	0	158	326	41	142	201
normalized size	1	1.00	0.28	0.00	1.14	2.36	0.30	1.03	1.46
time (sec)	N/A	0.093	0.008	0.273	2.973	0.802	1.372	1.133	5.063

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	580	580	79	0	0	0	27	0	-1
normalized size	1	1.00	0.14	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.366	0.026	0.282	0.000	0.931	0.825	0.000	0.000

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	62	0	0	0	27	0	-1
normalized size	1	1.00	0.11	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.309	0.019	0.285	0.000	0.976	0.777	0.000	0.000

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	46	0	0	0	24	0	37
normalized size	1	1.00	0.09	0.00	0.00	0.00	0.05	0.00	0.07
time (sec)	N/A	0.250	0.007	0.295	0.000	0.978	0.748	0.000	4.634

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	49	0	0	0	27	0	40
normalized size	1	1.00	0.09	0.00	0.00	0.00	0.05	0.00	0.07
time (sec)	N/A	0.300	0.009	0.339	0.000	0.919	0.821	0.000	4.830

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	578	578	51	0	0	0	32	0	-1
normalized size	1	1.00	0.09	0.00	0.00	0.00	0.06	0.00	-0.00
time (sec)	N/A	0.357	0.009	0.286	0.000	0.934	0.941	0.000	0.000

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	46	1690	61	48
normalized size	1	1.00	0.62	0.59	0.80	0.58	21.12	0.76	0.60
time (sec)	N/A	0.046	0.024	0.006	1.337	0.815	2.744	0.575	4.749

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	35	631	47	36
normalized size	1	1.00	0.66	0.61	0.80	0.59	10.69	0.80	0.61
time (sec)	N/A	0.034	0.019	0.007	1.329	0.996	1.801	0.573	4.766

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	25	30	23	178	30	24
normalized size	1	1.00	0.71	0.66	0.79	0.61	4.68	0.79	0.63
time (sec)	N/A	0.024	0.014	0.003	1.339	0.507	1.140	0.586	4.786

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	24	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	1.33	0.78	0.78
time (sec)	N/A	0.004	0.003	0.003	1.239	0.761	0.414	0.568	4.695

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	101	0	86	123	41	87	102
normalized size	1	1.00	1.17	0.00	1.00	1.43	0.48	1.01	1.19
time (sec)	N/A	0.052	0.031	0.280	3.093	0.603	1.059	1.126	4.843

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	37	0	118	182	41	118	130
normalized size	1	1.00	0.35	0.00	1.10	1.70	0.38	1.10	1.21
time (sec)	N/A	0.066	0.008	0.284	2.958	0.628	1.237	1.107	5.052

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	39	0	158	174	41	142	193
normalized size	1	1.00	0.28	0.00	1.14	1.26	0.30	1.03	1.40
time (sec)	N/A	0.088	0.008	0.286	2.980	1.008	1.447	1.132	5.144

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	79	0	0	0	27	0	-1
normalized size	1	1.00	0.27	0.00	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.158	0.027	0.280	0.000	1.040	0.847	0.000	0.000

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	62	0	0	0	27	0	-1
normalized size	1	1.00	0.23	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.123	0.022	0.278	0.000	0.930	0.785	0.000	0.000

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	46	0	0	0	24	0	37
normalized size	1	1.00	0.19	0.00	0.00	0.00	0.10	0.00	0.15
time (sec)	N/A	0.096	0.008	0.289	0.000	0.881	0.757	0.000	5.366

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	49	0	0	0	27	0	40
normalized size	1	1.00	0.18	0.00	0.00	0.00	0.10	0.00	0.15
time (sec)	N/A	0.121	0.009	0.277	0.000	0.786	0.901	0.000	5.458

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	51	0	0	0	32	0	-1
normalized size	1	1.00	0.17	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.150	0.010	0.280	0.000	0.704	1.009	0.000	0.000

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	58	1584	70	55
normalized size	1	1.00	0.62	0.59	0.80	0.72	19.80	0.88	0.69
time (sec)	N/A	0.044	0.023	0.007	1.338	0.762	2.841	0.587	5.440

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	38	36	47	46	561	52	41
normalized size	1	1.00	0.64	0.61	0.80	0.78	9.51	0.88	0.69
time (sec)	N/A	0.033	0.017	0.006	1.283	0.831	1.823	0.584	5.361

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	30	35	46	34	24
normalized size	1	1.00	0.71	0.63	0.79	0.92	1.21	0.89	0.63
time (sec)	N/A	0.023	0.012	0.005	1.353	0.859	0.704	0.573	5.591

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	24	26	14	14
normalized size	1	1.00	1.00	0.83	0.78	1.33	1.44	0.78	0.78
time (sec)	N/A	0.004	0.003	0.003	1.316	0.702	0.675	0.580	5.393

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	36	0	100	327	41	101	123
normalized size	1	1.00	0.35	0.00	0.96	3.14	0.39	0.97	1.18
time (sec)	N/A	0.065	0.007	0.287	3.020	1.011	1.135	1.103	5.590

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	125	37	0	136	453	41	134	178
normalized size	1	1.02	0.30	0.00	1.11	3.68	0.33	1.09	1.45
time (sec)	N/A	0.080	0.008	0.302	3.010	0.982	1.350	1.113	5.645

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	39	0	176	437	41	154	224
normalized size	1	1.00	0.25	0.00	1.11	2.75	0.26	0.97	1.41
time (sec)	N/A	0.106	0.008	0.304	2.972	0.815	1.622	1.103	5.672

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	65	0	0	0	27	0	-1
normalized size	1	1.00	0.11	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.365	0.025	0.299	0.000	0.928	0.885	0.000	0.000

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	55	0	0	0	27	0	-1
normalized size	1	1.00	0.10	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.303	0.019	0.286	0.000	0.696	0.880	0.000	0.000

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	58	0	0	0	24	0	37
normalized size	1	1.00	0.11	0.00	0.00	0.00	0.04	0.00	0.07
time (sec)	N/A	0.298	0.013	0.305	0.000	0.782	0.818	0.000	5.416

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	571	571	52	0	0	0	27	0	40
normalized size	1	1.00	0.09	0.00	0.00	0.00	0.05	0.00	0.07
time (sec)	N/A	0.352	0.010	0.314	0.000	0.937	1.018	0.000	5.575

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	599	599	54	0	0	0	32	0	-1
normalized size	1	1.00	0.09	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.418	0.010	0.304	0.000	0.947	1.147	0.000	0.000

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	275	102	0	0	0	0	0	-1
normalized size	1	1.41	0.52	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.391	0.074	0.079	0.000	0.000	0.000	0.000	0.000

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	244	85	0	0	0	46	0	-1
normalized size	1	1.49	0.52	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.302	0.050	0.041	0.000	0.000	33.290	0.000	0.000

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	211	56	0	0	0	46	0	-1
normalized size	1	1.59	0.42	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.274	0.014	0.040	0.000	0.000	1.522	0.000	0.000

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	208	56	0	0	0	49	0	-1
normalized size	1	1.59	0.43	0.00	0.00	0.00	0.37	0.00	-0.01
time (sec)	N/A	0.274	0.014	0.041	0.000	0.000	3.052	0.000	0.000

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	35	25	78	0	-1
normalized size	1	1.00	0.93	0.75	1.25	0.89	2.79	0.00	-0.04
time (sec)	N/A	0.006	0.009	0.003	1.453	1.265	56.586	0.000	0.000

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	41	31	0	46	0	0	-1
normalized size	1	1.00	0.72	0.54	0.00	0.81	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.016	0.006	0.000	0.764	0.000	0.000	0.000

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	52	42	0	57	0	0	-1
normalized size	1	1.00	0.61	0.49	0.00	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.019	0.006	0.000	1.251	0.000	0.000	0.000

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	63	53	64	68	0	0	-1
normalized size	1	1.00	0.56	0.47	0.57	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.018	0.013	1.474	1.214	0.000	0.000	0.000

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	103	0	0	0	46	0	-1
normalized size	1	1.00	0.23	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.971	0.061	0.046	0.000	1.311	127.899	0.000	0.000

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	85	0	0	0	46	0	-1
normalized size	1	1.00	0.20	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.713	0.044	0.041	0.000	0.897	6.720	0.000	0.000

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	54	0	0	0	46	0	-1
normalized size	1	1.00	0.14	0.00	0.00	0.00	0.12	0.00	-0.00
time (sec)	N/A	0.659	0.012	0.038	0.000	0.910	1.313	0.000	0.000

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	56	0	0	0	32	0	-1
normalized size	1	1.00	0.14	0.00	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.665	0.014	0.045	0.000	0.644	12.231	0.000	0.000

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	56	0	0	0	0	0	-1
normalized size	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.720	0.014	0.046	0.000	0.791	0.000	0.000	0.000

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	46	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	-0.02
time (sec)	N/A	0.017	0.012	0.065	0.000	0.927	1.896	0.000	0.000

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	46	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	-0.02
time (sec)	N/A	0.017	0.012	0.043	0.000	0.631	1.093	0.000	0.000

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	49	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.88	0.00	-0.02
time (sec)	N/A	0.018	0.012	0.042	0.000	0.815	2.021	0.000	0.000

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	303	102	0	0	0	0	0	-1
normalized size	1	1.36	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	0.081	0.039	0.000	0.000	0.000	0.000	0.000

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	272	89	0	0	0	46	0	-1
normalized size	1	1.42	0.46	0.00	0.00	0.00	0.24	0.00	-0.01
time (sec)	N/A	0.323	0.066	0.042	0.000	0.000	64.864	0.000	0.000

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	243	57	0	0	0	46	0	-1
normalized size	1	1.49	0.35	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.290	0.015	0.039	0.000	0.000	7.090	0.000	0.000

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	233	57	0	0	0	49	0	-1
normalized size	1	1.52	0.37	0.00	0.00	0.00	0.32	0.00	-0.01
time (sec)	N/A	0.287	0.012	0.046	0.000	0.000	7.232	0.000	0.000

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	234	57	0	0	0	53	0	-1
normalized size	1	1.49	0.36	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.291	0.016	0.051	0.000	0.000	56.366	0.000	0.000

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	0	43	0	0	-1
normalized size	1	1.00	0.93	0.75	0.00	1.54	0.00	0.00	-0.04
time (sec)	N/A	0.006	0.011	0.004	0.000	1.169	0.000	0.000	0.000

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	41	31	0	57	0	0	-1
normalized size	1	1.00	0.72	0.54	0.00	1.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.019	0.006	0.000	1.325	0.000	0.000	0.000

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	52	42	0	68	0	0	-1
normalized size	1	1.00	0.61	0.49	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.016	0.005	0.000	0.720	0.000	0.000	0.000

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	102	0	0	0	0	0	-1
normalized size	1	1.00	0.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.830	0.074	0.038	0.000	1.356	0.000	0.000	0.000

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	89	0	0	0	46	0	-1
normalized size	1	1.00	0.20	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.746	0.051	0.043	0.000	0.726	14.949	0.000	0.000

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	55	0	0	0	46	0	-1
normalized size	1	1.00	0.13	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.695	0.012	0.038	0.000	1.020	5.039	0.000	0.000

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	57	0	0	0	32	0	-1
normalized size	1	1.00	0.14	0.00	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.698	0.013	0.046	0.000	0.945	15.928	0.000	0.000

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	57	0	0	0	0	0	-1
normalized size	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.702	0.014	0.046	0.000	0.959	0.000	0.000	0.000

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	57	0	0	0	0	0	-1
normalized size	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.767	0.014	0.054	0.000	0.954	0.000	0.000	0.000

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	46	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.78	0.00	-0.02
time (sec)	N/A	0.019	0.011	0.038	0.000	0.835	8.787	0.000	0.000

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	46	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.78	0.00	-0.02
time (sec)	N/A	0.019	0.012	0.036	0.000	0.640	5.112	0.000	0.000

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	49	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.019	0.012	0.043	0.000	0.980	5.071	0.000	0.000

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	278	87	0	0	0	0	0	-1
normalized size	1	1.40	0.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.338	0.029	0.045	0.000	0.000	0.000	0.000	0.000

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	247	76	0	0	0	0	0	-1
normalized size	1	1.48	0.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.023	0.042	0.000	0.000	0.000	0.000	0.000

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	209	58	0	0	0	44	0	-1
normalized size	1	1.60	0.44	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.262	0.020	0.039	0.000	0.000	30.011	0.000	0.000

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	183	45	0	0	0	44	0	-1
normalized size	1	1.73	0.42	0.00	0.00	0.00	0.42	0.00	-0.01
time (sec)	N/A	0.245	0.010	0.273	0.000	0.000	1.525	0.000	0.000

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	35	25	36	0	-1
normalized size	1	1.00	0.93	0.75	1.25	0.89	1.29	0.00	-0.04
time (sec)	N/A	0.006	0.006	0.003	1.473	1.137	4.166	0.000	0.000

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	34	29	0	35	78	0	-1
normalized size	1	1.00	0.60	0.51	0.00	0.61	1.37	0.00	-0.02
time (sec)	N/A	0.015	0.019	0.004	0.000	1.139	109.454	0.000	0.000

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	52	42	0	46	0	0	-1
normalized size	1	1.00	0.61	0.49	0.00	0.54	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.025	0.006	0.000	0.871	0.000	0.000	0.000

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	63	53	64	57	0	0	-1
normalized size	1	1.00	0.56	0.47	0.57	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.031	0.004	1.536	0.829	0.000	0.000	0.000

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	87	0	0	0	44	0	-1
normalized size	1	1.00	0.21	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.689	0.031	0.043	0.000	0.918	126.930	0.000	0.000

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	66	0	0	0	44	0	-1
normalized size	1	1.00	0.17	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.640	0.023	0.036	0.000	0.984	5.435	0.000	0.000

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	54	0	0	0	31	0	-1
normalized size	1	1.00	0.15	0.00	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.591	0.015	0.279	0.000	0.871	1.786	0.000	0.000

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	56	0	0	0	48	0	-1
normalized size	1	1.00	0.14	0.00	0.00	0.00	0.12	0.00	-0.00
time (sec)	N/A	0.642	0.013	0.044	0.000	0.909	25.885	0.000	0.000

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	56	0	0	0	0	0	-1
normalized size	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.692	0.013	0.048	0.000	0.614	0.000	0.000	0.000

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	-0.02
time (sec)	N/A	0.018	0.013	0.293	0.000	0.942	1.593	0.000	0.000

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	46	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	-0.02
time (sec)	N/A	0.019	0.011	0.273	0.000	0.849	1.381	0.000	0.000

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	48	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.019	0.012	0.045	0.000	0.910	3.073	0.000	0.000

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	93	0	0	0	29	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.24	0.00	-0.01
time (sec)	N/A	0.049	0.053	0.321	0.000	0.883	0.967	0.000	0.000

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	62	0	0	0	29	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.033	0.049	0.386	0.000	0.903	0.857	0.000	0.000

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	46	0	0	0	26	0	37
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.35	0.00	0.49
time (sec)	N/A	0.018	0.006	0.306	0.000	0.904	0.811	0.000	4.657

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	49	0	0	0	29	0	40
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.40	0.00	0.56
time (sec)	N/A	0.021	0.009	0.290	0.000	0.941	0.859	0.000	4.805

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	51	0	0	0	34	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.031	0.009	0.288	0.000	0.815	0.977	0.000	0.000

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	51	0	0	0	34	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.046	0.010	0.291	0.000	0.950	1.102	0.000	0.000

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	95	0	0	0	31	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.046	0.058	0.063	0.000	0.593	0.976	0.000	0.000

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	64	0	0	0	31	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.033	0.056	0.036	0.000	0.823	0.901	0.000	0.000

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	47	0	0	0	27	0	38
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.35	0.00	0.49
time (sec)	N/A	0.018	0.007	0.302	0.000	0.616	0.826	0.000	4.802

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	50	0	0	0	31	0	41
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.41	0.00	0.54
time (sec)	N/A	0.020	0.010	0.036	0.000	0.804	0.916	0.000	4.980

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	52	0	0	0	36	0	-1
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.032	0.011	0.039	0.000	0.775	1.006	0.000	0.000

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	52	0	0	0	36	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.046	0.011	0.040	0.000	0.669	1.128	0.000	0.000

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	93	0	0	0	29	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.20	0.00	-0.01
time (sec)	N/A	0.051	0.056	0.299	0.000	0.669	1.197	0.000	0.000

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	62	0	0	0	29	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.24	0.00	-0.01
time (sec)	N/A	0.038	0.055	0.291	0.000	0.765	1.059	0.000	0.000

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	46	0	0	0	26	0	37
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.28	0.00	0.40
time (sec)	N/A	0.021	0.006	0.303	0.000	0.826	0.941	0.000	4.777

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	49	0	0	0	29	0	40
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.33	0.00	0.45
time (sec)	N/A	0.024	0.009	0.284	0.000	1.022	1.027	0.000	5.029

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	51	0	0	0	34	0	-1
normalized size	1	1.00	0.42	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.038	0.010	0.286	0.000	0.979	1.088	0.000	0.000

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	51	0	0	0	34	0	-1
normalized size	1	1.00	0.35	0.00	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.051	0.009	0.291	0.000	1.043	1.234	0.000	0.000

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	95	0	0	0	31	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.045	0.060	0.303	0.000	0.672	1.205	0.000	0.000

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	64	0	0	0	31	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.033	0.055	0.314	0.000	0.676	1.079	0.000	0.000

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	47	0	0	0	27	0	38
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.35	0.00	0.49
time (sec)	N/A	0.019	0.007	0.335	0.000	1.071	0.970	0.000	4.849

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	50	0	0	0	31	0	41
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.41	0.00	0.54
time (sec)	N/A	0.020	0.009	0.288	0.000	1.304	1.048	0.000	5.170

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	52	0	0	0	36	0	-1
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.032	0.010	0.286	0.000	1.043	1.121	0.000	0.000

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	52	0	0	0	36	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.045	0.010	0.288	0.000	1.154	1.278	0.000	0.000

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	47	0	0	0	26	0	37
normalized size	1	1.00	0.51	0.00	0.00	0.00	0.28	0.00	0.40
time (sec)	N/A	0.024	0.008	0.309	0.000	0.999	1.156	0.000	4.859

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	48	0	0	0	27	0	38
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.28	0.00	0.40
time (sec)	N/A	0.025	0.010	0.300	0.000	0.957	1.185	0.000	4.838

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	47	0	0	0	26	0	37
normalized size	1	1.00	0.42	0.00	0.00	0.00	0.23	0.00	0.33
time (sec)	N/A	0.029	0.007	0.305	0.000	0.686	1.537	0.000	4.823

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	48	0	0	0	27	0	38
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.28	0.00	0.40
time (sec)	N/A	0.026	0.008	0.309	0.000	0.628	1.542	0.000	4.816

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	90	0	0	0	27	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.18	0.00	-0.01
time (sec)	N/A	0.052	0.033	0.283	0.000	0.869	0.973	0.000	0.000

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	79	0	0	0	27	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.036	0.023	0.290	0.000	0.797	0.864	0.000	0.000

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	62	0	0	0	27	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.024	0.018	0.302	0.000	0.948	0.796	0.000	0.000

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	46	0	0	0	24	0	37
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.34	0.00	0.52
time (sec)	N/A	0.014	0.007	0.308	0.000	0.933	0.776	0.000	4.845

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	49	0	0	0	27	0	40
normalized size	1	1.00	0.53	0.00	0.00	0.00	0.29	0.00	0.43
time (sec)	N/A	0.025	0.009	0.289	0.000	1.142	0.854	0.000	5.088

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	51	0	0	0	32	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.036	0.009	0.289	0.000	1.123	0.999	0.000	0.000

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	51	0	0	0	32	0	-1
normalized size	1	1.00	0.34	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.051	0.008	0.287	0.000	1.124	1.107	0.000	0.000

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	89	0	0	0	29	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.045	0.034	0.291	0.000	0.991	1.006	0.000	0.000

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	79	0	0	0	29	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.034	0.026	0.289	0.000	1.089	0.912	0.000	0.000

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	0	0	0	29	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.36	0.00	-0.01
time (sec)	N/A	0.022	0.021	0.289	0.000	0.856	0.824	0.000	0.000

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	0	0	26	0	38
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.45	0.00	0.66
time (sec)	N/A	0.011	0.008	0.302	0.000	0.915	0.786	0.000	4.872

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	50	0	0	0	29	0	41
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.37	0.00	0.52
time (sec)	N/A	0.020	0.010	0.293	0.000	1.200	0.896	0.000	5.085

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	52	0	0	0	34	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.00	0.32	0.00	-0.01
time (sec)	N/A	0.031	0.010	0.293	0.000	1.129	1.012	0.000	0.000

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	52	0	0	0	34	0	-1
normalized size	1	1.00	0.40	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.044	0.010	0.289	0.000	0.759	1.132	0.000	0.000

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	90	0	0	0	27	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.043	0.035	0.298	0.000	0.899	0.964	0.000	0.000

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	78	0	0	0	27	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.031	0.030	0.297	0.000	1.009	0.855	0.000	0.000

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	62	0	0	0	27	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.020	0.020	0.290	0.000	0.881	0.819	0.000	0.000

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	46	0	0	0	24	0	37
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.43	0.00	0.66
time (sec)	N/A	0.010	0.007	0.304	0.000	0.940	0.781	0.000	4.883

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	49	0	0	0	27	0	40
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.36	0.00	0.53
time (sec)	N/A	0.019	0.010	0.289	0.000	1.441	0.963	0.000	5.070

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	51	0	0	0	32	0	-1
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.031	0.009	0.288	0.000	0.941	1.087	0.000	0.000

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	51	0	0	0	32	0	-1
normalized size	1	1.00	0.40	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.043	0.010	0.290	0.000	0.922	1.229	0.000	0.000

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	91	0	0	0	29	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.046	0.036	0.037	0.000	1.007	0.993	0.000	0.000

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	77	0	0	0	29	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.032	0.030	0.036	0.000	1.099	0.905	0.000	0.000

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	0	0	0	29	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.36	0.00	-0.01
time (sec)	N/A	0.022	0.021	0.035	0.000	1.050	0.838	0.000	0.000

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	0	0	26	0	38
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.45	0.00	0.66
time (sec)	N/A	0.011	0.008	0.306	0.000	1.049	0.805	0.000	4.909

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	50	0	0	0	29	0	41
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.37	0.00	0.53
time (sec)	N/A	0.021	0.010	0.037	0.000	1.161	0.982	0.000	5.100

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	52	0	0	0	34	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.00	0.32	0.00	-0.01
time (sec)	N/A	0.032	0.010	0.037	0.000	0.877	1.102	0.000	0.000

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	52	0	0	0	34	0	-1
normalized size	1	1.00	0.40	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.047	0.010	0.039	0.000	1.065	1.253	0.000	0.000

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	78	0	0	0	27	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.043	0.033	0.312	0.000	1.100	0.974	0.000	0.000

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	65	0	0	0	27	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.031	0.021	0.310	0.000	0.925	0.928	0.000	0.000

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	53	0	0	0	27	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.36	0.00	-0.01
time (sec)	N/A	0.019	0.019	0.301	0.000	0.918	0.917	0.000	0.000

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	24	0	37
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.43	0.00	0.66
time (sec)	N/A	0.010	0.014	0.313	0.000	0.917	0.889	0.000	4.921

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	52	0	0	0	27	0	40
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.36	0.00	0.53
time (sec)	N/A	0.020	0.009	0.309	0.000	0.848	1.067	0.000	5.156

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	54	0	0	0	32	0	-1
normalized size	1	1.00	0.53	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.030	0.010	0.310	0.000	0.968	1.227	0.000	0.000

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	54	0	0	0	32	0	-1
normalized size	1	1.00	0.43	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.042	0.010	0.308	0.000	0.805	1.404	0.000	0.000

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	78	0	0	0	29	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.047	0.032	0.313	0.000	0.753	0.986	0.000	0.000

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	66	0	0	0	29	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.031	0.023	0.382	0.000	1.198	0.949	0.000	0.000

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	0	0	0	29	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.022	0.018	0.334	0.000	1.020	0.934	0.000	0.000

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	0	0	0	26	0	38
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.34	0.00	0.49
time (sec)	N/A	0.019	0.015	0.355	0.000	1.036	0.892	0.000	4.913

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	53	0	0	0	29	0	41
normalized size	1	1.00	0.54	0.00	0.00	0.00	0.29	0.00	0.41
time (sec)	N/A	0.032	0.010	0.315	0.000	0.600	1.089	0.000	5.097

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	55	0	0	0	34	0	-1
normalized size	1	1.00	0.44	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.045	0.011	0.314	0.000	0.945	1.246	0.000	0.000

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	55	0	0	0	34	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.061	0.011	0.307	0.000	1.186	1.438	0.000	0.000

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	55	0	0	0	24	0	37
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.31	0.00	0.47
time (sec)	N/A	0.017	0.023	0.316	0.000	0.813	1.042	0.000	4.879

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	24	0	37
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.31	0.00	0.47
time (sec)	N/A	0.017	0.034	0.326	0.000	0.867	1.358	0.000	4.882

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	0	0	0	24	0	37
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.25	0.00	0.38
time (sec)	N/A	0.024	0.035	0.319	0.000	0.611	1.814	0.000	4.894

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	56	0	0	0	26	0	38
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.32	0.00	0.47
time (sec)	N/A	0.018	0.023	0.316	0.000	0.826	1.062	0.000	4.870

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	74	0	0	0	26	0	38
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.26	0.00	0.38
time (sec)	N/A	0.027	0.036	0.323	0.000	0.571	1.364	0.000	4.864

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	77	0	0	0	26	0	38
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.26	0.00	0.38
time (sec)	N/A	0.026	0.036	0.326	0.000	0.900	1.840	0.000	4.812

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	54	43	0	0	27	0	-1
normalized size	1	1.00	0.55	0.43	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.028	0.029	0.298	0.000	0.900	0.830	0.000	0.000

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	49	38	0	0	27	0	-1
normalized size	1	1.00	0.60	0.47	0.00	0.00	0.33	0.00	-0.01
time (sec)	N/A	0.020	0.019	0.271	0.000	0.901	0.755	0.000	0.000

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	41	31	0	0	27	0	-1
normalized size	1	1.00	0.65	0.49	0.00	0.00	0.43	0.00	-0.02
time (sec)	N/A	0.013	0.011	0.270	0.000	0.815	0.695	0.000	0.000

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	24	18	0	0	26	0	16
normalized size	1	1.00	0.56	0.42	0.00	0.00	0.60	0.00	0.37
time (sec)	N/A	0.007	0.004	0.268	0.000	0.616	0.672	0.000	0.094

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	27	33	0	0	29	0	36
normalized size	1	1.00	0.43	0.52	0.00	0.00	0.46	0.00	0.57
time (sec)	N/A	0.013	0.004	0.269	0.000	0.737	0.740	0.000	5.024

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	29	45	0	0	32	0	-1
normalized size	1	1.00	0.35	0.54	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.019	0.006	0.270	0.000	0.829	0.841	0.000	0.000

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	29	50	0	0	32	0	-1
normalized size	1	1.00	0.29	0.50	0.00	0.00	0.32	0.00	-0.01
time (sec)	N/A	0.030	0.006	0.272	0.000	0.552	0.953	0.000	0.000

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	54	50	0	0	29	0	-1
normalized size	1	1.00	0.65	0.60	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.022	0.026	0.285	0.000	0.874	0.855	0.000	0.000

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	45	0	0	29	0	-1
normalized size	1	1.00	0.75	0.69	0.00	0.00	0.45	0.00	-0.02
time (sec)	N/A	0.014	0.022	0.280	0.000	0.922	0.771	0.000	0.000

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	38	0	0	29	0	-1
normalized size	1	1.00	0.87	0.81	0.00	0.00	0.62	0.00	-0.02
time (sec)	N/A	0.008	0.010	0.286	0.000	0.798	0.716	0.000	0.000

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	18	0	0	27	0	16
normalized size	1	1.00	0.86	0.64	0.00	0.00	0.96	0.00	0.57
time (sec)	N/A	0.003	0.003	0.283	0.000	0.898	0.688	0.000	0.091

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	27	40	0	0	31	0	36
normalized size	1	1.00	0.57	0.85	0.00	0.00	0.66	0.00	0.77
time (sec)	N/A	0.008	0.005	0.283	0.000	0.626	0.765	0.000	5.047

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	29	45	0	0	34	0	-1
normalized size	1	1.00	0.43	0.67	0.00	0.00	0.51	0.00	-0.01
time (sec)	N/A	0.014	0.005	0.275	0.000	0.870	0.867	0.000	0.000

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	29	50	0	0	34	0	-1
normalized size	1	1.00	0.34	0.59	0.00	0.00	0.40	0.00	-0.01
time (sec)	N/A	0.021	0.005	0.280	0.000	0.828	0.975	0.000	0.000

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	54	43	0	0	27	0	-1
normalized size	1	1.00	0.65	0.52	0.00	0.00	0.33	0.00	-0.01
time (sec)	N/A	0.025	0.021	0.294	0.000	0.897	0.815	0.000	0.000

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	38	0	0	27	0	-1
normalized size	1	1.00	0.75	0.58	0.00	0.00	0.42	0.00	-0.02
time (sec)	N/A	0.015	0.020	0.288	0.000	0.649	0.738	0.000	0.000

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	31	0	0	27	0	-1
normalized size	1	1.00	0.87	0.66	0.00	0.00	0.57	0.00	-0.02
time (sec)	N/A	0.009	0.009	0.292	0.000	0.664	0.709	0.000	0.000

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	18	0	0	26	0	16
normalized size	1	1.00	0.89	0.67	0.00	0.00	0.96	0.00	0.59
time (sec)	N/A	0.003	0.005	0.273	0.000	0.768	0.678	0.000	0.082

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	27	33	0	0	29	0	36
normalized size	1	1.00	0.55	0.67	0.00	0.00	0.59	0.00	0.73
time (sec)	N/A	0.008	0.005	0.272	0.000	0.871	0.824	0.000	4.989

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	29	45	0	0	32	0	-1
normalized size	1	1.00	0.43	0.67	0.00	0.00	0.48	0.00	-0.01
time (sec)	N/A	0.015	0.006	0.276	0.000	0.839	0.934	0.000	0.000

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	29	50	0	0	32	0	-1
normalized size	1	1.00	0.34	0.59	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.023	0.005	0.271	0.000	0.951	1.077	0.000	0.000

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	59	0	0	0	29	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.023	0.037	0.056	0.000	1.053	0.840	0.000	0.000

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	54	0	0	0	29	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.45	0.00	-0.02
time (sec)	N/A	0.016	0.024	0.034	0.000	0.593	0.767	0.000	0.000

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	29	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.62	0.00	-0.02
time (sec)	N/A	0.009	0.015	0.033	0.000	0.983	0.733	0.000	0.000

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	18	0	0	27	0	16
normalized size	1	1.00	1.00	0.67	0.00	0.00	1.00	0.00	0.59
time (sec)	N/A	0.003	0.003	0.289	0.000	0.792	0.693	0.000	4.762

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	27	0	0	0	31	0	36
normalized size	1	1.00	0.55	0.00	0.00	0.00	0.63	0.00	0.73
time (sec)	N/A	0.009	0.005	0.034	0.000	0.594	0.841	0.000	5.028

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	29	0	0	0	34	0	-1
normalized size	1	1.00	0.43	0.00	0.00	0.00	0.51	0.00	-0.01
time (sec)	N/A	0.015	0.005	0.037	0.000	0.651	0.945	0.000	0.000

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	29	0	0	0	34	0	-1
normalized size	1	1.00	0.34	0.00	0.00	0.00	0.40	0.00	-0.01
time (sec)	N/A	0.022	0.005	0.036	0.000	0.666	1.106	0.000	0.000

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	68	65	0	0	29	0	-1
normalized size	1	1.00	0.26	0.25	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.144	0.020	0.292	0.000	0.802	0.851	0.000	0.000

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	63	60	0	0	29	0	-1
normalized size	1	1.00	0.26	0.25	0.00	0.00	0.12	0.00	-0.00
time (sec)	N/A	0.112	0.017	0.294	0.000	0.663	0.776	0.000	0.000

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	57	53	0	0	29	0	-1
normalized size	1	1.00	0.26	0.24	0.00	0.00	0.13	0.00	-0.00
time (sec)	N/A	0.091	0.014	0.292	0.000	0.641	0.727	0.000	0.000

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	43	40	0	0	27	0	34
normalized size	1	1.00	0.22	0.20	0.00	0.00	0.14	0.00	0.17
time (sec)	N/A	0.074	0.006	0.292	0.000	0.556	0.695	0.000	4.743

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	46	55	0	0	31	0	36
normalized size	1	1.00	0.21	0.25	0.00	0.00	0.14	0.00	0.16
time (sec)	N/A	0.090	0.006	0.296	0.000	0.677	0.759	0.000	4.886

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	48	67	0	0	34	0	-1
normalized size	1	1.00	0.20	0.28	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.109	0.008	0.289	0.000	0.669	0.866	0.000	0.000

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	48	72	0	0	34	0	-1
normalized size	1	1.00	0.18	0.28	0.00	0.00	0.13	0.00	-0.00
time (sec)	N/A	0.126	0.007	0.291	0.000	0.677	0.977	0.000	0.000

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	68	53	0	0	34	0	-1
normalized size	1	1.00	0.26	0.20	0.00	0.00	0.13	0.00	-0.00
time (sec)	N/A	0.134	0.020	0.274	0.000	0.699	0.841	0.000	0.000

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	63	48	0	0	34	0	-1
normalized size	1	1.00	0.26	0.20	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.110	0.017	0.273	0.000	0.670	0.767	0.000	0.000

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	58	41	0	0	34	0	-1
normalized size	1	1.00	0.26	0.18	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.094	0.014	0.272	0.000	0.622	0.716	0.000	0.000

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	43	21	0	0	32	0	34
normalized size	1	1.00	0.21	0.10	0.00	0.00	0.16	0.00	0.17
time (sec)	N/A	0.078	0.008	0.274	0.000	0.609	0.687	0.000	4.879

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	46	43	0	0	36	0	36
normalized size	1	1.00	0.21	0.19	0.00	0.00	0.16	0.00	0.16
time (sec)	N/A	0.093	0.008	0.278	0.000	0.548	0.758	0.000	5.038

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	48	48	0	0	39	0	-1
normalized size	1	1.00	0.20	0.20	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.111	0.008	0.275	0.000	0.567	0.863	0.000	0.000

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	48	53	0	0	39	0	-1
normalized size	1	1.00	0.18	0.20	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.129	0.008	0.296	0.000	0.650	0.971	0.000	0.000

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	68	65	0	0	31	0	-1
normalized size	1	1.00	0.49	0.47	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.057	0.021	0.301	0.000	0.611	0.856	0.000	0.000

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	63	60	0	0	31	0	-1
normalized size	1	1.00	0.52	0.50	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.046	0.018	0.315	0.000	0.618	0.771	0.000	0.000

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	57	53	0	0	31	0	-1
normalized size	1	1.00	0.56	0.52	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.037	0.015	0.290	0.000	0.561	0.742	0.000	0.000

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	43	40	0	0	29	0	34
normalized size	1	1.00	0.52	0.49	0.00	0.00	0.35	0.00	0.41
time (sec)	N/A	0.026	0.007	0.293	0.000	0.778	0.709	0.000	4.871

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	46	55	0	0	29	0	23
normalized size	1	1.00	0.44	0.53	0.00	0.00	0.28	0.00	0.22
time (sec)	N/A	0.035	0.008	0.293	0.000	0.610	0.845	0.000	5.068

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	48	67	0	0	32	0	-1
normalized size	1	1.00	0.39	0.55	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.046	0.008	0.292	0.000	0.643	0.963	0.000	0.000

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	48	72	0	0	32	0	-1
normalized size	1	1.00	0.34	0.51	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.057	0.008	0.287	0.000	0.506	1.112	0.000	0.000

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	68	0	0	0	36	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.058	0.020	0.055	0.000	0.623	0.838	0.000	0.000

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	63	0	0	0	36	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.046	0.019	0.033	0.000	0.586	0.764	0.000	0.000

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	58	0	0	0	36	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.036	0.014	0.034	0.000	0.716	0.721	0.000	0.000

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	43	21	0	0	34	0	34
normalized size	1	1.00	0.51	0.25	0.00	0.00	0.40	0.00	0.40
time (sec)	N/A	0.027	0.008	0.274	0.000	0.593	0.694	0.000	4.612

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	46	0	0	0	34	0	36
normalized size	1	1.00	0.44	0.00	0.00	0.00	0.32	0.00	0.34
time (sec)	N/A	0.035	0.008	0.037	0.000	0.686	0.836	0.000	4.731

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	48	0	0	0	37	0	-1
normalized size	1	1.00	0.39	0.00	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.049	0.009	0.036	0.000	0.560	0.941	0.000	0.000

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	48	0	0	0	37	0	-1
normalized size	1	1.00	0.34	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.058	0.008	0.038	0.000	0.868	1.118	0.000	0.000

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	102	0	0	0	46	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.108	0.068	0.106	0.000	0.634	25.911	0.000	0.000

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	85	0	0	0	46	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.085	0.045	0.055	0.000	0.808	2.709	0.000	0.000

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	54	0	0	0	46	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.52	0.00	-0.01
time (sec)	N/A	0.073	0.013	0.067	0.000	0.615	1.307	0.000	0.000

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	56	0	0	0	32	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.074	0.013	0.056	0.000	0.726	3.713	0.000	0.000

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	56	0	0	0	36	0	-1
normalized size	1	1.00	0.46	0.00	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.089	0.014	0.062	0.000	0.631	27.113	0.000	0.000

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	56	0	0	0	0	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.014	0.061	0.000	0.570	0.000	0.000	0.000

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	85	0	0	0	46	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.092	0.046	0.058	0.000	0.000	9.471	0.000	0.000

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	56	0	0	0	46	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.40	0.00	-0.01
time (sec)	N/A	0.070	0.012	0.052	0.000	0.000	1.806	0.000	0.000

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	54	0	0	0	49	0	-1
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.46	0.00	-0.01
time (sec)	N/A	0.069	0.013	0.056	0.000	0.000	2.315	0.000	0.000

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	0	25	78	0	37
normalized size	1	1.00	0.93	0.75	0.00	0.89	2.79	0.00	1.32
time (sec)	N/A	0.006	0.008	0.004	0.000	1.138	9.132	0.000	4.916

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	41	31	0	46	124	0	51
normalized size	1	1.00	0.72	0.54	0.00	0.81	2.18	0.00	0.89
time (sec)	N/A	0.015	0.015	0.006	0.000	1.097	69.581	0.000	4.976

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	52	42	0	57	0	0	65
normalized size	1	1.00	0.61	0.49	0.00	0.67	0.00	0.00	0.76
time (sec)	N/A	0.024	0.018	0.007	0.000	1.042	0.000	0.000	5.002

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	63	53	0	68	0	0	79
normalized size	1	1.00	0.56	0.47	0.00	0.60	0.00	0.00	0.70
time (sec)	N/A	0.038	0.016	0.008	0.000	1.161	0.000	0.000	5.022

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	88	0	0	0	48	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.092	0.055	0.105	0.000	1.111	2.639	0.000	0.000

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	55	0	0	0	39	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.42	0.00	-0.01
time (sec)	N/A	0.079	0.014	0.056	0.000	1.097	1.336	0.000	0.000

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	57	0	0	0	36	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.37	0.00	-0.01
time (sec)	N/A	0.079	0.016	0.066	0.000	0.707	3.705	0.000	0.000

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	57	0	0	0	39	0	-1
normalized size	1	1.00	0.45	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.096	0.015	0.064	0.000	0.722	26.579	0.000	0.000

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	57	0	0	0	0	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.016	0.064	0.000	0.553	0.000	0.000	0.000

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	88	0	0	0	48	0	-1
normalized size	1	1.00	0.26	0.00	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.368	0.050	0.064	0.000	0.000	9.288	0.000	0.000

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	57	0	0	0	48	0	-1
normalized size	1	1.00	0.19	0.00	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.268	0.012	0.058	0.000	0.000	1.782	0.000	0.000

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	55	0	0	0	51	0	-1
normalized size	1	1.00	0.19	0.00	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.275	0.013	0.058	0.000	0.000	2.324	0.000	0.000

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	22	0	35	178	0	38
normalized size	1	1.00	0.93	0.76	0.00	1.21	6.14	0.00	1.31
time (sec)	N/A	0.006	0.009	0.003	0.000	1.272	8.763	0.000	4.847

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	42	32	0	46	462	0	51
normalized size	1	1.00	0.71	0.54	0.00	0.78	7.83	0.00	0.86
time (sec)	N/A	0.016	0.017	0.005	0.000	0.856	68.247	0.000	4.892

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	53	43	0	58	0	0	65
normalized size	1	1.00	0.60	0.49	0.00	0.66	0.00	0.00	0.74
time (sec)	N/A	0.028	0.019	0.005	0.000	1.078	0.000	0.000	4.911

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	64	54	0	69	0	0	79
normalized size	1	1.00	0.55	0.46	0.00	0.59	0.00	0.00	0.68
time (sec)	N/A	0.040	0.017	0.006	0.000	1.006	0.000	0.000	4.930

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	97	0	0	314	44	0	-1
normalized size	1	1.00	0.83	0.00	0.00	2.68	0.38	0.00	-0.01
time (sec)	N/A	0.065	0.037	0.321	0.000	1.115	2.175	0.000	0.000

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	65	0	0	241	44	0	-1
normalized size	1	1.00	0.78	0.00	0.00	2.90	0.53	0.00	-0.01
time (sec)	N/A	0.050	0.011	0.283	0.000	1.343	1.505	0.000	0.000

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	0	25	36	0	25
normalized size	1	1.00	0.93	0.75	0.00	0.89	1.29	0.00	0.89
time (sec)	N/A	0.006	0.007	0.005	0.000	1.336	3.583	0.000	4.946

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	41	31	0	35	80	0	40
normalized size	1	1.00	0.72	0.54	0.00	0.61	1.40	0.00	0.70
time (sec)	N/A	0.016	0.018	0.006	0.000	0.712	34.720	0.000	4.997

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	52	42	0	46	0	0	54
normalized size	1	1.00	0.61	0.49	0.00	0.54	0.00	0.00	0.64
time (sec)	N/A	0.026	0.024	0.006	0.000	0.768	0.000	0.000	5.005

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	87	0	0	0	44	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.067	0.032	0.305	0.000	0.942	51.813	0.000	0.000

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	69	0	0	0	44	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.047	0.025	0.303	0.000	0.826	6.737	0.000	0.000

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	56	0	0	0	44	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.53	0.00	-0.01
time (sec)	N/A	0.032	0.012	0.280	0.000	0.946	1.008	0.000	0.000

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	54	0	0	0	31	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.035	0.013	0.296	0.000	0.522	2.038	0.000	0.000

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	56	0	0	0	34	0	-1
normalized size	1	1.00	0.44	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.049	0.013	0.303	0.000	0.813	12.233	0.000	0.000

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	56	0	0	0	34	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.068	0.013	0.366	0.000	0.698	89.345	0.000	0.000

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	241	0	0	340	46	0	-1
normalized size	1	1.00	0.78	0.00	0.00	1.10	0.15	0.00	-0.00
time (sec)	N/A	0.265	0.152	0.323	0.000	0.621	2.205	0.000	0.000

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	197	0	0	267	46	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.98	0.17	0.00	-0.00
time (sec)	N/A	0.228	0.043	0.325	0.000	0.657	1.523	0.000	0.000

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	22	0	26	88	0	26
normalized size	1	1.00	0.93	0.76	0.00	0.90	3.03	0.00	0.90
time (sec)	N/A	0.006	0.007	0.004	0.000	0.695	3.617	0.000	5.124

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	42	32	0	36	343	0	41
normalized size	1	1.00	0.71	0.54	0.00	0.61	5.81	0.00	0.69
time (sec)	N/A	0.015	0.019	0.004	0.000	0.585	34.343	0.000	5.145

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	53	43	0	47	0	0	55
normalized size	1	1.00	0.60	0.49	0.00	0.53	0.00	0.00	0.62
time (sec)	N/A	0.027	0.026	0.006	0.000	0.543	0.000	0.000	5.251

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	71	0	0	0	46	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.00	0.36	0.00	-0.01
time (sec)	N/A	0.049	0.029	0.306	0.000	0.614	6.785	0.000	0.000

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	57	0	0	0	46	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.51	0.00	-0.01
time (sec)	N/A	0.036	0.014	0.282	0.000	0.624	1.036	0.000	0.000

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	0	0	0	32	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.47	0.00	-0.01
time (sec)	N/A	0.024	0.014	0.306	0.000	0.641	2.063	0.000	0.000

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	57	0	0	0	39	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.036	0.014	0.302	0.000	0.615	12.195	0.000	0.000

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	57	0	0	0	36	0	-1
normalized size	1	1.00	0.44	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.051	0.014	0.303	0.000	0.765	87.775	0.000	0.000

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	66	0	0	0	44	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.51	0.00	-0.01
time (sec)	N/A	0.070	0.026	0.050	0.000	0.646	2.030	0.000	0.000

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	54	0	0	0	31	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.47	0.00	-0.02
time (sec)	N/A	0.063	0.012	0.326	0.000	0.706	1.714	0.000	0.000

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	56	0	0	0	48	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.49	0.00	-0.01
time (sec)	N/A	0.073	0.013	0.059	0.000	0.687	7.835	0.000	0.000

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	56	0	0	0	34	0	-1
normalized size	1	1.00	0.44	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.088	0.014	0.058	0.000	0.675	66.188	0.000	0.000

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	56	0	0	0	0	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.014	0.061	0.000	0.635	0.000	0.000	0.000

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	97	0	0	0	44	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.067	0.039	0.058	0.000	0.000	7.779	0.000	0.000

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	67	0	0	0	44	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.52	0.00	-0.01
time (sec)	N/A	0.056	0.014	0.278	0.000	0.000	1.501	0.000	0.000

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	21	0	25	36	0	22
normalized size	1	1.00	0.92	0.81	0.00	0.96	1.38	0.00	0.85
time (sec)	N/A	0.006	0.006	0.005	0.000	0.968	2.924	0.000	4.903

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	34	29	0	35	78	0	40
normalized size	1	1.00	0.62	0.53	0.00	0.64	1.42	0.00	0.73
time (sec)	N/A	0.014	0.018	0.006	0.000	0.927	24.762	0.000	4.982

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	52	42	0	46	483	0	54
normalized size	1	1.00	0.63	0.51	0.00	0.55	5.82	0.00	0.65
time (sec)	N/A	0.025	0.024	0.005	0.000	0.607	172.060	0.000	5.172

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	0	0	0	46	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.51	0.00	-0.01
time (sec)	N/A	0.076	0.028	0.055	0.000	0.578	2.199	0.000	0.000

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	0	0	0	32	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.47	0.00	-0.01
time (sec)	N/A	0.064	0.013	0.278	0.000	0.584	1.894	0.000	0.000

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	57	0	0	0	39	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.076	0.014	0.063	0.000	0.503	8.315	0.000	0.000

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	57	0	0	0	36	0	-1
normalized size	1	1.00	0.44	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.094	0.013	0.062	0.000	0.512	68.934	0.000	0.000

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	57	0	0	0	0	0	-1
normalized size	1	1.00	0.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.013	0.069	0.000	0.649	0.000	0.000	0.000

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	112	0	0	0	46	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.262	0.057	0.061	0.000	0.000	7.817	0.000	0.000

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	75	0	0	0	46	0	-1
normalized size	1	1.00	0.28	0.00	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.232	0.024	0.280	0.000	0.000	1.564	0.000	0.000

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	22	0	26	90	0	23
normalized size	1	1.00	0.93	0.81	0.00	0.96	3.33	0.00	0.85
time (sec)	N/A	0.007	0.006	0.004	0.000	0.972	3.001	0.000	5.076

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	30	0	34	352	0	41
normalized size	1	1.00	0.61	0.53	0.00	0.60	6.18	0.00	0.72
time (sec)	N/A	0.014	0.018	0.006	0.000	0.890	25.445	0.000	5.096

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	53	43	0	47	1263	0	55
normalized size	1	1.00	0.62	0.50	0.00	0.55	14.69	0.00	0.64
time (sec)	N/A	0.025	0.024	0.006	0.000	1.050	171.234	0.000	5.153

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	63	0	0	389	44	0	-1
normalized size	1	1.00	0.43	0.00	0.00	2.66	0.30	0.00	-0.01
time (sec)	N/A	0.076	0.036	0.328	0.000	0.869	24.494	0.000	0.000

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	59	0	0	319	44	0	-1
normalized size	1	1.00	0.55	0.00	0.00	2.98	0.41	0.00	-0.01
time (sec)	N/A	0.059	0.013	0.280	0.000	0.948	4.257	0.000	0.000

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	21	0	31	34	0	29
normalized size	1	1.00	0.92	0.81	0.00	1.19	1.31	0.00	1.12
time (sec)	N/A	0.006	0.006	0.005	0.000	0.810	3.141	0.000	4.968

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	34	29	0	48	78	0	57
normalized size	1	1.00	0.62	0.53	0.00	0.87	1.42	0.00	1.04
time (sec)	N/A	0.015	0.010	0.006	0.000	0.942	14.989	0.000	5.097

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	47	42	0	61	384	0	70
normalized size	1	1.00	0.57	0.51	0.00	0.73	4.63	0.00	0.84
time (sec)	N/A	0.024	0.011	0.007	0.000	0.549	109.044	0.000	5.154

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	58	53	0	72	0	0	85
normalized size	1	1.00	0.53	0.49	0.00	0.66	0.00	0.00	0.78
time (sec)	N/A	0.037	0.013	0.007	0.000	0.534	0.000	0.000	5.197

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	87	0	0	0	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.037	0.329	0.000	0.799	0.000	0.000	0.000

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	74	0	0	0	44	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.049	0.032	0.327	0.000	0.607	64.024	0.000	0.000

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	60	0	0	0	44	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.49	0.00	-0.01
time (sec)	N/A	0.033	0.031	0.280	0.000	0.754	7.776	0.000	0.000

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	0	0	0	44	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.70	0.00	-0.02
time (sec)	N/A	0.020	0.012	0.281	0.000	0.639	2.102	0.000	0.000

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	57	0	0	0	48	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.52	0.00	-0.01
time (sec)	N/A	0.035	0.012	0.328	0.000	0.705	6.013	0.000	0.000

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	59	0	0	0	34	0	-1
normalized size	1	1.00	0.47	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.051	0.013	0.330	0.000	0.619	41.486	0.000	0.000

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	59	0	0	0	0	0	-1
normalized size	1	1.00	0.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.012	0.336	0.000	0.893	0.000	0.000	0.000

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	-0.02
time (sec)	N/A	0.018	0.012	0.323	0.000	0.953	12.123	0.000	0.000

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	-0.02
time (sec)	N/A	0.017	0.010	0.280	0.000	0.893	3.690	0.000	0.000

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	-0.02
time (sec)	N/A	0.017	0.012	0.282	0.000	0.676	1.229	0.000	0.000

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	-0.02
time (sec)	N/A	0.018	0.012	0.286	0.000	0.721	1.300	0.000	0.000

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	44	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.79	0.00	-0.02
time (sec)	N/A	0.017	0.011	0.285	0.000	0.719	2.086	0.000	0.000

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	48	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.017	0.011	0.379	0.000	0.800	3.765	0.000	0.000

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.72	0.00	-0.02
time (sec)	N/A	0.019	0.013	0.335	0.000	0.857	23.695	0.000	0.000

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.72	0.00	-0.02
time (sec)	N/A	0.018	0.012	0.304	0.000	0.742	13.712	0.000	0.000

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.72	0.00	-0.02
time (sec)	N/A	0.019	0.011	0.281	0.000	0.644	7.478	0.000	0.000

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.72	0.00	-0.02
time (sec)	N/A	0.018	0.011	0.286	0.000	0.746	8.138	0.000	0.000

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	44	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.75	0.00	-0.02
time (sec)	N/A	0.018	0.012	0.281	0.000	0.579	14.678	0.000	0.000

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	48	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.81	0.00	-0.02
time (sec)	N/A	0.019	0.011	0.362	0.000	0.632	25.454	0.000	0.000

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	105	0	0	0	29	0	-1
normalized size	1	1.00	0.30	0.00	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.425	0.062	0.335	0.000	0.817	1.189	0.000	0.000

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	93	0	0	0	29	0	-1
normalized size	1	1.00	0.29	0.00	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.291	0.049	0.303	0.000	0.641	1.061	0.000	0.000

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	62	0	0	0	29	0	-1
normalized size	1	1.00	0.21	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.247	0.049	0.305	0.000	0.727	0.977	0.000	0.000

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	46	0	0	0	26	0	37
normalized size	1	1.00	0.17	0.00	0.00	0.00	0.10	0.00	0.14
time (sec)	N/A	0.209	0.007	0.330	0.000	0.541	0.893	0.000	4.884

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	49	0	0	0	29	0	40
normalized size	1	1.00	0.18	0.00	0.00	0.00	0.11	0.00	0.15
time (sec)	N/A	0.210	0.009	0.308	0.000	0.613	0.985	0.000	5.079

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	51	0	0	0	34	0	-1
normalized size	1	1.00	0.17	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.248	0.009	0.309	0.000	0.653	1.108	0.000	0.000

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	51	0	0	0	34	0	-1
normalized size	1	1.00	0.16	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.283	0.009	0.308	0.000	0.875	1.293	0.000	0.000

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	51	0	0	0	34	0	-1
normalized size	1	1.00	0.15	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.318	0.009	0.309	0.000	0.545	1.463	0.000	0.000

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	659	90	0	0	0	27	0	-1
normalized size	1	1.00	0.14	0.00	0.00	0.00	0.04	0.00	-0.00
time (sec)	N/A	0.693	0.032	0.313	0.000	0.652	1.106	0.000	0.000

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	635	635	79	0	0	0	27	0	-1
normalized size	1	1.00	0.12	0.00	0.00	0.00	0.04	0.00	-0.00
time (sec)	N/A	0.609	0.024	0.303	0.000	0.859	0.961	0.000	0.000

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	611	611	62	0	0	0	27	0	-1
normalized size	1	1.00	0.10	0.00	0.00	0.00	0.04	0.00	-0.00
time (sec)	N/A	0.547	0.020	0.302	0.000	0.582	0.843	0.000	0.000

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	46	0	0	0	24	0	37
normalized size	1	1.00	0.08	0.00	0.00	0.00	0.04	0.00	0.06
time (sec)	N/A	0.467	0.007	0.314	0.000	0.753	0.849	0.000	4.879

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	586	586	49	0	0	0	27	0	40
normalized size	1	1.00	0.08	0.00	0.00	0.00	0.05	0.00	0.07
time (sec)	N/A	0.533	0.010	0.302	0.000	0.573	0.974	0.000	5.087

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	633	633	51	0	0	0	32	0	-1
normalized size	1	1.00	0.08	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.595	0.009	0.306	0.000	0.604	1.121	0.000	0.000

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	661	661	51	0	0	0	32	0	-1
normalized size	1	1.00	0.08	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.672	0.010	0.306	0.000	0.656	1.301	0.000	0.000

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	89	0	0	0	27	0	-1
normalized size	1	1.00	0.27	0.00	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.292	0.033	0.313	0.000	0.724	0.992	0.000	0.000

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	79	0	0	0	27	0	-1
normalized size	1	1.00	0.26	0.00	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.251	0.024	0.312	0.000	0.777	0.919	0.000	0.000

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	62	0	0	0	27	0	-1
normalized size	1	1.00	0.22	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.212	0.020	0.322	0.000	0.581	0.937	0.000	0.000

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	46	0	0	0	24	0	37
normalized size	1	1.00	0.18	0.00	0.00	0.00	0.10	0.00	0.15
time (sec)	N/A	0.184	0.008	0.327	0.000	0.701	0.870	0.000	4.906

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	49	0	0	0	27	0	40
normalized size	1	1.00	0.18	0.00	0.00	0.00	0.10	0.00	0.15
time (sec)	N/A	0.212	0.010	0.326	0.000	0.644	1.151	0.000	5.100

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	51	0	0	0	32	0	-1
normalized size	1	1.00	0.17	0.00	0.00	0.00	0.11	0.00	-0.00
time (sec)	N/A	0.243	0.010	0.309	0.000	0.763	1.463	0.000	0.000

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	51	0	0	0	32	0	-1
normalized size	1	1.00	0.16	0.00	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.279	0.010	0.310	0.000	0.635	1.490	0.000	0.000

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	654	654	79	0	0	0	27	0	-1
normalized size	1	1.00	0.12	0.00	0.00	0.00	0.04	0.00	-0.00
time (sec)	N/A	0.669	0.024	0.332	0.000	0.639	1.052	0.000	0.000

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	65	0	0	0	27	0	-1
normalized size	1	1.00	0.10	0.00	0.00	0.00	0.04	0.00	-0.00
time (sec)	N/A	0.592	0.023	0.335	0.000	0.929	1.039	0.000	0.000

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	583	583	58	0	0	0	27	0	-1
normalized size	1	1.00	0.10	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.522	0.016	0.324	0.000	0.751	1.026	0.000	0.000

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	49	0	0	0	24	0	37
normalized size	1	1.00	0.09	0.00	0.00	0.00	0.04	0.00	0.07
time (sec)	N/A	0.394	0.007	0.327	0.000	0.673	0.977	0.000	4.943

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	52	0	0	0	27	0	40
normalized size	1	1.00	0.08	0.00	0.00	0.00	0.04	0.00	0.07
time (sec)	N/A	0.574	0.009	0.335	0.000	0.755	1.240	0.000	5.121

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	652	652	54	0	0	0	32	0	-1
normalized size	1	1.00	0.08	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.654	0.010	0.335	0.000	0.634	1.447	0.000	0.000

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	54	0	0	0	32	0	-1
normalized size	1	1.00	0.08	0.00	0.00	0.00	0.05	0.00	-0.00
time (sec)	N/A	0.741	0.010	0.325	0.000	0.735	1.704	0.000	0.000

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	95	132	106	148	2025	410	183
normalized size	1	1.00	0.95	1.32	1.06	1.48	20.25	4.10	1.83
time (sec)	N/A	0.064	0.055	0.006	1.445	0.483	11.065	0.585	4.966

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	80	73	98	981	231	117
normalized size	1	1.00	0.89	1.11	1.01	1.36	13.62	3.21	1.62
time (sec)	N/A	0.043	0.031	0.006	1.412	0.679	5.045	0.585	4.902

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	42	47	58	364	94	68
normalized size	1	1.00	0.83	0.88	0.98	1.21	7.58	1.96	1.42
time (sec)	N/A	0.029	0.019	0.004	1.288	0.733	2.003	0.579	4.887

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	21	25	97	21	21
normalized size	1	1.00	0.96	0.96	0.91	1.09	4.22	0.91	0.91
time (sec)	N/A	0.005	0.003	0.002	1.381	0.667	0.732	0.571	4.892

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	0	0	0	39	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.95	0.00	-0.02
time (sec)	N/A	0.022	0.011	0.288	0.000	0.677	2.487	0.000	0.000

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	42	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	1.00	0.00	-0.02
time (sec)	N/A	0.023	0.008	0.282	0.000	0.564	5.133	0.000	0.000

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	26	0	-1
normalized size	1	1.22	1.22	0.00	0.00	0.00	0.65	0.00	-0.02
time (sec)	N/A	0.013	0.008	0.282	0.000	0.534	11.851	0.000	0.000

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	26	0	-1
normalized size	1	1.22	1.22	0.00	0.00	0.00	0.65	0.00	-0.02
time (sec)	N/A	0.013	0.006	0.283	0.000	0.653	6.608	0.000	0.000

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	26	0	-1
normalized size	1	1.22	1.22	0.00	0.00	0.00	0.65	0.00	-0.02
time (sec)	N/A	0.014	0.006	0.290	0.000	0.687	3.813	0.000	0.000

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	44	44	0	0	0	22	0	41
normalized size	1	1.26	1.26	0.00	0.00	0.00	0.63	0.00	1.17
time (sec)	N/A	0.009	0.005	0.284	0.000	0.564	2.325	0.000	5.410

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	47	47	0	0	0	26	0	58
normalized size	1	1.24	1.24	0.00	0.00	0.00	0.68	0.00	1.53
time (sec)	N/A	0.013	0.006	0.284	0.000	0.698	3.746	0.000	5.029

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	-1
normalized size	1	1.21	1.21	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.008	0.285	0.000	0.722	0.000	0.000	0.000

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	-1
normalized size	1	1.21	1.21	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.008	0.282	0.000	0.683	0.000	0.000	0.000

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	-1
normalized size	1	1.21	1.21	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.008	0.306	0.000	0.584	0.000	0.000	0.000

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	37	0	-1
normalized size	1	1.21	1.21	0.00	0.00	0.00	0.88	0.00	-0.02
time (sec)	N/A	0.012	0.008	0.286	0.000	0.628	57.087	0.000	0.000

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	37	0	-1
normalized size	1	1.22	1.22	0.00	0.00	0.00	0.92	0.00	-0.02
time (sec)	N/A	0.013	0.007	0.294	0.000	0.607	38.997	0.000	0.000

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	0	0	-1
normalized size	1	1.22	1.22	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.007	0.283	0.000	0.635	0.000	0.000	0.000

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	-1
normalized size	1	1.21	1.21	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.008	0.291	0.000	0.656	0.000	0.000	0.000

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	-1
normalized size	1	1.21	1.21	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.008	0.283	0.000	0.762	0.000	0.000	0.000

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	61	63	0	0	0	51	0	-1
normalized size	1	1.15	1.19	0.00	0.00	0.00	0.96	0.00	-0.02
time (sec)	N/A	0.017	0.011	0.282	0.000	0.570	21.284	0.000	0.000

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	0	54	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	-0.02
time (sec)	N/A	0.019	0.011	0.283	0.000	0.603	21.457	0.000	0.000

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0	-1
normalized size	1	1.32	1.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.015	0.283	0.000	0.616	0.000	0.000	0.000

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	62	81	84	106	0	0	154
normalized size	1	1.00	0.59	0.77	0.80	1.01	0.00	0.00	1.47
time (sec)	N/A	0.057	0.014	0.006	1.458	0.713	0.000	0.000	5.089

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0	-1
normalized size	1	1.32	1.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.016	0.280	0.000	0.834	0.000	0.000	0.000

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	62	45	59	67	0	0	96
normalized size	1	1.00	0.93	0.67	0.88	1.00	0.00	0.00	1.43
time (sec)	N/A	0.020	0.015	0.004	1.405	0.695	0.000	0.000	5.020

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0	-1
normalized size	1	1.32	1.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.015	0.333	0.000	0.732	0.000	0.000	0.000

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	29	37	34	0	0	52
normalized size	1	1.00	0.97	0.97	1.23	1.13	0.00	0.00	1.73
time (sec)	N/A	0.006	0.012	0.003	1.478	0.667	0.000	0.000	5.053

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0	-1
normalized size	1	1.32	1.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.015	0.286	0.000	0.636	0.000	0.000	0.000

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	56	56	0	0	0	0	0	-1
normalized size	1	1.30	1.30	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.007	0.279	0.000	0.732	0.000	0.000	0.000

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	69	65	0	0	0	24	0	-1
normalized size	1	1.33	1.25	0.00	0.00	0.00	0.46	0.00	-0.02
time (sec)	N/A	0.019	0.017	0.282	0.000	0.645	15.080	0.000	0.000

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	64	61	0	0	0	0	0	-1
normalized size	1	1.31	1.24	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.017	0.282	0.000	0.648	0.000	0.000	0.000

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	69	65	0	0	0	0	0	-1
normalized size	1	1.33	1.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.018	0.283	0.000	0.779	0.000	0.000	0.000

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	64	61	0	0	0	0	0	-1
normalized size	1	1.31	1.24	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.017	0.281	0.000	0.682	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [321] had the largest ratio of [.6923]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	11	0.091
2	A	2	1	1.00	11	0.091
3	A	2	1	1.00	11	0.091
4	A	2	1	1.00	9	0.111
5	A	1	0	1.00	7	0.000
6	A	2	1	1.00	11	0.091
7	A	2	1	1.00	11	0.091
8	A	2	1	1.00	11	0.091
9	A	2	1	1.00	11	0.091
10	A	2	1	1.00	11	0.091
11	A	2	1	1.00	11	0.091
12	A	2	1	1.00	11	0.091
13	A	3	2	1.00	13	0.154
14	A	2	1	1.00	13	0.077
15	A	3	2	1.00	13	0.154
16	A	2	1	1.00	13	0.077
17	A	1	1	1.00	11	0.091
18	A	2	1	1.00	9	0.111
19	A	3	2	1.00	13	0.154
20	A	2	1	1.00	13	0.077
21	A	3	2	1.00	13	0.154
22	A	2	1	1.00	13	0.077
23	A	3	2	1.00	13	0.154
24	A	2	1	1.00	13	0.077
25	A	1	1	1.00	13	0.077
26	A	2	1	1.00	13	0.077
27	A	3	2	1.00	13	0.154
28	A	2	1	1.00	13	0.077
29	A	3	2	1.00	13	0.154
30	A	3	2	1.00	13	0.154
31	A	3	2	1.00	13	0.154
32	A	3	2	1.00	13	0.154
33	A	1	1	1.00	11	0.091
34	A	3	2	1.00	13	0.154
35	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	3	2	1.00	13	0.154
37	A	3	2	1.00	13	0.154
38	A	1	1	1.00	13	0.077
39	A	3	3	1.00	13	0.231
40	A	3	2	1.00	13	0.154
41	A	3	2	1.00	13	0.154
42	A	2	1	1.00	13	0.077
43	A	2	1	1.00	13	0.077
44	A	2	1	1.00	13	0.077
45	A	2	1	1.00	9	0.111
46	A	2	1	1.00	13	0.077
47	A	2	1	1.00	13	0.077
48	A	2	1	1.00	13	0.077
49	A	2	1	1.00	13	0.077
50	A	2	1	1.00	13	0.077
51	A	2	1	1.00	13	0.077
52	A	3	2	1.00	13	0.154
53	A	3	2	1.00	13	0.154
54	A	3	2	1.00	13	0.154
55	A	3	2	1.00	13	0.154
56	A	3	2	1.00	13	0.154
57	A	3	2	1.00	13	0.154
58	A	1	1	1.00	11	0.091
59	A	3	2	1.00	13	0.154
60	A	3	2	1.00	13	0.154
61	A	3	2	1.00	13	0.154
62	A	3	2	1.00	13	0.154
63	A	3	2	1.00	13	0.154
64	A	3	2	1.00	13	0.154
65	A	1	1	1.00	13	0.077
66	A	3	3	1.00	13	0.231
67	A	4	3	1.00	13	0.231
68	A	3	2	1.00	13	0.154
69	A	3	2	1.00	13	0.154
70	A	2	1	1.00	13	0.077
71	A	2	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	2	1	1.00	13	0.077
73	A	2	1	1.00	13	0.077
74	A	2	1	1.00	9	0.111
75	A	2	1	1.00	13	0.077
76	A	2	1	1.00	13	0.077
77	A	2	1	1.00	13	0.077
78	A	2	1	1.00	13	0.077
79	A	2	1	1.00	13	0.077
80	A	2	1	1.00	13	0.077
81	A	2	1	1.00	13	0.077
82	A	2	1	1.00	13	0.077
83	A	2	1	1.00	13	0.077
84	A	2	1	1.00	13	0.077
85	A	3	2	1.00	13	0.154
86	A	3	2	1.00	13	0.154
87	A	3	2	1.00	13	0.154
88	A	3	2	1.00	13	0.154
89	A	3	2	1.00	13	0.154
90	A	3	2	1.00	13	0.154
91	A	1	1	1.00	11	0.091
92	A	3	2	1.00	13	0.154
93	A	3	2	1.00	13	0.154
94	A	3	2	1.00	13	0.154
95	A	3	2	1.00	13	0.154
96	A	3	2	1.00	13	0.154
97	A	3	2	1.00	13	0.154
98	A	3	2	1.00	13	0.154
99	A	3	2	1.00	13	0.154
100	A	3	2	1.00	13	0.154
101	A	1	1	1.00	13	0.077
102	A	3	3	1.00	13	0.231
103	A	4	3	1.00	13	0.231
104	A	5	3	1.00	13	0.231
105	A	6	3	1.00	13	0.231
106	A	3	2	1.00	13	0.154
107	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	3	2	1.00	13	0.154
109	A	2	1	1.00	13	0.077
110	A	2	1	1.00	13	0.077
111	A	2	1	1.00	13	0.077
112	A	2	1	1.00	13	0.077
113	A	2	1	1.00	9	0.111
114	A	2	1	1.00	13	0.077
115	A	2	1	1.00	13	0.077
116	A	2	1	1.00	13	0.077
117	A	2	1	1.00	13	0.077
118	A	2	1	1.00	13	0.077
119	A	2	1	1.00	13	0.077
120	A	2	1	1.00	13	0.077
121	A	2	1	1.00	13	0.077
122	A	2	1	1.00	13	0.077
123	A	2	1	1.00	13	0.077
124	A	3	2	1.00	13	0.154
125	A	3	2	1.00	13	0.154
126	A	3	2	1.00	13	0.154
127	A	3	2	1.00	13	0.154
128	A	3	2	1.00	13	0.154
129	A	3	2	1.00	13	0.154
130	A	3	2	1.00	13	0.154
131	A	3	2	1.00	13	0.154
132	A	3	2	1.00	13	0.154
133	A	2	2	1.00	13	0.154
134	A	1	1	1.00	11	0.091
135	A	1	1	1.00	9	0.111
136	A	4	4	1.00	13	0.308
137	A	2	2	1.00	13	0.154
138	A	3	2	1.00	13	0.154
139	A	3	2	1.00	13	0.154
140	A	3	2	1.00	13	0.154
141	A	4	2	1.00	13	0.154
142	A	3	2	1.00	13	0.154
143	A	5	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	3	2	1.00	13	0.154
145	A	3	2	1.00	13	0.154
146	A	4	3	1.00	13	0.231
147	A	3	2	1.00	13	0.154
148	A	4	3	1.00	13	0.231
149	A	3	2	1.00	13	0.154
150	A	4	3	1.00	13	0.231
151	A	3	2	1.00	13	0.154
152	A	4	3	1.00	13	0.231
153	A	3	2	1.00	13	0.154
154	A	3	3	1.00	13	0.231
155	A	3	2	1.00	13	0.154
156	A	2	2	1.00	13	0.154
157	A	1	1	1.00	11	0.091
158	A	2	2	1.00	9	0.222
159	A	3	2	1.00	13	0.154
160	A	3	3	1.00	13	0.231
161	A	3	2	1.00	13	0.154
162	A	4	3	1.00	13	0.231
163	A	3	2	1.00	13	0.154
164	A	5	3	1.00	13	0.231
165	A	3	2	1.00	13	0.154
166	A	6	3	1.00	13	0.231
167	A	3	2	1.00	13	0.154
168	A	3	2	1.00	13	0.154
169	A	3	2	1.00	13	0.154
170	A	3	2	1.00	13	0.154
171	A	3	2	1.00	13	0.154
172	A	3	2	1.00	13	0.154
173	A	3	2	1.00	13	0.154
174	A	1	1	1.00	13	0.077
175	A	1	1	1.00	11	0.091
176	A	3	2	1.00	13	0.154
177	A	3	2	1.00	13	0.154
178	A	3	2	1.00	13	0.154
179	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	3	2	1.00	13	0.154
181	A	5	3	1.00	13	0.231
182	A	5	3	1.00	13	0.231
183	A	5	3	1.00	13	0.231
184	A	4	3	1.00	13	0.231
185	A	3	2	1.00	13	0.154
186	A	3	3	1.00	13	0.231
187	A	3	2	1.00	9	0.222
188	A	4	3	1.00	13	0.231
189	A	5	3	1.00	13	0.231
190	A	6	3	1.00	13	0.231
191	A	7	3	1.00	13	0.231
192	A	3	2	1.00	13	0.154
193	A	3	2	1.00	13	0.154
194	A	3	2	1.00	13	0.154
195	A	3	2	1.00	13	0.154
196	A	1	1	1.00	13	0.077
197	A	3	3	1.00	13	0.231
198	A	4	3	1.00	13	0.231
199	A	5	3	1.00	13	0.231
200	A	3	2	1.00	13	0.154
201	A	3	2	1.00	13	0.154
202	A	3	2	1.00	13	0.154
203	A	3	2	1.00	13	0.154
204	A	1	1	1.00	11	0.091
205	A	3	2	1.00	13	0.154
206	A	3	2	1.00	13	0.154
207	A	3	2	1.00	13	0.154
208	A	3	2	1.00	13	0.154
209	A	12	3	1.00	13	0.231
210	A	12	3	1.00	13	0.231
211	A	11	3	1.00	13	0.231
212	A	10	2	1.00	13	0.154
213	A	10	3	1.00	13	0.231
214	A	10	3	1.00	13	0.231
215	A	10	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	10	3	1.00	13	0.231
217	A	10	3	1.00	13	0.231
218	A	10	3	1.00	13	0.231
219	A	10	3	1.00	13	0.231
220	A	10	3	1.00	13	0.231
221	A	10	2	1.00	9	0.222
222	A	11	3	1.00	13	0.231
223	A	12	3	1.00	13	0.231
224	A	13	3	1.00	13	0.231
225	A	3	2	1.00	14	0.143
226	A	2	2	1.00	14	0.143
227	A	1	1	1.00	12	0.083
228	A	1	1	1.00	10	0.100
229	A	4	4	1.00	14	0.286
230	A	2	2	1.00	14	0.143
231	A	3	2	1.00	14	0.143
232	A	3	2	1.00	14	0.143
233	A	2	2	1.00	14	0.143
234	A	1	1	1.00	12	0.083
235	A	2	2	1.00	10	0.200
236	A	3	2	1.00	14	0.143
237	A	3	3	1.00	14	0.214
238	A	3	2	1.00	14	0.143
239	A	1	1	1.00	14	0.071
240	A	3	3	1.00	14	0.214
241	A	1	1	1.00	12	0.083
242	A	3	2	1.00	10	0.200
243	A	3	2	1.00	14	0.143
244	A	4	3	1.00	14	0.214
245	A	3	2	1.00	14	0.143
246	A	3	2	1.00	14	0.143
247	A	5	3	1.00	14	0.214
248	A	1	1	1.00	12	0.083
249	A	5	2	1.00	10	0.200
250	A	3	2	1.00	14	0.143
251	A	6	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
252	A	3	2	1.00	14	0.143
253	A	4	4	1.00	13	0.308
254	A	4	4	1.00	13	0.308
255	A	3	2	1.00	13	0.154
256	A	3	2	1.00	13	0.154
257	A	1	1	1.00	10	0.100
258	A	1	1	1.00	18	0.056
259	A	3	2	1.00	13	0.154
260	A	3	2	1.00	13	0.154
261	A	1	1	1.00	14	0.071
262	A	1	1	1.00	15	0.067
263	A	1	1	1.00	20	0.050
264	A	2	1	1.00	13	0.077
265	A	2	1	1.00	13	0.077
266	A	2	1	1.00	13	0.077
267	A	2	1	1.00	13	0.077
268	A	2	1	1.00	13	0.077
269	A	2	1	1.00	13	0.077
270	A	2	1	1.00	13	0.077
271	A	2	1	1.00	13	0.077
272	A	2	1	1.00	15	0.067
273	A	2	1	1.00	15	0.067
274	A	2	1	1.00	15	0.067
275	A	2	1	1.00	15	0.067
276	A	2	1	1.00	15	0.067
277	A	2	1	1.00	15	0.067
278	A	2	1	1.00	15	0.067
279	A	2	1	1.00	15	0.067
280	A	2	1	1.00	15	0.067
281	A	2	1	1.00	15	0.067
282	A	2	1	1.00	15	0.067
283	A	2	1	1.00	15	0.067
284	A	2	1	1.00	15	0.067
285	A	2	1	1.00	15	0.067
286	A	2	1	1.00	15	0.067
287	A	2	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	12	8	1.00	15	0.533
289	A	11	8	1.00	15	0.533
290	A	11	8	1.00	15	0.533
291	A	10	7	1.00	15	0.467
292	A	10	7	1.00	15	0.467
293	A	11	8	1.00	15	0.533
294	A	11	8	1.00	15	0.533
295	A	12	8	1.00	15	0.533
296	A	12	9	1.00	15	0.600
297	A	11	8	1.00	15	0.533
298	A	11	8	1.00	15	0.533
299	A	11	8	1.00	15	0.533
300	A	11	8	1.00	15	0.533
301	A	12	9	1.00	15	0.600
302	A	12	9	1.00	15	0.600
303	A	13	9	1.00	15	0.600
304	A	12	8	1.00	15	0.533
305	A	12	9	1.00	15	0.600
306	A	12	9	1.00	15	0.600
307	A	12	8	1.00	15	0.533
308	A	12	8	1.00	15	0.533
309	A	13	9	1.00	15	0.600
310	A	13	9	1.00	15	0.600
311	A	14	9	1.00	15	0.600
312	A	4	4	1.00	16	0.250
313	A	12	8	1.00	13	0.615
314	A	11	8	1.00	13	0.615
315	A	11	8	1.00	13	0.615
316	A	10	7	1.00	13	0.538
317	A	10	7	1.00	13	0.538
318	A	11	8	1.00	13	0.615
319	A	11	8	1.00	13	0.615
320	A	12	8	1.00	13	0.615
321	A	12	9	1.00	13	0.692
322	A	11	8	1.00	13	0.615
323	A	11	8	1.00	13	0.615

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
324	A	11	8	1.00	13	0.615
325	A	11	8	1.00	13	0.615
326	A	12	9	1.00	13	0.692
327	A	12	9	1.00	13	0.692
328	A	13	9	1.00	13	0.692
329	A	12	8	1.00	13	0.615
330	A	12	9	1.00	13	0.692
331	A	12	9	1.00	13	0.692
332	A	12	8	1.00	13	0.615
333	A	12	8	1.00	13	0.615
334	A	13	9	1.00	13	0.692
335	A	13	9	1.00	13	0.692
336	A	14	9	1.00	13	0.692
337	A	4	4	1.00	15	0.267
338	A	11	7	1.37	13	0.538
339	A	2	1	1.00	13	0.077
340	A	2	1	1.00	13	0.077
341	A	2	1	1.00	13	0.077
342	A	2	1	1.00	13	0.077
343	A	2	1	1.00	11	0.091
344	A	1	1	1.00	13	0.077
345	A	1	1	1.00	13	0.077
346	A	1	1	1.00	13	0.077
347	A	1	1	1.00	17	0.059
348	A	1	1	1.00	15	0.067
349	A	1	1	1.00	17	0.059
350	A	1	1	1.00	17	0.059
351	A	1	1	1.00	17	0.059
352	A	1	1	1.00	16	0.062
353	A	3	2	1.00	15	0.133
354	A	3	2	1.00	15	0.133
355	A	3	2	1.00	15	0.133
356	A	1	1	1.00	13	0.077
357	A	4	4	1.00	15	0.267
358	A	4	4	1.00	15	0.267
359	A	5	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
360	A	6	5	1.00	15	0.333
361	A	5	4	1.00	15	0.267
362	A	4	4	1.00	15	0.267
363	A	3	3	1.00	11	0.273
364	A	3	3	1.00	15	0.200
365	A	1	1	1.00	15	0.067
366	A	2	2	1.00	15	0.133
367	A	3	2	1.00	15	0.133
368	A	4	2	1.00	15	0.133
369	A	3	2	1.00	15	0.133
370	A	3	2	1.00	15	0.133
371	A	3	2	1.00	15	0.133
372	A	1	1	1.00	13	0.077
373	A	5	4	1.00	15	0.267
374	A	5	5	1.00	15	0.333
375	A	5	4	1.00	15	0.267
376	A	6	5	1.00	15	0.333
377	A	7	5	1.00	15	0.333
378	A	6	4	1.00	15	0.267
379	A	5	4	1.00	15	0.267
380	A	4	3	1.00	11	0.273
381	A	4	4	1.00	15	0.267
382	A	4	3	1.00	15	0.200
383	A	1	1	1.00	15	0.067
384	A	2	2	1.00	15	0.133
385	A	3	2	1.00	15	0.133
386	A	4	2	1.00	15	0.133
387	A	3	2	1.00	15	0.133
388	A	3	2	1.00	15	0.133
389	A	3	2	1.00	15	0.133
390	A	1	1	1.00	13	0.077
391	A	6	4	1.00	15	0.267
392	A	6	5	1.00	15	0.333
393	A	6	5	1.00	15	0.333
394	A	6	4	1.00	15	0.267
395	A	7	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	8	5	1.00	15	0.333
397	A	7	4	1.00	15	0.267
398	A	6	4	1.00	15	0.267
399	A	5	3	1.00	11	0.273
400	A	5	4	1.00	15	0.267
401	A	5	4	1.00	15	0.267
402	A	5	3	1.00	15	0.200
403	A	1	1	1.00	15	0.067
404	A	2	2	1.00	15	0.133
405	A	3	2	1.00	15	0.133
406	A	4	2	1.00	15	0.133
407	A	5	2	1.00	15	0.133
408	A	6	2	1.00	15	0.133
409	A	3	2	1.00	15	0.133
410	A	3	2	1.00	15	0.133
411	A	3	2	1.00	15	0.133
412	A	3	2	1.00	15	0.133
413	A	3	2	1.00	15	0.133
414	A	3	2	1.00	15	0.133
415	A	3	2	1.00	15	0.133
416	A	1	1	1.00	13	0.077
417	A	8	4	1.00	15	0.267
418	A	8	5	1.00	15	0.333
419	A	8	5	1.00	15	0.333
420	A	8	5	1.00	15	0.333
421	A	8	5	1.00	15	0.333
422	A	8	4	1.00	15	0.267
423	A	9	5	1.00	15	0.333
424	A	10	5	1.00	15	0.333
425	A	10	4	1.00	15	0.267
426	A	9	4	1.00	15	0.267
427	A	8	4	1.00	15	0.267
428	A	7	3	1.00	11	0.273
429	A	7	4	1.00	15	0.267
430	A	7	4	1.00	15	0.267
431	A	7	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
432	A	7	4	1.00	15	0.267
433	A	7	3	1.00	15	0.200
434	A	1	1	1.00	15	0.067
435	A	2	2	1.00	15	0.133
436	A	3	2	1.00	15	0.133
437	A	4	2	1.00	15	0.133
438	A	5	2	1.00	15	0.133
439	A	6	2	1.00	15	0.133
440	A	7	2	1.00	15	0.133
441	A	3	2	1.00	15	0.133
442	A	4	3	1.00	15	0.200
443	A	3	2	1.00	15	0.133
444	A	3	3	1.00	15	0.200
445	A	1	1	1.00	13	0.077
446	A	2	2	1.00	11	0.182
447	A	4	4	1.00	15	0.267
448	A	2	2	1.00	15	0.133
449	A	4	4	1.00	15	0.267
450	A	1	1	1.00	15	0.067
451	A	5	5	1.00	15	0.333
452	A	3	2	1.00	15	0.133
453	A	4	3	1.00	15	0.200
454	A	3	2	1.00	15	0.133
455	A	3	3	1.00	15	0.200
456	A	1	1	1.00	13	0.077
457	A	2	2	1.00	11	0.182
458	A	4	4	1.00	15	0.267
459	A	2	2	1.00	15	0.133
460	A	4	4	1.00	15	0.267
461	A	1	1	1.00	15	0.067
462	A	5	5	1.00	15	0.333
463	A	3	2	1.00	15	0.133
464	A	5	4	1.00	15	0.267
465	A	3	2	1.00	15	0.133
466	A	4	4	1.00	15	0.267
467	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
468	A	3	3	1.00	11	0.273
469	A	4	4	1.00	15	0.267
470	A	3	3	1.00	15	0.200
471	A	4	4	1.00	15	0.267
472	A	1	1	1.00	15	0.067
473	A	5	5	1.00	15	0.333
474	A	3	2	1.00	15	0.133
475	A	5	4	1.00	15	0.267
476	A	3	2	1.00	15	0.133
477	A	4	4	1.00	15	0.267
478	A	1	1	1.00	13	0.077
479	A	3	3	1.00	11	0.273
480	A	4	4	1.00	15	0.267
481	A	3	3	1.00	15	0.200
482	A	4	4	1.00	15	0.267
483	A	1	1	1.00	15	0.067
484	A	5	5	1.00	15	0.333
485	A	3	2	1.00	15	0.133
486	A	4	3	1.00	15	0.200
487	A	3	2	1.00	15	0.133
488	A	3	3	1.00	15	0.200
489	A	1	1	1.00	13	0.077
490	A	2	2	1.00	11	0.182
491	A	3	3	1.00	15	0.200
492	A	1	1	1.00	15	0.067
493	A	4	4	1.00	15	0.267
494	A	2	2	1.00	15	0.133
495	A	5	4	1.00	15	0.267
496	A	3	2	1.00	15	0.133
497	A	4	4	1.00	15	0.267
498	A	3	2	1.00	15	0.133
499	A	3	3	1.00	15	0.200
500	A	1	1	1.00	13	0.077
501	A	1	1	1.00	11	0.091
502	A	4	4	1.00	15	0.267
503	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
504	A	5	4	0.99	15	0.267
505	A	3	2	1.00	15	0.133
506	A	5	4	1.00	15	0.267
507	A	3	2	1.00	15	0.133
508	A	4	3	1.00	15	0.200
509	A	3	2	1.00	15	0.133
510	A	1	1	1.00	15	0.067
511	A	1	1	1.00	13	0.077
512	A	2	2	1.00	11	0.182
513	A	5	4	1.00	15	0.267
514	A	3	3	1.00	15	0.200
515	A	6	4	1.05	15	0.267
516	A	4	3	1.00	15	0.200
517	A	7	4	1.00	15	0.267
518	A	3	2	1.00	15	0.133
519	A	6	3	1.00	15	0.200
520	A	3	2	1.00	15	0.133
521	A	1	1	1.00	15	0.067
522	A	3	2	1.00	15	0.133
523	A	2	2	1.00	15	0.133
524	A	3	2	1.00	15	0.133
525	A	3	2	1.00	15	0.133
526	A	1	1	1.00	13	0.077
527	A	4	2	1.00	11	0.182
528	A	7	4	1.00	15	0.267
529	A	5	3	1.00	15	0.200
530	A	8	4	1.05	15	0.267
531	A	6	3	1.00	15	0.200
532	A	3	2	1.00	15	0.133
533	A	3	2	1.00	15	0.133
534	A	3	2	1.00	15	0.133
535	A	2	2	1.00	15	0.133
536	A	1	1	1.00	13	0.077
537	A	1	1	1.00	11	0.091
538	A	3	3	1.00	15	0.200
539	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
540	A	4	4	1.00	15	0.267
541	A	2	2	1.00	15	0.133
542	A	5	4	1.00	15	0.267
543	A	3	2	1.00	15	0.133
544	A	3	2	1.00	15	0.133
545	A	3	2	1.00	15	0.133
546	A	2	2	1.00	15	0.133
547	A	1	1	1.00	13	0.077
548	A	1	1	1.00	11	0.091
549	A	3	3	1.00	15	0.200
550	A	1	1	1.00	15	0.067
551	A	4	4	1.00	15	0.267
552	A	2	2	1.00	15	0.133
553	A	5	4	1.00	15	0.267
554	A	3	2	1.00	15	0.133
555	A	4	3	1.00	15	0.200
556	A	3	2	1.00	15	0.133
557	A	3	3	1.00	15	0.200
558	A	1	1	1.00	13	0.077
559	A	2	2	1.00	11	0.182
560	A	3	3	1.00	15	0.200
561	A	1	1	1.00	15	0.067
562	A	4	4	1.00	15	0.267
563	A	2	2	1.00	15	0.133
564	A	5	4	1.00	15	0.267
565	A	3	2	1.00	15	0.133
566	A	4	3	1.00	15	0.200
567	A	3	2	1.00	15	0.133
568	A	3	3	1.00	15	0.200
569	A	1	1	1.00	13	0.077
570	A	2	2	1.00	11	0.182
571	A	3	3	1.00	15	0.200
572	A	1	1	1.00	15	0.067
573	A	4	4	1.00	15	0.267
574	A	2	2	1.00	15	0.133
575	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
576	A	1	1	1.00	11	0.091
577	A	1	1	1.00	12	0.083
578	A	2	2	1.00	11	0.182
579	A	2	2	1.00	12	0.167
580	A	1	1	1.00	11	0.091
581	A	1	1	1.00	12	0.083
582	A	2	2	1.00	13	0.154
583	A	2	2	1.00	14	0.143
584	A	2	2	1.00	11	0.182
585	A	2	2	1.00	12	0.167
586	A	2	2	1.00	13	0.154
587	A	2	2	1.00	14	0.143
588	A	2	2	1.00	13	0.154
589	A	5	4	1.00	19	0.210
590	A	6	6	1.00	19	0.316
591	A	4	4	1.00	19	0.210
592	A	5	5	1.00	19	0.263
593	A	3	3	1.00	19	0.158
594	A	5	5	1.00	19	0.263
595	A	3	3	1.00	19	0.158
596	A	6	6	1.00	19	0.316
597	A	6	4	1.00	19	0.210
598	A	7	6	1.00	19	0.316
599	A	5	4	1.00	19	0.210
600	A	6	5	1.00	19	0.263
601	A	4	3	1.00	19	0.158
602	A	6	6	1.00	19	0.316
603	A	4	4	1.00	19	0.210
604	A	6	5	1.00	19	0.263
605	A	4	3	1.00	19	0.158
606	A	7	6	1.00	19	0.316
607	A	6	6	1.00	22	0.273
608	A	5	5	1.00	22	0.227
609	A	5	5	1.00	22	0.227
610	A	4	4	1.00	22	0.182
611	A	5	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
612	A	4	4	1.00	22	0.182
613	A	4	3	1.00	19	0.158
614	A	5	5	1.00	19	0.263
615	A	3	3	1.00	19	0.158
616	A	4	4	1.00	19	0.210
617	A	2	2	1.00	19	0.105
618	A	5	5	1.00	19	0.263
619	A	3	3	1.00	19	0.158
620	A	6	5	1.00	19	0.263
621	A	4	4	1.00	19	0.210
622	A	5	5	1.00	19	0.263
623	A	3	3	1.00	19	0.158
624	A	5	5	1.00	19	0.263
625	A	3	3	1.00	19	0.158
626	A	6	6	1.00	19	0.316
627	A	4	4	1.00	19	0.210
628	A	7	6	1.00	19	0.316
629	A	4	3	1.00	19	0.158
630	A	6	6	1.00	19	0.316
631	A	4	4	1.00	19	0.210
632	A	6	5	1.00	19	0.263
633	A	4	3	1.00	19	0.158
634	A	7	6	1.00	19	0.316
635	A	5	4	1.00	19	0.210
636	A	8	6	1.00	19	0.316
637	A	5	5	1.00	22	0.227
638	A	4	4	1.00	22	0.182
639	A	4	4	1.00	22	0.182
640	A	3	3	1.00	22	0.136
641	A	5	5	1.00	22	0.227
642	A	4	4	1.00	22	0.182
643	A	5	5	1.00	22	0.227
644	A	4	4	1.00	22	0.182
645	A	5	5	1.00	22	0.227
646	A	4	4	1.00	22	0.182
647	A	6	6	1.00	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
648	A	5	5	1.00	22	0.227
649	A	2	2	1.00	20	0.100
650	A	2	2	1.00	17	0.118
651	A	2	2	1.28	15	0.133
652	A	2	2	1.26	15	0.133
653	A	2	2	1.26	15	0.133
654	A	2	2	1.38	15	0.133
655	A	2	2	1.32	15	0.133
656	A	2	2	1.26	17	0.118
657	A	2	2	1.26	17	0.118
658	A	2	2	1.26	15	0.133
659	A	2	2	1.24	17	0.118
660	A	2	2	1.29	17	0.118
661	A	1	1	1.00	31	0.032
662	C	5	2	7.47	43	0.047
663	A	1	1	1.00	29	0.034
664	C	5	2	8.20	38	0.053
665	A	3	2	1.00	15	0.133
666	A	3	2	1.00	15	0.133
667	A	3	2	1.00	15	0.133
668	A	1	1	1.00	13	0.077
669	A	6	6	1.00	15	0.400
670	A	6	6	1.00	15	0.400
671	A	7	7	1.00	15	0.467
672	A	5	4	1.00	15	0.267
673	A	4	4	1.00	15	0.267
674	A	3	3	1.00	11	0.273
675	A	3	3	1.00	15	0.200
676	A	4	4	1.00	15	0.267
677	A	3	2	1.00	15	0.133
678	A	3	2	1.00	15	0.133
679	A	3	2	1.00	15	0.133
680	A	1	1	1.00	13	0.077
681	A	6	6	1.00	15	0.400
682	A	6	6	1.00	15	0.400
683	A	7	7	1.00	15	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
684	A	7	6	1.00	15	0.400
685	A	6	6	1.00	15	0.400
686	A	5	5	1.00	11	0.454
687	A	5	5	1.00	15	0.333
688	A	6	6	1.00	15	0.400
689	A	3	2	1.00	15	0.133
690	A	3	2	1.00	15	0.133
691	A	3	2	1.00	15	0.133
692	A	1	1	1.00	13	0.077
693	A	7	6	1.00	15	0.400
694	A	7	7	1.00	15	0.467
695	A	7	6	1.00	15	0.400
696	A	6	4	1.00	15	0.267
697	A	5	4	1.00	15	0.267
698	A	4	3	1.00	11	0.273
699	A	4	4	1.00	15	0.267
700	A	4	3	1.00	15	0.200
701	A	1	1	1.00	11	0.091
702	A	3	2	1.00	15	0.133
703	A	3	2	1.00	15	0.133
704	A	3	2	1.00	15	0.133
705	A	1	1	1.00	13	0.077
706	A	5	5	1.00	15	0.333
707	A	6	6	1.00	15	0.400
708	A	7	6	1.00	15	0.400
709	A	6	5	1.00	15	0.333
710	A	5	5	1.00	15	0.333
711	A	4	4	1.00	11	0.364
712	A	5	5	1.00	15	0.333
713	A	6	5	1.00	15	0.333
714	A	3	2	1.00	15	0.133
715	A	3	2	1.00	15	0.133
716	A	3	2	1.00	15	0.133
717	A	1	1	1.00	13	0.077
718	A	5	5	1.00	15	0.333
719	A	6	6	1.00	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
720	A	7	6	1.00	15	0.400
721	A	4	3	1.00	15	0.200
722	A	3	3	1.00	15	0.200
723	A	2	2	1.00	11	0.182
724	A	3	3	1.00	15	0.200
725	A	4	3	1.00	15	0.200
726	A	3	2	1.00	15	0.133
727	A	3	2	1.00	15	0.133
728	A	3	2	1.00	15	0.133
729	A	1	1	1.00	13	0.077
730	A	6	6	1.00	15	0.400
731	A	7	6	1.02	15	0.400
732	A	8	6	1.00	15	0.400
733	A	6	6	1.00	15	0.400
734	A	5	5	1.00	15	0.333
735	A	5	5	1.00	11	0.454
736	A	6	6	1.00	15	0.400
737	A	7	6	1.00	15	0.400
738	A	12	11	1.41	19	0.579
739	A	11	11	1.49	19	0.579
740	A	10	10	1.59	19	0.526
741	A	10	10	1.59	19	0.526
742	A	1	1	1.00	19	0.053
743	A	2	2	1.00	19	0.105
744	A	3	2	1.00	19	0.105
745	A	4	2	1.00	19	0.105
746	A	6	5	1.00	19	0.263
747	A	5	5	1.00	19	0.263
748	A	4	4	1.00	19	0.210
749	A	4	4	1.00	19	0.210
750	A	5	5	1.00	19	0.263
751	A	2	2	1.00	19	0.105
752	A	2	2	1.00	19	0.105
753	A	2	2	1.00	19	0.105
754	A	13	11	1.36	19	0.579
755	A	12	11	1.42	19	0.579

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
756	A	11	10	1.49	19	0.526
757	A	11	11	1.52	19	0.579
758	A	11	10	1.49	19	0.526
759	A	1	1	1.00	19	0.053
760	A	2	2	1.00	19	0.105
761	A	3	2	1.00	19	0.105
762	A	7	5	1.00	19	0.263
763	A	6	5	1.00	19	0.263
764	A	5	4	1.00	19	0.210
765	A	5	5	1.00	19	0.263
766	A	5	4	1.00	19	0.210
767	A	6	5	1.00	19	0.263
768	A	2	2	1.00	19	0.105
769	A	2	2	1.00	19	0.105
770	A	2	2	1.00	19	0.105
771	A	12	10	1.40	19	0.526
772	A	11	10	1.48	19	0.526
773	A	10	10	1.60	19	0.526
774	A	9	9	1.73	19	0.474
775	A	1	1	1.00	19	0.053
776	A	2	2	1.00	19	0.105
777	A	3	2	1.00	19	0.105
778	A	4	2	1.00	19	0.105
779	A	5	4	1.00	19	0.210
780	A	4	4	1.00	19	0.210
781	A	3	3	1.00	19	0.158
782	A	4	4	1.00	19	0.210
783	A	5	4	1.00	19	0.210
784	A	2	2	1.00	19	0.105
785	A	2	2	1.00	19	0.105
786	A	2	2	1.00	19	0.105
787	A	5	4	1.00	15	0.267
788	A	4	4	1.00	15	0.267
789	A	3	3	1.00	11	0.273
790	A	3	3	1.00	15	0.200
791	A	4	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
792	A	5	4	1.00	15	0.267
793	A	5	4	1.00	16	0.250
794	A	4	4	1.00	16	0.250
795	A	3	3	1.00	12	0.250
796	A	3	3	1.00	16	0.188
797	A	4	4	1.00	16	0.250
798	A	5	4	1.00	16	0.250
799	A	6	5	1.00	15	0.333
800	A	5	5	1.00	15	0.333
801	A	4	4	1.00	11	0.364
802	A	4	4	1.00	15	0.267
803	A	5	5	1.00	15	0.333
804	A	6	5	1.00	15	0.333
805	A	5	4	1.00	16	0.250
806	A	4	4	1.00	16	0.250
807	A	3	3	1.00	12	0.250
808	A	3	3	1.00	16	0.188
809	A	4	4	1.00	16	0.250
810	A	5	4	1.00	16	0.250
811	A	4	3	1.00	11	0.273
812	A	4	3	1.00	12	0.250
813	A	5	4	1.00	11	0.364
814	A	4	3	1.00	12	0.250
815	A	6	4	1.00	15	0.267
816	A	5	4	1.00	15	0.267
817	A	4	4	1.00	15	0.267
818	A	3	3	1.00	11	0.273
819	A	4	4	1.00	15	0.267
820	A	5	4	1.00	15	0.267
821	A	6	4	1.00	15	0.267
822	A	5	3	1.00	16	0.188
823	A	4	3	1.00	16	0.188
824	A	3	3	1.00	16	0.188
825	A	2	2	1.00	12	0.167
826	A	3	3	1.00	16	0.188
827	A	4	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
828	A	5	3	1.00	16	0.188
829	A	5	3	1.00	15	0.200
830	A	4	3	1.00	15	0.200
831	A	3	3	1.00	15	0.200
832	A	2	2	1.00	11	0.182
833	A	3	3	1.00	15	0.200
834	A	4	3	1.00	15	0.200
835	A	5	3	1.00	15	0.200
836	A	5	3	1.00	16	0.188
837	A	4	3	1.00	16	0.188
838	A	3	3	1.00	16	0.188
839	A	2	2	1.00	12	0.167
840	A	3	3	1.00	16	0.188
841	A	4	3	1.00	16	0.188
842	A	5	3	1.00	16	0.188
843	A	5	3	1.00	15	0.200
844	A	4	3	1.00	15	0.200
845	A	3	3	1.00	15	0.200
846	A	2	2	1.00	11	0.182
847	A	3	3	1.00	15	0.200
848	A	4	3	1.00	15	0.200
849	A	5	3	1.00	15	0.200
850	A	5	4	1.00	16	0.250
851	A	4	4	1.00	16	0.250
852	A	3	3	1.00	16	0.188
853	A	3	3	1.00	12	0.250
854	A	4	4	1.00	16	0.250
855	A	5	4	1.00	16	0.250
856	A	6	4	1.00	16	0.250
857	A	3	3	1.00	11	0.273
858	A	3	3	1.00	11	0.273
859	A	4	3	1.00	11	0.273
860	A	3	3	1.00	12	0.250
861	A	4	3	1.00	12	0.250
862	A	4	3	1.00	12	0.250
863	A	5	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
864	A	4	3	1.00	15	0.200
865	A	3	3	1.00	15	0.200
866	A	2	2	1.00	11	0.182
867	A	3	3	1.00	15	0.200
868	A	4	3	1.00	15	0.200
869	A	5	3	1.00	15	0.200
870	A	4	2	1.00	15	0.133
871	A	3	2	1.00	15	0.133
872	A	2	2	1.00	15	0.133
873	A	1	1	1.00	11	0.091
874	A	2	2	1.00	15	0.133
875	A	3	2	1.00	15	0.133
876	A	4	2	1.00	15	0.133
877	A	4	2	1.00	15	0.133
878	A	3	2	1.00	15	0.133
879	A	2	2	1.00	15	0.133
880	A	1	1	1.00	11	0.091
881	A	2	2	1.00	15	0.133
882	A	3	2	1.00	15	0.133
883	A	4	2	1.00	15	0.133
884	A	4	2	1.00	15	0.133
885	A	3	2	1.00	15	0.133
886	A	2	2	1.00	15	0.133
887	A	1	1	1.00	11	0.091
888	A	2	2	1.00	15	0.133
889	A	3	2	1.00	15	0.133
890	A	4	2	1.00	15	0.133
891	A	7	5	1.00	15	0.333
892	A	6	5	1.00	15	0.333
893	A	5	5	1.00	15	0.333
894	A	4	4	1.00	11	0.364
895	A	5	5	1.00	15	0.333
896	A	6	5	1.00	15	0.333
897	A	7	5	1.00	15	0.333
898	A	7	5	1.00	15	0.333
899	A	6	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
900	A	5	5	1.00	15	0.333
901	A	4	4	1.00	11	0.364
902	A	5	5	1.00	15	0.333
903	A	6	5	1.00	15	0.333
904	A	7	5	1.00	15	0.333
905	A	5	3	1.00	15	0.200
906	A	4	3	1.00	15	0.200
907	A	3	3	1.00	15	0.200
908	A	2	2	1.00	11	0.182
909	A	3	3	1.00	15	0.200
910	A	4	3	1.00	15	0.200
911	A	5	3	1.00	15	0.200
912	A	5	3	1.00	15	0.200
913	A	4	3	1.00	15	0.200
914	A	3	3	1.00	15	0.200
915	A	2	2	1.00	11	0.182
916	A	3	3	1.00	15	0.200
917	A	4	3	1.00	15	0.200
918	A	5	3	1.00	15	0.200
919	A	8	7	1.00	19	0.368
920	A	7	7	1.00	19	0.368
921	A	6	6	1.00	19	0.316
922	A	6	6	1.00	19	0.316
923	A	7	7	1.00	19	0.368
924	A	8	7	1.00	19	0.368
925	A	7	7	1.00	19	0.368
926	A	6	6	1.00	19	0.316
927	A	6	6	1.00	19	0.316
928	A	1	1	1.00	19	0.053
929	A	2	2	1.00	19	0.105
930	A	3	2	1.00	19	0.105
931	A	4	2	1.00	19	0.105
932	A	7	7	1.00	20	0.350
933	A	6	6	1.00	20	0.300
934	A	6	6	1.00	20	0.300
935	A	7	7	1.00	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
936	A	8	7	1.00	20	0.350
937	A	13	10	1.00	20	0.500
938	A	12	9	1.00	20	0.450
939	A	12	9	1.00	20	0.450
940	A	1	1	1.00	20	0.050
941	A	2	2	1.00	20	0.100
942	A	3	2	1.00	20	0.100
943	A	4	2	1.00	20	0.100
944	A	6	6	1.00	19	0.316
945	A	5	5	1.00	19	0.263
946	A	1	1	1.00	19	0.053
947	A	2	2	1.00	19	0.105
948	A	3	2	1.00	19	0.105
949	A	6	5	1.00	19	0.263
950	A	5	5	1.00	19	0.263
951	A	4	4	1.00	19	0.210
952	A	4	4	1.00	19	0.210
953	A	5	5	1.00	19	0.263
954	A	6	5	1.00	19	0.263
955	A	12	9	1.00	20	0.450
956	A	11	8	1.00	20	0.400
957	A	1	1	1.00	20	0.050
958	A	2	2	1.00	20	0.100
959	A	3	2	1.00	20	0.100
960	A	5	5	1.00	20	0.250
961	A	4	4	1.00	20	0.200
962	A	3	3	1.00	20	0.150
963	A	4	4	1.00	20	0.200
964	A	5	4	1.00	20	0.200
965	A	6	6	1.00	19	0.316
966	A	5	5	1.00	19	0.263
967	A	6	6	1.00	19	0.316
968	A	7	6	1.00	19	0.316
969	A	8	6	1.00	19	0.316
970	A	6	6	1.00	19	0.316
971	A	5	5	1.00	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
972	A	1	1	1.00	19	0.053
973	A	2	2	1.00	19	0.105
974	A	3	2	1.00	19	0.105
975	A	6	6	1.00	20	0.300
976	A	5	5	1.00	20	0.250
977	A	6	6	1.00	20	0.300
978	A	7	6	1.00	20	0.300
979	A	8	6	1.00	20	0.300
980	A	12	9	1.00	20	0.450
981	A	11	8	1.00	20	0.400
982	A	1	1	1.00	20	0.050
983	A	2	2	1.00	20	0.100
984	A	3	2	1.00	20	0.100
985	A	7	7	1.00	19	0.368
986	A	6	6	1.00	19	0.316
987	A	1	1	1.00	19	0.053
988	A	2	2	1.00	19	0.105
989	A	3	2	1.00	19	0.105
990	A	4	2	1.00	19	0.105
991	A	6	4	1.00	19	0.210
992	A	5	4	1.00	19	0.210
993	A	4	4	1.00	19	0.210
994	A	3	3	1.00	19	0.158
995	A	4	4	1.00	19	0.210
996	A	5	4	1.00	19	0.210
997	A	6	4	1.00	19	0.210
998	A	2	2	1.00	19	0.105
999	A	2	2	1.00	19	0.105
1000	A	2	2	1.00	19	0.105
1001	A	2	2	1.00	19	0.105
1002	A	2	2	1.00	19	0.105
1003	A	2	2	1.00	19	0.105
1004	A	2	2	1.00	19	0.105
1005	A	2	2	1.00	19	0.105
1006	A	2	2	1.00	19	0.105
1007	A	2	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1008	A	2	2	1.00	19	0.105
1009	A	2	2	1.00	19	0.105
1010	A	7	5	1.00	15	0.333
1011	A	6	5	1.00	15	0.333
1012	A	5	5	1.00	15	0.333
1013	A	4	4	1.00	11	0.364
1014	A	4	4	1.00	15	0.267
1015	A	5	5	1.00	15	0.333
1016	A	6	5	1.00	15	0.333
1017	A	7	5	1.00	15	0.333
1018	A	9	7	1.00	15	0.467
1019	A	8	7	1.00	15	0.467
1020	A	7	7	1.00	15	0.467
1021	A	6	6	1.00	11	0.546
1022	A	7	7	1.00	15	0.467
1023	A	8	7	1.00	15	0.467
1024	A	9	7	1.00	15	0.467
1025	A	6	4	1.00	15	0.267
1026	A	5	4	1.00	15	0.267
1027	A	4	4	1.00	15	0.267
1028	A	3	3	1.00	11	0.273
1029	A	4	4	1.00	15	0.267
1030	A	5	4	1.00	15	0.267
1031	A	6	4	1.00	15	0.267
1032	A	9	8	1.00	15	0.533
1033	A	8	8	1.00	15	0.533
1034	A	7	7	1.00	15	0.467
1035	A	5	5	1.00	11	0.454
1036	A	8	8	1.00	15	0.533
1037	A	9	8	1.00	15	0.533
1038	A	10	8	1.00	15	0.533
1039	A	3	2	1.00	13	0.154
1040	A	3	2	1.00	13	0.154
1041	A	3	2	1.00	13	0.154
1042	A	1	1	1.00	11	0.091
1043	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1044	A	2	2	1.00	13	0.154
1045	A	2	2	1.22	13	0.154
1046	A	2	2	1.22	13	0.154
1047	A	2	2	1.22	13	0.154
1048	A	2	2	1.26	9	0.222
1049	A	2	2	1.24	13	0.154
1050	A	2	2	1.21	15	0.133
1051	A	2	2	1.21	15	0.133
1052	A	2	2	1.21	15	0.133
1053	A	2	2	1.21	15	0.133
1054	A	2	2	1.22	15	0.133
1055	A	2	2	1.22	15	0.133
1056	A	2	2	1.21	15	0.133
1057	A	2	2	1.21	15	0.133
1058	A	2	2	1.15	13	0.154
1059	A	2	2	1.00	15	0.133
1060	A	2	2	1.32	17	0.118
1061	A	3	2	1.00	17	0.118
1062	A	2	2	1.32	17	0.118
1063	A	2	2	1.00	17	0.118
1064	A	2	2	1.32	17	0.118
1065	A	1	1	1.00	17	0.059
1066	A	2	2	1.32	17	0.118
1067	A	2	2	1.30	17	0.118
1068	A	2	2	1.33	15	0.133
1069	A	2	2	1.31	17	0.118
1070	A	2	2	1.33	17	0.118
1071	A	2	2	1.31	17	0.118

Chapter 3

Listing of integrals

3.1 $\int x^4 \left(a+bx^2\right) dx$

Optimal. Leaf size=17

$$\frac{ax^5}{5}+\frac{bx^7}{7}$$

```
[Out] 1/5*a*x^5+1/7*b*x^7
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^5}{5}+\frac{bx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2),x]
```

```
[Out] (a*x^5)/5 + (b*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a+bx^2\right) dx &= \int \left(ax^4+bx^6\right) dx\\ &= \frac{ax^5}{5}+\frac{bx^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^5}{5}+\frac{bx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2),x]
```

```
[Out] (a*x^5)/5 + (b*x^7)/7
```

fricas [A] time = 0.54, size = 13, normalized size = 0.76

$$\frac{1}{7}x^7b + \frac{1}{5}x^5a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b + 1/5*x^5*a
```

giac [A] time = 1.09, size = 13, normalized size = 0.76

$$\frac{1}{7}\,bx^7 + \frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{7}b\,x^7 + \frac{1}{5}a\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a),x)
```

```
[Out] 1/5*a*x^5+1/7*b*x^7
```

maxima [A] time = 1.31, size = 13, normalized size = 0.76

$$\frac{1}{7}\,bx^7 + \frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$\frac{b\,x^7}{7} + \frac{a\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2),x)
```

```
[Out] (a*x^5)/5 + (b*x^7)/7
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a),x)
```

```
[Out] a*x**5/5 + b*x**7/7
```

3.2 $\int x^3 \left(a+bx^2\right) dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

```
[Out] 1/4*a*x^4+1/6*b*x^6
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)dx&=\int\left(ax^3+bx^5\right)dx\\&=\frac{ax^4}{4}+\frac{bx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

fricas [A] time = 0.71, size = 13, normalized size = 0.76

$$\frac{1}{6}x^6b+\frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b + 1/4*x^4*a
```

giac [A] time = 0.92, size = 13, normalized size = 0.76

$$\frac{1}{6}bx^6+\frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{6}b\,x^{6}+\frac{1}{4}a\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a),x)
```

```
[Out] 1/4*a*x^4+1/6*b*x^6
```

maxima [A] time = 1.35, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^{6}+\frac{1}{4}\,ax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^{6}}{6}+\frac{a\,x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2),x)
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^{4}}{4}+\frac{bx^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a),x)
```

```
[Out] a*x**4/4 + b*x**6/6
```

3.3 $\int x^2 \left(a + bx^2\right) dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

```
[Out] 1/3*a*x^3+1/5*b*x^5
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(a + bx^2\right) dx &= \int \left(ax^2 + bx^4\right) dx \\ &= \frac{ax^3}{3} + \frac{bx^5}{5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

fricas [A] time = 0.74, size = 13, normalized size = 0.76

$$\frac{1}{5}x^5b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b + 1/3*x^3*a
```

giac [A] time = 1.11, size = 13, normalized size = 0.76

$$\frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{5} b\, x^{5}+\frac{1}{3} a\, x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a),x)
```

```
[Out] 1/3*a*x^3+1/5*b*x^5
```

maxima [A] time = 1.34, size = 13, normalized size = 0.76

$$\frac{1}{5}\, b x^{5} + \frac{1}{3}\, a x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\, x^5}{5}+\frac{a\, x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2),x)
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^{3}}{3} + \frac{bx^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a),x)
```

```
[Out] a*x**3/3 + b*x**5/5
```

3.4 $\int x\left(a+bx^2\right)\,dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

```
[Out] 1/2*a*x^2+1/4*b*x^4
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {14}

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2),x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)\,dx &= \int \left(ax+bx^3\right)\,dx\\ &= \frac{ax^2}{2}+\frac{bx^4}{4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2),x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

fricas [A] time = 0.71, size = 13, normalized size = 0.76

$$\frac{1}{4}x^4b+\frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b + 1/2*x^2*a
```

giac [A] time = 1.03, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a),x, algorithm="giac")
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{4}b\,x^{4}+\frac{1}{2}a\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a),x)
[Out] 1/2*a*x^2+1/4*b*x^4
```

maxima [A] time = 1.34, size = 14, normalized size = 0.82

$$\frac{\left(bx^{2}+a\right)^{2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a),x, algorithm="maxima")
[Out] 1/4*(b*x^2 + a)^2/b
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^{4}}{4}+\frac{a\,x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2),x)
[Out] (a*x^2)/2 + (b*x^4)/4
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^{2}}{2}+\frac{bx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a),x)
[Out] a*x**2/2 + b*x**4/4
```

3.5 $\int (a + bx^2)\, dx$

Optimal. Leaf size=12

$$ax + \frac{bx^3}{3}$$

```
[Out] a*x+1/3*b*x^3
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x^2,x]
[Out] a*x + (b*x^3)/3
```

Rubi steps

$$\int \left(a + bx^2\right) dx = ax + \frac{bx^3}{3}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x^2,x]
[Out] a*x + (b*x^3)/3
```

fricas [A] time = 0.82, size = 10, normalized size = 0.83

$$\frac{1}{3}x^3b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^2+a,x, algorithm="fricas")
[Out] 1/3*x^3*b + x*a
```

giac [A] time = 0.94, size = 10, normalized size = 0.83

$$\frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^2+a,x, algorithm="giac")
[Out] 1/3*b*x^3 + a*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{3}b\,x^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^2+a,x)
```

```
[Out] a*x+1/3*b*x^3
```

maxima [A] time = 1.34, size = 10, normalized size = 0.83

$$\frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^2+a,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + a*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^3}{3} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x^2,x)
```

```
[Out] a*x + (b*x^3)/3
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$ax + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**2+a,x)
```

```
[Out] a*x + b*x**3/3
```

3.6 $\int \frac{a+bx^2}{x}\,dx$

Optimal. Leaf size=13

$$a\log(x) + \frac{bx^2}{2}$$

[Out] 1/2*b*x^2+a*ln(x)

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{14\}$

$$a\log(x) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)/x,x]

[Out] (b*x^2)/2 + a*Log[x]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x}\,dx &= \int \left(\frac{a}{x}+bx\right) dx\\ &= \frac{bx^2}{2}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$a\log(x) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)/x,x]

[Out] (b*x^2)/2 + a*Log[x]

fricas [A] time = 0.80, size = 11, normalized size = 0.85

$$\frac{1}{2}\,bx^2 + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/x,x, algorithm="fricas")

[Out] 1/2*b*x^2 + a*log(x)

giac [A] time = 1.02, size = 14, normalized size = 1.08

$$\frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{b\,x^{2}}{2}+a\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x,x)
```

```
[Out] 1/2*b*x^2+a*ln(x)
```

maxima [A] time = 1.35, size = 14, normalized size = 1.08

$$\frac{1}{2}\,bx^{2}+\frac{1}{2}\,a\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.85

$$\frac{b\,x^{2}}{2}+a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x,x)
```

```
[Out] (b*x^2)/2 + a*log(x)
```

sympy [A] time = 0.08, size = 10, normalized size = 0.77

$$a\log{(x)}+\frac{bx^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x,x)
```

```
[Out] a*log(x) + b*x**2/2
```

3.7 $\int \frac{a+bx^2}{x^2} dx$

Optimal. Leaf size=10

$$bx - \frac{a}{x}$$

```
[Out] -a/x+b*x
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$bx - \frac{a}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^2,x]
```

```
[Out] -(a/x) + b*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x^2} dx &= \int \left(b+\frac{a}{x^2}\right) dx \\ &= -\frac{a}{x}+bx \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$bx - \frac{a}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^2,x]
```

```
[Out] -(a/x) + b*x
```

fricas [A] time = 0.56, size = 13, normalized size = 1.30

$$\frac{bx^2 - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] (b*x^2 - a)/x
```

giac [A] time = 0.98, size = 10, normalized size = 1.00

$$bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] b*x - a/x
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2,x)
```

```
[Out] -a/x+b*x
```

maxima [A] time = 1.33, size = 10, normalized size = 1.00

$$bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] b*x - a/x
```

mupad [B] time = 0.02, size = 10, normalized size = 1.00

$$b\,x - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^2,x)
```

```
[Out] b*x - a/x
```

sympy [A] time = 0.08, size = 5, normalized size = 0.50

$$-\frac{a}{x} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2,x)
```

```
[Out] -a/x + b*x
```

3.8 $\int \frac{a+bx^2}{x^3}\,dx$

Optimal. Leaf size=13

$$b\log(x)-\frac{a}{2x^2}$$

```
[Out] -1/2*a/x^2+b*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$b\log(x)-\frac{a}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^3,x]
```

```
[Out] -a/(2*x^2) + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^3}\,dx &= \int \left(\frac{a}{x^3}+\frac{b}{x}\right)dx\\ &= -\frac{a}{2x^2}+b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$b\log(x)-\frac{a}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^3,x]
```

```
[Out] -1/2*a/x^2 + b*Log[x]
```

fricas [A] time = 0.85, size = 17, normalized size = 1.31

$$\frac{2\,bx^2\log(x)-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^2*log(x) - a)/x^2
```

giac [A] time = 1.06, size = 20, normalized size = 1.54

$$\frac{1}{2}\,b\log\left(x^2\right)-\frac{bx^2+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*log(x^2) - 1/2*(b*x^2 + a)/x^2
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$b\ln(x) - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3,x)
```

```
[Out] -1/2*a/x^2+b*ln(x)
```

maxima [A] time = 1.36, size = 14, normalized size = 1.08

$$\frac{1}{2}\,b\log\left(x^2\right) - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*log(x^2) - 1/2*a/x^2
```

mupad [B] time = 4.93, size = 11, normalized size = 0.85

$$b\,\ln(x) - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^3,x)
```

```
[Out] b*log(x) - a/(2*x^2)
```

sympy [A] time = 0.11, size = 10, normalized size = 0.77

$$-\frac{a}{2x^2} + b\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*log(x)
```

3.9 $\int \frac{a+bx^2}{x^4} \, dx$

Optimal. Leaf size=15

$$-\frac{a}{3x^3}-\frac{b}{x}$$

[Out] -1/3*a/x^3-b/x

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$-\frac{a}{3x^3}-\frac{b}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)/x^4,x]

[Out] -a/(3*x^3) - b/x

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^4} \, dx &= \int \left(\frac{a}{x^4}+\frac{b}{x^2}\right) \, dx\\ &= -\frac{a}{3x^3}-\frac{b}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{a}{3x^3}-\frac{b}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)/x^4,x]

[Out] -1/3*a/x^3 - b/x

fricas [A] time = 0.63, size = 13, normalized size = 0.87

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/x^4,x, algorithm="fricas")

[Out] -1/3*(3*b*x^2 + a)/x^3

giac [A] time = 1.11, size = 13, normalized size = 0.87

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b*x^2 + a)/x^3
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$-\frac{b}{x}-\frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4,x)
```

```
[Out] -1/3*a/x^3-b/x
```

maxima [A] time = 1.29, size = 13, normalized size = 0.87

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b*x^2 + a)/x^3
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$-\frac{3\,b\,x^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^4,x)
```

```
[Out] -(a + 3*b*x^2)/(3*x^3)
```

sympy [A] time = 0.11, size = 14, normalized size = 0.93

$$\frac{-a-3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4,x)
```

```
[Out] (-a - 3*b*x**2)/(3*x**3)
```

3.10 $\int \frac{a+bx^2}{x^5}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{4x^4}-\frac{b}{2x^2}$$

```
[Out] -1/4*a/x^4-1/2*b/x^2
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$-\frac{a}{4x^4}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^5,x]
```

```
[Out] -a/(4*x^4) - b/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^5}\,dx &= \int \left(\frac{a}{x^5}+\frac{b}{x^3}\right)dx\\ &= -\frac{a}{4x^4}-\frac{b}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{4x^4}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(2*x^2)
```

fricas [A] time = 0.82, size = 13, normalized size = 0.76

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^2 + a)/x^4
```

giac [A] time = 0.99, size = 13, normalized size = 0.76

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5,x, algorithm="giac")
[Out] -1/4*(2*b*x^2 + a)/x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{b}{2x^2}-\frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^5,x)
[Out] -1/4*a/x^4-1/2*b/x^2
```

maxima [A] time = 1.33, size = 13, normalized size = 0.76

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5,x, algorithm="maxima")
[Out] -1/4*(2*b*x^2 + a)/x^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$-\frac{2\,b\,x^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^5,x)
[Out] -(a + 2*b*x^2)/(4*x^4)
```

sympy [A] time = 0.12, size = 14, normalized size = 0.82

$$\frac{-a-2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5,x)
[Out] (-a - 2*b*x**2)/(4*x**4)
```

3.11 $\int \frac{a+bx^2}{x^6} dx$

Optimal. Leaf size=17

$$-\frac{a}{5x^5}-\frac{b}{3x^3}$$

```
[Out] -1/5*a/x^5-1/3*b/x^3
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$-\frac{a}{5x^5}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^6,x]
```

```
[Out] -a/(5*x^5) - b/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^6} dx &= \int \left(\frac{a}{x^6}+\frac{b}{x^4}\right) dx\\ &= -\frac{a}{5x^5}-\frac{b}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{5x^5}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^6,x]
```

```
[Out] -1/5*a/x^5 - b/(3*x^3)
```

fricas [A] time = 0.91, size = 15, normalized size = 0.88

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

giac [A] time = 1.09, size = 15, normalized size = 0.88

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$-\frac{b}{3x^3}-\frac{a}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^6,x)
```

```
[Out] -1/5*a/x^5-1/3*b/x^3
```

maxima [A] time = 1.30, size = 15, normalized size = 0.88

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$-\frac{5\,b\,x^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^6,x)
```

```
[Out] -(3*a + 5*b*x^2)/(15*x^5)
```

sympy [A] time = 0.13, size = 15, normalized size = 0.88

$$\frac{-3a-5bx^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**6,x)
```

```
[Out] (-3*a - 5*b*x**2)/(15*x**5)
```

3.12 $\int \frac{a+bx^2}{x^7} dx$

Optimal. Leaf size=17

$$-\frac{a}{6x^6} - \frac{b}{4x^4}$$

[Out] -1/6*a/x^6-1/4*b/x^4

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$-\frac{a}{6x^6} - \frac{b}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)/x^7,x]

[Out] -a/(6*x^6) - b/(4*x^4)

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^7} dx &= \int \left(\frac{a}{x^7} + \frac{b}{x^5}\right) dx\\ &= -\frac{a}{6x^6} - \frac{b}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{6x^6} - \frac{b}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)/x^7,x]

[Out] -1/6*a/x^6 - b/(4*x^4)

fricas [A] time = 0.83, size = 15, normalized size = 0.88

$$-\frac{3\,bx^2 + 2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/x^7,x, algorithm="fricas")

[Out] -1/12*(3*b*x^2 + 2*a)/x^6

giac [A] time = 1.03, size = 15, normalized size = 0.88

$$-\frac{3\,bx^2 + 2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$-\frac{b}{4x^4}-\frac{a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^7,x)
```

```
[Out] -1/6*a/x^6-1/4*b/x^4
```

maxima [A] time = 1.42, size = 15, normalized size = 0.88

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$-\frac{3\,b\,x^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^7,x)
```

```
[Out] -(2*a + 3*b*x^2)/(12*x^6)
```

sympy [A] time = 0.14, size = 15, normalized size = 0.88

$$\frac{-2a-3bx^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**7,x)
```

```
[Out] (-2*a - 3*b*x**2)/(12*x**6)
```

3.13 $\int x^5 \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

```
[Out] 1/6*a^2*x^6+1/4*a*b*x^8+1/10*b^2*x^10
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^2,x]
[Out] (a^2*x^6)/6 + (a*b*x^8)/4 + (b^2*x^10)/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^2\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(a^2x^2+2abx^3+b^2x^4\right)\,dx,x,x^2\right)\\ &= \frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^2,x]
[Out] (a^2*x^6)/6 + (a*b*x^8)/4 + (b^2*x^10)/10
```

fricas [A] time = 0.61, size = 24, normalized size = 0.80

$$\frac{1}{10}x^{10}b^2+\frac{1}{4}x^8ba+\frac{1}{6}x^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^2 + 1/4*x^8*b*a + 1/6*x^6*a^2
```

giac [A] time = 1.07, size = 24, normalized size = 0.80

$$\frac{1}{10}\,b^2x^{10}+\frac{1}{4}\,abx^8+\frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{10}b^2x^{10}+\frac{1}{4}ab\,x^8+\frac{1}{6}a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^2,x)
```

```
[Out] 1/6*a^2*x^6+1/4*a*b*x^8+1/10*b^2*x^10
```

maxima [A] time = 1.34, size = 24, normalized size = 0.80

$$\frac{1}{10}\,b^2x^{10}+\frac{1}{4}\,abx^8+\frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^6}{6}+\frac{a\,b\,x^8}{4}+\frac{b^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^6)/6 + (b^2*x^10)/10 + (a*b*x^8)/4
```

sympy [A] time = 0.06, size = 24, normalized size = 0.80

$$\frac{a^2x^6}{6}+\frac{abx^8}{4}+\frac{b^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**6/6 + a*b*x**8/4 + b**2*x**10/10
```

3.14 $\int x^4 \left(a+bx^2\right)^2 dx$

Optimal. Leaf size=30

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^2 dx &= \int \left(a^2x^4+2abx^6+b^2x^8\right) dx\\ &= \frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

fricas [A] time = 0.69, size = 24, normalized size = 0.80

$$\frac{1}{9}x^9b^2+\frac{2}{7}x^7ba+\frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^2 + 2/7*x^7*b*a + 1/5*x^5*a^2
```

giac [A] time = 1.29, size = 24, normalized size = 0.80

$$\frac{1}{9}b^2x^9+\frac{2}{7}abx^7+\frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{9}b^{2}x^{9}+\frac{2}{7}ab\,x^{7}+\frac{1}{5}a^{2}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2,x)
```

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

maxima [A] time = 1.31, size = 24, normalized size = 0.80

$$\frac{1}{9}\,b^{2}x^{9}+\frac{2}{7}\,abx^{7}+\frac{1}{5}\,a^{2}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^5}{5}+\frac{2\,a\,b\,x^7}{7}+\frac{b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^5)/5 + (b^2*x^9)/9 + (2*a*b*x^7)/7
```

sympy [A] time = 0.06, size = 26, normalized size = 0.87

$$\frac{a^{2}x^{5}}{5}+\frac{2abx^{7}}{7}+\frac{b^{2}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9
```

3.15 $\int x^3 \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a+bx^2\right)^2 \, dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^2 \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^2x+2abx^2+b^2x^3\right) \, dx, x, x^2\right)\\ &= \frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

fricas [A] time = 0.62, size = 24, normalized size = 0.80

$$\frac{1}{8}x^8b^2+\frac{1}{3}x^6ba+\frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^2 + 1/3*x^6*b*a + 1/4*x^4*a^2
```

giac [A] time = 1.04, size = 24, normalized size = 0.80

$$\frac{1}{8}b^2x^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{8}b^2x^8 + \frac{1}{3}ab\,x^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2,x)
```

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

maxima [A] time = 1.35, size = 24, normalized size = 0.80

$$\frac{1}{8}b^2x^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4} + \frac{a\,b\,x^6}{3} + \frac{b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^8)/8 + (a*b*x^6)/3
```

sympy [A] time = 0.06, size = 24, normalized size = 0.80

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

3.16 $\int x^2 \left(a+bx^2\right)^2 dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2 dx &= \int \left(a^2x^2+2abx^4+b^2x^6\right) dx\\ &= \frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

fricas [A] time = 0.74, size = 24, normalized size = 0.80

$$\frac{1}{7}x^7b^2+\frac{2}{5}x^5ba+\frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^2 + 2/5*x^5*b*a + 1/3*x^3*a^2
```

giac [A] time = 1.13, size = 24, normalized size = 0.80

$$\frac{1}{7}b^2x^7+\frac{2}{5}abx^5+\frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{7}b^{2}x^{7}+\frac{2}{5}ab\,x^{5}+\frac{1}{3}a^{2}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2,x)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

maxima [A] time = 1.35, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^{2}x^{7}+\frac{2}{5}\,abx^{5}+\frac{1}{3}\,a^{2}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^{2}\,x^{3}}{3}+\frac{2\,a\,b\,x^{5}}{5}+\frac{b^{2}\,x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^7)/7 + (2*a*b*x^5)/5
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^{2}x^{3}}{3}+\frac{2abx^{5}}{5}+\frac{b^{2}x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

3.17 $\int x\left(a+bx^2\right)^2\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^3}{6b}$$

[Out] 1/6*(b*x^2+a)^3/b

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^2)^2,x]

[Out] (a + b*x^2)^3/(6*b)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^2\,dx=\frac{\left(a+bx^2\right)^3}{6b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)^2,x]

[Out] (a + b*x^2)^3/(6*b)

fricas [A] time = 0.55, size = 24, normalized size = 1.50

$$\frac{1}{6}x^6b^2+\frac{1}{2}x^4ba+\frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/6*x^6*b^2 + 1/2*x^4*b*a + 1/2*x^2*a^2

giac [A] time = 1.05, size = 14, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^3}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*(b*x^2 + a)^3/b
```

maple [A] time = 0.00, size = 25, normalized size = 1.56

$$\frac{1}{6}b^2x^6 + \frac{1}{2}ab\,x^4 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2,x)
```

```
[Out] 1/6*b^2*x^6+1/2*a*b*x^4+1/2*a^2*x^2
```

maxima [A] time = 1.35, size = 14, normalized size = 0.88

$$\frac{\left(bx^2 + a\right)^3}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^3/b
```

mupad [B] time = 0.03, size = 24, normalized size = 1.50

$$\frac{a^2\,x^2}{2} + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^6)/6 + (a*b*x^4)/2
```

sympy [B] time = 0.06, size = 24, normalized size = 1.50

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

3.18 $\int \left(a+bx^2\right)^2 dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2 dx &= \int \left(a^2+2abx^2+b^2x^4\right) dx\\ &= a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

fricas [A] time = 0.49, size = 21, normalized size = 0.84

$$\frac{1}{5}x^5b^2+\frac{2}{3}x^3ba+xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b^2 + 2/3*x^3*b*a + x*a^2
```

giac [A] time = 1.05, size = 21, normalized size = 0.84

$$\frac{1}{5}b^2x^5+\frac{2}{3}abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{5}b^2x^5 + \frac{2}{3}ab\,x^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2,x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

maxima [A] time = 1.31, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$a^2\,x + \frac{2\,a\,b\,x^3}{3} + \frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2,x)
```

```
[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

sympy [A] time = 0.06, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

3.19 $\int \frac{(a+bx^2)^2}{x}\,dx$

Optimal. Leaf size=23

$$a^2\log(x)+abx^2+\frac{b^2x^4}{4}$$

[Out] a*b*x^2+1/4*b^2*x^4+a^2*ln(x)

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$a^2\log(x)+abx^2+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^2/x,x]

[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx,x,x^2\right)\\
&= abx^2+\frac{b^2x^4}{4}+a^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$a^2\log(x)+abx^2+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^2/x,x]

[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]

fricas [A] time = 0.56, size = 21, normalized size = 0.91

$$\frac{1}{4}b^2x^4+abx^2+a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + a^2*log(x)
```

giac [A] time = 1.07, size = 24, normalized size = 1.04

$$\frac{1}{4}\,b^2x^4 + abx^2 + \frac{1}{2}\,a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{b^2x^4}{4} + ab\,x^2 + a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x,x)
```

```
[Out] a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

maxima [A] time = 1.39, size = 24, normalized size = 1.04

$$\frac{1}{4}\,b^2x^4 + abx^2 + \frac{1}{2}\,a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.91

$$a^2\,\ln(x) + \frac{b^2\,x^4}{4} + a\,b\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x,x)
```

```
[Out] a^2*log(x) + (b^2*x^4)/4 + a*b*x^2
```

sympy [A] time = 0.10, size = 20, normalized size = 0.87

$$a^2\log\left(x\right) + abx^2 + \frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x,x)
```

```
[Out] a**2*log(x) + a*b*x**2 + b**2*x**4/4
```

3.20 $\int \frac{(a+bx^2)^2}{x^2} \, dx$

Optimal. Leaf size=24

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^2/x^2,x]

[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^2} \, dx &= \int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right) dx\\ &= -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^2/x^2,x]

[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3

fricas [A] time = 0.86, size = 25, normalized size = 1.04

$$\frac{b^2x^4+6\,abx^2-3\,a^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^2,x, algorithm="fricas")

[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x

giac [A] time = 1.07, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3+2\,abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$\frac{b^2x^3}{3}+2abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2,x)
```

```
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

maxima [A] time = 1.36, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3+2\,abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

mupad [B] time = 0.03, size = 22, normalized size = 0.92

$$\frac{b^2\,x^3}{3}-\frac{a^2}{x}+2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^2,x)
```

```
[Out] (b^2*x^3)/3 - a^2/x + 2*a*b*x
```

sympy [A] time = 0.10, size = 19, normalized size = 0.79

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

3.21 $\int \frac{(a+bx^2)^2}{x^3} dx$

Optimal. Leaf size=27

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

```
[Out] -1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^2} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(b^2 + \frac{a^2}{x^2} + \frac{2ab}{x}\right) dx, x, x^2\right)\\
&= -\frac{a^2}{2x^2} + \frac{b^2x^2}{2} + 2ab\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 + (b^2*x^2)/2 + 2*a*b*Log[x]
```

fricas [A] time = 0.81, size = 27, normalized size = 1.00

$$\frac{b^2x^4 + 4\,abx^2\log(x) - a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^4 + 4*a*b*x^2*log(x) - a^2)/x^2
```

giac [A] time = 1.14, size = 32, normalized size = 1.19

$$\frac{1}{2}\,b^2x^2 + ab\log\left(x^2\right) - \frac{2\,abx^2 + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*(2*a*b*x^2 + a^2)/x^2
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$\frac{b^2x^2}{2} + 2ab\ln(x) - \frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

maxima [A] time = 1.39, size = 24, normalized size = 0.89

$$\frac{1}{2}\,b^2x^2 + ab\log\left(x^2\right) - \frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*a^2/x^2
```

mupad [B] time = 4.94, size = 23, normalized size = 0.85

$$\frac{b^2\,x^2}{2} - \frac{a^2}{2\,x^2} + 2\,a\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^3,x)
```

```
[Out] (b^2*x^2)/2 - a^2/(2*x^2) + 2*a*b*log(x)
```

sympy [A] time = 0.13, size = 24, normalized size = 0.89

$$-\frac{a^2}{2x^2} + 2ab\log{(x)} + \frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2
```

3.22 $\int \frac{(a+bx^2)^2}{x^4} \, dx$

Optimal. Leaf size=23

$$-\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

```
[Out] -1/3*a^2/x^3-2*a*b/x+b^2*x
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + b^2*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^4} \, dx &= \int \left(b^2 + \frac{a^2}{x^4} + \frac{2ab}{x^2}\right) dx\\ &= -\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (2*a*b)/x + b^2*x
```

fricas [A] time = 0.86, size = 26, normalized size = 1.13

$$\frac{3\,b^2x^4 - 6\,abx^2 - a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*x^4 - 6*a*b*x^2 - a^2)/x^3
```

giac [A] time = 0.88, size = 22, normalized size = 0.96

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4,x, algorithm="giac")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

maple [A] time = 0.01, size = 22, normalized size = 0.96

$$b^2x - \frac{2ab}{x} - \frac{a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4,x)
```

```
[Out] -1/3*a^2/x^3-2*a*b/x+b^2*x
```

maxima [A] time = 1.32, size = 22, normalized size = 0.96

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 1.04

$$b^2\,x - \frac{\frac{a^2}{3} + 2\,b\,a\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^4,x)
```

```
[Out] b^2*x - (a^2/3 + 2*a*b*x^2)/x^3
```

sympy [A] time = 0.14, size = 22, normalized size = 0.96

$$b^2x + \frac{-a^2 - 6abx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4,x)
```

```
[Out] b**2*x + (-a**2 - 6*a*b*x**2)/(3*x**3)
```

3.23 $\int \frac{(a+bx^2)^2}{x^5} \, dx$

Optimal. Leaf size=24

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

[Out] -1/4*a^2/x^4-a*b/x^2+b^2*ln(x)

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{266, 43\}$

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^2/x^5,x]

[Out] -a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^5} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^3} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right) dx, x, x^2\right)\\
&= -\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^2/x^5,x]

[Out] -1/4*a^2/x^4 - (a*b)/x^2 + b^2*Log[x]

fricas [A] time = 0.80, size = 28, normalized size = 1.17

$$\frac{4\,b^2x^4\log(x)-4\,abx^2-a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*x^4*log(x) - 4*a*b*x^2 - a^2)/x^4
```

giac [A] time = 1.14, size = 34, normalized size = 1.42

$$\frac{1}{2}\,b^2\log\left(x^2\right) - \frac{3\,b^2x^4 + 4\,abx^2 + a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2) - 1/4*(3*b^2*x^4 + 4*a*b*x^2 + a^2)/x^4
```

maple [A] time = 0.01, size = 23, normalized size = 0.96

$$b^2\ln(x) - \frac{ab}{x^2} - \frac{a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5,x)
```

```
[Out] -1/4*a^2/x^4-a*b/x^2+b^2*ln(x)
```

maxima [A] time = 1.28, size = 26, normalized size = 1.08

$$\frac{1}{2}\,b^2\log\left(x^2\right) - \frac{4\,abx^2 + a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/x^4
```

mupad [B] time = 0.04, size = 24, normalized size = 1.00

$$b^2\,\ln(x) - \frac{\frac{a^2}{4} + b\,a\,x^2}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^5,x)
```

```
[Out] b^2*log(x) - (a^2/4 + a*b*x^2)/x^4
```

sympy [A] time = 0.17, size = 24, normalized size = 1.00

$$b^2\log(x) + \frac{-a^2 - 4abx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5,x)
```

```
[Out] b**2*log(x) + (-a**2 - 4*a*b*x**2)/(4*x**4)
```

3.24 $\int \frac{(a+bx^2)^2}{x^6} dx$

Optimal. Leaf size=28

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

```
[Out] -1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^6,x]
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^6} dx &= \int \left(\frac{a^2}{x^6} + \frac{2ab}{x^4} + \frac{b^2}{x^2}\right) dx \\
&= -\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^6,x]
[Out] -1/5*a^2/x^5 - (2*a*b)/(3*x^3) - b^2/x
```

fricas [A] time = 0.79, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6,x, algorithm="fricas")
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

giac [A] time = 1.05, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.89

$$-\frac{b^2}{x}-\frac{2ab}{3x^3}-\frac{a^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6,x)
```

```
[Out] -1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

maxima [A] time = 1.30, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

mupad [B] time = 0.03, size = 25, normalized size = 0.89

$$-\frac{\frac{a^2}{5}+\frac{2\,a\,b\,x^2}{3}+b^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^6,x)
```

```
[Out] -(a^2/5 + b^2*x^4 + (2*a*b*x^2)/3)/x^5
```

sympy [A] time = 0.17, size = 27, normalized size = 0.96

$$\frac{-3a^2-10abx^2-15b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6,x)
```

```
[Out] (-3*a**2 - 10*a*b*x**2 - 15*b**2*x**4)/(15*x**5)
```

3.25 $\int \frac{(a+bx^2)^2}{x^7} dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^3}{6ax^6}$$

```
[Out] -1/6*(b*x^2+a)^3/a/x^6
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{(a+bx^2)^3}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^7,x]
```

```
[Out] -(a + b*x^2)^3/(6*a*x^6)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^7} dx = -\frac{(a+bx^2)^3}{6ax^6}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.58

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^7,x]
```

```
[Out] -1/6*a^2/x^6 - (a*b)/(2*x^4) - b^2/(2*x^2)
```

fricas [A] time = 0.85, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

giac [A] time = 1.11, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

maple [A] time = 0.01, size = 25, normalized size = 1.32

$$-\frac{b^2}{2x^2}-\frac{ab}{2x^4}-\frac{a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^7,x)
```

```
[Out] -1/6*a^2/x^6-1/2*b^2/x^2-1/2*a*b/x^4
```

maxima [A] time = 1.38, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

mupad [B] time = 0.03, size = 26, normalized size = 1.37

$$-\frac{\frac{a^2}{6}+\frac{a\,b\,x^2}{2}+\frac{b^2\,x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^7,x)
```

```
[Out] -(a^2/6 + (b^2*x^4)/2 + (a*b*x^2)/2)/x^6
```

sympy [A] time = 0.19, size = 26, normalized size = 1.37

$$\frac{-a^2-3abx^2-3b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**7,x)
```

```
[Out] (-a**2 - 3*a*b*x**2 - 3*b**2*x**4)/(6*x**6)
```

3.26 $\int \frac{(a+bx^2)^2}{x^8} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

```
[Out] -1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^8}\,dx &= \int \left(\frac{a^2}{x^8}+\frac{2ab}{x^6}+\frac{b^2}{x^4}\right)dx\\ &= -\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^8,x]
```

```
[Out] -1/7*a^2/x^7 - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

fricas [A] time = 0.79, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

giac [A] time = 1.06, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{3x^3} - \frac{2ab}{5x^5} - \frac{a^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^8,x)
```

```
[Out] -1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

maxima [A] time = 1.38, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

mupad [B] time = 0.04, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{7} + \frac{2\,a\,b\,x^2}{5} + \frac{b^2\,x^4}{3}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^8,x)
```

```
[Out] -(a^2/7 + (b^2*x^4)/3 + (2*a*b*x^2)/5)/x^7
```

sympy [A] time = 0.20, size = 27, normalized size = 0.90

$$\frac{-15a^2 - 42abx^2 - 35b^2x^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**8,x)
```

```
[Out] (-15*a**2 - 42*a*b*x**2 - 35*b**2*x**4)/(105*x**7)
```

3.27 $\int \frac{(a+bx^2)^2}{x^9} \, dx$

Optimal. Leaf size=30

$$-\frac{a^2}{8x^8} - \frac{ab}{3x^6} - \frac{b^2}{4x^4}$$

```
[Out] -1/8*a^2/x^8-1/3*a*b/x^6-1/4*b^2/x^4
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2}{8x^8} - \frac{ab}{3x^6} - \frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^9,x]
```

```
[Out] -a^2/(8*x^8) - (a*b)/(3*x^6) - b^2/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^9} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{(a+bx)^2}{x^5} \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a^2}{x^5} + \frac{2ab}{x^4} + \frac{b^2}{x^3}\right) \, dx, x, x^2\right) \\
&= -\frac{a^2}{8x^8} - \frac{ab}{3x^6} - \frac{b^2}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{8x^8} - \frac{ab}{3x^6} - \frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^9,x]
```

```
[Out] -1/8*a^2/x^8 - (a*b)/(3*x^6) - b^2/(4*x^4)
```

fricas [A] time = 0.70, size = 26, normalized size = 0.87

$$-\frac{6\,b^2x^4 + 8\,abx^2 + 3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^9,x, algorithm="fricas")
[Out] -1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

giac [A] time = 1.09, size = 26, normalized size = 0.87

$$-\frac{6\,b^2x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^9,x, algorithm="giac")
[Out] -1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{b^2}{4x^4}-\frac{ab}{3x^6}-\frac{a^2}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^9,x)
[Out] -1/8*a^2/x^8-1/3*a*b/x^6-1/4*b^2/x^4
```

maxima [A] time = 1.38, size = 26, normalized size = 0.87

$$-\frac{6\,b^2x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^9,x, algorithm="maxima")
[Out] -1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

mupad [B] time = 0.04, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{8}+\frac{a\,b\,x^2}{3}+\frac{b^2\,x^4}{4}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^9,x)
[Out] -(a^2/8 + (b^2*x^4)/4 + (a*b*x^2)/3)/x^8
```

sympy [A] time = 0.21, size = 27, normalized size = 0.90

$$\frac{-3a^2-8abx^2-6b^2x^4}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**9,x)
[Out] (-3*a**2 - 8*a*b*x**2 - 6*b**2*x**4)/(24*x**8)
```

3.28 $\int \frac{(a+bx^2)^2}{x^{10}} \, dx$

Optimal. Leaf size=30

$$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}$$

[Out] -1/9*a^2/x^9-2/7*a*b/x^7-1/5*b^2/x^5

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^2/x^10,x]

[Out] -a^2/(9*x^9) - (2*a*b)/(7*x^7) - b^2/(5*x^5)

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{10}} \, dx &= \int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^8}+\frac{b^2}{x^6}\right) dx\\ &= -\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^2/x^10,x]

[Out] -1/9*a^2/x^9 - (2*a*b)/(7*x^7) - b^2/(5*x^5)

fricas [A] time = 0.83, size = 26, normalized size = 0.87

$$-\frac{63\,b^2x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^10,x, algorithm="fricas")

[Out] -1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9

giac [A] time = 1.05, size = 26, normalized size = 0.87

$$-\frac{63\,b^2x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{5x^5}-\frac{2ab}{7x^7}-\frac{a^2}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^10,x)
```

```
[Out] -1/9*a^2/x^9-2/7*a*b/x^7-1/5*b^2/x^5
```

maxima [A] time = 1.34, size = 26, normalized size = 0.87

$$-\frac{63\,b^2x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9
```

mupad [B] time = 0.03, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{9}+\frac{2\,a\,b\,x^2}{7}+\frac{b^2\,x^4}{5}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^10,x)
```

```
[Out] -(a^2/9 + (b^2*x^4)/5 + (2*a*b*x^2)/7)/x^9
```

sympy [A] time = 0.22, size = 27, normalized size = 0.90

$$\frac{-35a^2-90abx^2-63b^2x^4}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**10,x)
```

```
[Out] (-35*a**2 - 90*a*b*x**2 - 63*b**2*x**4)/(315*x**9)
```

3.29 $\int x^9 \left(a+bx^2\right)^3 dx$

Optimal. Leaf size=43

$$\frac{a^3x^{10}}{10}+\frac{1}{4}a^2bx^{12}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{16}}{16}$$

```
[Out] 1/10*a^3*x^10+1/4*a^2*b*x^12+3/14*a*b^2*x^14+1/16*b^3*x^16
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{4}a^2bx^{12}+\frac{a^3x^{10}}{10}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^10)/10 + (a^2*b*x^12)/4 + (3*a*b^2*x^14)/14 + (b^3*x^16)/16
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^2\right)^3 dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^4(a+bx)^3\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(a^3x^4+3a^2bx^5+3ab^2x^6+b^3x^7\right)dx,x,x^2\right)\\ &= \frac{a^3x^{10}}{10}+\frac{1}{4}a^2bx^{12}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{16}}{16}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^{10}}{10}+\frac{1}{4}a^2bx^{12}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^10)/10 + (a^2*b*x^12)/4 + (3*a*b^2*x^14)/14 + (b^3*x^16)/16
```

fricas [A] time = 0.82, size = 35, normalized size = 0.81

$$\frac{1}{16}x^{16}b^3+\frac{3}{14}x^{14}b^2a+\frac{1}{4}x^{12}ba^2+\frac{1}{10}x^{10}a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^3 + 3/14*x^14*b^2*a + 1/4*x^12*b*a^2 + 1/10*x^10*a^3
```

giac [A] time = 1.17, size = 35, normalized size = 0.81

$$\frac{1}{16}\,b^{3}x^{16}+\frac{3}{14}\,ab^{2}x^{14}+\frac{1}{4}\,a^{2}bx^{12}+\frac{1}{10}\,a^{3}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{16}b^{3}x^{16}+\frac{3}{14}a\,b^{2}x^{14}+\frac{1}{4}a^{2}b\,x^{12}+\frac{1}{10}a^{3}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^3,x)
```

```
[Out] 1/10*a^3*x^10+1/4*a^2*b*x^12+3/14*a*b^2*x^14+1/16*b^3*x^16
```

maxima [A] time = 1.36, size = 35, normalized size = 0.81

$$\frac{1}{16}\,b^{3}x^{16}+\frac{3}{14}\,ab^{2}x^{14}+\frac{1}{4}\,a^{2}bx^{12}+\frac{1}{10}\,a^{3}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^{10}}{10}+\frac{a^2\,b\,x^{12}}{4}+\frac{3\,a\,b^2\,x^{14}}{14}+\frac{b^3\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^10)/10 + (b^3*x^16)/16 + (a^2*b*x^12)/4 + (3*a*b^2*x^14)/14
```

sympy [A] time = 0.07, size = 37, normalized size = 0.86

$$\frac{a^{3}x^{10}}{10}+\frac{a^{2}bx^{12}}{4}+\frac{3ab^{2}x^{14}}{14}+\frac{b^{3}x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**10/10 + a**2*b*x**12/4 + 3*a*b**2*x**14/14 + b**3*x**16/16
```

3.30 $\int x^7 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^8}{8}+\frac{3}{10}a^2bx^{10}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}$$

```
[Out] 1/8*a^3*x^8+3/10*a^2*b*x^10+1/4*a*b^2*x^12+1/14*b^3*x^14
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{10}a^2bx^{10}+\frac{a^3x^8}{8}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^8)/8 + (3*a^2*b*x^10)/10 + (a*b^2*x^12)/4 + (b^3*x^14)/14
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a+bx^2\right)^3 \, dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^3 \, dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^3x^3+3a^2bx^4+3ab^2x^5+b^3x^6\right) \, dx,x,x^2\right)\\
&= \frac{a^3x^8}{8}+\frac{3}{10}a^2bx^{10}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^8}{8}+\frac{3}{10}a^2bx^{10}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^8)/8 + (3*a^2*b*x^10)/10 + (a*b^2*x^12)/4 + (b^3*x^14)/14
```

fricas [A] time = 0.48, size = 35, normalized size = 0.81

$$\frac{1}{14}x^{14}b^3+\frac{1}{4}x^{12}b^2a+\frac{3}{10}x^{10}ba^2+\frac{1}{8}x^8a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^3 + 1/4*x^12*b^2*a + 3/10*x^10*b*a^2 + 1/8*x^8*a^3
```

giac [A] time = 1.18, size = 35, normalized size = 0.81

$$\frac{1}{14}\,b^3x^{14} + \frac{1}{4}\,ab^2x^{12} + \frac{3}{10}\,a^2bx^{10} + \frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{14}b^3x^{14} + \frac{1}{4}a\,b^2x^{12} + \frac{3}{10}a^2b\,x^{10} + \frac{1}{8}a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^3,x)
```

```
[Out] 1/8*a^3*x^8+3/10*a^2*b*x^10+1/4*a*b^2*x^12+1/14*b^3*x^14
```

maxima [A] time = 1.32, size = 35, normalized size = 0.81

$$\frac{1}{14}\,b^3x^{14} + \frac{1}{4}\,ab^2x^{12} + \frac{3}{10}\,a^2bx^{10} + \frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^8}{8} + \frac{3\,a^2\,b\,x^{10}}{10} + \frac{a\,b^2\,x^{12}}{4} + \frac{b^3\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^8)/8 + (b^3*x^14)/14 + (3*a^2*b*x^10)/10 + (a*b^2*x^12)/4
```

sympy [A] time = 0.07, size = 37, normalized size = 0.86

$$\frac{a^3x^8}{8} + \frac{3a^2bx^{10}}{10} + \frac{ab^2x^{12}}{4} + \frac{b^3x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**8/8 + 3*a**2*b*x**10/10 + a*b**2*x**12/4 + b**3*x**14/14
```

3.31 $\int x^5 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^6}{6}+\frac{3}{8}a^2bx^8+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}$$

```
[Out] 1/6*a^3*x^6+3/8*a^2*b*x^8+3/10*a*b^2*x^10+1/12*b^3*x^12
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{8}a^2bx^8+\frac{a^3x^6}{6}+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^6)/6 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^10)/10 + (b^3*x^12)/12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^3\,dx&=\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^3\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(a^3x^2+3a^2bx^3+3ab^2x^4+b^3x^5\right)dx,x,x^2\right)\\&=\frac{a^3x^6}{6}+\frac{3}{8}a^2bx^8+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^6}{6}+\frac{3}{8}a^2bx^8+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^6)/6 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^10)/10 + (b^3*x^12)/12
```

fricas [A] time = 0.58, size = 35, normalized size = 0.81

$$\frac{1}{12}x^{12}b^3+\frac{3}{10}x^{10}b^2a+\frac{3}{8}x^8ba^2+\frac{1}{6}x^6a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^3 + 3/10*x^10*b^2*a + 3/8*x^8*b*a^2 + 1/6*x^6*a^3
```

giac [A] time = 0.97, size = 35, normalized size = 0.81

$$\frac{1}{12}\,b^3x^{12}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{12}b^3x^{12}+\frac{3}{10}a\,b^2x^{10}+\frac{3}{8}a^2b\,x^8+\frac{1}{6}a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^3,x)
```

```
[Out] 1/6*a^3*x^6+3/8*a^2*b*x^8+3/10*a*b^2*x^10+1/12*b^3*x^12
```

maxima [A] time = 1.35, size = 35, normalized size = 0.81

$$\frac{1}{12}\,b^3x^{12}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^6}{6}+\frac{3\,a^2\,b\,x^8}{8}+\frac{3\,a\,b^2\,x^{10}}{10}+\frac{b^3\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^6)/6 + (b^3*x^12)/12 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^10)/10
```

sympy [A] time = 0.07, size = 39, normalized size = 0.91

$$\frac{a^3x^6}{6}+\frac{3a^2bx^8}{8}+\frac{3ab^2x^{10}}{10}+\frac{b^3x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**6/6 + 3*a**2*b*x**8/8 + 3*a*b**2*x**10/10 + b**3*x**12/12
```

3.32 $\int x^3 \left(a+bx^2\right)^3 dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^5}{10b^2}-\frac{a\left(a+bx^2\right)^4}{8b^2}$$

```
[Out] -1/8*a*(b*x^2+a)^4/b^2+1/10*(b*x^2+a)^5/b^2
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^5}{10b^2}-\frac{a\left(a+bx^2\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^3,x]
```

```
[Out] -(a*(a + b*x^2)^4)/(8*b^2) + (a + b*x^2)^5/(10*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^2\right)^3 dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^3 dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right) dx, x, x^2\right)\\
&= -\frac{a\left(a+bx^2\right)^4}{8b^2}+\frac{\left(a+bx^2\right)^5}{10b^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.26

$$\frac{a^3x^4}{4}+\frac{1}{2}a^2bx^6+\frac{3}{8}ab^2x^8+\frac{b^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^4)/4 + (a^2*b*x^6)/2 + (3*a*b^2*x^8)/8 + (b^3*x^10)/10
```

fricas [A] time = 0.71, size = 35, normalized size = 1.03

$$\frac{1}{10}x^{10}b^3+\frac{3}{8}x^8b^2a+\frac{1}{2}x^6ba^2+\frac{1}{4}x^4a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^3 + 3/8*x^8*b^2*a + 1/2*x^6*b*a^2 + 1/4*x^4*a^3
```

giac [A] time = 0.95, size = 35, normalized size = 1.03

$$\frac{1}{10}\,b^3x^{10}+\frac{3}{8}\,ab^2x^8+\frac{1}{2}\,a^2bx^6+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

maple [A] time = 0.00, size = 36, normalized size = 1.06

$$\frac{1}{10}b^3x^{10}+\frac{3}{8}a\,b^2x^8+\frac{1}{2}a^2b\,x^6+\frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^3,x)
```

```
[Out] 1/10*b^3*x^10+3/8*a*b^2*x^8+1/2*a^2*b*x^6+1/4*a^3*x^4
```

maxima [A] time = 1.37, size = 35, normalized size = 1.03

$$\frac{1}{10}\,b^3x^{10}+\frac{3}{8}\,ab^2x^8+\frac{1}{2}\,a^2bx^6+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

mupad [B] time = 0.04, size = 35, normalized size = 1.03

$$\frac{a^3\,x^4}{4}+\frac{a^2\,b\,x^6}{2}+\frac{3\,a\,b^2\,x^8}{8}+\frac{b^3\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^10)/10 + (a^2*b*x^6)/2 + (3*a*b^2*x^8)/8
```

sympy [A] time = 0.07, size = 37, normalized size = 1.09

$$\frac{a^3x^4}{4}+\frac{a^2bx^6}{2}+\frac{3ab^2x^8}{8}+\frac{b^3x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**4/4 + a**2*b*x**6/2 + 3*a*b**2*x**8/8 + b**3*x**10/10
```

3.33 $\int x\left(a+bx^2\right)^3\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^4}{8b}$$

[Out] 1/8*(b*x^2+a)^4/b

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^4}{8b}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^2)^3,x]

[Out] (a + b*x^2)^4/(8*b)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^3\,dx=\frac{\left(a+bx^2\right)^4}{8b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^4}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)^3,x]

[Out] (a + b*x^2)^4/(8*b)

fricas [B] time = 0.80, size = 35, normalized size = 2.19

$$\frac{1}{8}x^8b^3+\frac{1}{2}x^6b^2a+\frac{3}{4}x^4ba^2+\frac{1}{2}x^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/8*x^8*b^3 + 1/2*x^6*b^2*a + 3/4*x^4*b*a^2 + 1/2*x^2*a^3

giac [A] time = 1.00, size = 14, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^4}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(b*x^2 + a)^4/b
```

maple [B] time = 0.00, size = 36, normalized size = 2.25

$$\frac{1}{8}b^3x^8 + \frac{1}{2}a\,b^2x^6 + \frac{3}{4}a^2b\,x^4 + \frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^3,x)
```

```
[Out] 1/8*b^3*x^8+1/2*a*b^2*x^6+3/4*a^2*b*x^4+1/2*a^3*x^2
```

maxima [A] time = 1.34, size = 14, normalized size = 0.88

$$\frac{\left(bx^2 + a\right)^4}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^4/b
```

mupad [B] time = 0.06, size = 35, normalized size = 2.19

$$\frac{a^3\,x^2}{2} + \frac{3\,a^2\,b\,x^4}{4} + \frac{a\,b^2\,x^6}{2} + \frac{b^3\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^2)/2 + (b^3*x^8)/8 + (3*a^2*b*x^4)/4 + (a*b^2*x^6)/2
```

sympy [B] time = 0.07, size = 37, normalized size = 2.31

$$\frac{a^3x^2}{2} + \frac{3a^2bx^4}{4} + \frac{ab^2x^6}{2} + \frac{b^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**2/2 + 3*a**2*b*x**4/4 + a*b**2*x**6/2 + b**3*x**8/8
```

3.34 $\int \frac{(a+bx^2)^3}{x} \, dx$

Optimal. Leaf size=39

$$a^3 \log(x) + \frac{3}{2}a^2bx^2 + \frac{3}{4}ab^2x^4 + \frac{b^3x^6}{6}$$

```
[Out] 3/2*a^2*b*x^2+3/4*a*b^2*x^4+1/6*b^3*x^6+a^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{2}a^2bx^2 + a^3 \log(x) + \frac{3}{4}ab^2x^4 + \frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4 + (b^3*x^6)/6 + a^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^3}{x} \, dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(3a^2b + \frac{a^3}{x} + 3ab^2x + b^3x^2\right) dx, x, x^2\right) \\
&= \frac{3}{2}a^2bx^2 + \frac{3}{4}ab^2x^4 + \frac{b^3x^6}{6} + a^3 \log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$a^3 \log(x) + \frac{3}{2}a^2bx^2 + \frac{3}{4}ab^2x^4 + \frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4 + (b^3*x^6)/6 + a^3*Log[x]
```

fricas [A] time = 0.76, size = 33, normalized size = 0.85

$$\frac{1}{6}b^3x^6 + \frac{3}{4}ab^2x^4 + \frac{3}{2}a^2bx^2 + a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x,x, algorithm="fricas")
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

giac [A] time = 0.96, size = 36, normalized size = 0.92

$$\frac{1}{6}b^3x^6 + \frac{3}{4}ab^2x^4 + \frac{3}{2}a^2bx^2 + \frac{1}{2}a^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x,x, algorithm="giac")
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*log(x^2)
```

maple [A] time = 0.00, size = 34, normalized size = 0.87

$$\frac{b^3x^6}{6} + \frac{3a\,b^2x^4}{4} + \frac{3a^2b\,x^2}{2} + a^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x,x)
[Out] 3/2*a^2*b*x^2+3/4*a*b^2*x^4+1/6*b^3*x^6+a^3*ln(x)
```

maxima [A] time = 1.31, size = 36, normalized size = 0.92

$$\frac{1}{6}b^3x^6 + \frac{3}{4}ab^2x^4 + \frac{3}{2}a^2bx^2 + \frac{1}{2}a^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x,x, algorithm="maxima")
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*log(x^2)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.85

$$a^3\,\ln(x) + \frac{b^3\,x^6}{6} + \frac{3\,a^2\,b\,x^2}{2} + \frac{3\,a\,b^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x,x)
[Out] a^3*log(x) + (b^3*x^6)/6 + (3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4
```

sympy [A] time = 0.11, size = 37, normalized size = 0.95

$$a^3\log{(x)} + \frac{3a^2bx^2}{2} + \frac{3ab^2x^4}{4} + \frac{b^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x,x)
[Out] a**3*log(x) + 3*a**2*b*x**2/2 + 3*a*b**2*x**4/4 + b**3*x**6/6
```

3.35 $\int \frac{(a+bx^2)^3}{x^3} \, dx$

Optimal. Leaf size=40

$$-\frac{a^3}{2x^2} + 3a^2 b \log(x) + \frac{3}{2} a b^2 x^2 + \frac{b^3 x^4}{4}$$

```
[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$3a^2 b \log(x) - \frac{a^3}{2x^2} + \frac{3}{2} a b^2 x^2 + \frac{b^3 x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^3} \, dx &= \frac{1}{2} \operatorname{Subst}\left(\int \frac{(a+bx)^3}{x^2} \, dx, x, x^2\right) \\
&= \frac{1}{2} \operatorname{Subst}\left(\int \left(3ab^2 + \frac{a^3}{x^2} + \frac{3a^2 b}{x} + b^3 x\right) dx, x, x^2\right) \\
&= -\frac{a^3}{2x^2} + \frac{3}{2} a b^2 x^2 + \frac{b^3 x^4}{4} + 3a^2 b \log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$-\frac{a^3}{2x^2} + 3a^2 b \log(x) + \frac{3}{2} a b^2 x^2 + \frac{b^3 x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^3,x]
```

```
[Out] -1/2*a^3/x^2 + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

fricas [A] time = 0.73, size = 38, normalized size = 0.95

$$\frac{b^3x^6 + 6\,ab^2x^4 + 12\,a^2bx^2\log(x) - 2\,a^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

giac [A] time = 1.08, size = 46, normalized size = 1.15

$$\frac{1}{4}\,b^3x^4 + \frac{3}{2}\,ab^2x^2 + \frac{3}{2}\,a^2b\log\left(x^2\right) - \frac{3\,a^2bx^2 + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*(3*a^2*b*x^2 + a^3)/
x^2
```

maple [A] time = 0.01, size = 35, normalized size = 0.88

$$\frac{b^3x^4}{4} + \frac{3a\,b^2x^2}{2} + 3a^2b\ln(x) - \frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^3,x)
```

```
[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

maxima [A] time = 1.38, size = 36, normalized size = 0.90

$$\frac{1}{4}\,b^3x^4 + \frac{3}{2}\,ab^2x^2 + \frac{3}{2}\,a^2b\log\left(x^2\right) - \frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*a^3/x^2
```

mupad [B] time = 0.04, size = 34, normalized size = 0.85

$$\frac{b^3\,x^4}{4} - \frac{a^3}{2\,x^2} + \frac{3\,a\,b^2\,x^2}{2} + 3\,a^2\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^3,x)
```

```
[Out] (b^3*x^4)/4 - a^3/(2*x^2) + (3*a*b^2*x^2)/2 + 3*a^2*b*log(x)
```

sympy [A] time = 0.14, size = 37, normalized size = 0.92

$$-\frac{a^3}{2x^2} + 3a^2b\log(x) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2 + b**3*x**4/4
```

3.36 $\int \frac{(a+bx^2)^3}{x^5} dx$

Optimal. Leaf size=40

$$-\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + 3ab^2\log(x) + \frac{b^3x^2}{2}$$

```
[Out] -1/4*a^3/x^4-3/2*a^2*b/x^2+1/2*b^3*x^2+3*a*b^2*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{3a^2b}{2x^2} - \frac{a^3}{4x^4} + 3ab^2\log(x) + \frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^5,x]
```

```
[Out] -a^3/(4*x^4) - (3*a^2*b)/(2*x^2) + (b^3*x^2)/2 + 3*a*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^5} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^3} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(b^3 + \frac{a^3}{x^3} + \frac{3a^2b}{x^2} + \frac{3ab^2}{x}\right) dx, x, x^2\right) \\
&= -\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + \frac{b^3x^2}{2} + 3ab^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 40, normalized size = 1.00

$$-\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + 3ab^2\log(x) + \frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^5,x]
```

```
[Out] -1/4*a^3/x^4 - (3*a^2*b)/(2*x^2) + (b^3*x^2)/2 + 3*a*b^2*Log[x]
```

fricas [A] time = 0.79, size = 39, normalized size = 0.98

$$\frac{2\,b^3x^6 + 12\,ab^2x^4\log(x) - 6\,a^2bx^2 - a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^5,x, algorithm="fricas")
[Out] 1/4*(2*b^3*x^6 + 12*a*b^2*x^4*log(x) - 6*a^2*b*x^2 - a^3)/x^4
```

giac [A] time = 1.14, size = 46, normalized size = 1.15

$$\frac{1}{2}\,b^3x^2 + \frac{3}{2}\,ab^2\log\left(x^2\right) - \frac{9\,ab^2x^4 + 6\,a^2bx^2 + a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^5,x, algorithm="giac")
[Out] 1/2*b^3*x^2 + 3/2*a*b^2*log(x^2) - 1/4*(9*a*b^2*x^4 + 6*a^2*b*x^2 + a^3)/x^
4
```

maple [A] time = 0.01, size = 35, normalized size = 0.88

$$\frac{b^3x^2}{2} + 3a\,b^2\ln(x) - \frac{3a^2b}{2x^2} - \frac{a^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^5,x)
[Out] -1/4*a^3/x^4-3/2*a^2*b/x^2+1/2*b^3*x^2+3*a*b^2*ln(x)
```

maxima [A] time = 1.36, size = 37, normalized size = 0.92

$$\frac{1}{2}\,b^3x^2 + \frac{3}{2}\,ab^2\log\left(x^2\right) - \frac{6\,a^2bx^2 + a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^5,x, algorithm="maxima")
[Out] 1/2*b^3*x^2 + 3/2*a*b^2*log(x^2) - 1/4*(6*a^2*b*x^2 + a^3)/x^4
```

mupad [B] time = 4.90, size = 37, normalized size = 0.92

$$\frac{b^3\,x^2}{2} - \frac{\frac{a^3}{4} + \frac{3\,b\,a^2\,x^2}{2}}{x^4} + 3\,a\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^5,x)
[Out] (b^3*x^2)/2 - (a^3/4 + (3*a^2*b*x^2)/2)/x^4 + 3*a*b^2*log(x)
```

sympy [A] time = 0.19, size = 37, normalized size = 0.92

$$3ab^2\log(x) + \frac{b^3x^2}{2} + \frac{-a^3 - 6a^2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**5,x)
[Out] 3*a*b**2*log(x) + b**3*x**2/2 + (-a**3 - 6*a**2*b*x**2)/(4*x**4)
```

3.37 $\int \frac{(a+bx^2)^3}{x^7} dx$

Optimal. Leaf size=39

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}+b^3\log(x)$$

```
[Out] -1/6*a^3/x^6-3/4*a^2*b/x^4-3/2*a*b^2/x^2+b^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{3a^2b}{4x^4}-\frac{a^3}{6x^6}-\frac{3ab^2}{2x^2}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^7,x]
```

```
[Out] -a^3/(6*x^6) - (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) + b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^3}{x^4}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^3}+\frac{3ab^2}{x^2}+\frac{b^3}{x}\right)dx,x,x^2\right)\\ &= -\frac{a^3}{6x^6}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}+b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^7,x]
```

```
[Out] -1/6*a^3/x^6 - (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) + b^3*Log[x]
```

fricas [A] time = 0.79, size = 39, normalized size = 1.00

$$\frac{12\,b^3x^6\log(x)-18\,ab^2x^4-9\,a^2bx^2-2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^3*x^6*log(x) - 18*a*b^2*x^4 - 9*a^2*b*x^2 - 2*a^3)/x^6
```

giac [A] time = 1.05, size = 47, normalized size = 1.21

$$\frac{1}{2}\,b^3\log\left(x^2\right) - \frac{11\,b^3x^6 + 18\,ab^2x^4 + 9\,a^2bx^2 + 2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/2*b^3*log(x^2) - 1/12*(11*b^3*x^6 + 18*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)/x
^6
```

maple [A] time = 0.01, size = 34, normalized size = 0.87

$$b^3\ln(x) - \frac{3a\,b^2}{2x^2} - \frac{3a^2b}{4x^4} - \frac{a^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^7,x)
```

```
[Out] -1/6*a^3/x^6-3/4*a^2*b/x^4-3/2*a*b^2/x^2+b^3*ln(x)
```

maxima [A] time = 1.29, size = 39, normalized size = 1.00

$$\frac{1}{2}\,b^3\log\left(x^2\right) - \frac{18\,ab^2x^4 + 9\,a^2bx^2 + 2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*log(x^2) - 1/12*(18*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)/x^6
```

mupad [B] time = 0.05, size = 36, normalized size = 0.92

$$b^3\,\ln(x) - \frac{\frac{a^3}{6} + \frac{3\,a^2\,b\,x^2}{4} + \frac{3\,a\,b^2\,x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^7,x)
```

```
[Out] b^3*log(x) - (a^3/6 + (3*a^2*b*x^2)/4 + (3*a*b^2*x^4)/2)/x^6
```

sympy [A] time = 0.24, size = 37, normalized size = 0.95

$$b^3\log\left(x\right) + \frac{-2a^3 - 9a^2bx^2 - 18ab^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**7,x)
```

```
[Out] b**3*log(x) + (-2*a**3 - 9*a**2*b*x**2 - 18*a*b**2*x**4)/(12*x**6)
```

3.38 $\int \frac{(a+bx^2)^3}{x^9}\,dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^4}{8ax^8}$$

```
[Out] -1/8*(b*x^2+a)^4/a/x^8
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{(a+bx^2)^4}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^9,x]
```

```
[Out] -(a + b*x^2)^4/(8*a*x^8)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^3}{x^9}\,dx = -\frac{(a+bx^2)^4}{8ax^8}$$

Mathematica [B] time = 0.01, size = 43, normalized size = 2.26

$$-\frac{a^3}{8x^8}-\frac{a^2b}{2x^6}-\frac{3ab^2}{4x^4}-\frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^9,x]
```

```
[Out] -1/8*a^3/x^8 - (a^2*b)/(2*x^6) - (3*a*b^2)/(4*x^4) - b^3/(2*x^2)
```

fricas [B] time = 0.92, size = 35, normalized size = 1.84

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^9,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

giac [B] time = 1.04, size = 35, normalized size = 1.84

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^9,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

maple [B] time = 0.00, size = 36, normalized size = 1.89

$$-\frac{b^3}{2x^2}-\frac{3a\,b^2}{4x^4}-\frac{a^2b}{2x^6}-\frac{a^3}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^9,x)
```

```
[Out] -1/2*a^2*b/x^6-1/2*b^3/x^2-3/4*a*b^2/x^4-1/8*a^3/x^8
```

maxima [B] time = 1.32, size = 35, normalized size = 1.84

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^9,x, algorithm="maxima")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

mupad [B] time = 0.03, size = 37, normalized size = 1.95

$$-\frac{\frac{a^3}{8}+\frac{a^2\,b\,x^2}{2}+\frac{3\,a\,b^2\,x^4}{4}+\frac{b^3\,x^6}{2}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^9,x)
```

```
[Out] -(a^3/8 + (b^3*x^6)/2 + (a^2*b*x^2)/2 + (3*a*b^2*x^4)/4)/x^8
```

sympy [B] time = 0.27, size = 37, normalized size = 1.95

$$\frac{-a^3-4a^2bx^2-6ab^2x^4-4b^3x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**9,x)
```

```
[Out] (-a**3 - 4*a**2*b*x**2 - 6*a*b**2*x**4 - 4*b**3*x**6)/(8*x**8)
```

3.39 $\int \frac{\left(a+bx^2\right)^3}{x^{11}}\,dx$

Optimal. Leaf size=40

$$\frac{b\left(a+bx^2\right)^4}{40a^2x^8}-\frac{\left(a+bx^2\right)^4}{10ax^{10}}$$

```
[Out] -1/10*(b*x^2+a)^4/a/x^10+1/40*b*(b*x^2+a)^4/a^2/x^8
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{b\left(a+bx^2\right)^4}{40a^2x^8}-\frac{\left(a+bx^2\right)^4}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^11,x]
```

```
[Out] -(a + b*x^2)^4/(10*a*x^10) + (b*(a + b*x^2)^4)/(40*a^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^3}{x^{11}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^3}{x^6}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^4}{10ax^{10}}-\frac{b\operatorname{Subst}\left(\int \frac{(a+bx)^3}{x^5}\,dx,x,x^2\right)}{10a}\\
&= -\frac{\left(a+bx^2\right)^4}{10ax^{10}}+\frac{b\left(a+bx^2\right)^4}{40a^2x^8}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.08

$$-\frac{a^3}{10x^{10}}-\frac{3a^2b}{8x^8}-\frac{ab^2}{2x^6}-\frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^11,x]
[Out] -1/10*a^3/x^10 - (3*a^2*b)/(8*x^8) - (a*b^2)/(2*x^6) - b^3/(4*x^4)
```

fricas [A] time = 0.78, size = 37, normalized size = 0.92

$$-\frac{10\,b^3x^6+20\,ab^2x^4+15\,a^2bx^2+4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^11,x, algorithm="fricas")
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

giac [A] time = 1.02, size = 37, normalized size = 0.92

$$-\frac{10\,b^3x^6+20\,ab^2x^4+15\,a^2bx^2+4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^11,x, algorithm="giac")
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

maple [A] time = 0.00, size = 36, normalized size = 0.90

$$-\frac{b^3}{4x^4}-\frac{a\,b^2}{2x^6}-\frac{3a^2b}{8x^8}-\frac{a^3}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^11,x)
[Out] -1/2*a*b^2/x^6-1/4*b^3/x^4-1/10*a^3/x^10-3/8*a^2*b/x^8
```

maxima [A] time = 1.37, size = 37, normalized size = 0.92

$$-\frac{10\,b^3x^6+20\,ab^2x^4+15\,a^2bx^2+4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^11,x, algorithm="maxima")
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

mupad [B] time = 0.06, size = 37, normalized size = 0.92

$$-\frac{\frac{a^3}{10}+\frac{3\,a^2\,b\,x^2}{8}+\frac{a\,b^2\,x^4}{2}+\frac{b^3\,x^6}{4}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^11,x)
```

```
[Out] -(a^3/10 + (b^3*x^6)/4 + (3*a^2*b*x^2)/8 + (a*b^2*x^4)/2)/x^10
```

sympy [**A**] time = 0.29, size = 39, normalized size = 0.98

$$\frac{-4a^3 - 15a^2bx^2 - 20ab^2x^4 - 10b^3x^6}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**11,x)
```

```
[Out] (-4*a**3 - 15*a**2*b*x**2 - 20*a*b**2*x**4 - 10*b**3*x**6)/(40*x**10)
```

3.40 $\int \frac{(a+bx^2)^3}{x^{13}} dx$

Optimal. Leaf size=43

$$-\frac{a^3}{12x^{12}}-\frac{3a^2b}{10x^{10}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}$$

```
[Out] -1/12*a^3/x^12-3/10*a^2*b/x^10-3/8*a*b^2/x^8-1/6*b^3/x^6
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{3a^2b}{10x^{10}}-\frac{a^3}{12x^{12}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^13,x]
```

```
[Out] -a^3/(12*x^12) - (3*a^2*b)/(10*x^10) - (3*a*b^2)/(8*x^8) - b^3/(6*x^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^{13}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^3}{x^7}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^5}+\frac{b^3}{x^4}\right)dx, x, x^2\right)\\
&= -\frac{a^3}{12x^{12}}-\frac{3a^2b}{10x^{10}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{12x^{12}}-\frac{3a^2b}{10x^{10}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^13,x]
```

```
[Out] -1/12*a^3/x^12 - (3*a^2*b)/(10*x^10) - (3*a*b^2)/(8*x^8) - b^3/(6*x^6)
```

fricas [A] time = 0.80, size = 37, normalized size = 0.86

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^13,x, algorithm="fricas")
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

giac [A] time = 0.83, size = 37, normalized size = 0.86

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^13,x, algorithm="giac")
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$-\frac{b^3}{6x^6}-\frac{3a\,b^2}{8x^8}-\frac{3a^2b}{10x^{10}}-\frac{a^3}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^13,x)
[Out] -1/12*a^3/x^12-3/10*a^2*b/x^10-3/8*a*b^2/x^8-1/6*b^3/x^6
```

maxima [A] time = 1.29, size = 37, normalized size = 0.86

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^13,x, algorithm="maxima")
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

mupad [B] time = 0.05, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3}{12}+\frac{3\,a^2\,b\,x^2}{10}+\frac{3\,a\,b^2\,x^4}{8}+\frac{b^3\,x^6}{6}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^13,x)
[Out] -(a^3/12 + (b^3*x^6)/6 + (3*a^2*b*x^2)/10 + (3*a*b^2*x^4)/8)/x^12
```

sympy [A] time = 0.32, size = 39, normalized size = 0.91

$$\frac{-10a^3-36a^2bx^2-45ab^2x^4-20b^3x^6}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**13,x)
[Out] (-10*a**3 - 36*a**2*b*x**2 - 45*a*b**2*x**4 - 20*b**3*x**6)/(120*x**12)
```

3.41 $\int \frac{\left(a+bx^2\right)^3}{x^{15}}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{14x^{14}}-\frac{a^2b}{4x^{12}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}$$

```
[Out] -1/14*a^3/x^14-1/4*a^2*b/x^12-3/10*a*b^2/x^10-1/8*b^3/x^8
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2b}{4x^{12}}-\frac{a^3}{14x^{14}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^15,x]
```

```
[Out] -a^3/(14*x^14) - (a^2*b)/(4*x^12) - (3*a*b^2)/(10*x^10) - b^3/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^3}{x^{15}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^3}{x^8}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^7}+\frac{3ab^2}{x^6}+\frac{b^3}{x^5}\right)dx,x,x^2\right)\\ &= -\frac{a^3}{14x^{14}}-\frac{a^2b}{4x^{12}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$-\frac{a^3}{14x^{14}}-\frac{a^2b}{4x^{12}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^15,x]
```

```
[Out] -1/14*a^3/x^14 - (a^2*b)/(4*x^12) - (3*a*b^2)/(10*x^10) - b^3/(8*x^8)
```

fricas [A] time = 0.86, size = 37, normalized size = 0.86

$$-\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^15,x, algorithm="fricas")
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

giac [A] time = 1.04, size = 37, normalized size = 0.86

$$-\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^15,x, algorithm="giac")
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

maple [A] time = 0.01, size = 36, normalized size = 0.84

$$-\frac{b^3}{8x^8}-\frac{3a\,b^2}{10x^{10}}-\frac{a^2b}{4x^{12}}-\frac{a^3}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^15,x)
[Out] -1/14*a^3/x^14-1/4*a^2*b/x^12-3/10*a*b^2/x^10-1/8*b^3/x^8
```

maxima [A] time = 1.32, size = 37, normalized size = 0.86

$$-\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^15,x, algorithm="maxima")
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

mupad [B] time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3}{14}+\frac{a^2\,b\,x^2}{4}+\frac{3\,a\,b^2\,x^4}{10}+\frac{b^3\,x^6}{8}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^15,x)
[Out] -(a^3/14 + (b^3*x^6)/8 + (a^2*b*x^2)/4 + (3*a*b^2*x^4)/10)/x^14
```

sympy [A] time = 0.35, size = 39, normalized size = 0.91

$$\frac{-20a^3-70a^2bx^2-84ab^2x^4-35b^3x^6}{280x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**15,x)
[Out] (-20*a**3 - 70*a**2*b*x**2 - 84*a*b**2*x**4 - 35*b**3*x**6)/(280*x**14)
```

3.42 $\int x^6 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^7}{7}+\frac{1}{3}a^2bx^9+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}$$

```
[Out] 1/7*a^3*x^7+1/3*a^2*b*x^9+3/11*a*b^2*x^11+1/13*b^3*x^13
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{1}{3}a^2bx^9+\frac{a^3x^7}{7}+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^7)/7 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11 + (b^3*x^13)/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6 \left(a+bx^2\right)^3 \, dx &= \int \left(a^3x^6+3a^2bx^8+3ab^2x^{10}+b^3x^{12}\right) \, dx\\ &= \frac{a^3x^7}{7}+\frac{1}{3}a^2bx^9+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^7}{7}+\frac{1}{3}a^2bx^9+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^7)/7 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11 + (b^3*x^13)/13
```

fricas [A] time = 0.74, size = 35, normalized size = 0.81

$$\frac{1}{13}x^{13}b^3+\frac{3}{11}x^{11}b^2a+\frac{1}{3}x^9ba^2+\frac{1}{7}x^7a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^3 + 3/11*x^11*b^2*a + 1/3*x^9*b*a^2 + 1/7*x^7*a^3
```

giac [A] time = 0.93, size = 35, normalized size = 0.81

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{11}\,ab^2x^{11}+\frac{1}{3}\,a^2bx^9+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{13}b^3x^{13}+\frac{3}{11}a\,b^2x^{11}+\frac{1}{3}a^2b\,x^9+\frac{1}{7}a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^3,x)
```

```
[Out] 1/7*a^3*x^7+1/3*a^2*b*x^9+3/11*a*b^2*x^11+1/13*b^3*x^13
```

maxima [A] time = 1.35, size = 35, normalized size = 0.81

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{11}\,ab^2x^{11}+\frac{1}{3}\,a^2bx^9+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^7}{7}+\frac{a^2\,b\,x^9}{3}+\frac{3\,a\,b^2\,x^{11}}{11}+\frac{b^3\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^7)/7 + (b^3*x^13)/13 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11
```

sympy [A] time = 0.07, size = 37, normalized size = 0.86

$$\frac{a^3x^7}{7}+\frac{a^2bx^9}{3}+\frac{3ab^2x^{11}}{11}+\frac{b^3x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**7/7 + a**2*b*x**9/3 + 3*a*b**2*x**11/11 + b**3*x**13/13
```

3.43 $\int x^4 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

```
[Out] 1/5*a^3*x^5+3/7*a^2*b*x^7+1/3*a*b^2*x^9+1/11*b^3*x^11
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{3}{7}a^2bx^7+\frac{a^3x^5}{5}+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^5)/5 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3 + (b^3*x^11)/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a+bx^2\right)^3 \, dx &= \int \left(a^3x^4+3a^2bx^6+3ab^2x^8+b^3x^{10}\right) dx\\ &= \frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^5)/5 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3 + (b^3*x^11)/11
```

fricas [A] time = 0.73, size = 35, normalized size = 0.81

$$\frac{1}{11}x^{11}b^3+\frac{1}{3}x^9b^2a+\frac{3}{7}x^7ba^2+\frac{1}{5}x^5a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^3 + 1/3*x^9*b^2*a + 3/7*x^7*b*a^2 + 1/5*x^5*a^3
```

giac [A] time = 1.03, size = 35, normalized size = 0.81

$$\frac{1}{11}\,b^3x^{11}+\frac{1}{3}\,ab^2x^9+\frac{3}{7}\,a^2bx^7+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{11}b^3x^{11}+\frac{1}{3}a\,b^2x^9+\frac{3}{7}a^2b\,x^7+\frac{1}{5}a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^3,x)
```

```
[Out] 1/5*a^3*x^5+3/7*a^2*b*x^7+1/3*a*b^2*x^9+1/11*b^3*x^11
```

maxima [A] time = 1.38, size = 35, normalized size = 0.81

$$\frac{1}{11}\,b^3x^{11}+\frac{1}{3}\,ab^2x^9+\frac{3}{7}\,a^2bx^7+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^5}{5}+\frac{3\,a^2\,b\,x^7}{7}+\frac{a\,b^2\,x^9}{3}+\frac{b^3\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^5)/5 + (b^3*x^11)/11 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3
```

sympy [A] time = 0.07, size = 37, normalized size = 0.86

$$\frac{a^3x^5}{5}+\frac{3a^2bx^7}{7}+\frac{ab^2x^9}{3}+\frac{b^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**5/5 + 3*a**2*b*x**7/7 + a*b**2*x**9/3 + b**3*x**11/11
```

3.44 $\int x^2 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

```
[Out] 1/3*a^3*x^3+3/5*a^2*b*x^5+3/7*a*b^2*x^7+1/9*b^3*x^9
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{3}{5}a^2bx^5+\frac{a^3x^3}{3}+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7 + (b^3*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^3\,dx &= \int \left(a^3x^2+3a^2bx^4+3ab^2x^6+b^3x^8\right)dx\\ &= \frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7 + (b^3*x^9)/9
```

fricas [A] time = 0.75, size = 35, normalized size = 0.81

$$\frac{1}{9}x^9b^3+\frac{3}{7}x^7b^2a+\frac{3}{5}x^5ba^2+\frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^3 + 3/7*x^7*b^2*a + 3/5*x^5*b*a^2 + 1/3*x^3*a^3
```

giac [A] time = 1.03, size = 35, normalized size = 0.81

$$\frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{9}b^3x^9+\frac{3}{7}a\,b^2x^7+\frac{3}{5}a^2b\,x^5+\frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^3,x)
```

```
[Out] 1/3*a^3*x^3+3/5*a^2*b*x^5+3/7*a*b^2*x^7+1/9*b^3*x^9
```

maxima [A] time = 1.32, size = 35, normalized size = 0.81

$$\frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^3}{3}+\frac{3\,a^2\,b\,x^5}{5}+\frac{3\,a\,b^2\,x^7}{7}+\frac{b^3\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^9)/9 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7
```

sympy [A] time = 0.07, size = 39, normalized size = 0.91

$$\frac{a^3x^3}{3}+\frac{3a^2bx^5}{5}+\frac{3ab^2x^7}{7}+\frac{b^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**3/3 + 3*a**2*b*x**5/5 + 3*a*b**2*x**7/7 + b**3*x**9/9
```

3.45 $\int \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=35

$$a^3x + a^2bx^3 + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

[Out] a^3*x+a^2*b*x^3+3/5*a*b^2*x^5+1/7*b^3*x^7

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2bx^3 + a^3x + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^3,x]

[Out] a^3*x + a^2*b*x^3 + (3*a*b^2*x^5)/5 + (b^3*x^7)/7

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \left(a+bx^2\right)^3 \, dx &= \int \left(a^3+3a^2bx^2+3ab^2x^4+b^3x^6\right) \, dx \\ &= a^3x + a^2bx^3 + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$a^3x + a^2bx^3 + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^3,x]

[Out] a^3*x + a^2*b*x^3 + (3*a*b^2*x^5)/5 + (b^3*x^7)/7

fricas [A] time = 0.72, size = 31, normalized size = 0.89

$$\frac{1}{7}x^7b^3 + \frac{3}{5}x^5b^2a + x^3ba^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/7*x^7*b^3 + 3/5*x^5*b^2*a + x^3*b*a^2 + x*a^3

giac [A] time = 0.72, size = 31, normalized size = 0.89

$$\frac{1}{7}\,b^3x^7 + \frac{3}{5}\,ab^2x^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

maple [A] time = 0.00, size = 32, normalized size = 0.91

$$\frac{1}{7}b^3x^7 + \frac{3}{5}a\,b^2x^5 + a^2b\,x^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3,x)
```

```
[Out] a^3*x+a^2*b*x^3+3/5*a*b^2*x^5+1/7*b^3*x^7
```

maxima [A] time = 1.36, size = 31, normalized size = 0.89

$$\frac{1}{7}\,b^3x^7 + \frac{3}{5}\,ab^2x^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

mupad [B] time = 0.04, size = 31, normalized size = 0.89

$$a^3\,x + a^2\,b\,x^3 + \frac{3\,a\,b^2\,x^5}{5} + \frac{b^3\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3,x)
```

```
[Out] a^3*x + (b^3*x^7)/7 + a^2*b*x^3 + (3*a*b^2*x^5)/5
```

sympy [A] time = 0.06, size = 32, normalized size = 0.91

$$a^3x + a^2bx^3 + \frac{3ab^2x^5}{5} + \frac{b^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3,x)
```

```
[Out] a**3*x + a**2*b*x**3 + 3*a*b**2*x**5/5 + b**3*x**7/7
```

3.46 $\int \frac{(a+bx^2)^3}{x^2} dx$

Optimal. Leaf size=34

$$-\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}$$

```
[Out] -a^3/x+3*a^2*b*x+a*b^2*x^3+1/5*b^3*x^5
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$3a^2bx-\frac{a^3}{x}+ab^2x^3+\frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^2}\,dx &= \int \left(3a^2b+\frac{a^3}{x^2}+3ab^2x^2+b^3x^4\right)dx\\ &= -\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 1.00

$$-\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5
```

fricas [A] time = 0.83, size = 36, normalized size = 1.06

$$\frac{b^3x^6+5\,ab^2x^4+15\,a^2bx^2-5\,a^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(b^3*x^6 + 5*a*b^2*x^4 + 15*a^2*b*x^2 - 5*a^3)/x
```

giac [A] time = 1.04, size = 32, normalized size = 0.94

$$\frac{1}{5}\,b^3x^5+ab^2x^3+3\,a^2bx-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^2,x, algorithm="giac")
[Out] 1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x
```

maple [A] time = 0.00, size = 33, normalized size = 0.97

$$\frac{b^3x^5}{5} + a\,b^2x^3 + 3a^2bx - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^2,x)
[Out] -a^3/x+3*a^2*b*x+a*b^2*x^3+1/5*b^3*x^5
```

maxima [A] time = 1.38, size = 32, normalized size = 0.94

$$\frac{1}{5}\,b^3x^5 + ab^2x^3 + 3\,a^2bx - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^2,x, algorithm="maxima")
[Out] 1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x
```

mupad [B] time = 0.04, size = 32, normalized size = 0.94

$$\frac{b^3\,x^5}{5} - \frac{a^3}{x} + a\,b^2\,x^3 + 3\,a^2\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^2,x)
[Out] (b^3*x^5)/5 - a^3/x + a*b^2*x^3 + 3*a^2*b*x
```

sympy [A] time = 0.11, size = 29, normalized size = 0.85

$$-\frac{a^3}{x} + 3a^2bx + ab^2x^3 + \frac{b^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**2,x)
[Out] -a**3/x + 3*a**2*b*x + a*b**2*x**3 + b**3*x**5/5
```

3.47 $\int \frac{(a+bx^2)^3}{x^4} \, dx$

Optimal. Leaf size=37

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3ab^2x+\frac{b^3x^3}{3}$$

```
[Out] -1/3*a^3/x^3-3*a^2*b/x+3*a*b^2*x+1/3*b^3*x^3
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{3a^2b}{x}-\frac{a^3}{3x^3}+3ab^2x+\frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^4,x]
[Out] -a^3/(3*x^3) - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^4} \, dx &= \int \left(3ab^2+\frac{a^3}{x^4}+\frac{3a^2b}{x^2}+b^3x^2\right) dx\\ &= -\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3ab^2x+\frac{b^3x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 37, normalized size = 1.00

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3ab^2x+\frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^4,x]
[Out] -1/3*a^3/x^3 - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3
```

fricas [A] time = 0.79, size = 36, normalized size = 0.97

$$\frac{b^3x^6+9\,ab^2x^4-9\,a^2bx^2-a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^4,x, algorithm="fricas")
[Out] 1/3*(b^3*x^6 + 9*a*b^2*x^4 - 9*a^2*b*x^2 - a^3)/x^3
```

giac [A] time = 1.09, size = 34, normalized size = 0.92

$$\frac{1}{3}\,b^3x^3+3\,ab^2x-\frac{9\,a^2bx^2+a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^4,x, algorithm="giac")
[Out] 1/3*b^3*x^3 + 3*a*b^2*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

maple [A] time = 0.01, size = 34, normalized size = 0.92

$$\frac{b^3x^3}{3}+3a\,b^2x-\frac{3a^2b}{x}-\frac{a^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^4,x)
[Out] -1/3*a^3/x^3-3*a^2*b/x+3*a*b^2*x+1/3*b^3*x^3
```

maxima [A] time = 1.34, size = 34, normalized size = 0.92

$$\frac{1}{3}\,b^3x^3+3\,ab^2x-\frac{9\,a^2bx^2+a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^4,x, algorithm="maxima")
[Out] 1/3*b^3*x^3 + 3*a*b^2*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

mupad [B] time = 4.80, size = 36, normalized size = 0.97

$$\frac{b^3\,x^3}{3}-\frac{\frac{a^3}{3}+3\,b\,a^2\,x^2}{x^3}+3\,a\,b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^4,x)
[Out] (b^3*x^3)/3 - (a^3/3 + 3*a^2*b*x^2)/x^3 + 3*a*b^2*x
```

sympy [A] time = 0.15, size = 36, normalized size = 0.97

$$3ab^2x+\frac{b^3x^3}{3}+\frac{-a^3-9a^2bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**4,x)
[Out] 3*a*b**2*x + b**3*x**3/3 + (-a**3 - 9*a**2*b*x**2)/(3*x**3)
```

3.48 $\int \frac{(a+bx^2)^3}{x^6} \, dx$

Optimal. Leaf size=34

$$-\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3ab^2}{x}+b^3x$$

```
[Out] -1/5*a^3/x^5-a^2*b/x^3-3*a*b^2/x+b^3*x
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2b}{x^3}-\frac{a^3}{5x^5}-\frac{3ab^2}{x}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^6,x]
[Out] -a^3/(5*x^5) - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^6} \, dx &= \int \left(b^3+\frac{a^3}{x^6}+\frac{3a^2b}{x^4}+\frac{3ab^2}{x^2}\right) dx\\ &= -\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3ab^2}{x}+b^3x\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3ab^2}{x}+b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^6,x]
[Out] -1/5*a^3/x^5 - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x
```

fricas [A] time = 0.80, size = 37, normalized size = 1.09

$$\frac{5\,b^3x^6-15\,ab^2x^4-5\,a^2bx^2-a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^6,x, algorithm="fricas")
[Out] 1/5*(5*b^3*x^6 - 15*a*b^2*x^4 - 5*a^2*b*x^2 - a^3)/x^5
```

giac [A] time = 1.08, size = 33, normalized size = 0.97

$$b^3x-\frac{15\,ab^2x^4+5\,a^2bx^2+a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^6,x, algorithm="giac")
[Out] b^3*x - 1/5*(15*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/x^5
```

maple [A] time = 0.00, size = 33, normalized size = 0.97

$$b^3x - \frac{3a\,b^2}{x} - \frac{a^2b}{x^3} - \frac{a^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^6,x)
[Out] -1/5*a^3/x^5-a^2*b/x^3-3*a*b^2/x+b^3*x
```

maxima [A] time = 1.34, size = 33, normalized size = 0.97

$$b^3x - \frac{15\,ab^2x^4 + 5\,a^2bx^2 + a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^6,x, algorithm="maxima")
[Out] b^3*x - 1/5*(15*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/x^5
```

mupad [B] time = 0.03, size = 34, normalized size = 1.00

$$b^3\,x - \frac{\frac{a^3}{5} + a^2\,b\,x^2 + 3\,a\,b^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^6,x)
[Out] b^3*x - (a^3/5 + a^2*b*x^2 + 3*a*b^2*x^4)/x^5
```

sympy [A] time = 0.20, size = 34, normalized size = 1.00

$$b^3x + \frac{-a^3 - 5a^2bx^2 - 15ab^2x^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**6,x)
[Out] b**3*x + (-a**3 - 5*a**2*b*x**2 - 15*a*b**2*x**4)/(5*x**5)
```

3.49 $\int \frac{(a+bx^2)^3}{x^8}\,dx$

Optimal. Leaf size=39

$$-\frac{a^3}{7x^7}-\frac{3a^2b}{5x^5}-\frac{ab^2}{x^3}-\frac{b^3}{x}$$

```
[Out] -1/7*a^3/x^7-3/5*a^2*b/x^5-a*b^2/x^3-b^3/x
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{3a^2b}{5x^5}-\frac{a^3}{7x^7}-\frac{ab^2}{x^3}-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^8,x]
```

```
[Out] -a^3/(7*x^7) - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^8}\,dx &= \int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^4}+\frac{b^3}{x^2}\right)dx\\ &= -\frac{a^3}{7x^7}-\frac{3a^2b}{5x^5}-\frac{ab^2}{x^3}-\frac{b^3}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$-\frac{a^3}{7x^7}-\frac{3a^2b}{5x^5}-\frac{ab^2}{x^3}-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^8,x]
```

```
[Out] -1/7*a^3/x^7 - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x
```

fricas [A] time = 0.82, size = 37, normalized size = 0.95

$$-\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

giac [A] time = 1.09, size = 37, normalized size = 0.95

$$-\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^8,x, algorithm="giac")
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

maple [A] time = 0.00, size = 36, normalized size = 0.92

$$-\frac{b^3}{x} - \frac{a\,b^2}{x^3} - \frac{3a^2b}{5x^5} - \frac{a^3}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^8,x)
[Out] -1/7*a^3/x^7-3/5*a^2*b/x^5-a*b^2/x^3-b^3/x
```

maxima [A] time = 1.25, size = 37, normalized size = 0.95

$$-\frac{35\,b^3x^6 + 35\,ab^2x^4 + 21\,a^2bx^2 + 5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^8,x, algorithm="maxima")
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

mupad [B] time = 0.03, size = 35, normalized size = 0.90

$$-\frac{\frac{a^3}{7} + \frac{3\,a^2\,b\,x^2}{5} + a\,b^2\,x^4 + b^3\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^8,x)
[Out] -(a^3/7 + b^3*x^6 + (3*a^2*b*x^2)/5 + a*b^2*x^4)/x^7
```

sympy [A] time = 0.26, size = 39, normalized size = 1.00

$$\frac{-5a^3 - 21a^2bx^2 - 35ab^2x^4 - 35b^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**8,x)
[Out] (-5*a**3 - 21*a**2*b*x**2 - 35*a*b**2*x**4 - 35*b**3*x**6)/(35*x**7)
```

3.50 $\int \frac{\left(a+bx^2\right)^3}{x^{10}}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}$$

```
[Out] -1/9*a^3/x^9-3/7*a^2*b/x^7-3/5*a*b^2/x^5-1/3*b^3/x^3
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{3a^2b}{7x^7}-\frac{a^3}{9x^9}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^10,x]
[Out] -a^3/(9*x^9) - (3*a^2*b)/(7*x^7) - (3*a*b^2)/(5*x^5) - b^3/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^3}{x^{10}}\,dx &= \int \left(\frac{a^3}{x^{10}}+\frac{3a^2b}{x^8}+\frac{3ab^2}{x^6}+\frac{b^3}{x^4}\right)dx\\ &= -\frac{a^3}{9x^9}-\frac{3a^2b}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^10,x]
[Out] -1/9*a^3/x^9 - (3*a^2*b)/(7*x^7) - (3*a*b^2)/(5*x^5) - b^3/(3*x^3)
```

fricas [A] time = 0.82, size = 37, normalized size = 0.86

$$-\frac{105\,b^3x^6+189\,ab^2x^4+135\,a^2bx^2+35\,a^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^10,x, algorithm="fricas")
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

giac [A] time = 1.20, size = 37, normalized size = 0.86

$$-\frac{105\,b^3x^6+189\,ab^2x^4+135\,a^2bx^2+35\,a^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

maple [A] time = 0.01, size = 36, normalized size = 0.84

$$-\frac{b^3}{3x^3}-\frac{3a\,b^2}{5x^5}-\frac{3a^2b}{7x^7}-\frac{a^3}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^10,x)
```

```
[Out] -1/9*a^3/x^9-3/7*a^2*b/x^7-3/5*a*b^2/x^5-1/3*b^3/x^3
```

maxima [A] time = 1.49, size = 37, normalized size = 0.86

$$-\frac{105\,b^3x^6+189\,ab^2x^4+135\,a^2bx^2+35\,a^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

mupad [B] time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3}{9}+\frac{3\,a^2\,b\,x^2}{7}+\frac{3\,a\,b^2\,x^4}{5}+\frac{b^3\,x^6}{3}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^10,x)
```

```
[Out] -(a^3/9 + (b^3*x^6)/3 + (3*a^2*b*x^2)/7 + (3*a*b^2*x^4)/5)/x^9
```

sympy [A] time = 0.27, size = 39, normalized size = 0.91

$$\frac{-35a^3-135a^2bx^2-189ab^2x^4-105b^3x^6}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**10,x)
```

```
[Out] (-35*a**3 - 135*a**2*b*x**2 - 189*a*b**2*x**4 - 105*b**3*x**6)/(315*x**9)
```

3.51 $\int \frac{(a+bx^2)^3}{x^{12}}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{11x^{11}}-\frac{a^2b}{3x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}$$

```
[Out] -1/11*a^3/x^11-1/3*a^2*b/x^9-3/7*a*b^2/x^7-1/5*b^3/x^5
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2b}{3x^9}-\frac{a^3}{11x^{11}}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^12,x]
[Out] -a^3/(11*x^11) - (a^2*b)/(3*x^9) - (3*a*b^2)/(7*x^7) - b^3/(5*x^5)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^3}{x^{12}}\,dx &= \int \left(\frac{a^3}{x^{12}}+\frac{3a^2b}{x^{10}}+\frac{3ab^2}{x^8}+\frac{b^3}{x^6}\right)dx\\ &= -\frac{a^3}{11x^{11}}-\frac{a^2b}{3x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$-\frac{a^3}{11x^{11}}-\frac{a^2b}{3x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^12,x]
[Out] -1/11*a^3/x^11 - (a^2*b)/(3*x^9) - (3*a*b^2)/(7*x^7) - b^3/(5*x^5)
```

fricas [A] time = 0.74, size = 37, normalized size = 0.86

$$-\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^12,x, algorithm="fricas")
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

giac [A] time = 1.03, size = 37, normalized size = 0.86

$$-\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^12,x, algorithm="giac")
```

```
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$-\frac{b^3}{5x^5}-\frac{3a\,b^2}{7x^7}-\frac{a^2b}{3x^9}-\frac{a^3}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^12,x)
```

```
[Out] -1/11*a^3/x^11-1/3*a^2*b/x^9-3/7*a*b^2/x^7-1/5*b^3/x^5
```

maxima [A] time = 1.42, size = 37, normalized size = 0.86

$$-\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^12,x, algorithm="maxima")
```

```
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

mupad [B] time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3}{11}+\frac{a^2\,b\,x^2}{3}+\frac{3\,a\,b^2\,x^4}{7}+\frac{b^3\,x^6}{5}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^12,x)
```

```
[Out] -(a^3/11 + (b^3*x^6)/5 + (a^2*b*x^2)/3 + (3*a*b^2*x^4)/7)/x^11
```

sympy [A] time = 0.30, size = 39, normalized size = 0.91

$$\frac{-105a^3-385a^2bx^2-495ab^2x^4-231b^3x^6}{1155x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**12,x)
```

```
[Out] (-105*a**3 - 385*a**2*b*x**2 - 495*a*b**2*x**4 - 231*b**3*x**6)/(1155*x**11
)
```

3.52 $\int x^{13} \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=69

$$\frac{a^5x^{14}}{14}+\frac{5}{16}a^4bx^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}$$

```
[Out] 1/14*a^5*x^14+5/16*a^4*b*x^16+5/9*a^3*b^2*x^18+1/2*a^2*b^3*x^20+5/22*a*b^4*
x^22+1/24*b^5*x^24
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{2}a^2b^3x^{20}+\frac{5}{9}a^3b^2x^{18}+\frac{5}{16}a^4bx^{16}+\frac{a^5x^{14}}{14}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^14)/14 + (5*a^4*b*x^16)/16 + (5*a^3*b^2*x^18)/9 + (a^2*b^3*x^20)/2 +
 (5*a*b^4*x^22)/22 + (b^5*x^24)/24
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{13}\left(a+bx^2\right)^5\,dx &= \frac{1}{2}\text{Subst}\left(\int x^6(a+bx)^5\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^5x^6+5a^4bx^7+10a^3b^2x^8+10a^2b^3x^9+5ab^4x^{10}+b^5x^{11}\right)dx,x,x^2\right)\\ &= \frac{a^5x^{14}}{14}+\frac{5}{16}a^4bx^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^{14}}{14}+\frac{5}{16}a^4bx^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^14)/14 + (5*a^4*b*x^16)/16 + (5*a^3*b^2*x^18)/9 + (a^2*b^3*x^20)/2 +
 (5*a*b^4*x^22)/22 + (b^5*x^24)/24
```

fricas [A] time = 0.74, size = 57, normalized size = 0.83

$$\frac{1}{24}x^{24}b^5 + \frac{5}{22}x^{22}b^4a + \frac{1}{2}x^{20}b^3a^2 + \frac{5}{9}x^{18}b^2a^3 + \frac{5}{16}x^{16}ba^4 + \frac{1}{14}x^{14}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*b^5 + 5/22*x^22*b^4*a + 1/2*x^20*b^3*a^2 + 5/9*x^18*b^2*a^3 + 5/1
6*x^16*b*a^4 + 1/14*x^14*a^5
```

giac [A] time = 0.87, size = 57, normalized size = 0.83

$$\frac{1}{24}\,b^5x^{24} + \frac{5}{22}\,ab^4x^{22} + \frac{1}{2}\,a^2b^3x^{20} + \frac{5}{9}\,a^3b^2x^{18} + \frac{5}{16}\,a^4bx^{16} + \frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{24}b^5x^{24} + \frac{5}{22}a\,b^4x^{22} + \frac{1}{2}a^2b^3x^{20} + \frac{5}{9}a^3b^2x^{18} + \frac{5}{16}a^4b\,x^{16} + \frac{1}{14}a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b*x^2+a)^5,x)
```

```
[Out] 1/14*a^5*x^14+5/16*a^4*b*x^16+5/9*a^3*b^2*x^18+1/2*a^2*b^3*x^20+5/22*a*b^4*
x^22+1/24*b^5*x^24
```

maxima [A] time = 1.43, size = 57, normalized size = 0.83

$$\frac{1}{24}\,b^5x^{24} + \frac{5}{22}\,ab^4x^{22} + \frac{1}{2}\,a^2b^3x^{20} + \frac{5}{9}\,a^3b^2x^{18} + \frac{5}{16}\,a^4bx^{16} + \frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

mupad [B] time = 0.03, size = 57, normalized size = 0.83

$$\frac{a^5\,x^{14}}{14} + \frac{5\,a^4\,b\,x^{16}}{16} + \frac{5\,a^3\,b^2\,x^{18}}{9} + \frac{a^2\,b^3\,x^{20}}{2} + \frac{5\,a\,b^4\,x^{22}}{22} + \frac{b^5\,x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^14)/14 + (b^5*x^24)/24 + (5*a^4*b*x^16)/16 + (5*a*b^4*x^22)/22 + (5*
a^3*b^2*x^18)/9 + (a^2*b^3*x^20)/2
```

sympy [A] time = 0.08, size = 65, normalized size = 0.94

$$\frac{a^5x^{14}}{14} + \frac{5a^4bx^{16}}{16} + \frac{5a^3b^2x^{18}}{9} + \frac{a^2b^3x^{20}}{2} + \frac{5ab^4x^{22}}{22} + \frac{b^5x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**14/14 + 5*a**4*b*x**16/16 + 5*a**3*b**2*x**18/9 + a**2*b**3*x**20/2
 + 5*a*b**4*x**22/22 + b**5*x**24/24
```

3.53 $\int x^{11}\left(a+bx^2\right)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^{12}}{12}+\frac{5}{14}a^4bx^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}$$

```
[Out] 1/12*a^5*x^12+5/14*a^4*b*x^14+5/8*a^3*b^2*x^16+5/9*a^2*b^3*x^18+1/4*a*b^4*x
^20+1/22*b^5*x^22
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{5}{9}a^2b^3x^{18}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{14}a^4bx^{14}+\frac{a^5x^{12}}{12}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^12)/12 + (5*a^4*b*x^14)/14 + (5*a^3*b^2*x^16)/8 + (5*a^2*b^3*x^18)/9
 + (a*b^4*x^20)/4 + (b^5*x^22)/22
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^2\right)^5\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int x^5(a+bx)^5\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(a^5x^5+5a^4bx^6+10a^3b^2x^7+10a^2b^3x^8+5ab^4x^9+b^5x^{10}\right)dx,x,x^2\right)\\&=\frac{a^5x^{12}}{12}+\frac{5}{14}a^4bx^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^{12}}{12}+\frac{5}{14}a^4bx^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^12)/12 + (5*a^4*b*x^14)/14 + (5*a^3*b^2*x^16)/8 + (5*a^2*b^3*x^18)/9
 + (a*b^4*x^20)/4 + (b^5*x^22)/22
```

fricas [A] time = 0.72, size = 57, normalized size = 0.83

$$\frac{1}{22}x^{22}b^5+\frac{1}{4}x^{20}b^4a+\frac{5}{9}x^{18}b^3a^2+\frac{5}{8}x^{16}b^2a^3+\frac{5}{14}x^{14}ba^4+\frac{1}{12}x^{12}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^5 + 1/4*x^20*b^4*a + 5/9*x^18*b^3*a^2 + 5/8*x^16*b^2*a^3 + 5/14
*x^14*b*a^4 + 1/12*x^12*a^5
```

giac [A] time = 1.01, size = 57, normalized size = 0.83

$$\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14
*a^4*b*x^14 + 1/12*a^5*x^12
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{22}b^5x^{22}+\frac{1}{4}a\,b^4x^{20}+\frac{5}{9}a^2b^3x^{18}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{14}a^4b\,x^{14}+\frac{1}{12}a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^2+a)^5,x)
```

```
[Out] 1/12*a^5*x^12+5/14*a^4*b*x^14+5/8*a^3*b^2*x^16+5/9*a^2*b^3*x^18+1/4*a*b^4*x
^20+1/22*b^5*x^22
```

maxima [A] time = 1.36, size = 57, normalized size = 0.83

$$\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14
*a^4*b*x^14 + 1/12*a^5*x^12
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^{12}}{12}+\frac{5\,a^4\,b\,x^{14}}{14}+\frac{5\,a^3\,b^2\,x^{16}}{8}+\frac{5\,a^2\,b^3\,x^{18}}{9}+\frac{a\,b^4\,x^{20}}{4}+\frac{b^5\,x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^12)/12 + (b^5*x^22)/22 + (5*a^4*b*x^14)/14 + (a*b^4*x^20)/4 + (5*a^3
*b^2*x^16)/8 + (5*a^2*b^3*x^18)/9
```

sympy [A] time = 0.08, size = 65, normalized size = 0.94

$$\frac{a^5x^{12}}{12}+\frac{5a^4bx^{14}}{14}+\frac{5a^3b^2x^{16}}{8}+\frac{5a^2b^3x^{18}}{9}+\frac{ab^4x^{20}}{4}+\frac{b^5x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**12/12 + 5*a**4*b*x**14/14 + 5*a**3*b**2*x**16/8 + 5*a**2*b**3*x**18
/9 + a*b**4*x**20/4 + b**5*x**22/22
```

3.54 $\int x^9 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=69

$$\frac{a^5x^{10}}{10}+\frac{5}{12}a^4bx^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}$$

```
[Out] 1/10*a^5*x^10+5/12*a^4*b*x^12+5/7*a^3*b^2*x^14+5/8*a^2*b^3*x^16+5/18*a*b^4*
x^18+1/20*b^5*x^20
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{5}{8}a^2b^3x^{16}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{12}a^4bx^{12}+\frac{a^5x^{10}}{10}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^10)/10 + (5*a^4*b*x^12)/12 + (5*a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/8
 + (5*a*b^4*x^18)/18 + (b^5*x^20)/20
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^9\left(a+bx^2\right)^5 dx &= \frac{1}{2}\text{Subst}\left(\int x^4(a+bx)^5\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^5x^4+5a^4bx^5+10a^3b^2x^6+10a^2b^3x^7+5ab^4x^8+b^5x^9\right)dx,x,x^2\right)\\
&= \frac{a^5x^{10}}{10}+\frac{5}{12}a^4bx^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^{10}}{10}+\frac{5}{12}a^4bx^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^10)/10 + (5*a^4*b*x^12)/12 + (5*a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/8
 + (5*a*b^4*x^18)/18 + (b^5*x^20)/20
```

fricas [A] time = 0.61, size = 57, normalized size = 0.83

$$\frac{1}{20}x^{20}b^5 + \frac{5}{18}x^{18}b^4a + \frac{5}{8}x^{16}b^3a^2 + \frac{5}{7}x^{14}b^2a^3 + \frac{5}{12}x^{12}ba^4 + \frac{1}{10}x^{10}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^5 + 5/18*x^18*b^4*a + 5/8*x^16*b^3*a^2 + 5/7*x^14*b^2*a^3 + 5/1
2*x^12*b*a^4 + 1/10*x^10*a^5
```

giac [A] time = 1.16, size = 57, normalized size = 0.83

$$\frac{1}{20}\,b^5x^{20} + \frac{5}{18}\,ab^4x^{18} + \frac{5}{8}\,a^2b^3x^{16} + \frac{5}{7}\,a^3b^2x^{14} + \frac{5}{12}\,a^4bx^{12} + \frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{20}b^5x^{20} + \frac{5}{18}a\,b^4x^{18} + \frac{5}{8}a^2b^3x^{16} + \frac{5}{7}a^3b^2x^{14} + \frac{5}{12}a^4b\,x^{12} + \frac{1}{10}a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^5,x)
```

```
[Out] 1/10*a^5*x^10+5/12*a^4*b*x^12+5/7*a^3*b^2*x^14+5/8*a^2*b^3*x^16+5/18*a*b^4*
x^18+1/20*b^5*x^20
```

maxima [A] time = 1.37, size = 57, normalized size = 0.83

$$\frac{1}{20}\,b^5x^{20} + \frac{5}{18}\,ab^4x^{18} + \frac{5}{8}\,a^2b^3x^{16} + \frac{5}{7}\,a^3b^2x^{14} + \frac{5}{12}\,a^4bx^{12} + \frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^{10}}{10} + \frac{5\,a^4\,b\,x^{12}}{12} + \frac{5\,a^3\,b^2\,x^{14}}{7} + \frac{5\,a^2\,b^3\,x^{16}}{8} + \frac{5\,a\,b^4\,x^{18}}{18} + \frac{b^5\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^10)/10 + (b^5*x^20)/20 + (5*a^4*b*x^12)/12 + (5*a*b^4*x^18)/18 + (5*
a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/8
```

sympy [A] time = 0.08, size = 66, normalized size = 0.96

$$\frac{a^5x^{10}}{10} + \frac{5a^4bx^{12}}{12} + \frac{5a^3b^2x^{14}}{7} + \frac{5a^2b^3x^{16}}{8} + \frac{5ab^4x^{18}}{18} + \frac{b^5x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**10/10 + 5*a**4*b*x**12/12 + 5*a**3*b**2*x**14/7 + 5*a**2*b**3*x**16
/8 + 5*a*b**4*x**18/18 + b**5*x**20/20
```

3.55 $\int x^7 \left(a+bx^2\right)^5 dx$

Optimal. Leaf size=72

$$-\frac{a^3 \left(a+bx^2\right)^6}{12b^4}+\frac{3a^2 \left(a+bx^2\right)^7}{14b^4}+\frac{\left(a+bx^2\right)^9}{18b^4}-\frac{3a \left(a+bx^2\right)^8}{16b^4}$$

```
[Out] -1/12*a^3*(b*x^2+a)^6/b^4+3/14*a^2*(b*x^2+a)^7/b^4-3/16*a*(b*x^2+a)^8/b^4+1
/18*(b*x^2+a)^9/b^4
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2 \left(a+bx^2\right)^7}{14b^4}-\frac{a^3 \left(a+bx^2\right)^6}{12b^4}+\frac{\left(a+bx^2\right)^9}{18b^4}-\frac{3a \left(a+bx^2\right)^8}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^5,x]
```

```
[Out] -(a^3*(a + b*x^2)^6)/(12*b^4) + (3*a^2*(a + b*x^2)^7)/(14*b^4) - (3*a*(a +
b*x^2)^8)/(16*b^4) + (a + b*x^2)^9/(18*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a+bx^2\right)^5 dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^3(a+bx)^5 \, dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a^3(a+bx)^5}{b^3}+\frac{3a^2(a+bx)^6}{b^3}-\frac{3a(a+bx)^7}{b^3}+\frac{(a+bx)^8}{b^3}\right) dx, x, x^2\right)\\
&= -\frac{a^3 \left(a+bx^2\right)^6}{12b^4}+\frac{3a^2 \left(a+bx^2\right)^7}{14b^4}-\frac{3a \left(a+bx^2\right)^8}{16b^4}+\frac{\left(a+bx^2\right)^9}{18b^4}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 0.96

$$\frac{a^5x^8}{8}+\frac{1}{2}a^4bx^{10}+\frac{5}{6}a^3b^2x^{12}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{16}ab^4x^{16}+\frac{b^5x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^8)/8 + (a^4*b*x^10)/2 + (5*a^3*b^2*x^12)/6 + (5*a^2*b^3*x^14)/7 + (5
*a*b^4*x^16)/16 + (b^5*x^18)/18
```

fricas [A] time = 0.74, size = 57, normalized size = 0.79

$$\frac{1}{18}x^{18}b^5+\frac{5}{16}x^{16}b^4a+\frac{5}{7}x^{14}b^3a^2+\frac{5}{6}x^{12}b^2a^3+\frac{1}{2}x^{10}ba^4+\frac{1}{8}x^8a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^5 + 5/16*x^16*b^4*a + 5/7*x^14*b^3*a^2 + 5/6*x^12*b^2*a^3 + 1/2
*x^10*b*a^4 + 1/8*x^8*a^5
```

giac [A] time = 1.02, size = 57, normalized size = 0.79

$$\frac{1}{18}\,b^5x^{18}+\frac{5}{16}\,ab^4x^{16}+\frac{5}{7}\,a^2b^3x^{14}+\frac{5}{6}\,a^3b^2x^{12}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

maple [A] time = 0.00, size = 58, normalized size = 0.81

$$\frac{1}{18}b^5x^{18}+\frac{5}{16}a\,b^4x^{16}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{6}a^3b^2x^{12}+\frac{1}{2}a^4b\,x^{10}+\frac{1}{8}a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^5,x)
```

```
[Out] 1/18*b^5*x^18+5/16*a*b^4*x^16+5/7*a^2*b^3*x^14+5/6*a^3*b^2*x^12+1/2*a^4*b*x
^10+1/8*a^5*x^8
```

maxima [A] time = 1.37, size = 57, normalized size = 0.79

$$\frac{1}{18}\,b^5x^{18}+\frac{5}{16}\,ab^4x^{16}+\frac{5}{7}\,a^2b^3x^{14}+\frac{5}{6}\,a^3b^2x^{12}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

mupad [B] time = 0.02, size = 57, normalized size = 0.79

$$\frac{a^5\,x^8}{8}+\frac{a^4\,b\,x^{10}}{2}+\frac{5\,a^3\,b^2\,x^{12}}{6}+\frac{5\,a^2\,b^3\,x^{14}}{7}+\frac{5\,a\,b^4\,x^{16}}{16}+\frac{b^5\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^8)/8 + (b^5*x^18)/18 + (a^4*b*x^10)/2 + (5*a*b^4*x^16)/16 + (5*a^3*b
^2*x^12)/6 + (5*a^2*b^3*x^14)/7
```

sympy [A] time = 0.09, size = 65, normalized size = 0.90

$$\frac{a^5x^8}{8}+\frac{a^4bx^{10}}{2}+\frac{5a^3b^2x^{12}}{6}+\frac{5a^2b^3x^{14}}{7}+\frac{5ab^4x^{16}}{16}+\frac{b^5x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**8/8 + a**4*b*x**10/2 + 5*a**3*b**2*x**12/6 + 5*a**2*b**3*x**14/7 +
5*a*b**4*x**16/16 + b**5*x**18/18
```

3.56 $\int x^5 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=53

$$\frac{a^2 \left(a+bx^2\right)^6}{12b^3}+\frac{\left(a+bx^2\right)^8}{16b^3}-\frac{a \left(a+bx^2\right)^7}{7b^3}$$

```
[Out] 1/12*a^2*(b*x^2+a)^6/b^3-1/7*a*(b*x^2+a)^7/b^3+1/16*(b*x^2+a)^8/b^3
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^2 \left(a+bx^2\right)^6}{12b^3}+\frac{\left(a+bx^2\right)^8}{16b^3}-\frac{a \left(a+bx^2\right)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^5,x]
```

```
[Out] (a^2*(a + b*x^2)^6)/(12*b^3) - (a*(a + b*x^2)^7)/(7*b^3) + (a + b*x^2)^8/(1
6*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^2\right)^5 \, dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^5 \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2(a+bx)^5}{b^2}-\frac{2a(a+bx)^6}{b^2}+\frac{(a+bx)^7}{b^2}\right) dx, x, x^2\right)\\
&= \frac{a^2 \left(a+bx^2\right)^6}{12b^3}-\frac{a \left(a+bx^2\right)^7}{7b^3}+\frac{\left(a+bx^2\right)^8}{16b^3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.25

$$\frac{a^5x^6}{6}+\frac{5}{8}a^4bx^8+a^3b^2x^{10}+\frac{5}{6}a^2b^3x^{12}+\frac{5}{14}ab^4x^{14}+\frac{b^5x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^8)/8 + a^3*b^2*x^10 + (5*a^2*b^3*x^12)/6 + (5*a*b^
4*x^14)/14 + (b^5*x^16)/16
```

fricas [A] time = 0.68, size = 56, normalized size = 1.06

$$\frac{1}{16}x^{16}b^5+\frac{5}{14}x^{14}b^4a+\frac{5}{6}x^{12}b^3a^2+x^{10}b^2a^3+\frac{5}{8}x^8ba^4+\frac{1}{6}x^6a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^5 + 5/14*x^14*b^4*a + 5/6*x^12*b^3*a^2 + x^10*b^2*a^3 + 5/8*x^8
*b*a^4 + 1/6*x^6*a^5
```

giac [A] time = 1.06, size = 56, normalized size = 1.06

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

maple [A] time = 0.00, size = 57, normalized size = 1.08

$$\frac{1}{16}b^5x^{16}+\frac{5}{14}a\,b^4x^{14}+\frac{5}{6}a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}a^4b\,x^8+\frac{1}{6}a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^5,x)
```

```
[Out] 1/16*b^5*x^16+5/14*a*b^4*x^14+5/6*a^2*b^3*x^12+a^3*b^2*x^10+5/8*a^4*b*x^8+1
/6*a^5*x^6
```

maxima [A] time = 1.27, size = 56, normalized size = 1.06

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

mupad [B] time = 0.02, size = 56, normalized size = 1.06

$$\frac{a^5\,x^6}{6}+\frac{5\,a^4\,b\,x^8}{8}+a^3\,b^2\,x^{10}+\frac{5\,a^2\,b^3\,x^{12}}{6}+\frac{5\,a\,b^4\,x^{14}}{14}+\frac{b^5\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^6)/6 + (b^5*x^16)/16 + (5*a^4*b*x^8)/8 + (5*a*b^4*x^14)/14 + a^3*b^2
*x^10 + (5*a^2*b^3*x^12)/6
```

sympy [A] time = 0.08, size = 63, normalized size = 1.19

$$\frac{a^5x^6}{6}+\frac{5a^4bx^8}{8}+a^3b^2x^{10}+\frac{5a^2b^3x^{12}}{6}+\frac{5ab^4x^{14}}{14}+\frac{b^5x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**6/6 + 5*a**4*b*x**8/8 + a**3*b**2*x**10 + 5*a**2*b**3*x**12/6 + 5*a
*b**4*x**14/14 + b**5*x**16/16
```

3.57 $\int x^3 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^7}{14b^2}-\frac{a\left(a+bx^2\right)^6}{12b^2}$$

```
[Out] -1/12*a*(b*x^2+a)^6/b^2+1/14*(b*x^2+a)^7/b^2
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^7}{14b^2}-\frac{a\left(a+bx^2\right)^6}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^5,x]
```

```
[Out] -(a*(a + b*x^2)^6)/(12*b^2) + (a + b*x^2)^7/(14*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a+bx^2\right)^5 \, dx &= \frac{1}{2} \text{Subst}\left(\int x(a+bx)^5 \, dx, x, x^2\right)\\ &= \frac{1}{2} \text{Subst}\left(\int \left(-\frac{a(a+bx)^5}{b}+\frac{(a+bx)^6}{b}\right) dx, x, x^2\right)\\ &= -\frac{a\left(a+bx^2\right)^6}{12b^2}+\frac{\left(a+bx^2\right)^7}{14b^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.94

$$\frac{a^5x^4}{4}+\frac{5}{6}a^4bx^6+\frac{5}{4}a^3b^2x^8+a^2b^3x^{10}+\frac{5}{12}ab^4x^{12}+\frac{b^5x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^4)/4 + (5*a^4*b*x^6)/6 + (5*a^3*b^2*x^8)/4 + a^2*b^3*x^10 + (5*a*b^4
*x^12)/12 + (b^5*x^14)/14
```

fricas [A] time = 0.73, size = 56, normalized size = 1.65

$$\frac{1}{14}x^{14}b^5 + \frac{5}{12}x^{12}b^4a + x^{10}b^3a^2 + \frac{5}{4}x^8b^2a^3 + \frac{5}{6}x^6ba^4 + \frac{1}{4}x^4a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^5 + 5/12*x^12*b^4*a + x^10*b^3*a^2 + 5/4*x^8*b^2*a^3 + 5/6*x^6*
b*a^4 + 1/4*x^4*a^5
```

giac [A] time = 1.18, size = 56, normalized size = 1.65

$$\frac{1}{14}\,b^5x^{14} + \frac{5}{12}\,ab^4x^{12} + a^2b^3x^{10} + \frac{5}{4}\,a^3b^2x^8 + \frac{5}{6}\,a^4bx^6 + \frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

maple [A] time = 0.00, size = 57, normalized size = 1.68

$$\frac{1}{14}b^5x^{14} + \frac{5}{12}a\,b^4x^{12} + a^2b^3x^{10} + \frac{5}{4}a^3b^2x^8 + \frac{5}{6}a^4b\,x^6 + \frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^5,x)
```

```
[Out] 1/14*b^5*x^14+5/12*a*b^4*x^12+a^2*b^3*x^10+5/4*a^3*b^2*x^8+5/6*a^4*b*x^6+1/
4*a^5*x^4
```

maxima [A] time = 1.36, size = 56, normalized size = 1.65

$$\frac{1}{14}\,b^5x^{14} + \frac{5}{12}\,ab^4x^{12} + a^2b^3x^{10} + \frac{5}{4}\,a^3b^2x^8 + \frac{5}{6}\,a^4bx^6 + \frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

mupad [B] time = 0.02, size = 56, normalized size = 1.65

$$\frac{a^5\,x^4}{4} + \frac{5\,a^4\,b\,x^6}{6} + \frac{5\,a^3\,b^2\,x^8}{4} + a^2\,b^3\,x^{10} + \frac{5\,a\,b^4\,x^{12}}{12} + \frac{b^5\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^4)/4 + (b^5*x^14)/14 + (5*a^4*b*x^6)/6 + (5*a*b^4*x^12)/12 + (5*a^3*
b^2*x^8)/4 + a^2*b^3*x^10
```

sympy [B] time = 0.08, size = 63, normalized size = 1.85

$$\frac{a^5x^4}{4} + \frac{5a^4bx^6}{6} + \frac{5a^3b^2x^8}{4} + a^2b^3x^{10} + \frac{5ab^4x^{12}}{12} + \frac{b^5x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**4/4 + 5*a**4*b*x**6/6 + 5*a**3*b**2*x**8/4 + a**2*b**3*x**10 + 5*a*
b**4*x**12/12 + b**5*x**14/14
```

3.58 $\int x\left(a+bx^2\right)^5\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^6}{12b}$$

[Out] 1/12*(b*x^2+a)^6/b

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^6}{12b}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^2)^5,x]

[Out] (a + b*x^2)^6/(12*b)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^5\,dx=\frac{\left(a+bx^2\right)^6}{12b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^6}{12b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)^5,x]

[Out] (a + b*x^2)^6/(12*b)

fricas [B] time = 0.77, size = 57, normalized size = 3.56

$$\frac{1}{12}x^{12}b^5+\frac{1}{2}x^{10}b^4a+\frac{5}{4}x^8b^3a^2+\frac{5}{3}x^6b^2a^3+\frac{5}{4}x^4ba^4+\frac{1}{2}x^2a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^2+a)^5,x, algorithm="fricas")

[Out] 1/12*x^12*b^5 + 1/2*x^10*b^4*a + 5/4*x^8*b^3*a^2 + 5/3*x^6*b^2*a^3 + 5/4*x^4*b*a^4 + 1/2*x^2*a^5

giac [A] time = 1.06, size = 14, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^6}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/12*(b*x^2 + a)^6/b
```

maple [B] time = 0.00, size = 58, normalized size = 3.62

$$\frac{1}{12}b^5x^{12}+\frac{1}{2}a\,b^4x^{10}+\frac{5}{4}a^2b^3x^8+\frac{5}{3}a^3b^2x^6+\frac{5}{4}a^4b\,x^4+\frac{1}{2}a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^5,x)
```

```
[Out] 1/12*b^5*x^12+1/2*a*b^4*x^10+5/4*a^2*b^3*x^8+5/3*a^3*b^2*x^6+5/4*a^4*b*x^4+
1/2*a^5*x^2
```

maxima [A] time = 1.40, size = 14, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^6}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(b*x^2 + a)^6/b
```

mupad [B] time = 0.02, size = 57, normalized size = 3.56

$$\frac{a^5\,x^2}{2}+\frac{5\,a^4\,b\,x^4}{4}+\frac{5\,a^3\,b^2\,x^6}{3}+\frac{5\,a^2\,b^3\,x^8}{4}+\frac{a\,b^4\,x^{10}}{2}+\frac{b^5\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^2)/2 + (b^5*x^12)/12 + (5*a^4*b*x^4)/4 + (a*b^4*x^10)/2 + (5*a^3*b^2
*x^6)/3 + (5*a^2*b^3*x^8)/4
```

sympy [B] time = 0.08, size = 65, normalized size = 4.06

$$\frac{a^5x^2}{2}+\frac{5a^4bx^4}{4}+\frac{5a^3b^2x^6}{3}+\frac{5a^2b^3x^8}{4}+\frac{ab^4x^{10}}{2}+\frac{b^5x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**2/2 + 5*a**4*b*x**4/4 + 5*a**3*b**2*x**6/3 + 5*a**2*b**3*x**8/4 + a
*b**4*x**10/2 + b**5*x**12/12
```

3.59 $\int \frac{(a+bx^2)^5}{x} \, dx$

Optimal. Leaf size=65

$$a^5 \log(x) + \frac{5}{2}a^4bx^2 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^2b^3x^6 + \frac{5}{8}ab^4x^8 + \frac{b^5x^{10}}{10}$$

```
[Out] 5/2*a^4*b*x^2+5/2*a^3*b^2*x^4+5/3*a^2*b^3*x^6+5/8*a*b^4*x^8+1/10*b^5*x^10+a
^5*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{5}{3}a^2b^3x^6 + \frac{5}{2}a^3b^2x^4 + \frac{5}{2}a^4bx^2 + a^5 \log(x) + \frac{5}{8}ab^4x^8 + \frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x,x]
```

```
[Out] (5*a^4*b*x^2)/2 + (5*a^3*b^2*x^4)/2 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^8)/8 +
 (b^5*x^10)/10 + a^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{(a+bx)^5}{x} \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(5a^4b + \frac{a^5}{x} + 10a^3b^2x + 10a^2b^3x^2 + 5ab^4x^3 + b^5x^4\right) dx, x, x^2\right) \\
&= \frac{5}{2}a^4bx^2 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^2b^3x^6 + \frac{5}{8}ab^4x^8 + \frac{b^5x^{10}}{10} + a^5 \log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 65, normalized size = 1.00

$$a^5 \log(x) + \frac{5}{2}a^4bx^2 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^2b^3x^6 + \frac{5}{8}ab^4x^8 + \frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x,x]
```

```
[Out] (5*a^4*b*x^2)/2 + (5*a^3*b^2*x^4)/2 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^8)/8 +
 (b^5*x^10)/10 + a^5*Log[x]
```

fricas [A] time = 0.59, size = 55, normalized size = 0.85

$$\frac{1}{10}\,b^5x^{10} + \frac{5}{8}\,ab^4x^8 + \frac{5}{3}\,a^2b^3x^6 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{2}\,a^4bx^2 + a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + a^5*log(x)
```

giac [A] time = 0.92, size = 58, normalized size = 0.89

$$\frac{1}{10}\,b^5x^{10} + \frac{5}{8}\,ab^4x^8 + \frac{5}{3}\,a^2b^3x^6 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{2}\,a^4bx^2 + \frac{1}{2}\,a^5\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + 1/2*a^5*log(x^2)
```

maple [A] time = 0.00, size = 56, normalized size = 0.86

$$\frac{b^5x^{10}}{10} + \frac{5a\,b^4x^8}{8} + \frac{5a^2b^3x^6}{3} + \frac{5a^3b^2x^4}{2} + \frac{5a^4b\,x^2}{2} + a^5\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x,x)
```

```
[Out] 5/2*a^4*b*x^2+5/2*a^3*b^2*x^4+5/3*a^2*b^3*x^6+5/8*a*b^4*x^8+1/10*b^5*x^10+a
^5*ln(x)
```

maxima [A] time = 1.37, size = 58, normalized size = 0.89

$$\frac{1}{10}\,b^5x^{10} + \frac{5}{8}\,ab^4x^8 + \frac{5}{3}\,a^2b^3x^6 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{2}\,a^4bx^2 + \frac{1}{2}\,a^5\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + 1/2*a^5*log(x^2)
```

mupad [B] time = 0.03, size = 55, normalized size = 0.85

$$a^5\,\ln(x) + \frac{b^5\,x^{10}}{10} + \frac{5\,a^4\,b\,x^2}{2} + \frac{5\,a\,b^4\,x^8}{8} + \frac{5\,a^3\,b^2\,x^4}{2} + \frac{5\,a^2\,b^3\,x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x,x)
```

```
[Out] a^5*log(x) + (b^5*x^10)/10 + (5*a^4*b*x^2)/2 + (5*a*b^4*x^8)/8 + (5*a^3*b^2
*x^4)/2 + (5*a^2*b^3*x^6)/3
```

sympy [A] time = 0.14, size = 65, normalized size = 1.00

$$a^5\log\left(x\right) + \frac{5a^4bx^2}{2} + \frac{5a^3b^2x^4}{2} + \frac{5a^2b^3x^6}{3} + \frac{5ab^4x^8}{8} + \frac{b^5x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x,x)
```

```
[Out] a**5*log(x) + 5*a**4*b*x**2/2 + 5*a**3*b**2*x**4/2 + 5*a**2*b**3*x**6/3 + 5
*a*b**4*x**8/8 + b**5*x**10/10
```

3.60 $\int \frac{(a+bx^2)^5}{x^3} \, dx$

Optimal. Leaf size=64

$$-\frac{a^5}{2x^2}+5a^4b\log(x)+5a^3b^2x^2+\frac{5}{2}a^2b^3x^4+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}$$

```
[Out] -1/2*a^5/x^2+5*a^3*b^2*x^2+5/2*a^2*b^3*x^4+5/6*a*b^4*x^6+1/8*b^5*x^8+5*a^4*
b*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{5}{2}a^2b^3x^4+5a^3b^2x^2+5a^4b\log(x)-\frac{a^5}{2x^2}+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^3,x]
```

```
[Out] -a^5/(2*x^2) + 5*a^3*b^2*x^2 + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^6)/6 + (b^5*x
^8)/8 + 5*a^4*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^5}{x^3} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^2} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(10a^3b^2+\frac{a^5}{x^2}+\frac{5a^4b}{x}+10a^2b^3x+5ab^4x^2+b^5x^3\right) dx, x, x^2\right)\\
&= -\frac{a^5}{2x^2}+5a^3b^2x^2+\frac{5}{2}a^2b^3x^4+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}+5a^4b\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 64, normalized size = 1.00

$$-\frac{a^5}{2x^2}+5a^4b\log(x)+5a^3b^2x^2+\frac{5}{2}a^2b^3x^4+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^3,x]
```

```
[Out] -1/2*a^5/x^2 + 5*a^3*b^2*x^2 + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^6)/6 + (b^5*x
^8)/8 + 5*a^4*b*Log[x]
```

fricas [A] time = 0.79, size = 61, normalized size = 0.95

$$\frac{3\,b^5x^{10}+20\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4+120\,a^4bx^2\log(x)-12\,a^5}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 + 20*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4 + 120*a^
4*b*x^2*log(x) - 12*a^5)/x^2
```

giac [A] time = 1.06, size = 68, normalized size = 1.06

$$\frac{1}{8}\,b^5x^8+\frac{5}{6}\,ab^4x^6+\frac{5}{2}\,a^2b^3x^4+5\,a^3b^2x^2+\frac{5}{2}\,a^4b\log\left(x^2\right)-\frac{5\,a^4bx^2+a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5/2*a^4*b*l
og(x^2) - 1/2*(5*a^4*b*x^2 + a^5)/x^2
```

maple [A] time = 0.00, size = 57, normalized size = 0.89

$$\frac{b^5x^8}{8}+\frac{5a\,b^4x^6}{6}+\frac{5a^2b^3x^4}{2}+5a^3b^2x^2+5a^4b\ln(x)-\frac{a^5}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^3,x)
```

```
[Out] -1/2*a^5/x^2+5*a^3*b^2*x^2+5/2*a^2*b^3*x^4+5/6*a*b^4*x^6+1/8*b^5*x^8+5*a^4*
b*ln(x)
```

maxima [A] time = 1.37, size = 58, normalized size = 0.91

$$\frac{1}{8}\,b^5x^8+\frac{5}{6}\,ab^4x^6+\frac{5}{2}\,a^2b^3x^4+5\,a^3b^2x^2+\frac{5}{2}\,a^4b\log\left(x^2\right)-\frac{a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5/2*a^4*b*l
og(x^2) - 1/2*a^5/x^2
```

mupad [B] time = 0.03, size = 56, normalized size = 0.88

$$\frac{b^5\,x^8}{8}-\frac{a^5}{2\,x^2}+\frac{5\,a\,b^4\,x^6}{6}+5\,a^4\,b\,\ln(x)+5\,a^3\,b^2\,x^2+\frac{5\,a^2\,b^3\,x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^3,x)
```

```
[Out] (b^5*x^8)/8 - a^5/(2*x^2) + (5*a*b^4*x^6)/6 + 5*a^4*b*log(x) + 5*a^3*b^2*x^
2 + (5*a^2*b^3*x^4)/2
```

sympy [A] time = 0.17, size = 63, normalized size = 0.98

$$-\frac{a^5}{2x^2}+5a^4b\log{(x)}+5a^3b^2x^2+\frac{5a^2b^3x^4}{2}+\frac{5ab^4x^6}{6}+\frac{b^5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**3,x)
```

```
[Out] -a**5/(2*x**2) + 5*a**4*b*log(x) + 5*a**3*b**2*x**2 + 5*a**2*b**3*x**4/2 +
5*a*b**4*x**6/6 + b**5*x**8/8
```

3.61 $\int \frac{(a+bx^2)^5}{x^5}\,dx$

Optimal. Leaf size=64

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{2x^2}+10a^3b^2\log(x)+5a^2b^3x^2+\frac{5}{4}ab^4x^4+\frac{b^5x^6}{6}$$

```
[Out] -1/4*a^5/x^4-5/2*a^4*b/x^2+5*a^2*b^3*x^2+5/4*a*b^4*x^4+1/6*b^5*x^6+10*a^3*b
^2*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$5a^2b^3x^2+10a^3b^2\log(x)-\frac{5a^4b}{2x^2}-\frac{a^5}{4x^4}+\frac{5}{4}ab^4x^4+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^5,x]
```

```
[Out] -a^5/(4*x^4) - (5*a^4*b)/(2*x^2) + 5*a^2*b^3*x^2 + (5*a*b^4*x^4)/4 + (b^5*x
^6)/6 + 10*a^3*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(10a^2b^3+\frac{a^5}{x^3}+\frac{5a^4b}{x^2}+\frac{10a^3b^2}{x}+5ab^4x+b^5x^2\right)dx,x,x^2\right)\\
&= -\frac{a^5}{4x^4}-\frac{5a^4b}{2x^2}+5a^2b^3x^2+\frac{5}{4}ab^4x^4+\frac{b^5x^6}{6}+10a^3b^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 64, normalized size = 1.00

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{2x^2}+10a^3b^2\log(x)+5a^2b^3x^2+\frac{5}{4}ab^4x^4+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^5,x]
```

```
[Out] -1/4*a^5/x^4 - (5*a^4*b)/(2*x^2) + 5*a^2*b^3*x^2 + (5*a*b^4*x^4)/4 + (b^5*x
^6)/6 + 10*a^3*b^2*Log[x]
```

fricas [A] time = 0.91, size = 61, normalized size = 0.95

$$\frac{2\,b^5x^{10} + 15\,ab^4x^8 + 60\,a^2b^3x^6 + 120\,a^3b^2x^4\log(x) - 30\,a^4bx^2 - 3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^5*x^10 + 15*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4*log(x) -
 30*a^4*b*x^2 - 3*a^5)/x^4
```

giac [A] time = 0.88, size = 70, normalized size = 1.09

$$\frac{1}{6}\,b^5x^6 + \frac{5}{4}\,ab^4x^4 + 5\,a^2b^3x^2 + 5\,a^3b^2\log\left(x^2\right) - \frac{30\,a^3b^2x^4 + 10\,a^4bx^2 + a^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^5,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 5*a^3*b^2*log(x^2) - 1/4*(30*
a^3*b^2*x^4 + 10*a^4*b*x^2 + a^5)/x^4
```

maple [A] time = 0.00, size = 57, normalized size = 0.89

$$\frac{b^5x^6}{6} + \frac{5a\,b^4x^4}{4} + 5a^2b^3x^2 + 10a^3b^2\ln(x) - \frac{5a^4b}{2x^2} - \frac{a^5}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^5,x)
```

```
[Out] -1/4*a^5/x^4-5/2*a^4*b/x^2+5*a^2*b^3*x^2+5/4*a*b^4*x^4+1/6*b^5*x^6+10*a^3*b
^2*ln(x)
```

maxima [A] time = 1.33, size = 59, normalized size = 0.92

$$\frac{1}{6}\,b^5x^6 + \frac{5}{4}\,ab^4x^4 + 5\,a^2b^3x^2 + 5\,a^3b^2\log\left(x^2\right) - \frac{10\,a^4bx^2 + a^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 5*a^3*b^2*log(x^2) - 1/4*(10*
a^4*b*x^2 + a^5)/x^4
```

mupad [B] time = 0.03, size = 59, normalized size = 0.92

$$\frac{b^5\,x^6}{6} - \frac{\frac{a^5}{4} + \frac{5\,b\,a^4\,x^2}{2}}{x^4} + \frac{5\,a\,b^4\,x^4}{4} + 5\,a^2\,b^3\,x^2 + 10\,a^3\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^5,x)
```

```
[Out] (b^5*x^6)/6 - (a^5/4 + (5*a^4*b*x^2)/2)/x^4 + (5*a*b^4*x^4)/4 + 5*a^2*b^3*x
^2 + 10*a^3*b^2*log(x)
```

sympy [A] time = 0.22, size = 63, normalized size = 0.98

$$10a^3b^2\log{(x)} + 5a^2b^3x^2 + \frac{5ab^4x^4}{4} + \frac{b^5x^6}{6} + \frac{-a^5 - 10a^4bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**5,x)
```

```
[Out] 10*a**3*b**2*log(x) + 5*a**2*b**3*x**2 + 5*a*b**4*x**4/4 + b**5*x**6/6 + (-
a**5 - 10*a**4*b*x**2)/(4*x**4)
```

3.62 $\int \frac{(a+bx^2)^5}{x^7}\,dx$

Optimal. Leaf size=64

$$-\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+10a^2b^3\log(x)+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}$$

```
[Out] -1/6*a^5/x^6-5/4*a^4*b/x^4-5*a^3*b^2/x^2+5/2*a*b^4*x^2+1/4*b^5*x^4+10*a^2*b
^3*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{x^2}+10a^2b^3\log(x)-\frac{5a^4b}{4x^4}-\frac{a^5}{6x^6}+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^7,x]
```

```
[Out] -a^5/(6*x^6) - (5*a^4*b)/(4*x^4) - (5*a^3*b^2)/x^2 + (5*a*b^4*x^2)/2 + (b^5
*x^4)/4 + 10*a^2*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^4}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(5ab^4+\frac{a^5}{x^4}+\frac{5a^4b}{x^3}+\frac{10a^3b^2}{x^2}+\frac{10a^2b^3}{x}+b^5x\right)dx,x,x^2\right)\\
&= -\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}+10a^2b^3\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 64, normalized size = 1.00

$$-\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+10a^2b^3\log(x)+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^7,x]
```

```
[Out] -1/6*a^5/x^6 - (5*a^4*b)/(4*x^4) - (5*a^3*b^2)/x^2 + (5*a*b^4*x^2)/2 + (b^5
*x^4)/4 + 10*a^2*b^3*Log[x]
```

fricas [A] time = 0.82, size = 61, normalized size = 0.95

$$\frac{3\,b^5x^{10}+30\,ab^4x^8+120\,a^2b^3x^6\log(x)-60\,a^3b^2x^4-15\,a^4bx^2-2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^10 + 30*a*b^4*x^8 + 120*a^2*b^3*x^6*log(x) - 60*a^3*b^2*x^4 -
 15*a^4*b*x^2 - 2*a^5)/x^6
```

giac [A] time = 1.19, size = 72, normalized size = 1.12

$$\frac{1}{4}\,b^5x^4+\frac{5}{2}\,ab^4x^2+5\,a^2b^3\log\left(x^2\right)-\frac{110\,a^2b^3x^6+60\,a^3b^2x^4+15\,a^4bx^2+2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^7,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 5*a^2*b^3*log(x^2) - 1/12*(110*a^2*b^3*x^6 +
60*a^3*b^2*x^4 + 15*a^4*b*x^2 + 2*a^5)/x^6
```

maple [A] time = 0.01, size = 57, normalized size = 0.89

$$\frac{b^5x^4}{4}+\frac{5a\,b^4x^2}{2}+10a^2b^3\ln(x)-\frac{5a^3b^2}{x^2}-\frac{5a^4b}{4x^4}-\frac{a^5}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^7,x)
```

```
[Out] -1/6*a^5/x^6-5/4*a^4*b/x^4-5*a^3*b^2/x^2+5/2*a*b^4*x^2+1/4*b^5*x^4+10*a^2*b
^3*ln(x)
```

maxima [A] time = 1.40, size = 61, normalized size = 0.95

$$\frac{1}{4}\,b^5x^4+\frac{5}{2}\,ab^4x^2+5\,a^2b^3\log\left(x^2\right)-\frac{60\,a^3b^2x^4+15\,a^4bx^2+2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 5*a^2*b^3*log(x^2) - 1/12*(60*a^3*b^2*x^4 + 1
5*a^4*b*x^2 + 2*a^5)/x^6
```

mupad [B] time = 0.04, size = 59, normalized size = 0.92

$$\frac{b^5\,x^4}{4}-\frac{\frac{a^5}{6}+\frac{5\,a^4\,b\,x^2}{4}+5\,a^3\,b^2\,x^4}{x^6}+\frac{5\,a\,b^4\,x^2}{2}+10\,a^2\,b^3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^7,x)
```

```
[Out] (b^5*x^4)/4 - (a^5/6 + (5*a^4*b*x^2)/4 + 5*a^3*b^2*x^4)/x^6 + (5*a*b^4*x^2)
/2 + 10*a^2*b^3*log(x)
```

sympy [A] time = 0.28, size = 65, normalized size = 1.02

$$10a^2b^3\log{(x)}+\frac{5ab^4x^2}{2}+\frac{b^5x^4}{4}+\frac{-2a^5-15a^4bx^2-60a^3b^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**7,x)
```

```
[Out] 10*a**2*b**3*log(x) + 5*a*b**4*x**2/2 + b**5*x**4/4 + (-2*a**5 - 15*a**4*b*
x**2 - 60*a**3*b**2*x**4)/(12*x**6)
```

3.63 $\int \frac{(a+bx^2)^5}{x^9}\,dx$

Optimal. Leaf size=64

$$-\frac{a^5}{8x^8}-\frac{5a^4b}{6x^6}-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}+5ab^4\log(x)+\frac{b^5x^2}{2}$$

```
[Out] -1/8*a^5/x^8-5/6*a^4*b/x^6-5/2*a^3*b^2/x^4-5*a^2*b^3/x^2+1/2*b^5*x^2+5*a*b^
4*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}-\frac{5a^4b}{6x^6}-\frac{a^5}{8x^8}+5ab^4\log(x)+\frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^9,x]
```

```
[Out] -a^5/(8*x^8) - (5*a^4*b)/(6*x^6) - (5*a^3*b^2)/(2*x^4) - (5*a^2*b^3)/x^2 +
(b^5*x^2)/2 + 5*a*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^9}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^5}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(b^5+\frac{a^5}{x^5}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^3}+\frac{10a^2b^3}{x^2}+\frac{5ab^4}{x}\right)dx,x,x^2\right)\\
&= -\frac{a^5}{8x^8}-\frac{5a^4b}{6x^6}-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}+\frac{b^5x^2}{2}+5ab^4\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 64, normalized size = 1.00

$$-\frac{a^5}{8x^8}-\frac{5a^4b}{6x^6}-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}+5ab^4\log(x)+\frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^9,x]
```

```
[Out] -1/8*a^5/x^8 - (5*a^4*b)/(6*x^6) - (5*a^3*b^2)/(2*x^4) - (5*a^2*b^3)/x^2 +
(b^5*x^2)/2 + 5*a*b^4*Log[x]
```

fricas [A] time = 0.87, size = 61, normalized size = 0.95

$$\frac{12\,b^{5}x^{10}+120\,ab^{4}x^{8}\log(x)-120\,a^{2}b^{3}x^{6}-60\,a^{3}b^{2}x^{4}-20\,a^{4}bx^{2}-3\,a^{5}}{24\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^5*x^10 + 120*a*b^4*x^8*log(x) - 120*a^2*b^3*x^6 - 60*a^3*b^2*x^4
 - 20*a^4*b*x^2 - 3*a^5)/x^8
```

giac [A] time = 1.05, size = 70, normalized size = 1.09

$$\frac{1}{2}\,b^{5}x^{2}+\frac{5}{2}\,ab^{4}\log\left(x^{2}\right)-\frac{125\,ab^{4}x^{8}+120\,a^{2}b^{3}x^{6}+60\,a^{3}b^{2}x^{4}+20\,a^{4}bx^{2}+3\,a^{5}}{24\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2 + 5/2*a*b^4*log(x^2) - 1/24*(125*a*b^4*x^8 + 120*a^2*b^3*x^6 +
60*a^3*b^2*x^4 + 20*a^4*b*x^2 + 3*a^5)/x^8
```

maple [A] time = 0.01, size = 57, normalized size = 0.89

$$\frac{b^{5}x^{2}}{2}+5a\,b^{4}\ln(x)-\frac{5a^{2}b^{3}}{x^{2}}-\frac{5a^{3}b^{2}}{2x^{4}}-\frac{5a^{4}b}{6x^{6}}-\frac{a^{5}}{8x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^9,x)
```

```
[Out] -1/8*a^5/x^8-5/6*a^4*b/x^6-5/2*a^3*b^2/x^4-5*a^2*b^3/x^2+1/2*b^5*x^2+5*a*b^
4*ln(x)
```

maxima [A] time = 1.41, size = 61, normalized size = 0.95

$$\frac{1}{2}\,b^{5}x^{2}+\frac{5}{2}\,ab^{4}\log\left(x^{2}\right)-\frac{120\,a^{2}b^{3}x^{6}+60\,a^{3}b^{2}x^{4}+20\,a^{4}bx^{2}+3\,a^{5}}{24\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*x^2 + 5/2*a*b^4*log(x^2) - 1/24*(120*a^2*b^3*x^6 + 60*a^3*b^2*x^4 +
 20*a^4*b*x^2 + 3*a^5)/x^8
```

mupad [B] time = 0.04, size = 59, normalized size = 0.92

$$\frac{b^{5}\,x^{2}}{2}-\frac{\frac{a^{5}}{8}+\frac{5\,a^{4}\,b\,x^{2}}{6}+\frac{5\,a^{3}\,b^{2}\,x^{4}}{2}+5\,a^{2}\,b^{3}\,x^{6}}{x^{8}}+5\,a\,b^{4}\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^9,x)
```

```
[Out] (b^5*x^2)/2 - (a^5/8 + (5*a^4*b*x^2)/6 + (5*a^3*b^2*x^4)/2 + 5*a^2*b^3*x^6)
/x^8 + 5*a*b^4*log(x)
```

sympy [A] time = 0.36, size = 63, normalized size = 0.98

$$5ab^{4}\log{(x)}+\frac{b^{5}x^{2}}{2}+\frac{-3a^{5}-20a^{4}bx^{2}-60a^{3}b^{2}x^{4}-120a^{2}b^{3}x^{6}}{24x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**9,x)
```

```
[Out] 5*a*b**4*log(x) + b**5*x**2/2 + (-3*a**5 - 20*a**4*b*x**2 - 60*a**3*b**2*x*
*4 - 120*a**2*b**3*x**6)/(24*x**8)
```

3.64 $\int \frac{(a+bx^2)^5}{x^{11}} \, dx$

Optimal. Leaf size=65

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{8x^8}-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{2x^2}+b^5\log(x)$$

```
[Out] -1/10*a^5/x^10-5/8*a^4*b/x^8-5/3*a^3*b^2/x^6-5/2*a^2*b^3/x^4-5/2*a*b^4/x^2+
b^5*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5a^4b}{8x^8}-\frac{a^5}{10x^{10}}-\frac{5ab^4}{2x^2}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^11,x]
```

```
[Out] -a^5/(10*x^10) - (5*a^4*b)/(8*x^8) - (5*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/(2*x
^4) - (5*a*b^4)/(2*x^2) + b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^{11}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^6} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right) dx, x, x^2\right)\\
&= -\frac{a^5}{10x^{10}}-\frac{5a^4b}{8x^8}-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{2x^2}+b^5\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 65, normalized size = 1.00

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{8x^8}-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{2x^2}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^11,x]
```

```
[Out] -1/10*a^5/x^10 - (5*a^4*b)/(8*x^8) - (5*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/(2*x
^4) - (5*a*b^4)/(2*x^2) + b^5*Log[x]
```

fricas [A] time = 0.85, size = 61, normalized size = 0.94

$$\frac{120\,b^5x^{10}\log(x) - 300\,ab^4x^8 - 300\,a^2b^3x^6 - 200\,a^3b^2x^4 - 75\,a^4bx^2 - 12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(120*b^5*x^10*log(x) - 300*a*b^4*x^8 - 300*a^2*b^3*x^6 - 200*a^3*b^2*
x^4 - 75*a^4*b*x^2 - 12*a^5)/x^10
```

giac [A] time = 1.06, size = 69, normalized size = 1.06

$$\frac{1}{2}\,b^5\log\left(x^2\right) - \frac{137\,b^5x^{10} + 300\,ab^4x^8 + 300\,a^2b^3x^6 + 200\,a^3b^2x^4 + 75\,a^4bx^2 + 12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^11,x, algorithm="giac")
```

```
[Out] 1/2*b^5*log(x^2) - 1/120*(137*b^5*x^10 + 300*a*b^4*x^8 + 300*a^2*b^3*x^6 +
200*a^3*b^2*x^4 + 75*a^4*b*x^2 + 12*a^5)/x^10
```

maple [A] time = 0.01, size = 56, normalized size = 0.86

$$b^5\ln(x) - \frac{5a\,b^4}{2x^2} - \frac{5a^2b^3}{2x^4} - \frac{5a^3b^2}{3x^6} - \frac{5a^4b}{8x^8} - \frac{a^5}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^11,x)
```

```
[Out] -1/10*a^5/x^10-5/8*a^4*b/x^8-5/3*a^3*b^2/x^6-5/2*a^2*b^3/x^4-5/2*a*b^4/x^2+
b^5*ln(x)
```

maxima [A] time = 1.35, size = 61, normalized size = 0.94

$$\frac{1}{2}\,b^5\log\left(x^2\right) - \frac{300\,ab^4x^8 + 300\,a^2b^3x^6 + 200\,a^3b^2x^4 + 75\,a^4bx^2 + 12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*log(x^2) - 1/120*(300*a*b^4*x^8 + 300*a^2*b^3*x^6 + 200*a^3*b^2*x^4
 + 75*a^4*b*x^2 + 12*a^5)/x^10
```

mupad [B] time = 4.77, size = 58, normalized size = 0.89

$$b^5\,\ln(x) - \frac{\frac{a^5}{10} + \frac{5\,a^4\,b\,x^2}{8} + \frac{5\,a^3\,b^2\,x^4}{3} + \frac{5\,a^2\,b^3\,x^6}{2} + \frac{5\,a\,b^4\,x^8}{2}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^11,x)
```

```
[Out] b^5*log(x) - (a^5/10 + (5*a^4*b*x^2)/8 + (5*a*b^4*x^8)/2 + (5*a^3*b^2*x^4)/
3 + (5*a^2*b^3*x^6)/2)/x^10
```

sympy [A] time = 0.44, size = 61, normalized size = 0.94

$$b^5\log{(x)} + \frac{-12a^5 - 75a^4bx^2 - 200a^3b^2x^4 - 300a^2b^3x^6 - 300ab^4x^8}{120x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**11,x)
```

```
[Out] b**5*log(x) + (-12*a**5 - 75*a**4*b*x**2 - 200*a**3*b**2*x**4 - 300*a**2*b*
*3*x**6 - 300*a*b**4*x**8)/(120*x**10)
```

3.65 $\int \frac{(a+bx^2)^5}{x^{13}}\,dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^6}{12ax^{12}}$$

```
[Out] -1/12*(b*x^2+a)^6/a/x^12
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{(a+bx^2)^6}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^13,x]
```

```
[Out] -(a + b*x^2)^6/(12*a*x^12)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^5}{x^{13}}\,dx = -\frac{(a+bx^2)^6}{12ax^{12}}$$

Mathematica [B] time = 0.00, size = 69, normalized size = 3.63

$$-\frac{a^5}{12x^{12}}-\frac{a^4b}{2x^{10}}-\frac{5a^3b^2}{4x^8}-\frac{5a^2b^3}{3x^6}-\frac{5ab^4}{4x^4}-\frac{b^5}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^13,x]
```

```
[Out] -1/12*a^5/x^12 - (a^4*b)/(2*x^10) - (5*a^3*b^2)/(4*x^8) - (5*a^2*b^3)/(3*x^
6) - (5*a*b^4)/(4*x^4) - b^5/(2*x^2)
```

fricas [B] time = 0.69, size = 57, normalized size = 3.00

$$-\frac{6\,b^5x^{10}+15\,ab^4x^8+20\,a^2b^3x^6+15\,a^3b^2x^4+6\,a^4bx^2+a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

giac [B] time = 1.07, size = 57, normalized size = 3.00

$$-\frac{6 b^5 x^{10} + 15 ab^4 x^8 + 20 a^2 b^3 x^6 + 15 a^3 b^2 x^4 + 6 a^4 b x^2 + a^5}{12 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

maple [B] time = 0.01, size = 58, normalized size = 3.05

$$-\frac{b^5}{2x^2} - \frac{5a\,b^4}{4x^4} - \frac{5a^2b^3}{3x^6} - \frac{5a^3b^2}{4x^8} - \frac{a^4b}{2x^{10}} - \frac{a^5}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^13,x)
```

```
[Out] -5/3*a^2*b^3/x^6-1/2*b^5/x^2-5/4*a*b^4/x^4-1/2*a^4*b/x^10-1/12*a^5/x^12-5/4
*a^3*b^2/x^8
```

maxima [B] time = 1.37, size = 57, normalized size = 3.00

$$-\frac{6 b^5 x^{10} + 15 ab^4 x^8 + 20 a^2 b^3 x^6 + 15 a^3 b^2 x^4 + 6 a^4 b x^2 + a^5}{12 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^13,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

mupad [B] time = 4.75, size = 59, normalized size = 3.11

$$-\frac{\frac{a^5}{12} + \frac{a^4\,b\,x^2}{2} + \frac{5\,a^3\,b^2\,x^4}{4} + \frac{5\,a^2\,b^3\,x^6}{3} + \frac{5\,a\,b^4\,x^8}{4} + \frac{b^5\,x^{10}}{2}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^13,x)
```

```
[Out] -(a^5/12 + (b^5*x^10)/2 + (a^4*b*x^2)/2 + (5*a*b^4*x^8)/4 + (5*a^3*b^2*x^4)
/4 + (5*a^2*b^3*x^6)/3)/x^12
```

sympy [B] time = 0.47, size = 61, normalized size = 3.21

$$\frac{-a^5 - 6a^4bx^2 - 15a^3b^2x^4 - 20a^2b^3x^6 - 15ab^4x^8 - 6b^5x^{10}}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**13,x)
```

```
[Out] (-a**5 - 6*a**4*b*x**2 - 15*a**3*b**2*x**4 - 20*a**2*b**3*x**6 - 15*a*b**4*
x**8 - 6*b**5*x**10)/(12*x**12)
```

3.66 $\int \frac{\left(a+bx^2\right)^5}{x^{15}}\,dx$

Optimal. Leaf size=40

$$\frac{b\left(a+bx^2\right)^6}{84a^2x^{12}}-\frac{\left(a+bx^2\right)^6}{14ax^{14}}$$

```
[Out] -1/14*(b*x^2+a)^6/a/x^14+1/84*b*(b*x^2+a)^6/a^2/x^12
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{b\left(a+bx^2\right)^6}{84a^2x^{12}}-\frac{\left(a+bx^2\right)^6}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^15,x]
```

```
[Out] -(a + b*x^2)^6/(14*a*x^14) + (b*(a + b*x^2)^6)/(84*a^2*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^5}{x^{15}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^6}{14ax^{14}}-\frac{b\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx,x,x^2\right)}{14a}\\
&= -\frac{\left(a+bx^2\right)^6}{14ax^{14}}+\frac{b\left(a+bx^2\right)^6}{84a^2x^{12}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 67, normalized size = 1.68

$$-\frac{a^5}{14x^{14}}-\frac{5a^4b}{12x^{12}}-\frac{a^3b^2}{x^{10}}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{6x^6}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^15,x]
```

```
[Out] -1/14*a^5/x^14 - (5*a^4*b)/(12*x^12) - (a^3*b^2)/x^10 - (5*a^2*b^3)/(4*x^8)
 - (5*a*b^4)/(6*x^6) - b^5/(4*x^4)
```

fricas [A] time = 0.82, size = 59, normalized size = 1.48

$$-\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^15,x, algorithm="fricas")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

giac [A] time = 1.06, size = 59, normalized size = 1.48

$$-\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^15,x, algorithm="giac")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

maple [A] time = 0.00, size = 58, normalized size = 1.45

$$-\frac{b^5}{4x^4}-\frac{5a\,b^4}{6x^6}-\frac{5a^2b^3}{4x^8}-\frac{a^3b^2}{x^{10}}-\frac{5a^4b}{12x^{12}}-\frac{a^5}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^15,x)
```

```
[Out] -5/12*a^4*b/x^12-5/6*a*b^4/x^6-1/4*b^5/x^4-a^3*b^2/x^10-5/4*a^2*b^3/x^8-1/1
4*a^5/x^14
```

maxima [A] time = 1.34, size = 59, normalized size = 1.48

$$-\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^15,x, algorithm="maxima")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

mupad [B] time = 4.74, size = 58, normalized size = 1.45

$$-\frac{\frac{a^5}{14}+\frac{5\,a^4\,b\,x^2}{12}+a^3\,b^2\,x^4+\frac{5\,a^2\,b^3\,x^6}{4}+\frac{5\,a\,b^4\,x^8}{6}+\frac{b^5\,x^{10}}{4}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^15,x)
```

```
[Out] -(a^5/14 + (b^5*x^10)/4 + (5*a^4*b*x^2)/12 + (5*a*b^4*x^8)/6 + a^3*b^2*x^4
+ (5*a^2*b^3*x^6)/4)/x^14
```

sympy [A] time = 0.51, size = 63, normalized size = 1.58

$$\frac{-6a^5 - 35a^4bx^2 - 84a^3b^2x^4 - 105a^2b^3x^6 - 70ab^4x^8 - 21b^5x^{10}}{84x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**15,x)
```

```
[Out] (-6*a**5 - 35*a**4*b*x**2 - 84*a**3*b**2*x**4 - 105*a**2*b**3*x**6 - 70*a*b
**4*x**8 - 21*b**5*x**10)/(84*x**14)
```

3.67 $\int \frac{(a+bx^2)^5}{x^{17}}\,dx$

Optimal. Leaf size=62

$$-\frac{b^2\left(a+bx^2\right)^6}{336a^3x^{12}}+\frac{b\left(a+bx^2\right)^6}{56a^2x^{14}}-\frac{\left(a+bx^2\right)^6}{16ax^{16}}$$

```
[Out] -1/16*(b*x^2+a)^6/a/x^16+1/56*b*(b*x^2+a)^6/a^2/x^14-1/336*b^2*(b*x^2+a)^6/
a^3/x^12
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$-\frac{b^2\left(a+bx^2\right)^6}{336a^3x^{12}}+\frac{b\left(a+bx^2\right)^6}{56a^2x^{14}}-\frac{\left(a+bx^2\right)^6}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^17,x]
```

```
[Out] -(a + b*x^2)^6/(16*a*x^16) + (b*(a + b*x^2)^6)/(56*a^2*x^14) - (b^2*(a + b*
x^2)^6)/(336*a^3*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^5}{x^{17}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^9}\,dx, x, x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^6}{16ax^{16}} - \frac{b\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx, x, x^2\right)}{8a}$$
$$= -\frac{\left(a+bx^2\right)^6}{16ax^{16}} + \frac{b\left(a+bx^2\right)^6}{56a^2x^{14}} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx, x, x^2\right)}{56a^2}$$
$$= -\frac{\left(a+bx^2\right)^6}{16ax^{16}} + \frac{b\left(a+bx^2\right)^6}{56a^2x^{14}} - \frac{b^2\left(a+bx^2\right)^6}{336a^3x^{12}}$$

Mathematica [A] time = 0.00, size = 67, normalized size = 1.08

$$-\frac{a^5}{16x^{16}} - \frac{5a^4b}{14x^{14}} - \frac{5a^3b^2}{6x^{12}} - \frac{a^2b^3}{x^{10}} - \frac{5ab^4}{8x^8} - \frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^17,x]
```

```
[Out] -1/16*a^5/x^16 - (5*a^4*b)/(14*x^14) - (5*a^3*b^2)/(6*x^12) - (a^2*b^3)/x^1
0 - (5*a*b^4)/(8*x^8) - b^5/(6*x^6)
```

fricas [A] time = 0.85, size = 59, normalized size = 0.95

$$-\frac{56\,b^5x^{10} + 210\,ab^4x^8 + 336\,a^2b^3x^6 + 280\,a^3b^2x^4 + 120\,a^4bx^2 + 21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^17,x, algorithm="fricas")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

giac [A] time = 0.98, size = 59, normalized size = 0.95

$$-\frac{56\,b^5x^{10} + 210\,ab^4x^8 + 336\,a^2b^3x^6 + 280\,a^3b^2x^4 + 120\,a^4bx^2 + 21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^17,x, algorithm="giac")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

maple [A] time = 0.01, size = 58, normalized size = 0.94

$$-\frac{b^5}{6x^6} - \frac{5a\,b^4}{8x^8} - \frac{a^2b^3}{x^{10}} - \frac{5a^3b^2}{6x^{12}} - \frac{5a^4b}{14x^{14}} - \frac{a^5}{16x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^17,x)
```

```
[Out] -1/6*b^5/x^6-1/16*a^5/x^16-5/6*a^3*b^2/x^12-a^2*b^3/x^10-5/8*a*b^4/x^8-5/14
*a^4*b/x^14
```

maxima [A] time = 1.38, size = 59, normalized size = 0.95

$$-\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,a^3b^2x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^17,x, algorithm="maxima")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

mupad [B] time = 0.04, size = 58, normalized size = 0.94

$$-\frac{\frac{a^5}{16}+\frac{5\,a^4\,b\,x^2}{14}+\frac{5\,a^3\,b^2\,x^4}{6}+a^2\,b^3\,x^6+\frac{5\,a\,b^4\,x^8}{8}+\frac{b^5\,x^{10}}{6}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^17,x)
```

```
[Out] -(a^5/16 + (b^5*x^10)/6 + (5*a^4*b*x^2)/14 + (5*a*b^4*x^8)/8 + (5*a^3*b^2*x
^4)/6 + a^2*b^3*x^6)/x^16
```

sympy [A] time = 0.55, size = 63, normalized size = 1.02

$$\frac{-21a^5-120a^4bx^2-280a^3b^2x^4-336a^2b^3x^6-210ab^4x^8-56b^5x^{10}}{336x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**17,x)
```

```
[Out] (-21*a**5 - 120*a**4*b*x**2 - 280*a**3*b**2*x**4 - 336*a**2*b**3*x**6 - 210
*a*b**4*x**8 - 56*b**5*x**10)/(336*x**16)
```

3.68 $\int \frac{(a+bx^2)^5}{x^{19}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}$$

```
[Out] -1/18*a^5/x^18-5/16*a^4*b/x^16-5/7*a^3*b^2/x^14-5/6*a^2*b^3/x^12-1/2*a*b^4/
x^10-1/8*b^5/x^8
```

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{5a^4b}{16x^{16}}-\frac{a^5}{18x^{18}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^19,x]
```

```
[Out] -a^5/(18*x^18) - (5*a^4*b)/(16*x^16) - (5*a^3*b^2)/(7*x^14) - (5*a^2*b^3)/(
6*x^12) - (a*b^4)/(2*x^10) - b^5/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^{19}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^{10}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right)dx, x, x^2\right)\\
&= -\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^19,x]
```

```
[Out] -1/18*a^5/x^18 - (5*a^4*b)/(16*x^16) - (5*a^3*b^2)/(7*x^14) - (5*a^2*b^3)/(
6*x^12) - (a*b^4)/(2*x^10) - b^5/(8*x^8)
```

fricas [A]   time = 0.84, size = 59, normalized size = 0.86

$$-\frac{126\,b^{5}x^{10}+504\,ab^{4}x^{8}+840\,a^{2}b^{3}x^{6}+720\,a^{3}b^{2}x^{4}+315\,a^{4}bx^{2}+56\,a^{5}}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^19,x, algorithm="fricas")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

giac [A]   time = 0.94, size = 59, normalized size = 0.86

$$-\frac{126\,b^{5}x^{10}+504\,ab^{4}x^{8}+840\,a^{2}b^{3}x^{6}+720\,a^{3}b^{2}x^{4}+315\,a^{4}bx^{2}+56\,a^{5}}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^19,x, algorithm="giac")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

maple [A]   time = 0.01, size = 58, normalized size = 0.84

$$-\frac{b^{5}}{8x^{8}}-\frac{a\,b^{4}}{2x^{10}}-\frac{5a^{2}b^{3}}{6x^{12}}-\frac{5a^{3}b^{2}}{7x^{14}}-\frac{5a^{4}b}{16x^{16}}-\frac{a^{5}}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^19,x)
```

```
[Out] -1/18*a^5/x^18-5/16*a^4*b/x^16-5/7*a^3*b^2/x^14-5/6*a^2*b^3/x^12-1/2*a*b^4/
x^10-1/8*b^5/x^8
```

maxima [A]   time = 1.36, size = 59, normalized size = 0.86

$$-\frac{126\,b^{5}x^{10}+504\,ab^{4}x^{8}+840\,a^{2}b^{3}x^{6}+720\,a^{3}b^{2}x^{4}+315\,a^{4}bx^{2}+56\,a^{5}}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^19,x, algorithm="maxima")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

mupad [B]   time = 0.04, size = 59, normalized size = 0.86

$$-\frac{\frac{a^{5}}{18}+\frac{5\,a^{4}\,b\,x^{2}}{16}+\frac{5\,a^{3}\,b^{2}\,x^{4}}{7}+\frac{5\,a^{2}\,b^{3}\,x^{6}}{6}+\frac{a\,b^{4}\,x^{8}}{2}+\frac{b^{5}\,x^{10}}{8}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^19,x)
```

```
[Out] -(a^5/18 + (b^5*x^10)/8 + (5*a^4*b*x^2)/16 + (a*b^4*x^8)/2 + (5*a^3*b^2*x^4
)/7 + (5*a^2*b^3*x^6)/6)/x^18
```

sympy [A]   time = 0.59, size = 63, normalized size = 0.91

$$\frac{-56a^{5}-315a^{4}bx^{2}-720a^{3}b^{2}x^{4}-840a^{2}b^{3}x^{6}-504ab^{4}x^{8}-126b^{5}x^{10}}{1008x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**19,x)
```

```
[Out] (-56*a**5 - 315*a**4*b*x**2 - 720*a**3*b**2*x**4 - 840*a**2*b**3*x**6 - 504
*a*b**4*x**8 - 126*b**5*x**10)/(1008*x**18)
```

3.69 $\int \frac{\left(a+bx^2\right)^5}{x^{21}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{20x^{20}}-\frac{5a^4b}{18x^{18}}-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}$$

```
[Out] -1/20*a^5/x^20-5/18*a^4*b/x^18-5/8*a^3*b^2/x^16-5/7*a^2*b^3/x^14-5/12*a*b^4
/x^12-1/10*b^5/x^10
```

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5a^4b}{18x^{18}}-\frac{a^5}{20x^{20}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^21,x]
```

```
[Out] -a^5/(20*x^20) - (5*a^4*b)/(18*x^18) - (5*a^3*b^2)/(8*x^16) - (5*a^2*b^3)/(
7*x^14) - (5*a*b^4)/(12*x^12) - b^5/(10*x^10)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^5}{x^{21}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^{11}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^5}{x^{11}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^9}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^7}+\frac{b^5}{x^6}\right)dx,x,x^2\right)\\
&= -\frac{a^5}{20x^{20}}-\frac{5a^4b}{18x^{18}}-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{20x^{20}}-\frac{5a^4b}{18x^{18}}-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^21,x]
```

```
[Out] -1/20*a^5/x^20 - (5*a^4*b)/(18*x^18) - (5*a^3*b^2)/(8*x^16) - (5*a^2*b^3)/(
7*x^14) - (5*a*b^4)/(12*x^12) - b^5/(10*x^10)
```

fricas [A] time = 0.70, size = 59, normalized size = 0.86

$$-\frac{252\,b^{5}x^{10}+1050\,ab^{4}x^{8}+1800\,a^{2}b^{3}x^{6}+1575\,a^{3}b^{2}x^{4}+700\,a^{4}bx^{2}+126\,a^{5}}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^21,x, algorithm="fricas")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

giac [A] time = 1.02, size = 59, normalized size = 0.86

$$-\frac{252\,b^{5}x^{10}+1050\,ab^{4}x^{8}+1800\,a^{2}b^{3}x^{6}+1575\,a^{3}b^{2}x^{4}+700\,a^{4}bx^{2}+126\,a^{5}}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^21,x, algorithm="giac")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$-\frac{b^{5}}{10x^{10}}-\frac{5a\,b^{4}}{12x^{12}}-\frac{5a^{2}b^{3}}{7x^{14}}-\frac{5a^{3}b^{2}}{8x^{16}}-\frac{5a^{4}b}{18x^{18}}-\frac{a^{5}}{20x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^21,x)
```

```
[Out] -1/20*a^5/x^20-5/18*a^4*b/x^18-5/8*a^3*b^2/x^16-5/7*a^2*b^3/x^14-5/12*a*b^4
/x^12-1/10*b^5/x^10
```

maxima [A] time = 1.35, size = 59, normalized size = 0.86

$$-\frac{252\,b^{5}x^{10}+1050\,ab^{4}x^{8}+1800\,a^{2}b^{3}x^{6}+1575\,a^{3}b^{2}x^{4}+700\,a^{4}bx^{2}+126\,a^{5}}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^21,x, algorithm="maxima")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

mupad [B] time = 0.04, size = 59, normalized size = 0.86

$$-\frac{\frac{a^{5}}{20}+\frac{5\,a^{4}\,b\,x^{2}}{18}+\frac{5\,a^{3}\,b^{2}\,x^{4}}{8}+\frac{5\,a^{2}\,b^{3}\,x^{6}}{7}+\frac{5\,a\,b^{4}\,x^{8}}{12}+\frac{b^{5}\,x^{10}}{10}}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^21,x)
```

```
[Out] -(a^5/20 + (b^5*x^10)/10 + (5*a^4*b*x^2)/18 + (5*a*b^4*x^8)/12 + (5*a^3*b^2
*x^4)/8 + (5*a^2*b^3*x^6)/7)/x^20
```

sympy [A] time = 0.63, size = 63, normalized size = 0.91

$$\frac{-126a^{5}-700a^{4}bx^{2}-1575a^{3}b^{2}x^{4}-1800a^{2}b^{3}x^{6}-1050ab^{4}x^{8}-252b^{5}x^{10}}{2520x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**21,x)
```

```
[Out] (-126*a**5 - 700*a**4*b*x**2 - 1575*a**3*b**2*x**4 - 1800*a**2*b**3*x**6 -
1050*a*b**4*x**8 - 252*b**5*x**10)/(2520*x**20)
```

3.70 $\int x^8 \left(a+bx^2\right)^5 dx$

Optimal. Leaf size=69

$$\frac{a^5x^9}{9}+\frac{5}{11}a^4bx^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}$$

```
[Out] 1/9*a^5*x^9+5/11*a^4*b*x^11+10/13*a^3*b^2*x^13+2/3*a^2*b^3*x^15+5/17*a*b^4*
x^17+1/19*b^5*x^19
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{2}{3}a^2b^3x^{15}+\frac{10}{13}a^3b^2x^{13}+\frac{5}{11}a^4bx^{11}+\frac{a^5x^9}{9}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^9)/9 + (5*a^4*b*x^11)/11 + (10*a^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/3
 + (5*a*b^4*x^17)/17 + (b^5*x^19)/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^8 \left(a+bx^2\right)^5 dx &= \int \left(a^5x^8+5a^4bx^{10}+10a^3b^2x^{12}+10a^2b^3x^{14}+5ab^4x^{16}+b^5x^{18}\right) dx\\ &= \frac{a^5x^9}{9}+\frac{5}{11}a^4bx^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^9}{9}+\frac{5}{11}a^4bx^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^9)/9 + (5*a^4*b*x^11)/11 + (10*a^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/3
 + (5*a*b^4*x^17)/17 + (b^5*x^19)/19
```

fricas [A] time = 0.74, size = 57, normalized size = 0.83

$$\frac{1}{19}x^{19}b^5+\frac{5}{17}x^{17}b^4a+\frac{2}{3}x^{15}b^3a^2+\frac{10}{13}x^{13}b^2a^3+\frac{5}{11}x^{11}ba^4+\frac{1}{9}x^9a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^5 + 5/17*x^17*b^4*a + 2/3*x^15*b^3*a^2 + 10/13*x^13*b^2*a^3 + 5
/11*x^11*b*a^4 + 1/9*x^9*a^5
```

giac [A] time = 1.05, size = 57, normalized size = 0.83

$$\frac{1}{19}\,b^{5}x^{19} + \frac{5}{17}\,ab^{4}x^{17} + \frac{2}{3}\,a^{2}b^{3}x^{15} + \frac{10}{13}\,a^{3}b^{2}x^{13} + \frac{5}{11}\,a^{4}bx^{11} + \frac{1}{9}\,a^{5}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{19}b^{5}x^{19} + \frac{5}{17}a\,b^{4}x^{17} + \frac{2}{3}a^{2}b^{3}x^{15} + \frac{10}{13}a^{3}b^{2}x^{13} + \frac{5}{11}a^{4}b\,x^{11} + \frac{1}{9}a^{5}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^2+a)^5,x)
```

```
[Out] 1/9*a^5*x^9+5/11*a^4*b*x^11+10/13*a^3*b^2*x^13+2/3*a^2*b^3*x^15+5/17*a*b^4*
x^17+1/19*b^5*x^19
```

maxima [A] time = 1.32, size = 57, normalized size = 0.83

$$\frac{1}{19}\,b^{5}x^{19} + \frac{5}{17}\,ab^{4}x^{17} + \frac{2}{3}\,a^{2}b^{3}x^{15} + \frac{10}{13}\,a^{3}b^{2}x^{13} + \frac{5}{11}\,a^{4}bx^{11} + \frac{1}{9}\,a^{5}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^{5}\,x^{9}}{9} + \frac{5\,a^{4}\,b\,x^{11}}{11} + \frac{10\,a^{3}\,b^{2}\,x^{13}}{13} + \frac{2\,a^{2}\,b^{3}\,x^{15}}{3} + \frac{5\,a\,b^{4}\,x^{17}}{17} + \frac{b^{5}\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^9)/9 + (b^5*x^19)/19 + (5*a^4*b*x^11)/11 + (5*a*b^4*x^17)/17 + (10*a
^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/3
```

sympy [A] time = 0.08, size = 66, normalized size = 0.96

$$\frac{a^{5}x^{9}}{9} + \frac{5a^{4}bx^{11}}{11} + \frac{10a^{3}b^{2}x^{13}}{13} + \frac{2a^{2}b^{3}x^{15}}{3} + \frac{5ab^{4}x^{17}}{17} + \frac{b^{5}x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**9/9 + 5*a**4*b*x**11/11 + 10*a**3*b**2*x**13/13 + 2*a**2*b**3*x**15
/3 + 5*a*b**4*x**17/17 + b**5*x**19/19
```

3.71 $\int x^6 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=69

$$\frac{a^5x^7}{7}+\frac{5}{9}a^4bx^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}$$

```
[Out] 1/7*a^5*x^7+5/9*a^4*b*x^9+10/11*a^3*b^2*x^11+10/13*a^2*b^3*x^13+1/3*a*b^4*x
^15+1/17*b^5*x^17
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10}{13}a^2b^3x^{13}+\frac{10}{11}a^3b^2x^{11}+\frac{5}{9}a^4bx^9+\frac{a^5x^7}{7}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^9)/9 + (10*a^3*b^2*x^11)/11 + (10*a^2*b^3*x^13)/13
 + (a*b^4*x^15)/3 + (b^5*x^17)/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^6\left(a+bx^2\right)^5\,dx &= \int \left(a^5x^6+5a^4bx^8+10a^3b^2x^{10}+10a^2b^3x^{12}+5ab^4x^{14}+b^5x^{16}\right)dx\\
&= \frac{a^5x^7}{7}+\frac{5}{9}a^4bx^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^7}{7}+\frac{5}{9}a^4bx^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^9)/9 + (10*a^3*b^2*x^11)/11 + (10*a^2*b^3*x^13)/13
 + (a*b^4*x^15)/3 + (b^5*x^17)/17
```

fricas [A] time = 0.64, size = 57, normalized size = 0.83

$$\frac{1}{17}x^{17}b^5+\frac{1}{3}x^{15}b^4a+\frac{10}{13}x^{13}b^3a^2+\frac{10}{11}x^{11}b^2a^3+\frac{5}{9}x^9ba^4+\frac{1}{7}x^7a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^5 + 1/3*x^15*b^4*a + 10/13*x^13*b^3*a^2 + 10/11*x^11*b^2*a^3 +
5/9*x^9*b*a^4 + 1/7*x^7*a^5
```

giac [A] time = 0.98, size = 57, normalized size = 0.83

$$\frac{1}{17}\,b^5x^{17} + \frac{1}{3}\,ab^4x^{15} + \frac{10}{13}\,a^2b^3x^{13} + \frac{10}{11}\,a^3b^2x^{11} + \frac{5}{9}\,a^4bx^9 + \frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{17}b^5x^{17} + \frac{1}{3}a\,b^4x^{15} + \frac{10}{13}a^2b^3x^{13} + \frac{10}{11}a^3b^2x^{11} + \frac{5}{9}a^4b\,x^9 + \frac{1}{7}a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^5,x)
```

```
[Out] 1/7*a^5*x^7+5/9*a^4*b*x^9+10/11*a^3*b^2*x^11+10/13*a^2*b^3*x^13+1/3*a*b^4*x
^15+1/17*b^5*x^17
```

maxima [A] time = 1.31, size = 57, normalized size = 0.83

$$\frac{1}{17}\,b^5x^{17} + \frac{1}{3}\,ab^4x^{15} + \frac{10}{13}\,a^2b^3x^{13} + \frac{10}{11}\,a^3b^2x^{11} + \frac{5}{9}\,a^4bx^9 + \frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^7}{7} + \frac{5\,a^4\,b\,x^9}{9} + \frac{10\,a^3\,b^2\,x^{11}}{11} + \frac{10\,a^2\,b^3\,x^{13}}{13} + \frac{a\,b^4\,x^{15}}{3} + \frac{b^5\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^7)/7 + (b^5*x^17)/17 + (5*a^4*b*x^9)/9 + (a*b^4*x^15)/3 + (10*a^3*b^
2*x^11)/11 + (10*a^2*b^3*x^13)/13
```

sympy [A] time = 0.08, size = 65, normalized size = 0.94

$$\frac{a^5x^7}{7} + \frac{5a^4bx^9}{9} + \frac{10a^3b^2x^{11}}{11} + \frac{10a^2b^3x^{13}}{13} + \frac{ab^4x^{15}}{3} + \frac{b^5x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**7/7 + 5*a**4*b*x**9/9 + 10*a**3*b**2*x**11/11 + 10*a**2*b**3*x**13/
13 + a*b**4*x**15/3 + b**5*x**17/17
```

3.72 $\int x^4 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=69

$$\frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

```
[Out] 1/5*a^5*x^5+5/7*a^4*b*x^7+10/9*a^3*b^2*x^9+10/11*a^2*b^3*x^11+5/13*a*b^4*x^
13+1/15*b^5*x^15
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10}{11}a^2b^3x^{11}+\frac{10}{9}a^3b^2x^9+\frac{5}{7}a^4bx^7+\frac{a^5x^5}{5}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^7)/7 + (10*a^3*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11 +
 (5*a*b^4*x^13)/13 + (b^5*x^15)/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^5\,dx&=\int\left(a^5x^4+5a^4bx^6+10a^3b^2x^8+10a^2b^3x^{10}+5ab^4x^{12}+b^5x^{14}\right)dx\\&=\frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^7)/7 + (10*a^3*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11 +
 (5*a*b^4*x^13)/13 + (b^5*x^15)/15
```

fricas [A] time = 0.74, size = 57, normalized size = 0.83

$$\frac{1}{15}x^{15}b^5+\frac{5}{13}x^{13}b^4a+\frac{10}{11}x^{11}b^3a^2+\frac{10}{9}x^9b^2a^3+\frac{5}{7}x^7ba^4+\frac{1}{5}x^5a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^5 + 5/13*x^13*b^4*a + 10/11*x^11*b^3*a^2 + 10/9*x^9*b^2*a^3 + 5
/7*x^7*b*a^4 + 1/5*x^5*a^5
```

giac [A] time = 1.01, size = 57, normalized size = 0.83

$$\frac{1}{15}\,b^{5}x^{15} + \frac{5}{13}\,ab^{4}x^{13} + \frac{10}{11}\,a^{2}b^{3}x^{11} + \frac{10}{9}\,a^{3}b^{2}x^{9} + \frac{5}{7}\,a^{4}bx^{7} + \frac{1}{5}\,a^{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{15}b^{5}x^{15} + \frac{5}{13}a\,b^{4}x^{13} + \frac{10}{11}a^{2}b^{3}x^{11} + \frac{10}{9}a^{3}b^{2}x^{9} + \frac{5}{7}a^{4}b\,x^{7} + \frac{1}{5}a^{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^5,x)
```

```
[Out] 1/5*a^5*x^5+5/7*a^4*b*x^7+10/9*a^3*b^2*x^9+10/11*a^2*b^3*x^11+5/13*a*b^4*x^
13+1/15*b^5*x^15
```

maxima [A] time = 1.31, size = 57, normalized size = 0.83

$$\frac{1}{15}\,b^{5}x^{15} + \frac{5}{13}\,ab^{4}x^{13} + \frac{10}{11}\,a^{2}b^{3}x^{11} + \frac{10}{9}\,a^{3}b^{2}x^{9} + \frac{5}{7}\,a^{4}bx^{7} + \frac{1}{5}\,a^{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^{5}\,x^{5}}{5} + \frac{5\,a^{4}\,b\,x^{7}}{7} + \frac{10\,a^{3}\,b^{2}\,x^{9}}{9} + \frac{10\,a^{2}\,b^{3}\,x^{11}}{11} + \frac{5\,a\,b^{4}\,x^{13}}{13} + \frac{b^{5}\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^5)/5 + (b^5*x^15)/15 + (5*a^4*b*x^7)/7 + (5*a*b^4*x^13)/13 + (10*a^3
*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11
```

sympy [A] time = 0.08, size = 66, normalized size = 0.96

$$\frac{a^{5}x^{5}}{5} + \frac{5a^{4}bx^{7}}{7} + \frac{10a^{3}b^{2}x^{9}}{9} + \frac{10a^{2}b^{3}x^{11}}{11} + \frac{5ab^{4}x^{13}}{13} + \frac{b^{5}x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**5/5 + 5*a**4*b*x**7/7 + 10*a**3*b**2*x**9/9 + 10*a**2*b**3*x**11/11
 + 5*a*b**4*x**13/13 + b**5*x**15/15
```

3.73 $\int x^2 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=66

$$\frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

```
[Out] 1/3*a^5*x^3+a^4*b*x^5+10/7*a^3*b^2*x^7+10/9*a^2*b^3*x^9+5/11*a*b^4*x^11+1/1
3*b^5*x^13
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10}{9}a^2b^3x^9+\frac{10}{7}a^3b^2x^7+a^4bx^5+\frac{a^5x^3}{3}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^3)/3 + a^4*b*x^5 + (10*a^3*b^2*x^7)/7 + (10*a^2*b^3*x^9)/9 + (5*a*b^
4*x^11)/11 + (b^5*x^13)/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^5 dx &= \int \left(a^5x^2+5a^4bx^4+10a^3b^2x^6+10a^2b^3x^8+5ab^4x^{10}+b^5x^{12}\right) dx\\ &= \frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.00

$$\frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^3)/3 + a^4*b*x^5 + (10*a^3*b^2*x^7)/7 + (10*a^2*b^3*x^9)/9 + (5*a*b^
4*x^11)/11 + (b^5*x^13)/13
```

fricas [A] time = 0.81, size = 56, normalized size = 0.85

$$\frac{1}{13}x^{13}b^5+\frac{5}{11}x^{11}b^4a+\frac{10}{9}x^9b^3a^2+\frac{10}{7}x^7b^2a^3+x^5ba^4+\frac{1}{3}x^3a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^5 + 5/11*x^11*b^4*a + 10/9*x^9*b^3*a^2 + 10/7*x^7*b^2*a^3 + x^5
*b*a^4 + 1/3*x^3*a^5
```

giac [A] time = 1.09, size = 56, normalized size = 0.85

$$\frac{1}{13}\,b^{5}x^{13}+\frac{5}{11}\,ab^{4}x^{11}+\frac{10}{9}\,a^{2}b^{3}x^{9}+\frac{10}{7}\,a^{3}b^{2}x^{7}+a^{4}bx^{5}+\frac{1}{3}\,a^{5}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

maple [A] time = 0.00, size = 57, normalized size = 0.86

$$\frac{1}{13}b^{5}x^{13}+\frac{5}{11}a\,b^{4}x^{11}+\frac{10}{9}a^{2}b^{3}x^{9}+\frac{10}{7}a^{3}b^{2}x^{7}+a^{4}b\,x^{5}+\frac{1}{3}a^{5}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^5,x)
```

```
[Out] 1/3*a^5*x^3+a^4*b*x^5+10/7*a^3*b^2*x^7+10/9*a^2*b^3*x^9+5/11*a*b^4*x^11+1/1
3*b^5*x^13
```

maxima [A] time = 1.38, size = 56, normalized size = 0.85

$$\frac{1}{13}\,b^{5}x^{13}+\frac{5}{11}\,ab^{4}x^{11}+\frac{10}{9}\,a^{2}b^{3}x^{9}+\frac{10}{7}\,a^{3}b^{2}x^{7}+a^{4}bx^{5}+\frac{1}{3}\,a^{5}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

mupad [B] time = 0.02, size = 56, normalized size = 0.85

$$\frac{a^5\,x^3}{3}+a^4\,b\,x^5+\frac{10\,a^3\,b^2\,x^7}{7}+\frac{10\,a^2\,b^3\,x^9}{9}+\frac{5\,a\,b^4\,x^{11}}{11}+\frac{b^5\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^3)/3 + (b^5*x^13)/13 + a^4*b*x^5 + (5*a*b^4*x^11)/11 + (10*a^3*b^2*x
^7)/7 + (10*a^2*b^3*x^9)/9
```

sympy [A] time = 0.08, size = 63, normalized size = 0.95

$$\frac{a^{5}x^{3}}{3}+a^{4}bx^{5}+\frac{10a^{3}b^{2}x^{7}}{7}+\frac{10a^{2}b^{3}x^{9}}{9}+\frac{5ab^{4}x^{11}}{11}+\frac{b^{5}x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**3/3 + a**4*b*x**5 + 10*a**3*b**2*x**7/7 + 10*a**2*b**3*x**9/9 + 5*a
*b**4*x**11/11 + b**5*x**13/13
```

3.74 $\int \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=62

$$a^5x+\frac{5}{3}a^4bx^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}$$

```
[Out] a^5*x+5/3*a^4*b*x^3+2*a^3*b^2*x^5+10/7*a^2*b^3*x^7+5/9*a*b^4*x^9+1/11*b^5*x
^11
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$\frac{10}{7}a^2b^3x^7+2a^3b^2x^5+\frac{5}{3}a^4bx^3+a^5x+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^9
)/9 + (b^5*x^11)/11
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^5 \, dx &= \int \left(a^5+5a^4bx^2+10a^3b^2x^4+10a^2b^3x^6+5ab^4x^8+b^5x^{10}\right) \, dx\\ &= a^5x+\frac{5}{3}a^4bx^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 62, normalized size = 1.00

$$a^5x+\frac{5}{3}a^4bx^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^9
)/9 + (b^5*x^11)/11
```

fricas [A] time = 0.68, size = 54, normalized size = 0.87

$$\frac{1}{11}x^{11}b^5+\frac{5}{9}x^9b^4a+\frac{10}{7}x^7b^3a^2+2x^5b^2a^3+\frac{5}{3}x^3ba^4+xa^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^5 + 5/9*x^9*b^4*a + 10/7*x^7*b^3*a^2 + 2*x^5*b^2*a^3 + 5/3*x^3*
b*a^4 + x*a^5
```

giac [A] time = 1.14, size = 54, normalized size = 0.87

$$\frac{1}{11}\,b^{5}x^{11}+\frac{5}{9}\,ab^{4}x^{9}+\frac{10}{7}\,a^{2}b^{3}x^{7}+2\,a^{3}b^{2}x^{5}+\frac{5}{3}\,a^{4}bx^{3}+a^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

maple [A] time = 0.00, size = 55, normalized size = 0.89

$$\frac{1}{11}b^{5}x^{11}+\frac{5}{9}a\,b^{4}x^{9}+\frac{10}{7}a^{2}b^{3}x^{7}+2a^{3}b^{2}x^{5}+\frac{5}{3}a^{4}b\,x^{3}+a^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5,x)
```

```
[Out] a^5*x+5/3*a^4*b*x^3+2*a^3*b^2*x^5+10/7*a^2*b^3*x^7+5/9*a*b^4*x^9+1/11*b^5*x
^11
```

maxima [A] time = 1.38, size = 54, normalized size = 0.87

$$\frac{1}{11}\,b^{5}x^{11}+\frac{5}{9}\,ab^{4}x^{9}+\frac{10}{7}\,a^{2}b^{3}x^{7}+2\,a^{3}b^{2}x^{5}+\frac{5}{3}\,a^{4}bx^{3}+a^{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

mupad [B] time = 0.02, size = 54, normalized size = 0.87

$$a^5\,x+\frac{5\,a^4\,b\,x^3}{3}+2\,a^3\,b^2\,x^5+\frac{10\,a^2\,b^3\,x^7}{7}+\frac{5\,a\,b^4\,x^9}{9}+\frac{b^5\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5,x)
```

```
[Out] a^5*x + (b^5*x^11)/11 + (5*a^4*b*x^3)/3 + (5*a*b^4*x^9)/9 + 2*a^3*b^2*x^5 +
 (10*a^2*b^3*x^7)/7
```

sympy [A] time = 0.07, size = 61, normalized size = 0.98

$$a^{5}x+\frac{5a^{4}bx^{3}}{3}+2a^{3}b^{2}x^{5}+\frac{10a^{2}b^{3}x^{7}}{7}+\frac{5ab^{4}x^{9}}{9}+\frac{b^{5}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**3/3 + 2*a**3*b**2*x**5 + 10*a**2*b**3*x**7/7 + 5*a*b**
4*x**9/9 + b**5*x**11/11
```

3.75 $\int \frac{(a+bx^2)^5}{x^2}\,dx$

Optimal. Leaf size=61

$$-\frac{a^5}{x}+5a^4bx+\frac{10}{3}a^3b^2x^3+2a^2b^3x^5+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}$$

```
[Out] -a^5/x+5*a^4*b*x+10/3*a^3*b^2*x^3+2*a^2*b^3*x^5+5/7*a*b^4*x^7+1/9*b^5*x^9
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$2a^2b^3x^5+\frac{10}{3}a^3b^2x^3+5a^4bx-\frac{a^5}{x}+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^2,x]
```

```
[Out] -(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/7
 + (b^5*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^2}\,dx &= \int \left(5a^4b+\frac{a^5}{x^2}+10a^3b^2x^2+10a^2b^3x^4+5ab^4x^6+b^5x^8\right)dx\\ &= -\frac{a^5}{x}+5a^4bx+\frac{10}{3}a^3b^2x^3+2a^2b^3x^5+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 61, normalized size = 1.00

$$-\frac{a^5}{x}+5a^4bx+\frac{10}{3}a^3b^2x^3+2a^2b^3x^5+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^2,x]
```

```
[Out] -(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/7
 + (b^5*x^9)/9
```

fricas [A] time = 0.83, size = 59, normalized size = 0.97

$$\frac{7\,b^5x^{10}+45\,ab^4x^8+126\,a^2b^3x^6+210\,a^3b^2x^4+315\,a^4bx^2-63\,a^5}{63\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^5*x^10 + 45*a*b^4*x^8 + 126*a^2*b^3*x^6 + 210*a^3*b^2*x^4 + 315*a
^4*b*x^2 - 63*a^5)/x
```

giac [A] time = 1.13, size = 55, normalized size = 0.90

$$\frac{1}{9}\,b^{5}x^{9} + \frac{5}{7}\,ab^{4}x^{7} + 2\,a^{2}b^{3}x^{5} + \frac{10}{3}\,a^{3}b^{2}x^{3} + 5\,a^{4}bx - \frac{a^{5}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x
- a^5/x
```

maple [A] time = 0.00, size = 56, normalized size = 0.92

$$\frac{b^{5}x^{9}}{9} + \frac{5a\,b^{4}x^{7}}{7} + 2a^{2}b^{3}x^{5} + \frac{10a^{3}b^{2}x^{3}}{3} + 5a^{4}bx - \frac{a^{5}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^2,x)
```

```
[Out] -a^5/x+5*a^4*b*x+10/3*a^3*b^2*x^3+2*a^2*b^3*x^5+5/7*a*b^4*x^7+1/9*b^5*x^9
```

maxima [A] time = 1.39, size = 55, normalized size = 0.90

$$\frac{1}{9}\,b^{5}x^{9} + \frac{5}{7}\,ab^{4}x^{7} + 2\,a^{2}b^{3}x^{5} + \frac{10}{3}\,a^{3}b^{2}x^{3} + 5\,a^{4}bx - \frac{a^{5}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x
- a^5/x
```

mupad [B] time = 0.03, size = 55, normalized size = 0.90

$$\frac{b^5\,x^9}{9} - \frac{a^5}{x} + \frac{5\,a\,b^4\,x^7}{7} + \frac{10\,a^3\,b^2\,x^3}{3} + 2\,a^2\,b^3\,x^5 + 5\,a^4\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^2,x)
```

```
[Out] (b^5*x^9)/9 - a^5/x + (5*a*b^4*x^7)/7 + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5
+ 5*a^4*b*x
```

sympy [A] time = 0.13, size = 58, normalized size = 0.95

$$-\frac{a^{5}}{x} + 5a^{4}bx + \frac{10a^{3}b^{2}x^{3}}{3} + 2a^{2}b^{3}x^{5} + \frac{5ab^{4}x^{7}}{7} + \frac{b^{5}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**2,x)
```

```
[Out] -a**5/x + 5*a**4*b*x + 10*a**3*b**2*x**3/3 + 2*a**2*b**3*x**5 + 5*a*b**4*x*
*7/7 + b**5*x**9/9
```

3.76 $\int \frac{(a+bx^2)^5}{x^4} dx$

Optimal. Leaf size=60

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{x}+10a^3b^2x+\frac{10}{3}a^2b^3x^3+ab^4x^5+\frac{b^5x^7}{7}$$

```
[Out] -1/3*a^5/x^3-5*a^4*b/x+10*a^3*b^2*x+10/3*a^2*b^3*x^3+a*b^4*x^5+1/7*b^5*x^7
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10}{3}a^2b^3x^3+10a^3b^2x-\frac{5a^4b}{x}-\frac{a^5}{3x^3}+ab^4x^5+\frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^4,x]
```

```
[Out] -a^5/(3*x^3) - (5*a^4*b)/x + 10*a^3*b^2*x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5
+ (b^5*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^4}\,dx &= \int \left(10a^3b^2+\frac{a^5}{x^4}+\frac{5a^4b}{x^2}+10a^2b^3x^2+5ab^4x^4+b^5x^6\right)dx\\ &= -\frac{a^5}{3x^3}-\frac{5a^4b}{x}+10a^3b^2x+\frac{10}{3}a^2b^3x^3+ab^4x^5+\frac{b^5x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 60, normalized size = 1.00

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{x}+10a^3b^2x+\frac{10}{3}a^2b^3x^3+ab^4x^5+\frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^4,x]
```

```
[Out] -1/3*a^5/x^3 - (5*a^4*b)/x + 10*a^3*b^2*x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5
+ (b^5*x^7)/7
```

fricas [A] time = 0.80, size = 59, normalized size = 0.98

$$\frac{3\,b^5x^{10}+21\,ab^4x^8+70\,a^2b^3x^6+210\,a^3b^2x^4-105\,a^4bx^2-7\,a^5}{21\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^5*x^10 + 21*a*b^4*x^8 + 70*a^2*b^3*x^6 + 210*a^3*b^2*x^4 - 105*a^
4*b*x^2 - 7*a^5)/x^3
```

giac [A] time = 1.13, size = 55, normalized size = 0.92

$$\frac{1}{7}\,b^{5}x^{7} + ab^{4}x^{5} + \frac{10}{3}\,a^{2}b^{3}x^{3} + 10\,a^{3}b^{2}x - \frac{15\,a^{4}bx^{2} + a^{5}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 1/3*(15*a^4*b*x
^2 + a^5)/x^3
```

maple [A] time = 0.00, size = 55, normalized size = 0.92

$$\frac{b^{5}x^{7}}{7} + a\,b^{4}x^{5} + \frac{10a^{2}b^{3}x^{3}}{3} + 10a^{3}b^{2}x - \frac{5a^{4}b}{x} - \frac{a^{5}}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^4,x)
```

```
[Out] -1/3*a^5/x^3-5*a^4*b/x+10*a^3*b^2*x+10/3*a^2*b^3*x^3+a*b^4*x^5+1/7*b^5*x^7
```

maxima [A] time = 1.36, size = 55, normalized size = 0.92

$$\frac{1}{7}\,b^{5}x^{7} + ab^{4}x^{5} + \frac{10}{3}\,a^{2}b^{3}x^{3} + 10\,a^{3}b^{2}x - \frac{15\,a^{4}bx^{2} + a^{5}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 1/3*(15*a^4*b*x
^2 + a^5)/x^3
```

mupad [B] time = 0.03, size = 57, normalized size = 0.95

$$\frac{b^5\,x^7}{7} - \frac{\frac{a^5}{3} + 5\,b\,a^4\,x^2}{x^3} + 10\,a^3\,b^2\,x + a\,b^4\,x^5 + \frac{10\,a^2\,b^3\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^4,x)
```

```
[Out] (b^5*x^7)/7 - (a^5/3 + 5*a^4*b*x^2)/x^3 + 10*a^3*b^2*x + a*b^4*x^5 + (10*a^
2*b^3*x^3)/3
```

sympy [A] time = 0.17, size = 60, normalized size = 1.00

$$10a^{3}b^{2}x + \frac{10a^{2}b^{3}x^{3}}{3} + ab^{4}x^{5} + \frac{b^{5}x^{7}}{7} + \frac{-a^{5} - 15a^{4}bx^{2}}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**4,x)
```

```
[Out] 10*a**3*b**2*x + 10*a**2*b**3*x**3/3 + a*b**4*x**5 + b**5*x**7/7 + (-a**5 -
 15*a**4*b*x**2)/(3*x**3)
```

3.77 $\int \frac{(a+bx^2)^5}{x^6} \, dx$

Optimal. Leaf size=63

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}$$

```
[Out] -1/5*a^5/x^5-5/3*a^4*b/x^3-10*a^3*b^2/x+10*a^2*b^3*x+5/3*a*b^4*x^3+1/5*b^5*
x^5
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{x}+10a^2b^3x-\frac{5a^4b}{3x^3}-\frac{a^5}{5x^5}+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^6,x]
```

```
[Out] -a^5/(5*x^5) - (5*a^4*b)/(3*x^3) - (10*a^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^4
*x^3)/3 + (b^5*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5}{x^6} \, dx &= \int \left(10a^2b^3+\frac{a^5}{x^6}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^2}+5ab^4x^2+b^5x^4\right) dx\\ &= -\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 63, normalized size = 1.00

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^6,x]
```

```
[Out] -1/5*a^5/x^5 - (5*a^4*b)/(3*x^3) - (10*a^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^4
*x^3)/3 + (b^5*x^5)/5
```

fricas [A] time = 0.87, size = 59, normalized size = 0.94

$$\frac{3\,b^5x^{10}+25\,ab^4x^8+150\,a^2b^3x^6-150\,a^3b^2x^4-25\,a^4bx^2-3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^5*x^10 + 25*a*b^4*x^8 + 150*a^2*b^3*x^6 - 150*a^3*b^2*x^4 - 25*a^
4*b*x^2 - 3*a^5)/x^5
```

giac [A] time = 0.94, size = 58, normalized size = 0.92

$$\frac{1}{5}\,b^5x^5 + \frac{5}{3}\,ab^4x^3 + 10\,a^2b^3x - \frac{150\,a^3b^2x^4 + 25\,a^4bx^2 + 3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 1/15*(150*a^3*b^2*x^4 + 25*a^4
*b*x^2 + 3*a^5)/x^5
```

maple [A] time = 0.00, size = 56, normalized size = 0.89

$$\frac{b^5x^5}{5} + \frac{5a\,b^4x^3}{3} + 10a^2b^3x - \frac{10a^3b^2}{x} - \frac{5a^4b}{3x^3} - \frac{a^5}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^6,x)
```

```
[Out] -1/5*a^5/x^5-5/3*a^4*b/x^3-10*a^3*b^2/x+10*a^2*b^3*x+5/3*a*b^4*x^3+1/5*b^5*
x^5
```

maxima [A] time = 1.38, size = 58, normalized size = 0.92

$$\frac{1}{5}\,b^5x^5 + \frac{5}{3}\,ab^4x^3 + 10\,a^2b^3x - \frac{150\,a^3b^2x^4 + 25\,a^4bx^2 + 3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 1/15*(150*a^3*b^2*x^4 + 25*a^4
*b*x^2 + 3*a^5)/x^5
```

mupad [B] time = 0.05, size = 58, normalized size = 0.92

$$\frac{b^5\,x^5}{5} - \frac{\frac{a^5}{5} + \frac{5\,a^4\,b\,x^2}{3} + 10\,a^3\,b^2\,x^4}{x^5} + 10\,a^2\,b^3\,x + \frac{5\,a\,b^4\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^6,x)
```

```
[Out] (b^5*x^5)/5 - (a^5/5 + (5*a^4*b*x^2)/3 + 10*a^3*b^2*x^4)/x^5 + 10*a^2*b^3*x
 + (5*a*b^4*x^3)/3
```

sympy [A] time = 0.23, size = 63, normalized size = 1.00

$$10a^2b^3x + \frac{5ab^4x^3}{3} + \frac{b^5x^5}{5} + \frac{-3a^5 - 25a^4bx^2 - 150a^3b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**6,x)
```

```
[Out] 10*a**2*b**3*x + 5*a*b**4*x**3/3 + b**5*x**5/5 + (-3*a**5 - 25*a**4*b*x**2
- 150*a**3*b**2*x**4)/(15*x**5)
```

3.78 $\int \frac{(a+bx^2)^5}{x^8}\,dx$

Optimal. Leaf size=61

$$-\frac{a^5}{7x^7}-\frac{a^4b}{x^5}-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}+5ab^4x+\frac{b^5x^3}{3}$$

```
[Out] -1/7*a^5/x^7-a^4*b/x^5-10/3*a^3*b^2/x^3-10*a^2*b^3/x+5*a*b^4*x+1/3*b^5*x^3
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}-\frac{a^4b}{x^5}-\frac{a^5}{7x^7}+5ab^4x+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^8,x]
```

```
[Out] -a^5/(7*x^7) - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a*b^
4*x + (b^5*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^8}\,dx &= \int \left(5ab^4+\frac{a^5}{x^8}+\frac{5a^4b}{x^6}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^2}+b^5x^2\right)dx\\ &= -\frac{a^5}{7x^7}-\frac{a^4b}{x^5}-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}+5ab^4x+\frac{b^5x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 61, normalized size = 1.00

$$-\frac{a^5}{7x^7}-\frac{a^4b}{x^5}-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}+5ab^4x+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^8,x]
```

```
[Out] -1/7*a^5/x^7 - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a*b^
4*x + (b^5*x^3)/3
```

fricas [A] time = 0.79, size = 59, normalized size = 0.97

$$\frac{7\,b^5x^{10}+105\,ab^4x^8-210\,a^2b^3x^6-70\,a^3b^2x^4-21\,a^4bx^2-3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^5*x^10 + 105*a*b^4*x^8 - 210*a^2*b^3*x^6 - 70*a^3*b^2*x^4 - 21*a^
4*b*x^2 - 3*a^5)/x^7
```

giac [A] time = 1.12, size = 58, normalized size = 0.95

$$\frac{1}{3}\,b^5x^3 + 5\,ab^4x - \frac{210\,a^2b^3x^6 + 70\,a^3b^2x^4 + 21\,a^4bx^2 + 3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^8,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3 + 5*a*b^4*x - 1/21*(210*a^2*b^3*x^6 + 70*a^3*b^2*x^4 + 21*a^4*b
*x^2 + 3*a^5)/x^7
```

maple [A] time = 0.01, size = 56, normalized size = 0.92

$$\frac{b^5x^3}{3} + 5a\,b^4x - \frac{10a^2b^3}{x} - \frac{10a^3b^2}{3x^3} - \frac{a^4b}{x^5} - \frac{a^5}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^8,x)
```

```
[Out] -1/7*a^5/x^7-a^4*b/x^5-10/3*a^3*b^2/x^3-10*a^2*b^3/x+5*a*b^4*x+1/3*b^5*x^3
```

maxima [A] time = 1.44, size = 58, normalized size = 0.95

$$\frac{1}{3}\,b^5x^3 + 5\,ab^4x - \frac{210\,a^2b^3x^6 + 70\,a^3b^2x^4 + 21\,a^4bx^2 + 3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*b^5*x^3 + 5*a*b^4*x - 1/21*(210*a^2*b^3*x^6 + 70*a^3*b^2*x^4 + 21*a^4*b
*x^2 + 3*a^5)/x^7
```

mupad [B] time = 4.79, size = 59, normalized size = 0.97

$$-\frac{3\,a^5 + 21\,a^4\,b\,x^2 + 70\,a^3\,b^2\,x^4 + 210\,a^2\,b^3\,x^6 - 105\,a\,b^4\,x^8 - 7\,b^5\,x^{10}}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^8,x)
```

```
[Out] -(3*a^5 - 7*b^5*x^10 + 21*a^4*b*x^2 - 105*a*b^4*x^8 + 70*a^3*b^2*x^4 + 210*
a^2*b^3*x^6)/(21*x^7)
```

sympy [A] time = 0.29, size = 61, normalized size = 1.00

$$5ab^4x + \frac{b^5x^3}{3} + \frac{-3a^5 - 21a^4bx^2 - 70a^3b^2x^4 - 210a^2b^3x^6}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**8,x)
```

```
[Out] 5*a*b**4*x + b**5*x**3/3 + (-3*a**5 - 21*a**4*b*x**2 - 70*a**3*b**2*x**4 -
210*a**2*b**3*x**6)/(21*x**7)
```

3.79 $\int \frac{(a+bx^2)^5}{x^{10}}\,dx$

Optimal. Leaf size=60

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{x}+b^5x$$

```
[Out] -1/9*a^5/x^9-5/7*a^4*b/x^7-2*a^3*b^2/x^5-10/3*a^2*b^3/x^3-5*a*b^4/x+b^5*x
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5a^4b}{7x^7}-\frac{a^5}{9x^9}-\frac{5ab^4}{x}+b^5x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^10,x]
```

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3) -
 (5*a*b^4)/x + b^5*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{10}}\,dx &= \int \left(b^5+\frac{a^5}{x^{10}}+\frac{5a^4b}{x^8}+\frac{10a^3b^2}{x^6}+\frac{10a^2b^3}{x^4}+\frac{5ab^4}{x^2}\right)dx\\ &= -\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{x}+b^5x\end{aligned}$$

Mathematica [A] time = 0.01, size = 60, normalized size = 1.00

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{x}+b^5x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3) -
 (5*a*b^4)/x + b^5*x
```

fricas [A] time = 0.70, size = 59, normalized size = 0.98

$$\frac{63\,b^5x^{10}-315\,ab^4x^8-210\,a^2b^3x^6-126\,a^3b^2x^4-45\,a^4bx^2-7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(63*b^5*x^10 - 315*a*b^4*x^8 - 210*a^2*b^3*x^6 - 126*a^3*b^2*x^4 - 45*
a^4*b*x^2 - 7*a^5)/x^9
```

giac [A] time = 1.04, size = 57, normalized size = 0.95

$$b^5x - \frac{315\,ab^4x^8 + 210\,a^2b^3x^6 + 126\,a^3b^2x^4 + 45\,a^4bx^2 + 7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^10,x, algorithm="giac")
```

```
[Out] b^5*x - 1/63*(315*a*b^4*x^8 + 210*a^2*b^3*x^6 + 126*a^3*b^2*x^4 + 45*a^4*b*
x^2 + 7*a^5)/x^9
```

maple [A] time = 0.01, size = 55, normalized size = 0.92

$$b^5x - \frac{5a\,b^4}{x} - \frac{10a^2b^3}{3x^3} - \frac{2a^3b^2}{x^5} - \frac{5a^4b}{7x^7} - \frac{a^5}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^10,x)
```

```
[Out] -1/9*a^5/x^9-5/7*a^4*b/x^7-2*a^3*b^2/x^5-10/3*a^2*b^3/x^3-5*a*b^4/x+b^5*x
```

maxima [A] time = 1.44, size = 57, normalized size = 0.95

$$b^5x - \frac{315\,ab^4x^8 + 210\,a^2b^3x^6 + 126\,a^3b^2x^4 + 45\,a^4bx^2 + 7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] b^5*x - 1/63*(315*a*b^4*x^8 + 210*a^2*b^3*x^6 + 126*a^3*b^2*x^4 + 45*a^4*b*
x^2 + 7*a^5)/x^9
```

mupad [B] time = 0.04, size = 57, normalized size = 0.95

$$b^5\,x - \frac{\frac{a^5}{9} + \frac{5\,a^4\,b\,x^2}{7} + 2\,a^3\,b^2\,x^4 + \frac{10\,a^2\,b^3\,x^6}{3} + 5\,a\,b^4\,x^8}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^10,x)
```

```
[Out] b^5*x - (a^5/9 + (5*a^4*b*x^2)/7 + 5*a*b^4*x^8 + 2*a^3*b^2*x^4 + (10*a^2*b^
3*x^6)/3)/x^9
```

sympy [A] time = 0.36, size = 60, normalized size = 1.00

$$b^5x + \frac{-7a^5 - 45a^4bx^2 - 126a^3b^2x^4 - 210a^2b^3x^6 - 315ab^4x^8}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**10,x)
```

```
[Out] b**5*x + (-7*a**5 - 45*a**4*b*x**2 - 126*a**3*b**2*x**4 - 210*a**2*b**3*x**
6 - 315*a*b**4*x**8)/(63*x**9)
```

3.80 $\int \frac{(a+bx^2)^5}{x^{12}} \, dx$

Optimal. Leaf size=65

$$-\frac{a^5}{11x^{11}}-\frac{5a^4b}{9x^9}-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}$$

```
[Out] -1/11*a^5/x^11-5/9*a^4*b/x^9-10/7*a^3*b^2/x^7-2*a^2*b^3/x^5-5/3*a*b^4/x^3-b
^5/x
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5a^4b}{9x^9}-\frac{a^5}{11x^{11}}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^12,x]
```

```
[Out] -a^5/(11*x^11) - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/x^5
 - (5*a*b^4)/(3*x^3) - b^5/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^5}{x^{12}} \, dx &= \int \left(\frac{a^5}{x^{12}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^6}+\frac{5ab^4}{x^4}+\frac{b^5}{x^2}\right) dx \\
&= -\frac{a^5}{11x^{11}}-\frac{5a^4b}{9x^9}-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 65, normalized size = 1.00

$$-\frac{a^5}{11x^{11}}-\frac{5a^4b}{9x^9}-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^12,x]
```

```
[Out] -1/11*a^5/x^11 - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/x^5
 - (5*a*b^4)/(3*x^3) - b^5/x
```

fricas [A] time = 0.76, size = 59, normalized size = 0.91

$$-\frac{693\,b^5x^{10}+1155\,ab^4x^8+1386\,a^2b^3x^6+990\,a^3b^2x^4+385\,a^4bx^2+63\,a^5}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^12,x, algorithm="fricas")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

giac [A] time = 1.04, size = 59, normalized size = 0.91

$$-\frac{693\,b^{5}x^{10}+1155\,ab^{4}x^{8}+1386\,a^{2}b^{3}x^{6}+990\,a^{3}b^{2}x^{4}+385\,a^{4}bx^{2}+63\,a^{5}}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^12,x, algorithm="giac")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

maple [A] time = 0.01, size = 58, normalized size = 0.89

$$-\frac{b^{5}}{x}-\frac{5a\,b^{4}}{3x^{3}}-\frac{2a^{2}b^{3}}{x^{5}}-\frac{10a^{3}b^{2}}{7x^{7}}-\frac{5a^{4}b}{9x^{9}}-\frac{a^{5}}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^12,x)
```

```
[Out] -1/11*a^5/x^11-5/9*a^4*b/x^9-10/7*a^3*b^2/x^7-2*a^2*b^3/x^5-5/3*a*b^4/x^3-b
^5/x
```

maxima [A] time = 1.36, size = 59, normalized size = 0.91

$$-\frac{693\,b^{5}x^{10}+1155\,ab^{4}x^{8}+1386\,a^{2}b^{3}x^{6}+990\,a^{3}b^{2}x^{4}+385\,a^{4}bx^{2}+63\,a^{5}}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^12,x, algorithm="maxima")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

mupad [B] time = 0.04, size = 58, normalized size = 0.89

$$-\frac{\frac{a^{5}}{11}+\frac{5\,a^{4}\,b\,x^{2}}{9}+\frac{10\,a^{3}\,b^{2}\,x^{4}}{7}+2\,a^{2}\,b^{3}\,x^{6}+\frac{5\,a\,b^{4}\,x^{8}}{3}+b^{5}\,x^{10}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^12,x)
```

```
[Out] -(a^5/11 + b^5*x^10 + (5*a^4*b*x^2)/9 + (5*a*b^4*x^8)/3 + (10*a^3*b^2*x^4)/
7 + 2*a^2*b^3*x^6)/x^11
```

sympy [A] time = 0.43, size = 63, normalized size = 0.97

$$\frac{-63a^{5}-385a^{4}bx^{2}-990a^{3}b^{2}x^{4}-1386a^{2}b^{3}x^{6}-1155ab^{4}x^{8}-693b^{5}x^{10}}{693x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**12,x)
```

```
[Out] (-63*a**5 - 385*a**4*b*x**2 - 990*a**3*b**2*x**4 - 1386*a**2*b**3*x**6 - 11
55*a*b**4*x**8 - 693*b**5*x**10)/(693*x**11)
```

3.81 $\int \frac{(a+bx^2)^5}{x^{14}}\,dx$

Optimal. Leaf size=67

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{11x^{11}}-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}$$

```
[Out] -1/13*a^5/x^13-5/11*a^4*b/x^11-10/9*a^3*b^2/x^9-10/7*a^2*b^3/x^7-a*b^4/x^5-
1/3*b^5/x^3
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{5a^4b}{11x^{11}}-\frac{a^5}{13x^{13}}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^14,x]
```

```
[Out] -a^5/(13*x^13) - (5*a^4*b)/(11*x^11) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^3)/
(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{14}}\,dx &= \int \left(\frac{a^5}{x^{14}}+\frac{5a^4b}{x^{12}}+\frac{10a^3b^2}{x^{10}}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^6}+\frac{b^5}{x^4}\right)dx\\ &= -\frac{a^5}{13x^{13}}-\frac{5a^4b}{11x^{11}}-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 67, normalized size = 1.00

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{11x^{11}}-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^14,x]
```

```
[Out] -1/13*a^5/x^13 - (5*a^4*b)/(11*x^11) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^3)/
(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3)
```

fricas [A] time = 0.73, size = 59, normalized size = 0.88

$$-\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,a^3b^2x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^14,x, algorithm="fricas")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

giac [A] time = 1.09, size = 59, normalized size = 0.88

$$-\frac{3003\, b^{5}x^{10} + 9009\, ab^{4}x^{8} + 12870\, a^{2}b^{3}x^{6} + 10010\, a^{3}b^{2}x^{4} + 4095\, a^{4}bx^{2} + 693\, a^{5}}{9009\, x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^14,x, algorithm="giac")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

maple [A] time = 0.01, size = 58, normalized size = 0.87

$$-\frac{b^{5}}{3x^{3}} - \frac{a\, b^{4}}{x^{5}} - \frac{10a^{2}b^{3}}{7x^{7}} - \frac{10a^{3}b^{2}}{9x^{9}} - \frac{5a^{4}b}{11x^{11}} - \frac{a^{5}}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^14,x)
```

```
[Out] -1/13*a^5/x^13-5/11*a^4*b/x^11-10/9*a^3*b^2/x^9-10/7*a^2*b^3/x^7-a*b^4/x^5-
1/3*b^5/x^3
```

maxima [A] time = 1.36, size = 59, normalized size = 0.88

$$-\frac{3003\, b^{5}x^{10} + 9009\, ab^{4}x^{8} + 12870\, a^{2}b^{3}x^{6} + 10010\, a^{3}b^{2}x^{4} + 4095\, a^{4}bx^{2} + 693\, a^{5}}{9009\, x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^14,x, algorithm="maxima")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

mupad [B] time = 0.04, size = 58, normalized size = 0.87

$$-\frac{\frac{a^{5}}{13} + \frac{5\, a^{4}\, b\, x^{2}}{11} + \frac{10\, a^{3}\, b^{2}\, x^{4}}{9} + \frac{10\, a^{2}\, b^{3}\, x^{6}}{7} + a\, b^{4}\, x^{8} + \frac{b^{5}\, x^{10}}{3}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^14,x)
```

```
[Out] -(a^5/13 + (b^5*x^10)/3 + (5*a^4*b*x^2)/11 + a*b^4*x^8 + (10*a^3*b^2*x^4)/9
 + (10*a^2*b^3*x^6)/7)/x^13
```

sympy [A] time = 0.47, size = 63, normalized size = 0.94

$$\frac{-693a^{5} - 4095a^{4}bx^{2} - 10010a^{3}b^{2}x^{4} - 12870a^{2}b^{3}x^{6} - 9009ab^{4}x^{8} - 3003b^{5}x^{10}}{9009x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**14,x)
```

```
[Out] (-693*a**5 - 4095*a**4*b*x**2 - 10010*a**3*b**2*x**4 - 12870*a**2*b**3*x**6
 - 9009*a*b**4*x**8 - 3003*b**5*x**10)/(9009*x**13)
```

3.82 $\int \frac{(a+bx^2)^5}{x^{16}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{15x^{15}}-\frac{5a^4b}{13x^{13}}-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{7x^7}-\frac{b^5}{5x^5}$$

```
[Out] -1/15*a^5/x^15-5/13*a^4*b/x^13-10/11*a^3*b^2/x^11-10/9*a^2*b^3/x^9-5/7*a*b^
4/x^7-1/5*b^5/x^5
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5a^4b}{13x^{13}}-\frac{a^5}{15x^{15}}-\frac{5ab^4}{7x^7}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^16,x]
```

```
[Out] -a^5/(15*x^15) - (5*a^4*b)/(13*x^13) - (10*a^3*b^2)/(11*x^11) - (10*a^2*b^3
)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{16}}\,dx &= \int \left(\frac{a^5}{x^{16}}+\frac{5a^4b}{x^{14}}+\frac{10a^3b^2}{x^{12}}+\frac{10a^2b^3}{x^{10}}+\frac{5ab^4}{x^8}+\frac{b^5}{x^6}\right)dx\\ &= -\frac{a^5}{15x^{15}}-\frac{5a^4b}{13x^{13}}-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{7x^7}-\frac{b^5}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{15x^{15}}-\frac{5a^4b}{13x^{13}}-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{7x^7}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^16,x]
```

```
[Out] -1/15*a^5/x^15 - (5*a^4*b)/(13*x^13) - (10*a^3*b^2)/(11*x^11) - (10*a^2*b^3
)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5)
```

fricas [A] time = 0.70, size = 59, normalized size = 0.86

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

giac [A] time = 1.08, size = 59, normalized size = 0.86

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

maple [A] time = 0.01, size = 58, normalized size = 0.84

$$-\frac{b^5}{5x^5}-\frac{5a\,b^4}{7x^7}-\frac{10a^2b^3}{9x^9}-\frac{10a^3b^2}{11x^{11}}-\frac{5a^4b}{13x^{13}}-\frac{a^5}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^16,x)
```

```
[Out] -1/15*a^5/x^15-5/13*a^4*b/x^13-10/11*a^3*b^2/x^11-10/9*a^2*b^3/x^9-5/7*a*b^
4/x^7-1/5*b^5/x^5
```

maxima [A] time = 1.37, size = 59, normalized size = 0.86

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

mupad [B] time = 4.75, size = 59, normalized size = 0.86

$$-\frac{\frac{a^5}{15}+\frac{5\,a^4\,b\,x^2}{13}+\frac{10\,a^3\,b^2\,x^4}{11}+\frac{10\,a^2\,b^3\,x^6}{9}+\frac{5\,a\,b^4\,x^8}{7}+\frac{b^5\,x^{10}}{5}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^16,x)
```

```
[Out] -(a^5/15 + (b^5*x^10)/5 + (5*a^4*b*x^2)/13 + (5*a*b^4*x^8)/7 + (10*a^3*b^2*
x^4)/11 + (10*a^2*b^3*x^6)/9)/x^15
```

sympy [A] time = 0.51, size = 63, normalized size = 0.91

$$\frac{-3003a^5-17325a^4bx^2-40950a^3b^2x^4-50050a^2b^3x^6-32175ab^4x^8-9009b^5x^{10}}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**16,x)
```

```
[Out] (-3003*a**5 - 17325*a**4*b*x**2 - 40950*a**3*b**2*x**4 - 50050*a**2*b**3*x*
*6 - 32175*a*b**4*x**8 - 9009*b**5*x**10)/(45045*x**15)
```

3.83 $\int \frac{(a+bx^2)^5}{x^{18}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{17x^{17}}-\frac{a^4b}{3x^{15}}-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}$$

```
[Out] -1/17*a^5/x^17-1/3*a^4*b/x^15-10/13*a^3*b^2/x^13-10/11*a^2*b^3/x^11-5/9*a*b
^4/x^9-1/7*b^5/x^7
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{a^4b}{3x^{15}}-\frac{a^5}{17x^{17}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^18,x]
```

```
[Out] -a^5/(17*x^17) - (a^4*b)/(3*x^15) - (10*a^3*b^2)/(13*x^13) - (10*a^2*b^3)/(
11*x^11) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5}{x^{18}}\,dx &= \int \left(\frac{a^5}{x^{18}}+\frac{5a^4b}{x^{16}}+\frac{10a^3b^2}{x^{14}}+\frac{10a^2b^3}{x^{12}}+\frac{5ab^4}{x^{10}}+\frac{b^5}{x^8}\right)dx\\ &= -\frac{a^5}{17x^{17}}-\frac{a^4b}{3x^{15}}-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{17x^{17}}-\frac{a^4b}{3x^{15}}-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^18,x]
```

```
[Out] -1/17*a^5/x^17 - (a^4*b)/(3*x^15) - (10*a^3*b^2)/(13*x^13) - (10*a^2*b^3)/(
11*x^11) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7)
```

fricas [A] time = 0.61, size = 59, normalized size = 0.86

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^18,x, algorithm="fricas")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

giac [A] time = 1.10, size = 59, normalized size = 0.86

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^18,x, algorithm="giac")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$-\frac{b^5}{7x^7}-\frac{5a\,b^4}{9x^9}-\frac{10a^2b^3}{11x^{11}}-\frac{10a^3b^2}{13x^{13}}-\frac{a^4b}{3x^{15}}-\frac{a^5}{17x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^18,x)
```

```
[Out] -1/17*a^5/x^17-1/3*a^4*b/x^15-10/13*a^3*b^2/x^13-10/11*a^2*b^3/x^11-5/9*a*b
^4/x^9-1/7*b^5/x^7
```

maxima [A] time = 1.32, size = 59, normalized size = 0.86

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^18,x, algorithm="maxima")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

mupad [B] time = 0.04, size = 59, normalized size = 0.86

$$-\frac{\frac{a^5}{17}+\frac{a^4\,b\,x^2}{3}+\frac{10\,a^3\,b^2\,x^4}{13}+\frac{10\,a^2\,b^3\,x^6}{11}+\frac{5\,a\,b^4\,x^8}{9}+\frac{b^5\,x^{10}}{7}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^18,x)
```

```
[Out] -(a^5/17 + (b^5*x^10)/7 + (a^4*b*x^2)/3 + (5*a*b^4*x^8)/9 + (10*a^3*b^2*x^4
)/13 + (10*a^2*b^3*x^6)/11)/x^17
```

sympy [A] time = 0.54, size = 63, normalized size = 0.91

$$\frac{-9009a^5-51051a^4bx^2-117810a^3b^2x^4-139230a^2b^3x^6-85085ab^4x^8-21879b^5x^{10}}{153153x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**18,x)
```

```
[Out] (-9009*a**5 - 51051*a**4*b*x**2 - 117810*a**3*b**2*x**4 - 139230*a**2*b**3*
x**6 - 85085*a*b**4*x**8 - 21879*b**5*x**10)/(153153*x**17)
```

3.84 $\int \frac{(a+bx^2)^5}{x^{20}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

```
[Out] -1/19*a^5/x^19-5/17*a^4*b/x^17-2/3*a^3*b^2/x^15-10/13*a^2*b^3/x^13-5/11*a*b
^4/x^11-1/9*b^5/x^9
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5a^4b}{17x^{17}}-\frac{a^5}{19x^{19}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^20,x]
```

```
[Out] -a^5/(19*x^19) - (5*a^4*b)/(17*x^17) - (2*a^3*b^2)/(3*x^15) - (10*a^2*b^3)/
(13*x^13) - (5*a*b^4)/(11*x^11) - b^5/(9*x^9)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{20}}\,dx &= \int \left(\frac{a^5}{x^{20}}+\frac{5a^4b}{x^{18}}+\frac{10a^3b^2}{x^{16}}+\frac{10a^2b^3}{x^{14}}+\frac{5ab^4}{x^{12}}+\frac{b^5}{x^{10}}\right)dx\\ &= -\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 69, normalized size = 1.00

$$-\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^20,x]
```

```
[Out] -1/19*a^5/x^19 - (5*a^4*b)/(17*x^17) - (2*a^3*b^2)/(3*x^15) - (10*a^2*b^3)/
(13*x^13) - (5*a*b^4)/(11*x^11) - b^5/(9*x^9)
```

fricas [A] time = 0.53, size = 59, normalized size = 0.86

$$-\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,a^3b^2x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^20,x, algorithm="fricas")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

giac [A] time = 1.13, size = 59, normalized size = 0.86

$$-\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,a^3b^2x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^20,x, algorithm="giac")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

maple [A] time = 0.01, size = 58, normalized size = 0.84

$$-\frac{b^5}{9x^9}-\frac{5a\,b^4}{11x^{11}}-\frac{10a^2b^3}{13x^{13}}-\frac{2a^3b^2}{3x^{15}}-\frac{5a^4b}{17x^{17}}-\frac{a^5}{19x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^20,x)
```

```
[Out] -1/19*a^5/x^19-5/17*a^4*b/x^17-2/3*a^3*b^2/x^15-10/13*a^2*b^3/x^13-5/11*a*b
^4/x^11-1/9*b^5/x^9
```

maxima [A] time = 1.39, size = 59, normalized size = 0.86

$$-\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,a^3b^2x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^20,x, algorithm="maxima")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

mupad [B] time = 0.04, size = 59, normalized size = 0.86

$$-\frac{\frac{a^5}{19}+\frac{5\,a^4\,b\,x^2}{17}+\frac{2\,a^3\,b^2\,x^4}{3}+\frac{10\,a^2\,b^3\,x^6}{13}+\frac{5\,a\,b^4\,x^8}{11}+\frac{b^5\,x^{10}}{9}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^20,x)
```

```
[Out] -(a^5/19 + (b^5*x^10)/9 + (5*a^4*b*x^2)/17 + (5*a*b^4*x^8)/11 + (2*a^3*b^2*
x^4)/3 + (10*a^2*b^3*x^6)/13)/x^19
```

sympy [A] time = 0.57, size = 63, normalized size = 0.91

$$\frac{-21879a^5-122265a^4bx^2-277134a^3b^2x^4-319770a^2b^3x^6-188955ab^4x^8-46189b^5x^{10}}{415701x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**20,x)
```

```
[Out] (-21879*a**5 - 122265*a**4*b*x**2 - 277134*a**3*b**2*x**4 - 319770*a**2*b**
3*x**6 - 188955*a*b**4*x**8 - 46189*b**5*x**10)/(415701*x**19)
```

3.85 $\int x^{13} \left(a + bx^2\right)^8 \, dx$

Optimal. Leaf size=129

$$\frac{a^6 \left(a + bx^2\right)^9}{18b^7} - \frac{3a^5 \left(a + bx^2\right)^{10}}{10b^7} + \frac{15a^4 \left(a + bx^2\right)^{11}}{22b^7} - \frac{5a^3 \left(a + bx^2\right)^{12}}{6b^7} + \frac{15a^2 \left(a + bx^2\right)^{13}}{26b^7} + \frac{\left(a + bx^2\right)^{15}}{30b^7} - \frac{3a \left(a + bx^2\right)^{1}}{14b^7}$$

```
[Out] 1/18*a^6*(b*x^2+a)^9/b^7-3/10*a^5*(b*x^2+a)^10/b^7+15/22*a^4*(b*x^2+a)^11/b
^7-5/6*a^3*(b*x^2+a)^12/b^7+15/26*a^2*(b*x^2+a)^13/b^7-3/14*a*(b*x^2+a)^14/
b^7+1/30*(b*x^2+a)^15/b^7
```

Rubi [A] time = 0.21, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{15a^2 \left(a + bx^2\right)^{13}}{26b^7} - \frac{5a^3 \left(a + bx^2\right)^{12}}{6b^7} + \frac{15a^4 \left(a + bx^2\right)^{11}}{22b^7} - \frac{3a^5 \left(a + bx^2\right)^{10}}{10b^7} + \frac{a^6 \left(a + bx^2\right)^9}{18b^7} + \frac{\left(a + bx^2\right)^{15}}{30b^7} - \frac{3a \left(a + bx^2\right)^{1}}{14b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a + b*x^2)^8,x]
```

```
[Out] (a^6*(a + b*x^2)^9)/(18*b^7) - (3*a^5*(a + b*x^2)^10)/(10*b^7) + (15*a^4*(a
 + b*x^2)^11)/(22*b^7) - (5*a^3*(a + b*x^2)^12)/(6*b^7) + (15*a^2*(a + b*x^
2)^13)/(26*b^7) - (3*a*(a + b*x^2)^14)/(14*b^7) + (a + b*x^2)^15/(30*b^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^{13} \left(a + bx^2\right)^8 \, dx &= \frac{1}{2} \text{Subst}\left(\int x^6(a + bx)^8 \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a^6(a + bx)^8}{b^6} - \frac{6a^5(a + bx)^9}{b^6} + \frac{15a^4(a + bx)^{10}}{b^6} - \frac{20a^3(a + bx)^{11}}{b^6} + \frac{15a^2(a + l}{b^6}\right.\right. \\
&= \frac{a^6 \left(a + bx^2\right)^9}{18b^7} - \frac{3a^5 \left(a + bx^2\right)^{10}}{10b^7} + \frac{15a^4 \left(a + bx^2\right)^{11}}{22b^7} - \frac{5a^3 \left(a + bx^2\right)^{12}}{6b^7} + \frac{15a^2 \left(a + bx^2\right)^{13}}{26b^7}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 0.84

$$\frac{a^8x^{14}}{14} + \frac{1}{2}a^7bx^{16} + \frac{14}{9}a^6b^2x^{18} + \frac{14}{5}a^5b^3x^{20} + \frac{35}{11}a^4b^4x^{22} + \frac{7}{3}a^3b^5x^{24} + \frac{14}{13}a^2b^6x^{26} + \frac{2}{7}ab^7x^{28} + \frac{b^8x^{30}}{30}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^14)/14 + (a^7*b*x^16)/2 + (14*a^6*b^2*x^18)/9 + (14*a^5*b^3*x^20)/5
+ (35*a^4*b^4*x^22)/11 + (7*a^3*b^5*x^24)/3 + (14*a^2*b^6*x^26)/13 + (2*a*b
^7*x^28)/7 + (b^8*x^30)/30
```

fricas [A] time = 0.62, size = 90, normalized size = 0.70

$$\frac{1}{30}x^{30}b^8+\frac{2}{7}x^{28}b^7a+\frac{14}{13}x^{26}b^6a^2+\frac{7}{3}x^{24}b^5a^3+\frac{35}{11}x^{22}b^4a^4+\frac{14}{5}x^{20}b^3a^5+\frac{14}{9}x^{18}b^2a^6+\frac{1}{2}x^{16}ba^7+\frac{1}{14}x^{14}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/30*x^30*b^8 + 2/7*x^28*b^7*a + 14/13*x^26*b^6*a^2 + 7/3*x^24*b^5*a^3 + 35
/11*x^22*b^4*a^4 + 14/5*x^20*b^3*a^5 + 14/9*x^18*b^2*a^6 + 1/2*x^16*b*a^7 +
 1/14*x^14*a^8
```

giac [A] time = 1.06, size = 90, normalized size = 0.70

$$\frac{1}{30}\,b^8x^{30}+\frac{2}{7}\,ab^7x^{28}+\frac{14}{13}\,a^2b^6x^{26}+\frac{7}{3}\,a^3b^5x^{24}+\frac{35}{11}\,a^4b^4x^{22}+\frac{14}{5}\,a^5b^3x^{20}+\frac{14}{9}\,a^6b^2x^{18}+\frac{1}{2}\,a^7bx^{16}+\frac{1}{14}\,a^8x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/30*b^8*x^30 + 2/7*a*b^7*x^28 + 14/13*a^2*b^6*x^26 + 7/3*a^3*b^5*x^24 + 35
/11*a^4*b^4*x^22 + 14/5*a^5*b^3*x^20 + 14/9*a^6*b^2*x^18 + 1/2*a^7*b*x^16 +
 1/14*a^8*x^14
```

maple [A] time = 0.00, size = 91, normalized size = 0.71

$$\frac{1}{30}b^8x^{30}+\frac{2}{7}a\,b^7x^{28}+\frac{14}{13}a^2b^6x^{26}+\frac{7}{3}a^3b^5x^{24}+\frac{35}{11}a^4b^4x^{22}+\frac{14}{5}a^5b^3x^{20}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{2}a^7b\,x^{16}+\frac{1}{14}a^8x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b*x^2+a)^8,x)
```

```
[Out] 1/30*b^8*x^30+2/7*a*b^7*x^28+14/13*a^2*b^6*x^26+7/3*a^3*b^5*x^24+35/11*a^4*
b^4*x^22+14/5*a^5*b^3*x^20+14/9*a^6*b^2*x^18+1/2*a^7*b*x^16+1/14*a^8*x^14
```

maxima [A] time = 1.30, size = 90, normalized size = 0.70

$$\frac{1}{30}\,b^8x^{30}+\frac{2}{7}\,ab^7x^{28}+\frac{14}{13}\,a^2b^6x^{26}+\frac{7}{3}\,a^3b^5x^{24}+\frac{35}{11}\,a^4b^4x^{22}+\frac{14}{5}\,a^5b^3x^{20}+\frac{14}{9}\,a^6b^2x^{18}+\frac{1}{2}\,a^7bx^{16}+\frac{1}{14}\,a^8x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/30*b^8*x^30 + 2/7*a*b^7*x^28 + 14/13*a^2*b^6*x^26 + 7/3*a^3*b^5*x^24 + 35
/11*a^4*b^4*x^22 + 14/5*a^5*b^3*x^20 + 14/9*a^6*b^2*x^18 + 1/2*a^7*b*x^16 +
 1/14*a^8*x^14
```

mupad [B] time = 0.10, size = 90, normalized size = 0.70

$$\frac{a^8\,x^{14}}{14}+\frac{a^7\,b\,x^{16}}{2}+\frac{14\,a^6\,b^2\,x^{18}}{9}+\frac{14\,a^5\,b^3\,x^{20}}{5}+\frac{35\,a^4\,b^4\,x^{22}}{11}+\frac{7\,a^3\,b^5\,x^{24}}{3}+\frac{14\,a^2\,b^6\,x^{26}}{13}+\frac{2\,a\,b^7\,x^{28}}{7}+\frac{b^8\,x^{30}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^14)/14 + (b^8*x^30)/30 + (a^7*b*x^16)/2 + (2*a*b^7*x^28)/7 + (14*a^6
*b^2*x^18)/9 + (14*a^5*b^3*x^20)/5 + (35*a^4*b^4*x^22)/11 + (7*a^3*b^5*x^24
)/3 + (14*a^2*b^6*x^26)/13
```

sympy [A] time = 0.09, size = 105, normalized size = 0.81

$$\frac{a^8x^{14}}{14}+\frac{a^7bx^{16}}{2}+\frac{14a^6b^2x^{18}}{9}+\frac{14a^5b^3x^{20}}{5}+\frac{35a^4b^4x^{22}}{11}+\frac{7a^3b^5x^{24}}{3}+\frac{14a^2b^6x^{26}}{13}+\frac{2ab^7x^{28}}{7}+\frac{b^8x^{30}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**14/14 + a**7*b*x**16/2 + 14*a**6*b**2*x**18/9 + 14*a**5*b**3*x**20/
5 + 35*a**4*b**4*x**22/11 + 7*a**3*b**5*x**24/3 + 14*a**2*b**6*x**26/13 + 2
*a*b**7*x**28/7 + b**8*x**30/30
```

3.86 $\int x^{11} \left(a+bx^2\right)^8 dx$

Optimal. Leaf size=110

$$-\frac{a^5 \left(a+bx^2\right)^9}{18b^6}+\frac{a^4 \left(a+bx^2\right)^{10}}{4b^6}-\frac{5a^3 \left(a+bx^2\right)^{11}}{11b^6}+\frac{5a^2 \left(a+bx^2\right)^{12}}{12b^6}+\frac{\left(a+bx^2\right)^{14}}{28b^6}-\frac{5a \left(a+bx^2\right)^{13}}{26b^6}$$

```
[Out] -1/18*a^5*(b*x^2+a)^9/b^6+1/4*a^4*(b*x^2+a)^10/b^6-5/11*a^3*(b*x^2+a)^11/b^
6+5/12*a^2*(b*x^2+a)^12/b^6-5/26*a*(b*x^2+a)^13/b^6+1/28*(b*x^2+a)^14/b^6
```

Rubi [A] time = 0.17, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{5a^2 \left(a+bx^2\right)^{12}}{12b^6}-\frac{5a^3 \left(a+bx^2\right)^{11}}{11b^6}+\frac{a^4 \left(a+bx^2\right)^{10}}{4b^6}-\frac{a^5 \left(a+bx^2\right)^9}{18b^6}+\frac{\left(a+bx^2\right)^{14}}{28b^6}-\frac{5a \left(a+bx^2\right)^{13}}{26b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^2)^8,x]
```

```
[Out] -(a^5*(a + b*x^2)^9)/(18*b^6) + (a^4*(a + b*x^2)^10)/(4*b^6) - (5*a^3*(a +
b*x^2)^11)/(11*b^6) + (5*a^2*(a + b*x^2)^12)/(12*b^6) - (5*a*(a + b*x^2)^13
)/(26*b^6) + (a + b*x^2)^14/(28*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11} \left(a+bx^2\right)^8 dx &= \frac{1}{2}\text{Subst}\left(\int x^5(a+bx)^8\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^5(a+bx)^8}{b^5}+\frac{5a^4(a+bx)^9}{b^5}-\frac{10a^3(a+bx)^{10}}{b^5}+\frac{10a^2(a+bx)^{11}}{b^5}-\frac{5a(a\right.\right.\\ &= -\frac{a^5 \left(a+bx^2\right)^9}{18b^6}+\frac{a^4 \left(a+bx^2\right)^{10}}{4b^6}-\frac{5a^3 \left(a+bx^2\right)^{11}}{11b^6}+\frac{5a^2 \left(a+bx^2\right)^{12}}{12b^6}-\frac{5a \left(a+bx^2\right)^{13}}{26b^6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 0.98

$$\frac{a^8x^{12}}{12}+\frac{4}{7}a^7bx^{14}+\frac{7}{4}a^6b^2x^{16}+\frac{28}{9}a^5b^3x^{18}+\frac{7}{2}a^4b^4x^{20}+\frac{28}{11}a^3b^5x^{22}+\frac{7}{6}a^2b^6x^{24}+\frac{4}{13}ab^7x^{26}+\frac{b^8x^{28}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^12)/12 + (4*a^7*b*x^14)/7 + (7*a^6*b^2*x^16)/4 + (28*a^5*b^3*x^18)/9
 + (7*a^4*b^4*x^20)/2 + (28*a^3*b^5*x^22)/11 + (7*a^2*b^6*x^24)/6 + (4*a*b^
7*x^26)/13 + (b^8*x^28)/28
```

fricas [A] time = 0.60, size = 90, normalized size = 0.82

$$\frac{1}{28}x^{28}b^8+\frac{4}{13}x^{26}b^7a+\frac{7}{6}x^{24}b^6a^2+\frac{28}{11}x^{22}b^5a^3+\frac{7}{2}x^{20}b^4a^4+\frac{28}{9}x^{18}b^3a^5+\frac{7}{4}x^{16}b^2a^6+\frac{4}{7}x^{14}ba^7+\frac{1}{12}x^{12}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/28*x^28*b^8 + 4/13*x^26*b^7*a + 7/6*x^24*b^6*a^2 + 28/11*x^22*b^5*a^3 + 7
/2*x^20*b^4*a^4 + 28/9*x^18*b^3*a^5 + 7/4*x^16*b^2*a^6 + 4/7*x^14*b*a^7 + 1
/12*x^12*a^8
```

giac [A] time = 1.00, size = 90, normalized size = 0.82

$$\frac{1}{28}\,b^8x^{28}+\frac{4}{13}\,ab^7x^{26}+\frac{7}{6}\,a^2b^6x^{24}+\frac{28}{11}\,a^3b^5x^{22}+\frac{7}{2}\,a^4b^4x^{20}+\frac{28}{9}\,a^5b^3x^{18}+\frac{7}{4}\,a^6b^2x^{16}+\frac{4}{7}\,a^7bx^{14}+\frac{1}{12}\,a^8x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/28*b^8*x^28 + 4/13*a*b^7*x^26 + 7/6*a^2*b^6*x^24 + 28/11*a^3*b^5*x^22 + 7
/2*a^4*b^4*x^20 + 28/9*a^5*b^3*x^18 + 7/4*a^6*b^2*x^16 + 4/7*a^7*b*x^14 + 1
/12*a^8*x^12
```

maple [A] time = 0.00, size = 91, normalized size = 0.83

$$\frac{1}{28}b^8x^{28}+\frac{4}{13}a\,b^7x^{26}+\frac{7}{6}a^2b^6x^{24}+\frac{28}{11}a^3b^5x^{22}+\frac{7}{2}a^4b^4x^{20}+\frac{28}{9}a^5b^3x^{18}+\frac{7}{4}a^6b^2x^{16}+\frac{4}{7}a^7b\,x^{14}+\frac{1}{12}a^8x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^2+a)^8,x)
```

```
[Out] 1/28*b^8*x^28+4/13*a*b^7*x^26+7/6*a^2*b^6*x^24+28/11*a^3*b^5*x^22+7/2*a^4*b
^4*x^20+28/9*a^5*b^3*x^18+7/4*a^6*b^2*x^16+4/7*a^7*b*x^14+1/12*a^8*x^12
```

maxima [A] time = 1.35, size = 90, normalized size = 0.82

$$\frac{1}{28}\,b^8x^{28}+\frac{4}{13}\,ab^7x^{26}+\frac{7}{6}\,a^2b^6x^{24}+\frac{28}{11}\,a^3b^5x^{22}+\frac{7}{2}\,a^4b^4x^{20}+\frac{28}{9}\,a^5b^3x^{18}+\frac{7}{4}\,a^6b^2x^{16}+\frac{4}{7}\,a^7bx^{14}+\frac{1}{12}\,a^8x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/28*b^8*x^28 + 4/13*a*b^7*x^26 + 7/6*a^2*b^6*x^24 + 28/11*a^3*b^5*x^22 + 7
/2*a^4*b^4*x^20 + 28/9*a^5*b^3*x^18 + 7/4*a^6*b^2*x^16 + 4/7*a^7*b*x^14 + 1
/12*a^8*x^12
```

mupad [B] time = 4.57, size = 90, normalized size = 0.82

$$\frac{a^8\,x^{12}}{12}+\frac{4\,a^7\,b\,x^{14}}{7}+\frac{7\,a^6\,b^2\,x^{16}}{4}+\frac{28\,a^5\,b^3\,x^{18}}{9}+\frac{7\,a^4\,b^4\,x^{20}}{2}+\frac{28\,a^3\,b^5\,x^{22}}{11}+\frac{7\,a^2\,b^6\,x^{24}}{6}+\frac{4\,a\,b^7\,x^{26}}{13}+\frac{b^8\,x^{28}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^12)/12 + (b^8*x^28)/28 + (4*a^7*b*x^14)/7 + (4*a*b^7*x^26)/13 + (7*a
^6*b^2*x^16)/4 + (28*a^5*b^3*x^18)/9 + (7*a^4*b^4*x^20)/2 + (28*a^3*b^5*x^2
2)/11 + (7*a^2*b^6*x^24)/6
```

sympy [A] time = 0.09, size = 107, normalized size = 0.97

$$\frac{a^8x^{12}}{12}+\frac{4a^7bx^{14}}{7}+\frac{7a^6b^2x^{16}}{4}+\frac{28a^5b^3x^{18}}{9}+\frac{7a^4b^4x^{20}}{2}+\frac{28a^3b^5x^{22}}{11}+\frac{7a^2b^6x^{24}}{6}+\frac{4ab^7x^{26}}{13}+\frac{b^8x^{28}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**12/12 + 4*a**7*b*x**14/7 + 7*a**6*b**2*x**16/4 + 28*a**5*b**3*x**18
/9 + 7*a**4*b**4*x**20/2 + 28*a**3*b**5*x**22/11 + 7*a**2*b**6*x**24/6 + 4*
a*b**7*x**26/13 + b**8*x**28/28
```

3.87 $\int x^9 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=91

$$\frac{a^4\left(a+bx^2\right)^9}{18b^5}-\frac{a^3\left(a+bx^2\right)^{10}}{5b^5}+\frac{3a^2\left(a+bx^2\right)^{11}}{11b^5}+\frac{\left(a+bx^2\right)^{13}}{26b^5}-\frac{a\left(a+bx^2\right)^{12}}{6b^5}$$

```
[Out] 1/18*a^4*(b*x^2+a)^9/b^5-1/5*a^3*(b*x^2+a)^10/b^5+3/11*a^2*(b*x^2+a)^11/b^5
-1/6*a*(b*x^2+a)^12/b^5+1/26*(b*x^2+a)^13/b^5
```

Rubi [A] time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{11}}{11b^5}-\frac{a^3\left(a+bx^2\right)^{10}}{5b^5}+\frac{a^4\left(a+bx^2\right)^9}{18b^5}+\frac{\left(a+bx^2\right)^{13}}{26b^5}-\frac{a\left(a+bx^2\right)^{12}}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^8,x]
```

```
[Out] (a^4*(a + b*x^2)^9)/(18*b^5) - (a^3*(a + b*x^2)^10)/(5*b^5) + (3*a^2*(a + b
*x^2)^11)/(11*b^5) - (a*(a + b*x^2)^12)/(6*b^5) + (a + b*x^2)^13/(26*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^2\right)^8 \, dx &= \frac{1}{2}\text{Subst}\left(\int x^4(a+bx)^8 \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4(a+bx)^8}{b^4}-\frac{4a^3(a+bx)^9}{b^4}+\frac{6a^2(a+bx)^{10}}{b^4}-\frac{4a(a+bx)^{11}}{b^4}+\frac{(a+bx)^{12}}{b^4}\right) dx\\ &= \frac{a^4\left(a+bx^2\right)^9}{18b^5}-\frac{a^3\left(a+bx^2\right)^{10}}{5b^5}+\frac{3a^2\left(a+bx^2\right)^{11}}{11b^5}-\frac{a\left(a+bx^2\right)^{12}}{6b^5}+\frac{\left(a+bx^2\right)^{13}}{26b^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.16

$$\frac{a^8x^{10}}{10}+\frac{2}{3}a^7bx^{12}+2a^6b^2x^{14}+\frac{7}{2}a^5b^3x^{16}+\frac{35}{9}a^4b^4x^{18}+\frac{14}{5}a^3b^5x^{20}+\frac{14}{11}a^2b^6x^{22}+\frac{1}{3}ab^7x^{24}+\frac{b^8x^{26}}{26}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^10)/10 + (2*a^7*b*x^12)/3 + 2*a^6*b^2*x^14 + (7*a^5*b^3*x^16)/2 + (3
5*a^4*b^4*x^18)/9 + (14*a^3*b^5*x^20)/5 + (14*a^2*b^6*x^22)/11 + (a*b^7*x^2
4)/3 + (b^8*x^26)/26
```

fricas [A] time = 0.65, size = 90, normalized size = 0.99

$$\frac{1}{26}x^{26}b^8+\frac{1}{3}x^{24}b^7a+\frac{14}{11}x^{22}b^6a^2+\frac{14}{5}x^{20}b^5a^3+\frac{35}{9}x^{18}b^4a^4+\frac{7}{2}x^{16}b^3a^5+2x^{14}b^2a^6+\frac{2}{3}x^{12}ba^7+\frac{1}{10}x^{10}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/26*x^26*b^8 + 1/3*x^24*b^7*a + 14/11*x^22*b^6*a^2 + 14/5*x^20*b^5*a^3 + 3
5/9*x^18*b^4*a^4 + 7/2*x^16*b^3*a^5 + 2*x^14*b^2*a^6 + 2/3*x^12*b*a^7 + 1/1
0*x^10*a^8
```

giac [A] time = 1.00, size = 90, normalized size = 0.99

$$\frac{1}{26}\,b^8x^{26}+\frac{1}{3}\,ab^7x^{24}+\frac{14}{11}\,a^2b^6x^{22}+\frac{14}{5}\,a^3b^5x^{20}+\frac{35}{9}\,a^4b^4x^{18}+\frac{7}{2}\,a^5b^3x^{16}+2\,a^6b^2x^{14}+\frac{2}{3}\,a^7bx^{12}+\frac{1}{10}\,a^8x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/26*b^8*x^26 + 1/3*a*b^7*x^24 + 14/11*a^2*b^6*x^22 + 14/5*a^3*b^5*x^20 + 3
5/9*a^4*b^4*x^18 + 7/2*a^5*b^3*x^16 + 2*a^6*b^2*x^14 + 2/3*a^7*b*x^12 + 1/1
0*a^8*x^10
```

maple [A] time = 0.00, size = 91, normalized size = 1.00

$$\frac{1}{26}b^8x^{26}+\frac{1}{3}a\,b^7x^{24}+\frac{14}{11}a^2b^6x^{22}+\frac{14}{5}a^3b^5x^{20}+\frac{35}{9}a^4b^4x^{18}+\frac{7}{2}a^5b^3x^{16}+2a^6b^2x^{14}+\frac{2}{3}a^7b\,x^{12}+\frac{1}{10}a^8x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^8,x)
```

```
[Out] 1/26*b^8*x^26+1/3*a*b^7*x^24+14/11*a^2*b^6*x^22+14/5*a^3*b^5*x^20+35/9*a^4*
b^4*x^18+7/2*a^5*b^3*x^16+2*a^6*b^2*x^14+2/3*a^7*b*x^12+1/10*a^8*x^10
```

maxima [A] time = 1.28, size = 90, normalized size = 0.99

$$\frac{1}{26}\,b^8x^{26}+\frac{1}{3}\,ab^7x^{24}+\frac{14}{11}\,a^2b^6x^{22}+\frac{14}{5}\,a^3b^5x^{20}+\frac{35}{9}\,a^4b^4x^{18}+\frac{7}{2}\,a^5b^3x^{16}+2\,a^6b^2x^{14}+\frac{2}{3}\,a^7bx^{12}+\frac{1}{10}\,a^8x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/26*b^8*x^26 + 1/3*a*b^7*x^24 + 14/11*a^2*b^6*x^22 + 14/5*a^3*b^5*x^20 + 3
5/9*a^4*b^4*x^18 + 7/2*a^5*b^3*x^16 + 2*a^6*b^2*x^14 + 2/3*a^7*b*x^12 + 1/1
0*a^8*x^10
```

mupad [B] time = 4.59, size = 90, normalized size = 0.99

$$\frac{a^8\,x^{10}}{10}+\frac{2\,a^7\,b\,x^{12}}{3}+2\,a^6\,b^2\,x^{14}+\frac{7\,a^5\,b^3\,x^{16}}{2}+\frac{35\,a^4\,b^4\,x^{18}}{9}+\frac{14\,a^3\,b^5\,x^{20}}{5}+\frac{14\,a^2\,b^6\,x^{22}}{11}+\frac{a\,b^7\,x^{24}}{3}+\frac{b^8\,x^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^10)/10 + (b^8*x^26)/26 + (2*a^7*b*x^12)/3 + (a*b^7*x^24)/3 + 2*a^6*b
^2*x^14 + (7*a^5*b^3*x^16)/2 + (35*a^4*b^4*x^18)/9 + (14*a^3*b^5*x^20)/5 +
(14*a^2*b^6*x^22)/11
```

sympy [**A**] time = 0.09, size = 104, normalized size = 1.14

$$\frac{a^8x^{10}}{10}+\frac{2a^7bx^{12}}{3}+2a^6b^2x^{14}+\frac{7a^5b^3x^{16}}{2}+\frac{35a^4b^4x^{18}}{9}+\frac{14a^3b^5x^{20}}{5}+\frac{14a^2b^6x^{22}}{11}+\frac{ab^7x^{24}}{3}+\frac{b^8x^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**10/10 + 2*a**7*b*x**12/3 + 2*a**6*b**2*x**14 + 7*a**5*b**3*x**16/2
+ 35*a**4*b**4*x**18/9 + 14*a**3*b**5*x**20/5 + 14*a**2*b**6*x**22/11 + a*b
**7*x**24/3 + b**8*x**26/26
```

3.88 $\int x^7 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=72

$$-\frac{a^3\left(a+bx^2\right)^9}{18b^4}+\frac{3a^2\left(a+bx^2\right)^{10}}{20b^4}+\frac{\left(a+bx^2\right)^{12}}{24b^4}-\frac{3a\left(a+bx^2\right)^{11}}{22b^4}$$

```
[Out] -1/18*a^3*(b*x^2+a)^9/b^4+3/20*a^2*(b*x^2+a)^10/b^4-3/22*a*(b*x^2+a)^11/b^4
+1/24*(b*x^2+a)^12/b^4
```

Rubi [A] time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{10}}{20b^4}-\frac{a^3\left(a+bx^2\right)^9}{18b^4}+\frac{\left(a+bx^2\right)^{12}}{24b^4}-\frac{3a\left(a+bx^2\right)^{11}}{22b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^8,x]
```

```
[Out] -(a^3*(a + b*x^2)^9)/(18*b^4) + (3*a^2*(a + b*x^2)^10)/(20*b^4) - (3*a*(a +
 b*x^2)^11)/(22*b^4) + (a + b*x^2)^12/(24*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^8\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^8\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^8}{b^3}+\frac{3a^2(a+bx)^9}{b^3}-\frac{3a(a+bx)^{10}}{b^3}+\frac{(a+bx)^{11}}{b^3}\right)dx,x,x^2\right)\\ &= -\frac{a^3\left(a+bx^2\right)^9}{18b^4}+\frac{3a^2\left(a+bx^2\right)^{10}}{20b^4}-\frac{3a\left(a+bx^2\right)^{11}}{22b^4}+\frac{\left(a+bx^2\right)^{12}}{24b^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.47

$$\frac{a^8x^8}{8}+\frac{4}{5}a^7bx^{10}+\frac{7}{3}a^6b^2x^{12}+4a^5b^3x^{14}+\frac{35}{8}a^4b^4x^{16}+\frac{28}{9}a^3b^5x^{18}+\frac{7}{5}a^2b^6x^{20}+\frac{4}{11}ab^7x^{22}+\frac{b^8x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^8)/8 + (4*a^7*b*x^10)/5 + (7*a^6*b^2*x^12)/3 + 4*a^5*b^3*x^14 + (35*
a^4*b^4*x^16)/8 + (28*a^3*b^5*x^18)/9 + (7*a^2*b^6*x^20)/5 + (4*a*b^7*x^22)
/11 + (b^8*x^24)/24
```

fricas [A] time = 0.76, size = 90, normalized size = 1.25

$$\frac{1}{24}x^{24}b^8+\frac{4}{11}x^{22}b^7a+\frac{7}{5}x^{20}b^6a^2+\frac{28}{9}x^{18}b^5a^3+\frac{35}{8}x^{16}b^4a^4+4x^{14}b^3a^5+\frac{7}{3}x^{12}b^2a^6+\frac{4}{5}x^{10}ba^7+\frac{1}{8}x^8a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*b^8 + 4/11*x^22*b^7*a + 7/5*x^20*b^6*a^2 + 28/9*x^18*b^5*a^3 + 35
/8*x^16*b^4*a^4 + 4*x^14*b^3*a^5 + 7/3*x^12*b^2*a^6 + 4/5*x^10*b*a^7 + 1/8*
x^8*a^8
```

giac [A] time = 1.08, size = 90, normalized size = 1.25

$$\frac{1}{24}\,b^8x^{24}+\frac{4}{11}\,ab^7x^{22}+\frac{7}{5}\,a^2b^6x^{20}+\frac{28}{9}\,a^3b^5x^{18}+\frac{35}{8}\,a^4b^4x^{16}+4\,a^5b^3x^{14}+\frac{7}{3}\,a^6b^2x^{12}+\frac{4}{5}\,a^7bx^{10}+\frac{1}{8}\,a^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/24*b^8*x^24 + 4/11*a*b^7*x^22 + 7/5*a^2*b^6*x^20 + 28/9*a^3*b^5*x^18 + 35
/8*a^4*b^4*x^16 + 4*a^5*b^3*x^14 + 7/3*a^6*b^2*x^12 + 4/5*a^7*b*x^10 + 1/8*
a^8*x^8
```

maple [A] time = 0.00, size = 91, normalized size = 1.26

$$\frac{1}{24}b^8x^{24}+\frac{4}{11}a\,b^7x^{22}+\frac{7}{5}a^2b^6x^{20}+\frac{28}{9}a^3b^5x^{18}+\frac{35}{8}a^4b^4x^{16}+4a^5b^3x^{14}+\frac{7}{3}a^6b^2x^{12}+\frac{4}{5}a^7b\,x^{10}+\frac{1}{8}a^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^8,x)
```

```
[Out] 1/24*b^8*x^24+4/11*a*b^7*x^22+7/5*a^2*b^6*x^20+28/9*a^3*b^5*x^18+35/8*a^4*b
^4*x^16+4*a^5*b^3*x^14+7/3*a^6*b^2*x^12+4/5*a^7*b*x^10+1/8*a^8*x^8
```

maxima [A] time = 1.29, size = 90, normalized size = 1.25

$$\frac{1}{24}\,b^8x^{24}+\frac{4}{11}\,ab^7x^{22}+\frac{7}{5}\,a^2b^6x^{20}+\frac{28}{9}\,a^3b^5x^{18}+\frac{35}{8}\,a^4b^4x^{16}+4\,a^5b^3x^{14}+\frac{7}{3}\,a^6b^2x^{12}+\frac{4}{5}\,a^7bx^{10}+\frac{1}{8}\,a^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/24*b^8*x^24 + 4/11*a*b^7*x^22 + 7/5*a^2*b^6*x^20 + 28/9*a^3*b^5*x^18 + 35
/8*a^4*b^4*x^16 + 4*a^5*b^3*x^14 + 7/3*a^6*b^2*x^12 + 4/5*a^7*b*x^10 + 1/8*
a^8*x^8
```

mupad [B] time = 0.09, size = 90, normalized size = 1.25

$$\frac{a^8\,x^8}{8}+\frac{4\,a^7\,b\,x^{10}}{5}+\frac{7\,a^6\,b^2\,x^{12}}{3}+4\,a^5\,b^3\,x^{14}+\frac{35\,a^4\,b^4\,x^{16}}{8}+\frac{28\,a^3\,b^5\,x^{18}}{9}+\frac{7\,a^2\,b^6\,x^{20}}{5}+\frac{4\,a\,b^7\,x^{22}}{11}+\frac{b^8\,x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^8)/8 + (b^8*x^24)/24 + (4*a^7*b*x^10)/5 + (4*a*b^7*x^22)/11 + (7*a^6
*b^2*x^12)/3 + 4*a^5*b^3*x^14 + (35*a^4*b^4*x^16)/8 + (28*a^3*b^5*x^18)/9 +
 (7*a^2*b^6*x^20)/5
```

sympy [A] time = 0.09, size = 105, normalized size = 1.46

$$\frac{a^8x^8}{8}+\frac{4a^7bx^{10}}{5}+\frac{7a^6b^2x^{12}}{3}+4a^5b^3x^{14}+\frac{35a^4b^4x^{16}}{8}+\frac{28a^3b^5x^{18}}{9}+\frac{7a^2b^6x^{20}}{5}+\frac{4ab^7x^{22}}{11}+\frac{b^8x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**8/8 + 4*a**7*b*x**10/5 + 7*a**6*b**2*x**12/3 + 4*a**5*b**3*x**14 +
35*a**4*b**4*x**16/8 + 28*a**3*b**5*x**18/9 + 7*a**2*b**6*x**20/5 + 4*a*b**
7*x**22/11 + b**8*x**24/24
```

3.89 $\int x^5 \left(a+bx^2\right)^8 dx$

Optimal. Leaf size=53

$$\frac{a^2\left(a+bx^2\right)^9}{18b^3}+\frac{\left(a+bx^2\right)^{11}}{22b^3}-\frac{a\left(a+bx^2\right)^{10}}{10b^3}$$

```
[Out] 1/18*a^2*(b*x^2+a)^9/b^3-1/10*a*(b*x^2+a)^10/b^3+1/22*(b*x^2+a)^11/b^3
```

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^9}{18b^3}+\frac{\left(a+bx^2\right)^{11}}{22b^3}-\frac{a\left(a+bx^2\right)^{10}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^8,x]
```

```
[Out] (a^2*(a + b*x^2)^9)/(18*b^3) - (a*(a + b*x^2)^10)/(10*b^3) + (a + b*x^2)^11
/(22*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^2\right)^8 dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^8 \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2(a+bx)^8}{b^2}-\frac{2a(a+bx)^9}{b^2}+\frac{(a+bx)^{10}}{b^2}\right) dx, x, x^2\right)\\
&= \frac{a^2\left(a+bx^2\right)^9}{18b^3}-\frac{a\left(a+bx^2\right)^{10}}{10b^3}+\frac{\left(a+bx^2\right)^{11}}{22b^3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 103, normalized size = 1.94

$$\frac{a^8x^6}{6}+a^7bx^8+\frac{14}{5}a^6b^2x^{10}+\frac{14}{3}a^5b^3x^{12}+5a^4b^4x^{14}+\frac{7}{2}a^3b^5x^{16}+\frac{14}{9}a^2b^6x^{18}+\frac{2}{5}ab^7x^{20}+\frac{b^8x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^6)/6 + a^7*b*x^8 + (14*a^6*b^2*x^10)/5 + (14*a^5*b^3*x^12)/3 + 5*a^4
*b^4*x^14 + (7*a^3*b^5*x^16)/2 + (14*a^2*b^6*x^18)/9 + (2*a*b^7*x^20)/5 + (
b^8*x^22)/22
```

fricas [A] time = 0.84, size = 89, normalized size = 1.68

$$\frac{1}{22}x^{22}b^8+\frac{2}{5}x^{20}b^7a+\frac{14}{9}x^{18}b^6a^2+\frac{7}{2}x^{16}b^5a^3+5x^{14}b^4a^4+\frac{14}{3}x^{12}b^3a^5+\frac{14}{5}x^{10}b^2a^6+x^8ba^7+\frac{1}{6}x^6a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^8 + 2/5*x^20*b^7*a + 14/9*x^18*b^6*a^2 + 7/2*x^16*b^5*a^3 + 5*x
^14*b^4*a^4 + 14/3*x^12*b^3*a^5 + 14/5*x^10*b^2*a^6 + x^8*b*a^7 + 1/6*x^6*a
^8
```

giac [A] time = 1.13, size = 89, normalized size = 1.68

$$\frac{1}{22}\,b^8x^{22}+\frac{2}{5}\,ab^7x^{20}+\frac{14}{9}\,a^2b^6x^{18}+\frac{7}{2}\,a^3b^5x^{16}+5\,a^4b^4x^{14}+\frac{14}{3}\,a^5b^3x^{12}+\frac{14}{5}\,a^6b^2x^{10}+a^7bx^8+\frac{1}{6}\,a^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/22*b^8*x^22 + 2/5*a*b^7*x^20 + 14/9*a^2*b^6*x^18 + 7/2*a^3*b^5*x^16 + 5*a
^4*b^4*x^14 + 14/3*a^5*b^3*x^12 + 14/5*a^6*b^2*x^10 + a^7*b*x^8 + 1/6*a^8*x
^6
```

maple [A] time = 0.00, size = 90, normalized size = 1.70

$$\frac{1}{22}b^8x^{22}+\frac{2}{5}a\,b^7x^{20}+\frac{14}{9}a^2b^6x^{18}+\frac{7}{2}a^3b^5x^{16}+5a^4b^4x^{14}+\frac{14}{3}a^5b^3x^{12}+\frac{14}{5}a^6b^2x^{10}+a^7b\,x^8+\frac{1}{6}a^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^8,x)
```

```
[Out] 1/22*b^8*x^22+2/5*a*b^7*x^20+14/9*a^2*b^6*x^18+7/2*a^3*b^5*x^16+5*a^4*b^4*x
^14+14/3*a^5*b^3*x^12+14/5*a^6*b^2*x^10+a^7*b*x^8+1/6*a^8*x^6
```

maxima [A] time = 1.35, size = 89, normalized size = 1.68

$$\frac{1}{22}\,b^8x^{22}+\frac{2}{5}\,ab^7x^{20}+\frac{14}{9}\,a^2b^6x^{18}+\frac{7}{2}\,a^3b^5x^{16}+5\,a^4b^4x^{14}+\frac{14}{3}\,a^5b^3x^{12}+\frac{14}{5}\,a^6b^2x^{10}+a^7bx^8+\frac{1}{6}\,a^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/22*b^8*x^22 + 2/5*a*b^7*x^20 + 14/9*a^2*b^6*x^18 + 7/2*a^3*b^5*x^16 + 5*a
^4*b^4*x^14 + 14/3*a^5*b^3*x^12 + 14/5*a^6*b^2*x^10 + a^7*b*x^8 + 1/6*a^8*x
^6
```

mupad [B] time = 0.09, size = 89, normalized size = 1.68

$$\frac{a^8\,x^6}{6}+a^7\,b\,x^8+\frac{14\,a^6\,b^2\,x^{10}}{5}+\frac{14\,a^5\,b^3\,x^{12}}{3}+5\,a^4\,b^4\,x^{14}+\frac{7\,a^3\,b^5\,x^{16}}{2}+\frac{14\,a^2\,b^6\,x^{18}}{9}+\frac{2\,a\,b^7\,x^{20}}{5}+\frac{b^8\,x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^6)/6 + (b^8*x^22)/22 + a^7*b*x^8 + (2*a*b^7*x^20)/5 + (14*a^6*b^2*x^
10)/5 + (14*a^5*b^3*x^12)/3 + 5*a^4*b^4*x^14 + (7*a^3*b^5*x^16)/2 + (14*a^2
*b^6*x^18)/9
```

sympy [**B**] time = 0.09, size = 102, normalized size = 1.92

$$\frac{a^8x^6}{6} + a^7bx^8 + \frac{14a^6b^2x^{10}}{5} + \frac{14a^5b^3x^{12}}{3} + 5a^4b^4x^{14} + \frac{7a^3b^5x^{16}}{2} + \frac{14a^2b^6x^{18}}{9} + \frac{2ab^7x^{20}}{5} + \frac{b^8x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**6/6 + a**7*b*x**8 + 14*a**6*b**2*x**10/5 + 14*a**5*b**3*x**12/3 + 5
*a**4*b**4*x**14 + 7*a**3*b**5*x**16/2 + 14*a**2*b**6*x**18/9 + 2*a*b**7*x*
*20/5 + b**8*x**22/22
```

3.90 $\int x^3 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^{10}}{20b^2}-\frac{a\left(a+bx^2\right)^9}{18b^2}$$

```
[Out] -1/18*a*(b*x^2+a)^9/b^2+1/20*(b*x^2+a)^10/b^2
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{10}}{20b^2}-\frac{a\left(a+bx^2\right)^9}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^8,x]
```

```
[Out] -(a*(a + b*x^2)^9)/(18*b^2) + (a + b*x^2)^10/(20*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^2\right)^8 \, dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^8 \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a(a+bx)^8}{b}+\frac{(a+bx)^9}{b}\right) dx, x, x^2\right)\\
&= -\frac{a\left(a+bx^2\right)^9}{18b^2}+\frac{\left(a+bx^2\right)^{10}}{20b^2}
\end{aligned}$$

Mathematica [B] time = 0.00, size = 106, normalized size = 3.12

$$\frac{a^8x^4}{4}+\frac{4}{3}a^7bx^6+\frac{7}{2}a^6b^2x^8+\frac{28}{5}a^5b^3x^{10}+\frac{35}{6}a^4b^4x^{12}+4a^3b^5x^{14}+\frac{7}{4}a^2b^6x^{16}+\frac{4}{9}ab^7x^{18}+\frac{b^8x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^4)/4 + (4*a^7*b*x^6)/3 + (7*a^6*b^2*x^8)/2 + (28*a^5*b^3*x^10)/5 + (
35*a^4*b^4*x^12)/6 + 4*a^3*b^5*x^14 + (7*a^2*b^6*x^16)/4 + (4*a*b^7*x^18)/9
 + (b^8*x^20)/20
```

fricas [B] time = 0.77, size = 90, normalized size = 2.65

$$\frac{1}{20}x^{20}b^8+\frac{4}{9}x^{18}b^7a+\frac{7}{4}x^{16}b^6a^2+4x^{14}b^5a^3+\frac{35}{6}x^{12}b^4a^4+\frac{28}{5}x^{10}b^3a^5+\frac{7}{2}x^8b^2a^6+\frac{4}{3}x^6ba^7+\frac{1}{4}x^4a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^8 + 4/9*x^18*b^7*a + 7/4*x^16*b^6*a^2 + 4*x^14*b^5*a^3 + 35/6*x
^12*b^4*a^4 + 28/5*x^10*b^3*a^5 + 7/2*x^8*b^2*a^6 + 4/3*x^6*b*a^7 + 1/4*x^4
*a^8
```

giac [B] time = 1.18, size = 90, normalized size = 2.65

$$\frac{1}{20}\,b^8x^{20}+\frac{4}{9}\,ab^7x^{18}+\frac{7}{4}\,a^2b^6x^{16}+4\,a^3b^5x^{14}+\frac{35}{6}\,a^4b^4x^{12}+\frac{28}{5}\,a^5b^3x^{10}+\frac{7}{2}\,a^6b^2x^8+\frac{4}{3}\,a^7bx^6+\frac{1}{4}\,a^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/20*b^8*x^20 + 4/9*a*b^7*x^18 + 7/4*a^2*b^6*x^16 + 4*a^3*b^5*x^14 + 35/6*a
^4*b^4*x^12 + 28/5*a^5*b^3*x^10 + 7/2*a^6*b^2*x^8 + 4/3*a^7*b*x^6 + 1/4*a^8
*x^4
```

maple [B] time = 0.00, size = 91, normalized size = 2.68

$$\frac{1}{20}b^8x^{20}+\frac{4}{9}a\,b^7x^{18}+\frac{7}{4}a^2b^6x^{16}+4a^3b^5x^{14}+\frac{35}{6}a^4b^4x^{12}+\frac{28}{5}a^5b^3x^{10}+\frac{7}{2}a^6b^2x^8+\frac{4}{3}a^7b\,x^6+\frac{1}{4}a^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^8,x)
```

```
[Out] 1/20*b^8*x^20+4/9*a*b^7*x^18+7/4*a^2*b^6*x^16+4*a^3*b^5*x^14+35/6*a^4*b^4*x
^12+28/5*a^5*b^3*x^10+7/2*a^6*b^2*x^8+4/3*a^7*b*x^6+1/4*a^8*x^4
```

maxima [B] time = 1.43, size = 90, normalized size = 2.65

$$\frac{1}{20}\,b^8x^{20}+\frac{4}{9}\,ab^7x^{18}+\frac{7}{4}\,a^2b^6x^{16}+4\,a^3b^5x^{14}+\frac{35}{6}\,a^4b^4x^{12}+\frac{28}{5}\,a^5b^3x^{10}+\frac{7}{2}\,a^6b^2x^8+\frac{4}{3}\,a^7bx^6+\frac{1}{4}\,a^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/20*b^8*x^20 + 4/9*a*b^7*x^18 + 7/4*a^2*b^6*x^16 + 4*a^3*b^5*x^14 + 35/6*a
^4*b^4*x^12 + 28/5*a^5*b^3*x^10 + 7/2*a^6*b^2*x^8 + 4/3*a^7*b*x^6 + 1/4*a^8
*x^4
```

mupad [B] time = 0.09, size = 90, normalized size = 2.65

$$\frac{a^8\,x^4}{4}+\frac{4\,a^7\,b\,x^6}{3}+\frac{7\,a^6\,b^2\,x^8}{2}+\frac{28\,a^5\,b^3\,x^{10}}{5}+\frac{35\,a^4\,b^4\,x^{12}}{6}+4\,a^3\,b^5\,x^{14}+\frac{7\,a^2\,b^6\,x^{16}}{4}+\frac{4\,a\,b^7\,x^{18}}{9}+\frac{b^8\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^4)/4 + (b^8*x^20)/20 + (4*a^7*b*x^6)/3 + (4*a*b^7*x^18)/9 + (7*a^6*b
^2*x^8)/2 + (28*a^5*b^3*x^10)/5 + (35*a^4*b^4*x^12)/6 + 4*a^3*b^5*x^14 + (7
*a^2*b^6*x^16)/4
```

sympy [B] time = 0.08, size = 105, normalized size = 3.09

$$\frac{a^8x^4}{4}+\frac{4a^7bx^6}{3}+\frac{7a^6b^2x^8}{2}+\frac{28a^5b^3x^{10}}{5}+\frac{35a^4b^4x^{12}}{6}+4a^3b^5x^{14}+\frac{7a^2b^6x^{16}}{4}+\frac{4ab^7x^{18}}{9}+\frac{b^8x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**4/4 + 4*a**7*b*x**6/3 + 7*a**6*b**2*x**8/2 + 28*a**5*b**3*x**10/5 +
 35*a**4*b**4*x**12/6 + 4*a**3*b**5*x**14 + 7*a**2*b**6*x**16/4 + 4*a*b**7*
x**18/9 + b**8*x**20/20
```

3.91 $\int x\left(a+bx^2\right)^8\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^9}{18b}$$

```
[Out] 1/18*(b*x^2+a)^9/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^9}{18b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^8,x]
```

```
[Out] (a + b*x^2)^9/(18*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^8\,dx=\frac{\left(a+bx^2\right)^9}{18b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^9}{18b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^8,x]
```

```
[Out] (a + b*x^2)^9/(18*b)
```

fricas [B] time = 0.58, size = 90, normalized size = 5.62

$$\frac{1}{18}x^{18}b^8+\frac{1}{2}x^{16}b^7a+2x^{14}b^6a^2+\frac{14}{3}x^{12}b^5a^3+7x^{10}b^4a^4+7x^8b^3a^5+\frac{14}{3}x^6b^2a^6+2x^4ba^7+\frac{1}{2}x^2a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^8 + 1/2*x^16*b^7*a + 2*x^14*b^6*a^2 + 14/3*x^12*b^5*a^3 + 7*x^1
0*b^4*a^4 + 7*x^8*b^3*a^5 + 14/3*x^6*b^2*a^6 + 2*x^4*b*a^7 + 1/2*x^2*a^8
```

giac [A] time = 1.12, size = 14, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^9}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/18*(b*x^2 + a)^9/b
```

maple [B] time = 0.00, size = 91, normalized size = 5.69

$$\frac{1}{18}b^8x^{18}+\frac{1}{2}a\,b^7x^{16}+2a^2b^6x^{14}+\frac{14}{3}a^3b^5x^{12}+7a^4b^4x^{10}+7a^5b^3x^8+\frac{14}{3}a^6b^2x^6+2a^7b\,x^4+\frac{1}{2}a^8x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^8,x)
```

```
[Out] 1/18*b^8*x^18+1/2*a*b^7*x^16+2*a^2*b^6*x^14+14/3*a^3*b^5*x^12+7*a^4*b^4*x^1
0+7*a^5*b^3*x^8+14/3*a^6*b^2*x^6+2*a^7*b*x^4+1/2*a^8*x^2
```

maxima [A] time = 1.28, size = 14, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^9}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/18*(b*x^2 + a)^9/b
```

mupad [B] time = 4.61, size = 14, normalized size = 0.88

$$\frac{\left(b\,x^2+a\right)^9}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^8,x)
```

```
[Out] (a + b*x^2)^9/(18*b)
```

sympy [B] time = 0.08, size = 99, normalized size = 6.19

$$\frac{a^8x^2}{2}+2a^7bx^4+\frac{14a^6b^2x^6}{3}+7a^5b^3x^8+7a^4b^4x^{10}+\frac{14a^3b^5x^{12}}{3}+2a^2b^6x^{14}+\frac{ab^7x^{16}}{2}+\frac{b^8x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**2/2 + 2*a**7*b*x**4 + 14*a**6*b**2*x**6/3 + 7*a**5*b**3*x**8 + 7*a*
*4*b**4*x**10 + 14*a**3*b**5*x**12/3 + 2*a**2*b**6*x**14 + a*b**7*x**16/2 +
 b**8*x**18/18
```

3.92 $\int \frac{\left(a+bx^2\right)^8}{x}\,dx$

Optimal. Leaf size=100

$$a^8\log(x)+4a^7bx^2+7a^6b^2x^4+\frac{28}{3}a^5b^3x^6+\frac{35}{4}a^4b^4x^8+\frac{28}{5}a^3b^5x^{10}+\frac{7}{3}a^2b^6x^{12}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}$$

```
[Out] 4*a^7*b*x^2+7*a^6*b^2*x^4+28/3*a^5*b^3*x^6+35/4*a^4*b^4*x^8+28/5*a^3*b^5*x^
10+7/3*a^2*b^6*x^12+4/7*a*b^7*x^14+1/16*b^8*x^16+a^8*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{7}{3}a^2b^6x^{12}+\frac{28}{5}a^3b^5x^{10}+\frac{35}{4}a^4b^4x^8+\frac{28}{3}a^5b^3x^6+7a^6b^2x^4+4a^7bx^2+a^8\log(x)+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x,x]
```

```
[Out] 4*a^7*b*x^2 + 7*a^6*b^2*x^4 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28
*a^3*b^5*x^10)/5 + (7*a^2*b^6*x^12)/3 + (4*a*b^7*x^14)/7 + (b^8*x^16)/16 +
a^8*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^8}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^8}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(8a^7b+\frac{a^8}{x}+28a^6b^2x+56a^5b^3x^2+70a^4b^4x^3+56a^3b^5x^4+28a^2b^6x^5+8ab^7x^6\right.\right.\\ &= 4a^7bx^2+7a^6b^2x^4+\frac{28}{3}a^5b^3x^6+\frac{35}{4}a^4b^4x^8+\frac{28}{5}a^3b^5x^{10}+\frac{7}{3}a^2b^6x^{12}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}+a^8\end{aligned}$$

Mathematica [A] time = 0.00, size = 100, normalized size = 1.00

$$a^8\log(x)+4a^7bx^2+7a^6b^2x^4+\frac{28}{3}a^5b^3x^6+\frac{35}{4}a^4b^4x^8+\frac{28}{5}a^3b^5x^{10}+\frac{7}{3}a^2b^6x^{12}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x,x]
```

```
[Out] 4*a^7*b*x^2 + 7*a^6*b^2*x^4 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28
*a^3*b^5*x^10)/5 + (7*a^2*b^6*x^12)/3 + (4*a*b^7*x^14)/7 + (b^8*x^16)/16 +
a^8*Log[x]
```

fricas [A] time = 0.86, size = 88, normalized size = 0.88

$$\frac{1}{16}\,b^8x^{16}+\frac{4}{7}\,ab^7x^{14}+\frac{7}{3}\,a^2b^6x^{12}+\frac{28}{5}\,a^3b^5x^{10}+\frac{35}{4}\,a^4b^4x^8+\frac{28}{3}\,a^5b^3x^6+7\,a^6b^2x^4+4\,a^7bx^2+a^8\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x,x, algorithm="fricas")
```

```
[Out] 1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35/
4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8*log(x)
```

giac [A] time = 0.90, size = 91, normalized size = 0.91

$$\frac{1}{16}\,b^8x^{16}+\frac{4}{7}\,ab^7x^{14}+\frac{7}{3}\,a^2b^6x^{12}+\frac{28}{5}\,a^3b^5x^{10}+\frac{35}{4}\,a^4b^4x^8+\frac{28}{3}\,a^5b^3x^6+7\,a^6b^2x^4+4\,a^7bx^2+\frac{1}{2}\,a^8\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x,x, algorithm="giac")
```

```
[Out] 1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35/
4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + 1/2*a^8*lo
g(x^2)
```

maple [A] time = 0.00, size = 89, normalized size = 0.89

$$\frac{b^8x^{16}}{16}+\frac{4a\,b^7x^{14}}{7}+\frac{7a^2b^6x^{12}}{3}+\frac{28a^3b^5x^{10}}{5}+\frac{35a^4b^4x^8}{4}+\frac{28a^5b^3x^6}{3}+7a^6b^2x^4+4a^7b\,x^2+a^8\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x,x)
```

```
[Out] 4*a^7*b*x^2+7*a^6*b^2*x^4+28/3*a^5*b^3*x^6+35/4*a^4*b^4*x^8+28/5*a^3*b^5*x^
10+7/3*a^2*b^6*x^12+4/7*a*b^7*x^14+1/16*b^8*x^16+a^8*ln(x)
```

maxima [A] time = 1.30, size = 91, normalized size = 0.91

$$\frac{1}{16}\,b^8x^{16}+\frac{4}{7}\,ab^7x^{14}+\frac{7}{3}\,a^2b^6x^{12}+\frac{28}{5}\,a^3b^5x^{10}+\frac{35}{4}\,a^4b^4x^8+\frac{28}{3}\,a^5b^3x^6+7\,a^6b^2x^4+4\,a^7bx^2+\frac{1}{2}\,a^8\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x,x, algorithm="maxima")
```

```
[Out] 1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35/
4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + 1/2*a^8*lo
g(x^2)
```

mupad [B] time = 4.62, size = 88, normalized size = 0.88

$$a^8\,\ln(x)+\frac{b^8\,x^{16}}{16}+4\,a^7\,b\,x^2+\frac{4\,a\,b^7\,x^{14}}{7}+7\,a^6\,b^2\,x^4+\frac{28\,a^5\,b^3\,x^6}{3}+\frac{35\,a^4\,b^4\,x^8}{4}+\frac{28\,a^3\,b^5\,x^{10}}{5}+\frac{7\,a^2\,b^6\,x^{12}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x,x)
```

```
[Out] a^8*log(x) + (b^8*x^16)/16 + 4*a^7*b*x^2 + (4*a*b^7*x^14)/7 + 7*a^6*b^2*x^4
 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28*a^3*b^5*x^10)/5 + (7*a^2*b
^6*x^12)/3
```

sympy [A] time = 0.18, size = 102, normalized size = 1.02

$$a^8\log(x)+4a^7bx^2+7a^6b^2x^4+\frac{28a^5b^3x^6}{3}+\frac{35a^4b^4x^8}{4}+\frac{28a^3b^5x^{10}}{5}+\frac{7a^2b^6x^{12}}{3}+\frac{4ab^7x^{14}}{7}+\frac{b^8x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x,x)
```

```
[Out] a**8*log(x) + 4*a**7*b*x**2 + 7*a**6*b**2*x**4 + 28*a**5*b**3*x**6/3 + 35*a
**4*b**4*x**8/4 + 28*a**3*b**5*x**10/5 + 7*a**2*b**6*x**12/3 + 4*a*b**7*x**
14/7 + b**8*x**16/16
```

3.93 $\int \frac{(a+bx^2)^8}{x^3} \, dx$

Optimal. Leaf size=99

$$-\frac{a^8}{2x^2}+8a^7b\log(x)+14a^6b^2x^2+14a^5b^3x^4+\frac{35}{3}a^4b^4x^6+7a^3b^5x^8+\frac{14}{5}a^2b^6x^{10}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}$$

```
[Out] -1/2*a^8/x^2+14*a^6*b^2*x^2+14*a^5*b^3*x^4+35/3*a^4*b^4*x^6+7*a^3*b^5*x^8+1
4/5*a^2*b^6*x^10+2/3*a*b^7*x^12+1/14*b^8*x^14+8*a^7*b*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{14}{5}a^2b^6x^{10}+7a^3b^5x^8+\frac{35}{3}a^4b^4x^6+14a^5b^3x^4+14a^6b^2x^2+8a^7b\log(x)-\frac{a^8}{2x^2}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^3,x]
```

```
[Out] -a^8/(2*x^2) + 14*a^6*b^2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3
*b^5*x^8 + (14*a^2*b^6*x^10)/5 + (2*a*b^7*x^12)/3 + (b^8*x^14)/14 + 8*a^7*b
*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^8}{x^3} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^2} \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(28a^6b^2+\frac{a^8}{x^2}+\frac{8a^7b}{x}+56a^5b^3x+70a^4b^4x^2+56a^3b^5x^3+28a^2b^6x^4+8ab^7x^5\right.\right.\\ &= -\frac{a^8}{2x^2}+14a^6b^2x^2+14a^5b^3x^4+\frac{35}{3}a^4b^4x^6+7a^3b^5x^8+\frac{14}{5}a^2b^6x^{10}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}+8a\end{aligned}$$

Mathematica [A] time = 0.01, size = 99, normalized size = 1.00

$$-\frac{a^8}{2x^2}+8a^7b\log(x)+14a^6b^2x^2+14a^5b^3x^4+\frac{35}{3}a^4b^4x^6+7a^3b^5x^8+\frac{14}{5}a^2b^6x^{10}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^3,x]
```

```
[Out] -1/2*a^8/x^2 + 14*a^6*b^2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3
*b^5*x^8 + (14*a^2*b^6*x^10)/5 + (2*a*b^7*x^12)/3 + (b^8*x^14)/14 + 8*a^7*b
*Log[x]
```

fricas [A] time = 1.15, size = 94, normalized size = 0.95

$$\frac{15\,b^8x^{16}+140\,ab^7x^{14}+588\,a^2b^6x^{12}+1470\,a^3b^5x^{10}+2450\,a^4b^4x^8+2940\,a^5b^3x^6+2940\,a^6b^2x^4+1680\,a^7bx^2\log}{210\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^3,x, algorithm="fricas")
```

```
[Out] 1/210*(15*b^8*x^16 + 140*a*b^7*x^14 + 588*a^2*b^6*x^12 + 1470*a^3*b^5*x^10
+ 2450*a^4*b^4*x^8 + 2940*a^5*b^3*x^6 + 2940*a^6*b^2*x^4 + 1680*a^7*b*x^2*l
og(x) - 105*a^8)/x^2
```

giac [A] time = 1.06, size = 101, normalized size = 1.02

$$\frac{1}{14}\,b^8x^{14}+\frac{2}{3}\,ab^7x^{12}+\frac{14}{5}\,a^2b^6x^{10}+7\,a^3b^5x^8+\frac{35}{3}\,a^4b^4x^6+14\,a^5b^3x^4+14\,a^6b^2x^2+4\,a^7b\log\left(x^2\right)-\frac{8\,a^7bx^2+a^8}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^3,x, algorithm="giac")
```

```
[Out] 1/14*b^8*x^14 + 2/3*a*b^7*x^12 + 14/5*a^2*b^6*x^10 + 7*a^3*b^5*x^8 + 35/3*a
^4*b^4*x^6 + 14*a^5*b^3*x^4 + 14*a^6*b^2*x^2 + 4*a^7*b*log(x^2) - 1/2*(8*a^
7*b*x^2 + a^8)/x^2
```

maple [A] time = 0.01, size = 90, normalized size = 0.91

$$\frac{b^8x^{14}}{14}+\frac{2a\,b^7x^{12}}{3}+\frac{14a^2b^6x^{10}}{5}+7a^3b^5x^8+\frac{35a^4b^4x^6}{3}+14a^5b^3x^4+14a^6b^2x^2+8a^7b\ln(x)-\frac{a^8}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^3,x)
```

```
[Out] -1/2*a^8/x^2+14*a^6*b^2*x^2+14*a^5*b^3*x^4+35/3*a^4*b^4*x^6+7*a^3*b^5*x^8+1
4/5*a^2*b^6*x^10+2/3*a*b^7*x^12+1/14*b^8*x^14+8*a^7*b*ln(x)
```

maxima [A] time = 1.38, size = 91, normalized size = 0.92

$$\frac{1}{14}\,b^8x^{14}+\frac{2}{3}\,ab^7x^{12}+\frac{14}{5}\,a^2b^6x^{10}+7\,a^3b^5x^8+\frac{35}{3}\,a^4b^4x^6+14\,a^5b^3x^4+14\,a^6b^2x^2+4\,a^7b\log\left(x^2\right)-\frac{a^8}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^8*x^14 + 2/3*a*b^7*x^12 + 14/5*a^2*b^6*x^10 + 7*a^3*b^5*x^8 + 35/3*a
^4*b^4*x^6 + 14*a^5*b^3*x^4 + 14*a^6*b^2*x^2 + 4*a^7*b*log(x^2) - 1/2*a^8/x
^2
```

mupad [B] time = 0.06, size = 89, normalized size = 0.90

$$\frac{b^8\,x^{14}}{14}-\frac{a^8}{2\,x^2}+\frac{2\,a\,b^7\,x^{12}}{3}+8\,a^7\,b\,\ln(x)+14\,a^6\,b^2\,x^2+14\,a^5\,b^3\,x^4+\frac{35\,a^4\,b^4\,x^6}{3}+7\,a^3\,b^5\,x^8+\frac{14\,a^2\,b^6\,x^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^3,x)
```

```
[Out] (b^8*x^14)/14 - a^8/(2*x^2) + (2*a*b^7*x^12)/3 + 8*a^7*b*log(x) + 14*a^6*b^
2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3*b^5*x^8 + (14*a^2*b^6*x
^10)/5
```

sympy [A] time = 0.22, size = 100, normalized size = 1.01

$$-\frac{a^8}{2x^2}+8a^7b\log(x)+14a^6b^2x^2+14a^5b^3x^4+\frac{35a^4b^4x^6}{3}+7a^3b^5x^8+\frac{14a^2b^6x^{10}}{5}+\frac{2ab^7x^{12}}{3}+\frac{b^8x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**3,x)
```

```
[Out] -a**8/(2*x**2) + 8*a**7*b*log(x) + 14*a**6*b**2*x**2 + 14*a**5*b**3*x**4 +
35*a**4*b**4*x**6/3 + 7*a**3*b**5*x**8 + 14*a**2*b**6*x**10/5 + 2*a*b**7*x*
*12/3 + b**8*x**14/14
```

3.94 $\int \frac{(a+bx^2)^8}{x^5}\,dx$

Optimal. Leaf size=101

$$-\frac{a^8}{4x^4}-\frac{4a^7b}{x^2}+28a^6b^2\log(x)+28a^5b^3x^2+\frac{35}{2}a^4b^4x^4+\frac{28}{3}a^3b^5x^6+\frac{7}{2}a^2b^6x^8+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}$$

```
[Out] -1/4*a^8/x^4-4*a^7*b/x^2+28*a^5*b^3*x^2+35/2*a^4*b^4*x^4+28/3*a^3*b^5*x^6+7
/2*a^2*b^6*x^8+4/5*a*b^7*x^10+1/12*b^8*x^12+28*a^6*b^2*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{7}{2}a^2b^6x^8+\frac{28}{3}a^3b^5x^6+\frac{35}{2}a^4b^4x^4+28a^5b^3x^2+28a^6b^2\log(x)-\frac{4a^7b}{x^2}-\frac{a^8}{4x^4}+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^5,x]
```

```
[Out] -a^8/(4*x^4) - (4*a^7*b)/x^2 + 28*a^5*b^3*x^2 + (35*a^4*b^4*x^4)/2 + (28*a^
3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + (4*a*b^7*x^10)/5 + (b^8*x^12)/12 + 28*a^
6*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(56a^5b^3+\frac{a^8}{x^3}+\frac{8a^7b}{x^2}+\frac{28a^6b^2}{x}+70a^4b^4x+56a^3b^5x^2+28a^2b^6x^3+8ab^7x^4+b^8\right.\right.\\ &= -\frac{a^8}{4x^4}-\frac{4a^7b}{x^2}+28a^5b^3x^2+\frac{35}{2}a^4b^4x^4+\frac{28}{3}a^3b^5x^6+\frac{7}{2}a^2b^6x^8+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}+28a^6b^2\text{lo}\end{aligned}$$

Mathematica [A] time = 0.01, size = 101, normalized size = 1.00

$$-\frac{a^8}{4x^4}-\frac{4a^7b}{x^2}+28a^6b^2\log(x)+28a^5b^3x^2+\frac{35}{2}a^4b^4x^4+\frac{28}{3}a^3b^5x^6+\frac{7}{2}a^2b^6x^8+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^5,x]
```

```
[Out] -1/4*a^8/x^4 - (4*a^7*b)/x^2 + 28*a^5*b^3*x^2 + (35*a^4*b^4*x^4)/2 + (28*a^
3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + (4*a*b^7*x^10)/5 + (b^8*x^12)/12 + 28*a^
6*b^2*Log[x]
```

fricas [A] time = 0.79, size = 94, normalized size = 0.93

$$\frac{5\,b^8x^{16}+48\,ab^7x^{14}+210\,a^2b^6x^{12}+560\,a^3b^5x^{10}+1050\,a^4b^4x^8+1680\,a^5b^3x^6+1680\,a^6b^2x^4\log(x)-240\,a^7bx}{60\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^5,x, algorithm="fricas")
```

```
[Out] 1/60*(5*b^8*x^16 + 48*a*b^7*x^14 + 210*a^2*b^6*x^12 + 560*a^3*b^5*x^10 + 10
50*a^4*b^4*x^8 + 1680*a^5*b^3*x^6 + 1680*a^6*b^2*x^4*log(x) - 240*a^7*b*x^2
 - 15*a^8)/x^4
```

giac [A] time = 1.12, size = 103, normalized size = 1.02

$$\frac{1}{12}\,b^8x^{12}+\frac{4}{5}\,ab^7x^{10}+\frac{7}{2}\,a^2b^6x^8+\frac{28}{3}\,a^3b^5x^6+\frac{35}{2}\,a^4b^4x^4+28\,a^5b^3x^2+14\,a^6b^2\log\left(x^2\right)-\frac{84\,a^6b^2x^4+16\,a^7bx^2+a^8}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^5,x, algorithm="giac")
```

```
[Out] 1/12*b^8*x^12 + 4/5*a*b^7*x^10 + 7/2*a^2*b^6*x^8 + 28/3*a^3*b^5*x^6 + 35/2*
a^4*b^4*x^4 + 28*a^5*b^3*x^2 + 14*a^6*b^2*log(x^2) - 1/4*(84*a^6*b^2*x^4 +
16*a^7*b*x^2 + a^8)/x^4
```

maple [A] time = 0.01, size = 90, normalized size = 0.89

$$\frac{b^8x^{12}}{12}+\frac{4a\,b^7x^{10}}{5}+\frac{7a^2b^6x^8}{2}+\frac{28a^3b^5x^6}{3}+\frac{35a^4b^4x^4}{2}+28a^5b^3x^2+28a^6b^2\ln(x)-\frac{4a^7b}{x^2}-\frac{a^8}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^5,x)
```

```
[Out] -1/4*a^8/x^4-4*a^7*b/x^2+28*a^5*b^3*x^2+35/2*a^4*b^4*x^4+28/3*a^3*b^5*x^6+7
/2*a^2*b^6*x^8+4/5*a*b^7*x^10+1/12*b^8*x^12+28*a^6*b^2*ln(x)
```

maxima [A] time = 1.33, size = 92, normalized size = 0.91

$$\frac{1}{12}\,b^8x^{12}+\frac{4}{5}\,ab^7x^{10}+\frac{7}{2}\,a^2b^6x^8+\frac{28}{3}\,a^3b^5x^6+\frac{35}{2}\,a^4b^4x^4+28\,a^5b^3x^2+14\,a^6b^2\log\left(x^2\right)-\frac{16\,a^7bx^2+a^8}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*b^8*x^12 + 4/5*a*b^7*x^10 + 7/2*a^2*b^6*x^8 + 28/3*a^3*b^5*x^6 + 35/2*
a^4*b^4*x^4 + 28*a^5*b^3*x^2 + 14*a^6*b^2*log(x^2) - 1/4*(16*a^7*b*x^2 + a^
8)/x^4
```

mupad [B] time = 0.06, size = 92, normalized size = 0.91

$$\frac{b^8\,x^{12}}{12}-\frac{\frac{a^8}{4}+4\,b\,a^7\,x^2}{x^4}+\frac{4\,a\,b^7\,x^{10}}{5}+28\,a^5\,b^3\,x^2+\frac{35\,a^4\,b^4\,x^4}{2}+\frac{28\,a^3\,b^5\,x^6}{3}+\frac{7\,a^2\,b^6\,x^8}{2}+28\,a^6\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^5,x)
```

```
[Out] (b^8*x^12)/12 - (a^8/4 + 4*a^7*b*x^2)/x^4 + (4*a*b^7*x^10)/5 + 28*a^5*b^3*x
^2 + (35*a^4*b^4*x^4)/2 + (28*a^3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + 28*a^6*b
^2*log(x)
```

sympy [A] time = 0.27, size = 104, normalized size = 1.03

$$28a^6b^2\log(x)+28a^5b^3x^2+\frac{35a^4b^4x^4}{2}+\frac{28a^3b^5x^6}{3}+\frac{7a^2b^6x^8}{2}+\frac{4ab^7x^{10}}{5}+\frac{b^8x^{12}}{12}+\frac{-a^8-16a^7bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**5,x)
```

```
[Out] 28*a**6*b**2*log(x) + 28*a**5*b**3*x**2 + 35*a**4*b**4*x**4/2 + 28*a**3*b**
5*x**6/3 + 7*a**2*b**6*x**8/2 + 4*a*b**7*x**10/5 + b**8*x**12/12 + (-a**8 -
 16*a**7*b*x**2)/(4*x**4)
```

3.95 $\int \frac{(a+bx^2)^8}{x^7}\,dx$

Optimal. Leaf size=94

$$-\frac{a^8}{6x^6}-\frac{2a^7b}{x^4}-\frac{14a^6b^2}{x^2}+56a^5b^3\log(x)+35a^4b^4x^2+14a^3b^5x^4+\frac{14}{3}a^2b^6x^6+ab^7x^8+\frac{b^8x^{10}}{10}$$

```
[Out] -1/6*a^8/x^6-2*a^7*b/x^4-14*a^6*b^2/x^2+35*a^4*b^4*x^2+14*a^3*b^5*x^4+14/3*
a^2*b^6*x^6+a*b^7*x^8+1/10*b^8*x^10+56*a^5*b^3*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{14}{3}a^2b^6x^6+14a^3b^5x^4+35a^4b^4x^2-\frac{14a^6b^2}{x^2}+56a^5b^3\log(x)-\frac{2a^7b}{x^4}-\frac{a^8}{6x^6}+ab^7x^8+\frac{b^8x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^7,x]
```

```
[Out] -a^8/(6*x^6) - (2*a^7*b)/x^4 - (14*a^6*b^2)/x^2 + 35*a^4*b^4*x^2 + 14*a^3*b
^5*x^4 + (14*a^2*b^6*x^6)/3 + a*b^7*x^8 + (b^8*x^10)/10 + 56*a^5*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^4}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(70a^4b^4+\frac{a^8}{x^4}+\frac{8a^7b}{x^3}+\frac{28a^6b^2}{x^2}+\frac{56a^5b^3}{x}+56a^3b^5x+28a^2b^6x^2+8ab^7x^3+b\right.\right.\\ &= -\frac{a^8}{6x^6}-\frac{2a^7b}{x^4}-\frac{14a^6b^2}{x^2}+35a^4b^4x^2+14a^3b^5x^4+\frac{14}{3}a^2b^6x^6+ab^7x^8+\frac{b^8x^{10}}{10}+56a^5b^3\log(\end{aligned}$$

Mathematica [A] time = 0.01, size = 94, normalized size = 1.00

$$-\frac{a^8}{6x^6}-\frac{2a^7b}{x^4}-\frac{14a^6b^2}{x^2}+56a^5b^3\log(x)+35a^4b^4x^2+14a^3b^5x^4+\frac{14}{3}a^2b^6x^6+ab^7x^8+\frac{b^8x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^7,x]
```

```
[Out] -1/6*a^8/x^6 - (2*a^7*b)/x^4 - (14*a^6*b^2)/x^2 + 35*a^4*b^4*x^2 + 14*a^3*b
^5*x^4 + (14*a^2*b^6*x^6)/3 + a*b^7*x^8 + (b^8*x^10)/10 + 56*a^5*b^3*Log[x]
```

fricas [A] time = 0.87, size = 94, normalized size = 1.00

$$\frac{3\,b^8x^{16}+30\,ab^7x^{14}+140\,a^2b^6x^{12}+420\,a^3b^5x^{10}+1050\,a^4b^4x^8+1680\,a^5b^3x^6\log(x)-420\,a^6b^2x^4-60\,a^7bx^2-5}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^7,x, algorithm="fricas")
```

```
[Out] 1/30*(3*b^8*x^16 + 30*a*b^7*x^14 + 140*a^2*b^6*x^12 + 420*a^3*b^5*x^10 + 10
50*a^4*b^4*x^8 + 1680*a^5*b^3*x^6*log(x) - 420*a^6*b^2*x^4 - 60*a^7*b*x^2 -
 5*a^8)/x^6
```

giac [A] time = 1.05, size = 102, normalized size = 1.09

$$\frac{1}{10}\,b^8x^{10}+ab^7x^8+\frac{14}{3}\,a^2b^6x^6+14\,a^3b^5x^4+35\,a^4b^4x^2+28\,a^5b^3\log\left(x^2\right)-\frac{308\,a^5b^3x^6+84\,a^6b^2x^4+12\,a^7bx^2+a^8}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^7,x, algorithm="giac")
```

```
[Out] 1/10*b^8*x^10 + a*b^7*x^8 + 14/3*a^2*b^6*x^6 + 14*a^3*b^5*x^4 + 35*a^4*b^4*
x^2 + 28*a^5*b^3*log(x^2) - 1/6*(308*a^5*b^3*x^6 + 84*a^6*b^2*x^4 + 12*a^7*
b*x^2 + a^8)/x^6
```

maple [A] time = 0.01, size = 89, normalized size = 0.95

$$\frac{b^8x^{10}}{10}+a\,b^7x^8+\frac{14a^2b^6x^6}{3}+14a^3b^5x^4+35a^4b^4x^2+56a^5b^3\ln(x)-\frac{14a^6b^2}{x^2}-\frac{2a^7b}{x^4}-\frac{a^8}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^7,x)
```

```
[Out] -1/6*a^8/x^6-2*a^7*b/x^4-14*a^6*b^2/x^2+35*a^4*b^4*x^2+14*a^3*b^5*x^4+14/3*
a^2*b^6*x^6+a*b^7*x^8+1/10*b^8*x^10+56*a^5*b^3*ln(x)
```

maxima [A] time = 1.35, size = 91, normalized size = 0.97

$$\frac{1}{10}\,b^8x^{10}+ab^7x^8+\frac{14}{3}\,a^2b^6x^6+14\,a^3b^5x^4+35\,a^4b^4x^2+28\,a^5b^3\log\left(x^2\right)-\frac{84\,a^6b^2x^4+12\,a^7bx^2+a^8}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^7,x, algorithm="maxima")
```

```
[Out] 1/10*b^8*x^10 + a*b^7*x^8 + 14/3*a^2*b^6*x^6 + 14*a^3*b^5*x^4 + 35*a^4*b^4*
x^2 + 28*a^5*b^3*log(x^2) - 1/6*(84*a^6*b^2*x^4 + 12*a^7*b*x^2 + a^8)/x^6
```

mupad [B] time = 0.05, size = 91, normalized size = 0.97

$$\frac{b^8\,x^{10}}{10}-\frac{\frac{a^8}{6}+2\,a^7\,b\,x^2+14\,a^6\,b^2\,x^4}{x^6}+a\,b^7\,x^8+35\,a^4\,b^4\,x^2+14\,a^3\,b^5\,x^4+\frac{14\,a^2\,b^6\,x^6}{3}+56\,a^5\,b^3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^7,x)
```

```
[Out] (b^8*x^10)/10 - (a^8/6 + 2*a^7*b*x^2 + 14*a^6*b^2*x^4)/x^6 + a*b^7*x^8 + 35
*a^4*b^4*x^2 + 14*a^3*b^5*x^4 + (14*a^2*b^6*x^6)/3 + 56*a^5*b^3*log(x)
```

sympy [A] time = 0.33, size = 97, normalized size = 1.03

$$56a^5b^3\log(x)+35a^4b^4x^2+14a^3b^5x^4+\frac{14a^2b^6x^6}{3}+ab^7x^8+\frac{b^8x^{10}}{10}+\frac{-a^8-12a^7bx^2-84a^6b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**7,x)
```

```
[Out] 56*a**5*b**3*log(x) + 35*a**4*b**4*x**2 + 14*a**3*b**5*x**4 + 14*a**2*b**6*
x**6/3 + a*b**7*x**8 + b**8*x**10/10 + (-a**8 - 12*a**7*b*x**2 - 84*a**6*b*
*2*x**4)/(6*x**6)
```

3.96 $\int \frac{(a+bx^2)^8}{x^9}\,dx$

Optimal. Leaf size=97

$$-\frac{a^8}{8x^8}-\frac{4a^7b}{3x^6}-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+70a^4b^4\log(x)+28a^3b^5x^2+7a^2b^6x^4+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}$$

```
[Out] -1/8*a^8/x^8-4/3*a^7*b/x^6-7*a^6*b^2/x^4-28*a^5*b^3/x^2+28*a^3*b^5*x^2+7*a^
2*b^6*x^4+4/3*a*b^7*x^6+1/8*b^8*x^8+70*a^4*b^4*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+28a^3b^5x^2+7a^2b^6x^4+70a^4b^4\log(x)-\frac{4a^7b}{3x^6}-\frac{a^8}{8x^8}+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^9,x]
```

```
[Out] -a^8/(8*x^8) - (4*a^7*b)/(3*x^6) - (7*a^6*b^2)/x^4 - (28*a^5*b^3)/x^2 + 28*
a^3*b^5*x^2 + 7*a^2*b^6*x^4 + (4*a*b^7*x^6)/3 + (b^8*x^8)/8 + 70*a^4*b^4*Lo
g[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^9}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^5}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(56a^3b^5+\frac{a^8}{x^5}+\frac{8a^7b}{x^4}+\frac{28a^6b^2}{x^3}+\frac{56a^5b^3}{x^2}+\frac{70a^4b^4}{x}+28a^2b^6x+8ab^7x^2+b^8x^3\right)\right)\\ &= -\frac{a^8}{8x^8}-\frac{4a^7b}{3x^6}-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+28a^3b^5x^2+7a^2b^6x^4+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}+70a^4b^4\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 97, normalized size = 1.00

$$-\frac{a^8}{8x^8}-\frac{4a^7b}{3x^6}-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+70a^4b^4\log(x)+28a^3b^5x^2+7a^2b^6x^4+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^9,x]
```

[Out] -1/8*a^8/x^8 - (4*a^7*b)/(3*x^6) - (7*a^6*b^2)/x^4 - (28*a^5*b^3)/x^2 + 28*
a^3*b^5*x^2 + 7*a^2*b^6*x^4 + (4*a*b^7*x^6)/3 + (b^8*x^8)/8 + 70*a^4*b^4*Lo
g[x]

fricas [A] time = 0.88, size = 94, normalized size = 0.97

$$\frac{3\,b^8x^{16} + 32\,ab^7x^{14} + 168\,a^2b^6x^{12} + 672\,a^3b^5x^{10} + 1680\,a^4b^4x^8\log(x) - 672\,a^5b^3x^6 - 168\,a^6b^2x^4 - 32\,a^7bx^2 -}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^8*x^16 + 32*a*b^7*x^14 + 168*a^2*b^6*x^12 + 672*a^3*b^5*x^10 + 16
80*a^4*b^4*x^8*log(x) - 672*a^5*b^3*x^6 - 168*a^6*b^2*x^4 - 32*a^7*b*x^2 -
3*a^8)/x^8
```

giac [A] time = 1.00, size = 105, normalized size = 1.08

$$\frac{1}{8}\,b^8x^8+\frac{4}{3}\,ab^7x^6+7\,a^2b^6x^4+28\,a^3b^5x^2+35\,a^4b^4\log\left(x^2\right)-\frac{1750\,a^4b^4x^8 + 672\,a^5b^3x^6 + 168\,a^6b^2x^4 + 32\,a^7bx^2 + 3}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^9,x, algorithm="giac")
```

```
[Out] 1/8*b^8*x^8 + 4/3*a*b^7*x^6 + 7*a^2*b^6*x^4 + 28*a^3*b^5*x^2 + 35*a^4*b^4*l
og(x^2) - 1/24*(1750*a^4*b^4*x^8 + 672*a^5*b^3*x^6 + 168*a^6*b^2*x^4 + 32*a
^7*b*x^2 + 3*a^8)/x^8
```

maple [A] time = 0.01, size = 90, normalized size = 0.93

$$\frac{b^8x^8}{8} + \frac{4a\,b^7x^6}{3} + 7a^2b^6x^4 + 28a^3b^5x^2 + 70a^4b^4\ln(x) - \frac{28a^5b^3}{x^2} - \frac{7a^6b^2}{x^4} - \frac{4a^7b}{3x^6} - \frac{a^8}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^9,x)
```

```
[Out] -1/8*a^8/x^8-4/3*a^7*b/x^6-7*a^6*b^2/x^4-28*a^5*b^3/x^2+28*a^3*b^5*x^2+7*a^
2*b^6*x^4+4/3*a*b^7*x^6+1/8*b^8*x^8+70*a^4*b^4*ln(x)
```

maxima [A] time = 1.39, size = 94, normalized size = 0.97

$$\frac{1}{8}\,b^8x^8+\frac{4}{3}\,ab^7x^6+7\,a^2b^6x^4+28\,a^3b^5x^2+35\,a^4b^4\log\left(x^2\right)-\frac{672\,a^5b^3x^6 + 168\,a^6b^2x^4 + 32\,a^7bx^2 + 3\,a^8}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^9,x, algorithm="maxima")
```

```
[Out] 1/8*b^8*x^8 + 4/3*a*b^7*x^6 + 7*a^2*b^6*x^4 + 28*a^3*b^5*x^2 + 35*a^4*b^4*l
og(x^2) - 1/24*(672*a^5*b^3*x^6 + 168*a^6*b^2*x^4 + 32*a^7*b*x^2 + 3*a^8)/x
^8
```

mupad [B] time = 0.05, size = 92, normalized size = 0.95

$$\frac{b^8\,x^8}{8}-\frac{\frac{a^8}{8}+\frac{4\,a^7\,b\,x^2}{3}+7\,a^6\,b^2\,x^4+28\,a^5\,b^3\,x^6}{x^8}+\frac{4\,a\,b^7\,x^6}{3}+28\,a^3\,b^5\,x^2+7\,a^2\,b^6\,x^4+70\,a^4\,b^4\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^9,x)
```

```
[Out] (b^8*x^8)/8 - (a^8/8 + (4*a^7*b*x^2)/3 + 7*a^6*b^2*x^4 + 28*a^5*b^3*x^6)/x^
8 + (4*a*b^7*x^6)/3 + 28*a^3*b^5*x^2 + 7*a^2*b^6*x^4 + 70*a^4*b^4*log(x)
```

sympy [A] time = 0.42, size = 100, normalized size = 1.03

$$70a^4b^4\log(x)+28a^3b^5x^2+7a^2b^6x^4+\frac{4ab^7x^6}{3}+\frac{b^8x^8}{8}+\frac{-3a^8-32a^7bx^2-168a^6b^2x^4-672a^5b^3x^6}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**9,x)
```

```
[Out] 70*a**4*b**4*log(x) + 28*a**3*b**5*x**2 + 7*a**2*b**6*x**4 + 4*a*b**7*x**6/
3 + b**8*x**8/8 + (-3*a**8 - 32*a**7*b*x**2 - 168*a**6*b**2*x**4 - 672*a**5
*b**3*x**6)/(24*x**8)
```

3.97 $\int \frac{(a+bx^2)^8}{x^{11}} dx$

Optimal. Leaf size=95

$$-\frac{a^8}{10x^{10}}-\frac{a^7b}{x^8}-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+56a^3b^5\log(x)+14a^2b^6x^2+2ab^7x^4+\frac{b^8x^6}{6}$$

```
[Out] -1/10*a^8/x^10-a^7*b/x^8-14/3*a^6*b^2/x^6-14*a^5*b^3/x^4-35*a^4*b^4/x^2+14*
a^2*b^6*x^2+2*a*b^7*x^4+1/6*b^8*x^6+56*a^3*b^5*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+14a^2b^6x^2+56a^3b^5\log(x)-\frac{a^7b}{x^8}-\frac{a^8}{10x^{10}}+2ab^7x^4+\frac{b^8x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^11,x]
```

```
[Out] -a^8/(10*x^10) - (a^7*b)/x^8 - (14*a^6*b^2)/(3*x^6) - (14*a^5*b^3)/x^4 - (3
5*a^4*b^4)/x^2 + 14*a^2*b^6*x^2 + 2*a*b^7*x^4 + (b^8*x^6)/6 + 56*a^3*b^5*Lo
g[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{11}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^6}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(28a^2b^6+\frac{a^8}{x^6}+\frac{8a^7b}{x^5}+\frac{28a^6b^2}{x^4}+\frac{56a^5b^3}{x^3}+\frac{70a^4b^4}{x^2}+\frac{56a^3b^5}{x}+8ab^7x+b^8x^2\right.\right.\\ &= -\frac{a^8}{10x^{10}}-\frac{a^7b}{x^8}-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+14a^2b^6x^2+2ab^7x^4+\frac{b^8x^6}{6}+56a^3b^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 95, normalized size = 1.00

$$-\frac{a^8}{10x^{10}}-\frac{a^7b}{x^8}-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+56a^3b^5\log(x)+14a^2b^6x^2+2ab^7x^4+\frac{b^8x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^11,x]
```

```
[Out] -1/10*a^8/x^10 - (a^7*b)/x^8 - (14*a^6*b^2)/(3*x^6) - (14*a^5*b^3)/x^4 - (3
5*a^4*b^4)/x^2 + 14*a^2*b^6*x^2 + 2*a*b^7*x^4 + (b^8*x^6)/6 + 56*a^3*b^5*Lo
g[x]
```

fricas [A] time = 0.97, size = 94, normalized size = 0.99

$$\frac{5\,b^8x^{16} + 60\,ab^7x^{14} + 420\,a^2b^6x^{12} + 1680\,a^3b^5x^{10}\log(x) - 1050\,a^4b^4x^8 - 420\,a^5b^3x^6 - 140\,a^6b^2x^4 - 30\,a^7bx^2 - 3}{30\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^11,x, algorithm="fricas")
```

```
[Out] 1/30*(5*b^8*x^16 + 60*a*b^7*x^14 + 420*a^2*b^6*x^12 + 1680*a^3*b^5*x^10*log
(x) - 1050*a^4*b^4*x^8 - 420*a^5*b^3*x^6 - 140*a^6*b^2*x^4 - 30*a^7*b*x^2 -
 3*a^8)/x^10
```

giac [A] time = 1.13, size = 105, normalized size = 1.11

$$\frac{1}{6}\,b^8x^6{+}2\,ab^7x^4{+}14\,a^2b^6x^2{+}28\,a^3b^5\log\left(x^2\right){-}\frac{1918\,a^3b^5x^{10} + 1050\,a^4b^4x^8 + 420\,a^5b^3x^6 + 140\,a^6b^2x^4 + 30\,a^7bx^2 +}{30\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^11,x, algorithm="giac")
```

```
[Out] 1/6*b^8*x^6 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 28*a^3*b^5*log(x^2) - 1/30*(19
18*a^3*b^5*x^10 + 1050*a^4*b^4*x^8 + 420*a^5*b^3*x^6 + 140*a^6*b^2*x^4 + 30
*a^7*b*x^2 + 3*a^8)/x^10
```

maple [A] time = 0.01, size = 90, normalized size = 0.95

$$\frac{b^8x^6}{6} + 2a\,b^7x^4 + 14a^2b^6x^2 + 56a^3b^5\ln(x) - \frac{35a^4b^4}{x^2} - \frac{14a^5b^3}{x^4} - \frac{14a^6b^2}{3x^6} - \frac{a^7b}{x^8} - \frac{a^8}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^11,x)
```

```
[Out] -1/10*a^8/x^10-a^7*b/x^8-14/3*a^6*b^2/x^6-14*a^5*b^3/x^4-35*a^4*b^4/x^2+14*
a^2*b^6*x^2+2*a*b^7*x^4+1/6*b^8*x^6+56*a^3*b^5*ln(x)
```

maxima [A] time = 1.33, size = 94, normalized size = 0.99

$$\frac{1}{6}\,b^8x^6{+}2\,ab^7x^4{+}14\,a^2b^6x^2{+}28\,a^3b^5\log\left(x^2\right){-}\frac{1050\,a^4b^4x^8 + 420\,a^5b^3x^6 + 140\,a^6b^2x^4 + 30\,a^7bx^2 + 3\,a^8}{30\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^11,x, algorithm="maxima")
```

```
[Out] 1/6*b^8*x^6 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 28*a^3*b^5*log(x^2) - 1/30*(10
50*a^4*b^4*x^8 + 420*a^5*b^3*x^6 + 140*a^6*b^2*x^4 + 30*a^7*b*x^2 + 3*a^8)/
x^10
```

mupad [B] time = 5.11, size = 91, normalized size = 0.96

$$\frac{b^8\,x^6}{6}-\frac{\frac{a^8}{10}+a^7\,b\,x^2+\frac{14\,a^6\,b^2\,x^4}{3}+14\,a^5\,b^3\,x^6+35\,a^4\,b^4\,x^8}{x^{10}}{+}2\,a\,b^7\,x^4{+}14\,a^2\,b^6\,x^2{+}56\,a^3\,b^5\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^11,x)
```

```
[Out] (b^8*x^6)/6 - (a^8/10 + a^7*b*x^2 + (14*a^6*b^2*x^4)/3 + 14*a^5*b^3*x^6 + 3
5*a^4*b^4*x^8)/x^10 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 56*a^3*b^5*log(x)
```

sympy [A] time = 0.51, size = 99, normalized size = 1.04

$$56a^3b^5\log(x)+14a^2b^6x^2+2ab^7x^4+\frac{b^8x^6}{6}+\frac{-3a^8-30a^7bx^2-140a^6b^2x^4-420a^5b^3x^6-1050a^4b^4x^8}{30x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**11,x)
```

```
[Out] 56*a**3*b**5*log(x) + 14*a**2*b**6*x**2 + 2*a*b**7*x**4 + b**8*x**6/6 + (-3
*a**8 - 30*a**7*b*x**2 - 140*a**6*b**2*x**4 - 420*a**5*b**3*x**6 - 1050*a**
4*b**4*x**8)/(30*x**10)
```

3.98 $\int \frac{(a+bx^2)^8}{x^{13}}\,dx$

Optimal. Leaf size=101

$$-\frac{a^8}{12x^{12}}-\frac{4a^7b}{5x^{10}}-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+28a^2b^6\log(x)+4ab^7x^2+\frac{b^8x^4}{4}$$

```
[Out] -1/12*a^8/x^12-4/5*a^7*b/x^10-7/2*a^6*b^2/x^8-28/3*a^5*b^3/x^6-35/2*a^4*b^4
/x^4-28*a^3*b^5/x^2+4*a*b^7*x^2+1/4*b^8*x^4+28*a^2*b^6*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+28a^2b^6\log(x)-\frac{4a^7b}{5x^{10}}-\frac{a^8}{12x^{12}}+4ab^7x^2+\frac{b^8x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^13,x]
```

```
[Out] -a^8/(12*x^12) - (4*a^7*b)/(5*x^10) - (7*a^6*b^2)/(2*x^8) - (28*a^5*b^3)/(3
*x^6) - (35*a^4*b^4)/(2*x^4) - (28*a^3*b^5)/x^2 + 4*a*b^7*x^2 + (b^8*x^4)/4
 + 28*a^2*b^6*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{13}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^7}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(8ab^7+\frac{a^8}{x^7}+\frac{8a^7b}{x^6}+\frac{28a^6b^2}{x^5}+\frac{56a^5b^3}{x^4}+\frac{70a^4b^4}{x^3}+\frac{56a^3b^5}{x^2}+\frac{28a^2b^6}{x}+b^8x\right)dx\\ &= -\frac{a^8}{12x^{12}}-\frac{4a^7b}{5x^{10}}-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+4ab^7x^2+\frac{b^8x^4}{4}+28a^2b^6\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 101, normalized size = 1.00

$$-\frac{a^8}{12x^{12}}-\frac{4a^7b}{5x^{10}}-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+28a^2b^6\log(x)+4ab^7x^2+\frac{b^8x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^13,x]
```

```
[Out] -1/12*a^8/x^12 - (4*a^7*b)/(5*x^10) - (7*a^6*b^2)/(2*x^8) - (28*a^5*b^3)/(3
*x^6) - (35*a^4*b^4)/(2*x^4) - (28*a^3*b^5)/x^2 + 4*a*b^7*x^2 + (b^8*x^4)/4
 + 28*a^2*b^6*Log[x]
```

fricas [A] time = 0.75, size = 94, normalized size = 0.93

$$\frac{15\,b^{8}x^{16} + 240\,ab^{7}x^{14} + 1680\,a^{2}b^{6}x^{12}\log(x) - 1680\,a^{3}b^{5}x^{10} - 1050\,a^{4}b^{4}x^{8} - 560\,a^{5}b^{3}x^{6} - 210\,a^{6}b^{2}x^{4} - 48\,a^{7}b}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^13,x, algorithm="fricas")
```

```
[Out] 1/60*(15*b^8*x^16 + 240*a*b^7*x^14 + 1680*a^2*b^6*x^12*log(x) - 1680*a^3*b^
5*x^10 - 1050*a^4*b^4*x^8 - 560*a^5*b^3*x^6 - 210*a^6*b^2*x^4 - 48*a^7*b*x^
2 - 5*a^8)/x^12
```

giac [A] time = 1.21, size = 105, normalized size = 1.04

$$\frac{1}{4}\,b^{8}x^{4}+4\,ab^{7}x^{2}+14\,a^{2}b^{6}\log\left(x^{2}\right)-\frac{2058\,a^{2}b^{6}x^{12} + 1680\,a^{3}b^{5}x^{10} + 1050\,a^{4}b^{4}x^{8} + 560\,a^{5}b^{3}x^{6} + 210\,a^{6}b^{2}x^{4} + 48}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^13,x, algorithm="giac")
```

```
[Out] 1/4*b^8*x^4 + 4*a*b^7*x^2 + 14*a^2*b^6*log(x^2) - 1/60*(2058*a^2*b^6*x^12 +
 1680*a^3*b^5*x^10 + 1050*a^4*b^4*x^8 + 560*a^5*b^3*x^6 + 210*a^6*b^2*x^4 +
 48*a^7*b*x^2 + 5*a^8)/x^12
```

maple [A] time = 0.01, size = 90, normalized size = 0.89

$$\frac{b^{8}x^{4}}{4}+4a\,b^{7}x^{2}+28a^{2}b^{6}\ln(x)-\frac{28a^{3}b^{5}}{x^{2}}-\frac{35a^{4}b^{4}}{2x^{4}}-\frac{28a^{5}b^{3}}{3x^{6}}-\frac{7a^{6}b^{2}}{2x^{8}}-\frac{4a^{7}b}{5x^{10}}-\frac{a^{8}}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^13,x)
```

```
[Out] -1/12*a^8/x^12-4/5*a^7*b/x^10-7/2*a^6*b^2/x^8-28/3*a^5*b^3/x^6-35/2*a^4*b^4
/x^4-28*a^3*b^5/x^2+4*a*b^7*x^2+1/4*b^8*x^4+28*a^2*b^6*ln(x)
```

maxima [A] time = 1.38, size = 94, normalized size = 0.93

$$\frac{1}{4}\,b^{8}x^{4}+4\,ab^{7}x^{2}+14\,a^{2}b^{6}\log\left(x^{2}\right)-\frac{1680\,a^{3}b^{5}x^{10} + 1050\,a^{4}b^{4}x^{8} + 560\,a^{5}b^{3}x^{6} + 210\,a^{6}b^{2}x^{4} + 48\,a^{7}bx^{2} + 5\,a^{8}}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^13,x, algorithm="maxima")
```

```
[Out] 1/4*b^8*x^4 + 4*a*b^7*x^2 + 14*a^2*b^6*log(x^2) - 1/60*(1680*a^3*b^5*x^10 +
 1050*a^4*b^4*x^8 + 560*a^5*b^3*x^6 + 210*a^6*b^2*x^4 + 48*a^7*b*x^2 + 5*a^
8)/x^12
```

mupad [B] time = 0.06, size = 92, normalized size = 0.91

$$\frac{b^{8}\,x^{4}}{4}-\frac{\frac{a^{8}}{12}+\frac{4\,a^{7}\,b\,x^{2}}{5}+\frac{7\,a^{6}\,b^{2}\,x^{4}}{2}+\frac{28\,a^{5}\,b^{3}\,x^{6}}{3}+\frac{35\,a^{4}\,b^{4}\,x^{8}}{2}+28\,a^{3}\,b^{5}\,x^{10}}{x^{12}}+4\,a\,b^{7}\,x^{2}+28\,a^{2}\,b^{6}\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^13,x)
```

```
[Out] (b^8*x^4)/4 - (a^8/12 + (4*a^7*b*x^2)/5 + (7*a^6*b^2*x^4)/2 + (28*a^5*b^3*x
^6)/3 + (35*a^4*b^4*x^8)/2 + 28*a^3*b^5*x^10)/x^12 + 4*a*b^7*x^2 + 28*a^2*b
^6*log(x)
```

sympy [**A**] time = 0.62, size = 99, normalized size = 0.98

$$28a^2b^6\log(x)+4ab^7x^2+\frac{b^8x^4}{4}+\frac{-5a^8-48a^7bx^2-210a^6b^2x^4-560a^5b^3x^6-1050a^4b^4x^8-1680a^3b^5x^{10}}{60x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**13,x)
```

```
[Out] 28*a**2*b**6*log(x) + 4*a*b**7*x**2 + b**8*x**4/4 + (-5*a**8 - 48*a**7*b*x*
*2 - 210*a**6*b**2*x**4 - 560*a**5*b**3*x**6 - 1050*a**4*b**4*x**8 - 1680*a
**3*b**5*x**10)/(60*x**12)
```

3.99 $\int \frac{(a+bx^2)^8}{x^{15}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^8}{14x^{14}}-\frac{2a^7b}{3x^{12}}-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}+8ab^7\log(x)+\frac{b^8x^2}{2}$$

```
[Out] -1/14*a^8/x^14-2/3*a^7*b/x^12-14/5*a^6*b^2/x^10-7*a^5*b^3/x^8-35/3*a^4*b^4/
x^6-14*a^3*b^5/x^4-14*a^2*b^6/x^2+1/2*b^8*x^2+8*a*b^7*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}-\frac{2a^7b}{3x^{12}}-\frac{a^8}{14x^{14}}+8ab^7\log(x)+\frac{b^8x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^15,x]
```

```
[Out] -a^8/(14*x^14) - (2*a^7*b)/(3*x^12) - (14*a^6*b^2)/(5*x^10) - (7*a^5*b^3)/x
^8 - (35*a^4*b^4)/(3*x^6) - (14*a^3*b^5)/x^4 - (14*a^2*b^6)/x^2 + (b^8*x^2)
/2 + 8*a*b^7*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^{15}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^8}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(b^8+\frac{a^8}{x^8}+\frac{8a^7b}{x^7}+\frac{28a^6b^2}{x^6}+\frac{56a^5b^3}{x^5}+\frac{70a^4b^4}{x^4}+\frac{56a^3b^5}{x^3}+\frac{28a^2b^6}{x^2}+\frac{8ab^7}{x}\right)\right.\\
&= -\frac{a^8}{14x^{14}}-\frac{2a^7b}{3x^{12}}-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}+\frac{b^8x^2}{2}+8ab^7\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 99, normalized size = 1.00

$$-\frac{a^8}{14x^{14}}-\frac{2a^7b}{3x^{12}}-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}+8ab^7\log(x)+\frac{b^8x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^15,x]
```

```
[Out] -1/14*a^8/x^14 - (2*a^7*b)/(3*x^12) - (14*a^6*b^2)/(5*x^10) - (7*a^5*b^3)/x
^8 - (35*a^4*b^4)/(3*x^6) - (14*a^3*b^5)/x^4 - (14*a^2*b^6)/x^2 + (b^8*x^2)
/2 + 8*a*b^7*Log[x]
```

fricas [A] time = 0.73, size = 94, normalized size = 0.95

$$\frac{105\,b^8x^{16}+1680\,ab^7x^{14}\log(x)-2940\,a^2b^6x^{12}-2940\,a^3b^5x^{10}-2450\,a^4b^4x^8-1470\,a^5b^3x^6-588\,a^6b^2x^4-140\,a}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^15,x, algorithm="fricas")
```

```
[Out] 1/210*(105*b^8*x^16 + 1680*a*b^7*x^14*log(x) - 2940*a^2*b^6*x^12 - 2940*a^3
*b^5*x^10 - 2450*a^4*b^4*x^8 - 1470*a^5*b^3*x^6 - 588*a^6*b^2*x^4 - 140*a^7
*b*x^2 - 15*a^8)/x^14
```

giac [A] time = 1.08, size = 103, normalized size = 1.04

$$\frac{1}{2}\,b^8x^2+4\,ab^7\log\left(x^2\right)-\frac{2178\,ab^7x^{14}+2940\,a^2b^6x^{12}+2940\,a^3b^5x^{10}+2450\,a^4b^4x^8+1470\,a^5b^3x^6+588\,a^6b^2x^4+}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^15,x, algorithm="giac")
```

```
[Out] 1/2*b^8*x^2 + 4*a*b^7*log(x^2) - 1/210*(2178*a*b^7*x^14 + 2940*a^2*b^6*x^12
 + 2940*a^3*b^5*x^10 + 2450*a^4*b^4*x^8 + 1470*a^5*b^3*x^6 + 588*a^6*b^2*x^
4 + 140*a^7*b*x^2 + 15*a^8)/x^14
```

maple [A] time = 0.01, size = 90, normalized size = 0.91

$$\frac{b^8x^2}{2}+8a\,b^7\ln(x)-\frac{14a^2b^6}{x^2}-\frac{14a^3b^5}{x^4}-\frac{35a^4b^4}{3x^6}-\frac{7a^5b^3}{x^8}-\frac{14a^6b^2}{5x^{10}}-\frac{2a^7b}{3x^{12}}-\frac{a^8}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^15,x)
```

```
[Out] -1/14*a^8/x^14-2/3*a^7*b/x^12-14/5*a^6*b^2/x^10-7*a^5*b^3/x^8-35/3*a^4*b^4/
x^6-14*a^3*b^5/x^4-14*a^2*b^6/x^2+1/2*b^8*x^2+8*a*b^7*ln(x)
```

maxima [A] time = 1.35, size = 94, normalized size = 0.95

$$\frac{1}{2}\,b^8x^2+4\,ab^7\log\left(x^2\right)-\frac{2940\,a^2b^6x^{12}+2940\,a^3b^5x^{10}+2450\,a^4b^4x^8+1470\,a^5b^3x^6+588\,a^6b^2x^4+140\,a^7bx^2+15}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^15,x, algorithm="maxima")
```

```
[Out] 1/2*b^8*x^2 + 4*a*b^7*log(x^2) - 1/210*(2940*a^2*b^6*x^12 + 2940*a^3*b^5*x^
10 + 2450*a^4*b^4*x^8 + 1470*a^5*b^3*x^6 + 588*a^6*b^2*x^4 + 140*a^7*b*x^2
+ 15*a^8)/x^14
```

mupad [B] time = 5.15, size = 94, normalized size = 0.95

$$-\frac{\frac{a^8}{14}-\frac{b^8\,x^{16}}{2}+\frac{2\,a^7\,b\,x^2}{3}+\frac{14\,a^6\,b^2\,x^4}{5}+7\,a^5\,b^3\,x^6+\frac{35\,a^4\,b^4\,x^8}{3}+14\,a^3\,b^5\,x^{10}+14\,a^2\,b^6\,x^{12}-8\,a\,b^7\,x^{14}\,\ln(x)}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^15,x)
```

```
[Out] -(a^8/14 - (b^8*x^16)/2 + (2*a^7*b*x^2)/3 + (14*a^6*b^2*x^4)/5 + 7*a^5*b^3*
x^6 + (35*a^4*b^4*x^8)/3 + 14*a^3*b^5*x^10 + 14*a^2*b^6*x^12 - 8*a*b^7*x^14
*log(x))/x^14
```

sympy [A] time = 0.73, size = 99, normalized size = 1.00

$$8ab^7\log(x)+\frac{b^8x^2}{2}+\frac{-15a^8-140a^7bx^2-588a^6b^2x^4-1470a^5b^3x^6-2450a^4b^4x^8-2940a^3b^5x^{10}-2940a^2b^6x^{12}}{210x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**15,x)
```

```
[Out] 8*a*b**7*log(x) + b**8*x**2/2 + (-15*a**8 - 140*a**7*b*x**2 - 588*a**6*b**2
*x**4 - 1470*a**5*b**3*x**6 - 2450*a**4*b**4*x**8 - 2940*a**3*b**5*x**10 -
2940*a**2*b**6*x**12)/(210*x**14)
```

3.100 $\int \frac{(a+bx^2)^8}{x^{17}}\,dx$

Optimal. Leaf size=100

$$-\frac{a^8}{16x^{16}}-\frac{4a^7b}{7x^{14}}-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4ab^7}{x^2}+b^8\log(x)$$

```
[Out] -1/16*a^8/x^16-4/7*a^7*b/x^14-7/3*a^6*b^2/x^12-28/5*a^5*b^3/x^10-35/4*a^4*b
^4/x^8-28/3*a^3*b^5/x^6-7*a^2*b^6/x^4-4*a*b^7/x^2+b^8*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4a^7b}{7x^{14}}-\frac{a^8}{16x^{16}}-\frac{4ab^7}{x^2}+b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^17,x]
```

```
[Out] -a^8/(16*x^16) - (4*a^7*b)/(7*x^14) - (7*a^6*b^2)/(3*x^12) - (28*a^5*b^3)/(
5*x^10) - (35*a^4*b^4)/(4*x^8) - (28*a^3*b^5)/(3*x^6) - (7*a^2*b^6)/x^4 - (
4*a*b^7)/x^2 + b^8*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{17}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^9}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^8}{x^9}+\frac{8a^7b}{x^8}+\frac{28a^6b^2}{x^7}+\frac{56a^5b^3}{x^6}+\frac{70a^4b^4}{x^5}+\frac{56a^3b^5}{x^4}+\frac{28a^2b^6}{x^3}+\frac{8ab^7}{x^2}+\frac{b^8}{x}\right)dx,x,x^2\right)\\ &= -\frac{a^8}{16x^{16}}-\frac{4a^7b}{7x^{14}}-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4ab^7}{x^2}+b^8\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 100, normalized size = 1.00

$$-\frac{a^8}{16x^{16}}-\frac{4a^7b}{7x^{14}}-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4ab^7}{x^2}+b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^17,x]
```

```
[Out] -1/16*a^8/x^16 - (4*a^7*b)/(7*x^14) - (7*a^6*b^2)/(3*x^12) - (28*a^5*b^3)/(
5*x^10) - (35*a^4*b^4)/(4*x^8) - (28*a^3*b^5)/(3*x^6) - (7*a^2*b^6)/x^4 - (
4*a*b^7)/x^2 + b^8*Log[x]
```

fricas [A] time = 1.25, size = 94, normalized size = 0.94

$$\frac{1680\,b^{8}x^{16}\log(x) - 6720\,ab^{7}x^{14} - 11760\,a^{2}b^{6}x^{12} - 15680\,a^{3}b^{5}x^{10} - 14700\,a^{4}b^{4}x^{8} - 9408\,a^{5}b^{3}x^{6} - 3920\,a^{6}b^{2}x^{4}}{1680\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^17,x, algorithm="fricas")
```

```
[Out] 1/1680*(1680*b^8*x^16*log(x) - 6720*a*b^7*x^14 - 11760*a^2*b^6*x^12 - 15680
*a^3*b^5*x^10 - 14700*a^4*b^4*x^8 - 9408*a^5*b^3*x^6 - 3920*a^6*b^2*x^4 - 9
60*a^7*b*x^2 - 105*a^8)/x^16
```

giac [A] time = 1.06, size = 102, normalized size = 1.02

$$\frac{1}{2}\,b^{8}\log\left(x^{2}\right)-\frac{2283\,b^{8}x^{16} + 6720\,ab^{7}x^{14} + 11760\,a^{2}b^{6}x^{12} + 15680\,a^{3}b^{5}x^{10} + 14700\,a^{4}b^{4}x^{8} + 9408\,a^{5}b^{3}x^{6} + 39}{1680\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^17,x, algorithm="giac")
```

```
[Out] 1/2*b^8*log(x^2) - 1/1680*(2283*b^8*x^16 + 6720*a*b^7*x^14 + 11760*a^2*b^6*
x^12 + 15680*a^3*b^5*x^10 + 14700*a^4*b^4*x^8 + 9408*a^5*b^3*x^6 + 3920*a^6
*b^2*x^4 + 960*a^7*b*x^2 + 105*a^8)/x^16
```

maple [A] time = 0.01, size = 89, normalized size = 0.89

$$b^{8}\ln(x) - \frac{4a\,b^{7}}{x^{2}} - \frac{7a^{2}b^{6}}{x^{4}} - \frac{28a^{3}b^{5}}{3x^{6}} - \frac{35a^{4}b^{4}}{4x^{8}} - \frac{28a^{5}b^{3}}{5x^{10}} - \frac{7a^{6}b^{2}}{3x^{12}} - \frac{4a^{7}b}{7x^{14}} - \frac{a^{8}}{16x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^17,x)
```

```
[Out] -1/16*a^8/x^16-4/7*a^7*b/x^14-7/3*a^6*b^2/x^12-28/5*a^5*b^3/x^10-35/4*a^4*b
^4/x^8-28/3*a^3*b^5/x^6-7*a^2*b^6/x^4-4*a*b^7/x^2+b^8*ln(x)
```

maxima [A] time = 1.36, size = 94, normalized size = 0.94

$$\frac{1}{2}\,b^{8}\log\left(x^{2}\right)-\frac{6720\,ab^{7}x^{14} + 11760\,a^{2}b^{6}x^{12} + 15680\,a^{3}b^{5}x^{10} + 14700\,a^{4}b^{4}x^{8} + 9408\,a^{5}b^{3}x^{6} + 3920\,a^{6}b^{2}x^{4} + 9}{1680\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^17,x, algorithm="maxima")
```

```
[Out] 1/2*b^8*log(x^2) - 1/1680*(6720*a*b^7*x^14 + 11760*a^2*b^6*x^12 + 15680*a^3
*b^5*x^10 + 14700*a^4*b^4*x^8 + 9408*a^5*b^3*x^6 + 3920*a^6*b^2*x^4 + 960*a
^7*b*x^2 + 105*a^8)/x^16
```

mupad [B] time = 5.09, size = 91, normalized size = 0.91

$$b^{8}\,\ln(x) - \frac{\frac{a^{8}}{16} + \frac{4\,a^{7}\,b\,x^{2}}{7} + \frac{7\,a^{6}\,b^{2}\,x^{4}}{3} + \frac{28\,a^{5}\,b^{3}\,x^{6}}{5} + \frac{35\,a^{4}\,b^{4}\,x^{8}}{4} + \frac{28\,a^{3}\,b^{5}\,x^{10}}{3} + 7\,a^{2}\,b^{6}\,x^{12} + 4\,a\,b^{7}\,x^{14}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^17,x)
```

```
[Out] b^8*log(x) - (a^8/16 + (4*a^7*b*x^2)/7 + 4*a*b^7*x^14 + (7*a^6*b^2*x^4)/3 +
 (28*a^5*b^3*x^6)/5 + (35*a^4*b^4*x^8)/4 + (28*a^3*b^5*x^10)/3 + 7*a^2*b^6*
x^12)/x^16
```

sympy [A] time = 0.83, size = 97, normalized size = 0.97

$$b^8\log(x)+\frac{-105a^8-960a^7bx^2-3920a^6b^2x^4-9408a^5b^3x^6-14700a^4b^4x^8-15680a^3b^5x^{10}-11760a^2b^6x^{12}-672}{1680x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**17,x)
```

```
[Out] b**8*log(x) + (-105*a**8 - 960*a**7*b*x**2 - 3920*a**6*b**2*x**4 - 9408*a**
5*b**3*x**6 - 14700*a**4*b**4*x**8 - 15680*a**3*b**5*x**10 - 11760*a**2*b**
6*x**12 - 6720*a*b**7*x**14)/(1680*x**16)
```

3.101 $\int \frac{(a+bx^2)^8}{x^{19}}\,dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^9}{18ax^{18}}$$

```
[Out] -1/18*(b*x^2+a)^9/a/x^18
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{(a+bx^2)^9}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^19,x]
```

```
[Out] -(a + b*x^2)^9/(18*a*x^18)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^8}{x^{19}}\,dx = -\frac{(a+bx^2)^9}{18ax^{18}}$$

Mathematica [B] time = 0.00, size = 100, normalized size = 5.26

$$-\frac{a^8}{18x^{18}}-\frac{a^7b}{2x^{16}}-\frac{2a^6b^2}{x^{14}}-\frac{14a^5b^3}{3x^{12}}-\frac{7a^4b^4}{x^{10}}-\frac{7a^3b^5}{x^8}-\frac{14a^2b^6}{3x^6}-\frac{2ab^7}{x^4}-\frac{b^8}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^19,x]
```

```
[Out] -1/18*a^8/x^18 - (a^7*b)/(2*x^16) - (2*a^6*b^2)/x^14 - (14*a^5*b^3)/(3*x^12
) - (7*a^4*b^4)/x^10 - (7*a^3*b^5)/x^8 - (14*a^2*b^6)/(3*x^6) - (2*a*b^7)/x
^4 - b^8/(2*x^2)
```

fricas [B] time = 0.79, size = 90, normalized size = 4.74

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^19,x, algorithm="fricas")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

giac [B] time = 1.07, size = 90, normalized size = 4.74

$$-\frac{9\,b^{8}x^{16}+36\,ab^{7}x^{14}+84\,a^{2}b^{6}x^{12}+126\,a^{3}b^{5}x^{10}+126\,a^{4}b^{4}x^{8}+84\,a^{5}b^{3}x^{6}+36\,a^{6}b^{2}x^{4}+9\,a^{7}bx^{2}+a^{8}}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^19,x, algorithm="giac")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

maple [B] time = 0.01, size = 91, normalized size = 4.79

$$-\frac{b^{8}}{2x^{2}}-\frac{2a\,b^{7}}{x^{4}}-\frac{14a^{2}b^{6}}{3x^{6}}-\frac{7a^{3}b^{5}}{x^{8}}-\frac{7a^{4}b^{4}}{x^{10}}-\frac{14a^{5}b^{3}}{3x^{12}}-\frac{2a^{6}b^{2}}{x^{14}}-\frac{a^{7}b}{2x^{16}}-\frac{a^{8}}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^19,x)
```

```
[Out] -14/3*a^2*b^6/x^6-1/18*a^8/x^18-1/2*a^7*b/x^16-1/2*b^8/x^2-2*a*b^7/x^4-7*a^
4*b^4/x^10-14/3*a^5*b^3/x^12-7*a^3*b^5/x^8-2*a^6*b^2/x^14
```

maxima [B] time = 1.32, size = 90, normalized size = 4.74

$$-\frac{9\,b^{8}x^{16}+36\,ab^{7}x^{14}+84\,a^{2}b^{6}x^{12}+126\,a^{3}b^{5}x^{10}+126\,a^{4}b^{4}x^{8}+84\,a^{5}b^{3}x^{6}+36\,a^{6}b^{2}x^{4}+9\,a^{7}bx^{2}+a^{8}}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^19,x, algorithm="maxima")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

mupad [B] time = 0.08, size = 92, normalized size = 4.84

$$-\frac{\frac{a^{8}}{18}+\frac{a^{7}\,b\,x^{2}}{2}+2\,a^{6}\,b^{2}\,x^{4}+\frac{14\,a^{5}\,b^{3}\,x^{6}}{3}+7\,a^{4}\,b^{4}\,x^{8}+7\,a^{3}\,b^{5}\,x^{10}+\frac{14\,a^{2}\,b^{6}\,x^{12}}{3}+2\,a\,b^{7}\,x^{14}+\frac{b^{8}\,x^{16}}{2}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^19,x)
```

```
[Out] -(a^8/18 + (b^8*x^16)/2 + (a^7*b*x^2)/2 + 2*a*b^7*x^14 + 2*a^6*b^2*x^4 + (1
4*a^5*b^3*x^6)/3 + 7*a^4*b^4*x^8 + 7*a^3*b^5*x^10 + (14*a^2*b^6*x^12)/3)/x^
18
```

sympy [B] time = 0.90, size = 97, normalized size = 5.11

$$\frac{-a^{8}-9a^{7}bx^{2}-36a^{6}b^{2}x^{4}-84a^{5}b^{3}x^{6}-126a^{4}b^{4}x^{8}-126a^{3}b^{5}x^{10}-84a^{2}b^{6}x^{12}-36ab^{7}x^{14}-9b^{8}x^{16}}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**19,x)
```

```
[Out] (-a**8 - 9*a**7*b*x**2 - 36*a**6*b**2*x**4 - 84*a**5*b**3*x**6 - 126*a**4*b
**4*x**8 - 126*a**3*b**5*x**10 - 84*a**2*b**6*x**12 - 36*a*b**7*x**14 - 9*b
**8*x**16)/(18*x**18)
```

3.102 $\int \frac{(a+bx^2)^8}{x^{21}}\,dx$

Optimal. Leaf size=40

$$\frac{b(a+bx^2)^9}{180a^2x^{18}}-\frac{(a+bx^2)^9}{20ax^{20}}$$

[Out] -1/20*(b*x^2+a)^9/a/x^20+1/180*b*(b*x^2+a)^9/a^2/x^18

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{b(a+bx^2)^9}{180a^2x^{18}}-\frac{(a+bx^2)^9}{20ax^{20}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^8/x^21,x]

[Out] -(a + b*x^2)^9/(20*a*x^20) + (b*(a + b*x^2)^9)/(180*a^2*x^18)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^{21}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx, x, x^2\right)\\
&= -\frac{(a+bx^2)^9}{20ax^{20}}-\frac{b\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx, x, x^2\right)}{20a}\\
&= -\frac{(a+bx^2)^9}{20ax^{20}}+\frac{b(a+bx^2)^9}{180a^2x^{18}}
\end{aligned}$$

Mathematica [B] time = 0.00, size = 106, normalized size = 2.65

$$-\frac{a^8}{20x^{20}}-\frac{4a^7b}{9x^{18}}-\frac{7a^6b^2}{4x^{16}}-\frac{4a^5b^3}{x^{14}}-\frac{35a^4b^4}{6x^{12}}-\frac{28a^3b^5}{5x^{10}}-\frac{7a^2b^6}{2x^8}-\frac{4ab^7}{3x^6}-\frac{b^8}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^21,x]
```

```
[Out] -1/20*a^8/x^20 - (4*a^7*b)/(9*x^18) - (7*a^6*b^2)/(4*x^16) - (4*a^5*b^3)/x^
14 - (35*a^4*b^4)/(6*x^12) - (28*a^3*b^5)/(5*x^10) - (7*a^2*b^6)/(2*x^8) -
(4*a*b^7)/(3*x^6) - b^8/(4*x^4)
```

fricas [B] time = 0.84, size = 92, normalized size = 2.30

$$-\frac{45\,b^8x^{16}+240\,ab^7x^{14}+630\,a^2b^6x^{12}+1008\,a^3b^5x^{10}+1050\,a^4b^4x^8+720\,a^5b^3x^6+315\,a^6b^2x^4+80\,a^7bx^2+9\,a^8}{180\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^21,x, algorithm="fricas")
```

```
[Out] -1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^10
 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 + 9*
a^8)/x^20
```

giac [B] time = 1.00, size = 92, normalized size = 2.30

$$-\frac{45\,b^8x^{16}+240\,ab^7x^{14}+630\,a^2b^6x^{12}+1008\,a^3b^5x^{10}+1050\,a^4b^4x^8+720\,a^5b^3x^6+315\,a^6b^2x^4+80\,a^7bx^2+9\,a^8}{180\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^21,x, algorithm="giac")
```

```
[Out] -1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^10
 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 + 9*
a^8)/x^20
```

maple [B] time = 0.01, size = 91, normalized size = 2.28

$$-\frac{b^8}{4x^4}-\frac{4a\,b^7}{3x^6}-\frac{7a^2b^6}{2x^8}-\frac{28a^3b^5}{5x^{10}}-\frac{35a^4b^4}{6x^{12}}-\frac{4a^5b^3}{x^{14}}-\frac{7a^6b^2}{4x^{16}}-\frac{4a^7b}{9x^{18}}-\frac{a^8}{20x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^21,x)
```

```
[Out] -7/4*a^6*b^2/x^16-4/3*a*b^7/x^6-1/20*a^8/x^20-1/4*b^8/x^4-28/5*a^3*b^5/x^10
-4/9*a^7*b/x^18-35/6*a^4*b^4/x^12-7/2*a^2*b^6/x^8-4*a^5*b^3/x^14
```

maxima [B] time = 1.35, size = 92, normalized size = 2.30

$$-\frac{45\,b^8x^{16}+240\,ab^7x^{14}+630\,a^2b^6x^{12}+1008\,a^3b^5x^{10}+1050\,a^4b^4x^8+720\,a^5b^3x^6+315\,a^6b^2x^4+80\,a^7bx^2+9\,a^8}{180\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^21,x, algorithm="maxima")
```

```
[Out] -1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^10
 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 + 9*
a^8)/x^20
```

mupad [B] time = 0.08, size = 92, normalized size = 2.30

$$-\frac{\frac{a^8}{20}+\frac{4\,a^7\,b\,x^2}{9}+\frac{7\,a^6\,b^2\,x^4}{4}+4\,a^5\,b^3\,x^6+\frac{35\,a^4\,b^4\,x^8}{6}+\frac{28\,a^3\,b^5\,x^{10}}{5}+\frac{7\,a^2\,b^6\,x^{12}}{2}+\frac{4\,a\,b^7\,x^{14}}{3}+\frac{b^8\,x^{16}}{4}}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^21,x)
```

```
[Out] -(a^8/20 + (b^8*x^16)/4 + (4*a^7*b*x^2)/9 + (4*a*b^7*x^14)/3 + (7*a^6*b^2*x
^4)/4 + 4*a^5*b^3*x^6 + (35*a^4*b^4*x^8)/6 + (28*a^3*b^5*x^10)/5 + (7*a^2*b
^6*x^12)/2)/x^20
```

sympy [B] time = 0.94, size = 99, normalized size = 2.48

$$\frac{-9a^8-80a^7bx^2-315a^6b^2x^4-720a^5b^3x^6-1050a^4b^4x^8-1008a^3b^5x^{10}-630a^2b^6x^{12}-240ab^7x^{14}-45b^8x^{16}}{180x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**21,x)
```

```
[Out] (-9*a**8 - 80*a**7*b*x**2 - 315*a**6*b**2*x**4 - 720*a**5*b**3*x**6 - 1050*
a**4*b**4*x**8 - 1008*a**3*b**5*x**10 - 630*a**2*b**6*x**12 - 240*a*b**7*x*
*14 - 45*b**8*x**16)/(180*x**20)
```

3.103 $\int \frac{(a+bx^2)^8}{x^{23}}\,dx$

Optimal. Leaf size=62

$$-\frac{b^2(a+bx^2)^9}{990a^3x^{18}}+\frac{b(a+bx^2)^9}{110a^2x^{20}}-\frac{(a+bx^2)^9}{22ax^{22}}$$

```
[Out] -1/22*(b*x^2+a)^9/a/x^22+1/110*b*(b*x^2+a)^9/a^2/x^20-1/990*b^2*(b*x^2+a)^9
/a^3/x^18
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$-\frac{b^2(a+bx^2)^9}{990a^3x^{18}}+\frac{b(a+bx^2)^9}{110a^2x^{20}}-\frac{(a+bx^2)^9}{22ax^{22}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^23,x]
```

```
[Out] -(a + b*x^2)^9/(22*a*x^22) + (b*(a + b*x^2)^9)/(110*a^2*x^20) - (b^2*(a + b
*x^2)^9)/(990*a^3*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^8}{x^{23}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx, x, x^2\right)$$
$$= -\frac{(a+bx^2)^9}{22ax^{22}} - \frac{b\,\mathrm{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx, x, x^2\right)}{11a}$$
$$= -\frac{(a+bx^2)^9}{22ax^{22}} + \frac{b(a+bx^2)^9}{110a^2x^{20}} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx, x, x^2\right)}{110a^2}$$
$$= -\frac{(a+bx^2)^9}{22ax^{22}} + \frac{b(a+bx^2)^9}{110a^2x^{20}} - \frac{b^2(a+bx^2)^9}{990a^3x^{18}}$$

Mathematica [A] time = 0.00, size = 104, normalized size = 1.68

$$-\frac{a^8}{22x^{22}} - \frac{2a^7b}{5x^{20}} - \frac{14a^6b^2}{9x^{18}} - \frac{7a^5b^3}{2x^{16}} - \frac{5a^4b^4}{x^{14}} - \frac{14a^3b^5}{3x^{12}} - \frac{14a^2b^6}{5x^{10}} - \frac{ab^7}{x^8} - \frac{b^8}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^23,x]
```

```
[Out] -1/22*a^8/x^22 - (2*a^7*b)/(5*x^20) - (14*a^6*b^2)/(9*x^18) - (7*a^5*b^3)/(
2*x^16) - (5*a^4*b^4)/x^14 - (14*a^3*b^5)/(3*x^12) - (14*a^2*b^6)/(5*x^10)
- (a*b^7)/x^8 - b^8/(6*x^6)
```

fricas [A] time = 0.75, size = 92, normalized size = 1.48

$$-\frac{165\,b^8x^{16} + 990\,ab^7x^{14} + 2772\,a^2b^6x^{12} + 4620\,a^3b^5x^{10} + 4950\,a^4b^4x^8 + 3465\,a^5b^3x^6 + 1540\,a^6b^2x^4 + 396\,a^7b}{990\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^23,x, algorithm="fricas")
```

```
[Out] -1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x^
10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2
 + 45*a^8)/x^22
```

giac [A] time = 1.06, size = 92, normalized size = 1.48

$$-\frac{165\,b^8x^{16} + 990\,ab^7x^{14} + 2772\,a^2b^6x^{12} + 4620\,a^3b^5x^{10} + 4950\,a^4b^4x^8 + 3465\,a^5b^3x^6 + 1540\,a^6b^2x^4 + 396\,a^7b}{990\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^23,x, algorithm="giac")
```

```
[Out] -1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x^
10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2
 + 45*a^8)/x^22
```

maple [A] time = 0.01, size = 91, normalized size = 1.47

$$-\frac{b^8}{6x^6} - \frac{a\,b^7}{x^8} - \frac{14a^2b^6}{5x^{10}} - \frac{14a^3b^5}{3x^{12}} - \frac{5a^4b^4}{x^{14}} - \frac{7a^5b^3}{2x^{16}} - \frac{14a^6b^2}{9x^{18}} - \frac{2a^7b}{5x^{20}} - \frac{a^8}{22x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^23,x)
```

```
[Out] -1/6*b^8/x^6-7/2*a^5*b^3/x^16-2/5*a^7*b/x^20-14/9*a^6*b^2/x^18-14/5*a^2*b^6
/x^10-14/3*a^3*b^5/x^12-a*b^7/x^8-5*a^4*b^4/x^14-1/22*a^8/x^22
```

maxima [A] time = 1.36, size = 92, normalized size = 1.48

$$-\frac{165\,b^{8}x^{16}+990\,ab^{7}x^{14}+2772\,a^{2}b^{6}x^{12}+4620\,a^{3}b^{5}x^{10}+4950\,a^{4}b^{4}x^{8}+3465\,a^{5}b^{3}x^{6}+1540\,a^{6}b^{2}x^{4}+396\,a^{7}bx^{2}}{990\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^23,x, algorithm="maxima")
```

```
[Out] -1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x^
10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2
 + 45*a^8)/x^22
```

mupad [B] time = 0.08, size = 91, normalized size = 1.47

$$-\frac{\frac{a^8}{22}+\frac{2\,a^7\,b\,x^2}{5}+\frac{14\,a^6\,b^2\,x^4}{9}+\frac{7\,a^5\,b^3\,x^6}{2}+5\,a^4\,b^4\,x^8+\frac{14\,a^3\,b^5\,x^{10}}{3}+\frac{14\,a^2\,b^6\,x^{12}}{5}+a\,b^7\,x^{14}+\frac{b^8\,x^{16}}{6}}{x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^23,x)
```

```
[Out] -(a^8/22 + (b^8*x^16)/6 + (2*a^7*b*x^2)/5 + a*b^7*x^14 + (14*a^6*b^2*x^4)/9
 + (7*a^5*b^3*x^6)/2 + 5*a^4*b^4*x^8 + (14*a^3*b^5*x^10)/3 + (14*a^2*b^6*x^
12)/5)/x^22
```

sympy [A] time = 0.99, size = 99, normalized size = 1.60

$$\frac{-45a^{8}-396a^{7}bx^{2}-1540a^{6}b^{2}x^{4}-3465a^{5}b^{3}x^{6}-4950a^{4}b^{4}x^{8}-4620a^{3}b^{5}x^{10}-2772a^{2}b^{6}x^{12}-990ab^{7}x^{14}-165b^{8}}{990x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**23,x)
```

```
[Out] (-45*a**8 - 396*a**7*b*x**2 - 1540*a**6*b**2*x**4 - 3465*a**5*b**3*x**6 - 4
950*a**4*b**4*x**8 - 4620*a**3*b**5*x**10 - 2772*a**2*b**6*x**12 - 990*a*b*
*7*x**14 - 165*b**8*x**16)/(990*x**22)
```

3.104 $\int \frac{(a+bx^2)^8}{x^{25}}\,dx$

Optimal. Leaf size=84

$$\frac{b^3(a+bx^2)^9}{3960a^4x^{18}}-\frac{b^2(a+bx^2)^9}{440a^3x^{20}}+\frac{b(a+bx^2)^9}{88a^2x^{22}}-\frac{(a+bx^2)^9}{24ax^{24}}$$

```
[Out] -1/24*(b*x^2+a)^9/a/x^24+1/88*b*(b*x^2+a)^9/a^2/x^22-1/440*b^2*(b*x^2+a)^9/
a^3/x^20+1/3960*b^3*(b*x^2+a)^9/a^4/x^18
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{b^3(a+bx^2)^9}{3960a^4x^{18}}-\frac{b^2(a+bx^2)^9}{440a^3x^{20}}+\frac{b(a+bx^2)^9}{88a^2x^{22}}-\frac{(a+bx^2)^9}{24ax^{24}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^25,x]
```

```
[Out] -(a + b*x^2)^9/(24*a*x^24) + (b*(a + b*x^2)^9)/(88*a^2*x^22) - (b^2*(a + b*
x^2)^9)/(440*a^3*x^20) + (b^3*(a + b*x^2)^9)/(3960*a^4*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^8}{x^{25}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{13}}\,dx,x,x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^9}{24ax^{24}} - \frac{b\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx,x,x^2\right)}{8a}$$
$$= -\frac{\left(a+bx^2\right)^9}{24ax^{24}} + \frac{b\left(a+bx^2\right)^9}{88a^2x^{22}} + \frac{b^2\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx,x,x^2\right)}{44a^2}$$
$$= -\frac{\left(a+bx^2\right)^9}{24ax^{24}} + \frac{b\left(a+bx^2\right)^9}{88a^2x^{22}} - \frac{b^2\left(a+bx^2\right)^9}{440a^3x^{20}} - \frac{b^3\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx,x,x^2\right)}{440a^3}$$
$$= -\frac{\left(a+bx^2\right)^9}{24ax^{24}} + \frac{b\left(a+bx^2\right)^9}{88a^2x^{22}} - \frac{b^2\left(a+bx^2\right)^9}{440a^3x^{20}} + \frac{b^3\left(a+bx^2\right)^9}{3960a^4x^{18}}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.26

$$-\frac{a^8}{24x^{24}} - \frac{4a^7b}{11x^{22}} - \frac{7a^6b^2}{5x^{20}} - \frac{28a^5b^3}{9x^{18}} - \frac{35a^4b^4}{8x^{16}} - \frac{4a^3b^5}{x^{14}} - \frac{7a^2b^6}{3x^{12}} - \frac{4ab^7}{5x^{10}} - \frac{b^8}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^25,x]
```

```
[Out] -1/24*a^8/x^24 - (4*a^7*b)/(11*x^22) - (7*a^6*b^2)/(5*x^20) - (28*a^5*b^3)/
(9*x^18) - (35*a^4*b^4)/(8*x^16) - (4*a^3*b^5)/x^14 - (7*a^2*b^6)/(3*x^12)
- (4*a*b^7)/(5*x^10) - b^8/(8*x^8)
```

fricas [A] time = 0.88, size = 92, normalized size = 1.10

$$-\frac{495\,b^8x^{16} + 3168\,ab^7x^{14} + 9240\,a^2b^6x^{12} + 15840\,a^3b^5x^{10} + 17325\,a^4b^4x^8 + 12320\,a^5b^3x^6 + 5544\,a^6b^2x^4 + 1440}{3960\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^25,x, algorithm="fricas")
```

```
[Out] -1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^5
*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7
*b*x^2 + 165*a^8)/x^24
```

giac [A] time = 1.20, size = 92, normalized size = 1.10

$$-\frac{495\,b^8x^{16} + 3168\,ab^7x^{14} + 9240\,a^2b^6x^{12} + 15840\,a^3b^5x^{10} + 17325\,a^4b^4x^8 + 12320\,a^5b^3x^6 + 5544\,a^6b^2x^4 + 1440}{3960\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^25,x, algorithm="giac")
```

```
[Out] -1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^5
*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7
*b*x^2 + 165*a^8)/x^24
```

maple [A] time = 0.01, size = 91, normalized size = 1.08

$$-\frac{b^8}{8x^8} - \frac{4a\,b^7}{5x^{10}} - \frac{7a^2b^6}{3x^{12}} - \frac{4a^3b^5}{x^{14}} - \frac{35a^4b^4}{8x^{16}} - \frac{28a^5b^3}{9x^{18}} - \frac{7a^6b^2}{5x^{20}} - \frac{4a^7b}{11x^{22}} - \frac{a^8}{24x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^25,x)
```

```
[Out] -28/9*a^5*b^3/x^18-35/8*a^4*b^4/x^16-7/5*a^6*b^2/x^20-1/24*a^8/x^24-4/5*a*b
^7/x^10-7/3*a^2*b^6/x^12-1/8*b^8/x^8-4/11*a^7*b/x^22-4*a^3*b^5/x^14
```

maxima [A] time = 1.39, size = 92, normalized size = 1.10

$$-\frac{495\, b^{8} x^{16} + 3168\, a b^{7} x^{14} + 9240\, a^{2} b^{6} x^{12} + 15840\, a^{3} b^{5} x^{10} + 17325\, a^{4} b^{4} x^{8} + 12320\, a^{5} b^{3} x^{6} + 5544\, a^{6} b^{2} x^{4} + 14}{3960\, x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^25,x, algorithm="maxima")
```

```
[Out] -1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^5
*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7
*b*x^2 + 165*a^8)/x^24
```

mupad [B] time = 4.97, size = 92, normalized size = 1.10

$$-\frac{\frac{a^8}{24}+\frac{4\,a^7\,b\,x^2}{11}+\frac{7\,a^6\,b^2\,x^4}{5}+\frac{28\,a^5\,b^3\,x^6}{9}+\frac{35\,a^4\,b^4\,x^8}{8}+4\,a^3\,b^5\,x^{10}+\frac{7\,a^2\,b^6\,x^{12}}{3}+\frac{4\,a\,b^7\,x^{14}}{5}+\frac{b^8\,x^{16}}{8}}{x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^25,x)
```

```
[Out] -(a^8/24 + (b^8*x^16)/8 + (4*a^7*b*x^2)/11 + (4*a*b^7*x^14)/5 + (7*a^6*b^2*
x^4)/5 + (28*a^5*b^3*x^6)/9 + (35*a^4*b^4*x^8)/8 + 4*a^3*b^5*x^10 + (7*a^2*
b^6*x^12)/3)/x^24
```

sympy [A] time = 1.08, size = 99, normalized size = 1.18

$$\frac{-165a^{8} - 1440a^{7}bx^{2} - 5544a^{6}b^{2}x^{4} - 12320a^{5}b^{3}x^{6} - 17325a^{4}b^{4}x^{8} - 15840a^{3}b^{5}x^{10} - 9240a^{2}b^{6}x^{12} - 3168ab^{7}x^{1}}{3960x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**25,x)
```

```
[Out] (-165*a**8 - 1440*a**7*b*x**2 - 5544*a**6*b**2*x**4 - 12320*a**5*b**3*x**6
- 17325*a**4*b**4*x**8 - 15840*a**3*b**5*x**10 - 9240*a**2*b**6*x**12 - 316
8*a*b**7*x**14 - 495*b**8*x**16)/(3960*x**24)
```

3.105 $\int \frac{(a+bx^2)^8}{x^{27}} dx$

Optimal. Leaf size=106

$$-\frac{b^4(a+bx^2)^9}{12870a^5x^{18}}+\frac{b^3(a+bx^2)^9}{1430a^4x^{20}}-\frac{b^2(a+bx^2)^9}{286a^3x^{22}}+\frac{b(a+bx^2)^9}{78a^2x^{24}}-\frac{(a+bx^2)^9}{26ax^{26}}$$

```
[Out] -1/26*(b*x^2+a)^9/a/x^26+1/78*b*(b*x^2+a)^9/a^2/x^24-1/286*b^2*(b*x^2+a)^9/
a^3/x^22+1/1430*b^3*(b*x^2+a)^9/a^4/x^20-1/12870*b^4*(b*x^2+a)^9/a^5/x^18
```

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {266, 45, 37}

$$-\frac{b^4(a+bx^2)^9}{12870a^5x^{18}}+\frac{b^3(a+bx^2)^9}{1430a^4x^{20}}-\frac{b^2(a+bx^2)^9}{286a^3x^{22}}+\frac{b(a+bx^2)^9}{78a^2x^{24}}-\frac{(a+bx^2)^9}{26ax^{26}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^27,x]
```

```
[Out] -(a + b*x^2)^9/(26*a*x^26) + (b*(a + b*x^2)^9)/(78*a^2*x^24) - (b^2*(a + b*
x^2)^9)/(286*a^3*x^22) + (b^3*(a + b*x^2)^9)/(1430*a^4*x^20) - (b^4*(a + b*
x^2)^9)/(12870*a^5*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^8}{x^{27}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{14}}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}-\frac{(2b)\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{13}}\,dx,x,x^2\right)}{13a}\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}+\frac{b\left(a+bx^2\right)^9}{78a^2x^{24}}+\frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx,x,x^2\right)}{26a^2}\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}+\frac{b\left(a+bx^2\right)^9}{78a^2x^{24}}-\frac{b^2\left(a+bx^2\right)^9}{286a^3x^{22}}-\frac{b^3\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx,x,x^2\right)}{143a^3}\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}+\frac{b\left(a+bx^2\right)^9}{78a^2x^{24}}-\frac{b^2\left(a+bx^2\right)^9}{286a^3x^{22}}+\frac{b^3\left(a+bx^2\right)^9}{1430a^4x^{20}}+\frac{b^4\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx,x,x^2\right.}{1430a^4}\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}}+\frac{b\left(a+bx^2\right)^9}{78a^2x^{24}}-\frac{b^2\left(a+bx^2\right)^9}{286a^3x^{22}}+\frac{b^3\left(a+bx^2\right)^9}{1430a^4x^{20}}-\frac{b^4\left(a+bx^2\right)^9}{12870a^5x^{18}}
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.00

$$-\frac{a^8}{26x^{26}}-\frac{a^7b}{3x^{24}}-\frac{14a^6b^2}{11x^{22}}-\frac{14a^5b^3}{5x^{20}}-\frac{35a^4b^4}{9x^{18}}-\frac{7a^3b^5}{2x^{16}}-\frac{2a^2b^6}{x^{14}}-\frac{2ab^7}{3x^{12}}-\frac{b^8}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^27,x]
```

```
[Out] -1/26*a^8/x^26 - (a^7*b)/(3*x^24) - (14*a^6*b^2)/(11*x^22) - (14*a^5*b^3)/(
5*x^20) - (35*a^4*b^4)/(9*x^18) - (7*a^3*b^5)/(2*x^16) - (2*a^2*b^6)/x^14 -
 (2*a*b^7)/(3*x^12) - b^8/(10*x^10)
```

fricas [A] time = 0.89, size = 92, normalized size = 0.87

$$-\frac{1287\,b^8x^{16}+8580\,ab^7x^{14}+25740\,a^2b^6x^{12}+45045\,a^3b^5x^{10}+50050\,a^4b^4x^8+36036\,a^5b^3x^6+16380\,a^6b^2x^4}{12870\,x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^27,x, algorithm="fricas")
```

```
[Out] -1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3*
b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 4290
*a^7*b*x^2 + 495*a^8)/x^26
```

giac [A] time = 1.06, size = 92, normalized size = 0.87

$$-\frac{1287\,b^8x^{16}+8580\,ab^7x^{14}+25740\,a^2b^6x^{12}+45045\,a^3b^5x^{10}+50050\,a^4b^4x^8+36036\,a^5b^3x^6+16380\,a^6b^2x^4}{12870\,x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^27,x, algorithm="giac")
```

```
[Out] -1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3*
b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 4290
*a^7*b*x^2 + 495*a^8)/x^26
```

maple [A] time = 0.01, size = 91, normalized size = 0.86

$$-\frac{b^8}{10x^{10}}-\frac{2a\,b^7}{3x^{12}}-\frac{2a^2b^6}{x^{14}}-\frac{7a^3b^5}{2x^{16}}-\frac{35a^4b^4}{9x^{18}}-\frac{14a^5b^3}{5x^{20}}-\frac{14a^6b^2}{11x^{22}}-\frac{a^7b}{3x^{24}}-\frac{a^8}{26x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^27,x)
```

```
[Out] -1/3*a^7*b/x^24-35/9*a^4*b^4/x^18-14/5*a^5*b^3/x^20-1/26*a^8/x^26-14/11*a^6
*b^2/x^22-1/10*b^8/x^10-2/3*a*b^7/x^12-2*a^2*b^6/x^14-7/2*a^3*b^5/x^16
```

maxima [A] time = 1.30, size = 92, normalized size = 0.87

$$-\frac{1287\,b^8x^{16}+8580\,ab^7x^{14}+25740\,a^2b^6x^{12}+45045\,a^3b^5x^{10}+50050\,a^4b^4x^8+36036\,a^5b^3x^6+16380\,a^6b^2x^4+4}{12870\,x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^27,x, algorithm="maxima")
```

```
[Out] -1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3*
b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 4290
*a^7*b*x^2 + 495*a^8)/x^26
```

mupad [B] time = 0.08, size = 92, normalized size = 0.87

$$-\frac{\frac{a^8}{26}+\frac{a^7\,b\,x^2}{3}+\frac{14\,a^6\,b^2\,x^4}{11}+\frac{14\,a^5\,b^3\,x^6}{5}+\frac{35\,a^4\,b^4\,x^8}{9}+\frac{7\,a^3\,b^5\,x^{10}}{2}+2\,a^2\,b^6\,x^{12}+\frac{2\,a\,b^7\,x^{14}}{3}+\frac{b^8\,x^{16}}{10}}{x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^27,x)
```

```
[Out] -(a^8/26 + (b^8*x^16)/10 + (a^7*b*x^2)/3 + (2*a*b^7*x^14)/3 + (14*a^6*b^2*x
^4)/11 + (14*a^5*b^3*x^6)/5 + (35*a^4*b^4*x^8)/9 + (7*a^3*b^5*x^10)/2 + 2*a
^2*b^6*x^12)/x^26
```

sympy [A] time = 1.14, size = 99, normalized size = 0.93

$$\frac{-495a^8-4290a^7bx^2-16380a^6b^2x^4-36036a^5b^3x^6-50050a^4b^4x^8-45045a^3b^5x^{10}-25740a^2b^6x^{12}-8580ab^7x^1}{12870x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**27,x)
```

```
[Out] (-495*a**8 - 4290*a**7*b*x**2 - 16380*a**6*b**2*x**4 - 36036*a**5*b**3*x**6
 - 50050*a**4*b**4*x**8 - 45045*a**3*b**5*x**10 - 25740*a**2*b**6*x**12 - 8
580*a*b**7*x**14 - 1287*b**8*x**16)/(12870*x**26)
```

3.106 $\int \frac{(a+bx^2)^8}{x^{29}}\,dx$

Optimal. Leaf size=108

$$-\frac{a^8}{28x^{28}}-\frac{4a^7b}{13x^{26}}-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}$$

```
[Out] -1/28*a^8/x^28-4/13*a^7*b/x^26-7/6*a^6*b^2/x^24-28/11*a^5*b^3/x^22-7/2*a^4*
b^4/x^20-28/9*a^3*b^5/x^18-7/4*a^2*b^6/x^16-4/7*a*b^7/x^14-1/12*b^8/x^12
```

Rubi [A] time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4a^7b}{13x^{26}}-\frac{a^8}{28x^{28}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^29,x]
```

```
[Out] -a^8/(28*x^28) - (4*a^7*b)/(13*x^26) - (7*a^6*b^2)/(6*x^24) - (28*a^5*b^3)/
(11*x^22) - (7*a^4*b^4)/(2*x^20) - (28*a^3*b^5)/(9*x^18) - (7*a^2*b^6)/(4*x
^16) - (4*a*b^7)/(7*x^14) - b^8/(12*x^12)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^{29}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{15}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^8}{x^{15}}+\frac{8a^7b}{x^{14}}+\frac{28a^6b^2}{x^{13}}+\frac{56a^5b^3}{x^{12}}+\frac{70a^4b^4}{x^{11}}+\frac{56a^3b^5}{x^{10}}+\frac{28a^2b^6}{x^9}+\frac{8ab^7}{x^8}+\frac{b^8}{x^7}\right)\right.\\
&= -\frac{a^8}{28x^{28}}-\frac{4a^7b}{13x^{26}}-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 1.00

$$-\frac{a^8}{28x^{28}}-\frac{4a^7b}{13x^{26}}-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^29,x]
```

```
[Out] -1/28*a^8/x^28 - (4*a^7*b)/(13*x^26) - (7*a^6*b^2)/(6*x^24) - (28*a^5*b^3)/
(11*x^22) - (7*a^4*b^4)/(2*x^20) - (28*a^3*b^5)/(9*x^18) - (7*a^2*b^6)/(4*x
^16) - (4*a*b^7)/(7*x^14) - b^8/(12*x^12)
```

fricas [A] time = 0.88, size = 92, normalized size = 0.85

$$-\frac{3003\,b^8x^{16} + 20592\,ab^7x^{14} + 63063\,a^2b^6x^{12} + 112112\,a^3b^5x^{10} + 126126\,a^4b^4x^8 + 91728\,a^5b^3x^6 + 42042\,a^6b^2x^4}{36036\,x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^29,x, algorithm="fricas")
```

```
[Out] -1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a^
3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 + 1
1088*a^7*b*x^2 + 1287*a^8)/x^28
```

giac [A] time = 1.05, size = 92, normalized size = 0.85

$$-\frac{3003\,b^8x^{16} + 20592\,ab^7x^{14} + 63063\,a^2b^6x^{12} + 112112\,a^3b^5x^{10} + 126126\,a^4b^4x^8 + 91728\,a^5b^3x^6 + 42042\,a^6b^2x^4}{36036\,x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^29,x, algorithm="giac")
```

```
[Out] -1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a^
3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 + 1
1088*a^7*b*x^2 + 1287*a^8)/x^28
```

maple [A] time = 0.01, size = 91, normalized size = 0.84

$$-\frac{b^8}{12x^{12}} - \frac{4a\,b^7}{7x^{14}} - \frac{7a^2b^6}{4x^{16}} - \frac{28a^3b^5}{9x^{18}} - \frac{7a^4b^4}{2x^{20}} - \frac{28a^5b^3}{11x^{22}} - \frac{7a^6b^2}{6x^{24}} - \frac{4a^7b}{13x^{26}} - \frac{a^8}{28x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^29,x)
```

```
[Out] -1/28*a^8/x^28-4/13*a^7*b/x^26-7/6*a^6*b^2/x^24-28/11*a^5*b^3/x^22-7/2*a^4*
b^4/x^20-28/9*a^3*b^5/x^18-7/4*a^2*b^6/x^16-4/7*a*b^7/x^14-1/12*b^8/x^12
```

maxima [A] time = 1.41, size = 92, normalized size = 0.85

$$-\frac{3003\,b^8x^{16} + 20592\,ab^7x^{14} + 63063\,a^2b^6x^{12} + 112112\,a^3b^5x^{10} + 126126\,a^4b^4x^8 + 91728\,a^5b^3x^6 + 42042\,a^6b^2x^4}{36036\,x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^29,x, algorithm="maxima")
```

```
[Out] -1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a^
3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 + 1
1088*a^7*b*x^2 + 1287*a^8)/x^28
```

mupad [B] time = 4.89, size = 92, normalized size = 0.85

$$-\frac{\frac{a^8}{28} + \frac{4\,a^7\,b\,x^2}{13} + \frac{7\,a^6\,b^2\,x^4}{6} + \frac{28\,a^5\,b^3\,x^6}{11} + \frac{7\,a^4\,b^4\,x^8}{2} + \frac{28\,a^3\,b^5\,x^{10}}{9} + \frac{7\,a^2\,b^6\,x^{12}}{4} + \frac{4\,a\,b^7\,x^{14}}{7} + \frac{b^8\,x^{16}}{12}}{x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^29,x)
```

```
[Out] -(a^8/28 + (b^8*x^16)/12 + (4*a^7*b*x^2)/13 + (4*a*b^7*x^14)/7 + (7*a^6*b^2
*x^4)/6 + (28*a^5*b^3*x^6)/11 + (7*a^4*b^4*x^8)/2 + (28*a^3*b^5*x^10)/9 + (
7*a^2*b^6*x^12)/4)/x^28
```

sympy [A] time = 1.20, size = 99, normalized size = 0.92

$$\frac{-1287a^8 - 11088a^7bx^2 - 42042a^6b^2x^4 - 91728a^5b^3x^6 - 126126a^4b^4x^8 - 112112a^3b^5x^{10} - 63063a^2b^6x^{12} - 205}{36036x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**29,x)
```

```
[Out] (-1287*a**8 - 11088*a**7*b*x**2 - 42042*a**6*b**2*x**4 - 91728*a**5*b**3*x*
*6 - 126126*a**4*b**4*x**8 - 112112*a**3*b**5*x**10 - 63063*a**2*b**6*x**12
 - 20592*a*b**7*x**14 - 3003*b**8*x**16)/(36036*x**28)
```

3.107 $\int \frac{\left(a+bx^2\right)^8}{x^{31}}\,dx$

Optimal. Leaf size=108

$$-\frac{a^8}{30x^{30}}-\frac{2a^7b}{7x^{28}}-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}$$

```
[Out] -1/30*a^8/x^30-2/7*a^7*b/x^28-14/13*a^6*b^2/x^26-7/3*a^5*b^3/x^24-35/11*a^4
*b^4/x^22-14/5*a^3*b^5/x^20-14/9*a^2*b^6/x^18-1/2*a*b^7/x^16-1/14*b^8/x^14
```

Rubi [A] time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{2a^7b}{7x^{28}}-\frac{a^8}{30x^{30}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^31,x]
```

```
[Out] -a^8/(30*x^30) - (2*a^7*b)/(7*x^28) - (14*a^6*b^2)/(13*x^26) - (7*a^5*b^3)/
(3*x^24) - (35*a^4*b^4)/(11*x^22) - (14*a^3*b^5)/(5*x^20) - (14*a^2*b^6)/(9
*x^18) - (a*b^7)/(2*x^16) - b^8/(14*x^14)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^8}{x^{31}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{16}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^8}{x^{16}}+\frac{8a^7b}{x^{15}}+\frac{28a^6b^2}{x^{14}}+\frac{56a^5b^3}{x^{13}}+\frac{70a^4b^4}{x^{12}}+\frac{56a^3b^5}{x^{11}}+\frac{28a^2b^6}{x^{10}}+\frac{8ab^7}{x^9}+\frac{b^8}{x^8}\right) d
\\
&= -\frac{a^8}{30x^{30}}-\frac{2a^7b}{7x^{28}}-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 1.00

$$-\frac{a^8}{30x^{30}}-\frac{2a^7b}{7x^{28}}-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^31,x]
```

```
[Out] -1/30*a^8/x^30 - (2*a^7*b)/(7*x^28) - (14*a^6*b^2)/(13*x^26) - (7*a^5*b^3)/
(3*x^24) - (35*a^4*b^4)/(11*x^22) - (14*a^3*b^5)/(5*x^20) - (14*a^2*b^6)/(9
*x^18) - (a*b^7)/(2*x^16) - b^8/(14*x^14)
```

fricas [A] time = 0.77, size = 92, normalized size = 0.85

$$-\frac{6435\,b^8x^{16} + 45045\,ab^7x^{14} + 140140\,a^2b^6x^{12} + 252252\,a^3b^5x^{10} + 286650\,a^4b^4x^8 + 210210\,a^5b^3x^6 + 97020\,a}{90090\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^31,x, algorithm="fricas")
```

```
[Out] -1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*a
^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4 +
 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

giac [A] time = 1.22, size = 92, normalized size = 0.85

$$-\frac{6435\,b^8x^{16} + 45045\,ab^7x^{14} + 140140\,a^2b^6x^{12} + 252252\,a^3b^5x^{10} + 286650\,a^4b^4x^8 + 210210\,a^5b^3x^6 + 97020\,a}{90090\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^31,x, algorithm="giac")
```

```
[Out] -1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*a
^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4 +
 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

maple [A] time = 0.01, size = 91, normalized size = 0.84

$$-\frac{b^8}{14x^{14}} - \frac{a\,b^7}{2x^{16}} - \frac{14a^2b^6}{9x^{18}} - \frac{14a^3b^5}{5x^{20}} - \frac{35a^4b^4}{11x^{22}} - \frac{7a^5b^3}{3x^{24}} - \frac{14a^6b^2}{13x^{26}} - \frac{2a^7b}{7x^{28}} - \frac{a^8}{30x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^31,x)
```

```
[Out] -1/30*a^8/x^30-2/7*a^7*b/x^28-14/13*a^6*b^2/x^26-7/3*a^5*b^3/x^24-35/11*a^4
*b^4/x^22-14/5*a^3*b^5/x^20-14/9*a^2*b^6/x^18-1/2*a*b^7/x^16-1/14*b^8/x^14
```

maxima [A] time = 1.31, size = 92, normalized size = 0.85

$$-\frac{6435\,b^8x^{16} + 45045\,ab^7x^{14} + 140140\,a^2b^6x^{12} + 252252\,a^3b^5x^{10} + 286650\,a^4b^4x^8 + 210210\,a^5b^3x^6 + 97020\,a}{90090\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^31,x, algorithm="maxima")
```

```
[Out] -1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*a
^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4 +
 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

mupad [B] time = 4.86, size = 92, normalized size = 0.85

$$-\frac{\frac{a^8}{30} + \frac{2\,a^7\,b\,x^2}{7} + \frac{14\,a^6\,b^2\,x^4}{13} + \frac{7\,a^5\,b^3\,x^6}{3} + \frac{35\,a^4\,b^4\,x^8}{11} + \frac{14\,a^3\,b^5\,x^{10}}{5} + \frac{14\,a^2\,b^6\,x^{12}}{9} + \frac{a\,b^7\,x^{14}}{2} + \frac{b^8\,x^{16}}{14}}{x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^31,x)
```

```
[Out] -(a^8/30 + (b^8*x^16)/14 + (2*a^7*b*x^2)/7 + (a*b^7*x^14)/2 + (14*a^6*b^2*x
^4)/13 + (7*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/11 + (14*a^3*b^5*x^10)/5 + (1
4*a^2*b^6*x^12)/9)/x^30
```

sympy [**A**] time = 1.28, size = 99, normalized size = 0.92

$$\frac{-3003a^8 - 25740a^7bx^2 - 97020a^6b^2x^4 - 210210a^5b^3x^6 - 286650a^4b^4x^8 - 252252a^3b^5x^{10} - 140140a^2b^6x^{12} - 450}{90090x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**31,x)
```

```
[Out] (-3003*a**8 - 25740*a**7*b*x**2 - 97020*a**6*b**2*x**4 - 210210*a**5*b**3*x
**6 - 286650*a**4*b**4*x**8 - 252252*a**3*b**5*x**10 - 140140*a**2*b**6*x**
12 - 45045*a*b**7*x**14 - 6435*b**8*x**16)/(90090*x**30)
```

3.108 $\int \frac{(a+bx^2)^8}{x^{33}} \, dx$

Optimal. Leaf size=106

$$-\frac{a^8}{32x^{32}}-\frac{4a^7b}{15x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$$

```
[Out] -1/32*a^8/x^32-4/15*a^7*b/x^30-a^6*b^2/x^28-28/13*a^5*b^3/x^26-35/12*a^4*b^
4/x^24-28/11*a^3*b^5/x^22-7/5*a^2*b^6/x^20-4/9*a*b^7/x^18-1/16*b^8/x^16
```

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4a^7b}{15x^{30}}-\frac{a^8}{32x^{32}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^33,x]
```

```
[Out] -a^8/(32*x^32) - (4*a^7*b)/(15*x^30) - (a^6*b^2)/x^28 - (28*a^5*b^3)/(13*x^
26) - (35*a^4*b^4)/(12*x^24) - (28*a^3*b^5)/(11*x^22) - (7*a^2*b^6)/(5*x^20
) - (4*a*b^7)/(9*x^18) - b^8/(16*x^16)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^{33}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{17}} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^8}{x^{17}}+\frac{8a^7b}{x^{16}}+\frac{28a^6b^2}{x^{15}}+\frac{56a^5b^3}{x^{14}}+\frac{70a^4b^4}{x^{13}}+\frac{56a^3b^5}{x^{12}}+\frac{28a^2b^6}{x^{11}}+\frac{8ab^7}{x^{10}}+\frac{b^8}{x^9}\right)\right.\\
&= -\frac{a^8}{32x^{32}}-\frac{4a^7b}{15x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.00

$$-\frac{a^8}{32x^{32}}-\frac{4a^7b}{15x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^33,x]
```

```
[Out] -1/32*a^8/x^32 - (4*a^7*b)/(15*x^30) - (a^6*b^2)/x^28 - (28*a^5*b^3)/(13*x^
26) - (35*a^4*b^4)/(12*x^24) - (28*a^3*b^5)/(11*x^22) - (7*a^2*b^6)/(5*x^20
) - (4*a*b^7)/(9*x^18) - b^8/(16*x^16)
```

fricas [**A**] time = 0.60, size = 92, normalized size = 0.87

$$-\frac{12870\,b^{8}x^{16}+91520\,ab^{7}x^{14}+288288\,a^{2}b^{6}x^{12}+524160\,a^{3}b^{5}x^{10}+600600\,a^{4}b^{4}x^{8}+443520\,a^{5}b^{3}x^{6}+205920\,a^{6}}{205920\,x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^33,x, algorithm="fricas")
```

```
[Out] -1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160
*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^
4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

giac [**A**] time = 1.05, size = 92, normalized size = 0.87

$$-\frac{12870\,b^{8}x^{16}+91520\,ab^{7}x^{14}+288288\,a^{2}b^{6}x^{12}+524160\,a^{3}b^{5}x^{10}+600600\,a^{4}b^{4}x^{8}+443520\,a^{5}b^{3}x^{6}+205920\,a^{6}}{205920\,x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^33,x, algorithm="giac")
```

```
[Out] -1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160
*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^
4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

maple [**A**] time = 0.01, size = 91, normalized size = 0.86

$$-\frac{b^{8}}{16x^{16}}-\frac{4a\,b^{7}}{9x^{18}}-\frac{7a^{2}b^{6}}{5x^{20}}-\frac{28a^{3}b^{5}}{11x^{22}}-\frac{35a^{4}b^{4}}{12x^{24}}-\frac{28a^{5}b^{3}}{13x^{26}}-\frac{a^{6}b^{2}}{x^{28}}-\frac{4a^{7}b}{15x^{30}}-\frac{a^{8}}{32x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^33,x)
```

```
[Out] -1/32*a^8/x^32-4/15*a^7*b/x^30-a^6*b^2/x^28-28/13*a^5*b^3/x^26-35/12*a^4*b^
4/x^24-28/11*a^3*b^5/x^22-7/5*a^2*b^6/x^20-4/9*a*b^7/x^18-1/16*b^8/x^16
```

maxima [**A**] time = 1.34, size = 92, normalized size = 0.87

$$-\frac{12870\,b^{8}x^{16}+91520\,ab^{7}x^{14}+288288\,a^{2}b^{6}x^{12}+524160\,a^{3}b^{5}x^{10}+600600\,a^{4}b^{4}x^{8}+443520\,a^{5}b^{3}x^{6}+205920\,a^{6}}{205920\,x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^33,x, algorithm="maxima")
```

```
[Out] -1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160
*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^
4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

mupad [**B**] time = 0.08, size = 91, normalized size = 0.86

$$-\frac{\frac{a^8}{32}+\frac{4\,a^7\,b\,x^2}{15}+a^6\,b^2\,x^4+\frac{28\,a^5\,b^3\,x^6}{13}+\frac{35\,a^4\,b^4\,x^8}{12}+\frac{28\,a^3\,b^5\,x^{10}}{11}+\frac{7\,a^2\,b^6\,x^{12}}{5}+\frac{4\,a\,b^7\,x^{14}}{9}+\frac{b^8\,x^{16}}{16}}{x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^33,x)
```

```
[Out] -(a^8/32 + (b^8*x^16)/16 + (4*a^7*b*x^2)/15 + (4*a*b^7*x^14)/9 + a^6*b^2*x^
4 + (28*a^5*b^3*x^6)/13 + (35*a^4*b^4*x^8)/12 + (28*a^3*b^5*x^10)/11 + (7*a
^2*b^6*x^12)/5)/x^32
```

sympy [A] time = 1.34, size = 99, normalized size = 0.93

$$\frac{-6435a^8 - 54912a^7bx^2 - 205920a^6b^2x^4 - 443520a^5b^3x^6 - 600600a^4b^4x^8 - 524160a^3b^5x^{10} - 288288a^2b^6x^{12} -}{205920x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**33,x)
```

```
[Out] (-6435*a**8 - 54912*a**7*b*x**2 - 205920*a**6*b**2*x**4 - 443520*a**5*b**3*
x**6 - 600600*a**4*b**4*x**8 - 524160*a**3*b**5*x**10 - 288288*a**2*b**6*x*
*12 - 91520*a*b**7*x**14 - 12870*b**8*x**16)/(205920*x**32)
```

3.109 $\int x^8 \left(a+bx^2\right)^8 dx$

Optimal. Leaf size=108

$$\frac{a^8x^9}{9}+\frac{8}{11}a^7bx^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

```
[Out] 1/9*a^8*x^9+8/11*a^7*b*x^11+28/13*a^6*b^2*x^13+56/15*a^5*b^3*x^15+70/17*a^4
*b^4*x^17+56/19*a^3*b^5*x^19+4/3*a^2*b^6*x^21+8/23*a*b^7*x^23+1/25*b^8*x^25
```

Rubi [A] time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{4}{3}a^2b^6x^{21}+\frac{56}{19}a^3b^5x^{19}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{15}a^5b^3x^{15}+\frac{28}{13}a^6b^2x^{13}+\frac{8}{11}a^7bx^{11}+\frac{a^8x^9}{9}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^9)/9 + (8*a^7*b*x^11)/11 + (28*a^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)/
15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b^5*x^19)/19 + (4*a^2*b^6*x^21)/3 + (8*
a*b^7*x^23)/23 + (b^8*x^25)/25
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^8 \left(a+bx^2\right)^8 dx &= \int \left(a^8x^8+8a^7bx^{10}+28a^6b^2x^{12}+56a^5b^3x^{14}+70a^4b^4x^{16}+56a^3b^5x^{18}+28a^2b^6x^{20}+8ab^7\right.\\ &= \frac{a^8x^9}{9}+\frac{8}{11}a^7bx^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}ab\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 1.00

$$\frac{a^8x^9}{9}+\frac{8}{11}a^7bx^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^9)/9 + (8*a^7*b*x^11)/11 + (28*a^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)/
15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b^5*x^19)/19 + (4*a^2*b^6*x^21)/3 + (8*
a*b^7*x^23)/23 + (b^8*x^25)/25
```

fricas [A] time = 0.65, size = 90, normalized size = 0.83

$$\frac{1}{25}x^{25}b^8+\frac{8}{23}x^{23}b^7a+\frac{4}{3}x^{21}b^6a^2+\frac{56}{19}x^{19}b^5a^3+\frac{70}{17}x^{17}b^4a^4+\frac{56}{15}x^{15}b^3a^5+\frac{28}{13}x^{13}b^2a^6+\frac{8}{11}x^{11}ba^7+\frac{1}{9}x^9a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*b^8 + 8/23*x^23*b^7*a + 4/3*x^21*b^6*a^2 + 56/19*x^19*b^5*a^3 + 7
0/17*x^17*b^4*a^4 + 56/15*x^15*b^3*a^5 + 28/13*x^13*b^2*a^6 + 8/11*x^11*b*a
^7 + 1/9*x^9*a^8
```

giac [A] time = 1.10, size = 90, normalized size = 0.83

$$\frac{1}{25}\,b^8x^{25}+\frac{8}{23}\,ab^7x^{23}+\frac{4}{3}\,a^2b^6x^{21}+\frac{56}{19}\,a^3b^5x^{19}+\frac{70}{17}\,a^4b^4x^{17}+\frac{56}{15}\,a^5b^3x^{15}+\frac{28}{13}\,a^6b^2x^{13}+\frac{8}{11}\,a^7bx^{11}+\frac{1}{9}\,a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/25*b^8*x^25 + 8/23*a*b^7*x^23 + 4/3*a^2*b^6*x^21 + 56/19*a^3*b^5*x^19 + 7
0/17*a^4*b^4*x^17 + 56/15*a^5*b^3*x^15 + 28/13*a^6*b^2*x^13 + 8/11*a^7*b*x^
11 + 1/9*a^8*x^9
```

maple [A] time = 0.00, size = 91, normalized size = 0.84

$$\frac{1}{25}b^8x^{25}+\frac{8}{23}a\,b^7x^{23}+\frac{4}{3}a^2b^6x^{21}+\frac{56}{19}a^3b^5x^{19}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{15}a^5b^3x^{15}+\frac{28}{13}a^6b^2x^{13}+\frac{8}{11}a^7b\,x^{11}+\frac{1}{9}a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^2+a)^8,x)
```

```
[Out] 1/9*a^8*x^9+8/11*a^7*b*x^11+28/13*a^6*b^2*x^13+56/15*a^5*b^3*x^15+70/17*a^4
*b^4*x^17+56/19*a^3*b^5*x^19+4/3*a^2*b^6*x^21+8/23*a*b^7*x^23+1/25*b^8*x^25
```

maxima [A] time = 1.36, size = 90, normalized size = 0.83

$$\frac{1}{25}\,b^8x^{25}+\frac{8}{23}\,ab^7x^{23}+\frac{4}{3}\,a^2b^6x^{21}+\frac{56}{19}\,a^3b^5x^{19}+\frac{70}{17}\,a^4b^4x^{17}+\frac{56}{15}\,a^5b^3x^{15}+\frac{28}{13}\,a^6b^2x^{13}+\frac{8}{11}\,a^7bx^{11}+\frac{1}{9}\,a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/25*b^8*x^25 + 8/23*a*b^7*x^23 + 4/3*a^2*b^6*x^21 + 56/19*a^3*b^5*x^19 + 7
0/17*a^4*b^4*x^17 + 56/15*a^5*b^3*x^15 + 28/13*a^6*b^2*x^13 + 8/11*a^7*b*x^
11 + 1/9*a^8*x^9
```

mupad [B] time = 4.94, size = 90, normalized size = 0.83

$$\frac{a^8\,x^9}{9}+\frac{8\,a^7\,b\,x^{11}}{11}+\frac{28\,a^6\,b^2\,x^{13}}{13}+\frac{56\,a^5\,b^3\,x^{15}}{15}+\frac{70\,a^4\,b^4\,x^{17}}{17}+\frac{56\,a^3\,b^5\,x^{19}}{19}+\frac{4\,a^2\,b^6\,x^{21}}{3}+\frac{8\,a\,b^7\,x^{23}}{23}+\frac{b^8\,x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^9)/9 + (b^8*x^25)/25 + (8*a^7*b*x^11)/11 + (8*a*b^7*x^23)/23 + (28*a
^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)/15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b^5
*x^19)/19 + (4*a^2*b^6*x^21)/3
```

sympy [A] time = 0.09, size = 107, normalized size = 0.99

$$\frac{a^8x^9}{9}+\frac{8a^7bx^{11}}{11}+\frac{28a^6b^2x^{13}}{13}+\frac{56a^5b^3x^{15}}{15}+\frac{70a^4b^4x^{17}}{17}+\frac{56a^3b^5x^{19}}{19}+\frac{4a^2b^6x^{21}}{3}+\frac{8ab^7x^{23}}{23}+\frac{b^8x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**9/9 + 8*a**7*b*x**11/11 + 28*a**6*b**2*x**13/13 + 56*a**5*b**3*x**1
5/15 + 70*a**4*b**4*x**17/17 + 56*a**3*b**5*x**19/19 + 4*a**2*b**6*x**21/3
+ 8*a*b**7*x**23/23 + b**8*x**25/25
```

3.110 $\int x^6 \left(a+bx^2\right)^8 dx$

Optimal. Leaf size=108

$$\frac{a^8x^7}{7}+\frac{8}{9}a^7bx^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

```
[Out] 1/7*a^8*x^7+8/9*a^7*b*x^9+28/11*a^6*b^2*x^11+56/13*a^5*b^3*x^13+14/3*a^4*b^
4*x^15+56/17*a^3*b^5*x^17+28/19*a^2*b^6*x^19+8/21*a*b^7*x^21+1/23*b^8*x^23
```

Rubi [A] time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{19}a^2b^6x^{19}+\frac{56}{17}a^3b^5x^{17}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{13}a^5b^3x^{13}+\frac{28}{11}a^6b^2x^{11}+\frac{8}{9}a^7bx^9+\frac{a^8x^7}{7}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^7)/7 + (8*a^7*b*x^9)/9 + (28*a^6*b^2*x^11)/11 + (56*a^5*b^3*x^13)/13
 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*x^17)/17 + (28*a^2*b^6*x^19)/19 + (8*a
*b^7*x^21)/21 + (b^8*x^23)/23
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6 \left(a+bx^2\right)^8 dx &= \int \left(a^8x^6+8a^7bx^8+28a^6b^2x^{10}+56a^5b^3x^{12}+70a^4b^4x^{14}+56a^3b^5x^{16}+28a^2b^6x^{18}+8ab^7x\right.\\ &= \frac{a^8x^7}{7}+\frac{8}{9}a^7bx^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}ab^7\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 1.00

$$\frac{a^8x^7}{7}+\frac{8}{9}a^7bx^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^7)/7 + (8*a^7*b*x^9)/9 + (28*a^6*b^2*x^11)/11 + (56*a^5*b^3*x^13)/13
 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*x^17)/17 + (28*a^2*b^6*x^19)/19 + (8*a
*b^7*x^21)/21 + (b^8*x^23)/23
```

fricas [A] time = 0.79, size = 90, normalized size = 0.83

$$\frac{1}{23}x^{23}b^8+\frac{8}{21}x^{21}b^7a+\frac{28}{19}x^{19}b^6a^2+\frac{56}{17}x^{17}b^5a^3+\frac{14}{3}x^{15}b^4a^4+\frac{56}{13}x^{13}b^3a^5+\frac{28}{11}x^{11}b^2a^6+\frac{8}{9}x^9ba^7+\frac{1}{7}x^7a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*b^8 + 8/21*x^21*b^7*a + 28/19*x^19*b^6*a^2 + 56/17*x^17*b^5*a^3 +
 14/3*x^15*b^4*a^4 + 56/13*x^13*b^3*a^5 + 28/11*x^11*b^2*a^6 + 8/9*x^9*b*a^
7 + 1/7*x^7*a^8
```

giac [A] time = 1.14, size = 90, normalized size = 0.83

$$\frac{1}{23}\,b^8x^{23}+\frac{8}{21}\,ab^7x^{21}+\frac{28}{19}\,a^2b^6x^{19}+\frac{56}{17}\,a^3b^5x^{17}+\frac{14}{3}\,a^4b^4x^{15}+\frac{56}{13}\,a^5b^3x^{13}+\frac{28}{11}\,a^6b^2x^{11}+\frac{8}{9}\,a^7bx^9+\frac{1}{7}\,a^8x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/23*b^8*x^23 + 8/21*a*b^7*x^21 + 28/19*a^2*b^6*x^19 + 56/17*a^3*b^5*x^17 +
 14/3*a^4*b^4*x^15 + 56/13*a^5*b^3*x^13 + 28/11*a^6*b^2*x^11 + 8/9*a^7*b*x^
9 + 1/7*a^8*x^7
```

maple [A] time = 0.00, size = 91, normalized size = 0.84

$$\frac{1}{23}b^8x^{23}+\frac{8}{21}a\,b^7x^{21}+\frac{28}{19}a^2b^6x^{19}+\frac{56}{17}a^3b^5x^{17}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{13}a^5b^3x^{13}+\frac{28}{11}a^6b^2x^{11}+\frac{8}{9}a^7b\,x^9+\frac{1}{7}a^8x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^8,x)
```

```
[Out] 1/7*a^8*x^7+8/9*a^7*b*x^9+28/11*a^6*b^2*x^11+56/13*a^5*b^3*x^13+14/3*a^4*b^
4*x^15+56/17*a^3*b^5*x^17+28/19*a^2*b^6*x^19+8/21*a*b^7*x^21+1/23*b^8*x^23
```

maxima [A] time = 1.32, size = 90, normalized size = 0.83

$$\frac{1}{23}\,b^8x^{23}+\frac{8}{21}\,ab^7x^{21}+\frac{28}{19}\,a^2b^6x^{19}+\frac{56}{17}\,a^3b^5x^{17}+\frac{14}{3}\,a^4b^4x^{15}+\frac{56}{13}\,a^5b^3x^{13}+\frac{28}{11}\,a^6b^2x^{11}+\frac{8}{9}\,a^7bx^9+\frac{1}{7}\,a^8x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/23*b^8*x^23 + 8/21*a*b^7*x^21 + 28/19*a^2*b^6*x^19 + 56/17*a^3*b^5*x^17 +
 14/3*a^4*b^4*x^15 + 56/13*a^5*b^3*x^13 + 28/11*a^6*b^2*x^11 + 8/9*a^7*b*x^
9 + 1/7*a^8*x^7
```

mupad [B] time = 0.10, size = 90, normalized size = 0.83

$$\frac{a^8\,x^7}{7}+\frac{8\,a^7\,b\,x^9}{9}+\frac{28\,a^6\,b^2\,x^{11}}{11}+\frac{56\,a^5\,b^3\,x^{13}}{13}+\frac{14\,a^4\,b^4\,x^{15}}{3}+\frac{56\,a^3\,b^5\,x^{17}}{17}+\frac{28\,a^2\,b^6\,x^{19}}{19}+\frac{8\,a\,b^7\,x^{21}}{21}+\frac{b^8\,x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^7)/7 + (b^8*x^23)/23 + (8*a^7*b*x^9)/9 + (8*a*b^7*x^21)/21 + (28*a^6
*b^2*x^11)/11 + (56*a^5*b^3*x^13)/13 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*x^
17)/17 + (28*a^2*b^6*x^19)/19
```

sympy [A] time = 0.09, size = 107, normalized size = 0.99

$$\frac{a^8x^7}{7}+\frac{8a^7bx^9}{9}+\frac{28a^6b^2x^{11}}{11}+\frac{56a^5b^3x^{13}}{13}+\frac{14a^4b^4x^{15}}{3}+\frac{56a^3b^5x^{17}}{17}+\frac{28a^2b^6x^{19}}{19}+\frac{8ab^7x^{21}}{21}+\frac{b^8x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**7/7 + 8*a**7*b*x**9/9 + 28*a**6*b**2*x**11/11 + 56*a**5*b**3*x**13/
13 + 14*a**4*b**4*x**15/3 + 56*a**3*b**5*x**17/17 + 28*a**2*b**6*x**19/19 +
 8*a*b**7*x**21/21 + b**8*x**23/23
```

3.111 $\int x^4 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=108

$$\frac{a^8x^5}{5}+\frac{8}{7}a^7bx^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

```
[Out] 1/5*a^8*x^5+8/7*a^7*b*x^7+28/9*a^6*b^2*x^9+56/11*a^5*b^3*x^11+70/13*a^4*b^4
*x^13+56/15*a^3*b^5*x^15+28/17*a^2*b^6*x^17+8/19*a*b^7*x^19+1/21*b^8*x^21
```

Rubi [A] time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{17}a^2b^6x^{17}+\frac{56}{15}a^3b^5x^{15}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{11}a^5b^3x^{11}+\frac{28}{9}a^6b^2x^9+\frac{8}{7}a^7bx^7+\frac{a^8x^5}{5}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^5)/5 + (8*a^7*b*x^7)/7 + (28*a^6*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11 +
 (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x^15)/15 + (28*a^2*b^6*x^17)/17 + (8*a*
b^7*x^19)/19 + (b^8*x^21)/21
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^8\,dx &= \int \left(a^8x^4+8a^7bx^6+28a^6b^2x^8+56a^5b^3x^{10}+70a^4b^4x^{12}+56a^3b^5x^{14}+28a^2b^6x^{16}+8ab^7x^{1}\right.\\ &= \frac{a^8x^5}{5}+\frac{8}{7}a^7bx^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}ab^7x\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 1.00

$$\frac{a^8x^5}{5}+\frac{8}{7}a^7bx^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^5)/5 + (8*a^7*b*x^7)/7 + (28*a^6*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11 +
 (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x^15)/15 + (28*a^2*b^6*x^17)/17 + (8*a*
b^7*x^19)/19 + (b^8*x^21)/21
```

fricas [A] time = 0.80, size = 90, normalized size = 0.83

$$\frac{1}{21}x^{21}b^8+\frac{8}{19}x^{19}b^7a+\frac{28}{17}x^{17}b^6a^2+\frac{56}{15}x^{15}b^5a^3+\frac{70}{13}x^{13}b^4a^4+\frac{56}{11}x^{11}b^3a^5+\frac{28}{9}x^9b^2a^6+\frac{8}{7}x^7ba^7+\frac{1}{5}x^5a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^8 + 8/19*x^19*b^7*a + 28/17*x^17*b^6*a^2 + 56/15*x^15*b^5*a^3 +
 70/13*x^13*b^4*a^4 + 56/11*x^11*b^3*a^5 + 28/9*x^9*b^2*a^6 + 8/7*x^7*b*a^7
 + 1/5*x^5*a^8
```

giac [A] time = 1.10, size = 90, normalized size = 0.83

$$\frac{1}{21}\,b^8x^{21}+\frac{8}{19}\,ab^7x^{19}+\frac{28}{17}\,a^2b^6x^{17}+\frac{56}{15}\,a^3b^5x^{15}+\frac{70}{13}\,a^4b^4x^{13}+\frac{56}{11}\,a^5b^3x^{11}+\frac{28}{9}\,a^6b^2x^9+\frac{8}{7}\,a^7bx^7+\frac{1}{5}\,a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/21*b^8*x^21 + 8/19*a*b^7*x^19 + 28/17*a^2*b^6*x^17 + 56/15*a^3*b^5*x^15 +
 70/13*a^4*b^4*x^13 + 56/11*a^5*b^3*x^11 + 28/9*a^6*b^2*x^9 + 8/7*a^7*b*x^7
 + 1/5*a^8*x^5
```

maple [A] time = 0.00, size = 91, normalized size = 0.84

$$\frac{1}{21}b^8x^{21}+\frac{8}{19}a\,b^7x^{19}+\frac{28}{17}a^2b^6x^{17}+\frac{56}{15}a^3b^5x^{15}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{11}a^5b^3x^{11}+\frac{28}{9}a^6b^2x^9+\frac{8}{7}a^7b\,x^7+\frac{1}{5}a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^8,x)
```

```
[Out] 1/5*a^8*x^5+8/7*a^7*b*x^7+28/9*a^6*b^2*x^9+56/11*a^5*b^3*x^11+70/13*a^4*b^4
*x^13+56/15*a^3*b^5*x^15+28/17*a^2*b^6*x^17+8/19*a*b^7*x^19+1/21*b^8*x^21
```

maxima [A] time = 1.34, size = 90, normalized size = 0.83

$$\frac{1}{21}\,b^8x^{21}+\frac{8}{19}\,ab^7x^{19}+\frac{28}{17}\,a^2b^6x^{17}+\frac{56}{15}\,a^3b^5x^{15}+\frac{70}{13}\,a^4b^4x^{13}+\frac{56}{11}\,a^5b^3x^{11}+\frac{28}{9}\,a^6b^2x^9+\frac{8}{7}\,a^7bx^7+\frac{1}{5}\,a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/21*b^8*x^21 + 8/19*a*b^7*x^19 + 28/17*a^2*b^6*x^17 + 56/15*a^3*b^5*x^15 +
 70/13*a^4*b^4*x^13 + 56/11*a^5*b^3*x^11 + 28/9*a^6*b^2*x^9 + 8/7*a^7*b*x^7
 + 1/5*a^8*x^5
```

mupad [B] time = 0.10, size = 90, normalized size = 0.83

$$\frac{a^8\,x^5}{5}+\frac{8\,a^7\,b\,x^7}{7}+\frac{28\,a^6\,b^2\,x^9}{9}+\frac{56\,a^5\,b^3\,x^{11}}{11}+\frac{70\,a^4\,b^4\,x^{13}}{13}+\frac{56\,a^3\,b^5\,x^{15}}{15}+\frac{28\,a^2\,b^6\,x^{17}}{17}+\frac{8\,a\,b^7\,x^{19}}{19}+\frac{b^8\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^5)/5 + (b^8*x^21)/21 + (8*a^7*b*x^7)/7 + (8*a*b^7*x^19)/19 + (28*a^6
*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11 + (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x^1
5)/15 + (28*a^2*b^6*x^17)/17
```

sympy [A] time = 0.09, size = 107, normalized size = 0.99

$$\frac{a^8x^5}{5}+\frac{8a^7bx^7}{7}+\frac{28a^6b^2x^9}{9}+\frac{56a^5b^3x^{11}}{11}+\frac{70a^4b^4x^{13}}{13}+\frac{56a^3b^5x^{15}}{15}+\frac{28a^2b^6x^{17}}{17}+\frac{8ab^7x^{19}}{19}+\frac{b^8x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**5/5 + 8*a**7*b*x**7/7 + 28*a**6*b**2*x**9/9 + 56*a**5*b**3*x**11/11
 + 70*a**4*b**4*x**13/13 + 56*a**3*b**5*x**15/15 + 28*a**2*b**6*x**17/17 +
8*a*b**7*x**19/19 + b**8*x**21/21
```

3.112 $\int x^2 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=106

$$\frac{a^8x^3}{3}+\frac{8}{5}a^7bx^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

```
[Out] 1/3*a^8*x^3+8/5*a^7*b*x^5+4*a^6*b^2*x^7+56/9*a^5*b^3*x^9+70/11*a^4*b^4*x^11
+56/13*a^3*b^5*x^13+28/15*a^2*b^6*x^15+8/17*a*b^7*x^17+1/19*b^8*x^19
```

Rubi [A] time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{15}a^2b^6x^{15}+\frac{56}{13}a^3b^5x^{13}+\frac{70}{11}a^4b^4x^{11}+\frac{56}{9}a^5b^3x^9+4a^6b^2x^7+\frac{8}{5}a^7bx^5+\frac{a^8x^3}{3}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^3)/3 + (8*a^7*b*x^5)/5 + 4*a^6*b^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a^
4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13 + (28*a^2*b^6*x^15)/15 + (8*a*b^7*x^1
7)/17 + (b^8*x^19)/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^2\right)^8 \, dx &= \int \left(a^8x^2+8a^7bx^4+28a^6b^2x^6+56a^5b^3x^8+70a^4b^4x^{10}+56a^3b^5x^{12}+28a^2b^6x^{14}+8ab^7x^{16}\right.\\ &= \frac{a^8x^3}{3}+\frac{8}{5}a^7bx^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}ab^7x^{17}\end{aligned}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.00

$$\frac{a^8x^3}{3}+\frac{8}{5}a^7bx^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^3)/3 + (8*a^7*b*x^5)/5 + 4*a^6*b^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a^
4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13 + (28*a^2*b^6*x^15)/15 + (8*a*b^7*x^1
7)/17 + (b^8*x^19)/19
```

fricas [A] time = 0.59, size = 90, normalized size = 0.85

$$\frac{1}{19}x^{19}b^8+\frac{8}{17}x^{17}b^7a+\frac{28}{15}x^{15}b^6a^2+\frac{56}{13}x^{13}b^5a^3+\frac{70}{11}x^{11}b^4a^4+\frac{56}{9}x^9b^3a^5+4x^7b^2a^6+\frac{8}{5}x^5ba^7+\frac{1}{3}x^3a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^8 + 8/17*x^17*b^7*a + 28/15*x^15*b^6*a^2 + 56/13*x^13*b^5*a^3 +
 70/11*x^11*b^4*a^4 + 56/9*x^9*b^3*a^5 + 4*x^7*b^2*a^6 + 8/5*x^5*b*a^7 + 1/
3*x^3*a^8
```

giac [A] time = 0.91, size = 90, normalized size = 0.85

$$\frac{1}{19}\,b^8x^{19}+\frac{8}{17}\,ab^7x^{17}+\frac{28}{15}\,a^2b^6x^{15}+\frac{56}{13}\,a^3b^5x^{13}+\frac{70}{11}\,a^4b^4x^{11}+\frac{56}{9}\,a^5b^3x^9+4\,a^6b^2x^7+\frac{8}{5}\,a^7bx^5+\frac{1}{3}\,a^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/19*b^8*x^19 + 8/17*a*b^7*x^17 + 28/15*a^2*b^6*x^15 + 56/13*a^3*b^5*x^13 +
 70/11*a^4*b^4*x^11 + 56/9*a^5*b^3*x^9 + 4*a^6*b^2*x^7 + 8/5*a^7*b*x^5 + 1/
3*a^8*x^3
```

maple [A] time = 0.00, size = 91, normalized size = 0.86

$$\frac{1}{19}b^8x^{19}+\frac{8}{17}a\,b^7x^{17}+\frac{28}{15}a^2b^6x^{15}+\frac{56}{13}a^3b^5x^{13}+\frac{70}{11}a^4b^4x^{11}+\frac{56}{9}a^5b^3x^9+4a^6b^2x^7+\frac{8}{5}a^7b\,x^5+\frac{1}{3}a^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^8,x)
```

```
[Out] 1/3*a^8*x^3+8/5*a^7*b*x^5+4*a^6*b^2*x^7+56/9*a^5*b^3*x^9+70/11*a^4*b^4*x^11
+56/13*a^3*b^5*x^13+28/15*a^2*b^6*x^15+8/17*a*b^7*x^17+1/19*b^8*x^19
```

maxima [A] time = 1.37, size = 90, normalized size = 0.85

$$\frac{1}{19}\,b^8x^{19}+\frac{8}{17}\,ab^7x^{17}+\frac{28}{15}\,a^2b^6x^{15}+\frac{56}{13}\,a^3b^5x^{13}+\frac{70}{11}\,a^4b^4x^{11}+\frac{56}{9}\,a^5b^3x^9+4\,a^6b^2x^7+\frac{8}{5}\,a^7bx^5+\frac{1}{3}\,a^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/19*b^8*x^19 + 8/17*a*b^7*x^17 + 28/15*a^2*b^6*x^15 + 56/13*a^3*b^5*x^13 +
 70/11*a^4*b^4*x^11 + 56/9*a^5*b^3*x^9 + 4*a^6*b^2*x^7 + 8/5*a^7*b*x^5 + 1/
3*a^8*x^3
```

mupad [B] time = 4.97, size = 90, normalized size = 0.85

$$\frac{a^8\,x^3}{3}+\frac{8\,a^7\,b\,x^5}{5}+4\,a^6\,b^2\,x^7+\frac{56\,a^5\,b^3\,x^9}{9}+\frac{70\,a^4\,b^4\,x^{11}}{11}+\frac{56\,a^3\,b^5\,x^{13}}{13}+\frac{28\,a^2\,b^6\,x^{15}}{15}+\frac{8\,a\,b^7\,x^{17}}{17}+\frac{b^8\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^3)/3 + (b^8*x^19)/19 + (8*a^7*b*x^5)/5 + (8*a*b^7*x^17)/17 + 4*a^6*b
^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a^4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13 +
 (28*a^2*b^6*x^15)/15
```

sympy [A] time = 0.09, size = 105, normalized size = 0.99

$$\frac{a^8x^3}{3}+\frac{8a^7bx^5}{5}+4a^6b^2x^7+\frac{56a^5b^3x^9}{9}+\frac{70a^4b^4x^{11}}{11}+\frac{56a^3b^5x^{13}}{13}+\frac{28a^2b^6x^{15}}{15}+\frac{8ab^7x^{17}}{17}+\frac{b^8x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**3/3 + 8*a**7*b*x**5/5 + 4*a**6*b**2*x**7 + 56*a**5*b**3*x**9/9 + 70
*a**4*b**4*x**11/11 + 56*a**3*b**5*x**13/13 + 28*a**2*b**6*x**15/15 + 8*a*b
**7*x**17/17 + b**8*x**19/19
```

3.113 $\int (a+bx^2)^8 \, dx$

Optimal. Leaf size=101

$$a^8x+\frac{8}{3}a^7bx^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}$$

```
[Out] a^8*x+8/3*a^7*b*x^3+28/5*a^6*b^2*x^5+8*a^5*b^3*x^7+70/9*a^4*b^4*x^9+56/11*a
^3*b^5*x^11+28/13*a^2*b^6*x^13+8/15*a*b^7*x^15+1/17*b^8*x^17
```

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$\frac{28}{13}a^2b^6x^{13}+\frac{56}{11}a^3b^5x^{11}+\frac{70}{9}a^4b^4x^9+8a^5b^3x^7+\frac{28}{5}a^6b^2x^5+\frac{8}{3}a^7bx^3+a^8x+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8,x]
```

```
[Out] a^8*x + (8*a^7*b*x^3)/3 + (28*a^6*b^2*x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*
x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a^2*b^6*x^13)/13 + (8*a*b^7*x^15)/15 +
(b^8*x^17)/17
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^2)^8 \, dx &= \int \left(a^8+8a^7bx^2+28a^6b^2x^4+56a^5b^3x^6+70a^4b^4x^8+56a^3b^5x^{10}+28a^2b^6x^{12}+8ab^7x^{14}+b^8x^{16}\right) dx\\ &= a^8x+\frac{8}{3}a^7bx^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}\end{aligned}$$

Mathematica [A] time = 0.00, size = 101, normalized size = 1.00

$$a^8x+\frac{8}{3}a^7bx^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8,x]
```

```
[Out] a^8*x + (8*a^7*b*x^3)/3 + (28*a^6*b^2*x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*
x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a^2*b^6*x^13)/13 + (8*a*b^7*x^15)/15 +
(b^8*x^17)/17
```

fricas [A] time = 0.73, size = 87, normalized size = 0.86

$$\frac{1}{17}x^{17}b^8+\frac{8}{15}x^{15}b^7a+\frac{28}{13}x^{13}b^6a^2+\frac{56}{11}x^{11}b^5a^3+\frac{70}{9}x^9b^4a^4+8x^7b^3a^5+\frac{28}{5}x^5b^2a^6+\frac{8}{3}x^3ba^7+xa^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^8 + 8/15*x^15*b^7*a + 28/13*x^13*b^6*a^2 + 56/11*x^11*b^5*a^3 +
 70/9*x^9*b^4*a^4 + 8*x^7*b^3*a^5 + 28/5*x^5*b^2*a^6 + 8/3*x^3*b*a^7 + x*a^
8
```

giac [A] time = 1.18, size = 87, normalized size = 0.86

$$\frac{1}{17}\,b^8x^{17}+\frac{8}{15}\,ab^7x^{15}+\frac{28}{13}\,a^2b^6x^{13}+\frac{56}{11}\,a^3b^5x^{11}+\frac{70}{9}\,a^4b^4x^9+8\,a^5b^3x^7+\frac{28}{5}\,a^6b^2x^5+\frac{8}{3}\,a^7bx^3+a^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/17*b^8*x^17 + 8/15*a*b^7*x^15 + 28/13*a^2*b^6*x^13 + 56/11*a^3*b^5*x^11 +
 70/9*a^4*b^4*x^9 + 8*a^5*b^3*x^7 + 28/5*a^6*b^2*x^5 + 8/3*a^7*b*x^3 + a^8*
x
```

maple [A] time = 0.00, size = 88, normalized size = 0.87

$$\frac{1}{17}b^8x^{17}+\frac{8}{15}a\,b^7x^{15}+\frac{28}{13}a^2b^6x^{13}+\frac{56}{11}a^3b^5x^{11}+\frac{70}{9}a^4b^4x^9+8a^5b^3x^7+\frac{28}{5}a^6b^2x^5+\frac{8}{3}a^7b\,x^3+a^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8,x)
```

```
[Out] a^8*x+8/3*a^7*b*x^3+28/5*a^6*b^2*x^5+8*a^5*b^3*x^7+70/9*a^4*b^4*x^9+56/11*a
^3*b^5*x^11+28/13*a^2*b^6*x^13+8/15*a*b^7*x^15+1/17*b^8*x^17
```

maxima [A] time = 1.30, size = 87, normalized size = 0.86

$$\frac{1}{17}\,b^8x^{17}+\frac{8}{15}\,ab^7x^{15}+\frac{28}{13}\,a^2b^6x^{13}+\frac{56}{11}\,a^3b^5x^{11}+\frac{70}{9}\,a^4b^4x^9+8\,a^5b^3x^7+\frac{28}{5}\,a^6b^2x^5+\frac{8}{3}\,a^7bx^3+a^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/17*b^8*x^17 + 8/15*a*b^7*x^15 + 28/13*a^2*b^6*x^13 + 56/11*a^3*b^5*x^11 +
 70/9*a^4*b^4*x^9 + 8*a^5*b^3*x^7 + 28/5*a^6*b^2*x^5 + 8/3*a^7*b*x^3 + a^8*
x
```

mupad [B] time = 0.05, size = 87, normalized size = 0.86

$$a^8\,x+\frac{8\,a^7\,b\,x^3}{3}+\frac{28\,a^6\,b^2\,x^5}{5}+8\,a^5\,b^3\,x^7+\frac{70\,a^4\,b^4\,x^9}{9}+\frac{56\,a^3\,b^5\,x^{11}}{11}+\frac{28\,a^2\,b^6\,x^{13}}{13}+\frac{8\,a\,b^7\,x^{15}}{15}+\frac{b^8\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8,x)
```

```
[Out] a^8*x + (b^8*x^17)/17 + (8*a^7*b*x^3)/3 + (8*a*b^7*x^15)/15 + (28*a^6*b^2*x
^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a^2
*b^6*x^13)/13
```

sympy [A] time = 0.08, size = 102, normalized size = 1.01

$$a^8x+\frac{8a^7bx^3}{3}+\frac{28a^6b^2x^5}{5}+8a^5b^3x^7+\frac{70a^4b^4x^9}{9}+\frac{56a^3b^5x^{11}}{11}+\frac{28a^2b^6x^{13}}{13}+\frac{8ab^7x^{15}}{15}+\frac{b^8x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8,x)
```

```
[Out] a**8*x + 8*a**7*b*x**3/3 + 28*a**6*b**2*x**5/5 + 8*a**5*b**3*x**7 + 70*a**4
*b**4*x**9/9 + 56*a**3*b**5*x**11/11 + 28*a**2*b**6*x**13/13 + 8*a*b**7*x**
15/15 + b**8*x**17/17
```

3.114 $\int \frac{(a+bx^2)^8}{x^2} dx$

Optimal. Leaf size=100

$$-\frac{a^8}{x}+8a^7bx+\frac{28}{3}a^6b^2x^3+\frac{56}{5}a^5b^3x^5+10a^4b^4x^7+\frac{56}{9}a^3b^5x^9+\frac{28}{11}a^2b^6x^{11}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{15}}{15}$$

```
[Out] -a^8/x+8*a^7*b*x+28/3*a^6*b^2*x^3+56/5*a^5*b^3*x^5+10*a^4*b^4*x^7+56/9*a^3*
b^5*x^9+28/11*a^2*b^6*x^11+8/13*a*b^7*x^13+1/15*b^8*x^15
```

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{11}a^2b^6x^{11}+\frac{56}{9}a^3b^5x^9+10a^4b^4x^7+\frac{56}{5}a^5b^3x^5+\frac{28}{3}a^6b^2x^3+8a^7bx-\frac{a^8}{x}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^2,x]
```

```
[Out] -(a^8/x) + 8*a^7*b*x + (28*a^6*b^2*x^3)/3 + (56*a^5*b^3*x^5)/5 + 10*a^4*b^4
*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + (8*a*b^7*x^13)/13 + (b^8
*x^15)/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^8}{x^2}\,dx = \int \left(8a^7b+\frac{a^8}{x^2}+28a^6b^2x^2+56a^5b^3x^4+70a^4b^4x^6+56a^3b^5x^8+28a^2b^6x^{10}+8ab^7x^{12}+b^8x^{14}\right.$$

$$=-\frac{a^8}{x}+8a^7bx+\frac{28}{3}a^6b^2x^3+\frac{56}{5}a^5b^3x^5+10a^4b^4x^7+\frac{56}{9}a^3b^5x^9+\frac{28}{11}a^2b^6x^{11}+\frac{8}{13}ab^7x^{13}+\frac{b^8}{1}$$

Mathematica [A] time = 0.02, size = 100, normalized size = 1.00

$$-\frac{a^8}{x}+8a^7bx+\frac{28}{3}a^6b^2x^3+\frac{56}{5}a^5b^3x^5+10a^4b^4x^7+\frac{56}{9}a^3b^5x^9+\frac{28}{11}a^2b^6x^{11}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^2,x]
```

```
[Out] -(a^8/x) + 8*a^7*b*x + (28*a^6*b^2*x^3)/3 + (56*a^5*b^3*x^5)/5 + 10*a^4*b^4
*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + (8*a*b^7*x^13)/13 + (b^8
*x^15)/15
```

fricas [A] time = 0.93, size = 92, normalized size = 0.92

$$\frac{429\,b^8x^{16}+3960\,ab^7x^{14}+16380\,a^2b^6x^{12}+40040\,a^3b^5x^{10}+64350\,a^4b^4x^8+72072\,a^5b^3x^6+60060\,a^6b^2x^4+514}{6435\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^2,x, algorithm="fricas")
```

```
[Out] 1/6435*(429*b^8*x^16 + 3960*a*b^7*x^14 + 16380*a^2*b^6*x^12 + 40040*a^3*b^5
*x^10 + 64350*a^4*b^4*x^8 + 72072*a^5*b^3*x^6 + 60060*a^6*b^2*x^4 + 51480*a
^7*b*x^2 - 6435*a^8)/x
```

giac [A] time = 1.07, size = 88, normalized size = 0.88

$$\frac{1}{15}b^8x^{15}+\frac{8}{13}ab^7x^{13}+\frac{28}{11}a^2b^6x^{11}+\frac{56}{9}a^3b^5x^9+10\,a^4b^4x^7+\frac{56}{5}a^5b^3x^5+\frac{28}{3}a^6b^2x^3+8\,a^7bx-\frac{a^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^2,x, algorithm="giac")
```

```
[Out] 1/15*b^8*x^15 + 8/13*a*b^7*x^13 + 28/11*a^2*b^6*x^11 + 56/9*a^3*b^5*x^9 + 1
0*a^4*b^4*x^7 + 56/5*a^5*b^3*x^5 + 28/3*a^6*b^2*x^3 + 8*a^7*b*x - a^8/x
```

maple [A] time = 0.00, size = 89, normalized size = 0.89

$$\frac{b^8x^{15}}{15}+\frac{8a\,b^7x^{13}}{13}+\frac{28a^2b^6x^{11}}{11}+\frac{56a^3b^5x^9}{9}+10a^4b^4x^7+\frac{56a^5b^3x^5}{5}+\frac{28a^6b^2x^3}{3}+8a^7bx-\frac{a^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^2,x)
```

```
[Out] -a^8/x+8*a^7*b*x+28/3*a^6*b^2*x^3+56/5*a^5*b^3*x^5+10*a^4*b^4*x^7+56/9*a^3*
b^5*x^9+28/11*a^2*b^6*x^11+8/13*a*b^7*x^13+1/15*b^8*x^15
```

maxima [A] time = 1.30, size = 88, normalized size = 0.88

$$\frac{1}{15}b^8x^{15}+\frac{8}{13}ab^7x^{13}+\frac{28}{11}a^2b^6x^{11}+\frac{56}{9}a^3b^5x^9+10\,a^4b^4x^7+\frac{56}{5}a^5b^3x^5+\frac{28}{3}a^6b^2x^3+8\,a^7bx-\frac{a^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^8*x^15 + 8/13*a*b^7*x^13 + 28/11*a^2*b^6*x^11 + 56/9*a^3*b^5*x^9 + 1
0*a^4*b^4*x^7 + 56/5*a^5*b^3*x^5 + 28/3*a^6*b^2*x^3 + 8*a^7*b*x - a^8/x
```

mupad [B] time = 0.06, size = 88, normalized size = 0.88

$$\frac{b^8\,x^{15}}{15}-\frac{a^8}{x}+\frac{8\,a\,b^7\,x^{13}}{13}+\frac{28\,a^6\,b^2\,x^3}{3}+\frac{56\,a^5\,b^3\,x^5}{5}+10\,a^4\,b^4\,x^7+\frac{56\,a^3\,b^5\,x^9}{9}+\frac{28\,a^2\,b^6\,x^{11}}{11}+8\,a^7\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^2,x)
```

```
[Out] (b^8*x^15)/15 - a^8/x + (8*a*b^7*x^13)/13 + (28*a^6*b^2*x^3)/3 + (56*a^5*b^
3*x^5)/5 + 10*a^4*b^4*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + 8*a
^7*b*x
```

sympy [A] time = 0.18, size = 99, normalized size = 0.99

$$-\frac{a^8}{x}+8a^7bx+\frac{28a^6b^2x^3}{3}+\frac{56a^5b^3x^5}{5}+10a^4b^4x^7+\frac{56a^3b^5x^9}{9}+\frac{28a^2b^6x^{11}}{11}+\frac{8ab^7x^{13}}{13}+\frac{b^8x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**2,x)
```

```
[Out] -a**8/x + 8*a**7*b*x + 28*a**6*b**2*x**3/3 + 56*a**5*b**3*x**5/5 + 10*a**4*
b**4*x**7 + 56*a**3*b**5*x**9/9 + 28*a**2*b**6*x**11/11 + 8*a*b**7*x**13/13
 + b**8*x**15/15
```

3.115 $\int \frac{(a+bx^2)^8}{x^4} dx$

Optimal. Leaf size=98

$$-\frac{a^8}{3x^3}-\frac{8a^7b}{x}+28a^6b^2x+\frac{56}{3}a^5b^3x^3+14a^4b^4x^5+8a^3b^5x^7+\frac{28}{9}a^2b^6x^9+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}$$

```
[Out] -1/3*a^8/x^3-8*a^7*b/x+28*a^6*b^2*x+56/3*a^5*b^3*x^3+14*a^4*b^4*x^5+8*a^3*b
^5*x^7+28/9*a^2*b^6*x^9+8/11*a*b^7*x^11+1/13*b^8*x^13
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{9}a^2b^6x^9+8a^3b^5x^7+14a^4b^4x^5+\frac{56}{3}a^5b^3x^3+28a^6b^2x-\frac{8a^7b}{x}-\frac{a^8}{3x^3}+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^4,x]
```

```
[Out] -a^8/(3*x^3) - (8*a^7*b)/x + 28*a^6*b^2*x + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4
*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/9 + (8*a*b^7*x^11)/11 + (b^8*x^13)/
13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^4}dx &= \int \left(28a^6b^2+\frac{a^8}{x^4}+\frac{8a^7b}{x^2}+56a^5b^3x^2+70a^4b^4x^4+56a^3b^5x^6+28a^2b^6x^8+8ab^7x^{10}+b^8x^{12}\right) d\\ &= -\frac{a^8}{3x^3}-\frac{8a^7b}{x}+28a^6b^2x+\frac{56}{3}a^5b^3x^3+14a^4b^4x^5+8a^3b^5x^7+\frac{28}{9}a^2b^6x^9+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.01, size = 98, normalized size = 1.00

$$-\frac{a^8}{3x^3}-\frac{8a^7b}{x}+28a^6b^2x+\frac{56}{3}a^5b^3x^3+14a^4b^4x^5+8a^3b^5x^7+\frac{28}{9}a^2b^6x^9+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^4,x]
```

```
[Out] -1/3*a^8/x^3 - (8*a^7*b)/x + 28*a^6*b^2*x + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4
*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/9 + (8*a*b^7*x^11)/11 + (b^8*x^13)/
13
```

fricas [A] time = 0.53, size = 92, normalized size = 0.94

$$\frac{99\,b^8x^{16}+936\,ab^7x^{14}+4004\,a^2b^6x^{12}+10296\,a^3b^5x^{10}+18018\,a^4b^4x^8+24024\,a^5b^3x^6+36036\,a^6b^2x^4-10296\,a}{1287\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^4,x, algorithm="fricas")
```

```
[Out] 1/1287*(99*b^8*x^16 + 936*a*b^7*x^14 + 4004*a^2*b^6*x^12 + 10296*a^3*b^5*x^
10 + 18018*a^4*b^4*x^8 + 24024*a^5*b^3*x^6 + 36036*a^6*b^2*x^4 - 10296*a^7*
b*x^2 - 429*a^8)/x^3
```

giac [A] time = 1.10, size = 89, normalized size = 0.91

$$\frac{1}{13}\,b^8x^{13}+\frac{8}{11}\,ab^7x^{11}+\frac{28}{9}\,a^2b^6x^9+8\,a^3b^5x^7+14\,a^4b^4x^5+\frac{56}{3}\,a^5b^3x^3+28\,a^6b^2x-\frac{24\,a^7bx^2+a^8}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^4,x, algorithm="giac")
```

```
[Out] 1/13*b^8*x^13 + 8/11*a*b^7*x^11 + 28/9*a^2*b^6*x^9 + 8*a^3*b^5*x^7 + 14*a^4
*b^4*x^5 + 56/3*a^5*b^3*x^3 + 28*a^6*b^2*x - 1/3*(24*a^7*b*x^2 + a^8)/x^3
```

maple [A] time = 0.01, size = 89, normalized size = 0.91

$$\frac{b^8x^{13}}{13}+\frac{8a\,b^7x^{11}}{11}+\frac{28a^2b^6x^9}{9}+8a^3b^5x^7+14a^4b^4x^5+\frac{56a^5b^3x^3}{3}+28a^6b^2x-\frac{8a^7b}{x}-\frac{a^8}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^4,x)
```

```
[Out] -1/3*a^8/x^3-8*a^7*b/x+28*a^6*b^2*x+56/3*a^5*b^3*x^3+14*a^4*b^4*x^5+8*a^3*b
^5*x^7+28/9*a^2*b^6*x^9+8/11*a*b^7*x^11+1/13*b^8*x^13
```

maxima [A] time = 1.38, size = 89, normalized size = 0.91

$$\frac{1}{13}\,b^8x^{13}+\frac{8}{11}\,ab^7x^{11}+\frac{28}{9}\,a^2b^6x^9+8\,a^3b^5x^7+14\,a^4b^4x^5+\frac{56}{3}\,a^5b^3x^3+28\,a^6b^2x-\frac{24\,a^7bx^2+a^8}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^4,x, algorithm="maxima")
```

```
[Out] 1/13*b^8*x^13 + 8/11*a*b^7*x^11 + 28/9*a^2*b^6*x^9 + 8*a^3*b^5*x^7 + 14*a^4
*b^4*x^5 + 56/3*a^5*b^3*x^3 + 28*a^6*b^2*x - 1/3*(24*a^7*b*x^2 + a^8)/x^3
```

mupad [B] time = 0.05, size = 91, normalized size = 0.93

$$\frac{b^8\,x^{13}}{13}-\frac{\frac{a^8}{3}+8\,b\,a^7\,x^2}{x^3}+28\,a^6\,b^2\,x+\frac{8\,a\,b^7\,x^{11}}{11}+\frac{56\,a^5\,b^3\,x^3}{3}+14\,a^4\,b^4\,x^5+8\,a^3\,b^5\,x^7+\frac{28\,a^2\,b^6\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^4,x)
```

```
[Out] (b^8*x^13)/13 - (a^8/3 + 8*a^7*b*x^2)/x^3 + 28*a^6*b^2*x + (8*a*b^7*x^11)/1
1 + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/
9
```

sympy [A] time = 0.22, size = 100, normalized size = 1.02

$$28a^6b^2x+\frac{56a^5b^3x^3}{3}+14a^4b^4x^5+8a^3b^5x^7+\frac{28a^2b^6x^9}{9}+\frac{8ab^7x^{11}}{11}+\frac{b^8x^{13}}{13}+\frac{-a^8-24a^7bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**4,x)
```

```
[Out] 28*a**6*b**2*x + 56*a**5*b**3*x**3/3 + 14*a**4*b**4*x**5 + 8*a**3*b**5*x**7
 + 28*a**2*b**6*x**9/9 + 8*a*b**7*x**11/11 + b**8*x**13/13 + (-a**8 - 24*a*
*7*b*x**2)/(3*x**3)
```

3.116 $\int \frac{(a+bx^2)^8}{x^6}\,dx$

Optimal. Leaf size=100

$$-\frac{a^8}{5x^5}-\frac{8a^7b}{3x^3}-\frac{28a^6b^2}{x}+56a^5b^3x+\frac{70}{3}a^4b^4x^3+\frac{56}{5}a^3b^5x^5+4a^2b^6x^7+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}$$

```
[Out] -1/5*a^8/x^5-8/3*a^7*b/x^3-28*a^6*b^2/x+56*a^5*b^3*x+70/3*a^4*b^4*x^3+56/5*
a^3*b^5*x^5+4*a^2*b^6*x^7+8/9*a*b^7*x^9+1/11*b^8*x^11
```

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$4a^2b^6x^7+\frac{56}{5}a^3b^5x^5+\frac{70}{3}a^4b^4x^3+56a^5b^3x-\frac{28a^6b^2}{x}-\frac{8a^7b}{3x^3}-\frac{a^8}{5x^5}+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^6,x]
```

```
[Out] -a^8/(5*x^5) - (8*a^7*b)/(3*x^3) - (28*a^6*b^2)/x + 56*a^5*b^3*x + (70*a^4*
b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*x^7 + (8*a*b^7*x^9)/9 + (b^8*x^
11)/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^6}\,dx &= \int \left(56a^5b^3+\frac{a^8}{x^6}+\frac{8a^7b}{x^4}+\frac{28a^6b^2}{x^2}+70a^4b^4x^2+56a^3b^5x^4+28a^2b^6x^6+8ab^7x^8+b^8x^{10}\right)dx\\ &= -\frac{a^8}{5x^5}-\frac{8a^7b}{3x^3}-\frac{28a^6b^2}{x}+56a^5b^3x+\frac{70}{3}a^4b^4x^3+\frac{56}{5}a^3b^5x^5+4a^2b^6x^7+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.01, size = 100, normalized size = 1.00

$$-\frac{a^8}{5x^5}-\frac{8a^7b}{3x^3}-\frac{28a^6b^2}{x}+56a^5b^3x+\frac{70}{3}a^4b^4x^3+\frac{56}{5}a^3b^5x^5+4a^2b^6x^7+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^6,x]
```

```
[Out] -1/5*a^8/x^5 - (8*a^7*b)/(3*x^3) - (28*a^6*b^2)/x + 56*a^5*b^3*x + (70*a^4*
b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*x^7 + (8*a*b^7*x^9)/9 + (b^8*x^
11)/11
```

fricas [A] time = 0.62, size = 92, normalized size = 0.92

$$\frac{45\,b^8x^{16}+440\,ab^7x^{14}+1980\,a^2b^6x^{12}+5544\,a^3b^5x^{10}+11550\,a^4b^4x^8+27720\,a^5b^3x^6-13860\,a^6b^2x^4-1320\,a^7bx}{495\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^6,x, algorithm="fricas")
```

```
[Out] 1/495*(45*b^8*x^16 + 440*a*b^7*x^14 + 1980*a^2*b^6*x^12 + 5544*a^3*b^5*x^10
 + 11550*a^4*b^4*x^8 + 27720*a^5*b^3*x^6 - 13860*a^6*b^2*x^4 - 1320*a^7*b*x
^2 - 99*a^8)/x^5
```

giac [A] time = 0.97, size = 91, normalized size = 0.91

$$\frac{1}{11}\,b^8x^{11}+\frac{8}{9}\,ab^7x^9+4\,a^2b^6x^7+\frac{56}{5}\,a^3b^5x^5+\frac{70}{3}\,a^4b^4x^3+56\,a^5b^3x-\frac{420\,a^6b^2x^4+40\,a^7bx^2+3\,a^8}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^6,x, algorithm="giac")
```

```
[Out] 1/11*b^8*x^11 + 8/9*a*b^7*x^9 + 4*a^2*b^6*x^7 + 56/5*a^3*b^5*x^5 + 70/3*a^4
*b^4*x^3 + 56*a^5*b^3*x - 1/15*(420*a^6*b^2*x^4 + 40*a^7*b*x^2 + 3*a^8)/x^5
```

maple [A] time = 0.01, size = 89, normalized size = 0.89

$$\frac{b^8x^{11}}{11}+\frac{8a\,b^7x^9}{9}+4a^2b^6x^7+\frac{56a^3b^5x^5}{5}+\frac{70a^4b^4x^3}{3}+56a^5b^3x-\frac{28a^6b^2}{x}-\frac{8a^7b}{3x^3}-\frac{a^8}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^6,x)
```

```
[Out] -1/5*a^8/x^5-8/3*a^7*b/x^3-28*a^6*b^2/x+56*a^5*b^3*x+70/3*a^4*b^4*x^3+56/5*
a^3*b^5*x^5+4*a^2*b^6*x^7+8/9*a*b^7*x^9+1/11*b^8*x^11
```

maxima [A] time = 1.21, size = 91, normalized size = 0.91

$$\frac{1}{11}\,b^8x^{11}+\frac{8}{9}\,ab^7x^9+4\,a^2b^6x^7+\frac{56}{5}\,a^3b^5x^5+\frac{70}{3}\,a^4b^4x^3+56\,a^5b^3x-\frac{420\,a^6b^2x^4+40\,a^7bx^2+3\,a^8}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^6,x, algorithm="maxima")
```

```
[Out] 1/11*b^8*x^11 + 8/9*a*b^7*x^9 + 4*a^2*b^6*x^7 + 56/5*a^3*b^5*x^5 + 70/3*a^4
*b^4*x^3 + 56*a^5*b^3*x - 1/15*(420*a^6*b^2*x^4 + 40*a^7*b*x^2 + 3*a^8)/x^5
```

mupad [B] time = 4.96, size = 91, normalized size = 0.91

$$\frac{b^8\,x^{11}}{11}-\frac{\frac{a^8}{5}+\frac{8\,a^7\,b\,x^2}{3}+28\,a^6\,b^2\,x^4}{x^5}+56\,a^5\,b^3\,x+\frac{8\,a\,b^7\,x^9}{9}+\frac{70\,a^4\,b^4\,x^3}{3}+\frac{56\,a^3\,b^5\,x^5}{5}+4\,a^2\,b^6\,x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^6,x)
```

```
[Out] (b^8*x^11)/11 - (a^8/5 + (8*a^7*b*x^2)/3 + 28*a^6*b^2*x^4)/x^5 + 56*a^5*b^3
*x + (8*a*b^7*x^9)/9 + (70*a^4*b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*
x^7
```

sympy [A] time = 0.28, size = 102, normalized size = 1.02

$$56a^5b^3x+\frac{70a^4b^4x^3}{3}+\frac{56a^3b^5x^5}{5}+4a^2b^6x^7+\frac{8ab^7x^9}{9}+\frac{b^8x^{11}}{11}+\frac{-3a^8-40a^7bx^2-420a^6b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**6,x)
```

```
[Out] 56*a**5*b**3*x + 70*a**4*b**4*x**3/3 + 56*a**3*b**5*x**5/5 + 4*a**2*b**6*x*
*7 + 8*a*b**7*x**9/9 + b**8*x**11/11 + (-3*a**8 - 40*a**7*b*x**2 - 420*a**6
*b**2*x**4)/(15*x**5)
```

3.117 $\int \frac{(a+bx^2)^8}{x^8}\,dx$

Optimal. Leaf size=102

$$-\frac{a^8}{7x^7}-\frac{8a^7b}{5x^5}-\frac{28a^6b^2}{3x^3}-\frac{56a^5b^3}{x}+70a^4b^4x+\frac{56}{3}a^3b^5x^3+\frac{28}{5}a^2b^6x^5+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}$$

```
[Out] -1/7*a^8/x^7-8/5*a^7*b/x^5-28/3*a^6*b^2/x^3-56*a^5*b^3/x+70*a^4*b^4*x+56/3*
a^3*b^5*x^3+28/5*a^2*b^6*x^5+8/7*a*b^7*x^7+1/9*b^8*x^9
```

Rubi [A] time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{5}a^2b^6x^5+\frac{56}{3}a^3b^5x^3-\frac{28a^6b^2}{3x^3}+70a^4b^4x-\frac{56a^5b^3}{x}-\frac{8a^7b}{5x^5}-\frac{a^8}{7x^7}+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^8,x]
```

```
[Out] -a^8/(7*x^7) - (8*a^7*b)/(5*x^5) - (28*a^6*b^2)/(3*x^3) - (56*a^5*b^3)/x +
70*a^4*b^4*x + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*x^5)/5 + (8*a*b^7*x^7)/7 +
(b^8*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^8}\,dx &= \int \left(70a^4b^4+\frac{a^8}{x^8}+\frac{8a^7b}{x^6}+\frac{28a^6b^2}{x^4}+\frac{56a^5b^3}{x^2}+56a^3b^5x^2+28a^2b^6x^4+8ab^7x^6+b^8x^8\right)dx\\ &= -\frac{a^8}{7x^7}-\frac{8a^7b}{5x^5}-\frac{28a^6b^2}{3x^3}-\frac{56a^5b^3}{x}+70a^4b^4x+\frac{56}{3}a^3b^5x^3+\frac{28}{5}a^2b^6x^5+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 102, normalized size = 1.00

$$-\frac{a^8}{7x^7}-\frac{8a^7b}{5x^5}-\frac{28a^6b^2}{3x^3}-\frac{56a^5b^3}{x}+70a^4b^4x+\frac{56}{3}a^3b^5x^3+\frac{28}{5}a^2b^6x^5+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^8,x]
```

```
[Out] -1/7*a^8/x^7 - (8*a^7*b)/(5*x^5) - (28*a^6*b^2)/(3*x^3) - (56*a^5*b^3)/x +
70*a^4*b^4*x + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*x^5)/5 + (8*a*b^7*x^7)/7 +
(b^8*x^9)/9
```

fricas [A] time = 0.65, size = 92, normalized size = 0.90

$$\frac{35\,b^8x^{16}+360\,ab^7x^{14}+1764\,a^2b^6x^{12}+5880\,a^3b^5x^{10}+22050\,a^4b^4x^8-17640\,a^5b^3x^6-2940\,a^6b^2x^4-504\,a^7bx^2}{315\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^8,x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^8*x^16 + 360*a*b^7*x^14 + 1764*a^2*b^6*x^12 + 5880*a^3*b^5*x^10
 + 22050*a^4*b^4*x^8 - 17640*a^5*b^3*x^6 - 2940*a^6*b^2*x^4 - 504*a^7*b*x^2
 - 45*a^8)/x^7
```

giac [A] time = 0.93, size = 91, normalized size = 0.89

$$\frac{1}{9}\,b^8x^9+\frac{8}{7}\,ab^7x^7+\frac{28}{5}\,a^2b^6x^5+\frac{56}{3}\,a^3b^5x^3+70\,a^4b^4x-\frac{5880\,a^5b^3x^6+980\,a^6b^2x^4+168\,a^7bx^2+15\,a^8}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^8,x, algorithm="giac")
```

```
[Out] 1/9*b^8*x^9 + 8/7*a*b^7*x^7 + 28/5*a^2*b^6*x^5 + 56/3*a^3*b^5*x^3 + 70*a^4*
b^4*x - 1/105*(5880*a^5*b^3*x^6 + 980*a^6*b^2*x^4 + 168*a^7*b*x^2 + 15*a^8)
/x^7
```

maple [A] time = 0.00, size = 89, normalized size = 0.87

$$\frac{b^8x^9}{9}+\frac{8a\,b^7x^7}{7}+\frac{28a^2b^6x^5}{5}+\frac{56a^3b^5x^3}{3}+70a^4b^4x-\frac{56a^5b^3}{x}-\frac{28a^6b^2}{3x^3}-\frac{8a^7b}{5x^5}-\frac{a^8}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^8,x)
```

```
[Out] -1/7*a^8/x^7-8/5*a^7*b/x^5-28/3*a^6*b^2/x^3-56*a^5*b^3/x+70*a^4*b^4*x+56/3*
a^3*b^5*x^3+28/5*a^2*b^6*x^5+8/7*a*b^7*x^7+1/9*b^8*x^9
```

maxima [A] time = 1.36, size = 91, normalized size = 0.89

$$\frac{1}{9}\,b^8x^9+\frac{8}{7}\,ab^7x^7+\frac{28}{5}\,a^2b^6x^5+\frac{56}{3}\,a^3b^5x^3+70\,a^4b^4x-\frac{5880\,a^5b^3x^6+980\,a^6b^2x^4+168\,a^7bx^2+15\,a^8}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^8,x, algorithm="maxima")
```

```
[Out] 1/9*b^8*x^9 + 8/7*a*b^7*x^7 + 28/5*a^2*b^6*x^5 + 56/3*a^3*b^5*x^3 + 70*a^4*
b^4*x - 1/105*(5880*a^5*b^3*x^6 + 980*a^6*b^2*x^4 + 168*a^7*b*x^2 + 15*a^8)
/x^7
```

mupad [B] time = 4.80, size = 91, normalized size = 0.89

$$\frac{b^8\,x^9}{9}-\frac{\frac{a^8}{7}+\frac{8\,a^7\,b\,x^2}{5}+\frac{28\,a^6\,b^2\,x^4}{3}+56\,a^5\,b^3\,x^6}{x^7}+70\,a^4\,b^4\,x+\frac{8\,a\,b^7\,x^7}{7}+\frac{56\,a^3\,b^5\,x^3}{3}+\frac{28\,a^2\,b^6\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^8,x)
```

```
[Out] (b^8*x^9)/9 - (a^8/7 + (8*a^7*b*x^2)/5 + (28*a^6*b^2*x^4)/3 + 56*a^5*b^3*x^
6)/x^7 + 70*a^4*b^4*x + (8*a*b^7*x^7)/7 + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*
x^5)/5
```

sympy [A] time = 0.33, size = 102, normalized size = 1.00

$$70a^4b^4x+\frac{56a^3b^5x^3}{3}+\frac{28a^2b^6x^5}{5}+\frac{8ab^7x^7}{7}+\frac{b^8x^9}{9}+\frac{-15a^8-168a^7bx^2-980a^6b^2x^4-5880a^5b^3x^6}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**8,x)
```

```
[Out] 70*a**4*b**4*x + 56*a**3*b**5*x**3/3 + 28*a**2*b**6*x**5/5 + 8*a*b**7*x**7/
7 + b**8*x**9/9 + (-15*a**8 - 168*a**7*b*x**2 - 980*a**6*b**2*x**4 - 5880*a
**5*b**3*x**6)/(105*x**7)
```

3.118 $\int \frac{(a+bx^2)^8}{x^{10}}\, dx$

Optimal. Leaf size=102

$$-\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28}{3}a^2b^6x^3+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

```
[Out] -1/9*a^8/x^9-8/7*a^7*b/x^7-28/5*a^6*b^2/x^5-56/3*a^5*b^3/x^3-70*a^4*b^4/x+5
6*a^3*b^5*x+28/3*a^2*b^6*x^3+8/5*a*b^7*x^5+1/7*b^8*x^7
```

Rubi [A] time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}+\frac{28}{3}a^2b^6x^3-\frac{70a^4b^4}{x}+56a^3b^5x-\frac{8a^7b}{7x^7}-\frac{a^8}{9x^9}+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^10,x]
```

```
[Out] -a^8/(9*x^9) - (8*a^7*b)/(7*x^7) - (28*a^6*b^2)/(5*x^5) - (56*a^5*b^3)/(3*x
^3) - (70*a^4*b^4)/x + 56*a^3*b^5*x + (28*a^2*b^6*x^3)/3 + (8*a*b^7*x^5)/5
+ (b^8*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{10}}\, dx &= \int \left(56a^3b^5+\frac{a^8}{x^{10}}+\frac{8a^7b}{x^8}+\frac{28a^6b^2}{x^6}+\frac{56a^5b^3}{x^4}+\frac{70a^4b^4}{x^2}+28a^2b^6x^2+8ab^7x^4+b^8x^6\right) dx\\ &= -\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28}{3}a^2b^6x^3+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 102, normalized size = 1.00

$$-\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28}{3}a^2b^6x^3+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^10,x]
```

```
[Out] -1/9*a^8/x^9 - (8*a^7*b)/(7*x^7) - (28*a^6*b^2)/(5*x^5) - (56*a^5*b^3)/(3*x
^3) - (70*a^4*b^4)/x + 56*a^3*b^5*x + (28*a^2*b^6*x^3)/3 + (8*a*b^7*x^5)/5
+ (b^8*x^7)/7
```

fricas [A] time = 0.90, size = 92, normalized size = 0.90

$$\frac{45\,b^8x^{16}+504\,ab^7x^{14}+2940\,a^2b^6x^{12}+17640\,a^3b^5x^{10}-22050\,a^4b^4x^8-5880\,a^5b^3x^6-1764\,a^6b^2x^4-360\,a^7b}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(45*b^8*x^16 + 504*a*b^7*x^14 + 2940*a^2*b^6*x^12 + 17640*a^3*b^5*x^1
0 - 22050*a^4*b^4*x^8 - 5880*a^5*b^3*x^6 - 1764*a^6*b^2*x^4 - 360*a^7*b*x^2
 - 35*a^8)/x^9
```

giac [A] time = 0.94, size = 91, normalized size = 0.89

$$\frac{1}{7}b^8x^7+\frac{8}{5}ab^7x^5+\frac{28}{3}a^2b^6x^3+56a^3b^5x-\frac{22050a^4b^4x^8+5880a^5b^3x^6+1764a^6b^2x^4+360a^7bx^2+35a^8}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^10,x, algorithm="giac")
```

```
[Out] 1/7*b^8*x^7 + 8/5*a*b^7*x^5 + 28/3*a^2*b^6*x^3 + 56*a^3*b^5*x - 1/315*(2205
0*a^4*b^4*x^8 + 5880*a^5*b^3*x^6 + 1764*a^6*b^2*x^4 + 360*a^7*b*x^2 + 35*a^
8)/x^9
```

maple [A] time = 0.01, size = 89, normalized size = 0.87

$$\frac{b^8x^7}{7}+\frac{8a\,b^7x^5}{5}+\frac{28a^2b^6x^3}{3}+56a^3b^5x-\frac{70a^4b^4}{x}-\frac{56a^5b^3}{3x^3}-\frac{28a^6b^2}{5x^5}-\frac{8a^7b}{7x^7}-\frac{a^8}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^10,x)
```

```
[Out] -1/9*a^8/x^9-8/7*a^7*b/x^7-28/5*a^6*b^2/x^5-56/3*a^5*b^3/x^3-70*a^4*b^4/x+5
6*a^3*b^5*x+28/3*a^2*b^6*x^3+8/5*a*b^7*x^5+1/7*b^8*x^7
```

maxima [A] time = 1.36, size = 91, normalized size = 0.89

$$\frac{1}{7}b^8x^7+\frac{8}{5}ab^7x^5+\frac{28}{3}a^2b^6x^3+56a^3b^5x-\frac{22050a^4b^4x^8+5880a^5b^3x^6+1764a^6b^2x^4+360a^7bx^2+35a^8}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^10,x, algorithm="maxima")
```

```
[Out] 1/7*b^8*x^7 + 8/5*a*b^7*x^5 + 28/3*a^2*b^6*x^3 + 56*a^3*b^5*x - 1/315*(2205
0*a^4*b^4*x^8 + 5880*a^5*b^3*x^6 + 1764*a^6*b^2*x^4 + 360*a^7*b*x^2 + 35*a^
8)/x^9
```

mupad [B] time = 0.05, size = 91, normalized size = 0.89

$$\frac{b^8\,x^7}{7}-\frac{\frac{a^8}{9}+\frac{8\,a^7\,b\,x^2}{7}+\frac{28\,a^6\,b^2\,x^4}{5}+\frac{56\,a^5\,b^3\,x^6}{3}+70\,a^4\,b^4\,x^8}{x^9}+56\,a^3\,b^5\,x+\frac{8\,a\,b^7\,x^5}{5}+\frac{28\,a^2\,b^6\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^10,x)
```

```
[Out] (b^8*x^7)/7 - (a^8/9 + (8*a^7*b*x^2)/7 + (28*a^6*b^2*x^4)/5 + (56*a^5*b^3*x
^6)/3 + 70*a^4*b^4*x^8)/x^9 + 56*a^3*b^5*x + (8*a*b^7*x^5)/5 + (28*a^2*b^6*
x^3)/3
```

sympy [A] time = 0.42, size = 100, normalized size = 0.98

$$56a^3b^5x+\frac{28a^2b^6x^3}{3}+\frac{8ab^7x^5}{5}+\frac{b^8x^7}{7}+\frac{-35a^8-360a^7bx^2-1764a^6b^2x^4-5880a^5b^3x^6-22050a^4b^4x^8}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**10,x)
```

```
[Out] 56*a**3*b**5*x + 28*a**2*b**6*x**3/3 + 8*a*b**7*x**5/5 + b**8*x**7/7 + (-35
*a**8 - 360*a**7*b*x**2 - 1764*a**6*b**2*x**4 - 5880*a**5*b**3*x**6 - 22050
*a**4*b**4*x**8)/(315*x**9)
```

3.119 $\int \frac{(a+bx^2)^8}{x^{12}}\,dx$

Optimal. Leaf size=100

$$-\frac{a^8}{11x^{11}}-\frac{8a^7b}{9x^9}-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}$$

```
[Out] -1/11*a^8/x^11-8/9*a^7*b/x^9-4*a^6*b^2/x^7-56/5*a^5*b^3/x^5-70/3*a^4*b^4/x^
3-56*a^3*b^5/x+28*a^2*b^6*x+8/3*a*b^7*x^3+1/5*b^8*x^5
```

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x-\frac{8a^7b}{9x^9}-\frac{a^8}{11x^{11}}+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^12,x]
```

```
[Out] -a^8/(11*x^11) - (8*a^7*b)/(9*x^9) - (4*a^6*b^2)/x^7 - (56*a^5*b^3)/(5*x^5)
 - (70*a^4*b^4)/(3*x^3) - (56*a^3*b^5)/x + 28*a^2*b^6*x + (8*a*b^7*x^3)/3 +
 (b^8*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{12}}\,dx&=\int \left(28a^2b^6+\frac{a^8}{x^{12}}+\frac{8a^7b}{x^{10}}+\frac{28a^6b^2}{x^8}+\frac{56a^5b^3}{x^6}+\frac{70a^4b^4}{x^4}+\frac{56a^3b^5}{x^2}+8ab^7x^2+b^8x^4\right)dx\\&=-\frac{a^8}{11x^{11}}-\frac{8a^7b}{9x^9}-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 100, normalized size = 1.00

$$-\frac{a^8}{11x^{11}}-\frac{8a^7b}{9x^9}-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^12,x]
```

```
[Out] -1/11*a^8/x^11 - (8*a^7*b)/(9*x^9) - (4*a^6*b^2)/x^7 - (56*a^5*b^3)/(5*x^5)
 - (70*a^4*b^4)/(3*x^3) - (56*a^3*b^5)/x + 28*a^2*b^6*x + (8*a*b^7*x^3)/3 +
 (b^8*x^5)/5
```

fricas [A] time = 0.59, size = 92, normalized size = 0.92

$$\frac{99\,b^8x^{16}+1320\,ab^7x^{14}+13860\,a^2b^6x^{12}-27720\,a^3b^5x^{10}-11550\,a^4b^4x^8-5544\,a^5b^3x^6-1980\,a^6b^2x^4-440\,a^7bx}{495\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^12,x, algorithm="fricas")
```

```
[Out] 1/495*(99*b^8*x^16 + 1320*a*b^7*x^14 + 13860*a^2*b^6*x^12 - 27720*a^3*b^5*x
^10 - 11550*a^4*b^4*x^8 - 5544*a^5*b^3*x^6 - 1980*a^6*b^2*x^4 - 440*a^7*b*x
^2 - 45*a^8)/x^11
```

giac [A] time = 0.92, size = 91, normalized size = 0.91

$$\frac{1}{5}\,b^8x^5+\frac{8}{3}\,ab^7x^3+28\,a^2b^6x-\frac{27720\,a^3b^5x^{10}+11550\,a^4b^4x^8+5544\,a^5b^3x^6+1980\,a^6b^2x^4+440\,a^7bx^2+45\,a^8}{495\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^12,x, algorithm="giac")
```

```
[Out] 1/5*b^8*x^5 + 8/3*a*b^7*x^3 + 28*a^2*b^6*x - 1/495*(27720*a^3*b^5*x^10 + 11
550*a^4*b^4*x^8 + 5544*a^5*b^3*x^6 + 1980*a^6*b^2*x^4 + 440*a^7*b*x^2 + 45*
a^8)/x^11
```

maple [A] time = 0.01, size = 89, normalized size = 0.89

$$\frac{b^8x^5}{5}+\frac{8a\,b^7x^3}{3}+28a^2b^6x-\frac{56a^3b^5}{x}-\frac{70a^4b^4}{3x^3}-\frac{56a^5b^3}{5x^5}-\frac{4a^6b^2}{x^7}-\frac{8a^7b}{9x^9}-\frac{a^8}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^12,x)
```

```
[Out] -1/11*a^8/x^11-8/9*a^7*b/x^9-4*a^6*b^2/x^7-56/5*a^5*b^3/x^5-70/3*a^4*b^4/x^
3-56*a^3*b^5/x+28*a^2*b^6*x+8/3*a*b^7*x^3+1/5*b^8*x^5
```

maxima [A] time = 1.40, size = 91, normalized size = 0.91

$$\frac{1}{5}\,b^8x^5+\frac{8}{3}\,ab^7x^3+28\,a^2b^6x-\frac{27720\,a^3b^5x^{10}+11550\,a^4b^4x^8+5544\,a^5b^3x^6+1980\,a^6b^2x^4+440\,a^7bx^2+45\,a^8}{495\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^12,x, algorithm="maxima")
```

```
[Out] 1/5*b^8*x^5 + 8/3*a*b^7*x^3 + 28*a^2*b^6*x - 1/495*(27720*a^3*b^5*x^10 + 11
550*a^4*b^4*x^8 + 5544*a^5*b^3*x^6 + 1980*a^6*b^2*x^4 + 440*a^7*b*x^2 + 45*
a^8)/x^11
```

mupad [B] time = 4.58, size = 91, normalized size = 0.91

$$\frac{b^8\,x^5}{5}-\frac{\frac{a^8}{11}+\frac{8\,a^7\,b\,x^2}{9}+4\,a^6\,b^2\,x^4+\frac{56\,a^5\,b^3\,x^6}{5}+\frac{70\,a^4\,b^4\,x^8}{3}+56\,a^3\,b^5\,x^{10}}{x^{11}}+28\,a^2\,b^6\,x+\frac{8\,a\,b^7\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^12,x)
```

```
[Out] (b^8*x^5)/5 - (a^8/11 + (8*a^7*b*x^2)/9 + 4*a^6*b^2*x^4 + (56*a^5*b^3*x^6)/
5 + (70*a^4*b^4*x^8)/3 + 56*a^3*b^5*x^10)/x^11 + 28*a^2*b^6*x + (8*a*b^7*x^
3)/3
```

sympy [A] time = 0.50, size = 99, normalized size = 0.99

$$28a^2b^6x+\frac{8ab^7x^3}{3}+\frac{b^8x^5}{5}+\frac{-45a^8-440a^7bx^2-1980a^6b^2x^4-5544a^5b^3x^6-11550a^4b^4x^8-27720a^3b^5x^{10}}{495x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**12,x)
```

```
[Out] 28*a**2*b**6*x + 8*a*b**7*x**3/3 + b**8*x**5/5 + (-45*a**8 - 440*a**7*b*x**
2 - 1980*a**6*b**2*x**4 - 5544*a**5*b**3*x**6 - 11550*a**4*b**4*x**8 - 2772
0*a**3*b**5*x**10)/(495*x**11)
```

3.120 $\int \frac{(a+bx^2)^8}{x^{14}}\,dx$

Optimal. Leaf size=98

$$-\frac{a^8}{13x^{13}}-\frac{8a^7b}{11x^{11}}-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}+8ab^7x+\frac{b^8x^3}{3}$$

```
[Out] -1/13*a^8/x^13-8/11*a^7*b/x^11-28/9*a^6*b^2/x^9-8*a^5*b^3/x^7-14*a^4*b^4/x^
5-56/3*a^3*b^5/x^3-28*a^2*b^6/x+8*a*b^7*x+1/3*b^8*x^3
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}-\frac{8a^7b}{11x^{11}}-\frac{a^8}{13x^{13}}+8ab^7x+\frac{b^8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^14,x]
```

```
[Out] -a^8/(13*x^13) - (8*a^7*b)/(11*x^11) - (28*a^6*b^2)/(9*x^9) - (8*a^5*b^3)/x
^7 - (14*a^4*b^4)/x^5 - (56*a^3*b^5)/(3*x^3) - (28*a^2*b^6)/x + 8*a*b^7*x +
 (b^8*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{14}}\,dx &= \int \left(8ab^7+\frac{a^8}{x^{14}}+\frac{8a^7b}{x^{12}}+\frac{28a^6b^2}{x^{10}}+\frac{56a^5b^3}{x^8}+\frac{70a^4b^4}{x^6}+\frac{56a^3b^5}{x^4}+\frac{28a^2b^6}{x^2}+b^8x^2\right)dx\\ &= -\frac{a^8}{13x^{13}}-\frac{8a^7b}{11x^{11}}-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}+8ab^7x+\frac{b^8x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 98, normalized size = 1.00

$$-\frac{a^8}{13x^{13}}-\frac{8a^7b}{11x^{11}}-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}+8ab^7x+\frac{b^8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^14,x]
```

```
[Out] -1/13*a^8/x^13 - (8*a^7*b)/(11*x^11) - (28*a^6*b^2)/(9*x^9) - (8*a^5*b^3)/x
^7 - (14*a^4*b^4)/x^5 - (56*a^3*b^5)/(3*x^3) - (28*a^2*b^6)/x + 8*a*b^7*x +
 (b^8*x^3)/3
```

fricas [A] time = 0.84, size = 92, normalized size = 0.94

$$\frac{429\,b^8x^{16}+10296\,ab^7x^{14}-36036\,a^2b^6x^{12}-24024\,a^3b^5x^{10}-18018\,a^4b^4x^8-10296\,a^5b^3x^6-4004\,a^6b^2x^4-93}{1287\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^14,x, algorithm="fricas")
```

```
[Out] 1/1287*(429*b^8*x^16 + 10296*a*b^7*x^14 - 36036*a^2*b^6*x^12 - 24024*a^3*b^
5*x^10 - 18018*a^4*b^4*x^8 - 10296*a^5*b^3*x^6 - 4004*a^6*b^2*x^4 - 936*a^7
*b*x^2 - 99*a^8)/x^13
```

giac [A] time = 1.10, size = 91, normalized size = 0.93

$$\frac{1}{3}\,b^8x^3+8\,ab^7x-\frac{36036\,a^2b^6x^{12}+24024\,a^3b^5x^{10}+18018\,a^4b^4x^8+10296\,a^5b^3x^6+4004\,a^6b^2x^4+936\,a^7bx^2+99\,a}{1287\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^14,x, algorithm="giac")
```

```
[Out] 1/3*b^8*x^3 + 8*a*b^7*x - 1/1287*(36036*a^2*b^6*x^12 + 24024*a^3*b^5*x^10 +
 18018*a^4*b^4*x^8 + 10296*a^5*b^3*x^6 + 4004*a^6*b^2*x^4 + 936*a^7*b*x^2 +
 99*a^8)/x^13
```

maple [A] time = 0.01, size = 89, normalized size = 0.91

$$\frac{b^8x^3}{3}+8a\,b^7x-\frac{28a^2b^6}{x}-\frac{56a^3b^5}{3x^3}-\frac{14a^4b^4}{x^5}-\frac{8a^5b^3}{x^7}-\frac{28a^6b^2}{9x^9}-\frac{8a^7b}{11x^{11}}-\frac{a^8}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^14,x)
```

```
[Out] -1/13*a^8/x^13-8/11*a^7*b/x^11-28/9*a^6*b^2/x^9-8*a^5*b^3/x^7-14*a^4*b^4/x^
5-56/3*a^3*b^5/x^3-28*a^2*b^6/x+8*a*b^7*x+1/3*b^8*x^3
```

maxima [A] time = 1.33, size = 91, normalized size = 0.93

$$\frac{1}{3}\,b^8x^3+8\,ab^7x-\frac{36036\,a^2b^6x^{12}+24024\,a^3b^5x^{10}+18018\,a^4b^4x^8+10296\,a^5b^3x^6+4004\,a^6b^2x^4+936\,a^7bx^2+99\,a}{1287\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^14,x, algorithm="maxima")
```

```
[Out] 1/3*b^8*x^3 + 8*a*b^7*x - 1/1287*(36036*a^2*b^6*x^12 + 24024*a^3*b^5*x^10 +
 18018*a^4*b^4*x^8 + 10296*a^5*b^3*x^6 + 4004*a^6*b^2*x^4 + 936*a^7*b*x^2 +
 99*a^8)/x^13
```

mupad [B] time = 0.07, size = 92, normalized size = 0.94

$$-\frac{\frac{a^8}{13}+\frac{8\,a^7\,b\,x^2}{11}+\frac{28\,a^6\,b^2\,x^4}{9}+8\,a^5\,b^3\,x^6+14\,a^4\,b^4\,x^8+\frac{56\,a^3\,b^5\,x^{10}}{3}+28\,a^2\,b^6\,x^{12}-8\,a\,b^7\,x^{14}-\frac{b^8\,x^{16}}{3}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^14,x)
```

```
[Out] -(a^8/13 - (b^8*x^16)/3 + (8*a^7*b*x^2)/11 - 8*a*b^7*x^14 + (28*a^6*b^2*x^4
)/9 + 8*a^5*b^3*x^6 + 14*a^4*b^4*x^8 + (56*a^3*b^5*x^10)/3 + 28*a^2*b^6*x^1
2)/x^13
```

sympy [A] time = 0.57, size = 97, normalized size = 0.99

$$8ab^7x+\frac{b^8x^3}{3}+\frac{-99a^8-936a^7bx^2-4004a^6b^2x^4-10296a^5b^3x^6-18018a^4b^4x^8-24024a^3b^5x^{10}-36036a^2b^6x^{12}}{1287x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**14,x)
```

```
[Out] 8*a*b**7*x + b**8*x**3/3 + (-99*a**8 - 936*a**7*b*x**2 - 4004*a**6*b**2*x**
4 - 10296*a**5*b**3*x**6 - 18018*a**4*b**4*x**8 - 24024*a**3*b**5*x**10 - 3
6036*a**2*b**6*x**12)/(1287*x**13)
```

3.121 $\int \frac{(a+bx^2)^8}{x^{16}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^8}{15x^{15}}-\frac{8a^7b}{13x^{13}}-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8ab^7}{x}+b^8x$$

```
[Out] -1/15*a^8/x^15-8/13*a^7*b/x^13-28/11*a^6*b^2/x^11-56/9*a^5*b^3/x^9-10*a^4*b
^4/x^7-56/5*a^3*b^5/x^5-28/3*a^2*b^6/x^3-8*a*b^7/x+b^8*x
```

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8a^7b}{13x^{13}}-\frac{a^8}{15x^{15}}-\frac{8ab^7}{x}+b^8x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^16,x]
```

```
[Out] -a^8/(15*x^15) - (8*a^7*b)/(13*x^13) - (28*a^6*b^2)/(11*x^11) - (56*a^5*b^3
)/(9*x^9) - (10*a^4*b^4)/x^7 - (56*a^3*b^5)/(5*x^5) - (28*a^2*b^6)/(3*x^3)
- (8*a*b^7)/x + b^8*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{16}}\,dx &= \int \left(b^8+\frac{a^8}{x^{16}}+\frac{8a^7b}{x^{14}}+\frac{28a^6b^2}{x^{12}}+\frac{56a^5b^3}{x^{10}}+\frac{70a^4b^4}{x^8}+\frac{56a^3b^5}{x^6}+\frac{28a^2b^6}{x^4}+\frac{8ab^7}{x^2}\right)dx\\ &= -\frac{a^8}{15x^{15}}-\frac{8a^7b}{13x^{13}}-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8ab^7}{x}+b^8x\end{aligned}$$

Mathematica [A] time = 0.01, size = 99, normalized size = 1.00

$$-\frac{a^8}{15x^{15}}-\frac{8a^7b}{13x^{13}}-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8ab^7}{x}+b^8x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^16,x]
```

```
[Out] -1/15*a^8/x^15 - (8*a^7*b)/(13*x^13) - (28*a^6*b^2)/(11*x^11) - (56*a^5*b^3
)/(9*x^9) - (10*a^4*b^4)/x^7 - (56*a^3*b^5)/(5*x^5) - (28*a^2*b^6)/(3*x^3)
- (8*a*b^7)/x + b^8*x
```

fricas [A] time = 0.70, size = 92, normalized size = 0.93

$$\frac{6435\,b^8x^{16}-51480\,ab^7x^{14}-60060\,a^2b^6x^{12}-72072\,a^3b^5x^{10}-64350\,a^4b^4x^8-40040\,a^5b^3x^6-16380\,a^6b^2x^4-39}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^16,x, algorithm="fricas")
```

```
[Out] 1/6435*(6435*b^8*x^16 - 51480*a*b^7*x^14 - 60060*a^2*b^6*x^12 - 72072*a^3*b
^5*x^10 - 64350*a^4*b^4*x^8 - 40040*a^5*b^3*x^6 - 16380*a^6*b^2*x^4 - 3960*
a^7*b*x^2 - 429*a^8)/x^15
```

giac [A] time = 0.87, size = 90, normalized size = 0.91

$$b^{8}x - \frac{51480\,ab^{7}x^{14} + 60060\,a^{2}b^{6}x^{12} + 72072\,a^{3}b^{5}x^{10} + 64350\,a^{4}b^{4}x^{8} + 40040\,a^{5}b^{3}x^{6} + 16380\,a^{6}b^{2}x^{4} + 3960\,a^{7}}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^16,x, algorithm="giac")
```

```
[Out] b^8*x - 1/6435*(51480*a*b^7*x^14 + 60060*a^2*b^6*x^12 + 72072*a^3*b^5*x^10
+ 64350*a^4*b^4*x^8 + 40040*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 3960*a^7*b*x^
2 + 429*a^8)/x^15
```

maple [A] time = 0.01, size = 88, normalized size = 0.89

$$b^{8}x - \frac{8a\,b^{7}}{x} - \frac{28a^{2}b^{6}}{3x^{3}} - \frac{56a^{3}b^{5}}{5x^{5}} - \frac{10a^{4}b^{4}}{x^{7}} - \frac{56a^{5}b^{3}}{9x^{9}} - \frac{28a^{6}b^{2}}{11x^{11}} - \frac{8a^{7}b}{13x^{13}} - \frac{a^{8}}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^16,x)
```

```
[Out] -1/15*a^8/x^15-8/13*a^7*b/x^13-28/11*a^6*b^2/x^11-56/9*a^5*b^3/x^9-10*a^4*b
^4/x^7-56/5*a^3*b^5/x^5-28/3*a^2*b^6/x^3-8*a*b^7/x+b^8*x
```

maxima [A] time = 1.41, size = 90, normalized size = 0.91

$$b^{8}x - \frac{51480\,ab^{7}x^{14} + 60060\,a^{2}b^{6}x^{12} + 72072\,a^{3}b^{5}x^{10} + 64350\,a^{4}b^{4}x^{8} + 40040\,a^{5}b^{3}x^{6} + 16380\,a^{6}b^{2}x^{4} + 3960\,a^{7}}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^16,x, algorithm="maxima")
```

```
[Out] b^8*x - 1/6435*(51480*a*b^7*x^14 + 60060*a^2*b^6*x^12 + 72072*a^3*b^5*x^10
+ 64350*a^4*b^4*x^8 + 40040*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 3960*a^7*b*x^
2 + 429*a^8)/x^15
```

mupad [B] time = 4.52, size = 90, normalized size = 0.91

$$b^{8}\,x - \frac{\frac{a^{8}}{15} + \frac{8\,a^{7}\,b\,x^{2}}{13} + \frac{28\,a^{6}\,b^{2}\,x^{4}}{11} + \frac{56\,a^{5}\,b^{3}\,x^{6}}{9} + 10\,a^{4}\,b^{4}\,x^{8} + \frac{56\,a^{3}\,b^{5}\,x^{10}}{5} + \frac{28\,a^{2}\,b^{6}\,x^{12}}{3} + 8\,a\,b^{7}\,x^{14}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^16,x)
```

```
[Out] b^8*x - (a^8/15 + (8*a^7*b*x^2)/13 + 8*a*b^7*x^14 + (28*a^6*b^2*x^4)/11 + (
56*a^5*b^3*x^6)/9 + 10*a^4*b^4*x^8 + (56*a^3*b^5*x^10)/5 + (28*a^2*b^6*x^12
)/3)/x^15
```

sympy [A] time = 0.69, size = 95, normalized size = 0.96

$$b^{8}x + \frac{-429a^{8} - 3960a^{7}bx^{2} - 16380a^{6}b^{2}x^{4} - 40040a^{5}b^{3}x^{6} - 64350a^{4}b^{4}x^{8} - 72072a^{3}b^{5}x^{10} - 60060a^{2}b^{6}x^{12} - 51}{6435x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**16,x)
```

```
[Out] b**8*x + (-429*a**8 - 3960*a**7*b*x**2 - 16380*a**6*b**2*x**4 - 40040*a**5*
b**3*x**6 - 64350*a**4*b**4*x**8 - 72072*a**3*b**5*x**10 - 60060*a**2*b**6*
x**12 - 51480*a*b**7*x**14)/(6435*x**15)
```

3.122 $\int \frac{(a+bx^2)^8}{x^{18}}\,dx$

Optimal. Leaf size=104

$$-\frac{a^8}{17x^{17}}-\frac{8a^7b}{15x^{15}}-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}$$

```
[Out] -1/17*a^8/x^17-8/15*a^7*b/x^15-28/13*a^6*b^2/x^13-56/11*a^5*b^3/x^11-70/9*a
^4*b^4/x^9-8*a^3*b^5/x^7-28/5*a^2*b^6/x^5-8/3*a*b^7/x^3-b^8/x
```

Rubi [A] time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8a^7b}{15x^{15}}-\frac{a^8}{17x^{17}}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^18,x]
```

```
[Out] -a^8/(17*x^17) - (8*a^7*b)/(15*x^15) - (28*a^6*b^2)/(13*x^13) - (56*a^5*b^3
)/(11*x^11) - (70*a^4*b^4)/(9*x^9) - (8*a^3*b^5)/x^7 - (28*a^2*b^6)/(5*x^5)
 - (8*a*b^7)/(3*x^3) - b^8/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{18}}\,dx &= \int \left(\frac{a^8}{x^{18}}+\frac{8a^7b}{x^{16}}+\frac{28a^6b^2}{x^{14}}+\frac{56a^5b^3}{x^{12}}+\frac{70a^4b^4}{x^{10}}+\frac{56a^3b^5}{x^8}+\frac{28a^2b^6}{x^6}+\frac{8ab^7}{x^4}+\frac{b^8}{x^2}\right)dx\\ &= -\frac{a^8}{17x^{17}}-\frac{8a^7b}{15x^{15}}-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 104, normalized size = 1.00

$$-\frac{a^8}{17x^{17}}-\frac{8a^7b}{15x^{15}}-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^18,x]
```

```
[Out] -1/17*a^8/x^17 - (8*a^7*b)/(15*x^15) - (28*a^6*b^2)/(13*x^13) - (56*a^5*b^3
)/(11*x^11) - (70*a^4*b^4)/(9*x^9) - (8*a^3*b^5)/x^7 - (28*a^2*b^6)/(5*x^5)
 - (8*a*b^7)/(3*x^3) - b^8/x
```

fricas [A] time = 0.82, size = 92, normalized size = 0.88

$$-\frac{109395\,b^8x^{16}+291720\,ab^7x^{14}+612612\,a^2b^6x^{12}+875160\,a^3b^5x^{10}+850850\,a^4b^4x^8+556920\,a^5b^3x^6+2356}{109395\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^18,x, algorithm="fricas")
```

```
[Out] -1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 8751
60*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^2*
x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

giac [A] time = 0.82, size = 92, normalized size = 0.88

$$-\frac{109395\,b^8x^{16} + 291720\,ab^7x^{14} + 612612\,a^2b^6x^{12} + 875160\,a^3b^5x^{10} + 850850\,a^4b^4x^8 + 556920\,a^5b^3x^6 + 235620}{109395\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^18,x, algorithm="giac")
```

```
[Out] -1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 8751
60*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^2*
x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

maple [A] time = 0.01, size = 91, normalized size = 0.88

$$-\frac{b^8}{x}-\frac{8a\,b^7}{3x^3}-\frac{28a^2b^6}{5x^5}-\frac{8a^3b^5}{x^7}-\frac{70a^4b^4}{9x^9}-\frac{56a^5b^3}{11x^{11}}-\frac{28a^6b^2}{13x^{13}}-\frac{8a^7b}{15x^{15}}-\frac{a^8}{17x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^18,x)
```

```
[Out] -1/17*a^8/x^17-8/15*a^7*b/x^15-28/13*a^6*b^2/x^13-56/11*a^5*b^3/x^11-70/9*a
^4*b^4/x^9-8*a^3*b^5/x^7-28/5*a^2*b^6/x^5-8/3*a*b^7/x^3-b^8/x
```

maxima [A] time = 1.36, size = 92, normalized size = 0.88

$$-\frac{109395\,b^8x^{16} + 291720\,ab^7x^{14} + 612612\,a^2b^6x^{12} + 875160\,a^3b^5x^{10} + 850850\,a^4b^4x^8 + 556920\,a^5b^3x^6 + 235620}{109395\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^18,x, algorithm="maxima")
```

```
[Out] -1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 8751
60*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^2*
x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

mupad [B] time = 0.07, size = 91, normalized size = 0.88

$$-\frac{\frac{a^8}{17}+\frac{8\,a^7\,b\,x^2}{15}+\frac{28\,a^6\,b^2\,x^4}{13}+\frac{56\,a^5\,b^3\,x^6}{11}+\frac{70\,a^4\,b^4\,x^8}{9}+8\,a^3\,b^5\,x^{10}+\frac{28\,a^2\,b^6\,x^{12}}{5}+\frac{8\,a\,b^7\,x^{14}}{3}+b^8\,x^{16}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^18,x)
```

```
[Out] -(a^8/17 + b^8*x^16 + (8*a^7*b*x^2)/15 + (8*a*b^7*x^14)/3 + (28*a^6*b^2*x^4
)/13 + (56*a^5*b^3*x^6)/11 + (70*a^4*b^4*x^8)/9 + 8*a^3*b^5*x^10 + (28*a^2*
b^6*x^12)/5)/x^17
```

sympy [A] time = 0.79, size = 99, normalized size = 0.95

$$\frac{-6435a^8 - 58344a^7bx^2 - 235620a^6b^2x^4 - 556920a^5b^3x^6 - 850850a^4b^4x^8 - 875160a^3b^5x^{10} - 612612a^2b^6x^{12} - 2}{109395x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**18,x)
```

```
[Out] (-6435*a**8 - 58344*a**7*b*x**2 - 235620*a**6*b**2*x**4 - 556920*a**5*b**3*
x**6 - 850850*a**4*b**4*x**8 - 875160*a**3*b**5*x**10 - 612612*a**2*b**6*x*
*12 - 291720*a*b**7*x**14 - 109395*b**8*x**16)/(109395*x**17)
```

3.123 $\int \frac{(a+bx^2)^8}{x^{20}} dx$

Optimal. Leaf size=106

$$-\frac{a^8}{19x^{19}}-\frac{8a^7b}{17x^{17}}-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}$$

```
[Out] -1/19*a^8/x^19-8/17*a^7*b/x^17-28/15*a^6*b^2/x^15-56/13*a^5*b^3/x^13-70/11*
a^4*b^4/x^11-56/9*a^3*b^5/x^9-4*a^2*b^6/x^7-8/5*a*b^7/x^5-1/3*b^8/x^3
```

Rubi [A] time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8a^7b}{17x^{17}}-\frac{a^8}{19x^{19}}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^20,x]
```

```
[Out] -a^8/(19*x^19) - (8*a^7*b)/(17*x^17) - (28*a^6*b^2)/(15*x^15) - (56*a^5*b^3
)/(13*x^13) - (70*a^4*b^4)/(11*x^11) - (56*a^3*b^5)/(9*x^9) - (4*a^2*b^6)/x
^7 - (8*a*b^7)/(5*x^5) - b^8/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{20}}\,dx &= \int \left(\frac{a^8}{x^{20}}+\frac{8a^7b}{x^{18}}+\frac{28a^6b^2}{x^{16}}+\frac{56a^5b^3}{x^{14}}+\frac{70a^4b^4}{x^{12}}+\frac{56a^3b^5}{x^{10}}+\frac{28a^2b^6}{x^8}+\frac{8ab^7}{x^6}+\frac{b^8}{x^4}\right)dx\\ &= -\frac{a^8}{19x^{19}}-\frac{8a^7b}{17x^{17}}-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 106, normalized size = 1.00

$$-\frac{a^8}{19x^{19}}-\frac{8a^7b}{17x^{17}}-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^20,x]
```

```
[Out] -1/19*a^8/x^19 - (8*a^7*b)/(17*x^17) - (28*a^6*b^2)/(15*x^15) - (56*a^5*b^3
)/(13*x^13) - (70*a^4*b^4)/(11*x^11) - (56*a^3*b^5)/(9*x^9) - (4*a^2*b^6)/x
^7 - (8*a*b^7)/(5*x^5) - b^8/(3*x^3)
```

fricas [A] time = 0.85, size = 92, normalized size = 0.87

$$-\frac{692835\,b^8x^{16}+3325608\,ab^7x^{14}+8314020\,a^2b^6x^{12}+12932920\,a^3b^5x^{10}+13226850\,a^4b^4x^8+8953560\,a^5b^3x^6 -}{2078505\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^20,x, algorithm="fricas")
```

```
[Out] -1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 + 1
2932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876
*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

giac [A] time = 1.14, size = 92, normalized size = 0.87

$$-\frac{692835\,b^{8}x^{16} + 3325608\,ab^{7}x^{14} + 8314020\,a^{2}b^{6}x^{12} + 12932920\,a^{3}b^{5}x^{10} + 13226850\,a^{4}b^{4}x^{8} + 8953560\,a^{5}b^{3}}{2078505\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^20,x, algorithm="giac")
```

```
[Out] -1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 + 1
2932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876
*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

maple [A] time = 0.01, size = 91, normalized size = 0.86

$$-\frac{b^{8}}{3x^{3}}-\frac{8a\,b^{7}}{5x^{5}}-\frac{4a^{2}b^{6}}{x^{7}}-\frac{56a^{3}b^{5}}{9x^{9}}-\frac{70a^{4}b^{4}}{11x^{11}}-\frac{56a^{5}b^{3}}{13x^{13}}-\frac{28a^{6}b^{2}}{15x^{15}}-\frac{8a^{7}b}{17x^{17}}-\frac{a^{8}}{19x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^20,x)
```

```
[Out] -1/19*a^8/x^19-8/17*a^7*b/x^17-28/15*a^6*b^2/x^15-56/13*a^5*b^3/x^13-70/11*
a^4*b^4/x^11-56/9*a^3*b^5/x^9-4*a^2*b^6/x^7-8/5*a*b^7/x^5-1/3*b^8/x^3
```

maxima [A] time = 1.36, size = 92, normalized size = 0.87

$$-\frac{692835\,b^{8}x^{16} + 3325608\,ab^{7}x^{14} + 8314020\,a^{2}b^{6}x^{12} + 12932920\,a^{3}b^{5}x^{10} + 13226850\,a^{4}b^{4}x^{8} + 8953560\,a^{5}b^{3}}{2078505\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^20,x, algorithm="maxima")
```

```
[Out] -1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 + 1
2932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876
*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

mupad [B] time = 0.08, size = 92, normalized size = 0.87

$$-\frac{\frac{a^{8}}{19}+\frac{8\,a^{7}\,b\,x^{2}}{17}+\frac{28\,a^{6}\,b^{2}\,x^{4}}{15}+\frac{56\,a^{5}\,b^{3}\,x^{6}}{13}+\frac{70\,a^{4}\,b^{4}\,x^{8}}{11}+\frac{56\,a^{3}\,b^{5}\,x^{10}}{9}+4\,a^{2}\,b^{6}\,x^{12}+\frac{8\,a\,b^{7}\,x^{14}}{5}+\frac{b^{8}\,x^{16}}{3}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^20,x)
```

```
[Out] -(a^8/19 + (b^8*x^16)/3 + (8*a^7*b*x^2)/17 + (8*a*b^7*x^14)/5 + (28*a^6*b^2
*x^4)/15 + (56*a^5*b^3*x^6)/13 + (70*a^4*b^4*x^8)/11 + (56*a^3*b^5*x^10)/9
+ 4*a^2*b^6*x^12)/x^19
```

sympy [A] time = 0.84, size = 99, normalized size = 0.93

$$\frac{-109395a^{8}-978120a^{7}bx^{2}-3879876a^{6}b^{2}x^{4}-8953560a^{5}b^{3}x^{6}-13226850a^{4}b^{4}x^{8}-12932920a^{3}b^{5}x^{10}-8314}{2078505x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**20,x)
```

```
[Out] (-109395*a**8 - 978120*a**7*b*x**2 - 3879876*a**6*b**2*x**4 - 8953560*a**5*
b**3*x**6 - 13226850*a**4*b**4*x**8 - 12932920*a**3*b**5*x**10 - 8314020*a*
*2*b**6*x**12 - 3325608*a*b**7*x**14 - 692835*b**8*x**16)/(2078505*x**19)
```

3.124 $\int \frac{x^{11}}{a+bx^2} dx$

Optimal. Leaf size=79

$$-\frac{a^5 \log \left(a+bx^2\right)}{2b^6}+\frac{a^4x^2}{2b^5}-\frac{a^3x^4}{4b^4}+\frac{a^2x^6}{6b^3}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}$$

```
[Out] 1/2*a^4*x^2/b^5-1/4*a^3*x^4/b^4+1/6*a^2*x^6/b^3-1/8*a*x^8/b^2+1/10*x^10/b-1
/2*a^5*ln(b*x^2+a)/b^6
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^6}{6b^3}-\frac{a^3x^4}{4b^4}+\frac{a^4x^2}{2b^5}-\frac{a^5 \log \left(a+bx^2\right)}{2b^6}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2),x]
```

```
[Out] (a^4*x^2)/(2*b^5) - (a^3*x^4)/(4*b^4) + (a^2*x^6)/(6*b^3) - (a*x^8)/(8*b^2)
 + x^10/(10*b) - (a^5*Log[a + b*x^2])/(2*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{a+bx^2} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^5}{a+bx} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^4}{b}-\frac{a^5}{b^5(a+bx)}\right) dx, x, x^2\right)\\
&= \frac{a^4x^2}{2b^5}-\frac{a^3x^4}{4b^4}+\frac{a^2x^6}{6b^3}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}-\frac{a^5 \log \left(a+bx^2\right)}{2b^6}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 79, normalized size = 1.00

$$-\frac{a^5 \log \left(a+bx^2\right)}{2b^6}+\frac{a^4x^2}{2b^5}-\frac{a^3x^4}{4b^4}+\frac{a^2x^6}{6b^3}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2),x]
```

```
[Out] (a^4*x^2)/(2*b^5) - (a^3*x^4)/(4*b^4) + (a^2*x^6)/(6*b^3) - (a*x^8)/(8*b^2)
 + x^10/(10*b) - (a^5*Log[a + b*x^2])/(2*b^6)
```

fricas [A] time = 0.86, size = 67, normalized size = 0.85

$$\frac{12\,b^{5}x^{10} - 15\,ab^{4}x^{8} + 20\,a^{2}b^{3}x^{6} - 30\,a^{3}b^{2}x^{4} + 60\,a^{4}bx^{2} - 60\,a^{5}\log\left(bx^{2} + a\right)}{120\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/120*(12*b^5*x^10 - 15*a*b^4*x^8 + 20*a^2*b^3*x^6 - 30*a^3*b^2*x^4 + 60*a^
4*b*x^2 - 60*a^5*log(b*x^2 + a))/b^6
```

giac [A] time = 0.99, size = 69, normalized size = 0.87

$$-\frac{a^{5}\log\left(\left|bx^{2} + a\right|\right)}{2\,b^{6}} + \frac{12\,b^{4}x^{10} - 15\,ab^{3}x^{8} + 20\,a^{2}b^{2}x^{6} - 30\,a^{3}bx^{4} + 60\,a^{4}x^{2}}{120\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*a^5*log(abs(b*x^2 + a))/b^6 + 1/120*(12*b^4*x^10 - 15*a*b^3*x^8 + 20*a
^2*b^2*x^6 - 30*a^3*b*x^4 + 60*a^4*x^2)/b^5
```

maple [A] time = 0.00, size = 68, normalized size = 0.86

$$\frac{x^{10}}{10b} - \frac{a\,x^{8}}{8b^{2}} + \frac{a^{2}x^{6}}{6b^{3}} - \frac{a^{3}x^{4}}{4b^{4}} + \frac{a^{4}x^{2}}{2b^{5}} - \frac{a^{5}\ln\left(b\,x^{2} + a\right)}{2b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a),x)
```

```
[Out] 1/2*a^4*x^2/b^5-1/4*a^3*x^4/b^4+1/6*a^2*x^6/b^3-1/8*a*x^8/b^2+1/10*x^10/b-1
/2*a^5*ln(b*x^2+a)/b^6
```

maxima [A] time = 1.36, size = 68, normalized size = 0.86

$$-\frac{a^{5}\log\left(bx^{2} + a\right)}{2\,b^{6}} + \frac{12\,b^{4}x^{10} - 15\,ab^{3}x^{8} + 20\,a^{2}b^{2}x^{6} - 30\,a^{3}bx^{4} + 60\,a^{4}x^{2}}{120\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*a^5*log(b*x^2 + a)/b^6 + 1/120*(12*b^4*x^10 - 15*a*b^3*x^8 + 20*a^2*b^
2*x^6 - 30*a^3*b*x^4 + 60*a^4*x^2)/b^5
```

mupad [B] time = 0.06, size = 67, normalized size = 0.85

$$\frac{x^{10}}{10\,b} - \frac{a\,x^{8}}{8\,b^{2}} - \frac{a^{5}\,\ln\left(b\,x^{2} + a\right)}{2\,b^{6}} + \frac{a^{2}\,x^{6}}{6\,b^{3}} - \frac{a^{3}\,x^{4}}{4\,b^{4}} + \frac{a^{4}\,x^{2}}{2\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2),x)
```

```
[Out] x^10/(10*b) - (a*x^8)/(8*b^2) - (a^5*log(a + b*x^2))/(2*b^6) + (a^2*x^6)/(6
*b^3) - (a^3*x^4)/(4*b^4) + (a^4*x^2)/(2*b^5)
```

sympy [A] time = 0.18, size = 68, normalized size = 0.86

$$-\frac{a^{5}\log\left(a + bx^{2}\right)}{2b^{6}} + \frac{a^{4}x^{2}}{2b^{5}} - \frac{a^{3}x^{4}}{4b^{4}} + \frac{a^{2}x^{6}}{6b^{3}} - \frac{ax^{8}}{8b^{2}} + \frac{x^{10}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a),x)
```

```
[Out] -a**5*log(a + b*x**2)/(2*b**6) + a**4*x**2/(2*b**5) - a**3*x**4/(4*b**4) +
a**2*x**6/(6*b**3) - a*x**8/(8*b**2) + x**10/(10*b)
```

3.125 $\int \frac{x^{10}}{a+bx^2} dx$

Optimal. Leaf size=81

$$-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}$$

```
[Out] a^4*x/b^5-1/3*a^3*x^3/b^4+1/5*a^2*x^5/b^3-1/7*a*x^7/b^2+1/9*x^9/b-a^(9/2)*a
rctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {302, 205}

$$\frac{a^2x^5}{5b^3}-\frac{a^3x^3}{3b^4}+\frac{a^4x}{b^5}-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^3)/(3*b^4) + (a^2*x^5)/(5*b^3) - (a*x^7)/(7*b^2) + x^9
/(9*b) - (a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{a+bx^2}\,dx &= \int \left(\frac{a^4}{b^5}-\frac{a^3x^2}{b^4}+\frac{a^2x^4}{b^3}-\frac{ax^6}{b^2}+\frac{x^8}{b}-\frac{a^5}{b^5\left(a+bx^2\right)}\right)dx\\
&= \frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}-\frac{a^5\int\frac{1}{a+bx^2}\,dx}{b^5}\\
&= \frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 1.00

$$-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^3)/(3*b^4) + (a^2*x^5)/(5*b^3) - (a*x^7)/(7*b^2) + x^9
/(9*b) - (a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

fricas [A] time = 1.01, size = 170, normalized size = 2.10

$$\left[\frac{70\,b^4x^9-90\,ab^3x^7+126\,a^2b^2x^5-210\,a^3bx^3+315\,a^4\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+630\,a^4x}{630\,b^5},\ \frac{35\,b^4x^9-45\,ab^3x^7}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/630*(70*b^4*x^9 - 90*a*b^3*x^7 + 126*a^2*b^2*x^5 - 210*a^3*b*x^3 + 315*a
^4*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 630*a^4*x)/
b^5, 1/315*(35*b^4*x^9 - 45*a*b^3*x^7 + 63*a^2*b^2*x^5 - 105*a^3*b*x^3 - 31
5*a^4*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 315*a^4*x)/b^5]
```

giac [A] time = 1.08, size = 77, normalized size = 0.95

$$-\frac{a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^5}+\frac{35\,b^8x^9-45\,ab^7x^7+63\,a^2b^6x^5-105\,a^3b^5x^3+315\,a^4b^4x}{315\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^8*x^9 - 45*a*b^7*x
^7 + 63*a^2*b^6*x^5 - 105*a^3*b^5*x^3 + 315*a^4*b^4*x)/b^9
```

maple [A] time = 0.00, size = 71, normalized size = 0.88

$$\frac{x^9}{9b}-\frac{a\,x^7}{7b^2}+\frac{a^2x^5}{5b^3}-\frac{a^3x^3}{3b^4}-\frac{a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^5}+\frac{a^4x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a),x)
```

```
[Out] 1/9*x^9/b-1/7*a*x^7/b^2+1/5*a^2*x^5/b^3-1/3*a^3*x^3/b^4+a^4*x/b^5-a^5/b^5/(
a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

maxima [A] time = 2.92, size = 72, normalized size = 0.89

$$-\frac{a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^5}+\frac{35\,b^4x^9-45\,ab^3x^7+63\,a^2b^2x^5-105\,a^3bx^3+315\,a^4x}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^4*x^9 - 45*a*b^3*x
^7 + 63*a^2*b^2*x^5 - 105*a^3*b*x^3 + 315*a^4*x)/b^5
```

mupad [B] time = 0.06, size = 65, normalized size = 0.80

$$\frac{x^9}{9\,b}-\frac{a\,x^7}{7\,b^2}+\frac{a^4\,x}{b^5}-\frac{a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{a^2\,x^5}{5\,b^3}-\frac{a^3\,x^3}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2),x)
```

```
[Out] x^9/(9*b) - (a*x^7)/(7*b^2) + (a^4*x)/b^5 - (a^(9/2)*atan((b^(1/2)*x)/a^(1/
2)))/b^(11/2) + (a^2*x^5)/(5*b^3) - (a^3*x^3)/(3*b^4)
```

sympy [A] time = 0.20, size = 119, normalized size = 1.47

$$\frac{a^4x}{b^5} - \frac{a^3x^3}{3b^4} + \frac{a^2x^5}{5b^3} - \frac{ax^7}{7b^2} + \frac{\sqrt{-\frac{a^9}{b^{11}}}\log\left(x - \frac{b^5\sqrt{-\frac{a^9}{b^{11}}}}{a^4}\right)}{2} - \frac{\sqrt{-\frac{a^9}{b^{11}}}\log\left(x + \frac{b^5\sqrt{-\frac{a^9}{b^{11}}}}{a^4}\right)}{2} + \frac{x^9}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a),x)
```

```
[Out] a**4*x/b**5 - a**3*x**3/(3*b**4) + a**2*x**5/(5*b**3) - a*x**7/(7*b**2) + s
qrt(-a**9/b**11)*log(x - b**5*sqrt(-a**9/b**11)/a**4)/2 - sqrt(-a**9/b**11)
*log(x + b**5*sqrt(-a**9/b**11)/a**4)/2 + x**9/(9*b)
```

3.126 $\int \frac{x^9}{a+bx^2}\,dx$

Optimal. Leaf size=66

$$\frac{a^4\log\left(a+bx^2\right)}{2b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}$$

```
[Out] -1/2*a^3*x^2/b^4+1/4*a^2*x^4/b^3-1/6*a*x^6/b^2+1/8*x^8/b+1/2*a^4*ln(b*x^2+a
)/b^5
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = $\{266, 43\}$

$$\frac{a^2x^4}{4b^3}-\frac{a^3x^2}{2b^4}+\frac{a^4\log\left(a+bx^2\right)}{2b^5}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2),x]
```

```
[Out] -(a^3*x^2)/(2*b^4) + (a^2*x^4)/(4*b^3) - (a*x^6)/(6*b^2) + x^8/(8*b) + (a^4
*Log[a + b*x^2])/(2*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}+\frac{a^4\log\left(a+bx^2\right)}{2b^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 66, normalized size = 1.00

$$\frac{a^4\log\left(a+bx^2\right)}{2b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2),x]
```

```
[Out] -1/2*(a^3*x^2)/b^4 + (a^2*x^4)/(4*b^3) - (a*x^6)/(6*b^2) + x^8/(8*b) + (a^4
*Log[a + b*x^2])/(2*b^5)
```

fricas [A] time = 0.62, size = 56, normalized size = 0.85

$$\frac{3\,b^4x^8 - 4\,ab^3x^6 + 6\,a^2b^2x^4 - 12\,a^3bx^2 + 12\,a^4\log\left(bx^2 + a\right)}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 12*a^3*b*x^2 + 12*a^4*log(b
*x^2 + a))/b^5
```

giac [A] time = 1.08, size = 58, normalized size = 0.88

$$\frac{a^4\log\left(\left|bx^2 + a\right|\right)}{2\,b^5} + \frac{3\,b^3x^8 - 4\,ab^2x^6 + 6\,a^2bx^4 - 12\,a^3x^2}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*a^4*log(abs(b*x^2 + a))/b^5 + 1/24*(3*b^3*x^8 - 4*a*b^2*x^6 + 6*a^2*b*x
^4 - 12*a^3*x^2)/b^4
```

maple [A] time = 0.00, size = 57, normalized size = 0.86

$$\frac{x^8}{8b} - \frac{a\,x^6}{6b^2} + \frac{a^2x^4}{4b^3} - \frac{a^3x^2}{2b^4} + \frac{a^4\ln\left(b\,x^2 + a\right)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a),x)
```

```
[Out] -1/2*a^3*x^2/b^4+1/4*a^2*x^4/b^3-1/6*a*x^6/b^2+1/8*x^8/b+1/2*a^4*ln(b*x^2+a
)/b^5
```

maxima [A] time = 1.35, size = 57, normalized size = 0.86

$$\frac{a^4\log\left(bx^2 + a\right)}{2\,b^5} + \frac{3\,b^3x^8 - 4\,ab^2x^6 + 6\,a^2bx^4 - 12\,a^3x^2}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*a^4*log(b*x^2 + a)/b^5 + 1/24*(3*b^3*x^8 - 4*a*b^2*x^6 + 6*a^2*b*x^4 -
12*a^3*x^2)/b^4
```

mupad [B] time = 0.08, size = 56, normalized size = 0.85

$$\frac{x^8}{8\,b} - \frac{a\,x^6}{6\,b^2} + \frac{a^4\,\ln\left(b\,x^2 + a\right)}{2\,b^5} + \frac{a^2\,x^4}{4\,b^3} - \frac{a^3\,x^2}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2),x)
```

```
[Out] x^8/(8*b) - (a*x^6)/(6*b^2) + (a^4*log(a + b*x^2))/(2*b^5) + (a^2*x^4)/(4*b
^3) - (a^3*x^2)/(2*b^4)
```

sympy [A] time = 0.17, size = 56, normalized size = 0.85

$$\frac{a^4\log\left(a + bx^2\right)}{2b^5} - \frac{a^3x^2}{2b^4} + \frac{a^2x^4}{4b^3} - \frac{ax^6}{6b^2} + \frac{x^8}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a),x)
```

```
[Out] a**4*log(a + b*x**2)/(2*b**5) - a**3*x**2/(2*b**4) + a**2*x**4/(4*b**3) - a
*x**6/(6*b**2) + x**8/(8*b)
```

3.127 $\int \frac{x^8}{a+bx^2} \, dx$

Optimal. Leaf size=68

$$\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}$$

```
[Out] -a^3*x/b^4+1/3*a^2*x^3/b^3-1/5*a*x^5/b^2+1/7*x^7/b+a^(7/2)*arctan(x*b^(1/2)
/a^(1/2))/b^(9/2)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {302, 205}

$$\frac{a^2x^3}{3b^3}-\frac{a^3x}{b^4}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^3)/(3*b^3) - (a*x^5)/(5*b^2) + x^7/(7*b) + (a^(7/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{a+bx^2}\,dx &= \int \left(-\frac{a^3}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^4}{b^2}+\frac{x^6}{b}+\frac{a^4}{b^4\left(a+bx^2\right)}\right)dx\\
&= -\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}+\frac{a^4\int \frac{1}{a+bx^2}\,dx}{b^4}\\
&= -\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.00

$$\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^3)/(3*b^3) - (a*x^5)/(5*b^2) + x^7/(7*b) + (a^(7/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

fricas [A] time = 0.98, size = 148, normalized size = 2.18

$$\left[\frac{30\,b^3x^7-42\,ab^2x^5+70\,a^2bx^3+105\,a^3\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-210\,a^3x}{210\,b^4},\ \frac{15\,b^3x^7-21\,ab^2x^5+35\,a^2bx^3+1}{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*x^7 - 42*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*sqrt(-a/b)*log((
b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 210*a^3*x)/b^4, 1/105*(15*b^3*
x^7 - 21*a*b^2*x^5 + 35*a^2*b*x^3 + 105*a^3*sqrt(a/b)*arctan(b*x*sqrt(a/b)/
a) - 105*a^3*x)/b^4]
```

giac [A] time = 1.10, size = 65, normalized size = 0.96

$$\frac{a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{15\,b^6x^7-21\,ab^5x^5+35\,a^2b^4x^3-105\,a^3b^3x}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*x^7 - 21*a*b^5*x^
5 + 35*a^2*b^4*x^3 - 105*a^3*b^3*x)/b^7
```

maple [A] time = 0.00, size = 60, normalized size = 0.88

$$\frac{x^7}{7b}-\frac{a\,x^5}{5b^2}+\frac{a^2x^3}{3b^3}+\frac{a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^4}-\frac{a^3x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a),x)
```

```
[Out] 1/7*x^7/b-1/5*a*x^5/b^2+1/3*a^2*x^3/b^3-a^3*x/b^4+a^4/b^4/(a*b)^(1/2)*arcta
n(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.94, size = 60, normalized size = 0.88

$$\frac{a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{15\,b^3x^7-21\,ab^2x^5+35\,a^2bx^3-105\,a^3x}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^3*x^7 - 21*a*b^2*x^
5 + 35*a^2*b*x^3 - 105*a^3*x)/b^4
```

mupad [B] time = 0.05, size = 54, normalized size = 0.79

$$\frac{x^7}{7\,b}-\frac{a\,x^5}{5\,b^2}-\frac{a^3\,x}{b^4}+\frac{a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{a^2\,x^3}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2),x)
```

```
[Out] x^7/(7*b) - (a*x^5)/(5*b^2) - (a^3*x)/b^4 + (a^(7/2)*atan((b^(1/2)*x)/a^(1/
2)))/b^(9/2) + (a^2*x^3)/(3*b^3)
```

sympy [A] time = 0.19, size = 107, normalized size = 1.57

$$-\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}-\frac{\sqrt{-\frac{a^7}{b^9}}\log\left(x-\frac{b^4\sqrt{-\frac{a^7}{b^9}}}{a^3}\right)}{2}+\frac{\sqrt{-\frac{a^7}{b^9}}\log\left(x+\frac{b^4\sqrt{-\frac{a^7}{b^9}}}{a^3}\right)}{2}+\frac{x^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a),x)
```

```
[Out] -a**3*x/b**4 + a**2*x**3/(3*b**3) - a*x**5/(5*b**2) - sqrt(-a**7/b**9)*log(
x - b**4*sqrt(-a**7/b**9)/a**3)/2 + sqrt(-a**7/b**9)*log(x + b**4*sqrt(-a**
7/b**9)/a**3)/2 + x**7/(7*b)
```

3.128 $\int \frac{x^7}{a+bx^2}\,dx$

Optimal. Leaf size=53

$$-\frac{a^3\log\left(a+bx^2\right)}{2b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}$$

```
[Out] 1/2*a^2*x^2/b^3-1/4*a*x^4/b^2+1/6*x^6/b-1/2*a^3*ln(b*x^2+a)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^2}{2b^3}-\frac{a^3\log\left(a+bx^2\right)}{2b^4}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2),x]
```

```
[Out] (a^2*x^2)/(2*b^3) - (a*x^4)/(4*b^2) + x^6/(6*b) - (a^3*Log[a + b*x^2])/(2*b
^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{a^2x^2}{2b^3}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}-\frac{a^3\log\left(a+bx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.00

$$-\frac{a^3\log\left(a+bx^2\right)}{2b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2),x]
```

```
[Out] (a^2*x^2)/(2*b^3) - (a*x^4)/(4*b^2) + x^6/(6*b) - (a^3*Log[a + b*x^2])/(2*b
^4)
```

fricas [A] time = 0.70, size = 45, normalized size = 0.85

$$\frac{2\,b^3x^6 - 3\,ab^2x^4 + 6\,a^2bx^2 - 6\,a^3\log\left(bx^2+a\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^6 - 3*a*b^2*x^4 + 6*a^2*b*x^2 - 6*a^3*log(b*x^2 + a))/b^4
```

giac [A] time = 1.05, size = 47, normalized size = 0.89

$$-\frac{a^3\log\left(\left|bx^2+a\right|\right)}{2\,b^4} + \frac{2\,b^2x^6 - 3\,abx^4 + 6\,a^2x^2}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*a^3*log(abs(b*x^2 + a))/b^4 + 1/12*(2*b^2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)
/b^3
```

maple [A] time = 0.00, size = 46, normalized size = 0.87

$$\frac{x^6}{6b} - \frac{a\,x^4}{4b^2} + \frac{a^2x^2}{2b^3} - \frac{a^3\ln\left(b\,x^2+a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a),x)
```

```
[Out] 1/2*a^2*x^2/b^3-1/4*a*x^4/b^2+1/6*x^6/b-1/2*a^3*ln(b*x^2+a)/b^4
```

maxima [A] time = 1.36, size = 46, normalized size = 0.87

$$-\frac{a^3\log\left(bx^2+a\right)}{2\,b^4} + \frac{2\,b^2x^6 - 3\,abx^4 + 6\,a^2x^2}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*a^3*log(b*x^2 + a)/b^4 + 1/12*(2*b^2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3
```

mupad [B] time = 4.73, size = 45, normalized size = 0.85

$$\frac{x^6}{6\,b} - \frac{a\,x^4}{4\,b^2} - \frac{a^3\,\ln\left(b\,x^2+a\right)}{2\,b^4} + \frac{a^2\,x^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2),x)
```

```
[Out] x^6/(6*b) - (a*x^4)/(4*b^2) - (a^3*log(a + b*x^2))/(2*b^4) + (a^2*x^2)/(2*b
^3)
```

sympy [A] time = 0.16, size = 44, normalized size = 0.83

$$-\frac{a^3\log\left(a+bx^2\right)}{2b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^4}{4b^2} + \frac{x^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a),x)
```

```
[Out] -a**3*log(a + b*x**2)/(2*b**4) + a**2*x**2/(2*b**3) - a*x**4/(4*b**2) + x**
6/(6*b)
```

3.129 $\int \frac{x^6}{a+bx^2}\,dx$

Optimal. Leaf size=55

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}$$

```
[Out] a^2*x/b^3-1/3*a*x^3/b^2+1/5*x^5/b-a^(5/2)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {302, 205}

$$\frac{a^2x}{b^3}-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{a+bx^2}\,dx &= \int \left(\frac{a^2}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3\left(a+bx^2\right)}\right)dx\\ &= \frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}-\frac{a^3\int \frac{1}{a+bx^2}\,dx}{b^3}\\ &= \frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 1.00

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(7/2)
```

fricas [A] time = 1.18, size = 126, normalized size = 2.29

$$\left[\frac{6\,b^{2}x^{5} - 10\,abx^{3} + 15\,a^{2}\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2} - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^{2} + a}\right) + 30\,a^{2}x}{30\,b^{3}}, \frac{3\,b^{2}x^{5} - 5\,abx^{3} - 15\,a^{2}\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 15\,a^{2}x}{15\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*x^5 - 10*a*b*x^3 + 15*a^2*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-
a/b) - a)/(b*x^2 + a)) + 30*a^2*x)/b^3, 1/15*(3*b^2*x^5 - 5*a*b*x^3 - 15*a^
2*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 15*a^2*x)/b^3]
```

giac [A] time = 0.99, size = 55, normalized size = 1.00

$$-\frac{a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}} + \frac{3\,b^{4}x^{5} - 5\,ab^{3}x^{3} + 15\,a^{2}b^{2}x}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^4*x^5 - 5*a*b^3*x^3
+ 15*a^2*b^2*x)/b^5
```

maple [A] time = 0.00, size = 49, normalized size = 0.89

$$\frac{x^{5}}{5b} - \frac{a\,x^{3}}{3b^{2}} - \frac{a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^{3}} + \frac{a^{2}x}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a),x)
```

```
[Out] 1/5*x^5/b-1/3*a*x^3/b^2+a^2*x/b^3-a^3/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*
b*x)
```

maxima [A] time = 2.84, size = 50, normalized size = 0.91

$$-\frac{a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}} + \frac{3\,b^{2}x^{5} - 5\,abx^{3} + 15\,a^{2}x}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^2*x^5 - 5*a*b*x^3 +
15*a^2*x)/b^3
```

mupad [B] time = 0.07, size = 43, normalized size = 0.78

$$\frac{x^{5}}{5\,b} - \frac{a\,x^{3}}{3\,b^{2}} + \frac{a^{2}\,x}{b^{3}} - \frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2),x)
```

```
[Out] x^5/(5*b) - (a*x^3)/(3*b^2) + (a^2*x)/b^3 - (a^(5/2)*atan((b^(1/2)*x)/a^(1/
2)))/b^(7/2)
```

sympy [A] time = 0.18, size = 95, normalized size = 1.73

$$\frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{\sqrt{-\frac{a^5}{b^7}}\log\left(x - \frac{b^3\sqrt{-\frac{a^5}{b^7}}}{a^2}\right)}{2} - \frac{\sqrt{-\frac{a^5}{b^7}}\log\left(x + \frac{b^3\sqrt{-\frac{a^5}{b^7}}}{a^2}\right)}{2} + \frac{x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a),x)
```

```
[Out] a**2*x/b**3 - a*x**3/(3*b**2) + sqrt(-a**5/b**7)*log(x - b**3*sqrt(-a**5/b*
*7)/a**2)/2 - sqrt(-a**5/b**7)*log(x + b**3*sqrt(-a**5/b**7)/a**2)/2 + x**5
/(5*b)
```

3.130 $\int \frac{x^5}{a+bx^2}\,dx$

Optimal. Leaf size=40

$$\frac{a^2 \log\left(a+bx^2\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}$$

```
[Out] -1/2*a*x^2/b^2+1/4*x^4/b+1/2*a^2*ln(b*x^2+a)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2 \log\left(a+bx^2\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2),x]
```

```
[Out] -(a*x^2)/(2*b^2) + x^4/(4*b) + (a^2*Log[a + b*x^2])/(2*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx, x, x^2\right)\\
&= -\frac{ax^2}{2b^2}+\frac{x^4}{4b}+\frac{a^2\log\left(a+bx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{a^2 \log\left(a+bx^2\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2),x]
```

```
[Out] -1/2*(a*x^2)/b^2 + x^4/(4*b) + (a^2*Log[a + b*x^2])/(2*b^3)
```

fricas [A] time = 1.15, size = 33, normalized size = 0.82

$$\frac{b^2x^4-2\,abx^2+2\,a^2\log\left(bx^2+a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^4 - 2*a*b*x^2 + 2*a^2*log(b*x^2 + a))/b^3
```

giac [A] time = 1.12, size = 35, normalized size = 0.88

$$\frac{a^2 \log\left(\left|bx^2 + a\right|\right)}{2\,b^3} + \frac{bx^4 - 2\,ax^2}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(abs(b*x^2 + a))/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2
```

maple [A] time = 0.00, size = 35, normalized size = 0.88

$$\frac{x^4}{4b} - \frac{a\,x^2}{2b^2} + \frac{a^2 \ln\left(b\,x^2 + a\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a),x)
```

```
[Out] -1/2*a*x^2/b^2+1/4*x^4/b+1/2*a^2*ln(b*x^2+a)/b^3
```

maxima [A] time = 1.31, size = 34, normalized size = 0.85

$$\frac{a^2 \log\left(bx^2 + a\right)}{2\,b^3} + \frac{bx^4 - 2\,ax^2}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(b*x^2 + a)/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2
```

mupad [B] time = 4.65, size = 33, normalized size = 0.82

$$\frac{2\,a^2\,\ln\left(b\,x^2 + a\right) + b^2\,x^4 - 2\,a\,b\,x^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2),x)
```

```
[Out] (2*a^2*log(a + b*x^2) + b^2*x^4 - 2*a*b*x^2)/(4*b^3)
```

sympy [A] time = 0.15, size = 32, normalized size = 0.80

$$\frac{a^2 \log\left(a + bx^2\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a),x)
```

```
[Out] a**2*log(a + b*x**2)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b)
```

3.131 $\int \frac{x^4}{a+bx^2}\,dx$

Optimal. Leaf size=42

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}$$

```
[Out] -a*x/b^2+1/3*x^3/b+a^(3/2)*arctan(x*b^(1/2)/a^(1/2))/b^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {302, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2),x]
```

```
[Out] -((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a+bx^2}\,dx&=\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx\\&=-\frac{ax}{b^2}+\frac{x^3}{3b}+\frac{a^2\int \frac{1}{a+bx^2}\,dx}{b^2}\\&=-\frac{ax}{b^2}+\frac{x^3}{3b}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2),x]
```

```
[Out] -((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

fricas [A] time = 1.05, size = 99, normalized size = 2.36

$$\left[\frac{2\, bx^{3} + 3\, a\sqrt{-\frac{a}{b}} \log\left(\frac{bx^{2} + 2\, bx\sqrt{-\frac{a}{b}} - a}{bx^{2} + a}\right) - 6\, ax}{6\, b^{2}}, \frac{bx^{3} + 3\, a\sqrt{\frac{a}{b}} \arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - 3\, ax}{3\, b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*x^3 + 3*a*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
a)) - 6*a*x)/b^2, 1/3*(b*x^3 + 3*a*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*a*
x)/b^2]
```

giac [A] time = 1.01, size = 40, normalized size = 0.95

$$\frac{a^{2} \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, b^{2}} + \frac{b^{2} x^{3} - 3\, abx}{3\, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*x^3 - 3*a*b*x)/b^3
```

maple [A] time = 0.00, size = 38, normalized size = 0.90

$$\frac{x^{3}}{3b} + \frac{a^{2} \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\; b^{2}} - \frac{ax}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a),x)
```

```
[Out] 1/3*x^3/b-a*x/b^2+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.80, size = 37, normalized size = 0.88

$$\frac{a^{2} \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, b^{2}} + \frac{bx^{3} - 3\, ax}{3\, b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*x^3 - 3*a*x)/b^2
```

mupad [B] time = 0.07, size = 32, normalized size = 0.76

$$\frac{x^{3}}{3\, b} + \frac{a^{3/2} \operatorname{atan}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{b^{5/2}} - \frac{a\, x}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2),x)
```

```
[Out] x^3/(3*b) + (a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(5/2) - (a*x)/b^2
```

<u>**sympy**</u> **[B]** time = 0.17, size = 80, normalized size = 1.90

$$-\frac{ax}{b^2}-\frac{\sqrt{-\frac{a^3}{b^5}}\log\left(x-\frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2}+\frac{\sqrt{-\frac{a^3}{b^5}}\log\left(x+\frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2}+\frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a),x)
```

```
[Out] -a*x/b**2 - sqrt(-a**3/b**5)*log(x - b**2*sqrt(-a**3/b**5)/a)/2 + sqrt(-a**
3/b**5)*log(x + b**2*sqrt(-a**3/b**5)/a)/2 + x**3/(3*b)
```

3.132 $\int \frac{x^3}{a+bx^2} dx$

Optimal. Leaf size=27

$$\frac{x^2}{2b} - \frac{a\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] 1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{x^2}{2b} - \frac{a\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2),x]
```

```
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right)dx, x, x^2\right)\\
&= \frac{x^2}{2b} - \frac{a\log\left(a+bx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{x^2}{2b} - \frac{a\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2),x]
```

```
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

fricas [A] time = 1.07, size = 22, normalized size = 0.81

$$\frac{bx^2 - a\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

giac [A]   time = 1.09, size = 24, normalized size = 0.89

$$\frac{x^2}{2\,b} - \frac{a\log\left(\left|bx^2 + a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/b - 1/2*a*log(abs(b*x^2 + a))/b^2
```

maple [A]   time = 0.00, size = 24, normalized size = 0.89

$$\frac{x^2}{2b} - \frac{a\ln\left(b\,x^2 + a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a),x)
```

```
[Out] 1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2
```

maxima [A]   time = 1.32, size = 23, normalized size = 0.85

$$\frac{x^2}{2\,b} - \frac{a\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

mupad [B]   time = 0.04, size = 22, normalized size = 0.81

$$-\frac{a\,\ln\left(b\,x^2 + a\right) - b\,x^2}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2),x)
```

```
[Out] -(a*log(a + b*x^2) - b*x^2)/(2*b^2)
```

sympy [A]   time = 0.13, size = 20, normalized size = 0.74

$$-\frac{a\log\left(a + bx^2\right)}{2b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a),x)
```

```
[Out] -a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

3.133 $\int \frac{x^2}{a+bx^2} \, dx$

Optimal. Leaf size=31

$$\frac{x}{b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] x/b-arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = $\{321, 205\}$

$$\frac{x}{b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{a+bx^2} \, dx &= \frac{x}{b} - \frac{a \int \frac{1}{a+bx^2} \, dx}{b} \\ &= \frac{x}{b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{b^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{x}{b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

fricas [A] time = 0.77, size = 82, normalized size = 2.65

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2x}{2b}, -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
-(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

giac [A] time = 1.12, size = 26, normalized size = 0.84

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

maple [A] time = 0.00, size = 27, normalized size = 0.87

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\ b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a),x)
```

```
[Out] 1/b*x-a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 26, normalized size = 0.84

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

mupad [B] time = 0.03, size = 23, normalized size = 0.74

$$\frac{x}{b}-\frac{\sqrt{a}\ \mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2),x)
```

```
[Out] x/b - (a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
```

sympy [B] time = 0.15, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a),x)
```

```
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b**
3) + x)/2 + x/b
```

3.134 $\int \frac{x}{a+bx^2} \, dx$

Optimal. Leaf size=15

$$\frac{\log(a+bx^2)}{2b}$$

[Out] 1/2*ln(b*x^2+a)/b

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {260}

$$\frac{\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x^2),x]

[Out] Log[a + b*x^2]/(2*b)

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{a+bx^2} \, dx = \frac{\log(a+bx^2)}{2b}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x^2),x]

[Out] Log[a + b*x^2]/(2*b)

fricas [A] time = 0.68, size = 13, normalized size = 0.87

$$\frac{\log(bx^2+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^2+a),x, algorithm="fricas")

[Out] 1/2*log(b*x^2 + a)/b

giac [A] time = 1.07, size = 14, normalized size = 0.93

$$\frac{\log(|bx^2+a|)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a),x, algorithm="giac")
[Out] 1/2*log(abs(b*x^2 + a))/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln \left(b\,x^{2}+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a),x)
[Out] 1/2*ln(b*x^2+a)/b
```

maxima [A] time = 1.34, size = 13, normalized size = 0.87

$$\frac{\log \left(bx^{2}+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a),x, algorithm="maxima")
[Out] 1/2*log(b*x^2 + a)/b
```

mupad [B] time = 4.63, size = 13, normalized size = 0.87

$$\frac{\ln \left(b\,x^{2}+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2),x)
[Out] log(a + b*x^2)/(2*b)
```

sympy [A] time = 0.11, size = 10, normalized size = 0.67

$$\frac{\log \left(a+bx^{2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a),x)
[Out] log(a + b*x**2)/(2*b)
```

3.135 $\int \frac{1}{a+bx^2}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

[Out] arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(-1),x]

[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+bx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(-1),x]

[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])

fricas [A] time = 1.03, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab},\ \frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a),x, algorithm="fricas")

[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]

giac [A] time = 0.97, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="giac")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.00, size = 16, normalized size = 0.67

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a),x)
[Out] 1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.90, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="maxima")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 4.70, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2),x)
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

sympy [B] time = 0.13, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a),x)
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

3.136 $\int \frac{1}{x(a+bx^2)} dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

[Out] ln(x)/a-1/2*ln(b*x^2+a)/a

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^2)),x]

[Out] Log[x]/a - Log[a + b*x^2]/(2*a)

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)} dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, x^2\right)}{2a} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx} dx, x, x^2\right)}{2a}\\
&= \frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)),x]
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

fricas [A] time = 0.87, size = 18, normalized size = 0.82

$$-\frac{\log\left(bx^2+a\right)-2\,\log(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a),x, algorithm="fricas")
[Out] -1/2*(log(b*x^2 + a) - 2*log(x))/a
```

giac [A] time = 1.09, size = 24, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,a}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a),x, algorithm="giac")
[Out] 1/2*log(x^2)/a - 1/2*log(abs(b*x^2 + a))/a
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{a}-\frac{\ln\left(b\,x^2+a\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a),x)
[Out] ln(x)/a-1/2*ln(b*x^2+a)/a
```

maxima [A] time = 1.33, size = 23, normalized size = 1.05

$$-\frac{\log\left(bx^2+a\right)}{2\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 + a)/a + 1/2*log(x^2)/a
```

mupad [B] time = 0.08, size = 18, normalized size = 0.82

$$-\frac{\ln\left(b\,x^2+a\right)-2\,\ln(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)),x)
[Out] -(log(a + b*x^2) - 2*log(x))/(2*a)
```

sympy [A] time = 0.20, size = 15, normalized size = 0.68

$$\frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a),x)
```

```
[Out] log(x)/a - log(a/b + x**2)/(2*a)
```

3.137 $\int \frac{1}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

```
[Out] -1/a/x-arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {325, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2\right)}\,dx &= -\frac{1}{ax}-\frac{b\int \frac{1}{a+bx^2}\,dx}{a}\\ &= -\frac{1}{ax}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 0.79, size = 82, normalized size = 2.41

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax}, -\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a*
x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

giac [A] time = 1.13, size = 29, normalized size = 0.85

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

maple [A] time = 0.01, size = 30, normalized size = 0.88

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a),x)
```

```
[Out] -1/a/x-1/a*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.89, size = 29, normalized size = 0.85

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

mupad [B] time = 4.62, size = 26, normalized size = 0.76

$$-\frac{1}{a\,x}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)),x)
```

```
[Out] - 1/(a*x) - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

sympy [B] time = 0.18, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sqr
t(-b/a**3)/b + x)/2 - 1/(a*x)
```

3.138 $\int \frac{1}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=35

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

[Out] -1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a + b*x^2)),x]

[Out] -1/(2*a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a + b*x^2)),x]

[Out] -1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)

fricas [A] time = 0.91, size = 33, normalized size = 0.94

$$\frac{bx^2\log\left(bx^2+a\right)-2\,bx^2\log(x)-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

giac [A] time = 1.15, size = 43, normalized size = 1.23

$$-\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{b\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2)/a^2 + 1/2*b*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 - a)/(a^2*
x^2)
```

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(b\,x^2+a\right)}{2a^2}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a),x)
```

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

maxima [A] time = 1.37, size = 33, normalized size = 0.94

$$\frac{b\log\left(bx^2+a\right)}{2\,a^2}-\frac{b\log\left(x^2\right)}{2\,a^2}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)
```

mupad [B] time = 0.07, size = 31, normalized size = 0.89

$$\frac{b\,\ln\left(b\,x^2+a\right)}{2\,a^2}-\frac{1}{2\,a\,x^2}-\frac{b\,\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)),x)
```

```
[Out] (b*log(a + b*x^2))/(2*a^2) - 1/(2*a*x^2) - (b*log(x))/a^2
```

sympy [A] time = 0.26, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a),x)
```

```
[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

3.139 $\int \frac{1}{x^4\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=43

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

```
[Out] -1/3/a/x^3+b/a^2/x+b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {325, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^2\right)}\,dx &= -\frac{1}{3ax^3}-\frac{b\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^2\int \frac{1}{a+bx^2}\,dx}{a^2}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 1.03, size = 106, normalized size = 2.47

$$\left[\frac{3\,bx^3\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,bx^2-2\,a}{6\,a^2x^3},\ \frac{3\,bx^3\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,bx^2-a}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*b*
x^2 - a)/(a^2*x^3)]
```

giac [A] time = 1.12, size = 40, normalized size = 0.93

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,bx^2-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

maple [A] time = 0.01, size = 39, normalized size = 0.91

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2}+\frac{b}{a^2x}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a),x)
```

```
[Out] -1/3/a/x^3+b/a^2/x+b^2/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.88, size = 40, normalized size = 0.93

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,bx^2-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

mupad [B] time = 4.67, size = 37, normalized size = 0.86

$$\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{1}{3\,a}-\frac{b\,x^2}{a^2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)),x)
```

```
[Out] (b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(5/2) - (1/(3*a) - (b*x^2)/a^2)/x^3
```

sympy [B] time = 0.22, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{-a+3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5)
*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

3.140 $\int \frac{1}{x^5(a+bx^2)} dx$

Optimal. Leaf size=49

$$-\frac{b^2 \log \left(a+bx^2\right)}{2a^3}+\frac{b^2 \log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b^2 \log \left(a+bx^2\right)}{2a^3}+\frac{b^2 \log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)),x]
```

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2*a
^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(a+bx^2\right)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right) dx, x, x^2\right)\\
&= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2 \log(x)}{a^3}-\frac{b^2 \log \left(a+bx^2\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$-\frac{b^2 \log \left(a+bx^2\right)}{2a^3}+\frac{b^2 \log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2
*a^3)
```

fricas [A] time = 0.65, size = 45, normalized size = 0.92

$$-\frac{2\,b^2x^4\log\left(bx^2+a\right)-4\,b^2x^4\log(x)-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^4*log(b*x^2 + a) - 4*b^2*x^4*log(x) - 2*a*b*x^2 + a^2)/(a^3*x
^4)
```

giac [A] time = 1.19, size = 57, normalized size = 1.16

$$\frac{b^2\log\left(x^2\right)}{2\,a^3}-\frac{b^2\log\left(\left|bx^2+a\right|\right)}{2\,a^3}-\frac{3\,b^2x^4-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2)/a^3 - 1/2*b^2*log(abs(b*x^2 + a))/a^3 - 1/4*(3*b^2*x^4 - 2
*a*b*x^2 + a^2)/(a^3*x^4)
```

maple [A] time = 0.01, size = 44, normalized size = 0.90

$$\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln\left(b\,x^2+a\right)}{2a^3}+\frac{b}{2a^2x^2}-\frac{1}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a),x)
```

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

maxima [A] time = 1.36, size = 47, normalized size = 0.96

$$-\frac{b^2\log\left(bx^2+a\right)}{2\,a^3}+\frac{b^2\log\left(x^2\right)}{2\,a^3}+\frac{2\,bx^2-a}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(b*x^2 + a)/a^3 + 1/2*b^2*log(x^2)/a^3 + 1/4*(2*b*x^2 - a)/(a^2
*x^4)
```

mupad [B] time = 0.08, size = 46, normalized size = 0.94

$$\frac{b^2\,\ln(x)}{a^3}-\frac{b^2\,\ln\left(b\,x^2+a\right)}{2\,a^3}-\frac{\frac{1}{4\,a}-\frac{b\,x^2}{2\,a^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)),x)
```

```
[Out] (b^2*log(x))/a^3 - (b^2*log(a + b*x^2))/(2*a^3) - (1/(4*a) - (b*x^2)/(2*a^2
))/x^4
```

sympy [A] time = 0.30, size = 42, normalized size = 0.86

$$\frac{-a+2bx^2}{4a^2x^4}+\frac{b^2\log\left(x\right)}{a^3}-\frac{b^2\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a),x)
```

```
[Out] (-a + 2*b*x**2)/(4*a**2*x**4) + b**2*log(x)/a**3 - b**2*log(a/b + x**2)/(2*
a**3)
```

3.141 $\int \frac{1}{x^6(a+bx^2)}\,dx$

Optimal. Leaf size=58

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}-\frac{b^2}{a^3x}+\frac{b}{3a^2x^3}-\frac{1}{5ax^5}$$

```
[Out] -1/5/a/x^5+1/3*b/a^2/x^3-b^2/a^3/x-b^(5/2)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2
)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {325, 205}

$$-\frac{b^2}{a^3x}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}+\frac{b}{3a^2x^3}-\frac{1}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)),x]
```

```
[Out] -1/(5*a*x^5) + b/(3*a^2*x^3) - b^2/(a^3*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/a^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(a+bx^2\right)}\,dx &= -\frac{1}{5ax^5}-\frac{b\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{a}\\
&= -\frac{1}{5ax^5}+\frac{b}{3a^2x^3}+\frac{b^2\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a^2}\\
&= -\frac{1}{5ax^5}+\frac{b}{3a^2x^3}-\frac{b^2}{a^3x}-\frac{b^3\int \frac{1}{a+bx^2}\,dx}{a^3}\\
&= -\frac{1}{5ax^5}+\frac{b}{3a^2x^3}-\frac{b^2}{a^3x}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.00

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}-\frac{b^2}{a^3x}+\frac{b}{3a^2x^3}-\frac{1}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)),x]
```

```
[Out] -1/5*1/(a*x^5) + b/(3*a^2*x^3) - b^2/(a^3*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/a^(7/2)
```

fricas [A] time = 0.80, size = 132, normalized size = 2.28

$$\left[\frac{15\, b^2 x^5 \sqrt{-\frac{b}{a}} \log\left(\frac{bx^2 - 2\, ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) - 30\, b^2 x^4 + 10\, abx^2 - 6\, a^2}{30\, a^3 x^5}, -\frac{15\, b^2 x^5 \sqrt{\frac{b}{a}} \arctan\left(x\sqrt{\frac{b}{a}}\right) + 15\, b^2 x^4 - 5\, abx^2 +}{15\, a^3 x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(15*b^2*x^5*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)
) - 30*b^2*x^4 + 10*a*b*x^2 - 6*a^2)/(a^3*x^5), -1/15*(15*b^2*x^5*sqrt(b/a)
*arctan(x*sqrt(b/a)) + 15*b^2*x^4 - 5*a*b*x^2 + 3*a^2)/(a^3*x^5)]
```

giac [A] time = 0.62, size = 52, normalized size = 0.90

$$-\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, a^3} - \frac{15\, b^2 x^4 - 5\, abx^2 + 3\, a^2}{15\, a^3 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/15*(15*b^2*x^4 - 5*a*b*x^2 +
 3*a^2)/(a^3*x^5)
```

maple [A] time = 0.01, size = 52, normalized size = 0.90

$$-\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\; a^3} - \frac{b^2}{a^3 x} + \frac{b}{3a^2 x^3} - \frac{1}{5a\, x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a),x)
```

```
[Out] -1/5/a/x^5-b^2/a^3/x+1/3*b/a^2/x^3-b^3/a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)
*b*x)
```

maxima [A] time = 2.89, size = 52, normalized size = 0.90

$$-\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, a^3} - \frac{15\, b^2 x^4 - 5\, abx^2 + 3\, a^2}{15\, a^3 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/15*(15*b^2*x^4 - 5*a*b*x^2 +
 3*a^2)/(a^3*x^5)
```

mupad [B] time = 0.06, size = 48, normalized size = 0.83

$$-\frac{\frac{1}{5\,a}-\frac{b\,x^2}{3\,a^2}+\frac{b^2\,x^4}{a^3}}{x^5}-\frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)),x)
```

```
[Out] - (1/(5*a) - (b*x^2)/(3*a^2) + (b^2*x^4)/a^3)/x^5 - (b^(5/2)*atan((b^(1/2)*
x)/a^(1/2)))/a^(7/2)
```

sympy [B] time = 0.26, size = 100, normalized size = 1.72

$$\frac{\sqrt{-\frac{b^5}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^5}{a^7}}}{b^3}+x\right)}{2}-\frac{\sqrt{-\frac{b^5}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^5}{a^7}}}{b^3}+x\right)}{2}+\frac{-3a^2+5abx^2-15b^2x^4}{15a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a),x)
```

```
[Out] sqrt(-b**5/a**7)*log(-a**4*sqrt(-b**5/a**7)/b**3 + x)/2 - sqrt(-b**5/a**7)*
log(a**4*sqrt(-b**5/a**7)/b**3 + x)/2 + (-3*a**2 + 5*a*b*x**2 - 15*b**2*x**
4)/(15*a**3*x**5)
```

3.142 $\int \frac{1}{x^7(a+bx^2)}\,dx$

Optimal. Leaf size=63

$$\frac{b^3\log\left(a+bx^2\right)}{2a^4}-\frac{b^3\log(x)}{a^4}-\frac{b^2}{2a^3x^2}+\frac{b}{4a^2x^4}-\frac{1}{6ax^6}$$

```
[Out] -1/6/a/x^6+1/4*b/a^2/x^4-1/2*b^2/a^3/x^2-b^3*ln(x)/a^4+1/2*b^3*ln(b*x^2+a)/
a^4
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b^2}{2a^3x^2}+\frac{b^3\log\left(a+bx^2\right)}{2a^4}-\frac{b^3\log(x)}{a^4}+\frac{b}{4a^2x^4}-\frac{1}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)),x]
```

```
[Out] -1/(6*a*x^6) + b/(4*a^2*x^4) - b^2/(2*a^3*x^2) - (b^3*Log[x])/a^4 + (b^3*Lo
g[a + b*x^2])/(2*a^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(a+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{6ax^6}+\frac{b}{4a^2x^4}-\frac{b^2}{2a^3x^2}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log\left(a+bx^2\right)}{2a^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 63, normalized size = 1.00

$$\frac{b^3\log\left(a+bx^2\right)}{2a^4}-\frac{b^3\log(x)}{a^4}-\frac{b^2}{2a^3x^2}+\frac{b}{4a^2x^4}-\frac{1}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)),x]
```

```
[Out] -1/6*1/(a*x^6) + b/(4*a^2*x^4) - b^2/(2*a^3*x^2) - (b^3*Log[x])/a^4 + (b^3*
Log[a + b*x^2])/(2*a^4)
```

fricas [A] time = 0.88, size = 58, normalized size = 0.92

$$\frac{6 b^3 x^6 \log\left(bx^2+a\right) - 12 b^3 x^6 \log(x) - 6 ab^2 x^4 + 3 a^2 bx^2 - 2 a^3}{12 a^4 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b^3*x^6*log(b*x^2 + a) - 12*b^3*x^6*log(x) - 6*a*b^2*x^4 + 3*a^2*b*
x^2 - 2*a^3)/(a^4*x^6)
```

giac [A] time = 0.63, size = 70, normalized size = 1.11

$$-\frac{b^3 \log\left(x^2\right)}{2 a^4} + \frac{b^3 \log\left(\left|bx^2+a\right|\right)}{2 a^4} + \frac{11 b^3 x^6 - 6 ab^2 x^4 + 3 a^2 bx^2 - 2 a^3}{12 a^4 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b^3*log(x^2)/a^4 + 1/2*b^3*log(abs(b*x^2 + a))/a^4 + 1/12*(11*b^3*x^6
- 6*a*b^2*x^4 + 3*a^2*b*x^2 - 2*a^3)/(a^4*x^6)
```

maple [A] time = 0.01, size = 56, normalized size = 0.89

$$-\frac{b^3 \ln(x)}{a^4} + \frac{b^3 \ln\left(b\,x^2+a\right)}{2a^4} - \frac{b^2}{2a^3 x^2} + \frac{b}{4a^2 x^4} - \frac{1}{6a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a),x)
```

```
[Out] -1/6/a/x^6+1/4*b/a^2/x^4-1/2*b^2/a^3/x^2-b^3*ln(x)/a^4+1/2*b^3*ln(b*x^2+a)/
a^4
```

maxima [A] time = 1.36, size = 58, normalized size = 0.92

$$\frac{b^3 \log\left(bx^2+a\right)}{2 a^4} - \frac{b^3 \log\left(x^2\right)}{2 a^4} - \frac{6 b^2 x^4 - 3 abx^2 + 2 a^2}{12 a^3 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*b^3*log(b*x^2 + a)/a^4 - 1/2*b^3*log(x^2)/a^4 - 1/12*(6*b^2*x^4 - 3*a*b
*x^2 + 2*a^2)/(a^3*x^6)
```

mupad [B] time = 4.64, size = 58, normalized size = 0.92

$$\frac{b^3\,\ln\left(b\,x^2+a\right)}{2\,a^4} - \frac{\frac{1}{6\,a} - \frac{b\,x^2}{4\,a^2} + \frac{b^2\,x^4}{2\,a^3}}{x^6} - \frac{b^3\,\ln(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)),x)
```

```
[Out] (b^3*log(a + b*x^2))/(2*a^4) - (1/(6*a) - (b*x^2)/(4*a^2) + (b^2*x^4)/(2*a^
3))/x^6 - (b^3*log(x))/a^4
```

sympy [**A**] time = 0.35, size = 56, normalized size = 0.89

$$\frac{-2a^2+3abx^2-6b^2x^4}{12a^3x^6}-\frac{b^3\log\left(x\right)}{a^4}+\frac{b^3\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a),x)
```

```
[Out] (-2*a**2 + 3*a*b*x**2 - 6*b**2*x**4)/(12*a**3*x**6) - b**3*log(x)/a**4 + b*
*3*log(a/b + x**2)/(2*a**4)
```

3.143 $\int \frac{1}{x^8\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=69

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}+\frac{b^3}{a^4x}-\frac{b^2}{3a^3x^3}+\frac{b}{5a^2x^5}-\frac{1}{7ax^7}$$

```
[Out] -1/7/a/x^7+1/5*b/a^2/x^5-1/3*b^2/a^3/x^3+b^3/a^4/x+b^(7/2)*arctan(x*b^(1/2)
/a^(1/2))/a^(9/2)
```

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {325, 205}

$$-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}+\frac{b}{5a^2x^5}-\frac{1}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/(7*a*x^7) + b/(5*a^2*x^5) - b^2/(3*a^3*x^3) + b^3/(a^4*x) + (b^(7/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8\left(a+bx^2\right)}\,dx &= -\frac{1}{7ax^7}-\frac{b\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{a}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}+\frac{b^2\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{a^2}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}-\frac{b^3\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a^3}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^4\int \frac{1}{a+bx^2}\,dx}{a^4}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 69, normalized size = 1.00

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}+\frac{b^3}{a^4x}-\frac{b^2}{3a^3x^3}+\frac{b}{5a^2x^5}-\frac{1}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/7*1/(a*x^7) + b/(5*a^2*x^5) - b^2/(3*a^3*x^3) + b^3/(a^4*x) + (b^(7/2)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

fricas [A] time = 0.95, size = 154, normalized size = 2.23

$$\left[\frac{105\,b^3x^7\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+210\,b^3x^6-70\,ab^2x^4+42\,a^2bx^2-30\,a^3}{210\,a^4x^7}, \frac{105\,b^3x^7\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+10\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(105*b^3*x^7*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
a)) + 210*b^3*x^6 - 70*a*b^2*x^4 + 42*a^2*b*x^2 - 30*a^3)/(a^4*x^7), 1/105*
(105*b^3*x^7*sqrt(b/a)*arctan(x*sqrt(b/a)) + 105*b^3*x^6 - 35*a*b^2*x^4 + 2
1*a^2*b*x^2 - 15*a^3)/(a^4*x^7)]
```

giac [A] time = 0.64, size = 62, normalized size = 0.90

$$\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}+\frac{105\,b^3x^6-35\,ab^2x^4+21\,a^2bx^2-15\,a^3}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/105*(105*b^3*x^6 - 35*a*b^2*x
^4 + 21*a^2*b*x^2 - 15*a^3)/(a^4*x^7)
```

maple [A] time = 0.01, size = 61, normalized size = 0.88

$$\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^4}+\frac{b^3}{a^4x}-\frac{b^2}{3a^3x^3}+\frac{b}{5a^2x^5}-\frac{1}{7a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^2+a),x)
```

```
[Out] -1/7/a/x^7-1/3*b^2/a^3/x^3+1/5*b/a^2/x^5+b^3/a^4/x+b^4/a^4/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.93, size = 62, normalized size = 0.90

$$\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}+\frac{105\,b^3x^6-35\,ab^2x^4+21\,a^2bx^2-15\,a^3}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/105*(105*b^3*x^6 - 35*a*b^2*x
^4 + 21*a^2*b*x^2 - 15*a^3)/(a^4*x^7)
```

mupad [B] time = 0.06, size = 59, normalized size = 0.86

$$\frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}-\frac{\frac{1}{7\,a}-\frac{b\,x^2}{5\,a^2}+\frac{b^2\,x^4}{3\,a^3}-\frac{b^3\,x^6}{a^4}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^2)),x)
```

```
[Out] (b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(9/2) - (1/(7*a) - (b*x^2)/(5*a^2) +
(b^2*x^4)/(3*a^3) - (b^3*x^6)/a^4)/x^7
```

sympy [A] time = 0.30, size = 112, normalized size = 1.62

$$-\frac{\sqrt{-\frac{b^7}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^7}{a^9}}}{b^4}+x\right)}{2}+\frac{\sqrt{-\frac{b^7}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^7}{a^9}}}{b^4}+x\right)}{2}+\frac{-15a^3+21a^2bx^2-35ab^2x^4+105b^3x^6}{105a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**7/a**9)*log(-a**5*sqrt(-b**7/a**9)/b**4 + x)/2 + sqrt(-b**7/a**9)
*log(a**5*sqrt(-b**7/a**9)/b**4 + x)/2 + (-15*a**3 + 21*a**2*b*x**2 - 35*a*
b**2*x**4 + 105*b**3*x**6)/(105*a**4*x**7)
```

3.144 $\int \frac{1}{x^9(a+bx^2)} dx$

Optimal. Leaf size=75

$$-\frac{b^4 \log(a+bx^2)}{2a^5} + \frac{b^4 \log(x)}{a^5} + \frac{b^3}{2a^4x^2} - \frac{b^2}{4a^3x^4} + \frac{b}{6a^2x^6} - \frac{1}{8ax^8}$$

```
[Out] -1/8/a/x^8+1/6*b/a^2/x^6-1/4*b^2/a^3/x^4+1/2*b^3/a^4/x^2+b^4*ln(x)/a^5-1/2*
b^4*ln(b*x^2+a)/a^5
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{b^3}{2a^4x^2} - \frac{b^2}{4a^3x^4} - \frac{b^4 \log(a+bx^2)}{2a^5} + \frac{b^4 \log(x)}{a^5} + \frac{b}{6a^2x^6} - \frac{1}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^2)),x]
```

```
[Out] -1/(8*a*x^8) + b/(6*a^2*x^6) - b^2/(4*a^3*x^4) + b^3/(2*a^4*x^2) + (b^4*Log
[x])/a^5 - (b^4*Log[a + b*x^2])/(2*a^5)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^9(a+bx^2)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^5(a+bx)} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^5} - \frac{b}{a^2x^4} + \frac{b^2}{a^3x^3} - \frac{b^3}{a^4x^2} + \frac{b^4}{a^5x} - \frac{b^5}{a^5(a+bx)}\right) dx, x, x^2\right) \\
&= -\frac{1}{8ax^8} + \frac{b}{6a^2x^6} - \frac{b^2}{4a^3x^4} + \frac{b^3}{2a^4x^2} + \frac{b^4 \log(x)}{a^5} - \frac{b^4 \log(a+bx^2)}{2a^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 75, normalized size = 1.00

$$-\frac{b^4 \log(a+bx^2)}{2a^5} + \frac{b^4 \log(x)}{a^5} + \frac{b^3}{2a^4x^2} - \frac{b^2}{4a^3x^4} + \frac{b}{6a^2x^6} - \frac{1}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^2)),x]
```

```
[Out] -1/8*1/(a*x^8) + b/(6*a^2*x^6) - b^2/(4*a^3*x^4) + b^3/(2*a^4*x^2) + (b^4*L
og[x])/a^5 - (b^4*Log[a + b*x^2])/(2*a^5)
```

fricas [A] time = 0.89, size = 69, normalized size = 0.92

$$-\frac{12\,b^4x^8\log\left(bx^2+a\right)-24\,b^4x^8\log(x)-12\,ab^3x^6+6\,a^2b^2x^4-4\,a^3bx^2+3\,a^4}{24\,a^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/24*(12*b^4*x^8*log(b*x^2 + a) - 24*b^4*x^8*log(x) - 12*a*b^3*x^6 + 6*a^2
*b^2*x^4 - 4*a^3*b*x^2 + 3*a^4)/(a^5*x^8)
```

giac [A] time = 0.63, size = 81, normalized size = 1.08

$$\frac{b^4\log\left(x^2\right)}{2\,a^5}-\frac{b^4\log\left(\left|bx^2+a\right|\right)}{2\,a^5}-\frac{25\,b^4x^8-12\,ab^3x^6+6\,a^2b^2x^4-4\,a^3bx^2+3\,a^4}{24\,a^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b^4*log(x^2)/a^5 - 1/2*b^4*log(abs(b*x^2 + a))/a^5 - 1/24*(25*b^4*x^8 -
 12*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + 3*a^4)/(a^5*x^8)
```

maple [A] time = 0.01, size = 66, normalized size = 0.88

$$\frac{b^4\ln(x)}{a^5}-\frac{b^4\ln\left(b\,x^2+a\right)}{2a^5}+\frac{b^3}{2a^4x^2}-\frac{b^2}{4a^3x^4}+\frac{b}{6a^2x^6}-\frac{1}{8a\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^2+a),x)
```

```
[Out] -1/8/a/x^8+1/6*b/a^2/x^6-1/4*b^2/a^3/x^4+1/2*b^3/a^4/x^2+b^4*ln(x)/a^5-1/2*
b^4*ln(b*x^2+a)/a^5
```

maxima [A] time = 1.33, size = 69, normalized size = 0.92

$$-\frac{b^4\log\left(bx^2+a\right)}{2\,a^5}+\frac{b^4\log\left(x^2\right)}{2\,a^5}+\frac{12\,b^3x^6-6\,ab^2x^4+4\,a^2bx^2-3\,a^3}{24\,a^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*b^4*log(b*x^2 + a)/a^5 + 1/2*b^4*log(x^2)/a^5 + 1/24*(12*b^3*x^6 - 6*a
*b^2*x^4 + 4*a^2*b*x^2 - 3*a^3)/(a^4*x^8)
```

mupad [B] time = 4.67, size = 68, normalized size = 0.91

$$\frac{b^4\,\ln(x)}{a^5}-\frac{b^4\,\ln\left(b\,x^2+a\right)}{2\,a^5}-\frac{\frac{1}{8\,a}-\frac{b\,x^2}{6\,a^2}+\frac{b^2\,x^4}{4\,a^3}-\frac{b^3\,x^6}{2\,a^4}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^2)),x)
```

```
[Out] (b^4*log(x))/a^5 - (b^4*log(a + b*x^2))/(2*a^5) - (1/(8*a) - (b*x^2)/(6*a^2
) + (b^2*x^4)/(4*a^3) - (b^3*x^6)/(2*a^4))/x^8
```

sympy [A] time = 0.39, size = 68, normalized size = 0.91

$$\frac{-3a^3+4a^2bx^2-6ab^2x^4+12b^3x^6}{24a^4x^8}+\frac{b^4\log{(x)}}{a^5}-\frac{b^4\log{\left(\frac{a}{b}+x^2\right)}}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**2+a),x)
```

```
[Out] (-3*a**3 + 4*a**2*b*x**2 - 6*a*b**2*x**4 + 12*b**3*x**6)/(24*a**4*x**8) + b
**4*log(x)/a**5 - b**4*log(a/b + x**2)/(2*a**5)
```

3.145 $\int \frac{x^{13}}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=94

$$-\frac{a^6}{2b^7\left(a+bx^2\right)}-\frac{3a^5\log\left(a+bx^2\right)}{b^7}+\frac{5a^4x^2}{2b^6}-\frac{a^3x^4}{b^5}+\frac{a^2x^6}{2b^4}-\frac{ax^8}{4b^3}+\frac{x^{10}}{10b^2}$$

```
[Out] 5/2*a^4*x^2/b^6-a^3*x^4/b^5+1/2*a^2*x^6/b^4-1/4*a*x^8/b^3+1/10*x^10/b^2-1/2
*a^6/b^7/(b*x^2+a)-3*a^5*ln(b*x^2+a)/b^7
```

Rubi [A] time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^6}{2b^4}-\frac{a^3x^4}{b^5}+\frac{5a^4x^2}{2b^6}-\frac{a^6}{2b^7\left(a+bx^2\right)}-\frac{3a^5\log\left(a+bx^2\right)}{b^7}-\frac{ax^8}{4b^3}+\frac{x^{10}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^2)^2,x]
```

```
[Out] (5*a^4*x^2)/(2*b^6) - (a^3*x^4)/b^5 + (a^2*x^6)/(2*b^4) - (a*x^8)/(4*b^3) +
 x^10/(10*b^2) - a^6/(2*b^7*(a + b*x^2)) - (3*a^5*Log[a + b*x^2])/b^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13}}{\left(a+bx^2\right)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a+bx)^2} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{5a^4}{b^6}-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{b^4}-\frac{2ax^3}{b^3}+\frac{x^4}{b^2}+\frac{a^6}{b^6(a+bx)^2}-\frac{6a^5}{b^6(a+bx)}\right) dx, x, x^2\right)\\
&= \frac{5a^4x^2}{2b^6}-\frac{a^3x^4}{b^5}+\frac{a^2x^6}{2b^4}-\frac{ax^8}{4b^3}+\frac{x^{10}}{10b^2}-\frac{a^6}{2b^7\left(a+bx^2\right)}-\frac{3a^5\log\left(a+bx^2\right)}{b^7}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.88

$$\frac{-\frac{10a^6}{a+bx^2}-60a^5\log\left(a+bx^2\right)+50a^4bx^2-20a^3b^2x^4+10a^2b^3x^6-5ab^4x^8+2b^5x^{10}}{20b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^2)^2,x]
```

```
[Out] (50*a^4*b*x^2 - 20*a^3*b^2*x^4 + 10*a^2*b^3*x^6 - 5*a*b^4*x^8 + 2*b^5*x^10
- (10*a^6)/(a + b*x^2) - 60*a^5*Log[a + b*x^2])/(20*b^7)
```

fricas [A] time = 0.61, size = 104, normalized size = 1.11

$$\frac{2\,b^6x^{12} - 3\,ab^5x^{10} + 5\,a^2b^4x^8 - 10\,a^3b^3x^6 + 30\,a^4b^2x^4 + 50\,a^5bx^2 - 10\,a^6 - 60\left(a^5bx^2 + a^6\right)\log\left(bx^2 + a\right)}{20\left(b^8x^2 + ab^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^6*x^12 - 3*a*b^5*x^10 + 5*a^2*b^4*x^8 - 10*a^3*b^3*x^6 + 30*a^4*b
^2*x^4 + 50*a^5*b*x^2 - 10*a^6 - 60*(a^5*b*x^2 + a^6)*log(b*x^2 + a))/(b^8*
x^2 + a*b^7)
```

giac [A] time = 0.61, size = 103, normalized size = 1.10

$$-\frac{3\,a^5\log\left(\left|bx^2 + a\right|\right)}{b^7} + \frac{6\,a^5bx^2 + 5\,a^6}{2\left(bx^2 + a\right)b^7} + \frac{2\,b^8x^{10} - 5\,ab^7x^8 + 10\,a^2b^6x^6 - 20\,a^3b^5x^4 + 50\,a^4b^4x^2}{20\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3*a^5*log(abs(b*x^2 + a))/b^7 + 1/2*(6*a^5*b*x^2 + 5*a^6)/((b*x^2 + a)*b^7
) + 1/20*(2*b^8*x^10 - 5*a*b^7*x^8 + 10*a^2*b^6*x^6 - 20*a^3*b^5*x^4 + 50*a
^4*b^4*x^2)/b^10
```

maple [A] time = 0.01, size = 85, normalized size = 0.90

$$\frac{x^{10}}{10b^2} - \frac{a\,x^8}{4b^3} + \frac{a^2x^6}{2b^4} - \frac{a^3x^4}{b^5} + \frac{5a^4x^2}{2b^6} - \frac{a^6}{2\left(b\,x^2 + a\right)b^7} - \frac{3a^5\ln\left(b\,x^2 + a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2+a)^2,x)
```

```
[Out] 5/2*a^4*x^2/b^6-a^3*x^4/b^5+1/2*a^2*x^6/b^4-1/4*a*x^8/b^3+1/10*x^10/b^2-1/2
*a^6/b^7/(b*x^2+a)-3*a^5*ln(b*x^2+a)/b^7
```

maxima [A] time = 1.33, size = 88, normalized size = 0.94

$$-\frac{a^6}{2\left(b^8x^2 + ab^7\right)} - \frac{3\,a^5\log\left(bx^2 + a\right)}{b^7} + \frac{2\,b^4x^{10} - 5\,ab^3x^8 + 10\,a^2b^2x^6 - 20\,a^3bx^4 + 50\,a^4x^2}{20\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^6/(b^8*x^2 + a*b^7) - 3*a^5*log(b*x^2 + a)/b^7 + 1/20*(2*b^4*x^10 -
5*a*b^3*x^8 + 10*a^2*b^2*x^6 - 20*a^3*b*x^4 + 50*a^4*x^2)/b^6
```

mupad [B] time = 0.09, size = 90, normalized size = 0.96

$$\frac{x^{10}}{10\,b^2} - \frac{a^6}{2\,b\,\left(b^7\,x^2 + a\,b^6\right)} - \frac{a\,x^8}{4\,b^3} - \frac{3\,a^5\,\ln\left(b\,x^2 + a\right)}{b^7} + \frac{a^2\,x^6}{2\,b^4} - \frac{a^3\,x^4}{b^5} + \frac{5\,a^4\,x^2}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a + b*x^2)^2,x)
```

```
[Out] x^10/(10*b^2) - a^6/(2*b*(a*b^6 + b^7*x^2)) - (a*x^8)/(4*b^3) - (3*a^5*log(
a + b*x^2))/b^7 + (a^2*x^6)/(2*b^4) - (a^3*x^4)/b^5 + (5*a^4*x^2)/(2*b^6)
```

sympy [A] time = 0.31, size = 88, normalized size = 0.94

$$-\frac{a^6}{2ab^7 + 2b^8x^2} - \frac{3a^5 \log\left(a + bx^2\right)}{b^7} + \frac{5a^4x^2}{2b^6} - \frac{a^3x^4}{b^5} + \frac{a^2x^6}{2b^4} - \frac{ax^8}{4b^3} + \frac{x^{10}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**2+a)**2,x)
```

```
[Out] -a**6/(2*a*b**7 + 2*b**8*x**2) - 3*a**5*log(a + b*x**2)/b**7 + 5*a**4*x**2/
(2*b**6) - a**3*x**4/b**5 + a**2*x**6/(2*b**4) - a*x**8/(4*b**3) + x**10/(1
0*b**2)
```

3.146 $\int \frac{x^{12}}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=105

$$-\frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{13/2}}+\frac{11a^4x}{2b^6}-\frac{11a^3x^3}{6b^5}+\frac{11a^2x^5}{10b^4}-\frac{11ax^7}{14b^3}-\frac{x^{11}}{2b\left(a+bx^2\right)}+\frac{11x^9}{18b^2}$$

```
[Out] 11/2*a^4*x/b^6-11/6*a^3*x^3/b^5+11/10*a^2*x^5/b^4-11/14*a*x^7/b^3+11/18*x^9
/b^2-1/2*x^11/b/(b*x^2+a)-11/2*a^(9/2)*arctan(x*b^(1/2)/a^(1/2))/b^(13/2)
```

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{11a^2x^5}{10b^4}-\frac{11a^3x^3}{6b^5}+\frac{11a^4x}{2b^6}-\frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{13/2}}-\frac{11ax^7}{14b^3}-\frac{x^{11}}{2b\left(a+bx^2\right)}+\frac{11x^9}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^2)^2,x]
```

```
[Out] (11*a^4*x)/(2*b^6) - (11*a^3*x^3)/(6*b^5) + (11*a^2*x^5)/(10*b^4) - (11*a*x
^7)/(14*b^3) + (11*x^9)/(18*b^2) - x^11/(2*b*(a + b*x^2)) - (11*a^(9/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{12}}{\left(a+bx^2\right)^2}\,dx &= -\frac{x^{11}}{2b\left(a+bx^2\right)} + \frac{11\int \frac{x^{10}}{a+bx^2}\,dx}{2b} \\
&= -\frac{x^{11}}{2b\left(a+bx^2\right)} + \frac{11\int \left(\frac{a^4}{b^5} - \frac{a^3x^2}{b^4} + \frac{a^2x^4}{b^3} - \frac{ax^6}{b^2} + \frac{x^8}{b} - \frac{a^5}{b^5\left(a+bx^2\right)}\right)dx}{2b} \\
&= \frac{11a^4x}{2b^6} - \frac{11a^3x^3}{6b^5} + \frac{11a^2x^5}{10b^4} - \frac{11ax^7}{14b^3} + \frac{11x^9}{18b^2} - \frac{x^{11}}{2b\left(a+bx^2\right)} - \frac{\left(11a^5\right)\int \frac{1}{a+bx^2}\,dx}{2b^6} \\
&= \frac{11a^4x}{2b^6} - \frac{11a^3x^3}{6b^5} + \frac{11a^2x^5}{10b^4} - \frac{11ax^7}{14b^3} + \frac{11x^9}{18b^2} - \frac{x^{11}}{2b\left(a+bx^2\right)} - \frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{13/2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 93, normalized size = 0.89

$$
\frac{x\left(\frac{315a^5}{a+bx^2} + 3150a^4 - 840a^3bx^2 + 378a^2b^2x^4 - 180ab^3x^6 + 70b^4x^8\right)}{630b^6} - \frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{13/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^2)^2,x]
```

```
[Out] (x*(3150*a^4 - 840*a^3*b*x^2 + 378*a^2*b^2*x^4 - 180*a*b^3*x^6 + 70*b^4*x^8
 + (315*a^5)/(a + b*x^2)))/(630*b^6) - (11*a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(13/2))
```

fricas [A] time = 0.71, size = 234, normalized size = 2.23

$$
\left[\frac{140\,b^5x^{11} - 220\,ab^4x^9 + 396\,a^2b^3x^7 - 924\,a^3b^2x^5 + 4620\,a^4bx^3 + 6930\,a^5x + 3465\left(a^4bx^2 + a^5\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2b}{bx}\right.}{1260\left(b^7x^2 + ab^6\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/1260*(140*b^5*x^11 - 220*a*b^4*x^9 + 396*a^2*b^3*x^7 - 924*a^3*b^2*x^5 +
 4620*a^4*b*x^3 + 6930*a^5*x + 3465*(a^4*b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2
 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^7*x^2 + a*b^6), 1/630*(70*b^5*x^1
1 - 110*a*b^4*x^9 + 198*a^2*b^3*x^7 - 462*a^3*b^2*x^5 + 2310*a^4*b*x^3 + 34
65*a^5*x - 3465*(a^4*b*x^2 + a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^7*x
^2 + a*b^6)]
```

giac [A] time = 0.64, size = 95, normalized size = 0.90

$$
-\frac{11\,a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^6} + \frac{a^5x}{2\left(bx^2+a\right)b^6} + \frac{35\,b^{16}x^9 - 90\,ab^{15}x^7 + 189\,a^2b^{14}x^5 - 420\,a^3b^{13}x^3 + 1575\,a^4b^{12}x}{315\,b^{18}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -11/2*a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/2*a^5*x/((b*x^2 + a)*b^
6) + 1/315*(35*b^16*x^9 - 90*a*b^15*x^7 + 189*a^2*b^14*x^5 - 420*a^3*b^13*x
^3 + 1575*a^4*b^12*x)/b^18
```

maple [A] time = 0.01, size = 90, normalized size = 0.86

$$\frac{x^9}{9b^2}-\frac{2a\,x^7}{7b^3}+\frac{3a^2x^5}{5b^4}-\frac{4a^3x^3}{3b^5}+\frac{a^5x}{2\left(b\,x^2+a\right)b^6}-\frac{11a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^6}+\frac{5a^4x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2+a)^2,x)
```

```
[Out] 1/9*x^9/b^2-2/7*a*x^7/b^3+3/5*a^2*x^5/b^4-4/3*a^3*x^3/b^5+5*a^4*x/b^6+1/2/b
^6*a^5*x/(b*x^2+a)-11/2/b^6*a^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.92, size = 93, normalized size = 0.89

$$\frac{a^5x}{2\left(b^7x^2+ab^6\right)}-\frac{11\,a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^6}+\frac{35\,b^4x^9-90\,ab^3x^7+189\,a^2b^2x^5-420\,a^3bx^3+1575\,a^4x}{315\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^5*x/(b^7*x^2 + a*b^6) - 11/2*a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6
) + 1/315*(35*b^4*x^9 - 90*a*b^3*x^7 + 189*a^2*b^2*x^5 - 420*a^3*b*x^3 + 15
75*a^4*x)/b^6
```

mupad [B] time = 0.07, size = 88, normalized size = 0.84

$$\frac{x^9}{9\,b^2}-\frac{2\,a\,x^7}{7\,b^3}+\frac{5\,a^4\,x}{b^6}-\frac{11\,a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{13/2}}+\frac{3\,a^2\,x^5}{5\,b^4}-\frac{4\,a^3\,x^3}{3\,b^5}+\frac{a^5\,x}{2\,\left(b^7\,x^2+a\,b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x^2)^2,x)
```

```
[Out] x^9/(9*b^2) - (2*a*x^7)/(7*b^3) + (5*a^4*x)/b^6 - (11*a^(9/2)*atan((b^(1/2)
*x)/a^(1/2)))/(2*b^(13/2)) + (3*a^2*x^5)/(5*b^4) - (4*a^3*x^3)/(3*b^5) + (a
^5*x)/(2*(a*b^6 + b^7*x^2))
```

sympy [A] time = 0.34, size = 151, normalized size = 1.44

$$\frac{a^5x}{2ab^6+2b^7x^2}+\frac{5a^4x}{b^6}-\frac{4a^3x^3}{3b^5}+\frac{3a^2x^5}{5b^4}-\frac{2ax^7}{7b^3}+\frac{11\sqrt{-\frac{a^9}{b^{13}}}\log\left(x-\frac{b^6\sqrt{-\frac{a^9}{b^{13}}}}{a^4}\right)}{4}-\frac{11\sqrt{-\frac{a^9}{b^{13}}}\log\left(x+\frac{b^6\sqrt{-\frac{a^9}{b^{13}}}}{a^4}\right)}{4}+\frac{x^9}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x**2+a)**2,x)
```

```
[Out] a**5*x/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*x/b**6 - 4*a**3*x**3/(3*b**5) + 3*
a**2*x**5/(5*b**4) - 2*a*x**7/(7*b**3) + 11*sqrt(-a**9/b**13)*log(x - b**6*
sqrt(-a**9/b**13)/a**4)/4 - 11*sqrt(-a**9/b**13)*log(x + b**6*sqrt(-a**9/b*
*13)/a**4)/4 + x**9/(9*b**2)
```

3.147 $\int \frac{x^{11}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=83

$$\frac{a^5}{2b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

```
[Out] -2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^2
+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2x^4}{4b^4}-\frac{2a^3x^2}{b^5}+\frac{a^5}{2b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2)^2,x]
```

```
[Out] (-2*a^3*x^2)/b^5 + (3*a^2*x^4)/(4*b^4) - (a*x^6)/(3*b^3) + x^8/(8*b^2) + a^
5/(2*b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/(2*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^5}{(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{4a^3}{b^5}+\frac{3a^2x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a^5}{b^5(a+bx)^2}+\frac{5a^4}{b^5(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}+\frac{a^5}{2b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.87

$$\frac{\frac{12a^5}{a+bx^2}+60a^4\log\left(a+bx^2\right)-48a^3bx^2+18a^2b^2x^4-8ab^3x^6+3b^4x^8}{24b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2)^2,x]
```

```
[Out] (-48*a^3*b*x^2 + 18*a^2*b^2*x^4 - 8*a*b^3*x^6 + 3*b^4*x^8 + (12*a^5)/(a + b
*x^2) + 60*a^4*Log[a + b*x^2])/(24*b^6)
```

fricas [A] time = 0.92, size = 93, normalized size = 1.12

$$\frac{3 b^5x^{10} - 5 ab^4x^8 + 10 a^2b^3x^6 - 30 a^3b^2x^4 - 48 a^4bx^2 + 12 a^5 + 60 \left(a^4bx^2 + a^5\right) \log \left(bx^2 + a\right)}{24 \left(b^7x^2 + ab^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 - 5*a*b^4*x^8 + 10*a^2*b^3*x^6 - 30*a^3*b^2*x^4 - 48*a^4*b
*x^2 + 12*a^5 + 60*(a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^7*x^2 + a*b^6)
```

giac [A] time = 0.63, size = 92, normalized size = 1.11

$$\frac{5 a^4 \log \left(\left|bx^2 + a\right|\right)}{2 b^6} - \frac{5 a^4bx^2 + 4 a^5}{2 \left(bx^2 + a\right)b^6} + \frac{3 b^6x^8 - 8 ab^5x^6 + 18 a^2b^4x^4 - 48 a^3b^3x^2}{24 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*a^4*log(abs(b*x^2 + a))/b^6 - 1/2*(5*a^4*b*x^2 + 4*a^5)/((b*x^2 + a)*b^
6) + 1/24*(3*b^6*x^8 - 8*a*b^5*x^6 + 18*a^2*b^4*x^4 - 48*a^3*b^3*x^2)/b^8
```

maple [A] time = 0.01, size = 74, normalized size = 0.89

$$\frac{x^8}{8b^2} - \frac{a\,x^6}{3b^3} + \frac{3a^2x^4}{4b^4} - \frac{2a^3x^2}{b^5} + \frac{a^5}{2 \left(b\,x^2 + a\right) b^6} + \frac{5a^4 \ln \left(b\,x^2 + a\right)}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a)^2,x)
```

```
[Out] -2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^2
+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

maxima [A] time = 1.36, size = 77, normalized size = 0.93

$$\frac{a^5}{2 \left(b^7x^2 + ab^6\right)} + \frac{5 a^4 \log \left(bx^2 + a\right)}{2 b^6} + \frac{3 b^3x^8 - 8 ab^2x^6 + 18 a^2bx^4 - 48 a^3x^2}{24 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^5/(b^7*x^2 + a*b^6) + 5/2*a^4*log(b*x^2 + a)/b^6 + 1/24*(3*b^3*x^8 -
8*a*b^2*x^6 + 18*a^2*b*x^4 - 48*a^3*x^2)/b^5
```

mupad [B] time = 4.48, size = 79, normalized size = 0.95

$$\frac{x^8}{8\,b^2} + \frac{a^5}{2\,b\,\left(b^6\,x^2 + a\,b^5\right)} - \frac{a\,x^6}{3\,b^3} + \frac{5\,a^4\,\ln\left(b\,x^2 + a\right)}{2\,b^6} + \frac{3\,a^2\,x^4}{4\,b^4} - \frac{2\,a^3\,x^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2)^2,x)
```

```
[Out] x^8/(8*b^2) + a^5/(2*b*(a*b^5 + b^6*x^2)) - (a*x^6)/(3*b^3) + (5*a^4*log(a
+ b*x^2))/(2*b^6) + (3*a^2*x^4)/(4*b^4) - (2*a^3*x^2)/b^5
```

sympy [A] time = 0.29, size = 80, normalized size = 0.96

$$\frac{a^5}{2ab^6 + 2b^7x^2} + \frac{5a^4 \log\left(a + bx^2\right)}{2b^6} - \frac{2a^3x^2}{b^5} + \frac{3a^2x^4}{4b^4} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a)**2,x)
```

```
[Out] a**5/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*log(a + b*x**2)/(2*b**6) - 2*a**3*x*
*2/b**5 + 3*a**2*x**4/(4*b**4) - a*x**6/(3*b**3) + x**8/(8*b**2)
```

3.148 $\int \frac{x^{10}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=92

$$\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9x^7}{14b^2}$$

```
[Out] -9/2*a^3*x/b^5+3/2*a^2*x^3/b^4-9/10*a*x^5/b^3+9/14*x^7/b^2-1/2*x^9/b/(b*x^2
+a)+9/2*a^(7/2)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{3a^2x^3}{2b^4}-\frac{9a^3x}{2b^5}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9x^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^2,x]
```

```
[Out] (-9*a^3*x)/(2*b^5) + (3*a^2*x^3)/(2*b^4) - (9*a*x^5)/(10*b^3) + (9*x^7)/(14
*b^2) - x^9/(2*b*(a + b*x^2)) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^2\right)^2}\,dx = -\frac{x^9}{2b\left(a+bx^2\right)} + \frac{9\int \frac{x^8}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^9}{2b\left(a+bx^2\right)} + \frac{9\int \left(-\frac{a^3}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^4}{b^2}+\frac{x^6}{b}+\frac{a^4}{b^4\left(a+bx^2\right)}\right)dx}{2b}$$

$$= -\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{\left(9a^4\right)\int \frac{1}{a+bx^2}\,dx}{2b^5}$$

$$= -\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.89

$$\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{x\left(-\frac{35a^4}{a+bx^2}-280a^3+70a^2bx^2-28ab^2x^4+10b^3x^6\right)}{70b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^2,x]
```

```
[Out] (x*(-280*a^3 + 70*a^2*b*x^2 - 28*a*b^2*x^4 + 10*b^3*x^6 - (35*a^4)/(a + b*x
^2)))/(70*b^5) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

fricas [A] time = 1.00, size = 212, normalized size = 2.30

$$\left[\frac{20\,b^4x^9-36\,ab^3x^7+84\,a^2b^2x^5-420\,a^3bx^3-630\,a^4x+315\left(a^3bx^2+a^4\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{140\left(b^6x^2+ab^5\right)}, \frac{10\,b^4x^9-18\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/140*(20*b^4*x^9 - 36*a*b^3*x^7 + 84*a^2*b^2*x^5 - 420*a^3*b*x^3 - 630*a^
4*x + 315*(a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)))/(b^6*x^2 + a*b^5), 1/70*(10*b^4*x^9 - 18*a*b^3*x^7 + 42*a^2*b^
2*x^5 - 210*a^3*b*x^3 - 315*a^4*x + 315*(a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a))/(b^6*x^2 + a*b^5)]
```

giac [A] time = 0.64, size = 84, normalized size = 0.91

$$\frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^5}-\frac{a^4x}{2\left(bx^2+a\right)b^5}+\frac{5\,b^{12}x^7-14\,ab^{11}x^5+35\,a^2b^{10}x^3-140\,a^3b^9x}{35\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*a^4*x/((b*x^2 + a)*b^5)
 + 1/35*(5*b^12*x^7 - 14*a*b^11*x^5 + 35*a^2*b^10*x^3 - 140*a^3*b^9*x)/b^14
```

maple [A] time = 0.01, size = 78, normalized size = 0.85

$$\frac{x^7}{7b^2}-\frac{2a\,x^5}{5b^3}+\frac{a^2x^3}{b^4}-\frac{a^4x}{2\left(b\,x^2+a\right)b^5}+\frac{9a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^5}-\frac{4a^3x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^2,x)
```

```
[Out] 1/7*x^7/b^2-2/5*a*x^5/b^3+a^2*x^3/b^4-4*a^3*x/b^5-1/2/b^5*a^4*x/(b*x^2+a)+9
/2/b^5*a^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.01, size = 82, normalized size = 0.89

$$-\frac{a^4x}{2\left(b^6x^2+ab^5\right)}+\frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5}+\frac{5\,b^3x^7-14\,ab^2x^5+35\,a^2bx^3-140\,a^3x}{35\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^4*x/(b^6*x^2 + a*b^5) + 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5
) + 1/35*(5*b^3*x^7 - 14*a*b^2*x^5 + 35*a^2*b*x^3 - 140*a^3*x)/b^5
```

mupad [B] time = 4.56, size = 77, normalized size = 0.84

$$\frac{x^7}{7\,b^2}-\frac{2\,a\,x^5}{5\,b^3}-\frac{4\,a^3\,x}{b^5}+\frac{9\,a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{11/2}}+\frac{a^2\,x^3}{b^4}-\frac{a^4\,x}{2\,\left(b^6\,x^2+a\,b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2)^2,x)
```

```
[Out] x^7/(7*b^2) - (2*a*x^5)/(5*b^3) - (4*a^3*x)/b^5 + (9*a^(7/2)*atan((b^(1/2)*
x)/a^(1/2)))/(2*b^(11/2)) + (a^2*x^3)/b^4 - (a^4*x)/(2*(a*b^5 + b^6*x^2))
```

sympy [A] time = 0.33, size = 134, normalized size = 1.46

$$-\frac{a^4x}{2ab^5+2b^6x^2}-\frac{4a^3x}{b^5}+\frac{a^2x^3}{b^4}-\frac{2ax^5}{5b^3}-\frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{x^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**2,x)
```

```
[Out] -a**4*x/(2*a*b**5 + 2*b**6*x**2) - 4*a**3*x/b**5 + a**2*x**3/b**4 - 2*a*x**
5/(5*b**3) - 9*sqrt(-a**7/b**11)*log(x - b**5*sqrt(-a**7/b**11)/a**3)/4 + 9
*sqrt(-a**7/b**11)*log(x + b**5*sqrt(-a**7/b**11)/a**3)/4 + x**7/(7*b**2)
```

3.149 $\int \frac{x^9}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=70

$$-\frac{a^4}{2b^5\left(a+bx^2\right)}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

```
[Out] 3/2*a^2*x^2/b^4-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b*
x^2+a)/b^5
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2x^2}{2b^4}-\frac{a^4}{2b^5\left(a+bx^2\right)}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^2,x]
```

```
[Out] (3*a^2*x^2)/(2*b^4) - (a*x^4)/(2*b^3) + x^6/(6*b^2) - a^4/(2*b^5*(a + b*x^2
)) - (2*a^3*Log[a + b*x^2])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{\left(a+bx^2\right)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx)^2} \, dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right) \, dx,x,x^2\right)\\
&= \frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}-\frac{a^4}{2b^5\left(a+bx^2\right)}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.86

$$\frac{-\frac{3a^4}{a+bx^2}-12a^3\log\left(a+bx^2\right)+9a^2bx^2-3ab^2x^4+b^3x^6}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^2,x]
```

```
[Out] (9*a^2*b*x^2 - 3*a*b^2*x^4 + b^3*x^6 - (3*a^4)/(a + b*x^2) - 12*a^3*Log[a +
 b*x^2])/(6*b^5)
```

fricas [A] time = 0.57, size = 81, normalized size = 1.16

$$\frac{b^4x^8 - 2\,ab^3x^6 + 6\,a^2b^2x^4 + 9\,a^3bx^2 - 3\,a^4 - 12\left(a^3bx^2 + a^4\right)\log\left(bx^2 + a\right)}{6\left(b^6x^2 + ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*x^8 - 2*a*b^3*x^6 + 6*a^2*b^2*x^4 + 9*a^3*b*x^2 - 3*a^4 - 12*(a^3*
b*x^2 + a^4)*log(b*x^2 + a))/(b^6*x^2 + a*b^5)
```

giac [A] time = 0.65, size = 80, normalized size = 1.14

$$-\frac{2\,a^3\log\left(\left|bx^2 + a\right|\right)}{b^5} + \frac{b^4x^6 - 3\,ab^3x^4 + 9\,a^2b^2x^2}{6\,b^6} + \frac{4\,a^3bx^2 + 3\,a^4}{2\left(bx^2 + a\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(b*x^2 + a))/b^5 + 1/6*(b^4*x^6 - 3*a*b^3*x^4 + 9*a^2*b^2*x^2
)/b^6 + 1/2*(4*a^3*b*x^2 + 3*a^4)/((b*x^2 + a)*b^5)
```

maple [A] time = 0.01, size = 63, normalized size = 0.90

$$\frac{x^6}{6b^2} - \frac{a\,x^4}{2b^3} + \frac{3a^2x^2}{2b^4} - \frac{a^4}{2\left(b\,x^2 + a\right)b^5} - \frac{2a^3\ln\left(b\,x^2 + a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^2,x)
```

```
[Out] 3/2*a^2*x^2/b^4-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b*
x^2+a)/b^5
```

maxima [A] time = 1.35, size = 65, normalized size = 0.93

$$-\frac{a^4}{2\left(b^6x^2 + ab^5\right)} - \frac{2\,a^3\log\left(bx^2 + a\right)}{b^5} + \frac{b^2x^6 - 3\,abx^4 + 9\,a^2x^2}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^4/(b^6*x^2 + a*b^5) - 2*a^3*log(b*x^2 + a)/b^5 + 1/6*(b^2*x^6 - 3*a*
b*x^4 + 9*a^2*x^2)/b^4
```

mupad [B] time = 0.07, size = 68, normalized size = 0.97

$$\frac{x^6}{6\,b^2} - \frac{a^4}{2\,b\,\left(b^5\,x^2 + a\,b^4\right)} - \frac{a\,x^4}{2\,b^3} - \frac{2\,a^3\,\ln\left(b\,x^2 + a\right)}{b^5} + \frac{3\,a^2\,x^2}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2)^2,x)
```

```
[Out] x^6/(6*b^2) - a^4/(2*b*(a*b^4 + b^5*x^2)) - (a*x^4)/(2*b^3) - (2*a^3*log(a
+ b*x^2))/b^5 + (3*a^2*x^2)/(2*b^4)
```

sympy [A] time = 0.27, size = 66, normalized size = 0.94

$$-\frac{a^4}{2ab^5 + 2b^6x^2} - \frac{2a^3 \log\left(a + bx^2\right)}{b^5} + \frac{3a^2x^2}{2b^4} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**2,x)
```

```
[Out] -a**4/(2*a*b**5 + 2*b**6*x**2) - 2*a**3*log(a + b*x**2)/b**5 + 3*a**2*x**2/
(2*b**4) - a*x**4/(2*b**3) + x**6/(6*b**2)
```

3.150 $\int \frac{x^8}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=79

$$-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}-\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7x^5}{10b^2}$$

```
[Out] 7/2*a^2*x/b^4-7/6*a*x^3/b^3+7/10*x^5/b^2-1/2*x^7/b/(b*x^2+a)-7/2*a^(5/2)*ar
ctan(x*b^(1/2)/a^(1/2))/b^(9/2)
```

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{7a^2x}{2b^4}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}-\frac{7ax^3}{6b^3}-\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7x^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^2,x]
```

```
[Out] (7*a^2*x)/(2*b^4) - (7*a*x^3)/(6*b^3) + (7*x^5)/(10*b^2) - x^7/(2*b*(a + b*
x^2)) - (7*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2\right)^2}\,dx = -\frac{x^7}{2b\left(a+bx^2\right)} + \frac{7\int \frac{x^6}{a+bx^2}\,dx}{2b}$$
$$= -\frac{x^7}{2b\left(a+bx^2\right)} + \frac{7\int \left(\frac{a^2}{b^3} - \frac{ax^2}{b^2} + \frac{x^4}{b} - \frac{a^3}{b^3\left(a+bx^2\right)}\right)dx}{2b}$$
$$= \frac{7a^2x}{2b^4} - \frac{7ax^3}{6b^3} + \frac{7x^5}{10b^2} - \frac{x^7}{2b\left(a+bx^2\right)} - \frac{\left(7a^3\right)\int \frac{1}{a+bx^2}\,dx}{2b^4}$$
$$= \frac{7a^2x}{2b^4} - \frac{7ax^3}{6b^3} + \frac{7x^5}{10b^2} - \frac{x^7}{2b\left(a+bx^2\right)} - \frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.90

$$\frac{x\left(\frac{15a^3}{a+bx^2} + 90a^2 - 20abx^2 + 6b^2x^4\right)}{30b^4} - \frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^2,x]
```

```
[Out] (x*(90*a^2 - 20*a*b*x^2 + 6*b^2*x^4 + (15*a^3)/(a + b*x^2)))/(30*b^4) - (7*
a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

fricas [A] time = 0.95, size = 190, normalized size = 2.41

$$\left[\frac{12\,b^3x^7 - 28\,ab^2x^5 + 140\,a^2bx^3 + 210\,a^3x + 105\left(a^2bx^2 + a^3\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right)}{60\left(b^5x^2 + ab^4\right)}, \frac{6\,b^3x^7 - 14\,ab^2x^5 + 70\,a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^3*x^7 - 28*a*b^2*x^5 + 140*a^2*b*x^3 + 210*a^3*x + 105*(a^2*b*x
^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^5*
x^2 + a*b^4), 1/30*(6*b^3*x^7 - 14*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*x - 1
05*(a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^2 + a*b^4)]
```

giac [A] time = 0.63, size = 73, normalized size = 0.92

$$-\frac{7\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4} + \frac{a^3x}{2\left(bx^2 + a\right)b^4} + \frac{3\,b^8x^5 - 10\,ab^7x^3 + 45\,a^2b^6x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*a^3*x/((b*x^2 + a)*b^4
) + 1/15*(3*b^8*x^5 - 10*a*b^7*x^3 + 45*a^2*b^6*x)/b^10
```

maple [A] time = 0.01, size = 68, normalized size = 0.86

$$\frac{x^5}{5b^2} - \frac{2a\,x^3}{3b^3} + \frac{a^3x}{2\left(b\,x^2+a\right)b^4} - \frac{7a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^4} + \frac{3a^2x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^2,x)
```

```
[Out] 1/5*x^5/b^2-2/3*a*x^3/b^3+3*a^2*x/b^4+1/2/b^4*a^3*x/(b*x^2+a)-7/2/b^4*a^3/(
a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 71, normalized size = 0.90

$$\frac{a^3x}{2\left(b^5x^2+ab^4\right)} - \frac{7\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4} + \frac{3\,b^2x^5 - 10\,abx^3 + 45\,a^2x}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*x/(b^5*x^2 + a*b^4) - 7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4)
 + 1/15*(3*b^2*x^5 - 10*a*b*x^3 + 45*a^2*x)/b^4
```

mupad [B] time = 4.59, size = 66, normalized size = 0.84

$$\frac{x^5}{5\,b^2} - \frac{2\,a\,x^3}{3\,b^3} + \frac{3\,a^2\,x}{b^4} - \frac{7\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{9/2}} + \frac{a^3\,x}{2\,\left(b^5\,x^2+a\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2)^2,x)
```

```
[Out] x^5/(5*b^2) - (2*a*x^3)/(3*b^3) + (3*a^2*x)/b^4 - (7*a^(5/2)*atan((b^(1/2)*
x)/a^(1/2)))/(2*b^(9/2)) + (a^3*x)/(2*(a*b^4 + b^5*x^2))
```

sympy [A] time = 0.31, size = 124, normalized size = 1.57

$$\frac{a^3x}{2ab^4+2b^5x^2} + \frac{3a^2x}{b^4} - \frac{2ax^3}{3b^3} + \frac{7\sqrt{-\frac{a^5}{b^9}}\log\left(x - \frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4} - \frac{7\sqrt{-\frac{a^5}{b^9}}\log\left(x + \frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4} + \frac{x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**2,x)
```

```
[Out] a**3*x/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*x/b**4 - 2*a*x**3/(3*b**3) + 7*sqr
t(-a**5/b**9)*log(x - b**4*sqrt(-a**5/b**9)/a**2)/4 - 7*sqrt(-a**5/b**9)*lo
g(x + b**4*sqrt(-a**5/b**9)/a**2)/4 + x**5/(5*b**2)
```

3.151 $\int \frac{x^7}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=57

$$\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

```
[Out] -a*x^2/b^3+1/4*x^4/b^2+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^2,x]
```

```
[Out] -((a*x^2)/b^3) + x^4/(4*b^2) + a^3/(2*b^4*(a + b*x^2)) + (3*a^2*Log[a + b*x
^2])/(2*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a+bx^2\right)^2} dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{(a+bx)^2} dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right) dx, x, x^2\right)\\
&= -\frac{ax^2}{b^3}+\frac{x^4}{4b^2}+\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.86

$$\frac{\frac{2a^3}{a+bx^2}+6a^2\log\left(a+bx^2\right)-4abx^2+b^2x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^2,x]
```

```
[Out] (-4*a*b*x^2 + b^2*x^4 + (2*a^3)/(a + b*x^2) + 6*a^2*Log[a + b*x^2])/(4*b^4)
```

fricas [A] time = 0.75, size = 70, normalized size = 1.23

$$\frac{b^3x^6 - 3\,ab^2x^4 - 4\,a^2bx^2 + 2\,a^3 + 6\left(a^2bx^2 + a^3\right)\log\left(bx^2 + a\right)}{4\left(b^5x^2 + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/4*(b^3*x^6 - 3*a*b^2*x^4 - 4*a^2*b*x^2 + 2*a^3 + 6*(a^2*b*x^2 + a^3)*log(
b*x^2 + a))/(b^5*x^2 + a*b^4)
```

giac [A] time = 0.62, size = 67, normalized size = 1.18

$$\frac{3\,a^2\log\left(\left|bx^2 + a\right|\right)}{2\,b^4} + \frac{b^2x^4 - 4\,abx^2}{4\,b^4} - \frac{3\,a^2bx^2 + 2\,a^3}{2\left(bx^2 + a\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^2,x, algorithm="giac")
[Out] 3/2*a^2*log(abs(b*x^2 + a))/b^4 + 1/4*(b^2*x^4 - 4*a*b*x^2)/b^4 - 1/2*(3*a^
2*b*x^2 + 2*a^3)/((b*x^2 + a)*b^4)
```

maple [A] time = 0.01, size = 52, normalized size = 0.91

$$\frac{x^4}{4b^2} - \frac{a\,x^2}{b^3} + \frac{a^3}{2\left(b\,x^2 + a\right)b^4} + \frac{3a^2\ln\left(b\,x^2 + a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^2,x)
[Out] -a*x^2/b^3+1/4*x^4/b^2+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/b^4
```

maxima [A] time = 1.37, size = 54, normalized size = 0.95

$$\frac{a^3}{2\left(b^5x^2 + ab^4\right)} + \frac{3\,a^2\log\left(bx^2 + a\right)}{2\,b^4} + \frac{bx^4 - 4\,ax^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/2*a^3/(b^5*x^2 + a*b^4) + 3/2*a^2*log(b*x^2 + a)/b^4 + 1/4*(b*x^4 - 4*a*x
^2)/b^3
```

mupad [B] time = 0.08, size = 57, normalized size = 1.00

$$\frac{x^4}{4\,b^2} + \frac{a^3}{2\,b\,\left(b^4\,x^2 + a\,b^3\right)} - \frac{a\,x^2}{b^3} + \frac{3\,a^2\,\ln\left(b\,x^2 + a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^2,x)
[Out] x^4/(4*b^2) + a^3/(2*b*(a*b^3 + b^4*x^2)) - (a*x^2)/b^3 + (3*a^2*log(a + b*
x^2))/(2*b^4)
```

sympy [A] time = 0.26, size = 53, normalized size = 0.93

$$\frac{a^3}{2ab^4+2b^5x^2}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**2,x)
```

```
[Out] a**3/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*log(a + b*x**2)/(2*b**4) - a*x**2/b*
*3 + x**4/(4*b**2)
```

3.152 $\int \frac{x^6}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=66

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}} - \frac{5ax}{2b^3} - \frac{x^5}{2b\left(a+bx^2\right)} + \frac{5x^3}{6b^2}$$

```
[Out] -5/2*a*x/b^3+5/6*x^3/b^2-1/2*x^5/b/(b*x^2+a)+5/2*a^(3/2)*arctan(x*b^(1/2)/a
^(1/2))/b^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}} - \frac{5ax}{2b^3} - \frac{x^5}{2b\left(a+bx^2\right)} + \frac{5x^3}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^2,x]
```

```
[Out] (-5*a*x)/(2*b^3) + (5*x^3)/(6*b^2) - x^5/(2*b*(a + b*x^2)) + (5*a^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^2}\,dx = -\frac{x^5}{2b\left(a+bx^2\right)} + \frac{5\int \frac{x^4}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^5}{2b\left(a+bx^2\right)} + \frac{5\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx}{2b}$$

$$= -\frac{5ax}{2b^3} + \frac{5x^3}{6b^2} - \frac{x^5}{2b\left(a+bx^2\right)} + \frac{\left(5a^2\right)\int \frac{1}{a+bx^2}\,dx}{2b^3}$$

$$= -\frac{5ax}{2b^3} + \frac{5x^3}{6b^2} - \frac{x^5}{2b\left(a+bx^2\right)} + \frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.91

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}} + \frac{x\left(-\frac{3a^2}{a+bx^2}-12a+2bx^2\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^2,x]
```

```
[Out] (x*(-12*a + 2*b*x^2 - (3*a^2)/(a + b*x^2)))/(6*b^3) + (5*a^(3/2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

fricas [A] time = 0.95, size = 164, normalized size = 2.48

$$\left[\frac{4\,b^2x^5-20\,abx^3-30\,a^2x+15\left(abx^2+a^2\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{12\left(b^4x^2+ab^3\right)},\ \frac{2\,b^2x^5-10\,abx^3-15\,a^2x+15\left(abx^2+a^2\right)}{6\left(b^4x^2+ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^2*x^5 - 20*a*b*x^3 - 30*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(-a/b)*lo
g((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^4*x^2 + a*b^3), 1/6*(2*b^
2*x^5 - 10*a*b*x^3 - 15*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*sqr
t(a/b)/a))/(b^4*x^2 + a*b^3)]
```

giac [A] time = 0.64, size = 61, normalized size = 0.92

$$\frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3} - \frac{a^2x}{2\left(bx^2+a\right)b^3} + \frac{b^4x^3-6\,ab^3x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*a^2*x/((b*x^2 + a)*b^3)
 + 1/3*(b^4*x^3 - 6*a*b^3*x)/b^6
```

maple [A] time = 0.01, size = 57, normalized size = 0.86

$$\frac{x^3}{3b^2} - \frac{a^2x}{2\left(b\,x^2+a\right)b^3} + \frac{5a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^3} - \frac{2ax}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^2,x)
```

```
[Out] 1/3*x^3/b^2-2*a*x/b^3-1/2/b^3*a^2*x/(b*x^2+a)+5/2/b^3*a^2/(a*b)^(1/2)*arcta
n(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 59, normalized size = 0.89

$$-\frac{a^2x}{2\left(b^4x^2+ab^3\right)} + \frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3} + \frac{bx^3-6\,ax}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*x/(b^4*x^2 + a*b^3) + 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3
) + 1/3*(b*x^3 - 6*a*x)/b^3
```

mupad [B] time = 0.09, size = 56, normalized size = 0.85

$$\frac{x^3}{3\,b^2} + \frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{7/2}} - \frac{a^2\,x}{2\,\left(b^4\,x^2+a\,b^3\right)} - \frac{2\,a\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^2,x)
```

```
[Out] x^3/(3*b^2) + (5*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*b^(7/2)) - (a^2*x)/(
2*(a*b^3 + b^4*x^2)) - (2*a*x)/b^3
```

sympy [A] time = 0.29, size = 107, normalized size = 1.62

$$-\frac{a^2x}{2ab^3+2b^4x^2} - \frac{2ax}{b^3} - \frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x-\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4} + \frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x+\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4} + \frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**2,x)
```

```
[Out] -a**2*x/(2*a*b**3 + 2*b**4*x**2) - 2*a*x/b**3 - 5*sqrt(-a**3/b**7)*log(x -
b**3*sqrt(-a**3/b**7)/a)/4 + 5*sqrt(-a**3/b**7)*log(x + b**3*sqrt(-a**3/b**
7)/a)/4 + x**3/(3*b**2)
```

3.153 $\int \frac{x^5}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=44

$$-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

```
[Out] 1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^2,x]
```

```
[Out] x^2/(2*b^2) - a^2/(2*b^3*(a + b*x^2)) - (a*Log[a + b*x^2])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{x^2}{2b^2}-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.86

$$\frac{-\frac{a^2}{a+bx^2}-2a\log\left(a+bx^2\right)+bx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^2,x]
```

```
[Out] (b*x^2 - a^2/(a + b*x^2) - 2*a*Log[a + b*x^2])/(2*b^3)
```

fricas [A] time = 0.80, size = 56, normalized size = 1.27

$$\frac{b^2x^4 + abx^2 - a^2 - 2\left(abx^2 + a^2\right)\log\left(bx^2 + a\right)}{2\left(b^4x^2 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^4 + a*b*x^2 - a^2 - 2*(a*b*x^2 + a^2)*log(b*x^2 + a))/(b^4*x^2 +
 a*b^3)
```

giac [A] time = 0.62, size = 49, normalized size = 1.11

$$\frac{x^2}{2\,b^2} - \frac{a\log\left(\left|bx^2 + a\right|\right)}{b^3} + \frac{2\,abx^2 + a^2}{2\left(bx^2 + a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^2 - a*log(abs(b*x^2 + a))/b^3 + 1/2*(2*a*b*x^2 + a^2)/((b*x^2 + a
)*b^3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.93

$$\frac{x^2}{2b^2} - \frac{a^2}{2\left(b\,x^2 + a\right)b^3} - \frac{a\ln\left(b\,x^2 + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^2,x)
```

```
[Out] 1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

maxima [A] time = 1.39, size = 43, normalized size = 0.98

$$-\frac{a^2}{2\left(b^4x^2 + ab^3\right)} + \frac{x^2}{2\,b^2} - \frac{a\log\left(bx^2 + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2/(b^4*x^2 + a*b^3) + 1/2*x^2/b^2 - a*log(b*x^2 + a)/b^3
```

mupad [B] time = 0.05, size = 45, normalized size = 1.02

$$\frac{x^2}{2\,b^2} - \frac{a^2}{2\,\left(b^4\,x^2 + a\,b^3\right)} - \frac{a\,\ln\left(b\,x^2 + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^2,x)
```

```
[Out] x^2/(2*b^2) - a^2/(2*(a*b^3 + b^4*x^2)) - (a*log(a + b*x^2))/b^3
```

sympy [A] time = 0.23, size = 39, normalized size = 0.89

$$-\frac{a^2}{2ab^3+2b^4x^2}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**2,x)
```

```
[Out] -a**2/(2*a*b**3 + 2*b**4*x**2) - a*log(a + b*x**2)/b**3 + x**2/(2*b**2)
```

3.154 $\int \frac{x^4}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=55

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

```
[Out] 3/2*x/b^2-1/2*x^3/b/(b*x^2+a)-3/2*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 321, 205}

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^2,x]
```

```
[Out] (3*x)/(2*b^2) - x^3/(2*b*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(a+bx^2\right)^2}\,dx &= -\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3\int \frac{x^2}{a+bx^2}\,dx}{2b}\\
&= \frac{3x}{2b^2}-\frac{x^3}{2b\left(a+bx^2\right)}-\frac{(3a)\int \frac{1}{a+bx^2}\,dx}{2b^2}\\
&= \frac{3x}{2b^2}-\frac{x^3}{2b\left(a+bx^2\right)}-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.93

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}+\frac{ax}{2b^2\left(a+bx^2\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^2,x]
```

```
[Out] x/b^2 + (a*x)/(2*b^2*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(2*b^(5/2))
```

fricas [A] time = 0.83, size = 136, normalized size = 2.47

$$\left[\frac{4\,bx^3+3\left(bx^2+a\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,ax}{4\left(b^3x^2+ab^2\right)},\frac{2\,bx^3-3\left(bx^2+a\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+3\,ax}{2\left(b^3x^2+ab^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*x^3 + 3*(b*x^2 + a)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)
/(b*x^2 + a)) + 6*a*x)/(b^3*x^2 + a*b^2), 1/2*(2*b*x^3 - 3*(b*x^2 + a)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*a*x)/(b^3*x^2 + a*b^2)]
```

giac [A] time = 0.63, size = 42, normalized size = 0.76

$$-\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{ax}{2\left(bx^2+a\right)b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*a*x/((b*x^2 + a)*b^2) +
x/b^2
```

maple [A] time = 0.01, size = 43, normalized size = 0.78

$$\frac{ax}{2\left(b\,x^2+a\right)b^2}-\frac{3a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2,x)
```

```
[Out] x/b^2+1/2/b^2*a*x/(b*x^2+a)-3/2/b^2*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.94, size = 45, normalized size = 0.82

$$\frac{ax}{2\left(b^3x^2+ab^2\right)}-\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*x/(b^3*x^2 + a*b^2) - 3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + x
/b^2
```

mupad [B] time = 4.59, size = 43, normalized size = 0.78

$$\frac{x}{b^2} + \frac{a\,x}{2\,\left(b^3\,x^2 + a\,b^2\right)} - \frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^2,x)
```

```
[Out] x/b^2 + (a*x)/(2*(a*b^2 + b^3*x^2)) - (3*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))
/(2*b^(5/2))
```

sympy [A] time = 0.26, size = 83, normalized size = 1.51

$$\frac{ax}{2ab^2 + 2b^3x^2} + \frac{3\sqrt{-\frac{a}{b^5}}\log\left(-b^2\sqrt{-\frac{a}{b^5}} + x\right)}{4} - \frac{3\sqrt{-\frac{a}{b^5}}\log\left(b^2\sqrt{-\frac{a}{b^5}} + x\right)}{4} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2,x)
```

```
[Out] a*x/(2*a*b**2 + 2*b**3*x**2) + 3*sqrt(-a/b**5)*log(-b**2*sqrt(-a/b**5) + x)
/4 - 3*sqrt(-a/b**5)*log(b**2*sqrt(-a/b**5) + x)/4 + x/b**2
```

3.155 $\int \frac{x^3}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=33

$$\frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] 1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^2,x]
```

```
[Out] a/(2*b^2*(a + b*x^2)) + Log[a + b*x^2]/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^2} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^2} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right) dx, x, x^2\right)\\
&= \frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.82

$$\frac{\frac{a}{a+bx^2}+\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^2,x]
```

```
[Out] (a/(a + b*x^2) + Log[a + b*x^2])/(2*b^2)
```

fricas [A] time = 0.76, size = 35, normalized size = 1.06

$$\frac{\left(bx^2+a\right)\log\left(bx^2+a\right)+a}{2\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x^2 + a)*log(b*x^2 + a) + a)/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.64, size = 48, normalized size = 1.45

$$-\frac{\frac{\log\left(\frac{|bx^2+a|}{\left(bx^2+a\right)^2|b|}\right)}{b}-\frac{a}{\left(bx^2+a\right)b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b))/b
```

maple [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{a}{2\left(b\,x^2+a\right)b^2}+\frac{\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2,x)
```

```
[Out] 1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

maxima [A] time = 1.32, size = 32, normalized size = 0.97

$$\frac{a}{2\left(b^3x^2+ab^2\right)}+\frac{\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a/(b^3*x^2 + a*b^2) + 1/2*log(b*x^2 + a)/b^2
```

mupad [B] time = 0.05, size = 29, normalized size = 0.88

$$\frac{\ln\left(b\,x^2+a\right)}{2\,b^2}+\frac{a}{2\,b^2\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^2,x)
```

```
[Out] log(a + b*x^2)/(2*b^2) + a/(2*b^2*(a + b*x^2))
```

sympy [A] time = 0.19, size = 29, normalized size = 0.88

$$\frac{a}{2ab^2+2b^3x^2}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2,x)
```

```
[Out] a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2)
```

3.156 $\int \frac{x^2}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

```
[Out] -1/2*x/b/(b*x^2+a)+1/2*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {288, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^2,x]
```

```
[Out] -x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^2\right)^2}\,dx &= -\frac{x}{2b\left(a+bx^2\right)}+\frac{\int \frac{1}{a+bx^2}\,dx}{2b}\\ &= -\frac{x}{2b\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^2,x]
```

```
[Out] -1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

fricas [A] time = 0.65, size = 120, normalized size = 2.67

$$\left[-\frac{2\,abx+\left(bx^2+a\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\,\left(ab^3x^2+a^2b^2\right)},\,-\frac{abx-\left(bx^2+a\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,\left(ab^3x^2+a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x + (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a*b^3*x^2 + a^2*b^2), -1/2*(a*b*x - (b*x^2 + a)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a*b^3*x^2 + a^2*b^2)]
```

giac [A] time = 0.64, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b}-\frac{x}{2\,\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*x/((b*x^2 + a)*b)
```

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$-\frac{x}{2\left(b\,x^2+a\right)b}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2,x)
```

```
[Out] -1/2*x/b/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.96, size = 36, normalized size = 0.80

$$-\frac{x}{2\,\left(b^2x^2+ab\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(b^2*x^2 + a*b) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [B] time = 4.76, size = 33, normalized size = 0.73

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}-\frac{x}{2\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^2,x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(2*a^(1/2)*b^(3/2)) - x/(2*b*(a + b*x^2))
```

<u>**sympy**</u> **[B]** time = 0.21, size = 78, normalized size = 1.73

$$-\frac{x}{2ab+2b^2x^2}-\frac{\sqrt{-\frac{1}{ab^3}}\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^3}}\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2,x)
```

```
[Out] -x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3)) + x
)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/4
```

3.157 $\int \frac{x}{(a+bx^2)^2} dx$

Optimal. Leaf size=16

$$-\frac{1}{2b\left(a+bx^2\right)}$$

[Out] -1/2/b/(b*x^2+a)

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x^2)^2,x]

[Out] -1/(2*b*(a + b*x^2))

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^2} dx = -\frac{1}{2b\left(a+bx^2\right)}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x^2)^2,x]

[Out] -1/2*1/(b*(a + b*x^2))

fricas [A] time = 0.66, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^2+a)^2,x, algorithm="fricas")

[Out] -1/2/(b^2*x^2 + a*b)

giac [A] time = 0.64, size = 14, normalized size = 0.88

$$-\frac{1}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2/((b*x^2 + a)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(b\,x^{2}+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2,x)
```

```
[Out] -1/2/b/(b*x^2+a)
```

maxima [A] time = 1.30, size = 14, normalized size = 0.88

$$-\frac{1}{2\left(bx^{2}+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2/((b*x^2 + a)*b)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.88

$$-\frac{1}{2\,b\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^2,x)
```

```
[Out] -1/(2*b*(a + b*x^2))
```

sympy [A] time = 0.16, size = 15, normalized size = 0.94

$$-\frac{1}{2ab+2b^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2,x)
```

```
[Out] -1/(2*a*b + 2*b**2*x**2)
```

3.158 $\int \frac{1}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

[Out] 1/2*x/a/(b*x^2+a)+1/2*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(-2),x]

[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)^2} \, dx &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\int \frac{1}{a+bx^2} \, dx}{2a}\\ &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(-2),x]

[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])

fricas [**A**] time = 0.89, size = 120, normalized size = 2.67

$$\left[\frac{2\,abx - \left(bx^2 + a\right)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right)}{4\,\left(a^2b^2x^2 + a^3b\right)}, \frac{abx + \left(bx^2 + a\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,\left(a^2b^2x^2 + a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arctan
(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

giac [**A**] time = 0.65, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a} + \frac{x}{2\,\left(bx^2 + a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

maple [**A**] time = 0.00, size = 36, normalized size = 0.80

$$\frac{x}{2\,\left(b\,x^2 + a\right)a} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2,x)
```

```
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [**A**] time = 2.96, size = 35, normalized size = 0.78

$$\frac{x}{2\,\left(abx^2 + a^2\right)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

mupad [**B**] time = 4.74, size = 33, normalized size = 0.73

$$\frac{x}{2\,a\,\left(b\,x^2 + a\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^2,x)
```

```
[Out] x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

sympy [B] time = 0.21, size = 78, normalized size = 1.73

$$\frac{x}{2a^2+2abx^2}-\frac{\sqrt{-\frac{1}{a^3b}}\,\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b}}\,\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2,x)
```

```
[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x
)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

3.159 $\int \frac{1}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=38

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2),x]
```

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{1}{2a\left(a+bx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2}-\log\left(a+bx^2\right)+2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2),x]
```

```
[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

fricas [A] time = 0.96, size = 47, normalized size = 1.24

$$-\frac{\left(bx^2+a\right)\log\left(bx^2+a\right)-2\left(bx^2+a\right)\log(x)-a}{2\left(a^2bx^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 + a
^3)
```

giac [A] time = 0.63, size = 47, normalized size = 1.24

$$\frac{\log\left(x^2\right)}{2\,a^2}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bx^2+2\,a}{2\left(bx^2+a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
+ a)*a^2)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{1}{2\left(b\,x^2+a\right)a}+\frac{\ln(x)}{a^2}-\frac{\ln\left(b\,x^2+a\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

maxima [A] time = 1.39, size = 37, normalized size = 0.97

$$\frac{1}{2\left(abx^2+a^2\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^2}+\frac{\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

mupad [B] time = 4.70, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{a^2}+\frac{1}{2\,a\,\left(b\,x^2+a\right)}-\frac{\ln\left(b\,x^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^2),x)
```

```
[Out] log(x)/a^2 + 1/(2*a*(a + b*x^2)) - log(a + b*x^2)/(2*a^2)
```

sympy [A] time = 0.30, size = 34, normalized size = 0.89

$$\frac{1}{2a^2+2abx^2}+\frac{\log(x)}{a^2}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2,x)
```

```
[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

3.160 $\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

```
[Out] -3/2/a^2/x+1/2/a/x/(b*x^2+a)-3/2*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx &= \frac{1}{2ax\left(a+bx^2\right)}+\frac{3\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a}\\ &= -\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{(3b)\int \frac{1}{a+bx^2}\,dx}{2a^2}\\ &= -\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.95

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{bx}{2a^2(a+bx^2)}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*a^(5/2))
```

fricas [A] time = 0.98, size = 136, normalized size = 2.39

$$\left[-\frac{6\,bx^2-3\,(bx^3+ax)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+4\,a}{4\,(a^2bx^3+a^3x)},-\frac{3\,bx^2+3\,(bx^3+ax)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,a}{2\,(a^2bx^3+a^3x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x
)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

giac [A] time = 0.65, size = 47, normalized size = 0.82

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}-\frac{3\,bx^2+2\,a}{2\,(bx^3+ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
+ a*x)*a^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.81

$$-\frac{bx}{2\,(b\,x^2+a)\,a^2}-\frac{3b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^2}-\frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2,x)
```

```
[Out] -1/a^2/x-1/2/a^2*b*x/(b*x^2+a)-3/2/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b
*x)
```

maxima [A] time = 2.97, size = 49, normalized size = 0.86

$$-\frac{3\,bx^2+2\,a}{2\,(a^2bx^3+a^3x)}-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x^2 + 2*a)/(a^2*b*x^3 + a^3*x) - 3/2*b*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^2)
```

mupad [B] time = 0.07, size = 44, normalized size = 0.77

$$-\frac{\frac{1}{a}+\frac{3\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^2),x)
```

```
[Out] - (1/a + (3*b*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*atan((b^(1/2)*x)/a^(
1/2)))/(2*a^(5/2))
```

sympy [A] time = 0.29, size = 92, normalized size = 1.61

$$\frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{-2a-3bx^2}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2,x)
```

```
[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**3
*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

3.161 $\int \frac{1}{x^3(a+bx^2)^2}\,dx$

Optimal. Leaf size=49

$$\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{1}{2a^2x^2}$$

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b}{2a^2\left(a+bx^2\right)}+\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^
2])/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{2a^2x^2}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx^2\right)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.84

$$-\frac{a\left(\frac{b}{a+bx^2}+\frac{1}{x^2}\right)-2b\log\left(a+bx^2\right)+4b\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/2*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/a^3
```

fricas [A] time = 0.71, size = 73, normalized size = 1.49

$$-\frac{2\,abx^2 + a^2 - 2\left(b^2x^4 + abx^2\right)\log\left(bx^2 + a\right) + 4\left(b^2x^4 + abx^2\right)\log(x)}{2\left(a^3bx^4 + a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4 +
 a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

giac [A] time = 0.60, size = 51, normalized size = 1.04

$$-\frac{b\log\left(x^2\right)}{a^3} + \frac{b\log\left(\left|bx^2 + a\right|\right)}{a^3} - \frac{2\,bx^2 + a}{2\left(bx^4 + ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 + a
*x^2)*a^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$-\frac{b}{2\left(b\,x^2 + a\right)a^2} - \frac{2b\ln(x)}{a^3} + \frac{b\ln\left(b\,x^2 + a\right)}{a^3} - \frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

maxima [A] time = 1.37, size = 52, normalized size = 1.06

$$-\frac{2\,bx^2 + a}{2\left(a^2bx^4 + a^3x^2\right)} + \frac{b\log\left(bx^2 + a\right)}{a^3} - \frac{b\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - b*log(x^2
)/a^3
```

mupad [B] time = 0.08, size = 51, normalized size = 1.04

$$\frac{b\,\ln\left(b\,x^2 + a\right)}{a^3} - \frac{\frac{1}{2\,a} + \frac{b\,x^2}{a^2}}{b\,x^4 + a\,x^2} - \frac{2\,b\,\ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (b*log(a + b*x^2))/a^3 - (1/(2*a) + (b*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b*log
(x))/a^3
```

sympy [A] time = 0.36, size = 51, normalized size = 1.04

$$\frac{-a-2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/b
 + x**2)/a**3
```

3.162 $\int \frac{1}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=68

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

```
[Out] -5/6/a^2/x^3+5/2*b/a^3/x+1/2/a/x^3/(b*x^2+a)+5/2*b^(3/2)*arctan(x*b^(1/2)/a
^(1/2))/a^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^2}\,dx = \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{5\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{2a}$$

$$= -\frac{5}{6a^2x^3} + \frac{1}{2ax^3\left(a+bx^2\right)} - \frac{(5b)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a^2}$$

$$= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{\left(5b^2\right)\int \frac{1}{a+bx^2}\,dx}{2a^3}$$

$$= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.99

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}} + \frac{b^2x}{2a^3\left(a+bx^2\right)} + \frac{2b}{a^3x} - \frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*1/(a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

fricas [A] time = 0.89, size = 172, normalized size = 2.53

$$\left[\frac{30\,b^2x^4 + 20\,abx^2 + 15\left(b^2x^5 + abx^3\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) - 4\,a^2}{12\left(a^3bx^5 + a^4x^3\right)},\ \frac{15\,b^2x^4 + 10\,abx^2 + 15\left(b^2x^5 + abx^3\right)\sqrt{}}{6\left(a^3bx^5 + a^4x^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*x
^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6
*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

giac [A] time = 0.61, size = 59, normalized size = 0.87

$$\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3} + \frac{b^2x}{2\left(bx^2+a\right)a^3} + \frac{6\,bx^2 - a}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3)
 + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

maple [A] time = 0.01, size = 59, normalized size = 0.87

$$\frac{b^2x}{2\left(b\,x^2+a\right)a^3}+\frac{5b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^3}+\frac{2b}{a^3x}-\frac{1}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/3/a^2/x^3+2*b/a^3/x+1/2*b^2/a^3*x/(b*x^2+a)+5/2*b^2/a^3/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.01, size = 64, normalized size = 0.94

$$\frac{15\,b^2x^4+10\,abx^2-2\,a^2}{6\left(a^3bx^5+a^4x^3\right)}+\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*b^2*x^4 + 10*a*b*x^2 - 2*a^2)/(a^3*b*x^5 + a^4*x^3) + 5/2*b^2*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 4.73, size = 58, normalized size = 0.85

$$\frac{\frac{5\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{5\,b^2\,x^4}{2\,a^3}}{b\,x^5+a\,x^3}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2),x)
```

```
[Out] ((5*b*x^2)/(3*a^2) - 1/(3*a) + (5*b^2*x^4)/(2*a^3))/(a*x^3 + b*x^5) + (5*b^
(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(7/2))
```

sympy [A] time = 0.35, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{-2a^2+10abx^2+15b^2x^4}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2,x)
```

```
[Out] -5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/a
**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b*
*2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

3.163 $\int \frac{1}{x^5(a+bx^2)^2}\,dx$

Optimal. Leaf size=66

$$-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

```
[Out] -1/4/a^2/x^4+b/a^3/x^2+1/2*b^2/a^3/(b*x^2+a)+3*b^2*ln(x)/a^4-3/2*b^2*ln(b*x
^2+a)/a^4
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$\frac{b^2}{2a^3\left(a+bx^2\right)}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^2),x]
```

```
[Out] -1/(4*a^2*x^4) + b/(a^3*x^2) + b^2/(2*a^3*(a + b*x^2)) + (3*b^2*Log[x])/a^4
 - (3*b^2*Log[a + b*x^2])/(2*a^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{4a^2x^4}+\frac{b}{a^3x^2}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.86

$$\frac{-6b^2\log\left(a+bx^2\right)+a\left(\frac{2b^2}{a+bx^2}-\frac{a}{x^4}+\frac{4b}{x^2}\right)+12b^2\log(x)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^2),x]
```

```
[Out] (a*(-(a/x^4) + (4*b)/x^2 + (2*b^2)/(a + b*x^2)) + 12*b^2*Log[x] - 6*b^2*Log
[a + b*x^2])/(4*a^4)
```

fricas [A] time = 0.87, size = 90, normalized size = 1.36

$$\frac{6\,ab^2x^4 + 3\,a^2bx^2 - a^3 - 6\left(b^3x^6 + ab^2x^4\right)\log\left(bx^2 + a\right) + 12\left(b^3x^6 + ab^2x^4\right)\log(x)}{4\left(a^4bx^6 + a^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*a*b^2*x^4 + 3*a^2*b*x^2 - a^3 - 6*(b^3*x^6 + a*b^2*x^4)*log(b*x^2 +
a) + 12*(b^3*x^6 + a*b^2*x^4)*log(x))/(a^4*b*x^6 + a^5*x^4)
```

giac [A] time = 0.63, size = 86, normalized size = 1.30

$$\frac{3\,b^2\log\left(x^2\right)}{2\,a^4} - \frac{3\,b^2\log\left(\left|bx^2 + a\right|\right)}{2\,a^4} + \frac{3\,b^3x^2 + 4\,ab^2}{2\left(bx^2 + a\right)a^4} - \frac{9\,b^2x^4 - 4\,abx^2 + a^2}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 3/2*b^2*log(x^2)/a^4 - 3/2*b^2*log(abs(b*x^2 + a))/a^4 + 1/2*(3*b^3*x^2 + 4
*a*b^2)/((b*x^2 + a)*a^4) - 1/4*(9*b^2*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)
```

maple [A] time = 0.01, size = 61, normalized size = 0.92

$$\frac{b^2}{2\left(b\,x^2 + a\right)a^3} + \frac{3b^2\ln(x)}{a^4} - \frac{3b^2\ln\left(b\,x^2 + a\right)}{2a^4} + \frac{b}{a^3x^2} - \frac{1}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^2,x)
```

```
[Out] -1/4/a^2/x^4+b/a^3/x^2+1/2*b^2/a^3/(b*x^2+a)+3*b^2*ln(x)/a^4-3/2*b^2*ln(b*x
^2+a)/a^4
```

maxima [A] time = 1.32, size = 70, normalized size = 1.06

$$\frac{6\,b^2x^4 + 3\,abx^2 - a^2}{4\left(a^3bx^6 + a^4x^4\right)} - \frac{3\,b^2\log\left(bx^2 + a\right)}{2\,a^4} + \frac{3\,b^2\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*b^2*x^4 + 3*a*b*x^2 - a^2)/(a^3*b*x^6 + a^4*x^4) - 3/2*b^2*log(b*x^2
 + a)/a^4 + 3/2*b^2*log(x^2)/a^4
```

mupad [B] time = 4.80, size = 67, normalized size = 1.02

$$\frac{\frac{3\,b\,x^2}{4\,a^2} - \frac{1}{4\,a} + \frac{3\,b^2\,x^4}{2\,a^3}}{b\,x^6 + a\,x^4} - \frac{3\,b^2\,\ln\left(b\,x^2 + a\right)}{2\,a^4} + \frac{3\,b^2\,\ln(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^2),x)
```

```
[Out] ((3*b*x^2)/(4*a^2) - 1/(4*a) + (3*b^2*x^4)/(2*a^3))/(a*x^4 + b*x^6) - (3*b^
2*log(a + b*x^2))/(2*a^4) + (3*b^2*log(x))/a^4
```

sympy [A] time = 0.44, size = 68, normalized size = 1.03

$$\frac{-a^2 + 3abx^2 + 6b^2x^4}{4a^4x^4 + 4a^3bx^6} + \frac{3b^2\log(x)}{a^4} - \frac{3b^2\log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x**2 + 6*b**2*x**4)/(4*a**4*x**4 + 4*a**3*b*x**6) + 3*b**2*l
og(x)/a**4 - 3*b**2*log(a/b + x**2)/(2*a**4)
```

3.164 $\int \frac{1}{x^6(a+bx^2)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{7b^2}{2a^4x}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

```
[Out] -7/10/a^2/x^5+7/6*b/a^3/x^3-7/2*b^2/a^4/x+1/2/a/x^5/(b*x^2+a)-7/2*b^(5/2)*a
rctan(x*b^(1/2)/a^(1/2))/a^(9/2)
```

Rubi [A] time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{7b^2}{2a^4x}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -7/(10*a^2*x^5) + (7*b)/(6*a^3*x^3) - (7*b^2)/(2*a^4*x) + 1/(2*a*x^5*(a + b
*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^2\right)^2}\,dx = \frac{1}{2ax^5\left(a+bx^2\right)} + \frac{7\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{2a}$$
$$= -\frac{7}{10a^2x^5} + \frac{1}{2ax^5\left(a+bx^2\right)} - \frac{(7b)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{2a^2}$$
$$= -\frac{7}{10a^2x^5} + \frac{7b}{6a^3x^3} + \frac{1}{2ax^5\left(a+bx^2\right)} + \frac{\left(7b^2\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a^3}$$
$$= -\frac{7}{10a^2x^5} + \frac{7b}{6a^3x^3} - \frac{7b^2}{2a^4x} + \frac{1}{2ax^5\left(a+bx^2\right)} - \frac{\left(7b^3\right)\int \frac{1}{a+bx^2}\,dx}{2a^4}$$
$$= -\frac{7}{10a^2x^5} + \frac{7b}{6a^3x^3} - \frac{7b^2}{2a^4x} + \frac{1}{2ax^5\left(a+bx^2\right)} - \frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 0.99

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}} - \frac{b^3x}{2a^4\left(a+bx^2\right)} - \frac{3b^2}{a^4x} + \frac{2b}{3a^3x^3} - \frac{1}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -1/5*1/(a^2*x^5) + (2*b)/(3*a^3*x^3) - (3*b^2)/(a^4*x) - (b^3*x)/(2*a^4*(a
+ b*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

fricas [A] time = 0.91, size = 198, normalized size = 2.44

$$\left[-\frac{210\,b^3x^6 + 140\,ab^2x^4 - 28\,a^2bx^2 + 12\,a^3 - 105\left(b^3x^7 + ab^2x^5\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 - 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2+a}\right)}{60\left(a^4bx^7 + a^5x^5\right)}, -\frac{105\,b^3x^6 + 70\,ab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(210*b^3*x^6 + 140*a*b^2*x^4 - 28*a^2*b*x^2 + 12*a^3 - 105*(b^3*x^7
+ a*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a
^4*b*x^7 + a^5*x^5), -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a
^3 + 105*(b^3*x^7 + a*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b*x^7 +
a^5*x^5)]
```

giac [A] time = 0.65, size = 70, normalized size = 0.86

$$-\frac{7\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4} - \frac{b^3x}{2\left(bx^2+a\right)a^4} - \frac{45\,b^2x^4 - 10\,abx^2 + 3\,a^2}{15\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/2*b^3*x/((b*x^2 + a)*a^4
) - 1/15*(45*b^2*x^4 - 10*a*b*x^2 + 3*a^2)/(a^4*x^5)
```

maple [A] time = 0.01, size = 70, normalized size = 0.86

$$-\frac{b^3x}{2\left(b\,x^2+a\right)a^4}-\frac{7b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^4}-\frac{3b^2}{a^4x}+\frac{2b}{3a^3x^3}-\frac{1}{5a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^2,x)
```

```
[Out] -1/5/a^2/x^5-3*b^2/a^4/x+2/3*b/a^3/x^3-1/2*b^3/a^4*x/(b*x^2+a)-7/2*b^3/a^4/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.91, size = 75, normalized size = 0.93

$$-\frac{105\,b^3x^6+70\,ab^2x^4-14\,a^2bx^2+6\,a^3}{30\left(a^4bx^7+a^5x^5\right)}-\frac{7\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a^3)/(a^4*b*x^7 + a^5*
x^5) - 7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

mupad [B] time = 4.85, size = 70, normalized size = 0.86

$$-\frac{\frac{1}{5\,a}-\frac{7\,b\,x^2}{15\,a^2}+\frac{7\,b^2\,x^4}{3\,a^3}+\frac{7\,b^3\,x^6}{2\,a^4}}{b\,x^7+a\,x^5}-\frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^2),x)
```

```
[Out] - (1/(5*a) - (7*b*x^2)/(15*a^2) + (7*b^2*x^4)/(3*a^3) + (7*b^3*x^6)/(2*a^4)
)/(a*x^5 + b*x^7) - (7*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(9/2))
```

sympy [A] time = 0.40, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4}-\frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4}+\frac{-6a^3+14a^2bx^2-70ab^2x^4-105b^3x^6}{30a^5x^5+30a^4bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**2,x)
```

```
[Out] 7*sqrt(-b**5/a**9)*log(-a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - 7*sqrt(-b**5/a*
*9)*log(a**5*sqrt(-b**5/a**9)/b**3 + x)/4 + (-6*a**3 + 14*a**2*b*x**2 - 70*
a*b**2*x**4 - 105*b**3*x**6)/(30*a**5*x**5 + 30*a**4*b*x**7)
```

3.165 $\int \frac{1}{x^7\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=80

$$\frac{2b^3\log\left(a+bx^2\right)}{a^5}-\frac{4b^3\log(x)}{a^5}-\frac{b^3}{2a^4\left(a+bx^2\right)}-\frac{3b^2}{2a^4x^2}+\frac{b}{2a^3x^4}-\frac{1}{6a^2x^6}$$

```
[Out] -1/6/a^2/x^6+1/2*b/a^3/x^4-3/2*b^2/a^4/x^2-1/2*b^3/a^4/(b*x^2+a)-4*b^3*ln(x
)/a^5+2*b^3*ln(b*x^2+a)/a^5
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b^3}{2a^4\left(a+bx^2\right)}-\frac{3b^2}{2a^4x^2}+\frac{2b^3\log\left(a+bx^2\right)}{a^5}-\frac{4b^3\log(x)}{a^5}+\frac{b}{2a^3x^4}-\frac{1}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^2),x]
```

```
[Out] -1/(6*a^2*x^6) + b/(2*a^3*x^4) - (3*b^2)/(2*a^4*x^2) - b^3/(2*a^4*(a + b*x^
2)) - (4*b^3*Log[x])/a^5 + (2*b^3*Log[a + b*x^2])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{6a^2x^6}+\frac{b}{2a^3x^4}-\frac{3b^2}{2a^4x^2}-\frac{b^3}{2a^4\left(a+bx^2\right)}-\frac{4b^3\log(x)}{a^5}+\frac{2b^3\log\left(a+bx^2\right)}{a^5}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 0.85

$$\frac{a\left(-\frac{a^2}{x^6}-\frac{3b^3}{a+bx^2}+\frac{3ab}{x^4}-\frac{9b^2}{x^2}\right)+12b^3\log\left(a+bx^2\right)-24b^3\log(x)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^2),x]
```

```
[Out] (a*(-(a^2/x^6) + (3*a*b)/x^4 - (9*b^2)/x^2 - (3*b^3)/(a + b*x^2)) - 24*b^3*
Log[x] + 12*b^3*Log[a + b*x^2])/(6*a^5)
```

fricas [A] time = 0.61, size = 99, normalized size = 1.24

$$-\frac{12\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}-2\,a^{3}bx^{2}+a^{4}-12\,\left(b^{4}x^{8}+ab^{3}x^{6}\right)\log\left(bx^{2}+a\right)+24\,\left(b^{4}x^{8}+ab^{3}x^{6}\right)\log(x)}{6\,\left(a^{5}bx^{8}+a^{6}x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*b^3*x^6 + 6*a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4 - 12*(b^4*x^8 + a*b^
3*x^6)*log(b*x^2 + a) + 24*(b^4*x^8 + a*b^3*x^6)*log(x))/(a^5*b*x^8 + a^6*x
^6)
```

giac [A] time = 0.64, size = 99, normalized size = 1.24

$$-\frac{2\,b^{3}\log\left(x^{2}\right)}{a^{5}}+\frac{2\,b^{3}\log\left(\left|bx^{2}+a\right|\right)}{a^{5}}-\frac{4\,b^{4}x^{2}+5\,ab^{3}}{2\,\left(bx^{2}+a\right)a^{5}}+\frac{22\,b^{3}x^{6}-9\,ab^{2}x^{4}+3\,a^{2}bx^{2}-a^{3}}{6\,a^{5}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*b^3*log(x^2)/a^5 + 2*b^3*log(abs(b*x^2 + a))/a^5 - 1/2*(4*b^4*x^2 + 5*a*
b^3)/((b*x^2 + a)*a^5) + 1/6*(22*b^3*x^6 - 9*a*b^2*x^4 + 3*a^2*b*x^2 - a^3)
/(a^5*x^6)
```

maple [A] time = 0.01, size = 73, normalized size = 0.91

$$-\frac{b^{3}}{2\left(b\,x^{2}+a\right)a^{4}}-\frac{4b^{3}\ln(x)}{a^{5}}+\frac{2b^{3}\ln\left(b\,x^{2}+a\right)}{a^{5}}-\frac{3b^{2}}{2a^{4}x^{2}}+\frac{b}{2a^{3}x^{4}}-\frac{1}{6a^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^2,x)
```

```
[Out] -1/6/a^2/x^6+1/2*b/a^3/x^4-3/2*b^2/a^4/x^2-1/2*b^3/a^4/(b*x^2+a)-4*b^3*ln(x
)/a^5+2*b^3*ln(b*x^2+a)/a^5
```

maxima [A] time = 1.32, size = 79, normalized size = 0.99

$$-\frac{12\,b^{3}x^{6}+6\,ab^{2}x^{4}-2\,a^{2}bx^{2}+a^{3}}{6\,\left(a^{4}bx^{8}+a^{5}x^{6}\right)}+\frac{2\,b^{3}\log\left(bx^{2}+a\right)}{a^{5}}-\frac{2\,b^{3}\log\left(x^{2}\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(12*b^3*x^6 + 6*a*b^2*x^4 - 2*a^2*b*x^2 + a^3)/(a^4*b*x^8 + a^5*x^6) +
 2*b^3*log(b*x^2 + a)/a^5 - 2*b^3*log(x^2)/a^5
```

mupad [B] time = 0.12, size = 78, normalized size = 0.98

$$\frac{2\,b^{3}\,\ln\left(b\,x^{2}+a\right)}{a^{5}}-\frac{\frac{1}{6\,a}-\frac{b\,x^{2}}{3\,a^{2}}+\frac{b^{2}\,x^{4}}{a^{3}}+\frac{2\,b^{3}\,x^{6}}{a^{4}}}{b\,x^{8}+a\,x^{6}}-\frac{4\,b^{3}\,\ln(x)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)^2),x)
```

```
[Out] (2*b^3*log(a + b*x^2))/a^5 - (1/(6*a) - (b*x^2)/(3*a^2) + (b^2*x^4)/a^3 + (
2*b^3*x^6)/a^4)/(a*x^6 + b*x^8) - (4*b^3*log(x))/a^5
```

sympy [**A**] time = 0.47, size = 78, normalized size = 0.98

$$\frac{-a^3 + 2a^2bx^2 - 6ab^2x^4 - 12b^3x^6}{6a^5x^6 + 6a^4bx^8} - \frac{4b^3 \log(x)}{a^5} + \frac{2b^3 \log\left(\frac{a}{b} + x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**2,x)
```

```
[Out] (-a**3 + 2*a**2*b*x**2 - 6*a*b**2*x**4 - 12*b**3*x**6)/(6*a**5*x**6 + 6*a**
4*b*x**8) - 4*b**3*log(x)/a**5 + 2*b**3*log(a/b + x**2)/a**5
```

3.166 $\int \frac{1}{x^8\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=94

$$\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{11/2}}+\frac{9b^3}{2a^5x}-\frac{3b^2}{2a^4x^3}+\frac{9b}{10a^3x^5}-\frac{9}{14a^2x^7}+\frac{1}{2ax^7\left(a+bx^2\right)}$$

```
[Out] -9/14/a^2/x^7+9/10*b/a^3/x^5-3/2*b^2/a^4/x^3+9/2*b^3/a^5/x+1/2/a/x^7/(b*x^2
+a)+9/2*b^(7/2)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)
```

Rubi [A] time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{3b^2}{2a^4x^3}+\frac{9b^3}{2a^5x}+\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{11/2}}+\frac{9b}{10a^3x^5}-\frac{9}{14a^2x^7}+\frac{1}{2ax^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -9/(14*a^2*x^7) + (9*b)/(10*a^3*x^5) - (3*b^2)/(2*a^4*x^3) + (9*b^3)/(2*a^5
*x) + 1/(2*a*x^7*(a + b*x^2)) + (9*b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
a^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^8\left(a+bx^2\right)^2}\,dx = \frac{1}{2ax^7\left(a+bx^2\right)} + \frac{9\int \frac{1}{x^8\left(a+bx^2\right)}\,dx}{2a}$$

$$= -\frac{9}{14a^2x^7} + \frac{1}{2ax^7\left(a+bx^2\right)} - \frac{(9b)\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{2a^2}$$

$$= -\frac{9}{14a^2x^7} + \frac{9b}{10a^3x^5} + \frac{1}{2ax^7\left(a+bx^2\right)} + \frac{\left(9b^2\right)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{2a^3}$$

$$= -\frac{9}{14a^2x^7} + \frac{9b}{10a^3x^5} - \frac{3b^2}{2a^4x^3} + \frac{1}{2ax^7\left(a+bx^2\right)} - \frac{\left(9b^3\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a^4}$$

$$= -\frac{9}{14a^2x^7} + \frac{9b}{10a^3x^5} - \frac{3b^2}{2a^4x^3} + \frac{9b^3}{2a^5x} + \frac{1}{2ax^7\left(a+bx^2\right)} + \frac{\left(9b^4\right)\int \frac{1}{a+bx^2}\,dx}{2a^5}$$

$$= -\frac{9}{14a^2x^7} + \frac{9b}{10a^3x^5} - \frac{3b^2}{2a^4x^3} + \frac{9b^3}{2a^5x} + \frac{1}{2ax^7\left(a+bx^2\right)} + \frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{11/2}}$$

Mathematica [A] time = 0.06, size = 91, normalized size = 0.97

$$\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{11/2}} + \frac{b^4x}{2a^5\left(a+bx^2\right)} + \frac{4b^3}{a^5x} - \frac{b^2}{a^4x^3} + \frac{2b}{5a^3x^5} - \frac{1}{7a^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -1/7*1/(a^2*x^7) + (2*b)/(5*a^3*x^5) - b^2/(a^4*x^3) + (4*b^3)/(a^5*x) + (b
^4*x)/(2*a^5*(a + b*x^2)) + (9*b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(1
1/2))
```

fricas [A] time = 1.01, size = 220, normalized size = 2.34

$$\left[\frac{630\,b^4x^8 + 420\,ab^3x^6 - 84\,a^2b^2x^4 + 36\,a^3bx^2 - 20\,a^4 + 315\left(b^4x^9 + ab^3x^7\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{140\left(a^5bx^9 + a^6x^7\right)}, \frac{315\,b^4x^8}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/140*(630*b^4*x^8 + 420*a*b^3*x^6 - 84*a^2*b^2*x^4 + 36*a^3*b*x^2 - 20*a^
4 + 315*(b^4*x^9 + a*b^3*x^7)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)
/(b*x^2 + a)))/(a^5*b*x^9 + a^6*x^7), 1/70*(315*b^4*x^8 + 210*a*b^3*x^6 - 4
2*a^2*b^2*x^4 + 18*a^3*b*x^2 - 10*a^4 + 315*(b^4*x^9 + a*b^3*x^7)*sqrt(b/a)
*arctan(x*sqrt(b/a)))/(a^5*b*x^9 + a^6*x^7)]
```

giac [A] time = 0.62, size = 81, normalized size = 0.86

$$\frac{9\,b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^5} + \frac{b^4x}{2\left(bx^2+a\right)a^5} + \frac{140\,b^3x^6 - 35\,ab^2x^4 + 14\,a^2bx^2 - 5\,a^3}{35\,a^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 9/2*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/2*b^4*x/((b*x^2 + a)*a^5)
 + 1/35*(140*b^3*x^6 - 35*a*b^2*x^4 + 14*a^2*b*x^2 - 5*a^3)/(a^5*x^7)
```

maple [A] time = 0.01, size = 81, normalized size = 0.86

$$\frac{b^4x}{2\left(b\,x^2+a\right)a^5}+\frac{9b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^5}+\frac{4b^3}{a^5x}-\frac{b^2}{a^4x^3}+\frac{2b}{5a^3x^5}-\frac{1}{7a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^2+a)^2,x)
```

```
[Out] -1/7/a^2/x^7+4*b^3/a^5/x-b^2/a^4/x^3+2/5*b/a^3/x^5+1/2*b^4/a^5*x/(b*x^2+a)+
9/2*b^4/a^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 86, normalized size = 0.91

$$\frac{315\,b^4x^8+210\,ab^3x^6-42\,a^2b^2x^4+18\,a^3bx^2-10\,a^4}{70\left(a^5bx^9+a^6x^7\right)}+\frac{9\,b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/70*(315*b^4*x^8 + 210*a*b^3*x^6 - 42*a^2*b^2*x^4 + 18*a^3*b*x^2 - 10*a^4)
/(a^5*b*x^9 + a^6*x^7) + 9/2*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

mupad [B] time = 4.56, size = 80, normalized size = 0.85

$$\frac{\frac{9\,b\,x^2}{35\,a^2}-\frac{1}{7\,a}-\frac{3\,b^2\,x^4}{5\,a^3}+\frac{3\,b^3\,x^6}{a^4}+\frac{9\,b^4\,x^8}{2\,a^5}}{b\,x^9+a\,x^7}+\frac{9\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^2)^2),x)
```

```
[Out] ((9*b*x^2)/(35*a^2) - 1/(7*a) - (3*b^2*x^4)/(5*a^3) + (3*b^3*x^6)/a^4 + (9*
b^4*x^8)/(2*a^5))/(a*x^7 + b*x^9) + (9*b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))/(
2*a^(11/2))
```

sympy [A] time = 0.45, size = 138, normalized size = 1.47

$$-\frac{9\sqrt{-\frac{b^7}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^7}{a^{11}}}}{b^4}+x\right)}{4}+\frac{9\sqrt{-\frac{b^7}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^7}{a^{11}}}}{b^4}+x\right)}{4}+\frac{-10a^4+18a^3bx^2-42a^2b^2x^4+210ab^3x^6+315b^4x^8}{70a^6x^7+70a^5bx^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**2+a)**2,x)
```

```
[Out] -9*sqrt(-b**7/a**11)*log(-a**6*sqrt(-b**7/a**11)/b**4 + x)/4 + 9*sqrt(-b**7
/a**11)*log(a**6*sqrt(-b**7/a**11)/b**4 + x)/4 + (-10*a**4 + 18*a**3*b*x**2
 - 42*a**2*b**2*x**4 + 210*a*b**3*x**6 + 315*b**4*x**8)/(70*a**6*x**7 + 70*
a**5*b*x**9)
```

3.167 $\int \frac{1}{x^9(a+bx^2)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{5b^4\log\left(a+bx^2\right)}{2a^6}+\frac{5b^4\log(x)}{a^6}+\frac{b^4}{2a^5\left(a+bx^2\right)}+\frac{2b^3}{a^5x^2}-\frac{3b^2}{4a^4x^4}+\frac{b}{3a^3x^6}-\frac{1}{8a^2x^8}$$

```
[Out] -1/8/a^2/x^8+1/3*b/a^3/x^6-3/4*b^2/a^4/x^4+2*b^3/a^5/x^2+1/2*b^4/a^5/(b*x^2
+a)+5*b^4*ln(x)/a^6-5/2*b^4*ln(b*x^2+a)/a^6
```

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{b^4}{2a^5\left(a+bx^2\right)}+\frac{2b^3}{a^5x^2}-\frac{3b^2}{4a^4x^4}-\frac{5b^4\log\left(a+bx^2\right)}{2a^6}+\frac{5b^4\log(x)}{a^6}+\frac{b}{3a^3x^6}-\frac{1}{8a^2x^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^2)^2),x]
```

```
[Out] -1/(8*a^2*x^8) + b/(3*a^3*x^6) - (3*b^2)/(4*a^4*x^4) + (2*b^3)/(a^5*x^2) +
b^4/(2*a^5*(a + b*x^2)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x^2])/(2*a^
6)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^9\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^5(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^2x^5}-\frac{2b}{a^3x^4}+\frac{3b^2}{a^4x^3}-\frac{4b^3}{a^5x^2}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{5b^5}{a^6(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{8a^2x^8}+\frac{b}{3a^3x^6}-\frac{3b^2}{4a^4x^4}+\frac{2b^3}{a^5x^2}+\frac{b^4}{2a^5\left(a+bx^2\right)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log\left(a+bx^2\right)}{2a^6}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 79, normalized size = 0.85

$$\frac{a\left(-\frac{3a^3}{x^8}+\frac{8a^2b}{x^6}+12b^3\left(\frac{b}{a+bx^2}+\frac{4}{x^2}\right)-\frac{18ab^2}{x^4}\right)-60b^4\log\left(a+bx^2\right)+120b^4\log(x)}{24a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^2)^2),x]
```

```
[Out] (a*((-3*a^3)/x^8 + (8*a^2*b)/x^6 - (18*a*b^2)/x^4 + 12*b^3*(4/x^2 + b/(a +
b*x^2))) + 120*b^4*Log[x] - 60*b^4*Log[a + b*x^2])/(24*a^6)
```

fricas [A] time = 0.86, size = 112, normalized size = 1.20

$$\frac{60\,ab^4x^8+30\,a^2b^3x^6-10\,a^3b^2x^4+5\,a^4bx^2-3\,a^5-60\left(b^5x^{10}+ab^4x^8\right)\log\left(bx^2+a\right)+120\left(b^5x^{10}+ab^4x^8\right)\log(x}{24\left(a^6bx^{10}+a^7x^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(60*a*b^4*x^8 + 30*a^2*b^3*x^6 - 10*a^3*b^2*x^4 + 5*a^4*b*x^2 - 3*a^5
- 60*(b^5*x^10 + a*b^4*x^8)*log(b*x^2 + a) + 120*(b^5*x^10 + a*b^4*x^8)*log
(x))/(a^6*b*x^10 + a^7*x^8)
```

giac [A] time = 0.63, size = 110, normalized size = 1.18

$$\frac{5\,b^4\log\left(x^2\right)}{2\,a^6}-\frac{5\,b^4\log\left(\left|bx^2+a\right|\right)}{2\,a^6}+\frac{5\,b^5x^2+6\,ab^4}{2\left(bx^2+a\right)a^6}-\frac{125\,b^4x^8-48\,ab^3x^6+18\,a^2b^2x^4-8\,a^3bx^2+3\,a^4}{24\,a^6x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^4*log(x^2)/a^6 - 5/2*b^4*log(abs(b*x^2 + a))/a^6 + 1/2*(5*b^5*x^2 + 6
*a*b^4)/((b*x^2 + a)*a^6) - 1/24*(125*b^4*x^8 - 48*a*b^3*x^6 + 18*a^2*b^2*x
^4 - 8*a^3*b*x^2 + 3*a^4)/(a^6*x^8)
```

maple [A] time = 0.01, size = 84, normalized size = 0.90

$$\frac{b^4}{2\left(b\,x^2+a\right)a^5}+\frac{5b^4\ln(x)}{a^6}-\frac{5b^4\ln\left(b\,x^2+a\right)}{2a^6}+\frac{2b^3}{a^5x^2}-\frac{3b^2}{4a^4x^4}+\frac{b}{3a^3x^6}-\frac{1}{8a^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^2+a)^2,x)
```

```
[Out] -1/8/a^2/x^8+1/3*b/a^3/x^6-3/4*b^2/a^4/x^4+2*b^3/a^5/x^2+1/2*b^4/a^5/(b*x^2
+a)+5*b^4*ln(x)/a^6-5/2*b^4*ln(b*x^2+a)/a^6
```

maxima [A] time = 1.30, size = 92, normalized size = 0.99

$$\frac{60\,b^4x^8+30\,ab^3x^6-10\,a^2b^2x^4+5\,a^3bx^2-3\,a^4}{24\left(a^5bx^{10}+a^6x^8\right)}-\frac{5\,b^4\log\left(bx^2+a\right)}{2\,a^6}+\frac{5\,b^4\log\left(x^2\right)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(60*b^4*x^8 + 30*a*b^3*x^6 - 10*a^2*b^2*x^4 + 5*a^3*b*x^2 - 3*a^4)/(a^
5*b*x^10 + a^6*x^8) - 5/2*b^4*log(b*x^2 + a)/a^6 + 5/2*b^4*log(x^2)/a^6
```

mupad [B] time = 4.73, size = 89, normalized size = 0.96

$$\frac{\frac{5\,b\,x^2}{24\,a^2}-\frac{1}{8\,a}-\frac{5\,b^2\,x^4}{12\,a^3}+\frac{5\,b^3\,x^6}{4\,a^4}+\frac{5\,b^4\,x^8}{2\,a^5}}{b\,x^{10}+a\,x^8}-\frac{5\,b^4\,\ln\left(b\,x^2+a\right)}{2\,a^6}+\frac{5\,b^4\,\ln(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^2)^2),x)
```

```
[Out] ((5*b*x^2)/(24*a^2) - 1/(8*a) - (5*b^2*x^4)/(12*a^3) + (5*b^3*x^6)/(4*a^4)
+ (5*b^4*x^8)/(2*a^5))/(a*x^8 + b*x^10) - (5*b^4*log(a + b*x^2))/(2*a^6) +
(5*b^4*log(x))/a^6
```

sympy [A] time = 0.52, size = 94, normalized size = 1.01

$$\frac{-3a^4 + 5a^3bx^2 - 10a^2b^2x^4 + 30ab^3x^6 + 60b^4x^8}{24a^6x^8 + 24a^5bx^{10}} + \frac{5b^4 \log{(x)}}{a^6} - \frac{5b^4 \log{\left(\frac{a}{b} + x^2\right)}}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**2+a)**2,x)
```

```
[Out] (-3*a**4 + 5*a**3*b*x**2 - 10*a**2*b**2*x**4 + 30*a*b**3*x**6 + 60*b**4*x**
8)/(24*a**6*x**8 + 24*a**5*b*x**10) + 5*b**4*log(x)/a**6 - 5*b**4*log(a/b +
 x**2)/(2*a**6)
```

3.168 $\int \frac{x^{15}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=114

$$\frac{a^7}{4b^8\left(a+bx^2\right)^2}-\frac{7a^6}{2b^8\left(a+bx^2\right)}-\frac{21a^5\log\left(a+bx^2\right)}{2b^8}+\frac{15a^4x^2}{2b^7}-\frac{5a^3x^4}{2b^6}+\frac{a^2x^6}{b^5}-\frac{3ax^8}{8b^4}+\frac{x^{10}}{10b^3}$$

```
[Out] 15/2*a^4*x^2/b^7-5/2*a^3*x^4/b^6+a^2*x^6/b^5-3/8*a*x^8/b^4+1/10*x^10/b^3+1/
4*a^7/b^8/(b*x^2+a)^2-7/2*a^6/b^8/(b*x^2+a)-21/2*a^5*ln(b*x^2+a)/b^8
```

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^6}{b^5}-\frac{5a^3x^4}{2b^6}+\frac{15a^4x^2}{2b^7}-\frac{7a^6}{2b^8\left(a+bx^2\right)}+\frac{a^7}{4b^8\left(a+bx^2\right)^2}-\frac{21a^5\log\left(a+bx^2\right)}{2b^8}-\frac{3ax^8}{8b^4}+\frac{x^{10}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a + b*x^2)^3,x]
```

```
[Out] (15*a^4*x^2)/(2*b^7) - (5*a^3*x^4)/(2*b^6) + (a^2*x^6)/b^5 - (3*a*x^8)/(8*b
^4) + x^10/(10*b^3) + a^7/(4*b^8*(a + b*x^2)^2) - (7*a^6)/(2*b^8*(a + b*x^2
)) - (21*a^5*Log[a + b*x^2])/(2*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{15}}{\left(a+bx^2\right)^3}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^7}{(a+bx)^3}\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{15a^4}{b^7}-\frac{10a^3x}{b^6}+\frac{6a^2x^2}{b^5}-\frac{3ax^3}{b^4}+\frac{x^4}{b^3}-\frac{a^7}{b^7(a+bx)^3}+\frac{7a^6}{b^7(a+bx)^2}-\frac{21a^5}{b^7(a+bx)}\right)\right.\\&=\frac{15a^4x^2}{2b^7}-\frac{5a^3x^4}{2b^6}+\frac{a^2x^6}{b^5}-\frac{3ax^8}{8b^4}+\frac{x^{10}}{10b^3}+\frac{a^7}{4b^8\left(a+bx^2\right)^2}-\frac{7a^6}{2b^8\left(a+bx^2\right)}-\frac{21a^5\log\left(a+bx^2\right.}{2b^8}\end{aligned}$$

Mathematica [A] time = 0.03, size = 97, normalized size = 0.85

$$\frac{\frac{10a^7}{\left(a+bx^2\right)^2}-\frac{140a^6}{a+bx^2}-420a^5\log\left(a+bx^2\right)+300a^4bx^2-100a^3b^2x^4+40a^2b^3x^6-15ab^4x^8+4b^5x^{10}}{40b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a + b*x^2)^3,x]
```

```
[Out] (300*a^4*b*x^2 - 100*a^3*b^2*x^4 + 40*a^2*b^3*x^6 - 15*a*b^4*x^8 + 4*b^5*x^
10 + (10*a^7)/(a + b*x^2)^2 - (140*a^6)/(a + b*x^2) - 420*a^5*Log[a + b*x^2
])/(40*b^8)
```

fricas [A] time = 0.80, size = 137, normalized size = 1.20

$$\frac{4\,b^{7}x^{14}-7\,ab^{6}x^{12}+14\,a^{2}b^{5}x^{10}-35\,a^{3}b^{4}x^{8}+140\,a^{4}b^{3}x^{6}+500\,a^{5}b^{2}x^{4}+160\,a^{6}bx^{2}-130\,a^{7}-420\left(a^{5}b^{2}x^{4}+2\right.}{40\left(b^{10}x^{4}+2\,ab^{9}x^{2}+a^{2}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(4*b^7*x^14 - 7*a*b^6*x^12 + 14*a^2*b^5*x^10 - 35*a^3*b^4*x^8 + 140*a^
4*b^3*x^6 + 500*a^5*b^2*x^4 + 160*a^6*b*x^2 - 130*a^7 - 420*(a^5*b^2*x^4 +
2*a^6*b*x^2 + a^7)*log(b*x^2 + a))/(b^10*x^4 + 2*a*b^9*x^2 + a^2*b^8)
```

giac [A] time = 0.63, size = 114, normalized size = 1.00

$$-\frac{21\,a^{5}\log\left(\left|bx^{2}+a\right|\right)}{2\,b^{8}}+\frac{63\,a^{5}b^{2}x^{4}+112\,a^{6}bx^{2}+50\,a^{7}}{4\left(bx^{2}+a\right)^{2}b^{8}}+\frac{4\,b^{12}x^{10}-15\,ab^{11}x^{8}+40\,a^{2}b^{10}x^{6}-100\,a^{3}b^{9}x^{4}+300\,a}{40\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -21/2*a^5*log(abs(b*x^2 + a))/b^8 + 1/4*(63*a^5*b^2*x^4 + 112*a^6*b*x^2 + 5
0*a^7)/((b*x^2 + a)^2*b^8) + 1/40*(4*b^12*x^10 - 15*a*b^11*x^8 + 40*a^2*b^1
0*x^6 - 100*a^3*b^9*x^4 + 300*a^4*b^8*x^2)/b^15
```

maple [A] time = 0.01, size = 101, normalized size = 0.89

$$\frac{x^{10}}{10b^{3}}-\frac{3a\,x^{8}}{8b^{4}}+\frac{a^{2}x^{6}}{b^{5}}-\frac{5a^{3}x^{4}}{2b^{6}}+\frac{a^{7}}{4\left(b\,x^{2}+a\right)^{2}b^{8}}+\frac{15a^{4}x^{2}}{2b^{7}}-\frac{7a^{6}}{2\left(b\,x^{2}+a\right)b^{8}}-\frac{21a^{5}\ln\left(b\,x^{2}+a\right)}{2b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^2+a)^3,x)
```

```
[Out] 15/2*a^4*x^2/b^7-5/2*a^3*x^4/b^6+a^2*x^6/b^5-3/8*a*x^8/b^4+1/10*x^10/b^3+1/
4*a^7/b^8/(b*x^2+a)^2-7/2*a^6/b^8/(b*x^2+a)-21/2*a^5*ln(b*x^2+a)/b^8
```

maxima [A] time = 1.36, size = 111, normalized size = 0.97

$$-\frac{14\,a^{6}bx^{2}+13\,a^{7}}{4\left(b^{10}x^{4}+2\,ab^{9}x^{2}+a^{2}b^{8}\right)}-\frac{21\,a^{5}\log\left(bx^{2}+a\right)}{2\,b^{8}}+\frac{4\,b^{4}x^{10}-15\,ab^{3}x^{8}+40\,a^{2}b^{2}x^{6}-100\,a^{3}bx^{4}+300\,a^{4}x^{2}}{40\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(14*a^6*b*x^2 + 13*a^7)/(b^10*x^4 + 2*a*b^9*x^2 + a^2*b^8) - 21/2*a^5*
log(b*x^2 + a)/b^8 + 1/40*(4*b^4*x^10 - 15*a*b^3*x^8 + 40*a^2*b^2*x^6 - 100
*a^3*b*x^4 + 300*a^4*x^2)/b^7
```

mupad [B] time = 4.72, size = 111, normalized size = 0.97

$$\frac{x^{10}}{10\,b^3}-\frac{\frac{13\,a^7}{4\,b}+\frac{7\,a^6\,x^2}{2}}{a^2\,b^7+2\,a\,b^8\,x^2+b^9\,x^4}-\frac{3\,a\,x^8}{8\,b^4}-\frac{21\,a^5\,\ln\left(b\,x^2+a\right)}{2\,b^8}+\frac{a^2\,x^6}{b^5}-\frac{5\,a^3\,x^4}{2\,b^6}+\frac{15\,a^4\,x^2}{2\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(a + b*x^2)^3,x)
```

```
[Out] x^10/(10*b^3) - ((13*a^7)/(4*b) + (7*a^6*x^2)/2)/(a^2*b^7 + b^9*x^4 + 2*a*b
^8*x^2) - (3*a*x^8)/(8*b^4) - (21*a^5*log(a + b*x^2))/(2*b^8) + (a^2*x^6)/b
^5 - (5*a^3*x^4)/(2*b^6) + (15*a^4*x^2)/(2*b^7)
```

sympy [A] time = 0.48, size = 119, normalized size = 1.04

$$-\frac{21a^5\log\left(a+bx^2\right)}{2b^8}+\frac{15a^4x^2}{2b^7}-\frac{5a^3x^4}{2b^6}+\frac{a^2x^6}{b^5}-\frac{3ax^8}{8b^4}+\frac{-13a^7-14a^6bx^2}{4a^2b^8+8ab^9x^2+4b^{10}x^4}+\frac{x^{10}}{10b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**2+a)**3,x)
```

```
[Out] -21*a**5*log(a + b*x**2)/(2*b**8) + 15*a**4*x**2/(2*b**7) - 5*a**3*x**4/(2*
b**6) + a**2*x**6/b**5 - 3*a*x**8/(8*b**4) + (-13*a**7 - 14*a**6*b*x**2)/(4
*a**2*b**8 + 8*a*b**9*x**2 + 4*b**10*x**4) + x**10/(10*b**3)
```

3.169 $\int \frac{x^{13}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{a^6}{4b^7\left(a+bx^2\right)^2}+\frac{3a^5}{b^7\left(a+bx^2\right)}+\frac{15a^4\log\left(a+bx^2\right)}{2b^7}-\frac{5a^3x^2}{b^6}+\frac{3a^2x^4}{2b^5}-\frac{ax^6}{2b^4}+\frac{x^8}{8b^3}$$

```
[Out] -5*a^3*x^2/b^6+3/2*a^2*x^4/b^5-1/2*a*x^6/b^4+1/8*x^8/b^3-1/4*a^6/b^7/(b*x^2
+a)^2+3*a^5/b^7/(b*x^2+a)+15/2*a^4*ln(b*x^2+a)/b^7
```

Rubi [A] time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2x^4}{2b^5}-\frac{5a^3x^2}{b^6}+\frac{3a^5}{b^7\left(a+bx^2\right)}-\frac{a^6}{4b^7\left(a+bx^2\right)^2}+\frac{15a^4\log\left(a+bx^2\right)}{2b^7}-\frac{ax^6}{2b^4}+\frac{x^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^2)^3,x]
```

```
[Out] (-5*a^3*x^2)/b^6 + (3*a^2*x^4)/(2*b^5) - (a*x^6)/(2*b^4) + x^8/(8*b^3) - a^
6/(4*b^7*(a + b*x^2)^2) + (3*a^5)/(b^7*(a + b*x^2)) + (15*a^4*Log[a + b*x^2
])/(2*b^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13}}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^6}{(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{10a^3}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{b^4}+\frac{x^3}{b^3}+\frac{a^6}{b^6(a+bx)^3}-\frac{6a^5}{b^6(a+bx)^2}+\frac{15a^4}{b^6(a+bx)}\right)dx,x\right.\\
&= -\frac{5a^3x^2}{b^6}+\frac{3a^2x^4}{2b^5}-\frac{ax^6}{2b^4}+\frac{x^8}{8b^3}-\frac{a^6}{4b^7\left(a+bx^2\right)^2}+\frac{3a^5}{b^7\left(a+bx^2\right)}+\frac{15a^4\log\left(a+bx^2\right)}{2b^7}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.85

$$\frac{-\frac{2a^6}{\left(a+bx^2\right)^2}+\frac{24a^5}{a+bx^2}+60a^4\log\left(a+bx^2\right)-40a^3bx^2+12a^2b^2x^4-4ab^3x^6+b^4x^8}{8b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^2)^3,x]
```

```
[Out] (-40*a^3*b*x^2 + 12*a^2*b^2*x^4 - 4*a*b^3*x^6 + b^4*x^8 - (2*a^6)/(a + b*x^
2)^2 + (24*a^5)/(a + b*x^2) + 60*a^4*Log[a + b*x^2])/(8*b^7)
```

fricas [A] time = 0.82, size = 125, normalized size = 1.25

$$\frac{b^6x^{12} - 2\,ab^5x^{10} + 5\,a^2b^4x^8 - 20\,a^3b^3x^6 - 68\,a^4b^2x^4 - 16\,a^5bx^2 + 22\,a^6 + 60\left(a^4b^2x^4 + 2\,a^5bx^2 + a^6\right)\log\left(bx^2 + a\right.}{8\left(b^9x^4 + 2\,ab^8x^2 + a^2b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(b^6*x^12 - 2*a*b^5*x^10 + 5*a^2*b^4*x^8 - 20*a^3*b^3*x^6 - 68*a^4*b^2*
x^4 - 16*a^5*b*x^2 + 22*a^6 + 60*(a^4*b^2*x^4 + 2*a^5*b*x^2 + a^6)*log(b*x^
2 + a))/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)
```

giac [A] time = 0.61, size = 102, normalized size = 1.02

$$\frac{15\,a^4\log\left(\left|bx^2 + a\right|\right)}{2\,b^7} - \frac{45\,a^4b^2x^4 + 78\,a^5bx^2 + 34\,a^6}{4\left(bx^2 + a\right)^2b^7} + \frac{b^9x^8 - 4\,ab^8x^6 + 12\,a^2b^7x^4 - 40\,a^3b^6x^2}{8\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 15/2*a^4*log(abs(b*x^2 + a))/b^7 - 1/4*(45*a^4*b^2*x^4 + 78*a^5*b*x^2 + 34*
a^6)/((b*x^2 + a)^2*b^7) + 1/8*(b^9*x^8 - 4*a*b^8*x^6 + 12*a^2*b^7*x^4 - 40
*a^3*b^6*x^2)/b^12
```

maple [A] time = 0.01, size = 91, normalized size = 0.91

$$\frac{x^8}{8b^3} - \frac{a\,x^6}{2b^4} + \frac{3a^2x^4}{2b^5} - \frac{a^6}{4\left(b\,x^2 + a\right)^2 b^7} - \frac{5a^3x^2}{b^6} + \frac{3a^5}{\left(b\,x^2 + a\right)b^7} + \frac{15a^4\ln\left(b\,x^2 + a\right)}{2b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2+a)^3,x)
```

```
[Out] -5*a^3*x^2/b^6+3/2*a^2*x^4/b^5-1/2*a*x^6/b^4+1/8*x^8/b^3-1/4*a^6/b^7/(b*x^2
+a)^2+3*a^5/b^7/(b*x^2+a)+15/2*a^4*ln(b*x^2+a)/b^7
```

maxima [A] time = 1.29, size = 99, normalized size = 0.99

$$\frac{12\,a^5bx^2 + 11\,a^6}{4\left(b^9x^4 + 2\,ab^8x^2 + a^2b^7\right)} + \frac{15\,a^4\log\left(bx^2 + a\right)}{2\,b^7} + \frac{b^3x^8 - 4\,ab^2x^6 + 12\,a^2bx^4 - 40\,a^3x^2}{8\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*a^5*b*x^2 + 11*a^6)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7) + 15/2*a^4*lo
g(b*x^2 + a)/b^7 + 1/8*(b^3*x^8 - 4*a*b^2*x^6 + 12*a^2*b*x^4 - 40*a^3*x^2)/
b^6
```

mupad [B] time = 0.08, size = 100, normalized size = 1.00

$$\frac{\frac{11\,a^6}{4\,b} + 3\,a^5\,x^2}{a^2\,b^6 + 2\,a\,b^7\,x^2 + b^8\,x^4} + \frac{x^8}{8\,b^3} - \frac{a\,x^6}{2\,b^4} + \frac{15\,a^4\,\ln\left(b\,x^2 + a\right)}{2\,b^7} + \frac{3\,a^2\,x^4}{2\,b^5} - \frac{5\,a^3\,x^2}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a + b*x^2)^3,x)
```

```
[Out] ((11*a^6)/(4*b) + 3*a^5*x^2)/(a^2*b^6 + b^8*x^4 + 2*a*b^7*x^2) + x^8/(8*b^3
) - (a*x^6)/(2*b^4) + (15*a^4*log(a + b*x^2))/(2*b^7) + (3*a^2*x^4)/(2*b^5)
 - (5*a^3*x^2)/b^6
```

sympy [A] time = 0.47, size = 104, normalized size = 1.04

$$\frac{15a^4\log\left(a+bx^2\right)}{2b^7}-\frac{5a^3x^2}{b^6}+\frac{3a^2x^4}{2b^5}-\frac{ax^6}{2b^4}+\frac{11a^6+12a^5bx^2}{4a^2b^7+8ab^8x^2+4b^9x^4}+\frac{x^8}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**2+a)**3,x)
```

```
[Out] 15*a**4*log(a + b*x**2)/(2*b**7) - 5*a**3*x**2/b**6 + 3*a**2*x**4/(2*b**5)
- a*x**6/(2*b**4) + (11*a**6 + 12*a**5*b*x**2)/(4*a**2*b**7 + 8*a*b**8*x**2
 + 4*b**9*x**4) + x**8/(8*b**3)
```

3.170 $\int \frac{x^{11}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=87

$$\frac{a^5}{4b^6\left(a+bx^2\right)^2}-\frac{5a^4}{2b^6\left(a+bx^2\right)}-\frac{5a^3\log\left(a+bx^2\right)}{b^6}+\frac{3a^2x^2}{b^5}-\frac{3ax^4}{4b^4}+\frac{x^6}{6b^3}$$

```
[Out] 3*a^2*x^2/b^5-3/4*a*x^4/b^4+1/6*x^6/b^3+1/4*a^5/b^6/(b*x^2+a)^2-5/2*a^4/b^6
/(b*x^2+a)-5*a^3*ln(b*x^2+a)/b^6
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2x^2}{b^5}-\frac{5a^4}{2b^6\left(a+bx^2\right)}+\frac{a^5}{4b^6\left(a+bx^2\right)^2}-\frac{5a^3\log\left(a+bx^2\right)}{b^6}-\frac{3ax^4}{4b^4}+\frac{x^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2)^3,x]
```

```
[Out] (3*a^2*x^2)/b^5 - (3*a*x^4)/(4*b^4) + x^6/(6*b^3) + a^5/(4*b^6*(a + b*x^2)^
2) - (5*a^4)/(2*b^6*(a + b*x^2)) - (5*a^3*Log[a + b*x^2])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^5}{(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{6a^2}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{b^3}-\frac{a^5}{b^5(a+bx)^3}+\frac{5a^4}{b^5(a+bx)^2}-\frac{10a^3}{b^5(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{3a^2x^2}{b^5}-\frac{3ax^4}{4b^4}+\frac{x^6}{6b^3}+\frac{a^5}{4b^6\left(a+bx^2\right)^2}-\frac{5a^4}{2b^6\left(a+bx^2\right)}-\frac{5a^3\log\left(a+bx^2\right)}{b^6}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 0.86

$$\frac{\frac{3a^5}{\left(a+bx^2\right)^2}-\frac{30a^4}{a+bx^2}-60a^3\log\left(a+bx^2\right)+36a^2bx^2-9ab^2x^4+2b^3x^6}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2)^3,x]
```

```
[Out] (36*a^2*b*x^2 - 9*a*b^2*x^4 + 2*b^3*x^6 + (3*a^5)/(a + b*x^2)^2 - (30*a^4)/
(a + b*x^2) - 60*a^3*Log[a + b*x^2])/(12*b^6)
```

fricas [A] time = 0.78, size = 115, normalized size = 1.32

$$\frac{2\,b^5x^{10} - 5\,ab^4x^8 + 20\,a^2b^3x^6 + 63\,a^3b^2x^4 + 6\,a^4bx^2 - 27\,a^5 - 60\left(a^3b^2x^4 + 2\,a^4bx^2 + a^5\right)\log\left(bx^2 + a\right)}{12\left(b^8x^4 + 2\,ab^7x^2 + a^2b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^5*x^10 - 5*a*b^4*x^8 + 20*a^2*b^3*x^6 + 63*a^3*b^2*x^4 + 6*a^4*b*
x^2 - 27*a^5 - 60*(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^8*x^
4 + 2*a*b^7*x^2 + a^2*b^6)
```

giac [A] time = 0.63, size = 92, normalized size = 1.06

$$-\frac{5\,a^3\log\left(\left|bx^2 + a\right|\right)}{b^6} + \frac{30\,a^3b^2x^4 + 50\,a^4bx^2 + 21\,a^5}{4\left(bx^2 + a\right)^2b^6} + \frac{2\,b^6x^6 - 9\,ab^5x^4 + 36\,a^2b^4x^2}{12\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -5*a^3*log(abs(b*x^2 + a))/b^6 + 1/4*(30*a^3*b^2*x^4 + 50*a^4*b*x^2 + 21*a^
5)/((b*x^2 + a)^2*b^6) + 1/12*(2*b^6*x^6 - 9*a*b^5*x^4 + 36*a^2*b^4*x^2)/b^
9
```

maple [A] time = 0.01, size = 80, normalized size = 0.92

$$\frac{x^6}{6b^3} - \frac{3a\,x^4}{4b^4} + \frac{a^5}{4\left(b\,x^2 + a\right)^2 b^6} + \frac{3a^2x^2}{b^5} - \frac{5a^4}{2\left(b\,x^2 + a\right)b^6} - \frac{5a^3\ln\left(b\,x^2 + a\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a)^3,x)
```

```
[Out] 3*a^2*x^2/b^5-3/4*a*x^4/b^4+1/6*x^6/b^3+1/4*a^5/b^6/(b*x^2+a)^2-5/2*a^4/b^6
/(b*x^2+a)-5*a^3*ln(b*x^2+a)/b^6
```

maxima [A] time = 1.34, size = 89, normalized size = 1.02

$$-\frac{10\,a^4bx^2 + 9\,a^5}{4\left(b^8x^4 + 2\,ab^7x^2 + a^2b^6\right)} - \frac{5\,a^3\log\left(bx^2 + a\right)}{b^6} + \frac{2\,b^2x^6 - 9\,abx^4 + 36\,a^2x^2}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(10*a^4*b*x^2 + 9*a^5)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6) - 5*a^3*log(b
*x^2 + a)/b^6 + 1/12*(2*b^2*x^6 - 9*a*b*x^4 + 36*a^2*x^2)/b^5
```

mupad [B] time = 4.49, size = 90, normalized size = 1.03

$$\frac{x^6}{6\,b^3} - \frac{\frac{9\,a^5}{4\,b} + \frac{5\,a^4\,x^2}{2}}{a^2\,b^5 + 2\,a\,b^6\,x^2 + b^7\,x^4} - \frac{3\,a\,x^4}{4\,b^4} - \frac{5\,a^3\,\ln\left(b\,x^2 + a\right)}{b^6} + \frac{3\,a^2\,x^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2)^3,x)
```

```
[Out] x^6/(6*b^3) - ((9*a^5)/(4*b) + (5*a^4*x^2)/2)/(a^2*b^5 + b^7*x^4 + 2*a*b^6*
x^2) - (3*a*x^4)/(4*b^4) - (5*a^3*log(a + b*x^2))/b^6 + (3*a^2*x^2)/b^5
```

sympy [**A**] time = 0.44, size = 92, normalized size = 1.06

$$-\frac{5a^3\log\left(a+bx^2\right)}{b^6}+\frac{3a^2x^2}{b^5}-\frac{3ax^4}{4b^4}+\frac{-9a^5-10a^4bx^2}{4a^2b^6+8ab^7x^2+4b^8x^4}+\frac{x^6}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a)**3,x)
```

```
[Out] -5*a**3*log(a + b*x**2)/b**6 + 3*a**2*x**2/b**5 - 3*a*x**4/(4*b**4) + (-9*a
**5 - 10*a**4*b*x**2)/(4*a**2*b**6 + 8*a*b**7*x**2 + 4*b**8*x**4) + x**6/(6
*b**3)
```

3.171 $\int \frac{x^9}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=74

$$-\frac{a^4}{4b^5\left(a+bx^2\right)^2}+\frac{2a^3}{b^5\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{b^5}-\frac{3ax^2}{2b^4}+\frac{x^4}{4b^3}$$

```
[Out] -3/2*a*x^2/b^4+1/4*x^4/b^3-1/4*a^4/b^5/(b*x^2+a)^2+2*a^3/b^5/(b*x^2+a)+3*a^
2*ln(b*x^2+a)/b^5
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^4}{4b^5\left(a+bx^2\right)^2}+\frac{2a^3}{b^5\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{b^5}-\frac{3ax^2}{2b^4}+\frac{x^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^3,x]
```

```
[Out] (-3*a*x^2)/(2*b^4) + x^4/(4*b^3) - a^4/(4*b^5*(a + b*x^2)^2) + (2*a^3)/(b^5
*(a + b*x^2)) + (3*a^2*Log[a + b*x^2])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{3a}{b^4}+\frac{x}{b^3}+\frac{a^4}{b^4(a+bx)^3}-\frac{4a^3}{b^4(a+bx)^2}+\frac{6a^2}{b^4(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{3ax^2}{2b^4}+\frac{x^4}{4b^3}-\frac{a^4}{4b^5\left(a+bx^2\right)^2}+\frac{2a^3}{b^5\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.85

$$\frac{-\frac{a^4}{\left(a+bx^2\right)^2}+\frac{8a^3}{a+bx^2}+12a^2\log\left(a+bx^2\right)-6abx^2+b^2x^4}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^3,x]
```

```
[Out] (-6*a*b*x^2 + b^2*x^4 - a^4/(a + b*x^2)^2 + (8*a^3)/(a + b*x^2) + 12*a^2*Lo
g[a + b*x^2])/(4*b^5)
```

fricas [A] time = 0.79, size = 103, normalized size = 1.39

$$\frac{b^4x^8 - 4\,ab^3x^6 - 11\,a^2b^2x^4 + 2\,a^3bx^2 + 7\,a^4 + 12\left(a^2b^2x^4 + 2\,a^3bx^2 + a^4\right)\log\left(bx^2 + a\right)}{4\left(b^7x^4 + 2\,ab^6x^2 + a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*x^8 - 4*a*b^3*x^6 - 11*a^2*b^2*x^4 + 2*a^3*b*x^2 + 7*a^4 + 12*(a^2
*b^2*x^4 + 2*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*
b^5)
```

giac [A] time = 0.61, size = 80, normalized size = 1.08

$$\frac{3\,a^2\log\left(\left|bx^2 + a\right|\right)}{b^5} + \frac{b^3x^4 - 6\,ab^2x^2}{4\,b^6} - \frac{18\,a^2b^2x^4 + 28\,a^3bx^2 + 11\,a^4}{4\left(bx^2 + a\right)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3*a^2*log(abs(b*x^2 + a))/b^5 + 1/4*(b^3*x^4 - 6*a*b^2*x^2)/b^6 - 1/4*(18*a
^2*b^2*x^4 + 28*a^3*b*x^2 + 11*a^4)/((b*x^2 + a)^2*b^5)
```

maple [A] time = 0.01, size = 69, normalized size = 0.93

$$\frac{x^4}{4b^3} - \frac{a^4}{4\left(b\,x^2 + a\right)^2b^5} - \frac{3a\,x^2}{2b^4} + \frac{2a^3}{\left(b\,x^2 + a\right)b^5} + \frac{3a^2\ln\left(b\,x^2 + a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^3,x)
```

```
[Out] -3/2*a*x^2/b^4+1/4*x^4/b^3-1/4*a^4/b^5/(b*x^2+a)^2+2*a^3/b^5/(b*x^2+a)+3*a^
2*ln(b*x^2+a)/b^5
```

maxima [A] time = 1.34, size = 77, normalized size = 1.04

$$\frac{8\,a^3bx^2 + 7\,a^4}{4\left(b^7x^4 + 2\,ab^6x^2 + a^2b^5\right)} + \frac{3\,a^2\log\left(bx^2 + a\right)}{b^5} + \frac{bx^4 - 6\,ax^2}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^3*b*x^2 + 7*a^4)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5) + 3*a^2*log(b*x
^2 + a)/b^5 + 1/4*(b*x^4 - 6*a*x^2)/b^4
```

mupad [B] time = 0.08, size = 78, normalized size = 1.05

$$\frac{\frac{7\,a^4}{4\,b} + 2\,a^3\,x^2}{a^2\,b^4 + 2\,a\,b^5\,x^2 + b^6\,x^4} + \frac{x^4}{4\,b^3} - \frac{3\,a\,x^2}{2\,b^4} + \frac{3\,a^2\,\ln\left(b\,x^2 + a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2)^3,x)
```

```
[Out] ((7*a^4)/(4*b) + 2*a^3*x^2)/(a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + x^4/(4*b^3)
 - (3*a*x^2)/(2*b^4) + (3*a^2*log(a + b*x^2))/b^5
```

sympy [A] time = 0.42, size = 78, normalized size = 1.05

$$\frac{3a^2\log\left(a+bx^2\right)}{b^5}-\frac{3ax^2}{2b^4}+\frac{7a^4+8a^3bx^2}{4a^2b^5+8ab^6x^2+4b^7x^4}+\frac{x^4}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**3,x)
```

```
[Out] 3*a**2*log(a + b*x**2)/b**5 - 3*a*x**2/(2*b**4) + (7*a**4 + 8*a**3*b*x**2)/
(4*a**2*b**5 + 8*a*b**6*x**2 + 4*b**7*x**4) + x**4/(4*b**3)
```

3.172 $\int \frac{x^7}{(a+bx^2)^3} dx$

Optimal. Leaf size=65

$$\frac{a^3}{4b^4(a+bx^2)^2} - \frac{3a^2}{2b^4(a+bx^2)} - \frac{3a\log(a+bx^2)}{2b^4} + \frac{x^2}{2b^3}$$

```
[Out] 1/2*x^2/b^3+1/4*a^3/b^4/(b*x^2+a)^2-3/2*a^2/b^4/(b*x^2+a)-3/2*a*ln(b*x^2+a)
/b^4
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^3}{4b^4(a+bx^2)^2} - \frac{3a^2}{2b^4(a+bx^2)} - \frac{3a\log(a+bx^2)}{2b^4} + \frac{x^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^3,x]
```

```
[Out] x^2/(2*b^3) + a^3/(4*b^4*(a + b*x^2)^2) - (3*a^2)/(2*b^4*(a + b*x^2)) - (3*
a*Log[a + b*x^2])/(2*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(a+bx^2)^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx)^3} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^3} - \frac{a^3}{b^3(a+bx)^3} + \frac{3a^2}{b^3(a+bx)^2} - \frac{3a}{b^3(a+bx)}\right) dx, x, x^2\right) \\
&= \frac{x^2}{2b^3} + \frac{a^3}{4b^4(a+bx^2)^2} - \frac{3a^2}{2b^4(a+bx^2)} - \frac{3a\log(a+bx^2)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 0.74

$$-\frac{\frac{a^2(5a+6bx^2)}{(a+bx^2)^2} + 6a\log(a+bx^2) - 2bx^2}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(-2*b*x^2 + (a^2*(5*a + 6*b*x^2))/(a + b*x^2)^2 + 6*a*Log[a + b*x^2])/
b^4
```

fricas [A] time = 1.29, size = 91, normalized size = 1.40

$$\frac{2\,b^3x^6+4\,ab^2x^4-4\,a^2bx^2-5\,a^3-6\left(ab^2x^4+2\,a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{4\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^6 + 4*a*b^2*x^4 - 4*a^2*b*x^2 - 5*a^3 - 6*(a*b^2*x^4 + 2*a^2*b
*x^2 + a^3)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

giac [A] time = 0.66, size = 62, normalized size = 0.95

$$\frac{x^2}{2\,b^3}-\frac{3\,a\log\left(\left|bx^2+a\right|\right)}{2\,b^4}+\frac{9\,ab^2x^4+12\,a^2bx^2+4\,a^3}{4\left(bx^2+a\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^3 - 3/2*a*log(abs(b*x^2 + a))/b^4 + 1/4*(9*a*b^2*x^4 + 12*a^2*b*x
^2 + 4*a^3)/((b*x^2 + a)^2*b^4)
```

maple [A] time = 0.01, size = 58, normalized size = 0.89

$$\frac{a^3}{4\left(b\,x^2+a\right)^2b^4}+\frac{x^2}{2b^3}-\frac{3a^2}{2\left(b\,x^2+a\right)b^4}-\frac{3a\ln\left(b\,x^2+a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^3,x)
```

```
[Out] 1/2*x^2/b^3+1/4*a^3/b^4/(b*x^2+a)^2-3/2*a^2/b^4/(b*x^2+a)-3/2*a*ln(b*x^2+a)
/b^4
```

maxima [A] time = 1.28, size = 66, normalized size = 1.02

$$-\frac{6\,a^2bx^2+5\,a^3}{4\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}+\frac{x^2}{2\,b^3}-\frac{3\,a\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*a^2*b*x^2 + 5*a^3)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4) + 1/2*x^2/b^3
- 3/2*a*log(b*x^2 + a)/b^4
```

mupad [B] time = 4.75, size = 68, normalized size = 1.05

$$\frac{x^2}{2\,b^3}-\frac{\frac{5\,a^3}{4\,b}+\frac{3\,a^2\,x^2}{2}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}-\frac{3\,a\,\ln\left(b\,x^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^3,x)
```

```
[Out] x^2/(2*b^3) - ((5*a^3)/(4*b) + (3*a^2*x^2)/2)/(a^2*b^3 + b^5*x^4 + 2*a*b^4*
x^2) - (3*a*log(a + b*x^2))/(2*b^4)
```

sympy [A] time = 0.38, size = 68, normalized size = 1.05

$$-\frac{3a\log\left(a+bx^2\right)}{2b^4}+\frac{-5a^3-6a^2bx^2}{4a^2b^4+8ab^5x^2+4b^6x^4}+\frac{x^2}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**3,x)
```

```
[Out] -3*a*log(a + b*x**2)/(2*b**4) + (-5*a**3 - 6*a**2*b*x**2)/(4*a**2*b**4 + 8*
a*b**5*x**2 + 4*b**6*x**4) + x**2/(2*b**3)
```

3.173 $\int \frac{x^5}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=49

$$-\frac{a^2}{4b^3\left(a+bx^2\right)^2}+\frac{a}{b^3\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^3}$$

```
[Out] -1/4*a^2/b^3/(b*x^2+a)^2+a/b^3/(b*x^2+a)+1/2*ln(b*x^2+a)/b^3
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{4b^3\left(a+bx^2\right)^2}+\frac{a}{b^3\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^3,x]
```

```
[Out] -a^2/(4*b^3*(a + b*x^2)^2) + a/(b^3*(a + b*x^2)) + Log[a + b*x^2]/(2*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right)dx, x, x^2\right)\\
&= -\frac{a^2}{4b^3\left(a+bx^2\right)^2}+\frac{a}{b^3\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.80

$$\frac{\frac{a\left(3a+4bx^2\right)}{\left(a+bx^2\right)^2}+2\log\left(a+bx^2\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^3,x]
[Out] ((a*(3*a + 4*b*x^2))/(a + b*x^2)^2 + 2*Log[a + b*x^2])/(4*b^3)
```

fricas [A] time = 0.87, size = 69, normalized size = 1.41

$$\frac{4\,abx^2 + 3\,a^2 + 2\left(b^2x^4 + 2\,abx^2 + a^2\right)\log\left(bx^2 + a\right)}{4\left(b^5x^4 + 2\,ab^4x^2 + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/4*(4*a*b*x^2 + 3*a^2 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^5
*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

giac [A] time = 0.62, size = 42, normalized size = 0.86

$$\frac{\log\left(\left|bx^2 + a\right|\right)}{2\,b^3} - \frac{3\,bx^4 + 2\,ax^2}{4\left(bx^2 + a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/2*log(abs(b*x^2 + a))/b^3 - 1/4*(3*b*x^4 + 2*a*x^2)/((b*x^2 + a)^2*b^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$-\frac{a^2}{4\left(b\,x^2 + a\right)^2 b^3} + \frac{a}{\left(b\,x^2 + a\right)b^3} + \frac{\ln\left(b\,x^2 + a\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^3,x)
[Out] -1/4*a^2/b^3/(b*x^2+a)^2+a/b^3/(b*x^2+a)+1/2*ln(b*x^2+a)/b^3
```

maxima [A] time = 1.30, size = 55, normalized size = 1.12

$$\frac{4\,abx^2 + 3\,a^2}{4\left(b^5x^4 + 2\,ab^4x^2 + a^2b^3\right)} + \frac{\log\left(bx^2 + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/4*(4*a*b*x^2 + 3*a^2)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + 1/2*log(b*x^2 +
 a)/b^3
```

mupad [B] time = 0.06, size = 52, normalized size = 1.06

$$\frac{\frac{3\,a^2}{4\,b^3} + \frac{a\,x^2}{b^2}}{a^2 + 2\,a\,b\,x^2 + b^2\,x^4} + \frac{\ln\left(b\,x^2 + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^3,x)
[Out] ((3*a^2)/(4*b^3) + (a*x^2)/b^2)/(a^2 + b^2*x^4 + 2*a*b*x^2) + log(a + b*x^2
)/(2*b^3)
```

sympy [A] time = 0.32, size = 53, normalized size = 1.08

$$\frac{3a^2 + 4abx^2}{4a^2b^3 + 8ab^4x^2 + 4b^5x^4} + \frac{\log\left(a + bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**3,x)
```

```
[Out] (3*a**2 + 4*a*b*x**2)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + log(a +
 b*x**2)/(2*b**3)
```

3.174 $\int \frac{x^3}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=19

$$\frac{x^4}{4a(a+bx^2)^2}$$

[Out] 1/4*x^4/a/(b*x^2+a)^2

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$\frac{x^4}{4a(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*x^2)^3,x]

[Out] x^4/(4*a*(a + b*x^2)^2)

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2)^3} \, dx = \frac{x^4}{4a(a+bx^2)^2}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{a+2bx^2}{4b^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b*x^2)^3,x]

[Out] -1/4*(a + 2*b*x^2)/(b^2*(a + b*x^2)^2)

fricas [B] time = 0.89, size = 36, normalized size = 1.89

$$-\frac{2\,bx^2+a}{4\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^2+a)^3,x, algorithm="fricas")

[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)

giac [A] time = 0.62, size = 22, normalized size = 1.16

$$-\frac{2\,bx^2 + a}{4\,\left(bx^2 + a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*x^2 + a)/((b*x^2 + a)^2*b^2)
```

maple [A] time = 0.01, size = 31, normalized size = 1.63

$$\frac{a}{4\,\left(b\,x^2 + a\right)^2 b^2} - \frac{1}{2\,\left(b\,x^2 + a\right) b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^3,x)
```

```
[Out] -1/2/b^2/(b*x^2+a)+1/4*a/b^2/(b*x^2+a)^2
```

maxima [B] time = 1.35, size = 36, normalized size = 1.89

$$-\frac{2\,bx^2 + a}{4\,\left(b^4x^4 + 2\,ab^3x^2 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

mupad [B] time = 0.03, size = 37, normalized size = 1.95

$$-\frac{\frac{a}{4\,b^2} + \frac{x^2}{2\,b}}{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^3,x)
```

```
[Out] -(a/(4*b^2) + x^2/(2*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

sympy [B] time = 0.26, size = 36, normalized size = 1.89

$$\frac{-a - 2bx^2}{4a^2b^2 + 8ab^3x^2 + 4b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**3,x)
```

```
[Out] (-a - 2*b*x**2)/(4*a**2*b**2 + 8*a*b**3*x**2 + 4*b**4*x**4)
```

3.175 $\int \frac{x}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{4b\left(a+bx^2\right)^2}$$

```
[Out] -1/4/b/(b*x^2+a)^2
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$-\frac{1}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^3,x]
```

```
[Out] -1/(4*b*(a + b*x^2)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^3} \, dx = -\frac{1}{4b\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^3,x]
```

```
[Out] -1/4*1/(b*(a + b*x^2)^2)
```

fricas [A] time = 0.90, size = 26, normalized size = 1.62

$$-\frac{1}{4\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

giac [A] time = 0.63, size = 14, normalized size = 0.88

$$-\frac{1}{4\left(bx^2+a\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/4/((b*x^2 + a)^2*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{4\left(b\,x^{2}+a\right)^{2} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^3,x)
```

```
[Out] -1/4/b/(b*x^2+a)^2
```

maxima [A] time = 1.33, size = 14, normalized size = 0.88

$$-\frac{1}{4\left(bx^{2}+a\right)^{2} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4/((b*x^2 + a)^2*b)
```

mupad [B] time = 4.62, size = 28, normalized size = 1.75

$$-\frac{1}{4\,a^{2}\,b+8\,a\,b^{2}\,x^{2}+4\,b^{3}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^3,x)
```

```
[Out] -1/(4*a^2*b + 4*b^3*x^4 + 8*a*b^2*x^2)
```

sympy [A] time = 0.24, size = 27, normalized size = 1.69

$$-\frac{1}{4a^{2}b+8ab^{2}x^{2}+4b^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**3,x)
```

```
[Out] -1/(4*a**2*b + 8*a*b**2*x**2 + 4*b**3*x**4)
```

3.176 $\int \frac{1}{x\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=54

$$-\frac{\log\left(a+bx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{1}{2a^2\left(a+bx^2\right)}+\frac{1}{4a\left(a+bx^2\right)^2}$$

```
[Out] 1/4/a/(b*x^2+a)^2+1/2/a^2/(b*x^2+a)+ln(x)/a^3-1/2*ln(b*x^2+a)/a^3
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2a^2\left(a+bx^2\right)}-\frac{\log\left(a+bx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{1}{4a\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^3),x]
```

```
[Out] 1/(4*a*(a + b*x^2)^2) + 1/(2*a^2*(a + b*x^2)) + Log[x]/a^3 - Log[a + b*x^2]
/(2*a^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^3x}-\frac{b}{a(a+bx)^3}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{1}{4a\left(a+bx^2\right)^2}+\frac{1}{2a^2\left(a+bx^2\right)}+\frac{\log(x)}{a^3}-\frac{\log\left(a+bx^2\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.80

$$\frac{\frac{a\left(3a+2bx^2\right)}{\left(a+bx^2\right)^2}-2\log\left(a+bx^2\right)+4\log(x)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^3),x]
```

```
[Out] ((a*(3*a + 2*b*x^2))/(a + b*x^2)^2 + 4*Log[x] - 2*Log[a + b*x^2])/(4*a^3)
```

fricas [A] time = 0.91, size = 90, normalized size = 1.67

$$\frac{2\,abx^2+3\,a^2-2\left(b^2x^4+2\,abx^2+a^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+2\,abx^2+a^2\right)\log(x)}{4\left(a^3b^2x^4+2\,a^4bx^2+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*x^2 + 3*a^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a) + 4*(
b^2*x^4 + 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)
```

giac [A] time = 0.62, size = 59, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,a^3}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^3}+\frac{3\,b^2x^4+8\,abx^2+6\,a^2}{4\left(bx^2+a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^3 - 1/2*log(abs(b*x^2 + a))/a^3 + 1/4*(3*b^2*x^4 + 8*a*b*x^2
 + 6*a^2)/((b*x^2 + a)^2*a^3)
```

maple [A] time = 0.01, size = 49, normalized size = 0.91

$$\frac{1}{4\left(b\,x^2+a\right)^2a}+\frac{1}{2\left(b\,x^2+a\right)a^2}+\frac{\ln(x)}{a^3}-\frac{\ln\left(b\,x^2+a\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^3,x)
```

```
[Out] 1/4/a/(b*x^2+a)^2+1/2/a^2/(b*x^2+a)+ln(x)/a^3-1/2*ln(b*x^2+a)/a^3
```

maxima [A] time = 1.32, size = 60, normalized size = 1.11

$$\frac{2\,bx^2+3\,a}{4\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^3}+\frac{\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 1/2*log(b*x^2 + a)/
a^3 + 1/2*log(x^2)/a^3
```

mupad [B] time = 4.68, size = 56, normalized size = 1.04

$$\frac{\ln(x)}{a^3}+\frac{\frac{3}{4\,a}+\frac{b\,x^2}{2\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\ln\left(b\,x^2+a\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^3),x)
```

```
[Out] log(x)/a^3 + (3/(4*a) + (b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - log(
a + b*x^2)/(2*a^3)
```

sympy **[A]** time = 0.41, size = 56, normalized size = 1.04

$$\frac{3a+2bx^2}{4a^4+8a^3bx^2+4a^2b^2x^4}+\frac{\log(x)}{a^3}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**3,x)
```

```
[Out] (3*a + 2*b*x**2)/(4*a**4 + 8*a**3*b*x**2 + 4*a**2*b**2*x**4) + log(x)/a**3
- log(a/b + x**2)/(2*a**3)
```

3.177 $\int \frac{1}{x^3(a+bx^2)^3}\,dx$

Optimal. Leaf size=67

$$\frac{3b\log\left(a+bx^2\right)}{2a^4}-\frac{3b\log(x)}{a^4}-\frac{b}{a^3\left(a+bx^2\right)}-\frac{1}{2a^3x^2}-\frac{b}{4a^2\left(a+bx^2\right)^2}$$

```
[Out] -1/2/a^3/x^2-1/4*b/a^2/(b*x^2+a)^2-b/a^3/(b*x^2+a)-3*b*ln(x)/a^4+3/2*b*ln(b
*x^2+a)/a^4
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b}{a^3\left(a+bx^2\right)}-\frac{b}{4a^2\left(a+bx^2\right)^2}+\frac{3b\log\left(a+bx^2\right)}{2a^4}-\frac{3b\log(x)}{a^4}-\frac{1}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/(2*a^3*x^2) - b/(4*a^2*(a + b*x^2)^2) - b/(a^3*(a + b*x^2)) - (3*b*Log[x
])/a^4 + (3*b*Log[a + b*x^2])/(2*a^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x^2}-\frac{3b}{a^4x}+\frac{b^2}{a^2(a+bx)^3}+\frac{2b^2}{a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{2a^3x^2}-\frac{b}{4a^2\left(a+bx^2\right)^2}-\frac{b}{a^3\left(a+bx^2\right)}-\frac{3b\log(x)}{a^4}+\frac{3b\log\left(a+bx^2\right)}{2a^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 59, normalized size = 0.88

$$-\frac{\frac{a\left(2a^2+9abx^2+6b^2x^4\right)}{x^2\left(a+bx^2\right)^2}-6b\log\left(a+bx^2\right)+12b\log(x)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/4*((a*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4))/(x^2*(a + b*x^2)^2) + 12*b*Log[x]
 - 6*b*Log[a + b*x^2])/a^4
```

fricas [A] time = 0.88, size = 119, normalized size = 1.78

$$-\frac{6\,ab^2x^4 + 9\,a^2bx^2 + 2\,a^3 - 6\left(b^3x^6 + 2\,ab^2x^4 + a^2bx^2\right)\log\left(bx^2 + a\right) + 12\left(b^3x^6 + 2\,ab^2x^4 + a^2bx^2\right)\log(x)}{4\left(a^4b^2x^6 + 2\,a^5bx^4 + a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3 - 6*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*
x^2)*log(b*x^2 + a) + 12*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4*b
^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)
```

giac [A] time = 0.62, size = 82, normalized size = 1.22

$$-\frac{3\,b\log\left(x^2\right)}{2\,a^4} + \frac{3\,b\log\left(\left|bx^2 + a\right|\right)}{2\,a^4} - \frac{9\,b^3x^4 + 22\,ab^2x^2 + 14\,a^2b}{4\left(bx^2 + a\right)^2a^4} + \frac{3\,bx^2 - a}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/2*b*log(x^2)/a^4 + 3/2*b*log(abs(b*x^2 + a))/a^4 - 1/4*(9*b^3*x^4 + 22*a
*b^2*x^2 + 14*a^2*b)/((b*x^2 + a)^2*a^4) + 1/2*(3*b*x^2 - a)/(a^4*x^2)
```

maple [A] time = 0.01, size = 62, normalized size = 0.93

$$-\frac{b}{4\left(b\,x^2 + a\right)^2a^2} - \frac{b}{\left(b\,x^2 + a\right)a^3} - \frac{3b\ln(x)}{a^4} + \frac{3b\ln\left(b\,x^2 + a\right)}{2a^4} - \frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^3,x)
```

```
[Out] -1/2/a^3/x^2-1/4*b/a^2/(b*x^2+a)^2-b/a^3/(b*x^2+a)-3*b*ln(x)/a^4+3/2*b*ln(b
*x^2+a)/a^4
```

maxima [A] time = 1.34, size = 77, normalized size = 1.15

$$-\frac{6\,b^2x^4 + 9\,abx^2 + 2\,a^2}{4\left(a^3b^2x^6 + 2\,a^4bx^4 + a^5x^2\right)} + \frac{3\,b\log\left(bx^2 + a\right)}{2\,a^4} - \frac{3\,b\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)
+ 3/2*b*log(b*x^2 + a)/a^4 - 3/2*b*log(x^2)/a^4
```

mupad [B] time = 0.08, size = 75, normalized size = 1.12

$$\frac{3\,b\,\ln\left(b\,x^2 + a\right)}{2\,a^4} - \frac{\frac{1}{2\,a} + \frac{9\,b\,x^2}{4\,a^2} + \frac{3\,b^2\,x^4}{2\,a^3}}{a^2\,x^2 + 2\,a\,b\,x^4 + b^2\,x^6} - \frac{3\,b\,\ln(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^3),x)
```

```
[Out] (3*b*log(a + b*x^2))/(2*a^4) - (1/(2*a) + (9*b*x^2)/(4*a^2) + (3*b^2*x^4)/(
2*a^3))/(a^2*x^2 + b^2*x^6 + 2*a*b*x^4) - (3*b*log(x))/a^4
```

sympy [**A**] time = 0.51, size = 80, normalized size = 1.19

$$\frac{-2a^2 - 9abx^2 - 6b^2x^4}{4a^5x^2 + 8a^4bx^4 + 4a^3b^2x^6} - \frac{3b\log{(x)}}{a^4} + \frac{3b\log{\left(\frac{a}{b} + x^2\right)}}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**3,x)
```

```
[Out] (-2*a**2 - 9*a*b*x**2 - 6*b**2*x**4)/(4*a**5*x**2 + 8*a**4*b*x**4 + 4*a**3*
b**2*x**6) - 3*b*log(x)/a**4 + 3*b*log(a/b + x**2)/(2*a**4)
```

3.178 $\int \frac{1}{x^5(a+bx^2)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{3b^2\log\left(a+bx^2\right)}{a^5}+\frac{6b^2\log(x)}{a^5}+\frac{3b^2}{2a^4\left(a+bx^2\right)}+\frac{3b}{2a^4x^2}+\frac{b^2}{4a^3\left(a+bx^2\right)^2}-\frac{1}{4a^3x^4}$$

```
[Out] -1/4/a^3/x^4+3/2*b/a^4/x^2+1/4*b^2/a^3/(b*x^2+a)^2+3/2*b^2/a^4/(b*x^2+a)+6*
b^2*ln(x)/a^5-3*b^2*ln(b*x^2+a)/a^5
```

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$\frac{3b^2}{2a^4\left(a+bx^2\right)}+\frac{b^2}{4a^3\left(a+bx^2\right)^2}-\frac{3b^2\log\left(a+bx^2\right)}{a^5}+\frac{6b^2\log(x)}{a^5}+\frac{3b}{2a^4x^2}-\frac{1}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^3),x]
```

```
[Out] -1/(4*a^3*x^4) + (3*b)/(2*a^4*x^2) + b^2/(4*a^3*(a + b*x^2)^2) + (3*b^2)/(2
*a^4*(a + b*x^2)) + (6*b^2*Log[x])/a^5 - (3*b^2*Log[a + b*x^2])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^3x^3}-\frac{3b}{a^4x^2}+\frac{6b^2}{a^5x}-\frac{b^3}{a^3(a+bx)^3}-\frac{3b^3}{a^4(a+bx)^2}-\frac{6b^3}{a^5(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{4a^3x^4}+\frac{3b}{2a^4x^2}+\frac{b^2}{4a^3\left(a+bx^2\right)^2}+\frac{3b^2}{2a^4\left(a+bx^2\right)}+\frac{6b^2\log(x)}{a^5}-\frac{3b^2\log\left(a+bx^2\right)}{a^5}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.86

$$\frac{\frac{a\left(-a^3+4a^2bx^2+18ab^2x^4+12b^3x^6\right)}{x^4\left(a+bx^2\right)^2}-12b^2\log\left(a+bx^2\right)+24b^2\log(x)}{4a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^3),x]
```

```
[Out] ((a*(-a^3 + 4*a^2*b*x^2 + 18*a*b^2*x^4 + 12*b^3*x^6))/(x^4*(a + b*x^2)^2) +
 24*b^2*Log[x] - 12*b^2*Log[a + b*x^2])/(4*a^5)
```

fricas [A] time = 0.94, size = 134, normalized size = 1.56

$$\frac{12\,ab^3x^6 + 18\,a^2b^2x^4 + 4\,a^3bx^2 - a^4 - 12\left(b^4x^8 + 2\,ab^3x^6 + a^2b^2x^4\right)\log\left(bx^2 + a\right) + 24\left(b^4x^8 + 2\,ab^3x^6 + a^2b^2x\right.}{4\left(a^5b^2x^8 + 2\,a^6bx^6 + a^7x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(12*a*b^3*x^6 + 18*a^2*b^2*x^4 + 4*a^3*b*x^2 - a^4 - 12*(b^4*x^8 + 2*a*
b^3*x^6 + a^2*b^2*x^4)*log(b*x^2 + a) + 24*(b^4*x^8 + 2*a*b^3*x^6 + a^2*b^2
*x^4)*log(x))/(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4)
```

giac [A] time = 0.65, size = 80, normalized size = 0.93

$$\frac{3\,b^2\log\left(x^2\right)}{a^5} - \frac{3\,b^2\log\left(\left|bx^2 + a\right|\right)}{a^5} + \frac{12\,b^3x^6 + 18\,ab^2x^4 + 4\,a^2bx^2 - a^3}{4\left(bx^4 + ax^2\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3*b^2*log(x^2)/a^5 - 3*b^2*log(abs(b*x^2 + a))/a^5 + 1/4*(12*b^3*x^6 + 18*a
*b^2*x^4 + 4*a^2*b*x^2 - a^3)/((b*x^4 + a*x^2)^2*a^4)
```

maple [A] time = 0.02, size = 79, normalized size = 0.92

$$\frac{b^2}{4\left(b\,x^2 + a\right)^2a^3} + \frac{3b^2}{2\left(b\,x^2 + a\right)a^4} + \frac{6b^2\ln(x)}{a^5} - \frac{3b^2\ln\left(b\,x^2 + a\right)}{a^5} + \frac{3b}{2a^4x^2} - \frac{1}{4a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^3,x)
```

```
[Out] -1/4/a^3/x^4+3/2*b/a^4/x^2+1/4*b^2/a^3/(b*x^2+a)^2+3/2*b^2/a^4/(b*x^2+a)+6*
b^2*ln(x)/a^5-3*b^2*ln(b*x^2+a)/a^5
```

maxima [A] time = 1.42, size = 92, normalized size = 1.07

$$\frac{12\,b^3x^6 + 18\,ab^2x^4 + 4\,a^2bx^2 - a^3}{4\left(a^4b^2x^8 + 2\,a^5bx^6 + a^6x^4\right)} - \frac{3\,b^2\log\left(bx^2 + a\right)}{a^5} + \frac{3\,b^2\log\left(x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*b^3*x^6 + 18*a*b^2*x^4 + 4*a^2*b*x^2 - a^3)/(a^4*b^2*x^8 + 2*a^5*b*
x^6 + a^6*x^4) - 3*b^2*log(b*x^2 + a)/a^5 + 3*b^2*log(x^2)/a^5
```

mupad [B] time = 4.67, size = 88, normalized size = 1.02

$$\frac{\frac{b\,x^2}{a^2} - \frac{1}{4\,a} + \frac{9\,b^2\,x^4}{2\,a^3} + \frac{3\,b^3\,x^6}{a^4}}{a^2\,x^4 + 2\,a\,b\,x^6 + b^2\,x^8} - \frac{3\,b^2\,\ln\left(b\,x^2 + a\right)}{a^5} + \frac{6\,b^2\,\ln(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^3),x)
```

```
[Out] ((b*x^2)/a^2 - 1/(4*a) + (9*b^2*x^4)/(2*a^3) + (3*b^3*x^6)/a^4)/(a^2*x^4 +
b^2*x^8 + 2*a*b*x^6) - (3*b^2*log(a + b*x^2))/a^5 + (6*b^2*log(x))/a^5
```

sympy [A] time = 0.55, size = 90, normalized size = 1.05

$$\frac{-a^3+4a^2bx^2+18ab^2x^4+12b^3x^6}{4a^6x^4+8a^5bx^6+4a^4b^2x^8}+\frac{6b^2\log(x)}{a^5}-\frac{3b^2\log\left(\frac{a}{b}+x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**3,x)
```

```
[Out] (-a**3 + 4*a**2*b*x**2 + 18*a*b**2*x**4 + 12*b**3*x**6)/(4*a**6*x**4 + 8*a*
*5*b*x**6 + 4*a**4*b**2*x**8) + 6*b**2*log(x)/a**5 - 3*b**2*log(a/b + x**2)
/a**5
```

3.179 $\int \frac{1}{x^7(a+bx^2)^3}\,dx$

Optimal. Leaf size=95

$$\frac{5b^3\log\left(a+bx^2\right)}{a^6}-\frac{10b^3\log(x)}{a^6}-\frac{2b^3}{a^5\left(a+bx^2\right)}-\frac{3b^2}{a^5x^2}-\frac{b^3}{4a^4\left(a+bx^2\right)^2}+\frac{3b}{4a^4x^4}-\frac{1}{6a^3x^6}$$

```
[Out] -1/6/a^3/x^6+3/4*b/a^4/x^4-3*b^2/a^5/x^2-1/4*b^3/a^4/(b*x^2+a)^2-2*b^3/a^5/
(b*x^2+a)-10*b^3*ln(x)/a^6+5*b^3*ln(b*x^2+a)/a^6
```

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{2b^3}{a^5\left(a+bx^2\right)}-\frac{b^3}{4a^4\left(a+bx^2\right)^2}-\frac{3b^2}{a^5x^2}+\frac{5b^3\log\left(a+bx^2\right)}{a^6}-\frac{10b^3\log(x)}{a^6}+\frac{3b}{4a^4x^4}-\frac{1}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^3),x]
```

```
[Out] -1/(6*a^3*x^6) + (3*b)/(4*a^4*x^4) - (3*b^2)/(a^5*x^2) - b^3/(4*a^4*(a + b*
x^2)^2) - (2*b^3)/(a^5*(a + b*x^2)) - (10*b^3*Log[x])/a^6 + (5*b^3*Log[a +
b*x^2])/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4(a+bx)^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^3x^4}-\frac{3b}{a^4x^3}+\frac{6b^2}{a^5x^2}-\frac{10b^3}{a^6x}+\frac{b^4}{a^4(a+bx)^3}+\frac{4b^4}{a^5(a+bx)^2}+\frac{10b^4}{a^6(a+bx)}\right) d\right.\\ &= -\frac{1}{6a^3x^6}+\frac{3b}{4a^4x^4}-\frac{3b^2}{a^5x^2}-\frac{b^3}{4a^4\left(a+bx^2\right)^2}-\frac{2b^3}{a^5\left(a+bx^2\right)}-\frac{10b^3\log(x)}{a^6}+\frac{5b^3\log\left(a+\right.}{a^6}\end{aligned}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 0.89

$$-\frac{\frac{a\left(2a^4-5a^3bx^2+20a^2b^2x^4+90ab^3x^6+60b^4x^8\right)}{x^6\left(a+bx^2\right)^2}-60b^3\log\left(a+bx^2\right)+120b^3\log(x)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^3),x]
```

```
[Out] -1/12*((a*(2*a^4 - 5*a^3*b*x^2 + 20*a^2*b^2*x^4 + 90*a*b^3*x^6 + 60*b^4*x^8
))/(x^6*(a + b*x^2)^2) + 120*b^3*Log[x] - 60*b^3*Log[a + b*x^2])/a^6
```

fricas [A] time = 0.89, size = 145, normalized size = 1.53

$$-\frac{60\,ab^{4}x^{8}+90\,a^{2}b^{3}x^{6}+20\,a^{3}b^{2}x^{4}-5\,a^{4}bx^{2}+2\,a^{5}-60\left(b^{5}x^{10}+2\,ab^{4}x^{8}+a^{2}b^{3}x^{6}\right)\log\left(bx^{2}+a\right)+120\left(b^{5}x^{10}+\right.}{12\left(a^{6}b^{2}x^{10}+2\,a^{7}bx^{8}+a^{8}x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(60*a*b^4*x^8 + 90*a^2*b^3*x^6 + 20*a^3*b^2*x^4 - 5*a^4*b*x^2 + 2*a^5
 - 60*(b^5*x^10 + 2*a*b^4*x^8 + a^2*b^3*x^6)*log(b*x^2 + a) + 120*(b^5*x^10
 + 2*a*b^4*x^8 + a^2*b^3*x^6)*log(x))/(a^6*b^2*x^10 + 2*a^7*b*x^8 + a^8*x^6
)
```

giac [A] time = 0.65, size = 110, normalized size = 1.16

$$-\frac{5\,b^{3}\log\left(x^{2}\right)}{a^{6}}+\frac{5\,b^{3}\log\left(|bx^{2}+a|\right)}{a^{6}}-\frac{30\,b^{5}x^{4}+68\,ab^{4}x^{2}+39\,a^{2}b^{3}}{4\left(bx^{2}+a\right)^{2}a^{6}}+\frac{110\,b^{3}x^{6}-36\,ab^{2}x^{4}+9\,a^{2}bx^{2}-2\,a^{3}}{12\,a^{6}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -5*b^3*log(x^2)/a^6 + 5*b^3*log(abs(b*x^2 + a))/a^6 - 1/4*(30*b^5*x^4 + 68*
a*b^4*x^2 + 39*a^2*b^3)/((b*x^2 + a)^2*a^6) + 1/12*(110*b^3*x^6 - 36*a*b^2*
x^4 + 9*a^2*b*x^2 - 2*a^3)/(a^6*x^6)
```

maple [A] time = 0.01, size = 90, normalized size = 0.95

$$-\frac{b^{3}}{4\left(b\,x^{2}+a\right)^{2}a^{4}}-\frac{2b^{3}}{\left(b\,x^{2}+a\right)a^{5}}-\frac{10b^{3}\ln(x)}{a^{6}}+\frac{5b^{3}\ln\left(b\,x^{2}+a\right)}{a^{6}}-\frac{3b^{2}}{a^{5}x^{2}}+\frac{3b}{4a^{4}x^{4}}-\frac{1}{6a^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^3,x)
```

```
[Out] -1/6/a^3/x^6+3/4*b/a^4/x^4-3*b^2/a^5/x^2-1/4*b^3/a^4/(b*x^2+a)^2-2*b^3/a^5/
(b*x^2+a)-10*b^3*ln(x)/a^6+5*b^3*ln(b*x^2+a)/a^6
```

maxima [A] time = 1.36, size = 103, normalized size = 1.08

$$-\frac{60\,b^{4}x^{8}+90\,ab^{3}x^{6}+20\,a^{2}b^{2}x^{4}-5\,a^{3}bx^{2}+2\,a^{4}}{12\left(a^{5}b^{2}x^{10}+2\,a^{6}bx^{8}+a^{7}x^{6}\right)}+\frac{5\,b^{3}\log\left(bx^{2}+a\right)}{a^{6}}-\frac{5\,b^{3}\log\left(x^{2}\right)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(60*b^4*x^8 + 90*a*b^3*x^6 + 20*a^2*b^2*x^4 - 5*a^3*b*x^2 + 2*a^4)/(a
^5*b^2*x^10 + 2*a^6*b*x^8 + a^7*x^6) + 5*b^3*log(b*x^2 + a)/a^6 - 5*b^3*log
(x^2)/a^6
```

mupad [B] time = 4.69, size = 101, normalized size = 1.06

$$\frac{5\,b^3\,\ln\left(b\,x^2+a\right)}{a^6}-\frac{\frac{1}{6\,a}-\frac{5\,b\,x^2}{12\,a^2}+\frac{5\,b^2\,x^4}{3\,a^3}+\frac{15\,b^3\,x^6}{2\,a^4}+\frac{5\,b^4\,x^8}{a^5}}{a^2\,x^6+2\,a\,b\,x^8+b^2\,x^{10}}-\frac{10\,b^3\,\ln(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)^3),x)
```

```
[Out] (5*b^3*log(a + b*x^2))/a^6 - (1/(6*a) - (5*b*x^2)/(12*a^2) + (5*b^2*x^4)/(3
*a^3) + (15*b^3*x^6)/(2*a^4) + (5*b^4*x^8)/a^5)/(a^2*x^6 + b^2*x^10 + 2*a*b
*x^8) - (10*b^3*log(x))/a^6
```

sympy [A] time = 0.59, size = 104, normalized size = 1.09

$$\frac{-2a^4+5a^3bx^2-20a^2b^2x^4-90ab^3x^6-60b^4x^8}{12a^7x^6+24a^6bx^8+12a^5b^2x^{10}}-\frac{10b^3\log\left(x\right)}{a^6}+\frac{5b^3\log\left(\frac{a}{b}+x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**3,x)
```

```
[Out] (-2*a**4 + 5*a**3*b*x**2 - 20*a**2*b**2*x**4 - 90*a*b**3*x**6 - 60*b**4*x**
8)/(12*a**7*x**6 + 24*a**6*b*x**8 + 12*a**5*b**2*x**10) - 10*b**3*log(x)/a*
*6 + 5*b**3*log(a/b + x**2)/a**6
```

3.180 $\int \frac{1}{x^9\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=112

$$-\frac{15b^4\log\left(a+bx^2\right)}{2a^7}+\frac{15b^4\log(x)}{a^7}+\frac{5b^4}{2a^6\left(a+bx^2\right)}+\frac{5b^3}{a^6x^2}+\frac{b^4}{4a^5\left(a+bx^2\right)^2}-\frac{3b^2}{2a^5x^4}+\frac{b}{2a^4x^6}-\frac{1}{8a^3x^8}$$

```
[Out] -1/8/a^3/x^8+1/2*b/a^4/x^6-3/2*b^2/a^5/x^4+5*b^3/a^6/x^2+1/4*b^4/a^5/(b*x^2
+a)^2+5/2*b^4/a^6/(b*x^2+a)+15*b^4*ln(x)/a^7-15/2*b^4*ln(b*x^2+a)/a^7
```

Rubi [A] time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{5b^4}{2a^6\left(a+bx^2\right)}+\frac{b^4}{4a^5\left(a+bx^2\right)^2}+\frac{5b^3}{a^6x^2}-\frac{3b^2}{2a^5x^4}-\frac{15b^4\log\left(a+bx^2\right)}{2a^7}+\frac{15b^4\log(x)}{a^7}+\frac{b}{2a^4x^6}-\frac{1}{8a^3x^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^2)^3),x]
```

```
[Out] -1/(8*a^3*x^8) + b/(2*a^4*x^6) - (3*b^2)/(2*a^5*x^4) + (5*b^3)/(a^6*x^2) +
b^4/(4*a^5*(a + b*x^2)^2) + (5*b^4)/(2*a^6*(a + b*x^2)) + (15*b^4*Log[x])/a
^7 - (15*b^4*Log[a + b*x^2])/(2*a^7)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^9\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^5(a+bx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x^5}-\frac{3b}{a^4x^4}+\frac{6b^2}{a^5x^3}-\frac{10b^3}{a^6x^2}+\frac{15b^4}{a^7x}-\frac{b^5}{a^5(a+bx)^3}-\frac{5b^5}{a^6(a+bx)^2}-\frac{15b^5}{a^7(a+bx}\right.\right.\\ &= -\frac{1}{8a^3x^8}+\frac{b}{2a^4x^6}-\frac{3b^2}{2a^5x^4}+\frac{5b^3}{a^6x^2}+\frac{b^4}{4a^5\left(a+bx^2\right)^2}+\frac{5b^4}{2a^6\left(a+bx^2\right)}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\,\mathrm{l}}{}\end{aligned}$$

Mathematica [A] time = 0.06, size = 96, normalized size = 0.86

$$\frac{\frac{a\left(-a^5+2a^4bx^2-5a^3b^2x^4+20a^2b^3x^6+90ab^4x^8+60b^5x^{10}\right)}{x^8\left(a+bx^2\right)^2}-60b^4\log\left(a+bx^2\right)+120b^4\log(x)}{8a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^2)^3),x]
```

```
[Out] ((a*(-a^5 + 2*a^4*b*x^2 - 5*a^3*b^2*x^4 + 20*a^2*b^3*x^6 + 90*a*b^4*x^8 + 6
0*b^5*x^10))/(x^8*(a + b*x^2)^2) + 120*b^4*Log[x] - 60*b^4*Log[a + b*x^2])/
(8*a^7)
```

fricas [A] time = 0.95, size = 156, normalized size = 1.39

$$\frac{60\,ab^{5}x^{10}+90\,a^{2}b^{4}x^{8}+20\,a^{3}b^{3}x^{6}-5\,a^{4}b^{2}x^{4}+2\,a^{5}bx^{2}-a^{6}-60\left(b^{6}x^{12}+2\,ab^{5}x^{10}+a^{2}b^{4}x^{8}\right)\log\left(bx^{2}+a\right)+}{8\left(a^{7}b^{2}x^{12}+2\,a^{8}bx^{10}+a^{9}x^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(60*a*b^5*x^10 + 90*a^2*b^4*x^8 + 20*a^3*b^3*x^6 - 5*a^4*b^2*x^4 + 2*a^
5*b*x^2 - a^6 - 60*(b^6*x^12 + 2*a*b^5*x^10 + a^2*b^4*x^8)*log(b*x^2 + a) +
 120*(b^6*x^12 + 2*a*b^5*x^10 + a^2*b^4*x^8)*log(x))/(a^7*b^2*x^12 + 2*a^8*
b*x^10 + a^9*x^8)
```

giac [A] time = 0.63, size = 119, normalized size = 1.06

$$\frac{15\,b^{4}\log\left(x^{2}\right)}{2\,a^{7}}-\frac{15\,b^{4}\log\left(\left|bx^{2}+a\right|\right)}{2\,a^{7}}+\frac{45\,b^{6}x^{4}+100\,ab^{5}x^{2}+56\,a^{2}b^{4}}{4\left(bx^{2}+a\right)^{2}a^{7}}-\frac{125\,b^{4}x^{8}-40\,ab^{3}x^{6}+12\,a^{2}b^{2}x^{4}-4\,a^{3}bx}{8\,a^{7}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 15/2*b^4*log(x^2)/a^7 - 15/2*b^4*log(abs(b*x^2 + a))/a^7 + 1/4*(45*b^6*x^4
+ 100*a*b^5*x^2 + 56*a^2*b^4)/((b*x^2 + a)^2*a^7) - 1/8*(125*b^4*x^8 - 40*a
*b^3*x^6 + 12*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)/(a^7*x^8)
```

maple [A] time = 0.01, size = 101, normalized size = 0.90

$$\frac{b^{4}}{4\left(b\,x^{2}+a\right)^{2}a^{5}}+\frac{5b^{4}}{2\left(b\,x^{2}+a\right)a^{6}}+\frac{15b^{4}\ln(x)}{a^{7}}-\frac{15b^{4}\ln\left(b\,x^{2}+a\right)}{2a^{7}}+\frac{5b^{3}}{a^{6}x^{2}}-\frac{3b^{2}}{2a^{5}x^{4}}+\frac{b}{2a^{4}x^{6}}-\frac{1}{8a^{3}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^2+a)^3,x)
```

```
[Out] -1/8/a^3/x^8+1/2*b/a^4/x^6-3/2*b^2/a^5/x^4+5*b^3/a^6/x^2+1/4*b^4/a^5/(b*x^2
+a)^2+5/2*b^4/a^6/(b*x^2+a)+15*b^4*ln(x)/a^7-15/2*b^4*ln(b*x^2+a)/a^7
```

maxima [A] time = 1.34, size = 114, normalized size = 1.02

$$\frac{60\,b^{5}x^{10}+90\,ab^{4}x^{8}+20\,a^{2}b^{3}x^{6}-5\,a^{3}b^{2}x^{4}+2\,a^{4}bx^{2}-a^{5}}{8\left(a^{6}b^{2}x^{12}+2\,a^{7}bx^{10}+a^{8}x^{8}\right)}-\frac{15\,b^{4}\log\left(bx^{2}+a\right)}{2\,a^{7}}+\frac{15\,b^{4}\log\left(x^{2}\right)}{2\,a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(60*b^5*x^10 + 90*a*b^4*x^8 + 20*a^2*b^3*x^6 - 5*a^3*b^2*x^4 + 2*a^4*b*
x^2 - a^5)/(a^6*b^2*x^12 + 2*a^7*b*x^10 + a^8*x^8) - 15/2*b^4*log(b*x^2 + a
)/a^7 + 15/2*b^4*log(x^2)/a^7
```

mupad [B] time = 4.86, size = 111, normalized size = 0.99

$$\frac{\frac{b\,x^2}{4\,a^2}-\frac{1}{8\,a}-\frac{5\,b^2\,x^4}{8\,a^3}+\frac{5\,b^3\,x^6}{2\,a^4}+\frac{45\,b^4\,x^8}{4\,a^5}+\frac{15\,b^5\,x^{10}}{2\,a^6}}{a^2\,x^8+2\,a\,b\,x^{10}+b^2\,x^{12}}-\frac{15\,b^4\,\ln\left(b\,x^2+a\right)}{2\,a^7}+\frac{15\,b^4\,\ln(x)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^2)^3),x)
```

```
[Out] ((b*x^2)/(4*a^2) - 1/(8*a) - (5*b^2*x^4)/(8*a^3) + (5*b^3*x^6)/(2*a^4) + (4
5*b^4*x^8)/(4*a^5) + (15*b^5*x^10)/(2*a^6))/(a^2*x^8 + b^2*x^12 + 2*a*b*x^1
0) - (15*b^4*log(a + b*x^2))/(2*a^7) + (15*b^4*log(x))/a^7
```

sympy [A] time = 0.65, size = 116, normalized size = 1.04

$$\frac{-a^5+2a^4bx^2-5a^3b^2x^4+20a^2b^3x^6+90ab^4x^8+60b^5x^{10}}{8a^8x^8+16a^7bx^{10}+8a^6b^2x^{12}}+\frac{15b^4\log{(x)}}{a^7}-\frac{15b^4\log{\left(\frac{a}{b}+x^2\right)}}{2a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**2+a)**3,x)
```

```
[Out] (-a**5 + 2*a**4*b*x**2 - 5*a**3*b**2*x**4 + 20*a**2*b**3*x**6 + 90*a*b**4*x
**8 + 60*b**5*x**10)/(8*a**8*x**8 + 16*a**7*b*x**10 + 8*a**6*b**2*x**12) +
15*b**4*log(x)/a**7 - 15*b**4*log(a/b + x**2)/(2*a**7)
```

3.181 $\int \frac{x^{12}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=111

$$\frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}-\frac{99a^3x}{8b^6}+\frac{33a^2x^3}{8b^5}-\frac{99ax^5}{40b^4}-\frac{11x^9}{8b^2\left(a+bx^2\right)}-\frac{x^{11}}{4b\left(a+bx^2\right)^2}+\frac{99x^7}{56b^3}$$

```
[Out] -99/8*a^3*x/b^6+33/8*a^2*x^3/b^5-99/40*a*x^5/b^4+99/56*x^7/b^3-1/4*x^11/b/(
b*x^2+a)^2-11/8*x^9/b^2/(b*x^2+a)+99/8*a^(7/2)*arctan(x*b^(1/2)/a^(1/2))/b^
(13/2)
```

Rubi [A] time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{33a^2x^3}{8b^5}-\frac{99a^3x}{8b^6}+\frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}-\frac{11x^9}{8b^2\left(a+bx^2\right)}-\frac{99ax^5}{40b^4}-\frac{x^{11}}{4b\left(a+bx^2\right)^2}+\frac{99x^7}{56b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^2)^3,x]
```

```
[Out] (-99*a^3*x)/(8*b^6) + (33*a^2*x^3)/(8*b^5) - (99*a*x^5)/(40*b^4) + (99*x^7)
/(56*b^3) - x^11/(4*b*(a + b*x^2)^2) - (11*x^9)/(8*b^2*(a + b*x^2)) + (99*a
^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{12}}{\left(a+bx^2\right)^3}\,dx &= -\frac{x^{11}}{4b\left(a+bx^2\right)^2}+\frac{11\int \frac{x^{10}}{\left(a+bx^2\right)^2}\,dx}{4b}\\
&= -\frac{x^{11}}{4b\left(a+bx^2\right)^2}-\frac{11x^9}{8b^2\left(a+bx^2\right)}+\frac{99\int \frac{x^8}{a+bx^2}\,dx}{8b^2}\\
&= -\frac{x^{11}}{4b\left(a+bx^2\right)^2}-\frac{11x^9}{8b^2\left(a+bx^2\right)}+\frac{99\int \left(-\frac{a^3}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^4}{b^2}+\frac{x^6}{b}+\frac{a^4}{b^4\left(a+bx^2\right)}\right)dx}{8b^2}\\
&= -\frac{99a^3x}{8b^6}+\frac{33a^2x^3}{8b^5}-\frac{99ax^5}{40b^4}+\frac{99x^7}{56b^3}-\frac{x^{11}}{4b\left(a+bx^2\right)^2}-\frac{11x^9}{8b^2\left(a+bx^2\right)}+\frac{\left(99a^4\right)\int \frac{1}{a+bx^2}\,dx}{8b^6}\\
&= -\frac{99a^3x}{8b^6}+\frac{33a^2x^3}{8b^5}-\frac{99ax^5}{40b^4}+\frac{99x^7}{56b^3}-\frac{x^{11}}{4b\left(a+bx^2\right)^2}-\frac{11x^9}{8b^2\left(a+bx^2\right)}+\frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 99, normalized size = 0.89

$$
\frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}-\frac{3465a^5x+5775a^4bx^3+1848a^3b^2x^5-264a^2b^3x^7+88ab^4x^9-40b^5x^{11}}{280b^6\left(a+bx^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^2)^3,x]
```

```
[Out] -1/280*(3465*a^5*x + 5775*a^4*b*x^3 + 1848*a^3*b^2*x^5 - 264*a^2*b^3*x^7 +
88*a*b^4*x^9 - 40*b^5*x^11)/(b^6*(a + b*x^2)^2) + (99*a^(7/2)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(8*b^(13/2))
```

fricas [A] time = 0.86, size = 278, normalized size = 2.50

$$
\left[\frac{80\,b^5x^{11}-176\,ab^4x^9+528\,a^2b^3x^7-3696\,a^3b^2x^5-11550\,a^4bx^3-6930\,a^5x+3465\left(a^3b^2x^4+2\,a^4bx^2+a^5\right)\sqrt{-\frac{a}{b}}}{560\left(b^8x^4+2\,ab^7x^2+a^2b^6\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/560*(80*b^5*x^11 - 176*a*b^4*x^9 + 528*a^2*b^3*x^7 - 3696*a^3*b^2*x^5 -
11550*a^4*b*x^3 - 6930*a^5*x + 3465*(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)*sqrt(
-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^8*x^4 + 2*a*b^7*x
^2 + a^2*b^6), 1/280*(40*b^5*x^11 - 88*a*b^4*x^9 + 264*a^2*b^3*x^7 - 1848*a
^3*b^2*x^5 - 5775*a^4*b*x^3 - 3465*a^5*x + 3465*(a^3*b^2*x^4 + 2*a^4*b*x^2
+ a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6)
]
```

giac [A] time = 0.61, size = 96, normalized size = 0.86

$$
\frac{99\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^6}-\frac{21\,a^4bx^3+19\,a^5x}{8\left(bx^2+a\right)^2b^6}+\frac{5\,b^{18}x^7-21\,ab^{17}x^5+70\,a^2b^{16}x^3-350\,a^3b^{15}x}{35\,b^{21}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 99/8*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) - 1/8*(21*a^4*b*x^3 + 19*a^5
*x)/((b*x^2 + a)^2*b^6) + 1/35*(5*b^18*x^7 - 21*a*b^17*x^5 + 70*a^2*b^16*x^
3 - 350*a^3*b^15*x)/b^21
```

maple [A] time = 0.01, size = 99, normalized size = 0.89

$$\frac{x^7}{7b^3}-\frac{21a^4x^3}{8\left(b\,x^2+a\right)^2b^5}-\frac{3a\,x^5}{5b^4}-\frac{19a^5x}{8\left(b\,x^2+a\right)^2b^6}+\frac{2a^2x^3}{b^5}+\frac{99a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^6}-\frac{10a^3x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2+a)^3,x)
```

```
[Out] 1/7*x^7/b^3-3/5*a*x^5/b^4+2*a^2*x^3/b^5-10*a^3*x/b^6-21/8/b^5*a^4/(b*x^2+a)
^2*x^3-19/8/b^6*a^5/(b*x^2+a)^2*x+99/8/b^6*a^4/(a*b)^(1/2)*arctan(1/(a*b)^(
1/2)*b*x)
```

maxima [A] time = 2.95, size = 105, normalized size = 0.95

$$-\frac{21\,a^4bx^3+19\,a^5x}{8\left(b^8x^4+2\,ab^7x^2+a^2b^6\right)}+\frac{99\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^6}+\frac{5\,b^3x^7-21\,ab^2x^5+70\,a^2bx^3-350\,a^3x}{35\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(21*a^4*b*x^3 + 19*a^5*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6) + 99/8*a^4
*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/35*(5*b^3*x^7 - 21*a*b^2*x^5 + 7
0*a^2*b*x^3 - 350*a^3*x)/b^6
```

mupad [B] time = 0.08, size = 99, normalized size = 0.89

$$\frac{x^7}{7\,b^3}-\frac{\frac{19\,a^5\,x}{8}+\frac{21\,b\,a^4\,x^3}{8}}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4}-\frac{3\,a\,x^5}{5\,b^4}-\frac{10\,a^3\,x}{b^6}+\frac{99\,a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{13/2}}+\frac{2\,a^2\,x^3}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x^2)^3,x)
```

```
[Out] x^7/(7*b^3) - ((19*a^5*x)/8 + (21*a^4*b*x^3)/8)/(a^2*b^6 + b^8*x^4 + 2*a*b^
7*x^2) - (3*a*x^5)/(5*b^4) - (10*a^3*x)/b^6 + (99*a^(7/2)*atan((b^(1/2)*x)/
a^(1/2)))/(8*b^(13/2)) + (2*a^2*x^3)/b^5
```

sympy [A] time = 0.49, size = 162, normalized size = 1.46

$$-\frac{10a^3x}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^5}{5b^4}-\frac{99\sqrt{-\frac{a^7}{b^{13}}}\log\left(x-\frac{b^6\sqrt{-\frac{a^7}{b^{13}}}}{a^3}\right)}{16}+\frac{99\sqrt{-\frac{a^7}{b^{13}}}\log\left(x+\frac{b^6\sqrt{-\frac{a^7}{b^{13}}}}{a^3}\right)}{16}+\frac{-19a^5x-21a^4bx^3}{8a^2b^6+16ab^7x^2+8b^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x**2+a)**3,x)
```

```
[Out] -10*a**3*x/b**6 + 2*a**2*x**3/b**5 - 3*a*x**5/(5*b**4) - 99*sqrt(-a**7/b**1
3)*log(x - b**6*sqrt(-a**7/b**13)/a**3)/16 + 99*sqrt(-a**7/b**13)*log(x + b
**6*sqrt(-a**7/b**13)/a**3)/16 + (-19*a**5*x - 21*a**4*b*x**3)/(8*a**2*b**6
 + 16*a*b**7*x**2 + 8*b**8*x**4) + x**7/(7*b**3)
```

3.182 $\int \frac{x^{10}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=98

$$-\frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{11/2}}+\frac{63a^2x}{8b^5}-\frac{21ax^3}{8b^4}-\frac{9x^7}{8b^2\left(a+bx^2\right)}-\frac{x^9}{4b\left(a+bx^2\right)^2}+\frac{63x^5}{40b^3}$$

```
[Out] 63/8*a^2*x/b^5-21/8*a*x^3/b^4+63/40*x^5/b^3-1/4*x^9/b/(b*x^2+a)^2-9/8*x^7/b
^2/(b*x^2+a)-63/8*a^(5/2)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{63a^2x}{8b^5}-\frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{11/2}}-\frac{9x^7}{8b^2\left(a+bx^2\right)}-\frac{21ax^3}{8b^4}-\frac{x^9}{4b\left(a+bx^2\right)^2}+\frac{63x^5}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^3,x]
```

```
[Out] (63*a^2*x)/(8*b^5) - (21*a*x^3)/(8*b^4) + (63*x^5)/(40*b^3) - x^9/(4*b*(a +
 b*x^2)^2) - (9*x^7)/(8*b^2*(a + b*x^2)) - (63*a^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^2\right)^3}\,dx = -\frac{x^9}{4b\left(a+bx^2\right)^2} + \frac{9\int \frac{x^8}{\left(a+bx^2\right)^2}\,dx}{4b}$$
$$= -\frac{x^9}{4b\left(a+bx^2\right)^2} - \frac{9x^7}{8b^2\left(a+bx^2\right)} + \frac{63\int \frac{x^6}{a+bx^2}\,dx}{8b^2}$$
$$= -\frac{x^9}{4b\left(a+bx^2\right)^2} - \frac{9x^7}{8b^2\left(a+bx^2\right)} + \frac{63\int \left(\frac{a^2}{b^3} - \frac{ax^2}{b^2} + \frac{x^4}{b} - \frac{a^3}{b^3\left(a+bx^2\right)}\right)dx}{8b^2}$$
$$= \frac{63a^2x}{8b^5} - \frac{21ax^3}{8b^4} + \frac{63x^5}{40b^3} - \frac{x^9}{4b\left(a+bx^2\right)^2} - \frac{9x^7}{8b^2\left(a+bx^2\right)} - \frac{\left(63a^3\right)\int \frac{1}{a+bx^2}\,dx}{8b^5}$$
$$= \frac{63a^2x}{8b^5} - \frac{21ax^3}{8b^4} + \frac{63x^5}{40b^3} - \frac{x^9}{4b\left(a+bx^2\right)^2} - \frac{9x^7}{8b^2\left(a+bx^2\right)} - \frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.90

$$\frac{315a^4x + 525a^3bx^3 + 168a^2b^2x^5 - 24ab^3x^7 + 8b^4x^9}{40b^5\left(a+bx^2\right)^2} - \frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^3,x]
```

```
[Out] (315*a^4*x + 525*a^3*b*x^3 + 168*a^2*b^2*x^5 - 24*a*b^3*x^7 + 8*b^4*x^9)/(4
0*b^5*(a + b*x^2)^2) - (63*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(11/2)
)
```

fricas [A] time = 0.93, size = 256, normalized size = 2.61

$$\left[\frac{16\,b^4x^9 - 48\,ab^3x^7 + 336\,a^2b^2x^5 + 1050\,a^3bx^3 + 630\,a^4x + 315\left(a^2b^2x^4 + 2\,a^3bx^2 + a^4\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}}\right.}{bx^2+a}\right.}{80\left(b^7x^4 + 2\,ab^6x^2 + a^2b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/80*(16*b^4*x^9 - 48*a*b^3*x^7 + 336*a^2*b^2*x^5 + 1050*a^3*b*x^3 + 630*a
^4*x + 315*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 - 2*b*x*
sqrt(-a/b) - a)/(b*x^2 + a)))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5), 1/40*(8*b^
4*x^9 - 24*a*b^3*x^7 + 168*a^2*b^2*x^5 + 525*a^3*b*x^3 + 315*a^4*x - 315*(a
^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^7*x^4
 + 2*a*b^6*x^2 + a^2*b^5)]
```

giac [A] time = 0.64, size = 84, normalized size = 0.86

$$-\frac{63\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^5} + \frac{17\,a^3bx^3 + 15\,a^4x}{8\left(bx^2+a\right)^2b^5} + \frac{b^{12}x^5 - 5\,ab^{11}x^3 + 30\,a^2b^{10}x}{5\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -63/8*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/8*(17*a^3*b*x^3 + 15*a^
4*x)/((b*x^2 + a)^2*b^5) + 1/5*(b^12*x^5 - 5*a*b^11*x^3 + 30*a^2*b^10*x)/b^
15
```

maple [A] time = 0.01, size = 88, normalized size = 0.90

$$\frac{17a^3x^3}{8\left(b\,x^2+a\right)^2 b^4}+\frac{x^5}{5b^3}+\frac{15a^4x}{8\left(b\,x^2+a\right)^2 b^5}-\frac{a\,x^3}{b^4}-\frac{63a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^5}+\frac{6a^2x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^3,x)
```

```
[Out] 1/5*x^5/b^3-a*x^3/b^4+6*a^2*x/b^5+17/8/b^4*a^3/(b*x^2+a)^2*x^3+15/8/b^5*a^4
/(b*x^2+a)^2*x-63/8/b^5*a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.86, size = 93, normalized size = 0.95

$$\frac{17\,a^3bx^3+15\,a^4x}{8\left(b^7x^4+2\,ab^6x^2+a^2b^5\right)}-\frac{63\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^5}+\frac{b^2x^5-5\,abx^3+30\,a^2x}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(17*a^3*b*x^3 + 15*a^4*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5) - 63/8*a^3*
arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/5*(b^2*x^5 - 5*a*b*x^3 + 30*a^2*x
)/b^5
```

mupad [B] time = 0.07, size = 87, normalized size = 0.89

$$\frac{\frac{15\,a^4\,x}{8}+\frac{17\,b\,a^3\,x^3}{8}}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}+\frac{x^5}{5\,b^3}-\frac{a\,x^3}{b^4}+\frac{6\,a^2\,x}{b^5}-\frac{63\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2)^3,x)
```

```
[Out] ((15*a^4*x)/8 + (17*a^3*b*x^3)/8)/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) + x^5/(
5*b^3) - (a*x^3)/b^4 + (6*a^2*x)/b^5 - (63*a^(5/2)*atan((b^(1/2)*x)/a^(1/2)
))/(8*b^(11/2))
```

sympy [A] time = 0.47, size = 144, normalized size = 1.47

$$\frac{6a^2x}{b^5}-\frac{ax^3}{b^4}+\frac{63\sqrt{-\frac{a^5}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^5}{b^{11}}}}{a^2}\right)}{16}-\frac{63\sqrt{-\frac{a^5}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^5}{b^{11}}}}{a^2}\right)}{16}+\frac{15a^4x+17a^3bx^3}{8a^2b^5+16ab^6x^2+8b^7x^4}+\frac{x^5}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**3,x)
```

```
[Out] 6*a**2*x/b**5 - a*x**3/b**4 + 63*sqrt(-a**5/b**11)*log(x - b**5*sqrt(-a**5/
b**11)/a**2)/16 - 63*sqrt(-a**5/b**11)*log(x + b**5*sqrt(-a**5/b**11)/a**2)
/16 + (15*a**4*x + 17*a**3*b*x**3)/(8*a**2*b**5 + 16*a*b**6*x**2 + 8*b**7*x
**4) + x**5/(5*b**3)
```

3.183 $\int \frac{x^8}{(a+bx^2)^3} dx$

Optimal. Leaf size=85

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}-\frac{35ax}{8b^4}-\frac{7x^5}{8b^2(a+bx^2)}-\frac{x^7}{4b(a+bx^2)^2}+\frac{35x^3}{24b^3}$$

```
[Out] -35/8*a*x/b^4+35/24*x^3/b^3-1/4*x^7/b/(b*x^2+a)^2-7/8*x^5/b^2/(b*x^2+a)+35/
8*a^(3/2)*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}-\frac{7x^5}{8b^2(a+bx^2)}-\frac{35ax}{8b^4}-\frac{x^7}{4b(a+bx^2)^2}+\frac{35x^3}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^3,x]
```

```
[Out] (-35*a*x)/(8*b^4) + (35*x^3)/(24*b^3) - x^7/(4*b*(a + b*x^2)^2) - (7*x^5)/(
8*b^2*(a + b*x^2)) + (35*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2\right)^3}\,dx = -\frac{x^7}{4b\left(a+bx^2\right)^2} + \frac{7\int \frac{x^6}{\left(a+bx^2\right)^2}\,dx}{4b}$$

$$= -\frac{x^7}{4b\left(a+bx^2\right)^2} - \frac{7x^5}{8b^2\left(a+bx^2\right)} + \frac{35\int \frac{x^4}{a+bx^2}\,dx}{8b^2}$$

$$= -\frac{x^7}{4b\left(a+bx^2\right)^2} - \frac{7x^5}{8b^2\left(a+bx^2\right)} + \frac{35\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx}{8b^2}$$

$$= -\frac{35ax}{8b^4} + \frac{35x^3}{24b^3} - \frac{x^7}{4b\left(a+bx^2\right)^2} - \frac{7x^5}{8b^2\left(a+bx^2\right)} + \frac{\left(35a^2\right)\int \frac{1}{a+bx^2}\,dx}{8b^4}$$

$$= -\frac{35ax}{8b^4} + \frac{35x^3}{24b^3} - \frac{x^7}{4b\left(a+bx^2\right)^2} - \frac{7x^5}{8b^2\left(a+bx^2\right)} + \frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.91

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}} - \frac{105a^3x+175a^2bx^3+56ab^2x^5-8b^3x^7}{24b^4\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^3,x]
```

```
[Out] -1/24*(105*a^3*x + 175*a^2*b*x^3 + 56*a*b^2*x^5 - 8*b^3*x^7)/(b^4*(a + b*x^
2)^2) + (35*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

fricas [A] time = 0.85, size = 230, normalized size = 2.71

$$\left[\frac{16\,b^3x^7-112\,ab^2x^5-350\,a^2bx^3-210\,a^3x+105\left(ab^2x^4+2\,a^2bx^2+a^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{48\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}, \frac{8\,b^3x^7-56\,al\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*b^3*x^7 - 112*a*b^2*x^5 - 350*a^2*b*x^3 - 210*a^3*x + 105*(a*b^2*
x^4 + 2*a^2*b*x^2 + a^3)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x
^2 + a)))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/24*(8*b^3*x^7 - 56*a*b^2*x^5
 - 175*a^2*b*x^3 - 105*a^3*x + 105*(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*sqrt(a/b
)*arctan(b*x*sqrt(a/b)/a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

giac [A] time = 0.63, size = 73, normalized size = 0.86

$$\frac{35\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^4} - \frac{13\,a^2bx^3+11\,a^3x}{8\left(bx^2+a\right)^2b^4} + \frac{b^6x^3-9\,ab^5x}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 35/8*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/8*(13*a^2*b*x^3 + 11*a^3
*x)/((b*x^2 + a)^2*b^4) + 1/3*(b^6*x^3 - 9*a*b^5*x)/b^9
```

maple [A] time = 0.01, size = 77, normalized size = 0.91

$$-\frac{13a^2x^3}{8\left(bx^2+a\right)^2 b^3}-\frac{11a^3x}{8\left(bx^2+a\right)^2 b^4}+\frac{x^3}{3b^3}+\frac{35a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^4}-\frac{3ax}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^3,x)
```

```
[Out] 1/3*x^3/b^3-3*a*x/b^4-13/8/b^3*a^2/(b*x^2+a)^2*x^3-11/8/b^4*a^3/(b*x^2+a)^2
*x+35/8/b^4*a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 82, normalized size = 0.96

$$-\frac{13\,a^2bx^3+11\,a^3x}{8\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}+\frac{35\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^4}+\frac{bx^3-9\,ax}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(13*a^2*b*x^3 + 11*a^3*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4) + 35/8*a^2
*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/3*(b*x^3 - 9*a*x)/b^4
```

mupad [B] time = 4.72, size = 77, normalized size = 0.91

$$\frac{x^3}{3\,b^3}-\frac{\frac{11\,a^3\,x}{8}+\frac{13\,b\,a^2\,x^3}{8}}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4}+\frac{35\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{9/2}}-\frac{3\,a\,x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2)^3,x)
```

```
[Out] x^3/(3*b^3) - ((11*a^3*x)/8 + (13*a^2*b*x^3)/8)/(a^2*b^4 + b^6*x^4 + 2*a*b^
5*x^2) + (35*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*b^(9/2)) - (3*a*x)/b^4
```

sympy [A] time = 0.44, size = 133, normalized size = 1.56

$$-\frac{3ax}{b^4}-\frac{35\sqrt{-\frac{a^3}{b^9}}\log\left(x-\frac{b^4\sqrt{-\frac{a^3}{b^9}}}{a}\right)}{16}+\frac{35\sqrt{-\frac{a^3}{b^9}}\log\left(x+\frac{b^4\sqrt{-\frac{a^3}{b^9}}}{a}\right)}{16}+\frac{-11a^3x-13a^2bx^3}{8a^2b^4+16ab^5x^2+8b^6x^4}+\frac{x^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**3,x)
```

```
[Out] -3*a*x/b**4 - 35*sqrt(-a**3/b**9)*log(x - b**4*sqrt(-a**3/b**9)/a)/16 + 35*
sqrt(-a**3/b**9)*log(x + b**4*sqrt(-a**3/b**9)/a)/16 + (-11*a**3*x - 13*a**
2*b*x**3)/(8*a**2*b**4 + 16*a*b**5*x**2 + 8*b**6*x**4) + x**3/(3*b**3)
```

3.184 $\int \frac{x^6}{\left(a+bx^2\right)^3} dx$

Optimal. Leaf size=74

$$-\frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}}-\frac{5x^3}{8b^2\left(a+bx^2\right)}-\frac{x^5}{4b\left(a+bx^2\right)^2}+\frac{15x}{8b^3}$$

```
[Out] 15/8*x/b^3-1/4*x^5/b/(b*x^2+a)^2-5/8*x^3/b^2/(b*x^2+a)-15/8*arctan(x*b^(1/2
)/a^(1/2))*a^(1/2)/b^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 321, 205}

$$-\frac{5x^3}{8b^2\left(a+bx^2\right)}-\frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}}-\frac{x^5}{4b\left(a+bx^2\right)^2}+\frac{15x}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^3,x]
```

```
[Out] (15*x)/(8*b^3) - x^5/(4*b*(a + b*x^2)^2) - (5*x^3)/(8*b^2*(a + b*x^2)) - (1
5*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^3}\,dx = -\frac{x^5}{4b\left(a+bx^2\right)^2}+\frac{5\int \frac{x^4}{\left(a+bx^2\right)^2}\,dx}{4b}$$
$$= -\frac{x^5}{4b\left(a+bx^2\right)^2}-\frac{5x^3}{8b^2\left(a+bx^2\right)}+\frac{15\int \frac{x^2}{a+bx^2}\,dx}{8b^2}$$
$$= \frac{15x}{8b^3}-\frac{x^5}{4b\left(a+bx^2\right)^2}-\frac{5x^3}{8b^2\left(a+bx^2\right)}-\frac{(15a)\int \frac{1}{a+bx^2}\,dx}{8b^3}$$
$$= \frac{15x}{8b^3}-\frac{x^5}{4b\left(a+bx^2\right)^2}-\frac{5x^3}{8b^2\left(a+bx^2\right)}-\frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.89

$$\frac{15a^2x+25abx^3+8b^2x^5}{8b^3\left(a+bx^2\right)^2}-\frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^3,x]
```

```
[Out] (15*a^2*x + 25*a*b*x^3 + 8*b^2*x^5)/(8*b^3*(a + b*x^2)^2) - (15*Sqrt[a]*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(7/2))
```

fricas [A] time = 0.80, size = 202, normalized size = 2.73

$$\left[\frac{16\,b^2x^5+50\,abx^3+30\,a^2x+15\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{16\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)},\frac{8\,b^2x^5+25\,abx^3+15\,a^2x-1}{8\left(b^5x^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*b^2*x^5 + 50*a*b*x^3 + 30*a^2*x + 15*(b^2*x^4 + 2*a*b*x^2 + a^2)*
sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^5*x^4 + 2*a*
b^4*x^2 + a^2*b^3), 1/8*(8*b^2*x^5 + 25*a*b*x^3 + 15*a^2*x - 15*(b^2*x^4 +
2*a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^4 + 2*a*b^4*x^2
+ a^2*b^3)]
```

giac [A] time = 0.64, size = 54, normalized size = 0.73

$$-\frac{15\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}+\frac{x}{b^3}+\frac{9\,abx^3+7\,a^2x}{8\left(bx^2+a\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -15/8*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + x/b^3 + 1/8*(9*a*b*x^3 + 7*
a^2*x)/((b*x^2 + a)^2*b^3)
```

maple [A] time = 0.01, size = 63, normalized size = 0.85

$$\frac{9a\,x^3}{8\left(b\,x^2+a\right)^2 b^2}+\frac{7a^2x}{8\left(b\,x^2+a\right)^2 b^3}-\frac{15a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^3}+\frac{x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^3,x)
```

```
[Out] x/b^3+9/8/b^2*a/(b*x^2+a)^2*x^3+7/8/b^3*a^2/(b*x^2+a)^2*x-15/8/b^3*a/(a*b)^
(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.90, size = 68, normalized size = 0.92

$$\frac{9\,abx^3+7\,a^2x}{8\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}-\frac{15\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}+\frac{x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(9*a*b*x^3 + 7*a^2*x)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) - 15/8*a*arctan
(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + x/b^3
```

mupad [B] time = 4.75, size = 64, normalized size = 0.86

$$\frac{\frac{7\,a^2\,x}{8}+\frac{9\,b\,a\,x^3}{8}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{x}{b^3}-\frac{15\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^3,x)
```

```
[Out] ((7*a^2*x)/8 + (9*a*b*x^3)/8)/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) + x/b^3 - (
15*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*b^(7/2))
```

sympy [A] time = 0.40, size = 107, normalized size = 1.45

$$\frac{15\sqrt{-\frac{a}{b^7}}\log\left(-b^3\sqrt{-\frac{a}{b^7}}+x\right)}{16}-\frac{15\sqrt{-\frac{a}{b^7}}\log\left(b^3\sqrt{-\frac{a}{b^7}}+x\right)}{16}+\frac{7a^2x+9abx^3}{8a^2b^3+16ab^4x^2+8b^5x^4}+\frac{x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**3,x)
```

```
[Out] 15*sqrt(-a/b**7)*log(-b**3*sqrt(-a/b**7) + x)/16 - 15*sqrt(-a/b**7)*log(b**
3*sqrt(-a/b**7) + x)/16 + (7*a**2*x + 9*a*b*x**3)/(8*a**2*b**3 + 16*a*b**4*
x**2 + 8*b**5*x**4) + x/b**3
```

3.185 $\int \frac{x^4}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=64

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}}-\frac{3x}{8b^2\left(a+bx^2\right)}-\frac{x^3}{4b\left(a+bx^2\right)^2}$$

```
[Out] -1/4*x^3/b/(b*x^2+a)^2-3/8*x/b^2/(b*x^2+a)+3/8*arctan(x*b^(1/2)/a^(1/2))/b^
(5/2)/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {288, 205}

$$-\frac{3x}{8b^2\left(a+bx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}}-\frac{x^3}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^3,x]
```

```
[Out] -x^3/(4*b*(a + b*x^2)^2) - (3*x)/(8*b^2*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*Sqrt[a]*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(a+bx^2\right)^3}\,dx &= -\frac{x^3}{4b\left(a+bx^2\right)^2}+\frac{3\int \frac{x^2}{\left(a+bx^2\right)^2}\,dx}{4b}\\
&= -\frac{x^3}{4b\left(a+bx^2\right)^2}-\frac{3x}{8b^2\left(a+bx^2\right)}+\frac{3\int \frac{1}{a+bx^2}\,dx}{8b^2}\\
&= -\frac{x^3}{4b\left(a+bx^2\right)^2}-\frac{3x}{8b^2\left(a+bx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.86

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}}-\frac{3ax+5bx^3}{8b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^3,x]
```

```
[Out] -1/8*(3*a*x + 5*b*x^3)/(b^2*(a + b*x^2)^2) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(8*Sqrt[a]*b^(5/2))
```

fricas [A] time = 0.71, size = 188, normalized size = 2.94

$$\left[-\frac{10\,ab^2x^3+6\,a^2bx+3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{16\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)},\,-\frac{5\,ab^2x^3+3\,a^2bx-3\left(b^2x^4+2\,abx^2+a\right.}{8\left(ab^5x^4+2\,a^2b^4x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(10*a*b^2*x^3 + 6*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)
*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^5*x^4 + 2*a^2*b^4*x^2
+ a^3*b^3), -1/8*(5*a*b^2*x^3 + 3*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*s
qrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)]
```

giac [A] time = 0.60, size = 45, normalized size = 0.70

$$\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^2}-\frac{5\,bx^3+3\,ax}{8\,\left(bx^2+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/8*(5*b*x^3 + 3*a*x)/((b*x^2 +
 a)^2*b^2)
```

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^2}+\frac{-\frac{5x^3}{8b}-\frac{3ax}{8b^2}}{\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^3,x)
```

```
[Out] (-5/8/b*x^3-3/8*a/b^2*x)/(b*x^2+a)^2+3/8/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/
2)*b*x)
```

maxima [A] time = 3.00, size = 59, normalized size = 0.92

$$-\frac{5\,bx^3+3\,ax}{8\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(5*b*x^3 + 3*a*x)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) + 3/8*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*b^2)
```

mupad [B] time = 4.77, size = 56, normalized size = 0.88

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,b^{5/2}}-\frac{\frac{5\,x^3}{8\,b}+\frac{3\,a\,x}{8\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^3,x)
```

```
[Out] (3*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*b^(5/2)) - ((5*x^3)/(8*b) + (3*a*x
)/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

sympy [A] time = 0.33, size = 110, normalized size = 1.72

$$-\frac{3\sqrt{-\frac{1}{ab^5}}\,\log\left(-ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{ab^5}}\,\log\left(ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{16}+\frac{-3ax-5bx^3}{8a^2b^2+16ab^3x^2+8b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a*b**5))*log(-a*b**2*sqrt(-1/(a*b**5)) + x)/16 + 3*sqrt(-1/(a*b
**5))*log(a*b**2*sqrt(-1/(a*b**5)) + x)/16 + (-3*a*x - 5*b*x**3)/(8*a**2*b*
*2 + 16*a*b**3*x**2 + 8*b**4*x**4)
```

3.186 $\int \frac{x^2}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}+\frac{x}{8ab\left(a+bx^2\right)}-\frac{x}{4b\left(a+bx^2\right)^2}$$

```
[Out] -1/4*x/b/(b*x^2+a)^2+1/8*x/a/b/(b*x^2+a)+1/8*arctan(x*b^(1/2)/a^(1/2))/a^(3
/2)/b^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}+\frac{x}{8ab\left(a+bx^2\right)}-\frac{x}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^3,x]
```

```
[Out] -x/(4*b*(a + b*x^2)^2) + x/(8*a*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]
]/(8*a^(3/2)*b^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^3} dx = -\frac{x}{4b\left(a+bx^2\right)^2} + \frac{\int \frac{1}{\left(a+bx^2\right)^2} dx}{4b}$$
$$= -\frac{x}{4b\left(a+bx^2\right)^2} + \frac{x}{8ab\left(a+bx^2\right)} + \frac{\int \frac{1}{a+bx^2} dx}{8ab}$$
$$= -\frac{x}{4b\left(a+bx^2\right)^2} + \frac{x}{8ab\left(a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.89

$$\frac{\frac{\sqrt{a}\sqrt{b}\,x\left(bx^2-a\right)}{\left(a+bx^2\right)^2} + \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-a + b*x^2))/(a + b*x^2)^2 + ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(8*a^(3/2)*b^(3/2))
```

fricas [A] time = 0.71, size = 190, normalized size = 2.92

$$\left[\frac{2\,ab^2x^3 - 2\,a^2bx - \left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2+a}\right)}{16\left(a^2b^4x^4 + 2\,a^3b^3x^2 + a^4b^2\right)}, \frac{ab^2x^3 - a^2bx + \left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{ab}\,\text{a}}{8\left(a^2b^4x^4 + 2\,a^3b^3x^2 + a^4b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*a*b^2*x^3 - 2*a^2*b*x - (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*log
((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 +
a^4*b^2), 1/8*(a*b^2*x^3 - a^2*b*x + (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)]
```

giac [A] time = 0.62, size = 50, normalized size = 0.77

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab} + \frac{bx^3 - ax}{8\left(bx^2+a\right)^2 ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/8*(b*x^3 - a*x)/((b*x^2 + a)^
2*a*b)
```

maple [A] time = 0.01, size = 49, normalized size = 0.75

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab} + \frac{\frac{x^3}{8a} - \frac{x}{8b}}{\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^3,x)
```

```
[Out] (1/8/a*x^3-1/8/b*x)/(b*x^2+a)^2+1/8/b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*
x)
```

maxima [A] time = 3.04, size = 62, normalized size = 0.95

$$\frac{bx^3 - ax}{8\left(ab^3x^4 + 2\,a^2b^2x^2 + a^3b\right)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 1/8*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a*b)
```

mupad [B] time = 4.74, size = 55, normalized size = 0.85

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}} - \frac{\frac{x}{8\,b} - \frac{x^3}{8\,a}}{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^3,x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(8*a^(3/2)*b^(3/2)) - (x/(8*b) - x^3/(8*a))/(a^2
+ b^2*x^4 + 2*a*b*x^2)
```

sympy [B] time = 0.31, size = 110, normalized size = 1.69

$$-\frac{\sqrt{-\frac{1}{a^3b^3}}\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{a^3b^3}}\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}} + x\right)}{16} + \frac{-ax + bx^3}{8a^3b + 16a^2b^2x^2 + 8ab^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**3))*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/16 + sqrt(-1/(a
**3*b**3))*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/16 + (-a*x + b*x**3)/(8*a**
3*b + 16*a**2*b**2*x**2 + 8*a*b**3*x**4)
```

3.187 $\int \frac{1}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=62

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{3x}{8a^2\left(a+bx^2\right)}+\frac{x}{4a\left(a+bx^2\right)^2}$$

```
[Out] 1/4*x/a/(b*x^2+a)^2+3/8*x/a^2/(b*x^2+a)+3/8*arctan(x*b^(1/2)/a^(1/2))/a^(5/
2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 205}

$$\frac{3x}{8a^2\left(a+bx^2\right)}+\frac{3 \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{x}{4a\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-3),x]
```

```
[Out] x/(4*a*(a + b*x^2)^2) + (3*x)/(8*a^2*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*a^(5/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^3} \, dx &= \frac{x}{4a\left(a+bx^2\right)^2}+\frac{3\int \frac{1}{\left(a+bx^2\right)^2} \, dx}{4a}\\
&= \frac{x}{4a\left(a+bx^2\right)^2}+\frac{3x}{8a^2\left(a+bx^2\right)}+\frac{3\int \frac{1}{a+bx^2} \, dx}{8a^2}\\
&= \frac{x}{4a\left(a+bx^2\right)^2}+\frac{3x}{8a^2\left(a+bx^2\right)}+\frac{3 \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.89

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{5ax+3bx^3}{8a^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-3),x]
```

```
[Out] (5*a*x + 3*b*x^3)/(8*a^2*(a + b*x^2)^2) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*a^(5/2)*Sqrt[b])
```

fricas [A] time = 0.93, size = 188, normalized size = 3.03

$$\left[\frac{6\,ab^2x^3+10\,a^2bx-3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{16\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)},\ \frac{3\,ab^2x^3+5\,a^2bx+3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{}}{8\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*x^3 + 10*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2
 + a^5*b), 1/8*(3*a*b^2*x^3 + 5*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqr
t(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)]
```

giac [A] time = 0.63, size = 45, normalized size = 0.73

$$\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2}+\frac{3\,bx^3+5\,ax}{8\left(bx^2+a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/8*(3*b*x^3 + 5*a*x)/((b*x^2 +
 a)^2*a^2)
```

maple [A] time = 0.00, size = 51, normalized size = 0.82

$$\frac{x}{4\left(b\,x^2+a\right)^2a}+\frac{3x}{8\left(b\,x^2+a\right)a^2}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3,x)
```

```
[Out] 1/4*x/a/(b*x^2+a)^2+3/8*x/a^2/(b*x^2+a)+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^
(1/2)*b*x)
```

maxima [A] time = 3.03, size = 58, normalized size = 0.94

$$\frac{3\,bx^3+5\,ax}{8\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3/8*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^2)
```

mupad [B] time = 4.66, size = 55, normalized size = 0.89

$$\frac{\frac{5\,x}{8\,a}+\frac{3\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^3,x)
```

```
[Out] ((5*x)/(8*a) + (3*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (3*atan((b^
(1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2))
```

sympy [A] time = 0.32, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{a^5b}}\,\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5b}}\,\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{16}+\frac{5ax+3bx^3}{8a^4+16a^3bx^2+8a^2b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*b))*log(-a**3*sqrt(-1/(a**5*b)) + x)/16 + 3*sqrt(-1/(a**5*
b))*log(a**3*sqrt(-1/(a**5*b)) + x)/16 + (5*a*x + 3*b*x**3)/(8*a**4 + 16*a*
*3*b*x**2 + 8*a**2*b**2*x**4)
```

3.188 $\int \frac{1}{x^2(a+bx^2)^3}\,dx$

Optimal. Leaf size=76

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15}{8a^3x}+\frac{5}{8a^2x\left(a+bx^2\right)}+\frac{1}{4ax\left(a+bx^2\right)^2}$$

```
[Out] -15/8/a^3/x+1/4/a/x/(b*x^2+a)^2+5/8/a^2/x/(b*x^2+a)-15/8*arctan(x*b^(1/2)/a
^(1/2))*b^(1/2)/a^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$\frac{5}{8a^2x\left(a+bx^2\right)}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15}{8a^3x}+\frac{1}{4ax\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -15/(8*a^3*x) + 1/(4*a*x*(a + b*x^2)^2) + 5/(8*a^2*x*(a + b*x^2)) - (15*Sqr
t[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax\left(a+bx^2\right)^2} + \frac{5\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{1}{4ax\left(a+bx^2\right)^2} + \frac{5}{8a^2x\left(a+bx^2\right)} + \frac{15\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{8a^2}$$

$$= -\frac{15}{8a^3x} + \frac{1}{4ax\left(a+bx^2\right)^2} + \frac{5}{8a^2x\left(a+bx^2\right)} - \frac{(15b)\int \frac{1}{a+bx^2}\,dx}{8a^3}$$

$$= -\frac{15}{8a^3x} + \frac{1}{4ax\left(a+bx^2\right)^2} + \frac{5}{8a^2x\left(a+bx^2\right)} - \frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.89

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}} - \frac{8a^2+25abx^2+15b^2x^4}{8a^3x\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -1/8*(8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)/(a^3*x*(a + b*x^2)^2) - (15*Sqrt[b]*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

fricas [A] time = 0.94, size = 202, normalized size = 2.66

$$\left[-\frac{30\,b^2x^4+50\,abx^2-15\left(b^2x^5+2\,abx^3+a^2x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+16\,a^2}{16\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)}, -\frac{15\,b^2x^4+25\,abx^2+15\left(b^2\right.}{8\left(a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(30*b^2*x^4 + 50*a*b*x^2 - 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(-b/
a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 16*a^2)/(a^3*b^2*x^5 +
 2*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 15*(b^2*x^5 + 2*a*b*
x^3 + a^2*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*
x^3 + a^5*x)]
```

giac [A] time = 0.64, size = 57, normalized size = 0.75

$$-\frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3} - \frac{7\,b^2x^3+9\,abx}{8\left(bx^2+a\right)^2a^3} - \frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/8*(7*b^2*x^3 + 9*a*b*x)/(
(b*x^2 + a)^2*a^3) - 1/(a^3*x)
```

maple [A] time = 0.01, size = 66, normalized size = 0.87

$$-\frac{7b^{2}x^{3}}{8\left(b\,x^{2}+a\right)^{2}a^{3}}-\frac{9bx}{8\left(b\,x^{2}+a\right)^{2}a^{2}}-\frac{15b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^{3}}-\frac{1}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^3,x)
```

```
[Out] -1/a^3/x-7/8/a^3*b^2/(b*x^2+a)^2*x^3-9/8/a^2*b/(b*x^2+a)^2*x-15/8/a^3*b/(a*
b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.98, size = 71, normalized size = 0.93

$$-\frac{15\,b^{2}x^{4}+25\,abx^{2}+8\,a^{2}}{8\left(a^{3}b^{2}x^{5}+2\,a^{4}bx^{3}+a^{5}x\right)}-\frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
- 15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 4.67, size = 66, normalized size = 0.87

$$-\frac{\frac{1}{a}+\frac{25\,b\,x^{2}}{8\,a^{2}}+\frac{15\,b^{2}\,x^{4}}{8\,a^{3}}}{a^{2}\,x+2\,a\,b\,x^{3}+b^{2}\,x^{5}}-\frac{15\,\sqrt{b}\;\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^3),x)
```

```
[Out] - (1/a + (25*b*x^2)/(8*a^2) + (15*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 + 2*a*
b*x^3) - (15*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2))
```

sympy [A] time = 0.42, size = 116, normalized size = 1.53

$$\frac{15\sqrt{-\frac{b}{a^{7}}}\,\log\left(-\frac{a^{4}\sqrt{-\frac{b}{a^{7}}}}{b}+x\right)}{16}-\frac{15\sqrt{-\frac{b}{a^{7}}}\,\log\left(\frac{a^{4}\sqrt{-\frac{b}{a^{7}}}}{b}+x\right)}{16}+\frac{-8a^{2}-25abx^{2}-15b^{2}x^{4}}{8a^{5}x+16a^{4}bx^{3}+8a^{3}b^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**3,x)
```

```
[Out] 15*sqrt(-b/a**7)*log(-a**4*sqrt(-b/a**7)/b + x)/16 - 15*sqrt(-b/a**7)*log(a
**4*sqrt(-b/a**7)/b + x)/16 + (-8*a**2 - 25*a*b*x**2 - 15*b**2*x**4)/(8*a**
5*x + 16*a**4*b*x**3 + 8*a**3*b**2*x**5)
```

3.189 $\int \frac{1}{x^4(a+bx^2)^3}\,dx$

Optimal. Leaf size=87

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{35b}{8a^4x}-\frac{35}{24a^3x^3}+\frac{7}{8a^2x^3\left(a+bx^2\right)}+\frac{1}{4ax^3\left(a+bx^2\right)^2}$$

```
[Out] -35/24/a^3/x^3+35/8*b/a^4/x+1/4/a/x^3/(b*x^2+a)^2+7/8/a^2/x^3/(b*x^2+a)+35/
8*b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)
```

Rubi [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{7}{8a^2x^3\left(a+bx^2\right)}+\frac{35b}{8a^4x}-\frac{35}{24a^3x^3}+\frac{1}{4ax^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^3),x]
```

```
[Out] -35/(24*a^3*x^3) + (35*b)/(8*a^4*x) + 1/(4*a*x^3*(a + b*x^2)^2) + 7/(8*a^2*
x^3*(a + b*x^2)) + (35*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4(a+bx^2)^3}\,dx = \frac{1}{4ax^3(a+bx^2)^2} + \frac{7\int \frac{1}{x^4(a+bx^2)^2}\,dx}{4a}$$
$$= \frac{1}{4ax^3(a+bx^2)^2} + \frac{7}{8a^2x^3(a+bx^2)} + \frac{35\int \frac{1}{x^4(a+bx^2)}\,dx}{8a^2}$$
$$= -\frac{35}{24a^3x^3} + \frac{1}{4ax^3(a+bx^2)^2} + \frac{7}{8a^2x^3(a+bx^2)} - \frac{(35b)\int \frac{1}{x^2(a+bx^2)}\,dx}{8a^3}$$
$$= -\frac{35}{24a^3x^3} + \frac{35b}{8a^4x} + \frac{1}{4ax^3(a+bx^2)^2} + \frac{7}{8a^2x^3(a+bx^2)} + \frac{(35b^2)\int \frac{1}{a+bx^2}\,dx}{8a^4}$$
$$= -\frac{35}{24a^3x^3} + \frac{35b}{8a^4x} + \frac{1}{4ax^3(a+bx^2)^2} + \frac{7}{8a^2x^3(a+bx^2)} + \frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}$$

Mathematica [A] time = 0.05, size = 79, normalized size = 0.91

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}} + \frac{-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6}{24a^4x^3(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^3),x]
```

```
[Out] (-8*a^3 + 56*a^2*b*x^2 + 175*a*b^2*x^4 + 105*b^3*x^6)/(24*a^4*x^3*(a + b*x^
2)^2) + (35*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

fricas [A] time = 1.01, size = 238, normalized size = 2.74

$$\left[\frac{210\,b^3x^6+350\,ab^2x^4+112\,a^2bx^2-16\,a^3+105\left(b^3x^7+2\,ab^2x^5+a^2bx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{48\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)}, \frac{105\,b^3x^6+1}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(210*b^3*x^6 + 350*a*b^2*x^4 + 112*a^2*b*x^2 - 16*a^3 + 105*(b^3*x^7
+ 2*a*b^2*x^5 + a^2*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b
*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3), 1/24*(105*b^3*x^6 + 175*
a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3 + 105*(b^3*x^7 + 2*a*b^2*x^5 + a^2*b*x^3)*
sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

giac [A] time = 0.64, size = 71, normalized size = 0.82

$$\frac{35\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^4} + \frac{11\,b^3x^3+13\,ab^2x}{8\left(bx^2+a\right)^2a^4} + \frac{9\,bx^2-a}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/8*(11*b^3*x^3 + 13*a*b^2
*x)/((b*x^2 + a)^2*a^4) + 1/3*(9*b*x^2 - a)/(a^4*x^3)
```

maple [A] time = 0.02, size = 79, normalized size = 0.91

$$\frac{11b^3x^3}{8\left(b\,x^2+a\right)^2a^4}+\frac{13b^2x}{8\left(b\,x^2+a\right)^2a^3}+\frac{35b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^4}+\frac{3b}{a^4x}-\frac{1}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^3,x)
```

```
[Out] -1/3/a^3/x^3+3*b/a^4/x+11/8/a^4*b^3/(b*x^2+a)^2*x^3+13/8/a^3*b^2/(b*x^2+a)^
2*x+35/8/a^4*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.96, size = 86, normalized size = 0.99

$$\frac{105\,b^3x^6+175\,ab^2x^4+56\,a^2bx^2-8\,a^3}{24\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)}+\frac{35\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(105*b^3*x^6 + 175*a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3)/(a^4*b^2*x^7 + 2*
a^5*b*x^5 + a^6*x^3) + 35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

mupad [B] time = 4.67, size = 80, normalized size = 0.92

$$\frac{\frac{7\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{175\,b^2\,x^4}{24\,a^3}+\frac{35\,b^3\,x^6}{8\,a^4}}{a^2\,x^3+2\,a\,b\,x^5+b^2\,x^7}+\frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^3),x)
```

```
[Out] ((7*b*x^2)/(3*a^2) - 1/(3*a) + (175*b^2*x^4)/(24*a^3) + (35*b^3*x^6)/(8*a^4
))/(a^2*x^3 + b^2*x^7 + 2*a*b*x^5) + (35*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))
/(8*a^(9/2))
```

sympy [A] time = 0.48, size = 138, normalized size = 1.59

$$-\frac{35\sqrt{-\frac{b^3}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^3}{a^9}}}{b^2}+x\right)}{16}+\frac{35\sqrt{-\frac{b^3}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^3}{a^9}}}{b^2}+x\right)}{16}+\frac{-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6}{24a^6x^3+48a^5bx^5+24a^4b^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**3,x)
```

```
[Out] -35*sqrt(-b**3/a**9)*log(-a**5*sqrt(-b**3/a**9)/b**2 + x)/16 + 35*sqrt(-b**
3/a**9)*log(a**5*sqrt(-b**3/a**9)/b**2 + x)/16 + (-8*a**3 + 56*a**2*b*x**2
+ 175*a*b**2*x**4 + 105*b**3*x**6)/(24*a**6*x**3 + 48*a**5*b*x**5 + 24*a**4
*b**2*x**7)
```

3.190 $\int \frac{1}{x^6\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}-\frac{63b^2}{8a^5x}+\frac{21b}{8a^4x^3}-\frac{63}{40a^3x^5}+\frac{9}{8a^2x^5\left(a+bx^2\right)}+\frac{1}{4ax^5\left(a+bx^2\right)^2}$$

```
[Out] -63/40/a^3/x^5+21/8*b/a^4/x^3-63/8*b^2/a^5/x+1/4/a/x^5/(b*x^2+a)^2+9/8/a^2/
x^5/(b*x^2+a)-63/8*b^(5/2)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)
```

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{63b^2}{8a^5x}-\frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}+\frac{21b}{8a^4x^3}+\frac{9}{8a^2x^5\left(a+bx^2\right)}-\frac{63}{40a^3x^5}+\frac{1}{4ax^5\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -63/(40*a^3*x^5) + (21*b)/(8*a^4*x^3) - (63*b^2)/(8*a^5*x) + 1/(4*a*x^5*(a
+ b*x^2)^2) + 9/(8*a^2*x^5*(a + b*x^2)) - (63*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(8*a^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6 (a+bx^2)^3} dx = \frac{1}{4ax^5 (a+bx^2)^2} + \frac{9 \int \frac{1}{x^6 (a+bx^2)^2} dx}{4a}$$

$$= \frac{1}{4ax^5 (a+bx^2)^2} + \frac{9}{8a^2x^5 (a+bx^2)} + \frac{63 \int \frac{1}{x^6 (a+bx^2)} dx}{8a^2}$$

$$= -\frac{63}{40a^3x^5} + \frac{1}{4ax^5 (a+bx^2)^2} + \frac{9}{8a^2x^5 (a+bx^2)} - \frac{(63b) \int \frac{1}{x^4 (a+bx^2)} dx}{8a^3}$$

$$= -\frac{63}{40a^3x^5} + \frac{21b}{8a^4x^3} + \frac{1}{4ax^5 (a+bx^2)^2} + \frac{9}{8a^2x^5 (a+bx^2)} + \frac{(63b^2) \int \frac{1}{x^2 (a+bx^2)} dx}{8a^4}$$

$$= -\frac{63}{40a^3x^5} + \frac{21b}{8a^4x^3} - \frac{63b^2}{8a^5x} + \frac{1}{4ax^5 (a+bx^2)^2} + \frac{9}{8a^2x^5 (a+bx^2)} - \frac{(63b^3) \int \frac{1}{a+bx^2} dx}{8a^5}$$

$$= -\frac{63}{40a^3x^5} + \frac{21b}{8a^4x^3} - \frac{63b^2}{8a^5x} + \frac{1}{4ax^5 (a+bx^2)^2} + \frac{9}{8a^2x^5 (a+bx^2)} - \frac{63b^{5/2} \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}$$

Mathematica [**A**] time = 0.05, size = 90, normalized size = 0.90

$$-\frac{63b^{5/2} \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}} - \frac{8a^4 - 24a^3bx^2 + 168a^2b^2x^4 + 525ab^3x^6 + 315b^4x^8}{40a^5x^5 (a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -1/40*(8*a^4 - 24*a^3*b*x^2 + 168*a^2*b^2*x^4 + 525*a*b^3*x^6 + 315*b^4*x^8
)/(a^5*x^5*(a + b*x^2)^2) - (63*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(
11/2))
```

fricas [**A**] time = 0.89, size = 264, normalized size = 2.64

$$\left[-\frac{630\,b^4x^8 + 1050\,ab^3x^6 + 336\,a^2b^2x^4 - 48\,a^3bx^2 + 16\,a^4 - 315\left(b^4x^9 + 2\,ab^3x^7 + a^2b^2x^5\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 - 2\,ax\sqrt{-}}{bx^2 + a}\right.}{80\left(a^5b^2x^9 + 2\,a^6bx^7 + a^7x^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/80*(630*b^4*x^8 + 1050*a*b^3*x^6 + 336*a^2*b^2*x^4 - 48*a^3*b*x^2 + 16*
a^4 - 315*(b^4*x^9 + 2*a*b^3*x^7 + a^2*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a
*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5), -1/
40*(315*b^4*x^8 + 525*a*b^3*x^6 + 168*a^2*b^2*x^4 - 24*a^3*b*x^2 + 8*a^4 +
315*(b^4*x^9 + 2*a*b^3*x^7 + a^2*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a
^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5)]
```

giac [**A**] time = 0.64, size = 80, normalized size = 0.80

$$-\frac{63\,b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^5} - \frac{15\,b^4x^3 + 17\,ab^3x}{8\left(bx^2 + a\right)^2 a^5} - \frac{30\,b^2x^4 - 5\,abx^2 + a^2}{5\,a^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -63/8*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) - 1/8*(15*b^4*x^3 + 17*a*b^
3*x)/((b*x^2 + a)^2*a^5) - 1/5*(30*b^2*x^4 - 5*a*b*x^2 + a^2)/(a^5*x^5)
```

maple [A] time = 0.01, size = 89, normalized size = 0.89

$$-\frac{15b^4x^3}{8\left(bx^2+a\right)^2a^5}-\frac{17b^3x}{8\left(bx^2+a\right)^2a^4}-\frac{63b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^5}-\frac{6b^2}{a^5x}+\frac{b}{a^4x^3}-\frac{1}{5a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^3,x)
```

```
[Out] -1/5/a^3/x^5-6*b^2/a^5/x+b/a^4/x^3-15/8/a^5*b^4/(b*x^2+a)^2*x^3-17/8/a^4*b^
3/(b*x^2+a)^2*x-63/8/a^5*b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.91, size = 97, normalized size = 0.97

$$-\frac{315\,b^4x^8+525\,ab^3x^6+168\,a^2b^2x^4-24\,a^3bx^2+8\,a^4}{40\left(a^5b^2x^9+2\,a^6bx^7+a^7x^5\right)}-\frac{63\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(315*b^4*x^8 + 525*a*b^3*x^6 + 168*a^2*b^2*x^4 - 24*a^3*b*x^2 + 8*a^4
)/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5) - 63/8*b^3*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*a^5)
```

mupad [B] time = 5.02, size = 92, normalized size = 0.92

$$-\frac{\frac{1}{5\,a}-\frac{3\,b\,x^2}{5\,a^2}+\frac{21\,b^2\,x^4}{5\,a^3}+\frac{105\,b^3\,x^6}{8\,a^4}+\frac{63\,b^4\,x^8}{8\,a^5}}{a^2\,x^5+2\,a\,b\,x^7+b^2\,x^9}-\frac{63\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^3),x)
```

```
[Out] - (1/(5*a) - (3*b*x^2)/(5*a^2) + (21*b^2*x^4)/(5*a^3) + (105*b^3*x^6)/(8*a^
4) + (63*b^4*x^8)/(8*a^5))/(a^2*x^5 + b^2*x^9 + 2*a*b*x^7) - (63*b^(5/2)*at
an((b^(1/2)*x)/a^(1/2)))/(8*a^(11/2))
```

sympy [A] time = 0.53, size = 150, normalized size = 1.50

$$\frac{63\sqrt{-\frac{b^5}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^5}{a^{11}}}}{b^3}+x\right)}{16}-\frac{63\sqrt{-\frac{b^5}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^5}{a^{11}}}}{b^3}+x\right)}{16}+\frac{-8a^4+24a^3bx^2-168a^2b^2x^4-525ab^3x^6-315b^4x}{40a^7x^5+80a^6bx^7+40a^5b^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**3,x)
```

```
[Out] 63*sqrt(-b**5/a**11)*log(-a**6*sqrt(-b**5/a**11)/b**3 + x)/16 - 63*sqrt(-b*
*5/a**11)*log(a**6*sqrt(-b**5/a**11)/b**3 + x)/16 + (-8*a**4 + 24*a**3*b*x*
*2 - 168*a**2*b**2*x**4 - 525*a*b**3*x**6 - 315*b**4*x**8)/(40*a**7*x**5 +
80*a**6*b*x**7 + 40*a**5*b**2*x**9)
```

3.191 $\int \frac{1}{x^8\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=113

$$\frac{99b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{13/2}}+\frac{99b^3}{8a^6x}-\frac{33b^2}{8a^5x^3}+\frac{99b}{40a^4x^5}-\frac{99}{56a^3x^7}+\frac{11}{8a^2x^7\left(a+bx^2\right)}+\frac{1}{4ax^7\left(a+bx^2\right)^2}$$

```
[Out] -99/56/a^3/x^7+99/40*b/a^4/x^5-33/8*b^2/a^5/x^3+99/8*b^3/a^6/x+1/4/a/x^7/(b
*x^2+a)^2+11/8/a^2/x^7/(b*x^2+a)+99/8*b^(7/2)*arctan(x*b^(1/2)/a^(1/2))/a^(
13/2)
```

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{33b^2}{8a^5x^3}+\frac{99b^3}{8a^6x}+\frac{99b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{13/2}}+\frac{99b}{40a^4x^5}+\frac{11}{8a^2x^7\left(a+bx^2\right)}-\frac{99}{56a^3x^7}+\frac{1}{4ax^7\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -99/(56*a^3*x^7) + (99*b)/(40*a^4*x^5) - (33*b^2)/(8*a^5*x^3) + (99*b^3)/(8
*a^6*x) + 1/(4*a*x^7*(a + b*x^2)^2) + 11/(8*a^2*x^7*(a + b*x^2)) + (99*b^(7
/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(13/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^8\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax^7\left(a+bx^2\right)^2} + \frac{11\int \frac{1}{x^8\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{1}{4ax^7\left(a+bx^2\right)^2} + \frac{11}{8a^2x^7\left(a+bx^2\right)} + \frac{99\int \frac{1}{x^8\left(a+bx^2\right)}\,dx}{8a^2}$$

$$= -\frac{99}{56a^3x^7} + \frac{1}{4ax^7\left(a+bx^2\right)^2} + \frac{11}{8a^2x^7\left(a+bx^2\right)} - \frac{(99b)\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{8a^3}$$

$$= -\frac{99}{56a^3x^7} + \frac{99b}{40a^4x^5} + \frac{1}{4ax^7\left(a+bx^2\right)^2} + \frac{11}{8a^2x^7\left(a+bx^2\right)} + \frac{\left(99b^2\right)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{8a^4}$$

$$= -\frac{99}{56a^3x^7} + \frac{99b}{40a^4x^5} - \frac{33b^2}{8a^5x^3} + \frac{1}{4ax^7\left(a+bx^2\right)^2} + \frac{11}{8a^2x^7\left(a+bx^2\right)} - \frac{\left(99b^3\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{8a^5}$$

$$= -\frac{99}{56a^3x^7} + \frac{99b}{40a^4x^5} - \frac{33b^2}{8a^5x^3} + \frac{99b^3}{8a^6x} + \frac{1}{4ax^7\left(a+bx^2\right)^2} + \frac{11}{8a^2x^7\left(a+bx^2\right)} + \frac{\left(99b^4\right)\int \frac{1}{a+bx^2}}{8a^6}$$

$$= -\frac{99}{56a^3x^7} + \frac{99b}{40a^4x^5} - \frac{33b^2}{8a^5x^3} + \frac{99b^3}{8a^6x} + \frac{1}{4ax^7\left(a+bx^2\right)^2} + \frac{11}{8a^2x^7\left(a+bx^2\right)} + \frac{99b^{7/2}\tan^{-1}\left(\right.}{8a^{13/2}}$$

Mathematica [A] time = 0.06, size = 101, normalized size = 0.89

$$\frac{99b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{13/2}} + \frac{-40a^5+88a^4bx^2-264a^3b^2x^4+1848a^2b^3x^6+5775ab^4x^8+3465b^5x^{10}}{280a^6x^7\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^2)^3),x]
```

```
[Out] (-40*a^5 + 88*a^4*b*x^2 - 264*a^3*b^2*x^4 + 1848*a^2*b^3*x^6 + 5775*a*b^4*x
^8 + 3465*b^5*x^10)/(280*a^6*x^7*(a + b*x^2)^2) + (99*b^(7/2)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(8*a^(13/2))
```

fricas [A] time = 0.95, size = 286, normalized size = 2.53

$$\left[\frac{6930\,b^5x^{10}+11550\,ab^4x^8+3696\,a^2b^3x^6-528\,a^3b^2x^4+176\,a^4bx^2-80\,a^5+3465\left(b^5x^{11}+2\,ab^4x^9+a^2b^3x^7\right)\sqrt{-}}{560\left(a^6b^2x^{11}+2\,a^7bx^9+a^8x^7\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/560*(6930*b^5*x^10 + 11550*a*b^4*x^8 + 3696*a^2*b^3*x^6 - 528*a^3*b^2*x^
4 + 176*a^4*b*x^2 - 80*a^5 + 3465*(b^5*x^11 + 2*a*b^4*x^9 + a^2*b^3*x^7)*sq
rt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^6*b^2*x^11 + 2
*a^7*b*x^9 + a^8*x^7), 1/280*(3465*b^5*x^10 + 5775*a*b^4*x^8 + 1848*a^2*b^3
*x^6 - 264*a^3*b^2*x^4 + 88*a^4*b*x^2 - 40*a^5 + 3465*(b^5*x^11 + 2*a*b^4*x
```

```
^9 + a^2*b^3*x^7)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^2*x^11 + 2*a^7*b*x^
9 + a^8*x^7)]
```

giac [A] time = 0.61, size = 93, normalized size = 0.82

$$\frac{99\,b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^6}+\frac{19\,b^5x^3+21\,ab^4x}{8\,\left(bx^2+a\right)^2a^6}+\frac{350\,b^3x^6-70\,ab^2x^4+21\,a^2bx^2-5\,a^3}{35\,a^6x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 99/8*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) + 1/8*(19*b^5*x^3 + 21*a*b^4
*x)/((b*x^2 + a)^2*a^6) + 1/35*(350*b^3*x^6 - 70*a*b^2*x^4 + 21*a^2*b*x^2 -
 5*a^3)/(a^6*x^7)
```

maple [A] time = 0.01, size = 101, normalized size = 0.89

$$\frac{19b^5x^3}{8\left(b\,x^2+a\right)^2a^6}+\frac{21b^4x}{8\left(b\,x^2+a\right)^2a^5}+\frac{99b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^6}+\frac{10b^3}{a^6x}-\frac{2b^2}{a^5x^3}+\frac{3b}{5a^4x^5}-\frac{1}{7a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^2+a)^3,x)
```

```
[Out] -1/7/a^3/x^7+10*b^3/a^6/x-2*b^2/a^5/x^3+3/5*b/a^4/x^5+19/8/a^6*b^5/(b*x^2+a
)^2*x^3+21/8/a^5*b^4/(b*x^2+a)^2*x+99/8/a^6*b^4/(a*b)^(1/2)*arctan(1/(a*b)^
(1/2)*b*x)
```

maxima [A] time = 3.02, size = 108, normalized size = 0.96

$$\frac{3465\,b^5x^{10}+5775\,ab^4x^8+1848\,a^2b^3x^6-264\,a^3b^2x^4+88\,a^4bx^2-40\,a^5}{280\,\left(a^6b^2x^{11}+2\,a^7bx^9+a^8x^7\right)}+\frac{99\,b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/280*(3465*b^5*x^10 + 5775*a*b^4*x^8 + 1848*a^2*b^3*x^6 - 264*a^3*b^2*x^4
+ 88*a^4*b*x^2 - 40*a^5)/(a^6*b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7) + 99/8*b^4*
arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

mupad [B] time = 4.98, size = 102, normalized size = 0.90

$$\frac{\frac{11\,b\,x^2}{35\,a^2}-\frac{1}{7\,a}-\frac{33\,b^2\,x^4}{35\,a^3}+\frac{33\,b^3\,x^6}{5\,a^4}+\frac{165\,b^4\,x^8}{8\,a^5}+\frac{99\,b^5\,x^{10}}{8\,a^6}}{a^2\,x^7+2\,a\,b\,x^9+b^2\,x^{11}}+\frac{99\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^2)^3),x)
```

```
[Out] ((11*b*x^2)/(35*a^2) - 1/(7*a) - (33*b^2*x^4)/(35*a^3) + (33*b^3*x^6)/(5*a^
4) + (165*b^4*x^8)/(8*a^5) + (99*b^5*x^10)/(8*a^6))/(a^2*x^7 + b^2*x^11 + 2
*a*b*x^9) + (99*b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(13/2))
```

sympy [A] time = 0.58, size = 162, normalized size = 1.43

$$-\frac{99\sqrt{-\frac{b^7}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b^7}{a^{13}}}}{b^4}+x\right)}{16}+\frac{99\sqrt{-\frac{b^7}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b^7}{a^{13}}}}{b^4}+x\right)}{16}+\frac{-40a^5+88a^4bx^2-264a^3b^2x^4+1848a^2b^3x^6+}{280a^8x^7+560a^7bx^9+280a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**2+a)**3,x)
```

```
[Out] -99*sqrt(-b**7/a**13)*log(-a**7*sqrt(-b**7/a**13)/b**4 + x)/16 + 99*sqrt(-b
**7/a**13)*log(a**7*sqrt(-b**7/a**13)/b**4 + x)/16 + (-40*a**5 + 88*a**4*b*
x**2 - 264*a**3*b**2*x**4 + 1848*a**2*b**3*x**6 + 5775*a*b**4*x**8 + 3465*b
**5*x**10)/(280*a**8*x**7 + 560*a**7*b*x**9 + 280*a**6*b**2*x**11)
```

3.192 $\int \frac{x^{25}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=216

$$-\frac{a^{12}}{18b^{13}\left(a+bx^2\right)^9}+\frac{3a^{11}}{4b^{13}\left(a+bx^2\right)^8}-\frac{33a^{10}}{7b^{13}\left(a+bx^2\right)^7}+\frac{55a^9}{3b^{13}\left(a+bx^2\right)^6}-\frac{99a^8}{2b^{13}\left(a+bx^2\right)^5}+\frac{99a^7}{b^{13}\left(a+bx^2\right)^4}-\frac{1}{b^{13}(}$$

```
[Out] 55/2*a^2*x^2/b^12-5/2*a*x^4/b^11+1/6*x^6/b^10-1/18*a^12/b^13/(b*x^2+a)^9+3/
4*a^11/b^13/(b*x^2+a)^8-33/7*a^10/b^13/(b*x^2+a)^7+55/3*a^9/b^13/(b*x^2+a)^
6-99/2*a^8/b^13/(b*x^2+a)^5+99*a^7/b^13/(b*x^2+a)^4-154*a^6/b^13/(b*x^2+a)^
3+198*a^5/b^13/(b*x^2+a)^2-495/2*a^4/b^13/(b*x^2+a)-110*a^3*ln(b*x^2+a)/b^1
3
```

Rubi [A] time = 0.26, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^{12}}{18b^{13}\left(a+bx^2\right)^9}+\frac{3a^{11}}{4b^{13}\left(a+bx^2\right)^8}-\frac{33a^{10}}{7b^{13}\left(a+bx^2\right)^7}+\frac{55a^9}{3b^{13}\left(a+bx^2\right)^6}-\frac{99a^8}{2b^{13}\left(a+bx^2\right)^5}+\frac{99a^7}{b^{13}\left(a+bx^2\right)^4}-\frac{1}{b^{13}(}$$

Antiderivative was successfully verified.

```
[In] Int[x^25/(a + b*x^2)^10,x]
```

```
[Out] (55*a^2*x^2)/(2*b^12) - (5*a*x^4)/(2*b^11) + x^6/(6*b^10) - a^12/(18*b^13*(
a + b*x^2)^9) + (3*a^11)/(4*b^13*(a + b*x^2)^8) - (33*a^10)/(7*b^13*(a + b*
x^2)^7) + (55*a^9)/(3*b^13*(a + b*x^2)^6) - (99*a^8)/(2*b^13*(a + b*x^2)^5)
 + (99*a^7)/(b^13*(a + b*x^2)^4) - (154*a^6)/(b^13*(a + b*x^2)^3) + (198*a^
5)/(b^13*(a + b*x^2)^2) - (495*a^4)/(2*b^13*(a + b*x^2)) - (110*a^3*Log[a +
 b*x^2])/b^13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{25}}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^{12}}{(a+bx)^{10}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{55a^2}{b^{12}}-\frac{10ax}{b^{11}}+\frac{x^2}{b^{10}}+\frac{a^{12}}{b^{12}(a+bx)^{10}}-\frac{12a^{11}}{b^{12}(a+bx)^9}+\frac{66a^{10}}{b^{12}(a+bx)^8}-\frac{220}{b^{12}(a-}\right.\right.\\ &= \frac{55a^2x^2}{2b^{12}}-\frac{5ax^4}{2b^{11}}+\frac{x^6}{6b^{10}}-\frac{a^{12}}{18b^{13}\left(a+bx^2\right)^9}+\frac{3a^{11}}{4b^{13}\left(a+bx^2\right)^8}-\frac{33a^{10}}{7b^{13}\left(a+bx^2\right)^7}+\frac{55}{3b^{13}(a}\end{aligned}$$

Mathematica [A] time = 0.05, size = 169, normalized size = 0.78

$$-\frac{35201a^{12} + 289089a^{11}bx^2 + 1031616a^{10}b^2x^4 + 2074464a^9b^3x^6 + 2529576a^8b^4x^8 + 1831032a^7b^5x^{10} + 638568a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^25/(a + b*x^2)^10,x]
```

```
[Out] -1/252*(35201*a^12 + 289089*a^11*b*x^2 + 1031616*a^10*b^2*x^4 + 2074464*a^9
*b^3*x^6 + 2529576*a^8*b^4*x^8 + 1831032*a^7*b^5*x^10 + 638568*a^6*b^6*x^12
 - 58968*a^5*b^7*x^14 - 139482*a^4*b^8*x^16 - 43218*a^3*b^9*x^18 - 2772*a^2
*b^10*x^20 + 252*a*b^11*x^22 - 42*b^12*x^24 + 27720*a^3*(a + b*x^2)^9*Log[a
 + b*x^2])/(b^13*(a + b*x^2)^9)
```

fricas [A] time = 0.70, size = 346, normalized size = 1.60

$$\frac{42\,b^{12}x^{24} - 252\,ab^{11}x^{22} + 2772\,a^2b^{10}x^{20} + 43218\,a^3b^9x^{18} + 139482\,a^4b^8x^{16} + 58968\,a^5b^7x^{14} - 638568\,a^6b^6x^{12} -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^25/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/252*(42*b^12*x^24 - 252*a*b^11*x^22 + 2772*a^2*b^10*x^20 + 43218*a^3*b^9*
x^18 + 139482*a^4*b^8*x^16 + 58968*a^5*b^7*x^14 - 638568*a^6*b^6*x^12 - 183
1032*a^7*b^5*x^10 - 2529576*a^8*b^4*x^8 - 2074464*a^9*b^3*x^6 - 1031616*a^1
0*b^2*x^4 - 289089*a^11*b*x^2 - 35201*a^12 - 27720*(a^3*b^9*x^18 + 9*a^4*b^
8*x^16 + 36*a^5*b^7*x^14 + 84*a^6*b^6*x^12 + 126*a^7*b^5*x^10 + 126*a^8*b^4
*x^8 + 84*a^9*b^3*x^6 + 36*a^10*b^2*x^4 + 9*a^11*b*x^2 + a^12)*log(b*x^2 +
a))/(b^22*x^18 + 9*a*b^21*x^16 + 36*a^2*b^20*x^14 + 84*a^3*b^19*x^12 + 126*
a^4*b^18*x^10 + 126*a^5*b^17*x^8 + 84*a^6*b^16*x^6 + 36*a^7*b^15*x^4 + 9*a^
8*b^14*x^2 + a^9*b^13)
```

giac [A] time = 0.64, size = 168, normalized size = 0.78

$$-\frac{110\,a^3\log\left(\left|bx^2 + a\right|\right)}{b^{13}} + \frac{78419\,a^3b^9x^{18} + 643401\,a^4b^8x^{16} + 2374020\,a^5b^7x^{14} + 5151300\,a^6b^6x^{12} + 7227990\,a^7b^5x}{252\left(bx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^25/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -110*a^3*log(abs(b*x^2 + a))/b^13 + 1/252*(78419*a^3*b^9*x^18 + 643401*a^4*
b^8*x^16 + 2374020*a^5*b^7*x^14 + 5151300*a^6*b^6*x^12 + 7227990*a^7*b^5*x^
10 + 6791400*a^8*b^4*x^8 + 4268880*a^9*b^3*x^6 + 1729728*a^10*b^2*x^4 + 409
752*a^11*b*x^2 + 43218*a^12)/((b*x^2 + a)^9*b^13) + 1/6*(b^20*x^6 - 15*a*b^
19*x^4 + 165*a^2*b^18*x^2)/b^30
```

maple [A] time = 0.02, size = 199, normalized size = 0.92

$$-\frac{a^{12}}{18\left(b\,x^2 + a\right)^9 b^{13}} + \frac{3a^{11}}{4\left(b\,x^2 + a\right)^8 b^{13}} - \frac{33a^{10}}{7\left(b\,x^2 + a\right)^7 b^{13}} + \frac{55a^9}{3\left(b\,x^2 + a\right)^6 b^{13}} + \frac{x^6}{6b^{10}} - \frac{99a^8}{2\left(b\,x^2 + a\right)^5 b^{13}} + \frac{99a^7}{\left(b\,x^2 + a\right)^4 b^1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^25/(b*x^2+a)^10,x)
```

```
[Out] 55/2*a^2*x^2/b^12-5/2*a*x^4/b^11+1/6*x^6/b^10-1/18*a^12/b^13/(b*x^2+a)^9+3/
4*a^11/b^13/(b*x^2+a)^8-33/7*a^10/b^13/(b*x^2+a)^7+55/3*a^9/b^13/(b*x^2+a)^
```

```
6-99/2*a^8/b^13/(b*x^2+a)^5+99*a^7/b^13/(b*x^2+a)^4-154*a^6/b^13/(b*x^2+a)^
3+198*a^5/b^13/(b*x^2+a)^2-495/2*a^4/b^13/(b*x^2+a)-110*a^3*ln(b*x^2+a)/b^1
3
```

maxima [A] time = 1.62, size = 242, normalized size = 1.12

$$-\frac{62370\,a^4b^8x^{16}+449064\,a^5b^7x^{14}+1435896\,a^6b^6x^{12}+2652804\,a^7b^5x^{10}+3089394\,a^8b^4x^8+2318316\,a^9b^3x^6}{252\left(b^{22}x^{18}+9\,ab^{21}x^{16}+36\,a^2b^{20}x^{14}+84\,a^3b^{19}x^{12}+126\,a^4b^{18}x^{10}+126\,a^5b^{17}x^8+84\,a^6b^{16}x^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^25/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(62370*a^4*b^8*x^16 + 449064*a^5*b^7*x^14 + 1435896*a^6*b^6*x^12 + 2
652804*a^7*b^5*x^10 + 3089394*a^8*b^4*x^8 + 2318316*a^9*b^3*x^6 + 1093356*a
^10*b^2*x^4 + 296019*a^11*b*x^2 + 35201*a^12)/(b^22*x^18 + 9*a*b^21*x^16 +
36*a^2*b^20*x^14 + 84*a^3*b^19*x^12 + 126*a^4*b^18*x^10 + 126*a^5*b^17*x^8
+ 84*a^6*b^16*x^6 + 36*a^7*b^15*x^4 + 9*a^8*b^14*x^2 + a^9*b^13) - 110*a^3*
log(b*x^2 + a)/b^13 + 1/6*(b^2*x^6 - 15*a*b*x^4 + 165*a^2*x^2)/b^12
```

mupad [B] time = 5.29, size = 242, normalized size = 1.12

$$\frac{x^6}{6\,b^{10}}-\frac{\frac{35201\,a^{12}}{252\,b}+\frac{32891\,a^{11}\,x^2}{28}+\frac{30371\,a^{10}\,b\,x^4}{7}+\frac{27599\,a^9\,b^2\,x^6}{3}+\frac{24519\,a^8\,b^3\,x^8}{2}+10527\,a^7\,b^4\,x^{10}+5698\,a^6\,b^5\,x^{12}+17}{a^9\,b^{12}+9\,a^8\,b^{13}\,x^2+36\,a^7\,b^{14}\,x^4+84\,a^6\,b^{15}\,x^6+126\,a^5\,b^{16}\,x^8+126\,a^4\,b^{17}\,x^{10}+84\,a^3\,b^{18}\,x^{12}+36\,a^2\,b^1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^25/(a + b*x^2)^10,x)
```

```
[Out] x^6/(6*b^10) - ((35201*a^12)/(252*b) + (32891*a^11*x^2)/28 + (30371*a^10*b*
x^4)/7 + (27599*a^9*b^2*x^6)/3 + (24519*a^8*b^3*x^8)/2 + 10527*a^7*b^4*x^10
 + 5698*a^6*b^5*x^12 + 1782*a^5*b^6*x^14 + (495*a^4*b^7*x^16)/2)/(a^9*b^12
+ b^21*x^18 + 9*a*b^20*x^16 + 9*a^8*b^13*x^2 + 36*a^7*b^14*x^4 + 84*a^6*b^1
5*x^6 + 126*a^5*b^16*x^8 + 126*a^4*b^17*x^10 + 84*a^3*b^18*x^12 + 36*a^2*b^
19*x^14) - (5*a*x^4)/(2*b^11) - (110*a^3*log(a + b*x^2))/b^13 + (55*a^2*x^2
)/(2*b^12)
```

sympy [A] time = 2.15, size = 260, normalized size = 1.20

$$-\frac{110a^3\log\left(a+bx^2\right)}{b^{13}}+\frac{55a^2x^2}{2b^{12}}-\frac{5ax^4}{2b^{11}}+\frac{-35201a^{12}-296019a^{11}bx^2-1093356a^{10}b^2x^4-2318316a^9b^3x^6-3089}{252a^9b^{13}+2268a^8b^{14}x^2+9072a^7b^{15}x^4+21168a^6b^{16}x^6+31752a^5b^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**25/(b*x**2+a)**10,x)
```

```
[Out] -110*a**3*log(a + b*x**2)/b**13 + 55*a**2*x**2/(2*b**12) - 5*a*x**4/(2*b**1
1) + (-35201*a**12 - 296019*a**11*b*x**2 - 1093356*a**10*b**2*x**4 - 231831
6*a**9*b**3*x**6 - 3089394*a**8*b**4*x**8 - 2652804*a**7*b**5*x**10 - 14358
96*a**6*b**6*x**12 - 449064*a**5*b**7*x**14 - 62370*a**4*b**8*x**16)/(252*a
**9*b**13 + 2268*a**8*b**14*x**2 + 9072*a**7*b**15*x**4 + 21168*a**6*b**16*
x**6 + 31752*a**5*b**17*x**8 + 31752*a**4*b**18*x**10 + 21168*a**3*b**19*x*
*12 + 9072*a**2*b**20*x**14 + 2268*a*b**21*x**16 + 252*b**22*x**18) + x**6/
(6*b**10)
```

3.193 $\int \frac{x^{23}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=205

$$\frac{a^{11}}{18b^{12}\left(a+bx^2\right)^9}-\frac{11a^{10}}{16b^{12}\left(a+bx^2\right)^8}+\frac{55a^9}{14b^{12}\left(a+bx^2\right)^7}-\frac{55a^8}{4b^{12}\left(a+bx^2\right)^6}+\frac{33a^7}{b^{12}\left(a+bx^2\right)^5}-\frac{231a^6}{4b^{12}\left(a+bx^2\right)^4}+\frac{77}{b^{12}\left(a+\right.}$$

```
[Out] -5*a*x^2/b^11+1/4*x^4/b^10+1/18*a^11/b^12/(b*x^2+a)^9-11/16*a^10/b^12/(b*x^
2+a)^8+55/14*a^9/b^12/(b*x^2+a)^7-55/4*a^8/b^12/(b*x^2+a)^6+33*a^7/b^12/(b*
x^2+a)^5-231/4*a^6/b^12/(b*x^2+a)^4+77*a^5/b^12/(b*x^2+a)^3-165/2*a^4/b^12/
(b*x^2+a)^2+165/2*a^3/b^12/(b*x^2+a)+55/2*a^2*ln(b*x^2+a)/b^12
```

Rubi [A] time = 0.21, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^{11}}{18b^{12}\left(a+bx^2\right)^9}-\frac{11a^{10}}{16b^{12}\left(a+bx^2\right)^8}+\frac{55a^9}{14b^{12}\left(a+bx^2\right)^7}-\frac{55a^8}{4b^{12}\left(a+bx^2\right)^6}+\frac{33a^7}{b^{12}\left(a+bx^2\right)^5}-\frac{231a^6}{4b^{12}\left(a+bx^2\right)^4}+\frac{77}{b^{12}\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^23/(a + b*x^2)^10,x]
```

```
[Out] (-5*a*x^2)/b^11 + x^4/(4*b^10) + a^11/(18*b^12*(a + b*x^2)^9) - (11*a^10)/(
16*b^12*(a + b*x^2)^8) + (55*a^9)/(14*b^12*(a + b*x^2)^7) - (55*a^8)/(4*b^1
2*(a + b*x^2)^6) + (33*a^7)/(b^12*(a + b*x^2)^5) - (231*a^6)/(4*b^12*(a + b
*x^2)^4) + (77*a^5)/(b^12*(a + b*x^2)^3) - (165*a^4)/(2*b^12*(a + b*x^2)^2)
 + (165*a^3)/(2*b^12*(a + b*x^2)) + (55*a^2*Log[a + b*x^2])/(2*b^12)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{23}}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^{11}}{(a+bx)^{10}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{10a}{b^{11}}+\frac{x}{b^{10}}-\frac{a^{11}}{b^{11}(a+bx)^{10}}+\frac{11a^{10}}{b^{11}(a+bx)^9}-\frac{55a^9}{b^{11}(a+bx)^8}+\frac{165a^8}{b^{11}(a+bx)^7}-\right.\right.\\
&= -\frac{5ax^2}{b^{11}}+\frac{x^4}{4b^{10}}+\frac{a^{11}}{18b^{12}\left(a+bx^2\right)^9}-\frac{11a^{10}}{16b^{12}\left(a+bx^2\right)^8}+\frac{55a^9}{14b^{12}\left(a+bx^2\right)^7}-\frac{55a^8}{4b^{12}\left(a+bx^2\right)^6}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 158, normalized size = 0.77

$$\frac{42131a^{11} + 351459a^{10}bx^2 + 1281096a^9b^2x^4 + 2656584a^8b^3x^6 + 3402756a^7b^4x^8 + 2704212a^6b^5x^{10} + 1220688}{100}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^23/(a + b*x^2)^10,x]
```

```
[Out] (42131*a^11 + 351459*a^10*b*x^2 + 1281096*a^9*b^2*x^4 + 2656584*a^8*b^3*x^6
 + 3402756*a^7*b^4*x^8 + 2704212*a^6*b^5*x^10 + 1220688*a^5*b^6*x^12 + 1905
12*a^4*b^7*x^14 - 77112*a^3*b^8*x^16 - 36288*a^2*b^9*x^18 - 2772*a*b^10*x^2
0 + 252*b^11*x^22 + 27720*a^2*(a + b*x^2)^9*Log[a + b*x^2])/(1008*b^12*(a +
 b*x^2)^9)
```

fricas [A] time = 0.97, size = 335, normalized size = 1.63

$$\frac{252\,b^{11}x^{22} - 2772\,ab^{10}x^{20} - 36288\,a^2b^9x^{18} - 77112\,a^3b^8x^{16} + 190512\,a^4b^7x^{14} + 1220688\,a^5b^6x^{12} + 2704212\,a}{1008}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/1008*(252*b^11*x^22 - 2772*a*b^10*x^20 - 36288*a^2*b^9*x^18 - 77112*a^3*b
^8*x^16 + 190512*a^4*b^7*x^14 + 1220688*a^5*b^6*x^12 + 2704212*a^6*b^5*x^10
 + 3402756*a^7*b^4*x^8 + 2656584*a^8*b^3*x^6 + 1281096*a^9*b^2*x^4 + 351459
*a^10*b*x^2 + 42131*a^11 + 27720*(a^2*b^9*x^18 + 9*a^3*b^8*x^16 + 36*a^4*b^
7*x^14 + 84*a^5*b^6*x^12 + 126*a^6*b^5*x^10 + 126*a^7*b^4*x^8 + 84*a^8*b^3*
x^6 + 36*a^9*b^2*x^4 + 9*a^10*b*x^2 + a^11)*log(b*x^2 + a))/(b^21*x^18 + 9*
a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^18*x^12 + 126*a^4*b^17*x^10 + 126
*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 + 9*a^8*b^13*x^2 + a^9*b^
12)
```

giac [A] time = 0.63, size = 157, normalized size = 0.77

$$\frac{55\,a^2\log\left(\left|bx^2 + a\right|\right)}{2\,b^{12}} + \frac{b^{10}x^4 - 20\,ab^9x^2}{4\,b^{20}} - \frac{78419\,a^2b^9x^{18} + 622611\,a^3b^8x^{16} + 2240964\,a^4b^7x^{14} + 4763220\,a^5b^6x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/2*a^2*log(abs(b*x^2 + a))/b^12 + 1/4*(b^10*x^4 - 20*a*b^9*x^2)/b^20 - 1/
1008*(78419*a^2*b^9*x^18 + 622611*a^3*b^8*x^16 + 2240964*a^4*b^7*x^14 + 476
3220*a^5*b^6*x^12 + 6562710*a^6*b^5*x^10 + 6063750*a^7*b^4*x^8 + 3751440*a^
8*b^3*x^6 + 1496880*a^9*b^2*x^4 + 349272*a^10*b*x^2 + 36288*a^11)/((b*x^2 +
 a)^9*b^12)
```

maple [A] time = 0.02, size = 188, normalized size = 0.92

$$\frac{a^{11}}{18\left(b\,x^2 + a\right)^9 b^{12}} - \frac{11a^{10}}{16\left(b\,x^2 + a\right)^8 b^{12}} + \frac{55a^9}{14\left(b\,x^2 + a\right)^7 b^{12}} - \frac{55a^8}{4\left(b\,x^2 + a\right)^6 b^{12}} + \frac{33a^7}{\left(b\,x^2 + a\right)^5 b^{12}} - \frac{231a^6}{4\left(b\,x^2 + a\right)^4 b^{12}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(b*x^2+a)^10,x)
```

```
[Out] -5*a*x^2/b^11+1/4*x^4/b^10+1/18*a^11/b^12/(b*x^2+a)^9-11/16*a^10/b^12/(b*x^
2+a)^8+55/14*a^9/b^12/(b*x^2+a)^7-55/4*a^8/b^12/(b*x^2+a)^6+33*a^7/b^12/(b*
```

```
x^2+a)^5-231/4*a^6/b^12/(b*x^2+a)^4+77*a^5/b^12/(b*x^2+a)^3-165/2*a^4/b^12/
(b*x^2+a)^2+165/2*a^3/b^12/(b*x^2+a)+55/2*a^2*ln(b*x^2+a)/b^12
```

maxima [A] time = 1.58, size = 231, normalized size = 1.13

$$\frac{83160\,a^3b^8x^{16} + 582120\,a^4b^7x^{14} + 1823976\,a^5b^6x^{12} + 3318084\,a^6b^5x^{10} + 3817044\,a^7b^4x^8 + 2835756\,a^8b^3x^6 + 13}{1008\left(b^{21}x^{18} + 9\,ab^{20}x^{16} + 36\,a^2b^{19}x^{14} + 84\,a^3b^{18}x^{12} + 126\,a^4b^{17}x^{10} + 126\,a^5b^{16}x^8 + 84\,a^6b^{15}x^6 + 3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/1008*(83160*a^3*b^8*x^16 + 582120*a^4*b^7*x^14 + 1823976*a^5*b^6*x^12 + 3
318084*a^6*b^5*x^10 + 3817044*a^7*b^4*x^8 + 2835756*a^8*b^3*x^6 + 1326204*a
^9*b^2*x^4 + 356499*a^10*b*x^2 + 42131*a^11)/(b^21*x^18 + 9*a*b^20*x^16 + 3
6*a^2*b^19*x^14 + 84*a^3*b^18*x^12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 +
 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 + 9*a^8*b^13*x^2 + a^9*b^12) + 55/2*a^2*
log(b*x^2 + a)/b^12 + 1/4*(b*x^4 - 20*a*x^2)/b^11
```

mupad [B] time = 0.40, size = 230, normalized size = 1.12

$$\frac{\frac{42131\,a^{11}}{1008\,b} + \frac{39611\,a^{10}\,x^2}{112} + \frac{36839\,a^9\,b\,x^4}{28} + \frac{11253\,a^8\,b^2\,x^6}{4} + \frac{15147\,a^7\,b^3\,x^8}{4} + \frac{13167\,a^6\,b^4\,x^{10}}{4} + \frac{3619\,a^5\,b^5\,x^{12}}{2} + \frac{1155\,a^4\,b^6\,x^{14}}{2} + }{a^9\,b^{11} + 9\,a^8\,b^{12}\,x^2 + 36\,a^7\,b^{13}\,x^4 + 84\,a^6\,b^{14}\,x^6 + 126\,a^5\,b^{15}\,x^8 + 126\,a^4\,b^{16}\,x^{10} + 84\,a^3\,b^{17}\,x^{12} + 36\,a^2\,b^{18}\,x^{14} + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(a + b*x^2)^10,x)
```

```
[Out] ((42131*a^11)/(1008*b) + (39611*a^10*x^2)/112 + (36839*a^9*b*x^4)/28 + (112
53*a^8*b^2*x^6)/4 + (15147*a^7*b^3*x^8)/4 + (13167*a^6*b^4*x^10)/4 + (3619*
a^5*b^5*x^12)/2 + (1155*a^4*b^6*x^14)/2 + (165*a^3*b^7*x^16)/2)/(a^9*b^11 +
 b^20*x^18 + 9*a*b^19*x^16 + 9*a^8*b^12*x^2 + 36*a^7*b^13*x^4 + 84*a^6*b^14
*x^6 + 126*a^5*b^15*x^8 + 126*a^4*b^16*x^10 + 84*a^3*b^17*x^12 + 36*a^2*b^1
8*x^14) + x^4/(4*b^10) - (5*a*x^2)/b^11 + (55*a^2*log(a + b*x^2))/(2*b^12)
```

sympy [A] time = 2.06, size = 245, normalized size = 1.20

$$\frac{55a^2\log\left(a + bx^2\right)}{2b^{12}} - \frac{5ax^2}{b^{11}} + \frac{42131a^{11} + 356499a^{10}bx^2 + 1326204a^9b^2x^4 + 2835756a^8b^3x^6 + 3817044a^7b^4x^8 +}{1008a^9b^{12} + 9072a^8b^{13}x^2 + 36288a^7b^{14}x^4 + 84672a^6b^{15}x^6 + 127008a^5b^{16}x^8 + 127008}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(b*x**2+a)**10,x)
```

```
[Out] 55*a**2*log(a + b*x**2)/(2*b**12) - 5*a*x**2/b**11 + (42131*a**11 + 356499*
a**10*b*x**2 + 1326204*a**9*b**2*x**4 + 2835756*a**8*b**3*x**6 + 3817044*a*
*7*b**4*x**8 + 3318084*a**6*b**5*x**10 + 1823976*a**5*b**6*x**12 + 582120*a
**4*b**7*x**14 + 83160*a**3*b**8*x**16)/(1008*a**9*b**12 + 9072*a**8*b**13*
x**2 + 36288*a**7*b**14*x**4 + 84672*a**6*b**15*x**6 + 127008*a**5*b**16*x*
*8 + 127008*a**4*b**17*x**10 + 84672*a**3*b**18*x**12 + 36288*a**2*b**19*x*
*14 + 9072*a*b**20*x**16 + 1008*b**21*x**18) + x**4/(4*b**10)
```

3.194 $\int \frac{x^{21}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=188

$$-\frac{a^{10}}{18b^{11}\left(a+bx^2\right)^9}+\frac{5a^9}{8b^{11}\left(a+bx^2\right)^8}-\frac{45a^8}{14b^{11}\left(a+bx^2\right)^7}+\frac{10a^7}{b^{11}\left(a+bx^2\right)^6}-\frac{21a^6}{b^{11}\left(a+bx^2\right)^5}+\frac{63a^5}{2b^{11}\left(a+bx^2\right)^4}-\frac{3}{b^{11}\left(a\right.}$$

```
[Out] 1/2*x^2/b^10-1/18*a^10/b^11/(b*x^2+a)^9+5/8*a^9/b^11/(b*x^2+a)^8-45/14*a^8/
b^11/(b*x^2+a)^7+10*a^7/b^11/(b*x^2+a)^6-21*a^6/b^11/(b*x^2+a)^5+63/2*a^5/b
^11/(b*x^2+a)^4-35*a^4/b^11/(b*x^2+a)^3+30*a^3/b^11/(b*x^2+a)^2-45/2*a^2/b^
11/(b*x^2+a)-5*a*ln(b*x^2+a)/b^11
```

Rubi [A] time = 0.19, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^{10}}{18b^{11}\left(a+bx^2\right)^9}+\frac{5a^9}{8b^{11}\left(a+bx^2\right)^8}-\frac{45a^8}{14b^{11}\left(a+bx^2\right)^7}+\frac{10a^7}{b^{11}\left(a+bx^2\right)^6}-\frac{21a^6}{b^{11}\left(a+bx^2\right)^5}+\frac{63a^5}{2b^{11}\left(a+bx^2\right)^4}-\frac{3}{b^{11}\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^21/(a + b*x^2)^10,x]
```

```
[Out] x^2/(2*b^10) - a^10/(18*b^11*(a + b*x^2)^9) + (5*a^9)/(8*b^11*(a + b*x^2)^8
) - (45*a^8)/(14*b^11*(a + b*x^2)^7) + (10*a^7)/(b^11*(a + b*x^2)^6) - (21*
a^6)/(b^11*(a + b*x^2)^5) + (63*a^5)/(2*b^11*(a + b*x^2)^4) - (35*a^4)/(b^1
1*(a + b*x^2)^3) + (30*a^3)/(b^11*(a + b*x^2)^2) - (45*a^2)/(2*b^11*(a + b*
x^2)) - (5*a*Log[a + b*x^2])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{21}}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^{10}}{(a+bx)^{10}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^{10}}+\frac{a^{10}}{b^{10}(a+bx)^{10}}-\frac{10a^9}{b^{10}(a+bx)^9}+\frac{45a^8}{b^{10}(a+bx)^8}-\frac{120a^7}{b^{10}(a+bx)^7}+\frac{210}{b^{10}(a+\right.\right.\\ &= \frac{x^2}{2b^{10}}-\frac{a^{10}}{18b^{11}\left(a+bx^2\right)^9}+\frac{5a^9}{8b^{11}\left(a+bx^2\right)^8}-\frac{45a^8}{14b^{11}\left(a+bx^2\right)^7}+\frac{10a^7}{b^{11}\left(a+bx^2\right)^6}-\frac{21}{b^{11}\left(a+\right.}\end{aligned}$$

Mathematica [A] time = 0.04, size = 145, normalized size = 0.77

$$-\frac{4861a^{10}+41229a^9bx^2+153576a^8b^2x^4+328104a^7b^3x^6+439236a^6b^4x^8+375732a^5b^5x^{10}+197568a^4b^6x^{12}+}{504b^{11}\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^21/(a + b*x^2)^10,x]
```

```
[Out] -1/504*(4861*a^10 + 41229*a^9*b*x^2 + 153576*a^8*b^2*x^4 + 328104*a^7*b^3*x
^6 + 439236*a^6*b^4*x^8 + 375732*a^5*b^5*x^10 + 197568*a^4*b^6*x^12 + 54432
*a^3*b^7*x^14 + 2268*a^2*b^8*x^16 - 2268*a*b^9*x^18 - 252*b^10*x^20 + 2520*
a*(a + b*x^2)^9*Log[a + b*x^2])/(b^11*(a + b*x^2)^9)
```

fricas [A] time = 0.77, size = 322, normalized size = 1.71

$$\frac{252\,b^{10}x^{20}+2268\,ab^9x^{18}-2268\,a^2b^8x^{16}-54432\,a^3b^7x^{14}-197568\,a^4b^6x^{12}-375732\,a^5b^5x^{10}-439236\,a^6b^4x^8-}{504\left(b^{20}x^{18}+9\,ab^{19}x^{16}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^21/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/504*(252*b^10*x^20 + 2268*a*b^9*x^18 - 2268*a^2*b^8*x^16 - 54432*a^3*b^7*
x^14 - 197568*a^4*b^6*x^12 - 375732*a^5*b^5*x^10 - 439236*a^6*b^4*x^8 - 328
104*a^7*b^3*x^6 - 153576*a^8*b^2*x^4 - 41229*a^9*b*x^2 - 4861*a^10 - 2520*(
a*b^9*x^18 + 9*a^2*b^8*x^16 + 36*a^3*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a^5*b
^5*x^10 + 126*a^6*b^4*x^8 + 84*a^7*b^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 +
 a^10)*log(b*x^2 + a))/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^18*x^14 + 84*a
^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b^14*x^6 + 36*
a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11)
```

giac [A] time = 0.63, size = 139, normalized size = 0.74

$$\frac{x^2}{2\,b^{10}}-\frac{5\,a\log\left(\left|bx^2+a\right|\right)}{b^{11}}+\frac{7129\,ab^9x^{18}+52821\,a^2b^8x^{16}+181044\,a^3b^7x^{14}+369516\,a^4b^6x^{12}+490770\,a^5b^5x^{10}+}{504\left(bx^2+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^21/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^10 - 5*a*log(abs(b*x^2 + a))/b^11 + 1/504*(7129*a*b^9*x^18 + 5282
1*a^2*b^8*x^16 + 181044*a^3*b^7*x^14 + 369516*a^4*b^6*x^12 + 490770*a^5*b^5
*x^10 + 437850*a^6*b^4*x^8 + 261660*a^7*b^3*x^6 + 100800*a^8*b^2*x^4 + 2268
0*a^9*b*x^2 + 2268*a^10)/((b*x^2 + a)^9*b^11)
```

maple [A] time = 0.02, size = 177, normalized size = 0.94

$$-\frac{a^{10}}{18\left(b\,x^2+a\right)^9b^{11}}+\frac{5a^9}{8\left(b\,x^2+a\right)^8b^{11}}-\frac{45a^8}{14\left(b\,x^2+a\right)^7b^{11}}+\frac{10a^7}{\left(b\,x^2+a\right)^6b^{11}}-\frac{21a^6}{\left(b\,x^2+a\right)^5b^{11}}+\frac{63a^5}{2\left(b\,x^2+a\right)^4b^{11}}-\frac{}{\left(b\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^21/(b*x^2+a)^10,x)
```

```
[Out] 1/2*x^2/b^10-1/18*a^10/b^11/(b*x^2+a)^9+5/8*a^9/b^11/(b*x^2+a)^8-45/14*a^8/
b^11/(b*x^2+a)^7+10*a^7/b^11/(b*x^2+a)^6-21*a^6/b^11/(b*x^2+a)^5+63/2*a^5/b
^11/(b*x^2+a)^4-35*a^4/b^11/(b*x^2+a)^3+30*a^3/b^11/(b*x^2+a)^2-45/2*a^2/b^
11/(b*x^2+a)-5*a*ln(b*x^2+a)/b^11
```

maxima [A] time = 1.60, size = 220, normalized size = 1.17

$$-\frac{11340\,a^{2}b^{8}x^{16} + 75600\,a^{3}b^{7}x^{14} + 229320\,a^{4}b^{6}x^{12} + 407484\,a^{5}b^{5}x^{10} + 460404\,a^{6}b^{4}x^{8} + 337176\,a^{7}b^{3}x^{6} + 155}{504\left(b^{20}x^{18} + 9\,ab^{19}x^{16} + 36\,a^{2}b^{18}x^{14} + 84\,a^{3}b^{17}x^{12} + 126\,a^{4}b^{16}x^{10} + 126\,a^{5}b^{15}x^{8} + 84\,a^{6}b^{14}x^{6} + 36\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^21/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(11340*a^2*b^8*x^16 + 75600*a^3*b^7*x^14 + 229320*a^4*b^6*x^12 + 407
484*a^5*b^5*x^10 + 460404*a^6*b^4*x^8 + 337176*a^7*b^3*x^6 + 155844*a^8*b^2
*x^4 + 41481*a^9*b*x^2 + 4861*a^10)/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^1
8*x^14 + 84*a^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b
^14*x^6 + 36*a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11) + 1/2*x^2/b^10 - 5*a
*log(b*x^2 + a)/b^11
```

mupad [B] time = 0.43, size = 220, normalized size = 1.17

$$\frac{x^2}{2\,b^{10}}-\frac{\frac{4861\,a^{10}}{504\,b}+\frac{4609\,a^9\,x^2}{56}+\frac{4329\,a^8\,b\,x^4}{14}+669\,a^7\,b^2\,x^6+\frac{1827\,a^6\,b^3\,x^8}{2}+\frac{1617\,a^5\,b^4\,x^{10}}{2}+455\,a^4\,b^5\,x^{12}+150\,a}{a^9\,b^{10}+9\,a^8\,b^{11}\,x^2+36\,a^7\,b^{12}\,x^4+84\,a^6\,b^{13}\,x^6+126\,a^5\,b^{14}\,x^8+126\,a^4\,b^{15}\,x^{10}+84\,a^3\,b^{16}\,x^{12}+36\,a^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^21/(a + b*x^2)^10,x)
```

```
[Out] x^2/(2*b^10) - ((4861*a^10)/(504*b) + (4609*a^9*x^2)/56 + (4329*a^8*b*x^4)/
14 + 669*a^7*b^2*x^6 + (1827*a^6*b^3*x^8)/2 + (1617*a^5*b^4*x^10)/2 + 455*a
^4*b^5*x^12 + 150*a^3*b^6*x^14 + (45*a^2*b^7*x^16)/2)/(a^9*b^10 + b^19*x^18
 + 9*a*b^18*x^16 + 9*a^8*b^11*x^2 + 36*a^7*b^12*x^4 + 84*a^6*b^13*x^6 + 126
*a^5*b^14*x^8 + 126*a^4*b^15*x^10 + 84*a^3*b^16*x^12 + 36*a^2*b^17*x^14) -
(5*a*log(a + b*x^2))/b^11
```

sympy [A] time = 2.03, size = 233, normalized size = 1.24

$$-\frac{5a\log\left(a+bx^2\right)}{b^{11}}+\frac{-4861a^{10}-41481a^{9}bx^{2}-155844a^{8}b^{2}x^{4}-337176a^{7}b^{3}x^{6}-460404a^{6}b^{4}x^{8}-407484}{504a^{9}b^{11}+4536a^{8}b^{12}x^{2}+18144a^{7}b^{13}x^{4}+42336a^{6}b^{14}x^{6}+63504a^{5}b^{15}x^{8}+63504a^{4}b^{16}x^{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**21/(b*x**2+a)**10,x)
```

```
[Out] -5*a*log(a + b*x**2)/b**11 + (-4861*a**10 - 41481*a**9*b*x**2 - 155844*a**8
*b**2*x**4 - 337176*a**7*b**3*x**6 - 460404*a**6*b**4*x**8 - 407484*a**5*b*
*5*x**10 - 229320*a**4*b**6*x**12 - 75600*a**3*b**7*x**14 - 11340*a**2*b**8
*x**16)/(504*a**9*b**11 + 4536*a**8*b**12*x**2 + 18144*a**7*b**13*x**4 + 42
336*a**6*b**14*x**6 + 63504*a**5*b**15*x**8 + 63504*a**4*b**16*x**10 + 4233
6*a**3*b**17*x**12 + 18144*a**2*b**18*x**14 + 4536*a*b**19*x**16 + 504*b**2
0*x**18) + x**2/(2*b**10)
```

3.195 $\int \frac{x^{19}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=179

$$\frac{a^9}{18b^{10}\left(a+bx^2\right)^9}-\frac{9a^8}{16b^{10}\left(a+bx^2\right)^8}+\frac{18a^7}{7b^{10}\left(a+bx^2\right)^7}-\frac{7a^6}{b^{10}\left(a+bx^2\right)^6}+\frac{63a^5}{5b^{10}\left(a+bx^2\right)^5}-\frac{63a^4}{4b^{10}\left(a+bx^2\right)^4}+\frac{14a}{b^{10}\left(a+\right.}$$

```
[Out] 1/18*a^9/b^10/(b*x^2+a)^9-9/16*a^8/b^10/(b*x^2+a)^8+18/7*a^7/b^10/(b*x^2+a)
^7-7*a^6/b^10/(b*x^2+a)^6+63/5*a^5/b^10/(b*x^2+a)^5-63/4*a^4/b^10/(b*x^2+a)
^4+14*a^3/b^10/(b*x^2+a)^3-9*a^2/b^10/(b*x^2+a)^2+9/2*a/b^10/(b*x^2+a)+1/2*
ln(b*x^2+a)/b^10
```

Rubi [A] time = 0.17, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^9}{18b^{10}\left(a+bx^2\right)^9}-\frac{9a^8}{16b^{10}\left(a+bx^2\right)^8}+\frac{18a^7}{7b^{10}\left(a+bx^2\right)^7}-\frac{7a^6}{b^{10}\left(a+bx^2\right)^6}+\frac{63a^5}{5b^{10}\left(a+bx^2\right)^5}-\frac{63a^4}{4b^{10}\left(a+bx^2\right)^4}+\frac{14a}{b^{10}\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/(a + b*x^2)^10,x]
```

```
[Out] a^9/(18*b^10*(a + b*x^2)^9) - (9*a^8)/(16*b^10*(a + b*x^2)^8) + (18*a^7)/(7
*b^10*(a + b*x^2)^7) - (7*a^6)/(b^10*(a + b*x^2)^6) + (63*a^5)/(5*b^10*(a +
 b*x^2)^5) - (63*a^4)/(4*b^10*(a + b*x^2)^4) + (14*a^3)/(b^10*(a + b*x^2)^3
) - (9*a^2)/(b^10*(a + b*x^2)^2) + (9*a)/(2*b^10*(a + b*x^2)) + Log[a + b*x
^2]/(2*b^10)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{19}}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^9}{(a+bx)^{10}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^9}{b^9(a+bx)^{10}}+\frac{9a^8}{b^9(a+bx)^9}-\frac{36a^7}{b^9(a+bx)^8}+\frac{84a^6}{b^9(a+bx)^7}-\frac{126a^5}{b^9(a+bx)^6}+\frac{1}{b^9(\right.\right.\\ &= \frac{a^9}{18b^{10}\left(a+bx^2\right)^9}-\frac{9a^8}{16b^{10}\left(a+bx^2\right)^8}+\frac{18a^7}{7b^{10}\left(a+bx^2\right)^7}-\frac{7a^6}{b^{10}\left(a+bx^2\right)^6}+\frac{63a^5}{5b^{10}\left(a+bx^2\right)^5}-\end{aligned}$$

Mathematica [A] time = 0.03, size = 116, normalized size = 0.65

$$\frac{\frac{a\left(7129a^8+61641a^7bx^2+235224a^6b^2x^4+518616a^5b^3x^6+725004a^4b^4x^8+661500a^3b^5x^{10}+388080a^2b^6x^{12}+136080ab^7x^{14}+22680b^8x^{16}\right)}{\left(a+bx^2\right)^9}+2520\,\mathrm{lo}}{5040b^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/(a + b*x^2)^10,x]
```

```
[Out] ((a*(7129*a^8 + 61641*a^7*b*x^2 + 235224*a^6*b^2*x^4 + 518616*a^5*b^3*x^6 +
 725004*a^4*b^4*x^8 + 661500*a^3*b^5*x^10 + 388080*a^2*b^6*x^12 + 136080*a*
b^7*x^14 + 22680*b^8*x^16))/(a + b*x^2)^9 + 2520*Log[a + b*x^2])/(5040*b^10
)
```

fricas [A] time = 0.97, size = 300, normalized size = 1.68

$$\frac{22680\,ab^8x^{16}+136080\,a^2b^7x^{14}+388080\,a^3b^6x^{12}+661500\,a^4b^5x^{10}+725004\,a^5b^4x^8+518616\,a^6b^3x^6+2352}{5040\left(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a^2b^{17}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/5040*(22680*a*b^8*x^16 + 136080*a^2*b^7*x^14 + 388080*a^3*b^6*x^12 + 6615
00*a^4*b^5*x^10 + 725004*a^5*b^4*x^8 + 518616*a^6*b^3*x^6 + 235224*a^7*b^2*
x^4 + 61641*a^8*b*x^2 + 7129*a^9 + 2520*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b
^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3
*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log(b*x^2 + a))/(b^19*x^18 + 9*a
*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*
a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^1
0)
```

giac [A] time = 0.61, size = 119, normalized size = 0.66

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b^{10}}-\frac{7129\,b^8x^{18}+41481\,ab^7x^{16}+120564\,a^2b^6x^{14}+210756\,a^3b^5x^{12}+236754\,a^4b^4x^{10}+173250\,a^5}{5040\left(bx^2+a\right)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^10 - 1/5040*(7129*b^8*x^18 + 41481*a*b^7*x^16 + 1
20564*a^2*b^6*x^14 + 210756*a^3*b^5*x^12 + 236754*a^4*b^4*x^10 + 173250*a^5
*b^3*x^8 + 80220*a^6*b^2*x^6 + 21420*a^7*b*x^4 + 2520*a^8*x^2)/((b*x^2 + a)
^9*b^9)
```

maple [A] time = 0.01, size = 166, normalized size = 0.93

$$\frac{a^9}{18\left(b\,x^2+a\right)^9b^{10}}-\frac{9a^8}{16\left(b\,x^2+a\right)^8b^{10}}+\frac{18a^7}{7\left(b\,x^2+a\right)^7b^{10}}-\frac{7a^6}{\left(b\,x^2+a\right)^6b^{10}}+\frac{63a^5}{5\left(b\,x^2+a\right)^5b^{10}}-\frac{63a^4}{4\left(b\,x^2+a\right)^4b^{10}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^2+a)^10,x)
```

```
[Out] 1/18*a^9/b^10/(b*x^2+a)^9-9/16*a^8/b^10/(b*x^2+a)^8+18/7*a^7/b^10/(b*x^2+a)
^7-7*a^6/b^10/(b*x^2+a)^6+63/5*a^5/b^10/(b*x^2+a)^5-63/4*a^4/b^10/(b*x^2+a)
^4+14*a^3/b^10/(b*x^2+a)^3-9*a^2/b^10/(b*x^2+a)^2+9/2*a/b^10/(b*x^2+a)+1/2*
ln(b*x^2+a)/b^10
```

maxima [A] time = 1.52, size = 209, normalized size = 1.17

$$\frac{22680\, ab^{8}x^{16} + 136080\, a^{2}b^{7}x^{14} + 388080\, a^{3}b^{6}x^{12} + 661500\, a^{4}b^{5}x^{10} + 725004\, a^{5}b^{4}x^{8} + 518616\, a^{6}b^{3}x^{6} + 235224}{5040\left(b^{19}x^{18} + 9\, ab^{18}x^{16} + 36\, a^{2}b^{17}x^{14} + 84\, a^{3}b^{16}x^{12} + 126\, a^{4}b^{15}x^{10} + 126\, a^{5}b^{14}x^{8} + 84\, a^{6}b^{13}x^{6} + 36\, a^{7}b^{1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/5040*(22680*a*b^8*x^16 + 136080*a^2*b^7*x^14 + 388080*a^3*b^6*x^12 + 6615
00*a^4*b^5*x^10 + 725004*a^5*b^4*x^8 + 518616*a^6*b^3*x^6 + 235224*a^7*b^2*
x^4 + 61641*a^8*b*x^2 + 7129*a^9)/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*
x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^1
3*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^10) + 1/2*log(b*x^2 + a)/b
^10
```

mupad [B] time = 5.36, size = 207, normalized size = 1.16

$$\frac{\frac{7129\, a^{9}}{5040\, b^{10}} + \frac{9\, a\, x^{16}}{2\, b^{2}} + \frac{27\, a^{2}\, x^{14}}{b^{3}} + \frac{77\, a^{3}\, x^{12}}{b^{4}} + \frac{525\, a^{4}\, x^{10}}{4\, b^{5}} + \frac{2877\, a^{5}\, x^{8}}{20\, b^{6}} + \frac{1029\, a^{6}\, x^{6}}{10\, b^{7}} + \frac{3267\, a^{7}\, x^{4}}{70\, b^{8}} + \frac{6849\, a^{8}\, x^{2}}{560\, b^{9}}}{a^{9} + 9\, a^{8}\, b\, x^{2} + 36\, a^{7}\, b^{2}\, x^{4} + 84\, a^{6}\, b^{3}\, x^{6} + 126\, a^{5}\, b^{4}\, x^{8} + 126\, a^{4}\, b^{5}\, x^{10} + 84\, a^{3}\, b^{6}\, x^{12} + 36\, a^{2}\, b^{7}\, x^{14} + 9\, a\, b^{8}\, x^{16} + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(a + b*x^2)^10,x)
```

```
[Out] ((7129*a^9)/(5040*b^10) + (9*a*x^16)/(2*b^2) + (27*a^2*x^14)/b^3 + (77*a^3*
x^12)/b^4 + (525*a^4*x^10)/(4*b^5) + (2877*a^5*x^8)/(20*b^6) + (1029*a^6*x^
6)/(10*b^7) + (3267*a^7*x^4)/(70*b^8) + (6849*a^8*x^2)/(560*b^9))/(a^9 + b^
9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126
*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14) + log(
a + b*x^2)/(2*b^10)
```

sympy [A] time = 1.84, size = 219, normalized size = 1.22

$$\frac{7129a^{9} + 61641a^{8}bx^{2} + 235224a^{7}b^{2}x^{4} + 518616a^{6}b^{3}x^{6} + 725004a^{5}b^{4}x^{8} + 661500a^{4}b^{5}x^{10} + 388080}{5040a^{9}b^{10} + 45360a^{8}b^{11}x^{2} + 181440a^{7}b^{12}x^{4} + 423360a^{6}b^{13}x^{6} + 635040a^{5}b^{14}x^{8} + 635040a^{4}b^{15}x^{10} + 423360a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**2+a)**10,x)
```

```
[Out] (7129*a**9 + 61641*a**8*b*x**2 + 235224*a**7*b**2*x**4 + 518616*a**6*b**3*x
**6 + 725004*a**5*b**4*x**8 + 661500*a**4*b**5*x**10 + 388080*a**3*b**6*x**
12 + 136080*a**2*b**7*x**14 + 22680*a*b**8*x**16)/(5040*a**9*b**10 + 45360*
a**8*b**11*x**2 + 181440*a**7*b**12*x**4 + 423360*a**6*b**13*x**6 + 635040*
a**5*b**14*x**8 + 635040*a**4*b**15*x**10 + 423360*a**3*b**16*x**12 + 18144
0*a**2*b**17*x**14 + 45360*a*b**18*x**16 + 5040*b**19*x**18) + log(a + b*x*
*2)/(2*b**10)
```

3.196 $\int \frac{x^{17}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=19

$$\frac{x^{18}}{18a\left(a+bx^2\right)^9}$$

[Out] 1/18*x^18/a/(b*x^2+a)^9

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$\frac{x^{18}}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

[In] Int[x^17/(a + b*x^2)^10,x]

[Out] x^18/(18*a*(a + b*x^2)^9)

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{17}}{\left(a+bx^2\right)^{10}}\,dx=\frac{x^{18}}{18a\left(a+bx^2\right)^9}$$

Mathematica [B] time = 0.02, size = 101, normalized size = 5.32

$$-\frac{a^8+9a^7bx^2+36a^6b^2x^4+84a^5b^3x^6+126a^4b^4x^8+126a^3b^5x^{10}+84a^2b^6x^{12}+36ab^7x^{14}+9b^8x^{16}}{18b^9\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

[In] Integrate[x^17/(a + b*x^2)^10,x]

[Out] -1/18*(a^8 + 9*a^7*b*x^2 + 36*a^6*b^2*x^4 + 84*a^5*b^3*x^6 + 126*a^4*b^4*x^
8 + 126*a^3*b^5*x^10 + 84*a^2*b^6*x^12 + 36*a*b^7*x^14 + 9*b^8*x^16)/(b^9*(
a + b*x^2)^9)

fricas [B] time = 0.89, size = 190, normalized size = 10.00

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2}{18\left(b^{18}x^{18}+9\,ab^{17}x^{16}+36\,a^2b^{16}x^{14}+84\,a^3b^{15}x^{12}+126\,a^4b^{14}x^{10}+126\,a^5b^{13}x^8+84\,a^6b^{12}x^6+36\,a^7b^{11}x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^17/(b*x^2+a)^10,x, algorithm="fricas")

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/(b^18*
x^18 + 9*a*b^17*x^16 + 36*a^2*b^16*x^14 + 84*a^3*b^15*x^12 + 126*a^4*b^14*x
^10 + 126*a^5*b^13*x^8 + 84*a^6*b^12*x^6 + 36*a^7*b^11*x^4 + 9*a^8*b^10*x^2
 + a^9*b^9)
```

giac [B] time = 0.64, size = 99, normalized size = 5.21

$$-\frac{9\,b^8x^{16} + 36\,ab^7x^{14} + 84\,a^2b^6x^{12} + 126\,a^3b^5x^{10} + 126\,a^4b^4x^8 + 84\,a^5b^3x^6 + 36\,a^6b^2x^4 + 9\,a^7bx^2 + a^8}{18\,(bx^2 + a)^9\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/((b*x^
2 + a)^9*b^9)
```

maple [B] time = 0.01, size = 150, normalized size = 7.89

$$-\frac{a^8}{18\,(b\,x^2+a)^9\,b^9}+\frac{a^7}{2\,(b\,x^2+a)^8\,b^9}-\frac{2a^6}{(b\,x^2+a)^7\,b^9}+\frac{14a^5}{3\,(b\,x^2+a)^6\,b^9}-\frac{7a^4}{(b\,x^2+a)^5\,b^9}+\frac{7a^3}{(b\,x^2+a)^4\,b^9}-\frac{14a^2}{3\,(b\,x^2+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(b*x^2+a)^10,x)
```

```
[Out] 1/2*a^7/b^9/(b*x^2+a)^8-1/2/b^9/(b*x^2+a)-2*a^6/b^9/(b*x^2+a)^7-7*a^4/b^9/(
b*x^2+a)^5+7*a^3/b^9/(b*x^2+a)^4-14/3*a^2/b^9/(b*x^2+a)^3+14/3*a^5/b^9/(b*x
^2+a)^6-1/18*a^8/b^9/(b*x^2+a)^9+2*a/b^9/(b*x^2+a)^2
```

maxima [B] time = 1.50, size = 190, normalized size = 10.00

$$-\frac{9\,b^8x^{16} + 36\,ab^7x^{14} + 84\,a^2b^6x^{12} + 126\,a^3b^5x^{10} + 126\,a^4b^4x^8 + 84\,a^5b^3x^6 + 36\,a^6b^2x^4 + 9\,a^7bx^2 +}{18\,(b^{18}x^{18} + 9\,ab^{17}x^{16} + 36\,a^2b^{16}x^{14} + 84\,a^3b^{15}x^{12} + 126\,a^4b^{14}x^{10} + 126\,a^5b^{13}x^8 + 84\,a^6b^{12}x^6 + 36\,a^7b^{11}x^4 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/(b^18*
x^18 + 9*a*b^17*x^16 + 36*a^2*b^16*x^14 + 84*a^3*b^15*x^12 + 126*a^4*b^14*x
^10 + 126*a^5*b^13*x^8 + 84*a^6*b^12*x^6 + 36*a^7*b^11*x^4 + 9*a^8*b^10*x^2
 + a^9*b^9)
```

mupad [B] time = 0.12, size = 192, normalized size = 10.11

$$-\frac{a^8 + 9\,a^7\,b\,x^2 + 36\,a^6\,b^2\,x^4 + 84\,a^5\,b^3\,x^6 + 126\,a^4\,b^4\,x^8 + 126\,a^3\,b^5\,x^{10} + 84\,a^2\,b^6\,x^{12} + 36\,a}{18\,a^9\,b^9 + 162\,a^8\,b^{10}\,x^2 + 648\,a^7\,b^{11}\,x^4 + 1512\,a^6\,b^{12}\,x^6 + 2268\,a^5\,b^{13}\,x^8 + 2268\,a^4\,b^{14}\,x^{10} + 1512\,a^3\,b^{15}\,x^{12} + 64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(a + b*x^2)^10,x)
```

```
[Out] -(a^8 + 9*b^8*x^16 + 9*a^7*b*x^2 + 36*a*b^7*x^14 + 36*a^6*b^2*x^4 + 84*a^5*
b^3*x^6 + 126*a^4*b^4*x^8 + 126*a^3*b^5*x^10 + 84*a^2*b^6*x^12)/(18*a^9*b^9
 + 18*b^18*x^18 + 162*a*b^17*x^16 + 162*a^8*b^10*x^2 + 648*a^7*b^11*x^4 + 1
512*a^6*b^12*x^6 + 2268*a^5*b^13*x^8 + 2268*a^4*b^14*x^10 + 1512*a^3*b^15*x
^12 + 648*a^2*b^16*x^14)
```

sympy [B] time = 1.70, size = 202, normalized size = 10.63

$$\frac{-a^8 - 9a^7bx^2 - 36a^6b^2x^4 - 84a^5b^3x^6 - 126a^4b^4x^8 - 126a^3b^5x^{10} - 84a^2b^6x^{12} - 36ab^7x^{14} -}{18a^9b^9 + 162a^8b^{10}x^2 + 648a^7b^{11}x^4 + 1512a^6b^{12}x^6 + 2268a^5b^{13}x^8 + 2268a^4b^{14}x^{10} + 1512a^3b^{15}x^{12} + 648a^2b^{16}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(b*x**2+a)**10,x)
```

```
[Out] (-a**8 - 9*a**7*b*x**2 - 36*a**6*b**2*x**4 - 84*a**5*b**3*x**6 - 126*a**4*b
**4*x**8 - 126*a**3*b**5*x**10 - 84*a**2*b**6*x**12 - 36*a*b**7*x**14 - 9*b
**8*x**16)/(18*a**9*b**9 + 162*a**8*b**10*x**2 + 648*a**7*b**11*x**4 + 1512
*a**6*b**12*x**6 + 2268*a**5*b**13*x**8 + 2268*a**4*b**14*x**10 + 1512*a**3
*b**15*x**12 + 648*a**2*b**16*x**14 + 162*a*b**17*x**16 + 18*b**18*x**18)
```

3.197 $\int \frac{x^{15}}{\left(a+bx^2\right)^{10}} \, dx$

Optimal. Leaf size=39

$$\frac{x^{16}}{144a^2\left(a+bx^2\right)^8}+\frac{x^{16}}{18a\left(a+bx^2\right)^9}$$

```
[Out] 1/18*x^16/a/(b*x^2+a)^9+1/144*x^16/a^2/(b*x^2+a)^8
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{x^{16}}{144a^2\left(a+bx^2\right)^8}+\frac{x^{16}}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a + b*x^2)^10,x]
```

```
[Out] x^16/(18*a*(a + b*x^2)^9) + x^16/(144*a^2*(a + b*x^2)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{15}}{\left(a+bx^2\right)^{10}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^7}{(a+bx)^{10}} \, dx,x,x^2\right)\\
&= \frac{x^{16}}{18a\left(a+bx^2\right)^9}+\frac{\text{Subst}\left(\int \frac{x^7}{(a+bx)^9} \, dx,x,x^2\right)}{18a}\\
&= \frac{x^{16}}{18a\left(a+bx^2\right)^9}+\frac{x^{16}}{144a^2\left(a+bx^2\right)^8}
\end{aligned}$$

Mathematica [B] time = 0.02, size = 90, normalized size = 2.31

$$-\frac{a^7+9a^6bx^2+36a^5b^2x^4+84a^4b^3x^6+126a^3b^4x^8+126a^2b^5x^{10}+84ab^6x^{12}+36b^7x^{14}}{144b^8\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a + b*x^2)^10,x]
```

```
[Out] -1/144*(a^7 + 9*a^6*b*x^2 + 36*a^5*b^2*x^4 + 84*a^4*b^3*x^6 + 126*a^3*b^4*x
^8 + 126*a^2*b^5*x^10 + 84*a*b^6*x^12 + 36*b^7*x^14)/(b^8*(a + b*x^2)^9)
```

fricas [B] time = 0.89, size = 179, normalized size = 4.59

$$-\frac{36\,b^7x^{14}+84\,ab^6x^{12}+126\,a^2b^5x^{10}+126\,a^3b^4x^8+84\,a^4b^3x^6+36\,a^5b^2x^4+9\,a^6bx^2+a^7}{144\left(b^{17}x^{18}+9\,ab^{16}x^{16}+36\,a^2b^{15}x^{14}+84\,a^3b^{14}x^{12}+126\,a^4b^{13}x^{10}+126\,a^5b^{12}x^8+84\,a^6b^{11}x^6+36\,a^7b^{10}x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 +
84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/(b^17*x^18 + 9*a*b^16*
x^16 + 36*a^2*b^15*x^14 + 84*a^3*b^14*x^12 + 126*a^4*b^13*x^10 + 126*a^5*b^
12*x^8 + 84*a^6*b^11*x^6 + 36*a^7*b^10*x^4 + 9*a^8*b^9*x^2 + a^9*b^8)
```

giac [B] time = 0.63, size = 88, normalized size = 2.26

$$-\frac{36\,b^7x^{14}+84\,ab^6x^{12}+126\,a^2b^5x^{10}+126\,a^3b^4x^8+84\,a^4b^3x^6+36\,a^5b^2x^4+9\,a^6bx^2+a^7}{144\left(bx^2+a\right)^9b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 +
84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/((b*x^2 + a)^9*b^8)
```

maple [B] time = 0.01, size = 133, normalized size = 3.41

$$\frac{a^7}{18\left(b\,x^2+a\right)^9b^8}-\frac{7a^6}{16\left(b\,x^2+a\right)^8b^8}+\frac{3a^5}{2\left(b\,x^2+a\right)^7b^8}-\frac{35a^4}{12\left(b\,x^2+a\right)^6b^8}+\frac{7a^3}{2\left(b\,x^2+a\right)^5b^8}-\frac{21a^2}{8\left(b\,x^2+a\right)^4b^8}+\frac{}{6\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^2+a)^10,x)
```

```
[Out] -7/16*a^6/b^8/(b*x^2+a)^8+3/2*a^5/b^8/(b*x^2+a)^7+7/2*a^3/b^8/(b*x^2+a)^5-2
1/8*a^2/b^8/(b*x^2+a)^4+7/6*a/b^8/(b*x^2+a)^3+1/18*a^7/b^8/(b*x^2+a)^9-35/1
2*a^4/b^8/(b*x^2+a)^6-1/4/b^8/(b*x^2+a)^2
```

maxima [B] time = 1.47, size = 179, normalized size = 4.59

$$-\frac{36\,b^7x^{14}+84\,ab^6x^{12}+126\,a^2b^5x^{10}+126\,a^3b^4x^8+84\,a^4b^3x^6+36\,a^5b^2x^4+9\,a^6bx^2+a^7}{144\left(b^{17}x^{18}+9\,ab^{16}x^{16}+36\,a^2b^{15}x^{14}+84\,a^3b^{14}x^{12}+126\,a^4b^{13}x^{10}+126\,a^5b^{12}x^8+84\,a^6b^{11}x^6+36\,a^7b^{10}x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 +
84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/(b^17*x^18 + 9*a*b^16*
x^16 + 36*a^2*b^15*x^14 + 84*a^3*b^14*x^12 + 126*a^4*b^13*x^10 + 126*a^5*b^
12*x^8 + 84*a^6*b^11*x^6 + 36*a^7*b^10*x^4 + 9*a^8*b^9*x^2 + a^9*b^8)
```

mupad [B] time = 4.89, size = 181, normalized size = 4.64

$$-\frac{a^7+9\,a^6\,b\,x^2+36\,a^5\,b^2\,x^4+84\,a^4\,b^3\,x^6+126\,a^3\,b^4\,x^8+126\,a^2\,b^5\,x^{10}+84\,a\,b^6}{144\,a^9\,b^8+1296\,a^8\,b^9\,x^2+5184\,a^7\,b^{10}\,x^4+12096\,a^6\,b^{11}\,x^6+18144\,a^5\,b^{12}\,x^8+18144\,a^4\,b^{13}\,x^{10}+12096\,a^3\,b^{14}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(a + b*x^2)^10,x)
```

```
[Out] -(a^7 + 36*b^7*x^14 + 9*a^6*b*x^2 + 84*a*b^6*x^12 + 36*a^5*b^2*x^4 + 84*a^4
*b^3*x^6 + 126*a^3*b^4*x^8 + 126*a^2*b^5*x^10)/(144*a^9*b^8 + 144*b^17*x^18
 + 1296*a*b^16*x^16 + 1296*a^8*b^9*x^2 + 5184*a^7*b^10*x^4 + 12096*a^6*b^11
*x^6 + 18144*a^5*b^12*x^8 + 18144*a^4*b^13*x^10 + 12096*a^3*b^14*x^12 + 518
4*a^2*b^15*x^14)
```

sympy [B] time = 1.42, size = 190, normalized size = 4.87

$$\frac{-a^7-9a^6bx^2-36a^5b^2x^4-84a^4b^3x^6-126a^3b^4x^8-126a^2b^5x^{10}-84ab^6x^{12}-36b}{144a^9b^8+1296a^8b^9x^2+5184a^7b^{10}x^4+12096a^6b^{11}x^6+18144a^5b^{12}x^8+18144a^4b^{13}x^{10}+12096a^3b^{14}x^{12}+5184}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**2+a)**10,x)
```

```
[Out] (-a**7 - 9*a**6*b*x**2 - 36*a**5*b**2*x**4 - 84*a**4*b**3*x**6 - 126*a**3*b
**4*x**8 - 126*a**2*b**5*x**10 - 84*a*b**6*x**12 - 36*b**7*x**14)/(144*a**9
*b**8 + 1296*a**8*b**9*x**2 + 5184*a**7*b**10*x**4 + 12096*a**6*b**11*x**6
+ 18144*a**5*b**12*x**8 + 18144*a**4*b**13*x**10 + 12096*a**3*b**14*x**12 +
 5184*a**2*b**15*x**14 + 1296*a*b**16*x**16 + 144*b**17*x**18)
```

3.198 $\int \frac{x^{13}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=58

$$\frac{x^{14}}{504a^3\left(a+bx^2\right)^7}+\frac{x^{14}}{72a^2\left(a+bx^2\right)^8}+\frac{x^{14}}{18a\left(a+bx^2\right)^9}$$

```
[Out] 1/18*x^14/a/(b*x^2+a)^9+1/72*x^14/a^2/(b*x^2+a)^8+1/504*x^14/a^3/(b*x^2+a)^
7
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{x^{14}}{504a^3\left(a+bx^2\right)^7}+\frac{x^{14}}{72a^2\left(a+bx^2\right)^8}+\frac{x^{14}}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^2)^10,x]
```

```
[Out] x^14/(18*a*(a + b*x^2)^9) + x^14/(72*a^2*(a + b*x^2)^8) + x^14/(504*a^3*(a
+ b*x^2)^7)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{(a+bx)^{10}}\,dx, x, x^2\right)$$
$$= \frac{x^{14}}{18a\left(a+bx^2\right)^9} + \frac{\text{Subst}\left(\int \frac{x^6}{(a+bx)^9}\,dx, x, x^2\right)}{9a}$$
$$= \frac{x^{14}}{18a\left(a+bx^2\right)^9} + \frac{x^{14}}{72a^2\left(a+bx^2\right)^8} + \frac{\text{Subst}\left(\int \frac{x^6}{(a+bx)^8}\,dx, x, x^2\right)}{72a^2}$$
$$= \frac{x^{14}}{18a\left(a+bx^2\right)^9} + \frac{x^{14}}{72a^2\left(a+bx^2\right)^8} + \frac{x^{14}}{504a^3\left(a+bx^2\right)^7}$$

Mathematica [A] time = 0.02, size = 79, normalized size = 1.36

$$-\frac{a^6+9a^5bx^2+36a^4b^2x^4+84a^3b^3x^6+126a^2b^4x^8+126ab^5x^{10}+84b^6x^{12}}{504b^7\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^2)^10,x]
```

```
[Out] -1/504*(a^6 + 9*a^5*b*x^2 + 36*a^4*b^2*x^4 + 84*a^3*b^3*x^6 + 126*a^2*b^4*x
^8 + 126*a*b^5*x^10 + 84*b^6*x^12)/(b^7*(a + b*x^2)^9)
```

fricas [B] time = 0.86, size = 168, normalized size = 2.90

$$-\frac{84\,b^6x^{12}+126\,ab^5x^{10}+126\,a^2b^4x^8+84\,a^3b^3x^6+36\,a^4b^2x^4+9\,a^5bx^2+a^6}{504\left(b^{16}x^{18}+9\,ab^{15}x^{16}+36\,a^2b^{14}x^{14}+84\,a^3b^{13}x^{12}+126\,a^4b^{12}x^{10}+126\,a^5b^{11}x^8+84\,a^6b^{10}x^6+36\,a^7b^9x^4+9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 3
6*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/(b^16*x^18 + 9*a*b^15*x^16 + 36*a^2*b^14
*x^14 + 84*a^3*b^13*x^12 + 126*a^4*b^12*x^10 + 126*a^5*b^11*x^8 + 84*a^6*b^
10*x^6 + 36*a^7*b^9*x^4 + 9*a^8*b^8*x^2 + a^9*b^7)
```

giac [A] time = 0.63, size = 77, normalized size = 1.33

$$-\frac{84\,b^6x^{12}+126\,ab^5x^{10}+126\,a^2b^4x^8+84\,a^3b^3x^6+36\,a^4b^2x^4+9\,a^5bx^2+a^6}{504\left(bx^2+a\right)^9b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 3
6*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/((b*x^2 + a)^9*b^7)
```

maple [B] time = 0.01, size = 116, normalized size = 2.00

$$-\frac{a^6}{18\left(b\,x^2+a\right)^9b^7}+\frac{3a^5}{8\left(b\,x^2+a\right)^8b^7}-\frac{15a^4}{14\left(b\,x^2+a\right)^7b^7}+\frac{5a^3}{3\left(b\,x^2+a\right)^6b^7}-\frac{3a^2}{2\left(b\,x^2+a\right)^5b^7}+\frac{3a}{4\left(b\,x^2+a\right)^4b^7}-\frac{}{6\left(b\,x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2+a)^10,x)
```

```
[Out] 3/8*a^5/b^7/(b*x^2+a)^8-15/14*a^4/b^7/(b*x^2+a)^7-1/6/b^7/(b*x^2+a)^3-3/2*a
^2/b^7/(b*x^2+a)^5+3/4*a/b^7/(b*x^2+a)^4-1/18*a^6/b^7/(b*x^2+a)^9+5/3*a^3/b
^7/(b*x^2+a)^6
```

maxima [B] time = 1.46, size = 168, normalized size = 2.90

$$-\frac{84\,b^{6}x^{12}+126\,ab^{5}x^{10}+126\,a^{2}b^{4}x^{8}+84\,a^{3}b^{3}x^{6}+36\,a^{4}b^{2}x^{4}+9\,a^{5}bx^{2}+a^{6}}{504\left(b^{16}x^{18}+9\,ab^{15}x^{16}+36\,a^{2}b^{14}x^{14}+84\,a^{3}b^{13}x^{12}+126\,a^{4}b^{12}x^{10}+126\,a^{5}b^{11}x^{8}+84\,a^{6}b^{10}x^{6}+36\,a^{7}b^{9}x^{4}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 3
6*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/(b^16*x^18 + 9*a*b^15*x^16 + 36*a^2*b^14
*x^14 + 84*a^3*b^13*x^12 + 126*a^4*b^12*x^10 + 126*a^5*b^11*x^8 + 84*a^6*b^
10*x^6 + 36*a^7*b^9*x^4 + 9*a^8*b^8*x^2 + a^9*b^7)
```

mupad [B] time = 0.10, size = 170, normalized size = 2.93

$$-\frac{a^6+9\,a^5\,b\,x^2+36\,a^4\,b^2\,x^4+84\,a^3\,b^3\,x^6+126\,a^2\,b^4\,x^8+126\,a\,b^5\,x^{1\cdots}}{504\,a^9\,b^7+4536\,a^8\,b^8\,x^2+18144\,a^7\,b^9\,x^4+42336\,a^6\,b^{10}\,x^6+63504\,a^5\,b^{11}\,x^8+63504\,a^4\,b^{12}\,x^{10}+42336\,a^3\,b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a + b*x^2)^10,x)
```

```
[Out] -(a^6 + 84*b^6*x^12 + 9*a^5*b*x^2 + 126*a*b^5*x^10 + 36*a^4*b^2*x^4 + 84*a^
3*b^3*x^6 + 126*a^2*b^4*x^8)/(504*a^9*b^7 + 504*b^16*x^18 + 4536*a*b^15*x^1
6 + 4536*a^8*b^8*x^2 + 18144*a^7*b^9*x^4 + 42336*a^6*b^10*x^6 + 63504*a^5*b
^11*x^8 + 63504*a^4*b^12*x^10 + 42336*a^3*b^13*x^12 + 18144*a^2*b^14*x^14)
```

sympy [B] time = 1.33, size = 178, normalized size = 3.07

$$\frac{-a^6-9a^5bx^2-36a^4b^2x^4-84a^3b^3x^6-126a^2b^4x^8-126ab^5x^{10}-84b^6x\cdots}{504a^9b^7+4536a^8b^8x^2+18144a^7b^9x^4+42336a^6b^{10}x^6+63504a^5b^{11}x^8+63504a^4b^{12}x^{10}+42336a^3b^{13}x^{12}+1\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**2+a)**10,x)
```

```
[Out] (-a**6 - 9*a**5*b*x**2 - 36*a**4*b**2*x**4 - 84*a**3*b**3*x**6 - 126*a**2*b
**4*x**8 - 126*a*b**5*x**10 - 84*b**6*x**12)/(504*a**9*b**7 + 4536*a**8*b**
8*x**2 + 18144*a**7*b**9*x**4 + 42336*a**6*b**10*x**6 + 63504*a**5*b**11*x*
*8 + 63504*a**4*b**12*x**10 + 42336*a**3*b**13*x**12 + 18144*a**2*b**14*x**
14 + 4536*a*b**15*x**16 + 504*b**16*x**18)
```

3.199 $\int \frac{x^{11}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=77

$$\frac{x^{12}}{1008a^4\left(a+bx^2\right)^6}+\frac{x^{12}}{168a^3\left(a+bx^2\right)^7}+\frac{x^{12}}{48a^2\left(a+bx^2\right)^8}+\frac{x^{12}}{18a\left(a+bx^2\right)^9}$$

```
[Out] 1/18*x^12/a/(b*x^2+a)^9+1/48*x^12/a^2/(b*x^2+a)^8+1/168*x^12/a^3/(b*x^2+a)^
7+1/1008*x^12/a^4/(b*x^2+a)^6
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{x^{12}}{1008a^4\left(a+bx^2\right)^6}+\frac{x^{12}}{168a^3\left(a+bx^2\right)^7}+\frac{x^{12}}{48a^2\left(a+bx^2\right)^8}+\frac{x^{12}}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2)^10,x]
```

```
[Out] x^12/(18*a*(a + b*x^2)^9) + x^12/(48*a^2*(a + b*x^2)^8) + x^12/(168*a^3*(a
+ b*x^2)^7) + x^12/(1008*a^4*(a + b*x^2)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^5}{(a+bx)^{10}}\,dx, x, x^2\right)$$

$$= \frac{x^{12}}{18a\left(a+bx^2\right)^9} + \frac{\mathrm{Subst}\left(\int \frac{x^5}{(a+bx)^9}\,dx, x, x^2\right)}{6a}$$

$$= \frac{x^{12}}{18a\left(a+bx^2\right)^9} + \frac{x^{12}}{48a^2\left(a+bx^2\right)^8} + \frac{\mathrm{Subst}\left(\int \frac{x^5}{(a+bx)^8}\,dx, x, x^2\right)}{24a^2}$$

$$= \frac{x^{12}}{18a\left(a+bx^2\right)^9} + \frac{x^{12}}{48a^2\left(a+bx^2\right)^8} + \frac{x^{12}}{168a^3\left(a+bx^2\right)^7} + \frac{\mathrm{Subst}\left(\int \frac{x^5}{(a+bx)^7}\,dx, x, x^2\right)}{168a^3}$$

$$= \frac{x^{12}}{18a\left(a+bx^2\right)^9} + \frac{x^{12}}{48a^2\left(a+bx^2\right)^8} + \frac{x^{12}}{168a^3\left(a+bx^2\right)^7} + \frac{x^{12}}{1008a^4\left(a+bx^2\right)^6}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.88

$$-\frac{a^5+9a^4bx^2+36a^3b^2x^4+84a^2b^3x^6+126ab^4x^8+126b^5x^{10}}{1008b^6\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2)^10,x]
```

```
[Out] -1/1008*(a^5 + 9*a^4*b*x^2 + 36*a^3*b^2*x^4 + 84*a^2*b^3*x^6 + 126*a*b^4*x^
8 + 126*b^5*x^10)/(b^6*(a + b*x^2)^9)
```

fricas [B] time = 0.56, size = 157, normalized size = 2.04

$$-\frac{126\,b^5x^{10}+126\,ab^4x^8+84\,a^2b^3x^6+36\,a^3b^2x^4+9\,a^4bx^2+a^5}{1008\left(b^{15}x^{18}+9\,ab^{14}x^{16}+36\,a^2b^{13}x^{14}+84\,a^3b^{12}x^{12}+126\,a^4b^{11}x^{10}+126\,a^5b^{10}x^8+84\,a^6b^9x^6+36\,a^7b^8x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 + 9
*a^4*b*x^2 + a^5)/(b^15*x^18 + 9*a*b^14*x^16 + 36*a^2*b^13*x^14 + 84*a^3*b^
12*x^12 + 126*a^4*b^11*x^10 + 126*a^5*b^10*x^8 + 84*a^6*b^9*x^6 + 36*a^7*b^
8*x^4 + 9*a^8*b^7*x^2 + a^9*b^6)
```

giac [A] time = 0.63, size = 66, normalized size = 0.86

$$-\frac{126\,b^5x^{10}+126\,ab^4x^8+84\,a^2b^3x^6+36\,a^3b^2x^4+9\,a^4bx^2+a^5}{1008\left(bx^2+a\right)^9b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 + 9
*a^4*b*x^2 + a^5)/((b*x^2 + a)^9*b^6)
```

maple [A] time = 0.01, size = 99, normalized size = 1.29

$$\frac{a^5}{18\left(b\,x^2+a\right)^9b^6}-\frac{5a^4}{16\left(b\,x^2+a\right)^8b^6}+\frac{5a^3}{7\left(b\,x^2+a\right)^7b^6}-\frac{5a^2}{6\left(b\,x^2+a\right)^6b^6}+\frac{a}{2\left(b\,x^2+a\right)^5b^6}-\frac{1}{8\left(b\,x^2+a\right)^4b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a)^10,x)
```

```
[Out] -5/16*a^4/b^6/(b*x^2+a)^8+5/7*a^3/b^6/(b*x^2+a)^7+1/2*a/b^6/(b*x^2+a)^5-1/8
/b^6/(b*x^2+a)^4-5/6*a^2/b^6/(b*x^2+a)^6+1/18*a^5/b^6/(b*x^2+a)^9
```

maxima [B] time = 1.43, size = 157, normalized size = 2.04

$$-\frac{126\,b^5x^{10}+126\,ab^4x^8+84\,a^2b^3x^6+36\,a^3b^2x^4+9\,a^4bx^2+a^5}{1008\,\left(b^{15}x^{18}+9\,ab^{14}x^{16}+36\,a^2b^{13}x^{14}+84\,a^3b^{12}x^{12}+126\,a^4b^{11}x^{10}+126\,a^5b^{10}x^8+84\,a^6b^9x^6+36\,a^7b^8x^4+9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 + 9
*a^4*b*x^2 + a^5)/(b^15*x^18 + 9*a*b^14*x^16 + 36*a^2*b^13*x^14 + 84*a^3*b^
12*x^12 + 126*a^4*b^11*x^10 + 126*a^5*b^10*x^8 + 84*a^6*b^9*x^6 + 36*a^7*b^
8*x^4 + 9*a^8*b^7*x^2 + a^9*b^6)
```

mupad [B] time = 0.10, size = 159, normalized size = 2.06

$$-\frac{a^5+9\,a^4\,b\,x^2+36\,a^3\,b^2\,x^4+84\,a^2\,b^3\,x^6+126\,a\,b^4\,x^8+126\,b}{1008\,a^9\,b^6+9072\,a^8\,b^7\,x^2+36288\,a^7\,b^8\,x^4+84672\,a^6\,b^9\,x^6+127008\,a^5\,b^{10}\,x^8+127008\,a^4\,b^{11}\,x^{10}+84672\,a^3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2)^10,x)
```

```
[Out] -(a^5 + 126*b^5*x^10 + 9*a^4*b*x^2 + 126*a*b^4*x^8 + 36*a^3*b^2*x^4 + 84*a^
2*b^3*x^6)/(1008*a^9*b^6 + 1008*b^15*x^18 + 9072*a*b^14*x^16 + 9072*a^8*b^7
*x^2 + 36288*a^7*b^8*x^4 + 84672*a^6*b^9*x^6 + 127008*a^5*b^10*x^8 + 127008
*a^4*b^11*x^10 + 84672*a^3*b^12*x^12 + 36288*a^2*b^13*x^14)
```

sympy [B] time = 1.18, size = 167, normalized size = 2.17

$$\frac{-a^5-9a^4bx^2-36a^3b^2x^4-84a^2b^3x^6-126ab^4x^8-126b^5x^{10}}{1008a^9b^6+9072a^8b^7x^2+36288a^7b^8x^4+84672a^6b^9x^6+127008a^5b^{10}x^8+127008a^4b^{11}x^{10}+84672a^3b^{12}x^{12}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a)**10,x)
```

```
[Out] (-a**5 - 9*a**4*b*x**2 - 36*a**3*b**2*x**4 - 84*a**2*b**3*x**6 - 126*a*b**4
*x**8 - 126*b**5*x**10)/(1008*a**9*b**6 + 9072*a**8*b**7*x**2 + 36288*a**7*
b**8*x**4 + 84672*a**6*b**9*x**6 + 127008*a**5*b**10*x**8 + 127008*a**4*b**
11*x**10 + 84672*a**3*b**12*x**12 + 36288*a**2*b**13*x**14 + 9072*a*b**14*x
**16 + 1008*b**15*x**18)
```

3.200 $\int \frac{x^9}{(a+bx^2)^{10}} \, dx$

Optimal. Leaf size=91

$$-\frac{a^4}{18b^5\left(a+bx^2\right)^9}+\frac{a^3}{4b^5\left(a+bx^2\right)^8}-\frac{3a^2}{7b^5\left(a+bx^2\right)^7}+\frac{a}{3b^5\left(a+bx^2\right)^6}-\frac{1}{10b^5\left(a+bx^2\right)^5}$$

```
[Out] -1/18*a^4/b^5/(b*x^2+a)^9+1/4*a^3/b^5/(b*x^2+a)^8-3/7*a^2/b^5/(b*x^2+a)^7+1
/3*a/b^5/(b*x^2+a)^6-1/10/b^5/(b*x^2+a)^5
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^4}{18b^5\left(a+bx^2\right)^9}+\frac{a^3}{4b^5\left(a+bx^2\right)^8}-\frac{3a^2}{7b^5\left(a+bx^2\right)^7}+\frac{a}{3b^5\left(a+bx^2\right)^6}-\frac{1}{10b^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^10,x]
```

```
[Out] -a^4/(18*b^5*(a + b*x^2)^9) + a^3/(4*b^5*(a + b*x^2)^8) - (3*a^2)/(7*b^5*(a
 + b*x^2)^7) + a/(3*b^5*(a + b*x^2)^6) - 1/(10*b^5*(a + b*x^2)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{\left(a+bx^2\right)^{10}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx)^{10}} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4}{b^4(a+bx)^{10}}-\frac{4a^3}{b^4(a+bx)^9}+\frac{6a^2}{b^4(a+bx)^8}-\frac{4a}{b^4(a+bx)^7}+\frac{1}{b^4(a+bx)^6}\right) dx,\right.\\
&= -\frac{a^4}{18b^5\left(a+bx^2\right)^9}+\frac{a^3}{4b^5\left(a+bx^2\right)^8}-\frac{3a^2}{7b^5\left(a+bx^2\right)^7}+\frac{a}{3b^5\left(a+bx^2\right)^6}-\frac{1}{10b^5\left(a+bx^2\right)^5}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.63

$$-\frac{a^4+9a^3bx^2+36a^2b^2x^4+84ab^3x^6+126b^4x^8}{1260b^5\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^10,x]
```

```
[Out] -1/1260*(a^4 + 9*a^3*b*x^2 + 36*a^2*b^2*x^4 + 84*a*b^3*x^6 + 126*b^4*x^8)/(
b^5*(a + b*x^2)^9)
```

fricas [A] time = 1.00, size = 146, normalized size = 1.60

$$-\frac{126\,b^{4}x^{8}+84\,ab^{3}x^{6}+36\,a^{2}b^{2}x^{4}+9\,a^{3}bx^{2}+a^{4}}{1260\,\left(b^{14}x^{18}+9\,ab^{13}x^{16}+36\,a^{2}b^{12}x^{14}+84\,a^{3}b^{11}x^{12}+126\,a^{4}b^{10}x^{10}+126\,a^{5}b^{9}x^{8}+84\,a^{6}b^{8}x^{6}+36\,a^{7}b^{7}x^{4}+9\,a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/(
b^14*x^18 + 9*a*b^13*x^16 + 36*a^2*b^12*x^14 + 84*a^3*b^11*x^12 + 126*a^4*b
^10*x^10 + 126*a^5*b^9*x^8 + 84*a^6*b^8*x^6 + 36*a^7*b^7*x^4 + 9*a^8*b^6*x^
2 + a^9*b^5)
```

giac [A] time = 0.64, size = 55, normalized size = 0.60

$$-\frac{126\,b^{4}x^{8}+84\,ab^{3}x^{6}+36\,a^{2}b^{2}x^{4}+9\,a^{3}bx^{2}+a^{4}}{1260\,\left(bx^{2}+a\right)^{9}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/(
(b*x^2 + a)^9*b^5)
```

maple [A] time = 0.01, size = 82, normalized size = 0.90

$$-\frac{a^{4}}{18\left(b\,x^{2}+a\right)^{9}b^{5}}+\frac{a^{3}}{4\left(b\,x^{2}+a\right)^{8}b^{5}}-\frac{3a^{2}}{7\left(b\,x^{2}+a\right)^{7}b^{5}}+\frac{a}{3\left(b\,x^{2}+a\right)^{6}b^{5}}-\frac{1}{10\left(b\,x^{2}+a\right)^{5}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^10,x)
```

```
[Out] -1/18*a^4/b^5/(b*x^2+a)^9+1/4*a^3/b^5/(b*x^2+a)^8-3/7*a^2/b^5/(b*x^2+a)^7+1
/3*a/b^5/(b*x^2+a)^6-1/10/b^5/(b*x^2+a)^5
```

maxima [A] time = 1.44, size = 146, normalized size = 1.60

$$-\frac{126\,b^{4}x^{8}+84\,ab^{3}x^{6}+36\,a^{2}b^{2}x^{4}+9\,a^{3}bx^{2}+a^{4}}{1260\,\left(b^{14}x^{18}+9\,ab^{13}x^{16}+36\,a^{2}b^{12}x^{14}+84\,a^{3}b^{11}x^{12}+126\,a^{4}b^{10}x^{10}+126\,a^{5}b^{9}x^{8}+84\,a^{6}b^{8}x^{6}+36\,a^{7}b^{7}x^{4}+9\,a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/(
b^14*x^18 + 9*a*b^13*x^16 + 36*a^2*b^12*x^14 + 84*a^3*b^11*x^12 + 126*a^4*b
^10*x^10 + 126*a^5*b^9*x^8 + 84*a^6*b^8*x^6 + 36*a^7*b^7*x^4 + 9*a^8*b^6*x^
2 + a^9*b^5)
```

mupad [B] time = 4.82, size = 148, normalized size = 1.63

$$-\frac{a^{4}+9\,a^{3}\,b\,x^{2}+36\,a^{2}\,b^{2}\,x^{4}+84\,a\,b^{3}\,x^{6}+126\,b^{4}\,x^{8}}{1260\,a^{9}\,b^{5}+11340\,a^{8}\,b^{6}\,x^{2}+45360\,a^{7}\,b^{7}\,x^{4}+105840\,a^{6}\,b^{8}\,x^{6}+158760\,a^{5}\,b^{9}\,x^{8}+158760\,a^{4}\,b^{10}\,x^{10}+105840\,a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2)^10,x)
```

```
[Out] -(a^4 + 126*b^4*x^8 + 9*a^3*b*x^2 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4)/(1260*a^
9*b^5 + 1260*b^14*x^18 + 11340*a*b^13*x^16 + 11340*a^8*b^6*x^2 + 45360*a^7*
b^7*x^4 + 105840*a^6*b^8*x^6 + 158760*a^5*b^9*x^8 + 158760*a^4*b^10*x^10 +
105840*a^3*b^11*x^12 + 45360*a^2*b^12*x^14)
```

sympy **[A]** time = 1.08, size = 155, normalized size = 1.70

$$\frac{-a^4-9a^3bx^2-36a^2b^2x^4-84ab^3x^6-126b^4x^8}{1260a^9b^5+11340a^8b^6x^2+45360a^7b^7x^4+105840a^6b^8x^6+158760a^5b^9x^8+158760a^4b^{10}x^{10}+105840a^3b^{11}x^1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**10,x)
```

```
[Out] (-a**4 - 9*a**3*b*x**2 - 36*a**2*b**2*x**4 - 84*a*b**3*x**6 - 126*b**4*x**8
)/(1260*a**9*b**5 + 11340*a**8*b**6*x**2 + 45360*a**7*b**7*x**4 + 105840*a*
*6*b**8*x**6 + 158760*a**5*b**9*x**8 + 158760*a**4*b**10*x**10 + 105840*a**
3*b**11*x**12 + 45360*a**2*b**12*x**14 + 11340*a*b**13*x**16 + 1260*b**14*x
**18)
```

3.201 $\int \frac{x^7}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=72

$$\frac{a^3}{18b^4\left(a+bx^2\right)^9}-\frac{3a^2}{16b^4\left(a+bx^2\right)^8}+\frac{3a}{14b^4\left(a+bx^2\right)^7}-\frac{1}{12b^4\left(a+bx^2\right)^6}$$

```
[Out] 1/18*a^3/b^4/(b*x^2+a)^9-3/16*a^2/b^4/(b*x^2+a)^8+3/14*a/b^4/(b*x^2+a)^7-1/
12/b^4/(b*x^2+a)^6
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^3}{18b^4\left(a+bx^2\right)^9}-\frac{3a^2}{16b^4\left(a+bx^2\right)^8}+\frac{3a}{14b^4\left(a+bx^2\right)^7}-\frac{1}{12b^4\left(a+bx^2\right)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^10,x]
```

```
[Out] a^3/(18*b^4*(a + b*x^2)^9) - (3*a^2)/(16*b^4*(a + b*x^2)^8) + (3*a)/(14*b^4
*(a + b*x^2)^7) - 1/(12*b^4*(a + b*x^2)^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{(a+bx)^{10}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{10}}+\frac{3a^2}{b^3(a+bx)^9}-\frac{3a}{b^3(a+bx)^8}+\frac{1}{b^3(a+bx)^7}\right)dx,x,x^2\right)\\
&= \frac{a^3}{18b^4\left(a+bx^2\right)^9}-\frac{3a^2}{16b^4\left(a+bx^2\right)^8}+\frac{3a}{14b^4\left(a+bx^2\right)^7}-\frac{1}{12b^4\left(a+bx^2\right)^6}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.64

$$-\frac{a^3+9a^2bx^2+36ab^2x^4+84b^3x^6}{1008b^4\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^10,x]
```

```
[Out] -1/1008*(a^3 + 9*a^2*b*x^2 + 36*a*b^2*x^4 + 84*b^3*x^6)/(b^4*(a + b*x^2)^9)
```

fricas [B] time = 0.80, size = 135, normalized size = 1.88

$$-\frac{84\,b^{3}x^{6} + 36\,ab^{2}x^{4} + 9\,a^{2}bx^{2} + a^{3}}{1008\left(b^{13}x^{18} + 9\,ab^{12}x^{16} + 36\,a^{2}b^{11}x^{14} + 84\,a^{3}b^{10}x^{12} + 126\,a^{4}b^{9}x^{10} + 126\,a^{5}b^{8}x^{8} + 84\,a^{6}b^{7}x^{6} + 36\,a^{7}b^{6}x^{4} + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/(b^13*x^18 + 9*a*b^
12*x^16 + 36*a^2*b^11*x^14 + 84*a^3*b^10*x^12 + 126*a^4*b^9*x^10 + 126*a^5*
b^8*x^8 + 84*a^6*b^7*x^6 + 36*a^7*b^6*x^4 + 9*a^8*b^5*x^2 + a^9*b^4)
```

giac [A] time = 0.64, size = 44, normalized size = 0.61

$$-\frac{84\,b^{3}x^{6} + 36\,ab^{2}x^{4} + 9\,a^{2}bx^{2} + a^{3}}{1008\left(bx^{2} + a\right)^{9}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/((b*x^2 + a)^9*b^4)
```

maple [A] time = 0.01, size = 65, normalized size = 0.90

$$\frac{a^{3}}{18\left(b\,x^{2}+a\right)^{9}b^{4}} - \frac{3a^{2}}{16\left(b\,x^{2}+a\right)^{8}b^{4}} + \frac{3a}{14\left(b\,x^{2}+a\right)^{7}b^{4}} - \frac{1}{12\left(b\,x^{2}+a\right)^{6}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^10,x)
```

```
[Out] 1/18*a^3/b^4/(b*x^2+a)^9-3/16*a^2/b^4/(b*x^2+a)^8+3/14*a/b^4/(b*x^2+a)^7-1/
12/b^4/(b*x^2+a)^6
```

maxima [B] time = 1.45, size = 135, normalized size = 1.88

$$-\frac{84\,b^{3}x^{6} + 36\,ab^{2}x^{4} + 9\,a^{2}bx^{2} + a^{3}}{1008\left(b^{13}x^{18} + 9\,ab^{12}x^{16} + 36\,a^{2}b^{11}x^{14} + 84\,a^{3}b^{10}x^{12} + 126\,a^{4}b^{9}x^{10} + 126\,a^{5}b^{8}x^{8} + 84\,a^{6}b^{7}x^{6} + 36\,a^{7}b^{6}x^{4} + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/(b^13*x^18 + 9*a*b^
12*x^16 + 36*a^2*b^11*x^14 + 84*a^3*b^10*x^12 + 126*a^4*b^9*x^10 + 126*a^5*
b^8*x^8 + 84*a^6*b^7*x^6 + 36*a^7*b^6*x^4 + 9*a^8*b^5*x^2 + a^9*b^4)
```

mupad [B] time = 0.10, size = 136, normalized size = 1.89

$$-\frac{\frac{a^{3}}{1008\,b^{4}} + \frac{x^{6}}{12\,b} + \frac{a\,x^{4}}{28\,b^{2}} + \frac{a^{2}\,x^{2}}{112\,b^{3}}}{a^{9} + 9\,a^{8}\,b\,x^{2} + 36\,a^{7}\,b^{2}\,x^{4} + 84\,a^{6}\,b^{3}\,x^{6} + 126\,a^{5}\,b^{4}\,x^{8} + 126\,a^{4}\,b^{5}\,x^{10} + 84\,a^{3}\,b^{6}\,x^{12} + 36\,a^{2}\,b^{7}\,x^{14} + 9\,a\,b^{8}\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^10,x)
```

```
[Out] -(a^3/(1008*b^4) + x^6/(12*b) + (a*x^4)/(28*b^2) + (a^2*x^2)/(112*b^3))/(a^
9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6
 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

sympy [B] time = 1.01, size = 143, normalized size = 1.99

$$\frac{-a^3 - 9a^2bx^2 - 36ab^2x^4 - 84b^3x^6}{1008a^9b^4 + 9072a^8b^5x^2 + 36288a^7b^6x^4 + 84672a^6b^7x^6 + 127008a^5b^8x^8 + 127008a^4b^9x^{10} + 84672a^3b^{10}x^{12} + 362}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**10,x)
```

```
[Out] (-a**3 - 9*a**2*b*x**2 - 36*a*b**2*x**4 - 84*b**3*x**6)/(1008*a**9*b**4 + 9
072*a**8*b**5*x**2 + 36288*a**7*b**6*x**4 + 84672*a**6*b**7*x**6 + 127008*a
**5*b**8*x**8 + 127008*a**4*b**9*x**10 + 84672*a**3*b**10*x**12 + 36288*a**
2*b**11*x**14 + 9072*a*b**12*x**16 + 1008*b**13*x**18)
```

3.202 $\int \frac{x^5}{\left(a+bx^2\right)^{10}} \, dx$

Optimal. Leaf size=53

$$-\frac{a^2}{18b^3\left(a+bx^2\right)^9}+\frac{a}{8b^3\left(a+bx^2\right)^8}-\frac{1}{14b^3\left(a+bx^2\right)^7}$$

```
[Out] -1/18*a^2/b^3/(b*x^2+a)^9+1/8*a/b^3/(b*x^2+a)^8-1/14/b^3/(b*x^2+a)^7
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{18b^3\left(a+bx^2\right)^9}+\frac{a}{8b^3\left(a+bx^2\right)^8}-\frac{1}{14b^3\left(a+bx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^10,x]
```

```
[Out] -a^2/(18*b^3*(a + b*x^2)^9) + a/(8*b^3*(a + b*x^2)^8) - 1/(14*b^3*(a + b*x^
2)^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^{10}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{10}} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{10}}-\frac{2a}{b^2(a+bx)^9}+\frac{1}{b^2(a+bx)^8}\right) dx, x, x^2\right)\\
&= -\frac{a^2}{18b^3\left(a+bx^2\right)^9}+\frac{a}{8b^3\left(a+bx^2\right)^8}-\frac{1}{14b^3\left(a+bx^2\right)^7}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.66

$$-\frac{a^2+9abx^2+36b^2x^4}{504b^3\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^10,x]
```

```
[Out] -1/504*(a^2 + 9*a*b*x^2 + 36*b^2*x^4)/(b^3*(a + b*x^2)^9)
```

fricas [B] time = 0.87, size = 124, normalized size = 2.34

$$-\frac{36\,b^{2}x^{4}+9\,abx^{2}+a^{2}}{504\left(b^{12}x^{18}+9\,ab^{11}x^{16}+36\,a^{2}b^{10}x^{14}+84\,a^{3}b^{9}x^{12}+126\,a^{4}b^{8}x^{10}+126\,a^{5}b^{7}x^{8}+84\,a^{6}b^{6}x^{6}+36\,a^{7}b^{5}x^{4}+9\,a^{8}b^{4}x^{2}+a^{9}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/(b^12*x^18 + 9*a*b^11*x^16 + 36*a^2*b
^10*x^14 + 84*a^3*b^9*x^12 + 126*a^4*b^8*x^10 + 126*a^5*b^7*x^8 + 84*a^6*b^
6*x^6 + 36*a^7*b^5*x^4 + 9*a^8*b^4*x^2 + a^9*b^3)
```

giac [A] time = 0.64, size = 33, normalized size = 0.62

$$-\frac{36\,b^{2}x^{4}+9\,abx^{2}+a^{2}}{504\left(bx^{2}+a\right)^{9}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/((b*x^2 + a)^9*b^3)
```

maple [A] time = 0.01, size = 48, normalized size = 0.91

$$-\frac{a^{2}}{18\left(b\,x^{2}+a\right)^{9}b^{3}}+\frac{a}{8\left(b\,x^{2}+a\right)^{8}b^{3}}-\frac{1}{14\left(b\,x^{2}+a\right)^{7}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^10,x)
```

```
[Out] -1/18*a^2/b^3/(b*x^2+a)^9+1/8*a/b^3/(b*x^2+a)^8-1/14/b^3/(b*x^2+a)^7
```

maxima [B] time = 1.51, size = 124, normalized size = 2.34

$$-\frac{36\,b^{2}x^{4}+9\,abx^{2}+a^{2}}{504\left(b^{12}x^{18}+9\,ab^{11}x^{16}+36\,a^{2}b^{10}x^{14}+84\,a^{3}b^{9}x^{12}+126\,a^{4}b^{8}x^{10}+126\,a^{5}b^{7}x^{8}+84\,a^{6}b^{6}x^{6}+36\,a^{7}b^{5}x^{4}+9\,a^{8}b^{4}x^{2}+a^{9}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/(b^12*x^18 + 9*a*b^11*x^16 + 36*a^2*b
^10*x^14 + 84*a^3*b^9*x^12 + 126*a^4*b^8*x^10 + 126*a^5*b^7*x^8 + 84*a^6*b^
6*x^6 + 36*a^7*b^5*x^4 + 9*a^8*b^4*x^2 + a^9*b^3)
```

mupad [B] time = 4.83, size = 125, normalized size = 2.36

$$-\frac{\frac{a^{2}}{504\,b^{3}}+\frac{x^{4}}{14\,b}+\frac{a\,x^{2}}{56\,b^{2}}}{a^{9}+9\,a^{8}\,b\,x^{2}+36\,a^{7}\,b^{2}\,x^{4}+84\,a^{6}\,b^{3}\,x^{6}+126\,a^{5}\,b^{4}\,x^{8}+126\,a^{4}\,b^{5}\,x^{10}+84\,a^{3}\,b^{6}\,x^{12}+36\,a^{2}\,b^{7}\,x^{14}+9\,a\,b^{8}\,x^{16}+b^{9}\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^10,x)
```

```
[Out] -(a^2/(504*b^3) + x^4/(14*b) + (a*x^2)/(56*b^2))/(a^9 + b^9*x^18 + 9*a^8*b*
x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 12
6*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

sympy [B] time = 0.95, size = 131, normalized size = 2.47

$$\frac{-a^2 - 9abx^2 - 36b^2x^4}{504a^9b^3 + 4536a^8b^4x^2 + 18144a^7b^5x^4 + 42336a^6b^6x^6 + 63504a^5b^7x^8 + 63504a^4b^8x^{10} + 42336a^3b^9x^{12} + 1814}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**10,x)
```

```
[Out] (-a**2 - 9*a*b*x**2 - 36*b**2*x**4)/(504*a**9*b**3 + 4536*a**8*b**4*x**2 +
18144*a**7*b**5*x**4 + 42336*a**6*b**6*x**6 + 63504*a**5*b**7*x**8 + 63504*
a**4*b**8*x**10 + 42336*a**3*b**9*x**12 + 18144*a**2*b**10*x**14 + 4536*a*b
**11*x**16 + 504*b**12*x**18)
```

3.203 $\int \frac{x^3}{(a+bx^2)^{10}} \, dx$

Optimal. Leaf size=34

$$\frac{a}{18b^2(a+bx^2)^9} - \frac{1}{16b^2(a+bx^2)^8}$$

[Out] 1/18*a/b^2/(b*x^2+a)^9-1/16/b^2/(b*x^2+a)^8

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a}{18b^2(a+bx^2)^9} - \frac{1}{16b^2(a+bx^2)^8}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*x^2)^10,x]

[Out] a/(18*b^2*(a + b*x^2)^9) - 1/(16*b^2*(a + b*x^2)^8)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx^2)^{10}} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^{10}} \, dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{10}} + \frac{1}{b(a+bx)^9}\right) dx, x, x^2\right) \\
&= \frac{a}{18b^2(a+bx^2)^9} - \frac{1}{16b^2(a+bx^2)^8}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.71

$$-\frac{a+9bx^2}{144b^2(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b*x^2)^10,x]

[Out] -1/144*(a + 9*b*x^2)/(b^2*(a + b*x^2)^9)

fricas [B] time = 0.79, size = 113, normalized size = 3.32

$$-\frac{9\,bx^2 + a}{144\left(b^{11}x^{18} + 9\,ab^{10}x^{16} + 36\,a^2b^9x^{14} + 84\,a^3b^8x^{12} + 126\,a^4b^7x^{10} + 126\,a^5b^6x^8 + 84\,a^6b^5x^6 + 36\,a^7b^4x^4 + 9\,a^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/144*(9*b*x^2 + a)/(b^11*x^18 + 9*a*b^10*x^16 + 36*a^2*b^9*x^14 + 84*a^3*
b^8*x^12 + 126*a^4*b^7*x^10 + 126*a^5*b^6*x^8 + 84*a^6*b^5*x^6 + 36*a^7*b^4
*x^4 + 9*a^8*b^3*x^2 + a^9*b^2)
```

giac [A] time = 0.62, size = 22, normalized size = 0.65

$$-\frac{9\,bx^2 + a}{144\left(bx^2 + a\right)^9 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/144*(9*b*x^2 + a)/((b*x^2 + a)^9*b^2)
```

maple [A] time = 0.01, size = 31, normalized size = 0.91

$$\frac{a}{18\left(b\,x^2 + a\right)^9 b^2} - \frac{1}{16\left(b\,x^2 + a\right)^8 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^10,x)
```

```
[Out] 1/18*a/b^2/(b*x^2+a)^9-1/16/b^2/(b*x^2+a)^8
```

maxima [B] time = 1.39, size = 113, normalized size = 3.32

$$-\frac{9\,bx^2 + a}{144\left(b^{11}x^{18} + 9\,ab^{10}x^{16} + 36\,a^2b^9x^{14} + 84\,a^3b^8x^{12} + 126\,a^4b^7x^{10} + 126\,a^5b^6x^8 + 84\,a^6b^5x^6 + 36\,a^7b^4x^4 + 9\,a^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/144*(9*b*x^2 + a)/(b^11*x^18 + 9*a*b^10*x^16 + 36*a^2*b^9*x^14 + 84*a^3*
b^8*x^12 + 126*a^4*b^7*x^10 + 126*a^5*b^6*x^8 + 84*a^6*b^5*x^6 + 36*a^7*b^4
*x^4 + 9*a^8*b^3*x^2 + a^9*b^2)
```

mupad [B] time = 0.11, size = 114, normalized size = 3.35

$$-\frac{\frac{a}{144\,b^2} + \frac{x^2}{16\,b}}{a^9 + 9\,a^8\,b\,x^2 + 36\,a^7\,b^2\,x^4 + 84\,a^6\,b^3\,x^6 + 126\,a^5\,b^4\,x^8 + 126\,a^4\,b^5\,x^{10} + 84\,a^3\,b^6\,x^{12} + 36\,a^2\,b^7\,x^{14} + 9\,a\,b^8\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^10,x)
```

```
[Out] -(a/(144*b^2) + x^2/(16*b))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 +
36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a
^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

sympy [B] time = 0.93, size = 119, normalized size = 3.50

$$\frac{-a-9bx^{2}}{144a^{9}b^{2}+1296a^{8}b^{3}x^{2}+5184a^{7}b^{4}x^{4}+12096a^{6}b^{5}x^{6}+18144a^{5}b^{6}x^{8}+18144a^{4}b^{7}x^{10}+12096a^{3}b^{8}x^{12}+5184a^{2}b^{9}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**10,x)
```

```
[Out] (-a - 9*b*x**2)/(144*a**9*b**2 + 1296*a**8*b**3*x**2 + 5184*a**7*b**4*x**4
+ 12096*a**6*b**5*x**6 + 18144*a**5*b**6*x**8 + 18144*a**4*b**7*x**10 + 120
96*a**3*b**8*x**12 + 5184*a**2*b**9*x**14 + 1296*a*b**10*x**16 + 144*b**11*
x**18)
```

3.204 $\int \frac{x}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{18b\left(a+bx^2\right)^9}$$

```
[Out] -1/18/b/(b*x^2+a)^9
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$-\frac{1}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^10,x]
```

```
[Out] -1/(18*b*(a + b*x^2)^9)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{10}}\,dx=-\frac{1}{18b\left(a+bx^2\right)^9}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^10,x]
```

```
[Out] -1/18*1/(b*(a + b*x^2)^9)
```

fricas [B] time = 0.90, size = 103, normalized size = 6.44

$$-\frac{1}{18\left(b^{10}x^{18}+9\,ab^9x^{16}+36\,a^2b^8x^{14}+84\,a^3b^7x^{12}+126\,a^4b^6x^{10}+126\,a^5b^5x^8+84\,a^6b^4x^6+36\,a^7b^3x^4+9\,a^8b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/18/(b^10*x^18 + 9*a*b^9*x^16 + 36*a^2*b^8*x^14 + 84*a^3*b^7*x^12 + 126*a
^4*b^6*x^10 + 126*a^5*b^5*x^8 + 84*a^6*b^4*x^6 + 36*a^7*b^3*x^4 + 9*a^8*b^2
*x^2 + a^9*b)
```

giac [A] time = 0.59, size = 14, normalized size = 0.88

$$-\frac{1}{18\left(bx^2+a\right)^9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/18/((b*x^2 + a)^9*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{18\left(b\,x^2+a\right)^9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^10,x)
```

```
[Out] -1/18/b/(b*x^2+a)^9
```

maxima [A] time = 1.30, size = 14, normalized size = 0.88

$$-\frac{1}{18\left(bx^2+a\right)^9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/18/((b*x^2 + a)^9*b)
```

mupad [B] time = 0.13, size = 14, normalized size = 0.88

$$-\frac{1}{18\,b\left(b\,x^2+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^10,x)
```

```
[Out] -1/(18*b*(a + b*x^2)^9)
```

sympy [B] time = 0.89, size = 110, normalized size = 6.88

$$-\frac{1}{18a^9b+162a^8b^2x^2+648a^7b^3x^4+1512a^6b^4x^6+2268a^5b^5x^8+2268a^4b^6x^{10}+1512a^3b^7x^{12}+648a^2b^8x^{14}+162}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**10,x)
```

```
[Out] -1/(18*a**9*b + 162*a**8*b**2*x**2 + 648*a**7*b**3*x**4 + 1512*a**6*b**4*x*
*6 + 2268*a**5*b**5*x**8 + 2268*a**4*b**6*x**10 + 1512*a**3*b**7*x**12 + 64
8*a**2*b**8*x**14 + 162*a*b**9*x**16 + 18*b**10*x**18)
```

3.205 $\int \frac{1}{x(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=166

$$-\frac{\log\left(a+bx^2\right)}{2a^{10}}+\frac{\log(x)}{a^{10}}+\frac{1}{2a^9\left(a+bx^2\right)}+\frac{1}{4a^8\left(a+bx^2\right)^2}+\frac{1}{6a^7\left(a+bx^2\right)^3}+\frac{1}{8a^6\left(a+bx^2\right)^4}+\frac{1}{10a^5\left(a+bx^2\right)^5}+\frac{}{12}$$

```
[Out] 1/18/a/(b*x^2+a)^9+1/16/a^2/(b*x^2+a)^8+1/14/a^3/(b*x^2+a)^7+1/12/a^4/(b*x^
2+a)^6+1/10/a^5/(b*x^2+a)^5+1/8/a^6/(b*x^2+a)^4+1/6/a^7/(b*x^2+a)^3+1/4/a^8
/(b*x^2+a)^2+1/2/a^9/(b*x^2+a)+ln(x)/a^10-1/2*ln(b*x^2+a)/a^10
```

Rubi [A] time = 0.13, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$\frac{1}{2a^9\left(a+bx^2\right)}+\frac{1}{4a^8\left(a+bx^2\right)^2}+\frac{1}{6a^7\left(a+bx^2\right)^3}+\frac{1}{8a^6\left(a+bx^2\right)^4}+\frac{1}{10a^5\left(a+bx^2\right)^5}+\frac{1}{12a^4\left(a+bx^2\right)^6}+\frac{1}{14a^3\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^10),x]
```

```
[Out] 1/(18*a*(a + b*x^2)^9) + 1/(16*a^2*(a + b*x^2)^8) + 1/(14*a^3*(a + b*x^2)^7
) + 1/(12*a^4*(a + b*x^2)^6) + 1/(10*a^5*(a + b*x^2)^5) + 1/(8*a^6*(a + b*x
^2)^4) + 1/(6*a^7*(a + b*x^2)^3) + 1/(4*a^8*(a + b*x^2)^2) + 1/(2*a^9*(a +
b*x^2)) + Log[x]/a^10 - Log[a + b*x^2]/(2*a^10)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{10}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^{10}x}-\frac{b}{a(a+bx)^{10}}-\frac{b}{a^2(a+bx)^9}-\frac{b}{a^3(a+bx)^8}-\frac{b}{a^4(a+bx)^7}-\frac{b}{a^5(a+b\right.\right.\\
&= \frac{1}{18a\left(a+bx^2\right)^9}+\frac{1}{16a^2\left(a+bx^2\right)^8}+\frac{1}{14a^3\left(a+bx^2\right)^7}+\frac{1}{12a^4\left(a+bx^2\right)^6}+\frac{1}{10a^5\left(a+bx^2\right.}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 120, normalized size = 0.72

$$\frac{\frac{a\left(7129a^8+41481a^7bx^2+120564a^6b^2x^4+210756a^5b^3x^6+236754a^4b^4x^8+173250a^3b^5x^{10}+80220a^2b^6x^{12}+21420ab^7x^{14}+2520b^8x^{16}\right)}{\left(a+bx^2\right)^9}-2520\log(}{5040a^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^10),x]
```

```
[Out] ((a*(7129*a^8 + 41481*a^7*b*x^2 + 120564*a^6*b^2*x^4 + 210756*a^5*b^3*x^6 +
 236754*a^4*b^4*x^8 + 173250*a^3*b^5*x^10 + 80220*a^2*b^6*x^12 + 21420*a*b^
7*x^14 + 2520*b^8*x^16))/(a + b*x^2)^9 + 5040*Log[x] - 2520*Log[a + b*x^2])
/(5040*a^10)
```

fricas [B] time = 0.91, size = 398, normalized size = 2.40

$$\frac{2520\,ab^8x^{16} + 21420\,a^2b^7x^{14} + 80220\,a^3b^6x^{12} + 173250\,a^4b^5x^{10} + 236754\,a^5b^4x^8 + 210756\,a^6b^3x^6 + 120564\,a^7b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/5040*(2520*a*b^8*x^16 + 21420*a^2*b^7*x^14 + 80220*a^3*b^6*x^12 + 173250*
a^4*b^5*x^10 + 236754*a^5*b^4*x^8 + 210756*a^6*b^3*x^6 + 120564*a^7*b^2*x^4
 + 41481*a^8*b*x^2 + 7129*a^9 - 2520*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*
x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^
6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log(b*x^2 + a) + 5040*(b^9*x^18 + 9
*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^
5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log(x))/(a
^10*b^9*x^18 + 9*a^11*b^8*x^16 + 36*a^12*b^7*x^14 + 84*a^13*b^6*x^12 + 126*
a^14*b^5*x^10 + 126*a^15*b^4*x^8 + 84*a^16*b^3*x^6 + 36*a^17*b^2*x^4 + 9*a^
18*b*x^2 + a^19)
```

giac [A] time = 0.63, size = 136, normalized size = 0.82

$$\frac{\log\left(x^2\right)}{2\,a^{10}} - \frac{\log\left(\left|bx^2 + a\right|\right)}{2\,a^{10}} + \frac{7129\,b^9x^{18} + 66681\,ab^8x^{16} + 278064\,a^2b^7x^{14} + 679056\,a^3b^6x^{12} + 1071504\,a^4b^5x^{10} + 1}{5040\,\left(bx^2 + a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^10 - 1/2*log(abs(b*x^2 + a))/a^10 + 1/5040*(7129*b^9*x^18 +
66681*a*b^8*x^16 + 278064*a^2*b^7*x^14 + 679056*a^3*b^6*x^12 + 1071504*a^4*
b^5*x^10 + 1135008*a^5*b^4*x^8 + 809592*a^6*b^3*x^6 + 377208*a^7*b^2*x^4 +
105642*a^8*b*x^2 + 14258*a^9)/((b*x^2 + a)^9*a^10)
```

maple [A] time = 0.02, size = 147, normalized size = 0.89

$$\frac{1}{18\left(b\,x^2+a\right)^9 a} + \frac{1}{16\left(b\,x^2+a\right)^8 a^2} + \frac{1}{14\left(b\,x^2+a\right)^7 a^3} + \frac{1}{12\left(b\,x^2+a\right)^6 a^4} + \frac{1}{10\left(b\,x^2+a\right)^5 a^5} + \frac{1}{8\left(b\,x^2+a\right)^4 a^6} + \frac{}{6\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^10,x)
```

```
[Out] 1/18/a/(b*x^2+a)^9+1/16/a^2/(b*x^2+a)^8+1/14/a^3/(b*x^2+a)^7+1/12/a^4/(b*x^
2+a)^6+1/10/a^5/(b*x^2+a)^5+1/8/a^6/(b*x^2+a)^4+1/6/a^7/(b*x^2+a)^3+1/4/a^8
/(b*x^2+a)^2+1/2/a^9/(b*x^2+a)+ln(x)/a^10-1/2*ln(b*x^2+a)/a^10
```

maxima [A] time = 1.65, size = 214, normalized size = 1.29

$$\frac{2520\,b^8x^{16} + 21420\,ab^7x^{14} + 80220\,a^2b^6x^{12} + 173250\,a^3b^5x^{10} + 236754\,a^4b^4x^8 + 210756\,a^5b^3x^6 + 120564\,a^6b^2x}{5040\left(a^9b^9x^{18} + 9\,a^{10}b^8x^{16} + 36\,a^{11}b^7x^{14} + 84\,a^{12}b^6x^{12} + 126\,a^{13}b^5x^{10} + 126\,a^{14}b^4x^8 + 84\,a^{15}b^3x^6 + 36\,a^{16}b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/5040*(2520*b^8*x^16 + 21420*a*b^7*x^14 + 80220*a^2*b^6*x^12 + 173250*a^3*
b^5*x^10 + 236754*a^4*b^4*x^8 + 210756*a^5*b^3*x^6 + 120564*a^6*b^2*x^4 + 4
1481*a^7*b*x^2 + 7129*a^8)/(a^9*b^9*x^18 + 9*a^10*b^8*x^16 + 36*a^11*b^7*x^
14 + 84*a^12*b^6*x^12 + 126*a^13*b^5*x^10 + 126*a^14*b^4*x^8 + 84*a^15*b^3*
x^6 + 36*a^16*b^2*x^4 + 9*a^17*b*x^2 + a^18) - 1/2*log(b*x^2 + a)/a^10 + 1/
2*log(x^2)/a^10
```

mupad [B] time = 5.46, size = 210, normalized size = 1.27

$$\frac{\ln(x)}{a^{10}}+\frac{\frac{7129}{5040\,a}+\frac{4609\,b\,x^2}{560\,a^2}+\frac{3349\,b^2\,x^4}{140\,a^3}+\frac{2509\,b^3\,x^6}{60\,a^4}+\frac{1879\,b^4\,x^8}{40\,a^5}+\frac{275\,b^5\,x^{10}}{8\,a^6}+\frac{191\,b^6\,x^{12}}{12\,a^7}+\frac{17\,b^7\,x^{14}}{4\,a^8}+\frac{b^8\,x^{16}}{2\,a^9}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^10),x)
```

```
[Out] log(x)/a^10 + (7129/(5040*a) + (4609*b*x^2)/(560*a^2) + (3349*b^2*x^4)/(140
*a^3) + (2509*b^3*x^6)/(60*a^4) + (1879*b^4*x^8)/(40*a^5) + (275*b^5*x^10)/
(8*a^6) + (191*b^6*x^12)/(12*a^7) + (17*b^7*x^14)/(4*a^8) + (b^8*x^16)/(2*a
^9))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6
*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^
7*x^14) - log(a + b*x^2)/(2*a^10)
```

sympy [A] time = 1.31, size = 223, normalized size = 1.34

$$\frac{7129a^8+41481a^7bx^2+120564a^6b^2x^4+210756a^5b^3x^6+236754a^4b^4x^8+173250a^3b^5x^{10}+80}{5040a^{18}+45360a^{17}bx^2+181440a^{16}b^2x^4+423360a^{15}b^3x^6+635040a^{14}b^4x^8+635040a^{13}b^5x^{10}+423360a^{12}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**10,x)
```

```
[Out] (7129*a**8 + 41481*a**7*b*x**2 + 120564*a**6*b**2*x**4 + 210756*a**5*b**3*x
**6 + 236754*a**4*b**4*x**8 + 173250*a**3*b**5*x**10 + 80220*a**2*b**6*x**1
2 + 21420*a*b**7*x**14 + 2520*b**8*x**16)/(5040*a**18 + 45360*a**17*b*x**2
+ 181440*a**16*b**2*x**4 + 423360*a**15*b**3*x**6 + 635040*a**14*b**4*x**8
+ 635040*a**13*b**5*x**10 + 423360*a**12*b**6*x**12 + 181440*a**11*b**7*x**
14 + 45360*a**10*b**8*x**16 + 5040*a**9*b**9*x**18) + log(x)/a**10 - log(a/
b + x**2)/(2*a**10)
```

3.206 $\int \frac{1}{x^3\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=184

$$\frac{5b\log\left(a+bx^2\right)}{a^{11}}-\frac{10b\log(x)}{a^{11}}-\frac{9b}{2a^{10}\left(a+bx^2\right)}-\frac{1}{2a^{10}x^2}-\frac{2b}{a^9\left(a+bx^2\right)^2}-\frac{7b}{6a^8\left(a+bx^2\right)^3}-\frac{3b}{4a^7\left(a+bx^2\right)^4}-\frac{b}{2a^6\left(a+b\right.}$$

```
[Out] -1/2/a^10/x^2-1/18*b/a^2/(b*x^2+a)^9-1/8*b/a^3/(b*x^2+a)^8-3/14*b/a^4/(b*x^
2+a)^7-1/3*b/a^5/(b*x^2+a)^6-1/2*b/a^6/(b*x^2+a)^5-3/4*b/a^7/(b*x^2+a)^4-7/
6*b/a^8/(b*x^2+a)^3-2*b/a^9/(b*x^2+a)^2-9/2*b/a^10/(b*x^2+a)-10*b*ln(x)/a^1
1+5*b*ln(b*x^2+a)/a^11
```

Rubi [A] time = 0.19, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{9b}{2a^{10}\left(a+bx^2\right)}-\frac{2b}{a^9\left(a+bx^2\right)^2}-\frac{7b}{6a^8\left(a+bx^2\right)^3}-\frac{3b}{4a^7\left(a+bx^2\right)^4}-\frac{b}{2a^6\left(a+bx^2\right)^5}-\frac{b}{3a^5\left(a+bx^2\right)^6}-\frac{3b}{14a^4\left(a+bx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^10),x]
```

```
[Out] -1/(2*a^10*x^2) - b/(18*a^2*(a + b*x^2)^9) - b/(8*a^3*(a + b*x^2)^8) - (3*b
)/(14*a^4*(a + b*x^2)^7) - b/(3*a^5*(a + b*x^2)^6) - b/(2*a^6*(a + b*x^2)^5
) - (3*b)/(4*a^7*(a + b*x^2)^4) - (7*b)/(6*a^8*(a + b*x^2)^3) - (2*b)/(a^9*
(a + b*x^2)^2) - (9*b)/(2*a^10*(a + b*x^2)) - (10*b*Log[x])/a^11 + (5*b*Log
[a + b*x^2])/a^11
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{10}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^{10}x^2}-\frac{10b}{a^{11}x}+\frac{b^2}{a^2(a+bx)^{10}}+\frac{2b^2}{a^3(a+bx)^9}+\frac{3b^2}{a^4(a+bx)^8}+\frac{4b^2}{a^5(a+bx)^7}+\right.\right.\\ &= -\frac{1}{2a^{10}x^2}-\frac{b}{18a^2\left(a+bx^2\right)^9}-\frac{b}{8a^3\left(a+bx^2\right)^8}-\frac{3b}{14a^4\left(a+bx^2\right)^7}-\frac{b}{3a^5\left(a+bx^2\right)^6}-\frac{}{2a^6\left(a\right.}\end{aligned}$$

Mathematica [A] time = 0.13, size = 136, normalized size = 0.74

$$-\frac{\frac{a\left(252a^9+7129a^8bx^2+41481a^7b^2x^4+120564a^6b^3x^6+210756a^5b^4x^8+236754a^4b^5x^{10}+173250a^3b^6x^{12}+80220a^2b^7x^{14}+21420ab^8x^{16}+2520b^9x^{18}\right)}{x^2\left(a+bx^2\right)^9}}{504a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^10),x]
```

```
[Out] -1/504*((a*(252*a^9 + 7129*a^8*b*x^2 + 41481*a^7*b^2*x^4 + 120564*a^6*b^3*x
^6 + 210756*a^5*b^4*x^8 + 236754*a^4*b^5*x^10 + 173250*a^3*b^6*x^12 + 80220
*a^2*b^7*x^14 + 21420*a*b^8*x^16 + 2520*b^9*x^18))/(x^2*(a + b*x^2)^9) + 50
40*b*Log[x] - 2520*b*Log[a + b*x^2])/a^11
```

fricas [B] time = 0.95, size = 427, normalized size = 2.32

$$-\frac{2520\,ab^9x^{18} + 21420\,a^2b^8x^{16} + 80220\,a^3b^7x^{14} + 173250\,a^4b^6x^{12} + 236754\,a^5b^5x^{10} + 210756\,a^6b^4x^8 + 12056}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(2520*a*b^9*x^18 + 21420*a^2*b^8*x^16 + 80220*a^3*b^7*x^14 + 173250*
a^4*b^6*x^12 + 236754*a^5*b^5*x^10 + 210756*a^6*b^4*x^8 + 120564*a^7*b^3*x^
6 + 41481*a^8*b^2*x^4 + 7129*a^9*b*x^2 + 252*a^10 - 2520*(b^10*x^20 + 9*a*b
^9*x^18 + 36*a^2*b^8*x^16 + 84*a^3*b^7*x^14 + 126*a^4*b^6*x^12 + 126*a^5*b^
5*x^10 + 84*a^6*b^4*x^8 + 36*a^7*b^3*x^6 + 9*a^8*b^2*x^4 + a^9*b*x^2)*log(b
*x^2 + a) + 5040*(b^10*x^20 + 9*a*b^9*x^18 + 36*a^2*b^8*x^16 + 84*a^3*b^7*x
^14 + 126*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 84*a^6*b^4*x^8 + 36*a^7*b^3*x^6
 + 9*a^8*b^2*x^4 + a^9*b*x^2)*log(x))/(a^11*b^9*x^20 + 9*a^12*b^8*x^18 + 36
*a^13*b^7*x^16 + 84*a^14*b^6*x^14 + 126*a^15*b^5*x^12 + 126*a^16*b^4*x^10 +
 84*a^17*b^3*x^8 + 36*a^18*b^2*x^6 + 9*a^19*b*x^4 + a^20*x^2)
```

giac [A] time = 0.59, size = 159, normalized size = 0.86

$$-\frac{5\,b\log\left(x^2\right)}{a^{11}} + \frac{5\,b\log\left(\left|bx^2 + a\right|\right)}{a^{11}} + \frac{10\,bx^2 - a}{2\,a^{11}x^2} - \frac{7129\,b^{10}x^{18} + 66429\,ab^9x^{16} + 275796\,a^2b^8x^{14} + 669984\,a^3b^7x^1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -5*b*log(x^2)/a^11 + 5*b*log(abs(b*x^2 + a))/a^11 + 1/2*(10*b*x^2 - a)/(a^1
1*x^2) - 1/504*(7129*b^10*x^18 + 66429*a*b^9*x^16 + 275796*a^2*b^8*x^14 + 6
69984*a^3*b^7*x^12 + 1050336*a^4*b^6*x^10 + 1103256*a^5*b^5*x^8 + 777840*a^
6*b^4*x^6 + 356040*a^7*b^3*x^4 + 96570*a^8*b^2*x^2 + 11990*a^9*b)/((b*x^2 +
 a)^9*a^11)
```

maple [A] time = 0.02, size = 167, normalized size = 0.91

$$-\frac{b}{18\left(b\,x^2+a\right)^9a^2}-\frac{b}{8\left(b\,x^2+a\right)^8a^3}-\frac{3b}{14\left(b\,x^2+a\right)^7a^4}-\frac{b}{3\left(b\,x^2+a\right)^6a^5}-\frac{b}{2\left(b\,x^2+a\right)^5a^6}-\frac{3b}{4\left(b\,x^2+a\right)^4a^7}-\frac{}{6(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^10,x)
```

```
[Out] -1/2/a^10/x^2-1/18*b/a^2/(b*x^2+a)^9-1/8*b/a^3/(b*x^2+a)^8-3/14*b/a^4/(b*x^
2+a)^7-1/3*b/a^5/(b*x^2+a)^6-1/2*b/a^6/(b*x^2+a)^5-3/4*b/a^7/(b*x^2+a)^4-7/
```

```
6*b/a^8/(b*x^2+a)^3-2*b/a^9/(b*x^2+a)^2-9/2*b/a^10/(b*x^2+a)-10*b*ln(x)/a^1
1+5*b*ln(b*x^2+a)/a^11
```

maxima [A] time = 1.67, size = 231, normalized size = 1.26

$$-\frac{2520\,b^{9}x^{18}+21420\,ab^{8}x^{16}+80220\,a^{2}b^{7}x^{14}+173250\,a^{3}b^{6}x^{12}+236754\,a^{4}b^{5}x^{10}+210756\,a^{5}b^{4}x^{8}+120564\,a^{6}b^{}}{504\left(a^{10}b^{9}x^{20}+9\,a^{11}b^{8}x^{18}+36\,a^{12}b^{7}x^{16}+84\,a^{13}b^{6}x^{14}+126\,a^{14}b^{5}x^{12}+126\,a^{15}b^{4}x^{10}+84\,a^{16}b^{3}x^{8}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(2520*b^9*x^18 + 21420*a*b^8*x^16 + 80220*a^2*b^7*x^14 + 173250*a^3*
b^6*x^12 + 236754*a^4*b^5*x^10 + 210756*a^5*b^4*x^8 + 120564*a^6*b^3*x^6 +
41481*a^7*b^2*x^4 + 7129*a^8*b*x^2 + 252*a^9)/(a^10*b^9*x^20 + 9*a^11*b^8*x
^18 + 36*a^12*b^7*x^16 + 84*a^13*b^6*x^14 + 126*a^14*b^5*x^12 + 126*a^15*b^
4*x^10 + 84*a^16*b^3*x^8 + 36*a^17*b^2*x^6 + 9*a^18*b*x^4 + a^19*x^2) + 5*b
*log(b*x^2 + a)/a^11 - 5*b*log(x^2)/a^11
```

mupad [B] time = 0.52, size = 229, normalized size = 1.24

$$\frac{5\,b\,\ln\left(b\,x^2+a\right)}{a^{11}}-\frac{\frac{1}{2\,a}+\frac{7129\,b\,x^2}{504\,a^2}+\frac{4609\,b^2\,x^4}{56\,a^3}+\frac{3349\,b^3\,x^6}{14\,a^4}+\frac{2509\,b^4\,x^8}{6\,a^5}+\frac{1879\,b^5\,x^{10}}{4\,a^6}+\frac{1375\,b^6\,x^{12}}{4\,a^7}+\frac{955\,b^7\,x^{14}}{6\,a^8}+}{a^9\,x^2+9\,a^8\,b\,x^4+36\,a^7\,b^2\,x^6+84\,a^6\,b^3\,x^8+126\,a^5\,b^4\,x^{10}+126\,a^4\,b^5\,x^{12}+84\,a^3\,b^6\,x^{14}+36\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^10),x)
```

```
[Out] (5*b*log(a + b*x^2))/a^11 - (1/(2*a) + (7129*b*x^2)/(504*a^2) + (4609*b^2*x
^4)/(56*a^3) + (3349*b^3*x^6)/(14*a^4) + (2509*b^4*x^8)/(6*a^5) + (1879*b^5
*x^10)/(4*a^6) + (1375*b^6*x^12)/(4*a^7) + (955*b^7*x^14)/(6*a^8) + (85*b^8
*x^16)/(2*a^9) + (5*b^9*x^18)/a^10)/(a^9*x^2 + b^9*x^20 + 9*a^8*b*x^4 + 9*a
*b^8*x^18 + 36*a^7*b^2*x^6 + 84*a^6*b^3*x^8 + 126*a^5*b^4*x^10 + 126*a^4*b^
5*x^12 + 84*a^3*b^6*x^14 + 36*a^2*b^7*x^16) - (10*b*log(x))/a^11
```

sympy [A] time = 1.46, size = 245, normalized size = 1.33

$$\frac{-252a^{9}-7129a^{8}bx^{2}-41481a^{7}b^{2}x^{4}-120564a^{6}b^{3}x^{6}-210756a^{5}b^{4}x^{8}-236754a^{4}b^{5}x^{10}-173250a^{3}b^{6}x^{12}-8}{504a^{19}x^{2}+4536a^{18}bx^{4}+18144a^{17}b^{2}x^{6}+42336a^{16}b^{3}x^{8}+63504a^{15}b^{4}x^{10}+63504a^{14}b^{5}x^{12}+42336a^{13}b^{6}x^{14}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**10,x)
```

```
[Out] (-252*a**9 - 7129*a**8*b*x**2 - 41481*a**7*b**2*x**4 - 120564*a**6*b**3*x**
6 - 210756*a**5*b**4*x**8 - 236754*a**4*b**5*x**10 - 173250*a**3*b**6*x**12
 - 80220*a**2*b**7*x**14 - 21420*a*b**8*x**16 - 2520*b**9*x**18)/(504*a**19
*x**2 + 4536*a**18*b*x**4 + 18144*a**17*b**2*x**6 + 42336*a**16*b**3*x**8 +
 63504*a**15*b**4*x**10 + 63504*a**14*b**5*x**12 + 42336*a**13*b**6*x**14 +
 18144*a**12*b**7*x**16 + 4536*a**11*b**8*x**18 + 504*a**10*b**9*x**20) - 1
0*b*log(x)/a**11 + 5*b*log(a/b + x**2)/a**11
```

3.207 $\int \frac{1}{x^5(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=217

$$-\frac{55b^2\log\left(a+bx^2\right)}{2a^{12}}+\frac{55b^2\log(x)}{a^{12}}+\frac{45b^2}{2a^{11}\left(a+bx^2\right)}+\frac{5b}{a^{11}x^2}+\frac{9b^2}{a^{10}\left(a+bx^2\right)^2}-\frac{1}{4a^{10}x^4}+\frac{14b^2}{3a^9\left(a+bx^2\right)^3}+\frac{21b^2}{8a^8\left(a+\right.}$$

```
[Out] -1/4/a^10/x^4+5*b/a^11/x^2+1/18*b^2/a^3/(b*x^2+a)^9+3/16*b^2/a^4/(b*x^2+a)^
8+3/7*b^2/a^5/(b*x^2+a)^7+5/6*b^2/a^6/(b*x^2+a)^6+3/2*b^2/a^7/(b*x^2+a)^5+2
1/8*b^2/a^8/(b*x^2+a)^4+14/3*b^2/a^9/(b*x^2+a)^3+9*b^2/a^10/(b*x^2+a)^2+45/
2*b^2/a^11/(b*x^2+a)+55*b^2*ln(x)/a^12-55/2*b^2*ln(b*x^2+a)/a^12
```

Rubi [A] time = 0.22, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{45b^2}{2a^{11}\left(a+bx^2\right)}+\frac{9b^2}{a^{10}\left(a+bx^2\right)^2}+\frac{14b^2}{3a^9\left(a+bx^2\right)^3}+\frac{21b^2}{8a^8\left(a+bx^2\right)^4}+\frac{3b^2}{2a^7\left(a+bx^2\right)^5}+\frac{5b^2}{6a^6\left(a+bx^2\right)^6}+\frac{3b^2}{7a^5\left(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^10),x]
```

```
[Out] -1/(4*a^10*x^4) + (5*b)/(a^11*x^2) + b^2/(18*a^3*(a + b*x^2)^9) + (3*b^2)/(
16*a^4*(a + b*x^2)^8) + (3*b^2)/(7*a^5*(a + b*x^2)^7) + (5*b^2)/(6*a^6*(a +
 b*x^2)^6) + (3*b^2)/(2*a^7*(a + b*x^2)^5) + (21*b^2)/(8*a^8*(a + b*x^2)^4)
 + (14*b^2)/(3*a^9*(a + b*x^2)^3) + (9*b^2)/(a^10*(a + b*x^2)^2) + (45*b^2)
/(2*a^11*(a + b*x^2)) + (55*b^2*Log[x])/a^12 - (55*b^2*Log[a + b*x^2])/(2*a
^12)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(a+bx)^{10}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^{10}x^3}-\frac{10b}{a^{11}x^2}+\frac{55b^2}{a^{12}x}-\frac{b^3}{a^3(a+bx)^{10}}-\frac{3b^3}{a^4(a+bx)^9}-\frac{6b^3}{a^5(a+bx)^8}-\frac{}{a^6(}\right.\right.\\ &= -\frac{1}{4a^{10}x^4}+\frac{5b}{a^{11}x^2}+\frac{b^2}{18a^3\left(a+bx^2\right)^9}+\frac{3b^2}{16a^4\left(a+bx^2\right)^8}+\frac{3b^2}{7a^5\left(a+bx^2\right)^7}+\frac{5b^2}{6a^6\left(a+bx\right.}\end{aligned}$$

Mathematica [A] time = 0.10, size = 151, normalized size = 0.70

$$\frac{\frac{a\left(-252a^{10}+2772a^9bx^2+78419a^8b^2x^4+456291a^7b^3x^6+1326204a^6b^4x^8+2318316a^5b^5x^{10}+2604294a^4b^6x^{12}+1905750a^3b^7x^{14}+882420a^2b^8x^{16}+235620a\right.}{x^4\left(a+bx^2\right)^9}}{1008a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^10),x]
```

```
[Out] ((a*(-252*a^10 + 2772*a^9*b*x^2 + 78419*a^8*b^2*x^4 + 456291*a^7*b^3*x^6 +
1326204*a^6*b^4*x^8 + 2318316*a^5*b^5*x^10 + 2604294*a^4*b^6*x^12 + 1905750
*a^3*b^7*x^14 + 882420*a^2*b^8*x^16 + 235620*a*b^9*x^18 + 27720*b^10*x^20))
/(x^4*(a + b*x^2)^9) + 55440*b^2*Log[x] - 27720*b^2*Log[a + b*x^2])/(1008*a
^12)
```

fricas [B] time = 0.80, size = 442, normalized size = 2.04

$$\frac{27720\,ab^{10}x^{20}+235620\,a^2b^9x^{18}+882420\,a^3b^8x^{16}+1905750\,a^4b^7x^{14}+2604294\,a^5b^6x^{12}+2318316\,a^6b^5x^{10}+1\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/1008*(27720*a*b^10*x^20 + 235620*a^2*b^9*x^18 + 882420*a^3*b^8*x^16 + 190
5750*a^4*b^7*x^14 + 2604294*a^5*b^6*x^12 + 2318316*a^6*b^5*x^10 + 1326204*a
^7*b^4*x^8 + 456291*a^8*b^3*x^6 + 78419*a^9*b^2*x^4 + 2772*a^10*b*x^2 - 252
*a^11 - 27720*(b^11*x^22 + 9*a*b^10*x^20 + 36*a^2*b^9*x^18 + 84*a^3*b^8*x^1
6 + 126*a^4*b^7*x^14 + 126*a^5*b^6*x^12 + 84*a^6*b^5*x^10 + 36*a^7*b^4*x^8
+ 9*a^8*b^3*x^6 + a^9*b^2*x^4)*log(b*x^2 + a) + 55440*(b^11*x^22 + 9*a*b^10
*x^20 + 36*a^2*b^9*x^18 + 84*a^3*b^8*x^16 + 126*a^4*b^7*x^14 + 126*a^5*b^6*
x^12 + 84*a^6*b^5*x^10 + 36*a^7*b^4*x^8 + 9*a^8*b^3*x^6 + a^9*b^2*x^4)*log(
x))/(a^12*b^9*x^22 + 9*a^13*b^8*x^20 + 36*a^14*b^7*x^18 + 84*a^15*b^6*x^16
+ 126*a^16*b^5*x^14 + 126*a^17*b^4*x^12 + 84*a^18*b^3*x^10 + 36*a^19*b^2*x^
8 + 9*a^20*b*x^6 + a^21*x^4)
```

giac [A] time = 0.61, size = 174, normalized size = 0.80

$$\frac{55\,b^2\log\left(x^2\right)}{2\,a^{12}}-\frac{55\,b^2\log\left(\left|bx^2+a\right|\right)}{2\,a^{12}}-\frac{165\,b^2x^4-20\,abx^2+a^2}{4\,a^{12}x^4}+\frac{78419\,b^{11}x^{18}+728451\,ab^{10}x^{16}+3013596\,a^2b^9x^{1\ldots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/2*b^2*log(x^2)/a^12 - 55/2*b^2*log(abs(b*x^2 + a))/a^12 - 1/4*(165*b^2*x
^4 - 20*a*b*x^2 + a^2)/(a^12*x^4) + 1/1008*(78419*b^11*x^18 + 728451*a*b^10
*x^16 + 3013596*a^2*b^9*x^14 + 7290444*a^3*b^8*x^12 + 11372256*a^4*b^7*x^10
 + 11871216*a^5*b^6*x^8 + 8302224*a^6*b^5*x^6 + 3757680*a^7*b^4*x^4 + 10017
90*a^8*b^3*x^2 + 120550*a^9*b^2)/((b*x^2 + a)^9*a^12)
```

maple [A] time = 0.02, size = 198, normalized size = 0.91

$$\frac{b^2}{18\left(b\,x^2+a\right)^9a^3}+\frac{3b^2}{16\left(b\,x^2+a\right)^8a^4}+\frac{3b^2}{7\left(b\,x^2+a\right)^7a^5}+\frac{5b^2}{6\left(b\,x^2+a\right)^6a^6}+\frac{3b^2}{2\left(b\,x^2+a\right)^5a^7}+\frac{21b^2}{8\left(b\,x^2+a\right)^4a^8}+\frac{1\ldots}{3\left(b\,x^2\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^10,x)
```

```
[Out] -1/4/a^10/x^4+5*b/a^11/x^2+1/18*b^2/a^3/(b*x^2+a)^9+3/16*b^2/a^4/(b*x^2+a)^
8+3/7*b^2/a^5/(b*x^2+a)^7+5/6*b^2/a^6/(b*x^2+a)^6+3/2*b^2/a^7/(b*x^2+a)^5+2
1/8*b^2/a^8/(b*x^2+a)^4+14/3*b^2/a^9/(b*x^2+a)^3+9*b^2/a^10/(b*x^2+a)^2+45/
2*b^2/a^11/(b*x^2+a)+55*b^2*ln(x)/a^12-55/2*b^2*ln(b*x^2+a)/a^12
```

maxima [A] time = 1.71, size = 246, normalized size = 1.13

$$\frac{27720\,b^{10}x^{20} + 235620\,ab^{9}x^{18} + 882420\,a^{2}b^{8}x^{16} + 1905750\,a^{3}b^{7}x^{14} + 2604294\,a^{4}b^{6}x^{12} + 2318316\,a^{5}b^{5}x^{10} + 1}{1008\,\left(a^{11}b^{9}x^{22} + 9\,a^{12}b^{8}x^{20} + 36\,a^{13}b^{7}x^{18} + 84\,a^{14}b^{6}x^{16} + 126\,a^{15}b^{5}x^{14} + 126\,a^{16}b^{4}x^{12}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/1008*(27720*b^10*x^20 + 235620*a*b^9*x^18 + 882420*a^2*b^8*x^16 + 1905750
*a^3*b^7*x^14 + 2604294*a^4*b^6*x^12 + 2318316*a^5*b^5*x^10 + 1326204*a^6*b
^4*x^8 + 456291*a^7*b^3*x^6 + 78419*a^8*b^2*x^4 + 2772*a^9*b*x^2 - 252*a^10
)/(a^11*b^9*x^22 + 9*a^12*b^8*x^20 + 36*a^13*b^7*x^18 + 84*a^14*b^6*x^16 +
126*a^15*b^5*x^14 + 126*a^16*b^4*x^12 + 84*a^17*b^3*x^10 + 36*a^18*b^2*x^8
+ 9*a^19*b*x^6 + a^20*x^4) - 55/2*b^2*log(b*x^2 + a)/a^12 + 55/2*b^2*log(x^
2)/a^12
```

mupad [B] time = 5.78, size = 243, normalized size = 1.12

$$\frac{\frac{11\,b\,x^2}{4\,a^2} - \frac{1}{4\,a} + \frac{78419\,b^2\,x^4}{1008\,a^3} + \frac{50699\,b^3\,x^6}{112\,a^4} + \frac{36839\,b^4\,x^8}{28\,a^5} + \frac{27599\,b^5\,x^{10}}{12\,a^6} + \frac{20669\,b^6\,x^{12}}{8\,a^7} + \frac{15125\,b^7\,x^{14}}{8\,a^8} + \frac{10505\,b^8\,x^{16}}{12\,a^9} + \frac{935\,b^9\,x^{18}}{4\,a^{10}}}{a^9\,x^4 + 9\,a^8\,b\,x^6 + 36\,a^7\,b^2\,x^8 + 84\,a^6\,b^3\,x^{10} + 126\,a^5\,b^4\,x^{12} + 126\,a^4\,b^5\,x^{14} + 84\,a^3\,b^6\,x^{16} + 36\,a^2\,b^7\,x^{18} + 9\,a\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^10),x)
```

```
[Out] ((11*b*x^2)/(4*a^2) - 1/(4*a) + (78419*b^2*x^4)/(1008*a^3) + (50699*b^3*x^6
)/(112*a^4) + (36839*b^4*x^8)/(28*a^5) + (27599*b^5*x^10)/(12*a^6) + (20669
*b^6*x^12)/(8*a^7) + (15125*b^7*x^14)/(8*a^8) + (10505*b^8*x^16)/(12*a^9) +
 (935*b^9*x^18)/(4*a^10) + (55*b^10*x^20)/(2*a^11))/(a^9*x^4 + b^9*x^22 + 9
*a^8*b*x^6 + 9*a*b^8*x^20 + 36*a^7*b^2*x^8 + 84*a^6*b^3*x^10 + 126*a^5*b^4*
x^12 + 126*a^4*b^5*x^14 + 84*a^3*b^6*x^16 + 36*a^2*b^7*x^18) - (55*b^2*log(
a + b*x^2))/(2*a^12) + (55*b^2*log(x))/a^12
```

sympy [A] time = 1.55, size = 260, normalized size = 1.20

$$\frac{-252a^{10} + 2772a^{9}bx^{2} + 78419a^{8}b^{2}x^{4} + 456291a^{7}b^{3}x^{6} + 1326204a^{6}b^{4}x^{8} + 2318316a^{5}b^{5}x^{10} + 2604294a^{4}b^{6}x^{12}}{1008a^{20}x^{4} + 9072a^{19}bx^{6} + 36288a^{18}b^{2}x^{8} + 84672a^{17}b^{3}x^{10} + 127008a^{16}b^{4}x^{12} + 127008a^{15}b^{5}x^{14} + 846}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**10,x)
```

```
[Out] (-252*a**10 + 2772*a**9*b*x**2 + 78419*a**8*b**2*x**4 + 456291*a**7*b**3*x**
*6 + 1326204*a**6*b**4*x**8 + 2318316*a**5*b**5*x**10 + 2604294*a**4*b**6*x
**12 + 1905750*a**3*b**7*x**14 + 882420*a**2*b**8*x**16 + 235620*a*b**9*x**
18 + 27720*b**10*x**20)/(1008*a**20*x**4 + 9072*a**19*b*x**6 + 36288*a**18*
b**2*x**8 + 84672*a**17*b**3*x**10 + 127008*a**16*b**4*x**12 + 127008*a**15
*b**5*x**14 + 84672*a**14*b**6*x**16 + 36288*a**13*b**7*x**18 + 9072*a**12*
b**8*x**20 + 1008*a**11*b**9*x**22) + 55*b**2*log(x)/a**12 - 55*b**2*log(a/
b + x**2)/(2*a**12)
```

3.208 $\int \frac{1}{x^7\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=226

$$\frac{110b^3\log\left(a+bx^2\right)}{a^{13}}-\frac{220b^3\log(x)}{a^{13}}-\frac{165b^3}{2a^{12}\left(a+bx^2\right)}-\frac{55b^2}{2a^{12}x^2}-\frac{30b^3}{a^{11}\left(a+bx^2\right)^2}+\frac{5b}{2a^{11}x^4}-\frac{14b^3}{a^{10}\left(a+bx^2\right)^3}-\frac{1}{6a^{10}x^6}-\frac{}{a^9}$$

```
[Out] -1/6/a^10/x^6+5/2*b/a^11/x^4-55/2*b^2/a^12/x^2-1/18*b^3/a^4/(b*x^2+a)^9-1/4
*b^3/a^5/(b*x^2+a)^8-5/7*b^3/a^6/(b*x^2+a)^7-5/3*b^3/a^7/(b*x^2+a)^6-7/2*b^
3/a^8/(b*x^2+a)^5-7*b^3/a^9/(b*x^2+a)^4-14*b^3/a^10/(b*x^2+a)^3-30*b^3/a^11
/(b*x^2+a)^2-165/2*b^3/a^12/(b*x^2+a)-220*b^3*ln(x)/a^13+110*b^3*ln(b*x^2+a
)/a^13
```

Rubi [A] time = 0.23, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$-\frac{165b^3}{2a^{12}\left(a+bx^2\right)}-\frac{30b^3}{a^{11}\left(a+bx^2\right)^2}-\frac{14b^3}{a^{10}\left(a+bx^2\right)^3}-\frac{7b^3}{a^9\left(a+bx^2\right)^4}-\frac{7b^3}{2a^8\left(a+bx^2\right)^5}-\frac{5b^3}{3a^7\left(a+bx^2\right)^6}-\frac{5b^3}{7a^6\left(a+bx^2\right)^7}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^10),x]
```

```
[Out] -1/(6*a^10*x^6) + (5*b)/(2*a^11*x^4) - (55*b^2)/(2*a^12*x^2) - b^3/(18*a^4*
(a + b*x^2)^9) - b^3/(4*a^5*(a + b*x^2)^8) - (5*b^3)/(7*a^6*(a + b*x^2)^7)
- (5*b^3)/(3*a^7*(a + b*x^2)^6) - (7*b^3)/(2*a^8*(a + b*x^2)^5) - (7*b^3)/(
a^9*(a + b*x^2)^4) - (14*b^3)/(a^10*(a + b*x^2)^3) - (30*b^3)/(a^11*(a + b*
x^2)^2) - (165*b^3)/(2*a^12*(a + b*x^2)) - (220*b^3*Log[x])/a^13 + (110*b^3
*Log[a + b*x^2])/a^13
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4(a+bx)^{10}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^{10}x^4}-\frac{10b}{a^{11}x^3}+\frac{55b^2}{a^{12}x^2}-\frac{220b^3}{a^{13}x}+\frac{b^4}{a^4(a+bx)^{10}}+\frac{4b^4}{a^5(a+bx)^9}+\frac{10b^4}{a^6(a+bx)^8}\right.\right.\\
&= -\frac{1}{6a^{10}x^6}+\frac{5b}{2a^{11}x^4}-\frac{55b^2}{2a^{12}x^2}-\frac{b^3}{18a^4\left(a+bx^2\right)^9}-\frac{b^3}{4a^5\left(a+bx^2\right)^8}-\frac{5b^3}{7a^6\left(a+bx^2\right)^7}-\frac{}{3a^7(}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 162, normalized size = 0.72

$$-\frac{-27720b^3\log\left(a+bx^2\right)+\frac{a\left(42a^{11}-252a^{10}bx^2+2772a^9b^2x^4+78419a^8b^3x^6+456291a^7b^4x^8+1326204a^6b^5x^{10}+2318316a^5b^6x^{12}+2604294\right.}{x^6\left(a+bx^2\right)^9}}{252a^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^10),x]
```

```
[Out] -1/252*((a*(42*a^11 - 252*a^10*b*x^2 + 2772*a^9*b^2*x^4 + 78419*a^8*b^3*x^6
 + 456291*a^7*b^4*x^8 + 1326204*a^6*b^5*x^10 + 2318316*a^5*b^6*x^12 + 26042
94*a^4*b^7*x^14 + 1905750*a^3*b^8*x^16 + 882420*a^2*b^9*x^18 + 235620*a*b^1
0*x^20 + 27720*b^11*x^22))/(x^6*(a + b*x^2)^9) + 55440*b^3*Log[x] - 27720*b
^3*Log[a + b*x^2])/a^13
```

fricas [B] time = 1.11, size = 453, normalized size = 2.00

$$-\frac{27720\,ab^{11}x^{22}+235620\,a^2b^{10}x^{20}+882420\,a^3b^9x^{18}+1905750\,a^4b^8x^{16}+2604294\,a^5b^7x^{14}+2318316\,a^6b^6x^1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(27720*a*b^11*x^22 + 235620*a^2*b^10*x^20 + 882420*a^3*b^9*x^18 + 19
05750*a^4*b^8*x^16 + 2604294*a^5*b^7*x^14 + 2318316*a^6*b^6*x^12 + 1326204*
a^7*b^5*x^10 + 456291*a^8*b^4*x^8 + 78419*a^9*b^3*x^6 + 2772*a^10*b^2*x^4 -
 252*a^11*b*x^2 + 42*a^12 - 27720*(b^12*x^24 + 9*a*b^11*x^22 + 36*a^2*b^10*
x^20 + 84*a^3*b^9*x^18 + 126*a^4*b^8*x^16 + 126*a^5*b^7*x^14 + 84*a^6*b^6*x
^12 + 36*a^7*b^5*x^10 + 9*a^8*b^4*x^8 + a^9*b^3*x^6)*log(b*x^2 + a) + 55440
*(b^12*x^24 + 9*a*b^11*x^22 + 36*a^2*b^10*x^20 + 84*a^3*b^9*x^18 + 126*a^4*
b^8*x^16 + 126*a^5*b^7*x^14 + 84*a^6*b^6*x^12 + 36*a^7*b^5*x^10 + 9*a^8*b^4
*x^8 + a^9*b^3*x^6)*log(x))/(a^13*b^9*x^24 + 9*a^14*b^8*x^22 + 36*a^15*b^7*
x^20 + 84*a^16*b^6*x^18 + 126*a^17*b^5*x^16 + 126*a^18*b^4*x^14 + 84*a^19*b
^3*x^12 + 36*a^20*b^2*x^10 + 9*a^21*b*x^8 + a^22*x^6)
```

giac [A] time = 0.64, size = 187, normalized size = 0.83

$$-\frac{110\,b^3\log\left(x^2\right)}{a^{13}}+\frac{110\,b^3\log\left(\left|bx^2+a\right|\right)}{a^{13}}+\frac{1210\,b^3x^6-165\,ab^2x^4+15\,a^2bx^2-a^3}{6\,a^{13}x^6}-\frac{78419\,b^{12}x^{18}+726561\,ab^{11}x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -110*b^3*log(x^2)/a^13 + 110*b^3*log(abs(b*x^2 + a))/a^13 + 1/6*(1210*b^3*x
^6 - 165*a*b^2*x^4 + 15*a^2*b*x^2 - a^3)/(a^13*x^6) - 1/252*(78419*b^12*x^1
8 + 726561*a*b^11*x^16 + 2996964*a^2*b^10*x^14 + 7225764*a^3*b^9*x^12 + 112
26726*a^4*b^8*x^10 + 11663316*a^5*b^7*x^8 + 8108184*a^6*b^6*x^6 + 3641256*a
^7*b^5*x^4 + 960210*a^8*b^4*x^2 + 113620*a^9*b^3)/((b*x^2 + a)^9*a^13)
```

maple [A] time = 0.02, size = 209, normalized size = 0.92

$$-\frac{b^3}{18\left(b\,x^2+a\right)^9a^4}-\frac{b^3}{4\left(b\,x^2+a\right)^8a^5}-\frac{5b^3}{7\left(b\,x^2+a\right)^7a^6}-\frac{5b^3}{3\left(b\,x^2+a\right)^6a^7}-\frac{7b^3}{2\left(b\,x^2+a\right)^5a^8}-\frac{7b^3}{\left(b\,x^2+a\right)^4a^9}-\frac{1}{\left(b\,x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^10,x)
```

```
[Out] -1/6/a^10/x^6+5/2*b/a^11/x^4-55/2*b^2/a^12/x^2-1/18*b^3/a^4/(b*x^2+a)^9-1/4
*b^3/a^5/(b*x^2+a)^8-5/7*b^3/a^6/(b*x^2+a)^7-5/3*b^3/a^7/(b*x^2+a)^6-7/2*b^
3/a^8/(b*x^2+a)^5-7*b^3/a^9/(b*x^2+a)^4-14*b^3/a^10/(b*x^2+a)^3-30*b^3/a^11
/(b*x^2+a)^2-165/2*b^3/a^12/(b*x^2+a)-220*b^3*ln(x)/a^13+110*b^3*ln(b*x^2+a
)/a^13
```

maxima [A] time = 1.71, size = 257, normalized size = 1.14

$$-\frac{27720\,b^{11}x^{22}+235620\,ab^{10}x^{20}+882420\,a^{2}b^{9}x^{18}+1905750\,a^{3}b^{8}x^{16}+2604294\,a^{4}b^{7}x^{14}+2318316\,a^{5}b^{6}x^{12}+1}{252\left(a^{12}b^{9}x^{24}+9\,a^{13}b^{8}x^{22}+36\,a^{14}b^{7}x^{20}+84\,a^{15}b^{6}x^{18}+126\,a^{16}b^{5}x^{16}+126\,a^{17}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(27720*b^11*x^22 + 235620*a*b^10*x^20 + 882420*a^2*b^9*x^18 + 190575
0*a^3*b^8*x^16 + 2604294*a^4*b^7*x^14 + 2318316*a^5*b^6*x^12 + 1326204*a^6*
b^5*x^10 + 456291*a^7*b^4*x^8 + 78419*a^8*b^3*x^6 + 2772*a^9*b^2*x^4 - 252*
a^10*b*x^2 + 42*a^11)/(a^12*b^9*x^24 + 9*a^13*b^8*x^22 + 36*a^14*b^7*x^20 +
 84*a^15*b^6*x^18 + 126*a^16*b^5*x^16 + 126*a^17*b^4*x^14 + 84*a^18*b^3*x^1
2 + 36*a^19*b^2*x^10 + 9*a^20*b*x^8 + a^21*x^6) + 110*b^3*log(b*x^2 + a)/a^
13 - 110*b^3*log(x^2)/a^13
```

mupad [B] time = 1.09, size = 255, normalized size = 1.13

$$\frac{110\,b^{3}\,\ln\left(b\,x^{2}+a\right)}{a^{13}}-\frac{\frac{1}{6\,a}-\frac{b\,x^{2}}{a^{2}}+\frac{11\,b^{2}\,x^{4}}{a^{3}}+\frac{78419\,b^{3}\,x^{6}}{252\,a^{4}}+\frac{50699\,b^{4}\,x^{8}}{28\,a^{5}}+\frac{36839\,b^{5}\,x^{10}}{7\,a^{6}}+\frac{27599\,b^{6}\,x^{12}}{3\,a^{7}}+\frac{20669\,b^{7}\,x^{14}}{2\,a^{8}}+\frac{15125\,b^{8}\,x^{16}}{2\,a^{9}}}{a^{9}\,x^{6}+9\,a^{8}\,b\,x^{8}+36\,a^{7}\,b^{2}\,x^{10}+84\,a^{6}\,b^{3}\,x^{12}+126\,a^{5}\,b^{4}\,x^{14}+126\,a^{4}\,b^{5}\,x^{16}+84\,a^{3}\,b^{6}\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)^10),x)
```

```
[Out] (110*b^3*log(a + b*x^2))/a^13 - (1/(6*a) - (b*x^2)/a^2 + (11*b^2*x^4)/a^3 +
 (78419*b^3*x^6)/(252*a^4) + (50699*b^4*x^8)/(28*a^5) + (36839*b^5*x^10)/(7
*a^6) + (27599*b^6*x^12)/(3*a^7) + (20669*b^7*x^14)/(2*a^8) + (15125*b^8*x^
16)/(2*a^9) + (10505*b^9*x^18)/(3*a^10) + (935*b^10*x^20)/a^11 + (110*b^11*
x^22)/a^12)/(a^9*x^6 + b^9*x^24 + 9*a^8*b*x^8 + 9*a*b^8*x^22 + 36*a^7*b^2*x
^10 + 84*a^6*b^3*x^12 + 126*a^5*b^4*x^14 + 126*a^4*b^5*x^16 + 84*a^3*b^6*x^
18 + 36*a^2*b^7*x^20) - (220*b^3*log(x))/a^13
```

sympy [A] time = 1.60, size = 270, normalized size = 1.19

$$\frac{-42a^{11}+252a^{10}bx^{2}-2772a^{9}b^{2}x^{4}-78419a^{8}b^{3}x^{6}-456291a^{7}b^{4}x^{8}-1326204a^{6}b^{5}x^{10}-2318316a^{5}b^{6}x^{12}-26042}{252a^{21}x^{6}+2268a^{20}bx^{8}+9072a^{19}b^{2}x^{10}+21168a^{18}b^{3}x^{12}+31752a^{17}b^{4}x^{14}+31752a^{16}b^{5}x^{16}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**10,x)
```

```
[Out] (-42*a**11 + 252*a**10*b*x**2 - 2772*a**9*b**2*x**4 - 78419*a**8*b**3*x**6
- 456291*a**7*b**4*x**8 - 1326204*a**6*b**5*x**10 - 2318316*a**5*b**6*x**12
 - 2604294*a**4*b**7*x**14 - 1905750*a**3*b**8*x**16 - 882420*a**2*b**9*x**
18 - 235620*a*b**10*x**20 - 27720*b**11*x**22)/(252*a**21*x**6 + 2268*a**20
*b*x**8 + 9072*a**19*b**2*x**10 + 21168*a**18*b**3*x**12 + 31752*a**17*b**4
*x**14 + 31752*a**16*b**5*x**16 + 21168*a**15*b**6*x**18 + 9072*a**14*b**7*
x**20 + 2268*a**13*b**8*x**22 + 252*a**12*b**9*x**24) - 220*b**3*log(x)/a**
13 + 110*b**3*log(a/b + x**2)/a**13
```

3.209 $\int \frac{x^{24}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=231

$$-\frac{7436429a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536b^{25/2}}+\frac{7436429a^2x}{65536b^{12}}-\frac{7436429ax^3}{196608b^{11}}-\frac{1062347x^7}{65536b^9\left(a+bx^2\right)}-\frac{1062347x^9}{294912b^8\left(a+bx^2\right)^2}-\frac{9657}{73728b^7\,(}$$

```
[Out] 7436429/65536*a^2*x/b^12-7436429/196608*a*x^3/b^11+7436429/327680*x^5/b^10-
1/18*x^23/b/(b*x^2+a)^9-23/288*x^21/b^2/(b*x^2+a)^8-23/192*x^19/b^3/(b*x^2+
a)^7-437/2304*x^17/b^4/(b*x^2+a)^6-7429/23040*x^15/b^5/(b*x^2+a)^5-7429/122
88*x^13/b^6/(b*x^2+a)^4-96577/73728*x^11/b^7/(b*x^2+a)^3-1062347/294912*x^9
/b^8/(b*x^2+a)^2-1062347/65536*x^7/b^9/(b*x^2+a)-7436429/65536*a^(5/2)*arct
an(x*b^(1/2)/a^(1/2))/b^(25/2)
```

Rubi [A] time = 0.17, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{7436429a^2x}{65536b^{12}}-\frac{7436429a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536b^{25/2}}-\frac{23x^{21}}{288b^2\left(a+bx^2\right)^8}-\frac{23x^{19}}{192b^3\left(a+bx^2\right)^7}-\frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6}-\frac{7429}{23040b^5\,(a}$$

Antiderivative was successfully verified.

```
[In] Int[x^24/(a + b*x^2)^10,x]
```

```
[Out] (7436429*a^2*x)/(65536*b^12) - (7436429*a*x^3)/(196608*b^11) + (7436429*x^5
)/(327680*b^10) - x^23/(18*b*(a + b*x^2)^9) - (23*x^21)/(288*b^2*(a + b*x^2
)^8) - (23*x^19)/(192*b^3*(a + b*x^2)^7) - (437*x^17)/(2304*b^4*(a + b*x^2)
^6) - (7429*x^15)/(23040*b^5*(a + b*x^2)^5) - (7429*x^13)/(12288*b^6*(a + b
*x^2)^4) - (96577*x^11)/(73728*b^7*(a + b*x^2)^3) - (1062347*x^9)/(294912*b
^8*(a + b*x^2)^2) - (1062347*x^7)/(65536*b^9*(a + b*x^2)) - (7436429*a^(5/2
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(25/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{24}}{\left(a+bx^2\right)^{10}}dx = -\frac{x^{23}}{18b\left(a+bx^2\right)^9} + \frac{23\int \frac{x^{22}}{\left(a+bx^2\right)^9}dx}{18b}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} + \frac{161\int \frac{x^{20}}{\left(a+bx^2\right)^8}dx}{96b^2}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} + \frac{437\int \frac{x^{18}}{\left(a+bx^2\right)^7}dx}{192b^3}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} + \frac{7429\int \frac{x^{16}}{(a+bx}}{2304b^4}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} - \frac{7429x^1}{23040b^5(a+}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} - \frac{7429x^1}{23040b^5(a+}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} - \frac{7429x^1}{23040b^5(a+}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} - \frac{7429x^1}{23040b^5(a+}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} - \frac{7429x^1}{23040b^5(a+}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^{19}}{192b^3\left(a+bx^2\right)^7} - \frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6} - \frac{7429x^1}{23040b^5(a+}$$

$$= \frac{7436429a^2x}{65536b^{12}} - \frac{7436429ax^3}{196608b^{11}} + \frac{7436429x^5}{327680b^{10}} - \frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^1}{192b^3(a+}$$

$$= \frac{7436429a^2x}{65536b^{12}} - \frac{7436429ax^3}{196608b^{11}} + \frac{7436429x^5}{327680b^{10}} - \frac{x^{23}}{18b\left(a+bx^2\right)^9} - \frac{23x^{21}}{288b^2\left(a+bx^2\right)^8} - \frac{23x^1}{192b^3(a+}$$

Mathematica [A] time = 0.09, size = 166, normalized size = 0.72

$$\frac{\frac{\sqrt{b}x(334639305a^{11}+2900207310a^{10}bx^2+11110024926a^9b^2x^4+24648575094a^8b^3x^6+34810986496a^7b^4x^8+32314857354a^6b^5x^{10}+19562592546a^5b^6x^1}{\left(a+bx^2\right)^9}}{2949120b^{25}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24/(a + b*x^2)^10,x]
```

[Out] ((Sqrt[b]*x*(334639305*a^11 + 2900207310*a^10*b*x^2 + 11110024926*a^9*b^2*x^4 + 24648575094*a^8*b^3*x^6 + 34810986496*a^7*b^4*x^8 + 32314857354*a^6*b^5*x^10 + 19562592546*a^5*b^6*x^12 + 7323998514*a^4*b^7*x^14 + 1469632311*a^3*b^8*x^16 + 94961664*a^2*b^9*x^18 - 4521984*a*b^10*x^20 + 589824*b^11*x^22))/(a + b*x^2)^9 - 334639305*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2949120*b^(25/2))

fricas [A] time = 0.69, size = 718, normalized size = 3.11

$$\left[\frac{1179648\, b^{11}x^{23} - 9043968\, ab^{10}x^{21} + 189923328\, a^2b^9x^{19} + 2939264622\, a^3b^8x^{17} + 14647997028\, a^4b^7x^{15} + 39\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^24/(b*x^2+a)^10,x, algorithm="fricas")

[Out] [1/5898240*(1179648*b^11*x^23 - 9043968*a*b^10*x^21 + 189923328*a^2*b^9*x^19 + 2939264622*a^3*b^8*x^17 + 14647997028*a^4*b^7*x^15 + 39125185092*a^5*b^6*x^13 + 64629714708*a^6*b^5*x^11 + 69621972992*a^7*b^4*x^9 + 49297150188*a^8*b^3*x^7 + 22220049852*a^9*b^2*x^5 + 5800414620*a^10*b*x^3 + 669278610*a^11*x + 334639305*(a^2*b^9*x^18 + 9*a^3*b^8*x^16 + 36*a^4*b^7*x^14 + 84*a^5*b^6*x^12 + 126*a^6*b^5*x^10 + 126*a^7*b^4*x^8 + 84*a^8*b^3*x^6 + 36*a^9*b^2*x^4 + 9*a^10*b*x^2 + a^11)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^21*x^18 + 9*a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^18*x^12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 + 9*a^8*b^13*x^2 + a^9*b^12), 1/2949120*(589824*b^11*x^23 - 4521984*a*b^10*x^21 + 94961664*a^2*b^9*x^19 + 1469632311*a^3*b^8*x^17 + 7323998514*a^4*b^7*x^15 + 19562592546*a^5*b^6*x^13 + 32314857354*a^6*b^5*x^11 + 34810986496*a^7*b^4*x^9 + 24648575094*a^8*b^3*x^7 + 11110024926*a^9*b^2*x^5 + 2900207310*a^10*b*x^3 + 334639305*a^11*x - 334639305*(a^2*b^9*x^18 + 9*a^3*b^8*x^16 + 36*a^4*b^7*x^14 + 84*a^5*b^6*x^12 + 126*a^6*b^5*x^10 + 126*a^7*b^4*x^8 + 84*a^8*b^3*x^6 + 36*a^9*b^2*x^4 + 9*a^10*b*x^2 + a^11)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^21*x^18 + 9*a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^18*x^12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 + 9*a^8*b^13*x^2 + a^9*b^12)]

giac [A] time = 0.59, size = 162, normalized size = 0.70

$$-\frac{7436429\, a^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\, b^{12}} + \frac{314167095\, a^3b^8x^{17} + 2236176690\, a^4b^7x^{15} + 7101970722\, a^5b^6x^{13} + 13066540938\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^24/(b*x^2+a)^10,x, algorithm="giac")

[Out] -7436429/65536*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^12) + 1/2949120*(314167095*a^3*b^8*x^17 + 2236176690*a^4*b^7*x^15 + 7101970722*a^5*b^6*x^13 + 13066540938*a^6*b^5*x^11 + 15178104832*a^7*b^4*x^9 + 11372226678*a^8*b^3*x^7 + 5358651102*a^9*b^2*x^5 + 1450223310*a^10*b*x^3 + 172437705*a^11*x)/((b*x^2 + a)^9*b^12) + 1/15*(3*b^40*x^5 - 50*a*b^39*x^3 + 825*a^2*b^38*x)/b^50

maple [A] time = 0.02, size = 228, normalized size = 0.99

$$\frac{6981491 a^3 x^{17}}{65536\left(b\,x^2+a\right)^9 b^4} + \frac{74539223 a^4 x^{15}}{98304\left(b\,x^2+a\right)^9 b^5} + \frac{394553929 a^5 x^{13}}{163840\left(b\,x^2+a\right)^9 b^6} + \frac{725918941 a^6 x^{11}}{163840\left(b\,x^2+a\right)^9 b^7} + \frac{463199 a^7 x^9}{90\left(b\,x^2+a\right)^9 b^8} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24/(b*x^2+a)^10,x)
```

```
[Out] 1/5*x^5/b^10-10/3*a*x^3/b^11+55*a^2*x/b^12+3831949/65536/b^12*a^11/(b*x^2+a
)^9*x+48340777/98304/b^11*a^10/(b*x^2+a)^9*x^3+297702839/163840/b^10*a^9/(b
*x^2+a)^9*x^5+631790371/163840/b^9*a^8/(b*x^2+a)^9*x^7+463199/90/b^8*a^7/(b
*x^2+a)^9*x^9+725918941/163840/b^7*a^6/(b*x^2+a)^9*x^11+394553929/163840/b^
6*a^5/(b*x^2+a)^9*x^13+74539223/98304/b^5*a^4/(b*x^2+a)^9*x^15+6981491/6553
6/b^4*a^3/(b*x^2+a)^9*x^17-7436429/65536/b^12*a^3/(a*b)^(1/2)*arctan(1/(a*b
)^(1/2)*b*x)
```

maxima [A] time = 3.39, size = 248, normalized size = 1.07

$$\frac{314167095\,a^3b^8x^{17}+2236176690\,a^4b^7x^{15}+7101970722\,a^5b^6x^{13}+13066540938\,a^6b^5x^{11}+15178104832\,a^7b^4x^9}{2949120\left(b^{21}x^{18}+9\,ab^{20}x^{16}+36\,a^2b^{19}x^{14}+84\,a^3b^{18}x^{12}+126\,a^4b^{17}x^{10}+126\,a^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/2949120*(314167095*a^3*b^8*x^17 + 2236176690*a^4*b^7*x^15 + 7101970722*a^
5*b^6*x^13 + 13066540938*a^6*b^5*x^11 + 15178104832*a^7*b^4*x^9 + 113722266
78*a^8*b^3*x^7 + 5358651102*a^9*b^2*x^5 + 1450223310*a^10*b*x^3 + 172437705
*a^11*x)/(b^21*x^18 + 9*a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^18*x^12 +
 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 +
 9*a^8*b^13*x^2 + a^9*b^12) - 7436429/65536*a^3*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*b^12) + 1/15*(3*b^2*x^5 - 50*a*b*x^3 + 825*a^2*x)/b^12
```

mupad [B] time = 4.90, size = 241, normalized size = 1.04

$$\frac{\frac{3831949\,a^{11}\,x}{65536}+\frac{48340777\,a^{10}\,b\,x^3}{98304}+\frac{297702839\,a^9\,b^2\,x^5}{163840}+\frac{631790371\,a^8\,b^3\,x^7}{163840}+\frac{463199\,a^7\,b^4\,x^9}{90}+\frac{725918941\,a^6\,b^5\,x^{11}}{163840}+\frac{394553929\,a^5\,b^6\,x^{13}}{163840}}{a^9\,b^{12}+9\,a^8\,b^{13}\,x^2+36\,a^7\,b^{14}\,x^4+84\,a^6\,b^{15}\,x^6+126\,a^5\,b^{16}\,x^8+126\,a^4\,b^{17}\,x^{10}+84\,a^3\,b^{18}\,x^{12}+36\,a^2\,b^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24/(a + b*x^2)^10,x)
```

```
[Out] ((3831949*a^11*x)/65536 + (48340777*a^10*b*x^3)/98304 + (297702839*a^9*b^2*
x^5)/163840 + (631790371*a^8*b^3*x^7)/163840 + (463199*a^7*b^4*x^9)/90 + (7
25918941*a^6*b^5*x^11)/163840 + (394553929*a^5*b^6*x^13)/163840 + (74539223
*a^4*b^7*x^15)/98304 + (6981491*a^3*b^8*x^17)/65536)/(a^9*b^12 + b^21*x^18
+ 9*a*b^20*x^16 + 9*a^8*b^13*x^2 + 36*a^7*b^14*x^4 + 84*a^6*b^15*x^6 + 126*
a^5*b^16*x^8 + 126*a^4*b^17*x^10 + 84*a^3*b^18*x^12 + 36*a^2*b^19*x^14) + x
^5/(5*b^10) - (10*a*x^3)/(3*b^11) + (55*a^2*x)/b^12 - (7436429*a^(5/2)*atan
((b^(1/2)*x)/a^(1/2)))/(65536*b^(25/2))
```

sympy [A] time = 2.04, size = 314, normalized size = 1.36

$$\frac{55a^2x}{b^{12}}-\frac{10ax^3}{3b^{11}}+\frac{7436429\sqrt{-\frac{a^5}{b^{25}}}\log\left(x-\frac{b^{12}\sqrt{-\frac{a^5}{b^{25}}}}{a^2}\right)}{131072}-\frac{7436429\sqrt{-\frac{a^5}{b^{25}}}\log\left(x+\frac{b^{12}\sqrt{-\frac{a^5}{b^{25}}}}{a^2}\right)}{131072}+\frac{172437705a^{11}x+}{2949120a^9b^{12}+26542}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24/(b*x**2+a)**10,x)
```

```
[Out] 55*a**2*x/b**12 - 10*a*x**3/(3*b**11) + 7436429*sqrt(-a**5/b**25)*log(x - b
**12*sqrt(-a**5/b**25)/a**2)/131072 - 7436429*sqrt(-a**5/b**25)*log(x + b**
```

```
12*sqrt(-a**5/b**25)/a**2)/131072 + (172437705*a**11*x + 1450223310*a**10*b
*x**3 + 5358651102*a**9*b**2*x**5 + 11372226678*a**8*b**3*x**7 + 1517810483
2*a**7*b**4*x**9 + 13066540938*a**6*b**5*x**11 + 7101970722*a**5*b**6*x**13
 + 2236176690*a**4*b**7*x**15 + 314167095*a**3*b**8*x**17)/(2949120*a**9*b*
*12 + 26542080*a**8*b**13*x**2 + 106168320*a**7*b**14*x**4 + 247726080*a**6
*b**15*x**6 + 371589120*a**5*b**16*x**8 + 371589120*a**4*b**17*x**10 + 2477
26080*a**3*b**18*x**12 + 106168320*a**2*b**19*x**14 + 26542080*a*b**20*x**1
6 + 2949120*b**21*x**18) + x**5/(5*b**10)
```

3.210 $\int \frac{x^{22}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=218

$$\frac{1616615a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536b^{23/2}}-\frac{1616615ax}{65536b^{11}}-\frac{323323x^5}{65536b^9\left(a+bx^2\right)}-\frac{46189x^7}{32768b^8\left(a+bx^2\right)^2}-\frac{46189x^9}{73728b^7\left(a+bx^2\right)^3}-\frac{4199}{12288b^6\left(a\right.}$$

```
[Out] -1616615/65536*a*x/b^11+1616615/196608*x^3/b^10-1/18*x^21/b/(b*x^2+a)^9-7/9
6*x^19/b^2/(b*x^2+a)^8-19/192*x^17/b^3/(b*x^2+a)^7-323/2304*x^15/b^4/(b*x^2
+a)^6-323/1536*x^13/b^5/(b*x^2+a)^5-4199/12288*x^11/b^6/(b*x^2+a)^4-46189/7
3728*x^9/b^7/(b*x^2+a)^3-46189/32768*x^7/b^8/(b*x^2+a)^2-323323/65536*x^5/b
^9/(b*x^2+a)+1616615/65536*a^(3/2)*arctan(x*b^(1/2)/a^(1/2))/b^(23/2)
```

Rubi [A] time = 0.14, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{1616615a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536b^{23/2}}-\frac{7x^{19}}{96b^2\left(a+bx^2\right)^8}-\frac{19x^{17}}{192b^3\left(a+bx^2\right)^7}-\frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6}-\frac{323x^{13}}{1536b^5\left(a+bx^2\right)^5}-\frac{4199}{12288b^6\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^22/(a + b*x^2)^10,x]
```

```
[Out] (-1616615*a*x)/(65536*b^11) + (1616615*x^3)/(196608*b^10) - x^21/(18*b*(a +
 b*x^2)^9) - (7*x^19)/(96*b^2*(a + b*x^2)^8) - (19*x^17)/(192*b^3*(a + b*x^
2)^7) - (323*x^15)/(2304*b^4*(a + b*x^2)^6) - (323*x^13)/(1536*b^5*(a + b*x
^2)^5) - (4199*x^11)/(12288*b^6*(a + b*x^2)^4) - (46189*x^9)/(73728*b^7*(a
+ b*x^2)^3) - (46189*x^7)/(32768*b^8*(a + b*x^2)^2) - (323323*x^5)/(65536*b
^9*(a + b*x^2)) + (1616615*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(2
3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{22}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{21}}{18b\left(a+bx^2\right)^9} + \frac{7\int \frac{x^{20}}{\left(a+bx^2\right)^9}\,dx}{6b}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} + \frac{133\int \frac{x^{18}}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} + \frac{323\int \frac{x^{16}}{\left(a+bx^2\right)^7}\,dx}{192b^3}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} + \frac{1615\int \frac{x}{(a+}}{768b}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x}{1536b^5(a}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x}{1536b^5(a}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x}{1536b^5(a}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x}{1536b^5(a}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x}{1536b^5(a}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x}{1536b^5(a}$$

$$= -\frac{1616615ax}{65536b^{11}} + \frac{1616615x^3}{196608b^{10}} - \frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{}{230}$$

$$= -\frac{1616615ax}{65536b^{11}} + \frac{1616615x^3}{196608b^{10}} - \frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{}{230}$$

Mathematica [A] time = 0.08, size = 155, normalized size = 0.71

$$\frac{14549535a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right) + \frac{\sqrt{b}\,x\left(-14549535a^{10}-126095970a^9bx^2-483044562a^8b^2x^4-1071677178a^7b^3x^6-1513521152a^6b^4x^8-140499\right.}{(a+bx}}{589824b^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^22/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[b]*x*(-14549535*a^10 - 126095970*a^9*b*x^2 - 483044562*a^8*b^2*x^4 -
 1071677178*a^7*b^3*x^6 - 1513521152*a^6*b^4*x^8 - 1404993798*a^5*b^5*x^10
- 850547502*a^4*b^6*x^12 - 318434718*a^3*b^7*x^14 - 63897057*a^2*b^8*x^16 -
 4128768*a*b^9*x^18 + 196608*b^10*x^20))/(a + b*x^2)^9 + 14549535*a^(3/2)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(589824*b^(23/2))
```

fricas [A] time = 0.84, size = 692, normalized size = 3.17

$$\left[\frac{393216\, b^{10}x^{21} - 8257536\, ab^{9}x^{19} - 127794114\, a^{2}b^{8}x^{17} - 636869436\, a^{3}b^{7}x^{15} - 1701095004\, a^{4}b^{6}x^{13} - 280998759}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^22/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(393216*b^10*x^21 - 8257536*a*b^9*x^19 - 127794114*a^2*b^8*x^17
- 636869436*a^3*b^7*x^15 - 1701095004*a^4*b^6*x^13 - 2809987596*a^5*b^5*x^1
1 - 3027042304*a^6*b^4*x^9 - 2143354356*a^7*b^3*x^7 - 966089124*a^8*b^2*x^5
 - 252191940*a^9*b*x^3 - 29099070*a^10*x + 14549535*(a*b^9*x^18 + 9*a^2*b^8
*x^16 + 36*a^3*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 126*a^6*b^4*
x^8 + 84*a^7*b^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 + a^10)*sqrt(-a/b)*log(
(b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^20*x^18 + 9*a*b^19*x^16 + 3
6*a^2*b^18*x^14 + 84*a^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 +
 84*a^6*b^14*x^6 + 36*a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11), 1/589824*(
196608*b^10*x^21 - 4128768*a*b^9*x^19 - 63897057*a^2*b^8*x^17 - 318434718*a
^3*b^7*x^15 - 850547502*a^4*b^6*x^13 - 1404993798*a^5*b^5*x^11 - 1513521152
*a^6*b^4*x^9 - 1071677178*a^7*b^3*x^7 - 483044562*a^8*b^2*x^5 - 126095970*a
^9*b*x^3 - 14549535*a^10*x + 14549535*(a*b^9*x^18 + 9*a^2*b^8*x^16 + 36*a^3
*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 126*a^6*b^4*x^8 + 84*a^7*b
^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 + a^10)*sqrt(a/b)*arctan(b*x*sqrt(a/b
)/a))/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^18*x^14 + 84*a^3*b^17*x^12 + 12
6*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b^14*x^6 + 36*a^7*b^13*x^4 + 9*
a^8*b^12*x^2 + a^9*b^11)]
```

giac [A] time = 0.65, size = 150, normalized size = 0.69

$$\frac{1616615\, a^{2} \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\, \sqrt{ab}\, b^{11}} - \frac{17890785\, a^{2}b^{8}x^{17} + 122613150\, a^{3}b^{7}x^{15} + 379867950\, a^{4}b^{6}x^{13} + 686588166\, a^{5}b^{5}x^{11} +}{58982}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^22/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1616615/65536*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^11) - 1/589824*(178907
85*a^2*b^8*x^17 + 122613150*a^3*b^7*x^15 + 379867950*a^4*b^6*x^13 + 6865881
66*a^5*b^5*x^11 + 786857984*a^6*b^4*x^9 + 583302906*a^7*b^3*x^7 + 272477394
*a^8*b^2*x^5 + 73208418*a^9*b*x^3 + 8651295*a^10*x)/((b*x^2 + a)^9*b^11) +
1/3*(b^20*x^3 - 30*a*b^19*x)/b^30
```

maple [A] time = 0.02, size = 217, normalized size = 1.00

$$-\frac{1987865a^{2}x^{17}}{65536\left(b\,x^{2}+a\right)^{9} b^{3}} - \frac{20435525a^{3}x^{15}}{98304\left(b\,x^{2}+a\right)^{9} b^{4}} - \frac{21103775a^{4}x^{13}}{32768\left(b\,x^{2}+a\right)^{9} b^{5}} - \frac{38143787a^{5}x^{11}}{32768\left(b\,x^{2}+a\right)^{9} b^{6}} - \frac{24013a^{6}x^{9}}{18\left(b\,x^{2}+a\right)^{9} b^{7}} - \frac{32}{3276}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^22/(b*x^2+a)^10,x)
```

```
[Out] 1/3*x^3/b^10-10*a*x/b^11-961255/65536/b^11*a^10/(b*x^2+a)^9*x-12201403/9830
4/b^10*a^9/(b*x^2+a)^9*x^3-15137633/32768/b^9*a^8/(b*x^2+a)^9*x^5-32405717/
32768/b^8*a^7/(b*x^2+a)^9*x^7-24013/18/b^7*a^6/(b*x^2+a)^9*x^9-38143787/327
68/b^6*a^5/(b*x^2+a)^9*x^11-21103775/32768/b^5*a^4/(b*x^2+a)^9*x^13-2043552
5/98304/b^4*a^3/(b*x^2+a)^9*x^15-1987865/65536/b^3*a^2/(b*x^2+a)^9*x^17+161
6615/65536/b^11*a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.25, size = 236, normalized size = 1.08

$$-\frac{17890785\,a^2b^8x^{17}+122613150\,a^3b^7x^{15}+379867950\,a^4b^6x^{13}+686588166\,a^5b^5x^{11}+786857984\,a^6b^4x^9+5}{589824\left(b^{20}x^{18}+9\,ab^{19}x^{16}+36\,a^2b^{18}x^{14}+84\,a^3b^{17}x^{12}+126\,a^4b^{16}x^{10}+126\,a^5b^{15}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^22/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/589824*(17890785*a^2*b^8*x^17 + 122613150*a^3*b^7*x^15 + 379867950*a^4*b
^6*x^13 + 686588166*a^5*b^5*x^11 + 786857984*a^6*b^4*x^9 + 583302906*a^7*b^
3*x^7 + 272477394*a^8*b^2*x^5 + 73208418*a^9*b*x^3 + 8651295*a^10*x)/(b^20*
x^18 + 9*a*b^19*x^16 + 36*a^2*b^18*x^14 + 84*a^3*b^17*x^12 + 126*a^4*b^16*x
^10 + 126*a^5*b^15*x^8 + 84*a^6*b^14*x^6 + 36*a^7*b^13*x^4 + 9*a^8*b^12*x^2
 + a^9*b^11) + 1616615/65536*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^11) + 1
/3*(b*x^3 - 30*a*x)/b^11
```

mupad [B] time = 0.40, size = 231, normalized size = 1.06

$$\frac{x^3}{3\,b^{10}}-\frac{\frac{961255\,a^{10}\,x}{65536}+\frac{12201403\,a^9\,b\,x^3}{98304}+\frac{15137633\,a^8\,b^2\,x^5}{32768}+\frac{32405717\,a^7\,b^3\,x^7}{32768}+\frac{24013\,a^6\,b^4\,x^9}{18}+\frac{38143787\,a^5\,b^5\,x^{11}}{32768}+\frac{21103775\,a^4\,b}{32768}}{a^9\,b^{11}+9\,a^8\,b^{12}\,x^2+36\,a^7\,b^{13}\,x^4+84\,a^6\,b^{14}\,x^6+126\,a^5\,b^{15}\,x^8+126\,a^4\,b^{16}\,x^{10}+84\,a^3\,b^{17}\,x^{12}+36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^22/(a + b*x^2)^10,x)
```

```
[Out] x^3/(3*b^10) - ((961255*a^10*x)/65536 + (12201403*a^9*b*x^3)/98304 + (15137
633*a^8*b^2*x^5)/32768 + (32405717*a^7*b^3*x^7)/32768 + (24013*a^6*b^4*x^9)
/18 + (38143787*a^5*b^5*x^11)/32768 + (21103775*a^4*b^6*x^13)/32768 + (2043
5525*a^3*b^7*x^15)/98304 + (1987865*a^2*b^8*x^17)/65536)/(a^9*b^11 + b^20*x
^18 + 9*a*b^19*x^16 + 9*a^8*b^12*x^2 + 36*a^7*b^13*x^4 + 84*a^6*b^14*x^6 +
126*a^5*b^15*x^8 + 126*a^4*b^16*x^10 + 84*a^3*b^17*x^12 + 36*a^2*b^18*x^14)
 + (1616615*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*b^(23/2)) - (10*a*x)/
b^11
```

sympy [A] time = 1.92, size = 299, normalized size = 1.37

$$-\frac{10ax}{b^{11}}-\frac{1616615\sqrt{-\frac{a^3}{b^{23}}}\,\log\left(x-\frac{b^{11}\sqrt{-\frac{a^3}{b^{23}}}}{a}\right)}{131072}+\frac{1616615\sqrt{-\frac{a^3}{b^{23}}}\,\log\left(x+\frac{b^{11}\sqrt{-\frac{a^3}{b^{23}}}}{a}\right)}{131072}+\frac{-8651295a^{10}x-732}{589824a^9b^{11}+5308416a^8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**22/(b*x**2+a)**10,x)
```

```
[Out] -10*a*x/b**11 - 1616615*sqrt(-a**3/b**23)*log(x - b**11*sqrt(-a**3/b**23)/a
)/131072 + 1616615*sqrt(-a**3/b**23)*log(x + b**11*sqrt(-a**3/b**23)/a)/131
072 + (-8651295*a**10*x - 73208418*a**9*b*x**3 - 272477394*a**8*b**2*x**5 -
 583302906*a**7*b**3*x**7 - 786857984*a**6*b**4*x**9 - 686588166*a**5*b**5*
x**11 - 379867950*a**4*b**6*x**13 - 122613150*a**3*b**7*x**15 - 17890785*a*
```

```
*2*b**8*x**17)/(589824*a**9*b**11 + 5308416*a**8*b**12*x**2 + 21233664*a**7
*b**13*x**4 + 49545216*a**6*b**14*x**6 + 74317824*a**5*b**15*x**8 + 7431782
4*a**4*b**16*x**10 + 49545216*a**3*b**17*x**12 + 21233664*a**2*b**18*x**14
+ 5308416*a*b**19*x**16 + 589824*b**20*x**18) + x**3/(3*b**10)
```

3.211 $\int \frac{x^{20}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=207

$$-\frac{230945\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536b^{21/2}}-\frac{230945x^3}{196608b^9(a+bx^2)}-\frac{46189x^5}{98304b^8(a+bx^2)^2}-\frac{46189x^7}{172032b^7(a+bx^2)^3}-\frac{46189x^9}{258048b^6(a+bx^2)$$

```
[Out] 230945/65536*x/b^10-1/18*x^19/b/(b*x^2+a)^9-19/288*x^17/b^2/(b*x^2+a)^8-323
/4032*x^15/b^3/(b*x^2+a)^7-1615/16128*x^13/b^4/(b*x^2+a)^6-4199/32256*x^11/
b^5/(b*x^2+a)^5-46189/258048*x^9/b^6/(b*x^2+a)^4-46189/172032*x^7/b^7/(b*x^
2+a)^3-46189/98304*x^5/b^8/(b*x^2+a)^2-230945/196608*x^3/b^9/(b*x^2+a)-2309
45/65536*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(21/2)
```

Rubi [A] time = 0.12, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {288, 321, 205}

$$-\frac{19x^{17}}{288b^2(a+bx^2)^8}-\frac{323x^{15}}{4032b^3(a+bx^2)^7}-\frac{1615x^{13}}{16128b^4(a+bx^2)^6}-\frac{4199x^{11}}{32256b^5(a+bx^2)^5}-\frac{46189x^9}{258048b^6(a+bx^2)^4}-\frac{}{1720}$$

Antiderivative was successfully verified.

```
[In] Int[x^20/(a + b*x^2)^10,x]
```

```
[Out] (230945*x)/(65536*b^10) - x^19/(18*b*(a + b*x^2)^9) - (19*x^17)/(288*b^2*(a
 + b*x^2)^8) - (323*x^15)/(4032*b^3*(a + b*x^2)^7) - (1615*x^13)/(16128*b^4
*(a + b*x^2)^6) - (4199*x^11)/(32256*b^5*(a + b*x^2)^5) - (46189*x^9)/(2580
48*b^6*(a + b*x^2)^4) - (46189*x^7)/(172032*b^7*(a + b*x^2)^3) - (46189*x^5
)/(98304*b^8*(a + b*x^2)^2) - (230945*x^3)/(196608*b^9*(a + b*x^2)) - (2309
45*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(21/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{20}}{\left(a+bx^2\right)^{10}}\,dx &= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}+\frac{19\int \frac{x^{18}}{\left(a+bx^2\right)^9}\,dx}{18b} \\
&= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}+\frac{323\int \frac{x^{16}}{\left(a+bx^2\right)^8}\,dx}{288b^2} \\
&= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}+\frac{1615\int \frac{x^{14}}{\left(a+bx^2\right)^7}\,dx}{1344b^3} \\
&= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}+\frac{20995\int}{1612} \\
&= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{4199}{32256b^5(} \\
&= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{4199}{32256b^5(} \\
&= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{4199}{32256b^5(} \\
&= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{4199}{32256b^5(} \\
&= -\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{4199}{32256b^5(} \\
&= \frac{230945x}{65536b^{10}}-\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}- \\
&= \frac{230945x}{65536b^{10}}-\frac{x^{19}}{18b\left(a+bx^2\right)^9}-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 144, normalized size = 0.70

$$
\frac{\frac{\sqrt{b}\,x\left(14549535a^9+126095970a^8bx^2+483044562a^7b^2x^4+1071677178a^6b^3x^6+1513521152a^5b^4x^8+1404993798a^4b^5x^{10}+850547502a^3b^6x^{12}+31843471\right.}{\left(a+bx^2\right)^9}}{4128768b^{21/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^20/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[b]*x*(14549535*a^9 + 126095970*a^8*b*x^2 + 483044562*a^7*b^2*x^4 + 1
071677178*a^6*b^3*x^6 + 1513521152*a^5*b^4*x^8 + 1404993798*a^4*b^5*x^10 +
850547502*a^3*b^6*x^12 + 318434718*a^2*b^7*x^14 + 63897057*a*b^8*x^16 + 412
```

```
8768*b^9*x^18))/(a + b*x^2)^9 - 14549535*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(4128768*b^(21/2))
```

fricas [A] time = 1.05, size = 664, normalized size = 3.21

$$\left[\frac{8257536\, b^9 x^{19} + 127794114\, ab^8 x^{17} + 636869436\, a^2 b^7 x^{15} + 1701095004\, a^3 b^6 x^{13} + 2809987596\, a^4 b^5 x^{11} + 30\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(8257536*b^9*x^19 + 127794114*a*b^8*x^17 + 636869436*a^2*b^7*x^1
5 + 1701095004*a^3*b^6*x^13 + 2809987596*a^4*b^5*x^11 + 3027042304*a^5*b^4*
x^9 + 2143354356*a^6*b^3*x^7 + 966089124*a^7*b^2*x^5 + 252191940*a^8*b*x^3
+ 29099070*a^9*x + 14549535*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84
*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^
7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a
)/(b*x^2 + a)))/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a^3*b^16
*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36*a^7*b^1
2*x^4 + 9*a^8*b^11*x^2 + a^9*b^10), 1/4128768*(4128768*b^9*x^19 + 63897057*
a*b^8*x^17 + 318434718*a^2*b^7*x^15 + 850547502*a^3*b^6*x^13 + 1404993798*a
^4*b^5*x^11 + 1513521152*a^5*b^4*x^9 + 1071677178*a^6*b^3*x^7 + 483044562*a
^7*b^2*x^5 + 126095970*a^8*b*x^3 + 14549535*a^9*x - 14549535*(b^9*x^18 + 9*
a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5
*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a/b)*a
rctan(b*x*sqrt(a/b)/a))/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*
a^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36
*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^10)]
```

giac [A] time = 0.64, size = 131, normalized size = 0.63

$$-\frac{230945\, a \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\, \sqrt{ab}\, b^{10}} + \frac{x}{b^{10}} + \frac{26738145\, ab^8 x^{17} + 169799070\, a^2 b^7 x^{15} + 503730990\, a^3 b^6 x^{13} + 884769030\, a^4 b\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -230945/65536*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^10) + x/b^10 + 1/4128768
*(26738145*a*b^8*x^17 + 169799070*a^2*b^7*x^15 + 503730990*a^3*b^6*x^13 + 8
84769030*a^4*b^5*x^11 + 993296384*a^5*b^4*x^9 + 724860666*a^6*b^3*x^7 + 334
408914*a^7*b^2*x^5 + 88937058*a^8*b*x^3 + 10420767*a^9*x)/((b*x^2 + a)^9*b^
10)
```

maple [A] time = 0.02, size = 203, normalized size = 0.98

$$\frac{424415 a\, x^{17}}{65536 \left(b\, x^2+a\right)^9 b^2} + \frac{4042835 a^2 x^{15}}{98304 \left(b\, x^2+a\right)^9 b^3} + \frac{3997865 a^3 x^{13}}{32768 \left(b\, x^2+a\right)^9 b^4} + \frac{49153835 a^4 x^{11}}{229376 \left(b\, x^2+a\right)^9 b^5} + \frac{30313 a^5 x^9}{126 \left(b\, x^2+a\right)^9 b^6} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20/(b*x^2+a)^10,x)
```

```
[Out] x/b^10+165409/65536/b^10*a^9/(b*x^2+a)^9*x+2117549/98304/b^9*a^8/(b*x^2+a)^
9*x^3+2654039/32768/b^8*a^7/(b*x^2+a)^9*x^5+40270037/229376/b^7*a^6/(b*x^2+
```

```
a)^9*x^7+30313/126/b^6*a^5/(b*x^2+a)^9*x^9+49153835/229376/b^5*a^4/(b*x^2+a
)^9*x^11+3997865/32768/b^4*a^3/(b*x^2+a)^9*x^13+4042835/98304/b^3*a^2/(b*x^
2+a)^9*x^15+424415/65536/b^2*a/(b*x^2+a)^9*x^17-230945/65536/b^10*a/(a*b)^(
1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.17, size = 222, normalized size = 1.07

$$\frac{26738145\,ab^{8}x^{17}+169799070\,a^{2}b^{7}x^{15}+503730990\,a^{3}b^{6}x^{13}+884769030\,a^{4}b^{5}x^{11}+993296384\,a^{5}b^{4}x^{9}+72486}{4128768\left(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a^{2}b^{17}x^{14}+84\,a^{3}b^{16}x^{12}+126\,a^{4}b^{15}x^{10}+126\,a^{5}b^{14}x^{8}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(26738145*a*b^8*x^17 + 169799070*a^2*b^7*x^15 + 503730990*a^3*b^6
*x^13 + 884769030*a^4*b^5*x^11 + 993296384*a^5*b^4*x^9 + 724860666*a^6*b^3*
x^7 + 334408914*a^7*b^2*x^5 + 88937058*a^8*b*x^3 + 10420767*a^9*x)/(b^19*x^
18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15*x^1
0 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*x^2 +
 a^9*b^10) - 230945/65536*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^10) + x/b^10
```

mupad [B] time = 0.44, size = 218, normalized size = 1.05

$$\frac{\frac{165409\,a^9\,x}{65536}+\frac{2117549\,a^8\,b\,x^3}{98304}+\frac{2654039\,a^7\,b^2\,x^5}{32768}+\frac{40270037\,a^6\,b^3\,x^7}{229376}+\frac{30313\,a^5\,b^4\,x^9}{126}+\frac{49153835\,a^4\,b^5\,x^{11}}{229376}+\frac{3997865\,a^3\,b^6\,x^{13}}{32768}+\frac{404283}{98}}{a^9\,b^{10}+9\,a^8\,b^{11}\,x^2+36\,a^7\,b^{12}\,x^4+84\,a^6\,b^{13}\,x^6+126\,a^5\,b^{14}\,x^8+126\,a^4\,b^{15}\,x^{10}+84\,a^3\,b^{16}\,x^{12}+36\,a^2\,b^{17}\,x^{14}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20/(a + b*x^2)^10,x)
```

```
[Out] ((165409*a^9*x)/65536 + (2117549*a^8*b*x^3)/98304 + (424415*a*b^8*x^17)/655
36 + (2654039*a^7*b^2*x^5)/32768 + (40270037*a^6*b^3*x^7)/229376 + (30313*a
^5*b^4*x^9)/126 + (49153835*a^4*b^5*x^11)/229376 + (3997865*a^3*b^6*x^13)/3
2768 + (4042835*a^2*b^7*x^15)/98304)/(a^9*b^10 + b^19*x^18 + 9*a*b^18*x^16
+ 9*a^8*b^11*x^2 + 36*a^7*b^12*x^4 + 84*a^6*b^13*x^6 + 126*a^5*b^14*x^8 + 1
26*a^4*b^15*x^10 + 84*a^3*b^16*x^12 + 36*a^2*b^17*x^14) + x/b^10 - (230945*
a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*b^(21/2))
```

sympy [A] time = 1.82, size = 274, normalized size = 1.32

$$\frac{230945\sqrt{-\frac{a}{b^{21}}}\,\log\left(-b^{10}\sqrt{-\frac{a}{b^{21}}}+x\right)}{131072}-\frac{230945\sqrt{-\frac{a}{b^{21}}}\,\log\left(b^{10}\sqrt{-\frac{a}{b^{21}}}+x\right)}{131072}+\frac{10420767a^{9}x+8893705}{4128768a^{9}b^{10}+37158912a^{8}b^{11}x^{2}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**20/(b*x**2+a)**10,x)
```

```
[Out] 230945*sqrt(-a/b**21)*log(-b**10*sqrt(-a/b**21) + x)/131072 - 230945*sqrt(-
a/b**21)*log(b**10*sqrt(-a/b**21) + x)/131072 + (10420767*a**9*x + 88937058
*a**8*b*x**3 + 334408914*a**7*b**2*x**5 + 724860666*a**6*b**3*x**7 + 993296
384*a**5*b**4*x**9 + 884769030*a**4*b**5*x**11 + 503730990*a**3*b**6*x**13
+ 169799070*a**2*b**7*x**15 + 26738145*a*b**8*x**17)/(4128768*a**9*b**10 +
37158912*a**8*b**11*x**2 + 148635648*a**7*b**12*x**4 + 346816512*a**6*b**13
*x**6 + 520224768*a**5*b**14*x**8 + 520224768*a**4*b**15*x**10 + 346816512*
a**3*b**16*x**12 + 148635648*a**2*b**17*x**14 + 37158912*a*b**18*x**16 + 41
28768*b**19*x**18) + x/b**10
```

3.212 $\int \frac{x^{18}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=197

$$\frac{12155\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536\sqrt{a}\,b^{19/2}}-\frac{12155x}{65536b^9\left(a+bx^2\right)}-\frac{12155x^3}{98304b^8\left(a+bx^2\right)^2}-\frac{2431x^5}{24576b^7\left(a+bx^2\right)^3}-\frac{2431x^7}{28672b^6\left(a+bx^2\right)^4}-\frac{}{3225}$$

```
[Out] -1/18*x^17/b/(b*x^2+a)^9-17/288*x^15/b^2/(b*x^2+a)^8-85/1344*x^13/b^3/(b*x^
2+a)^7-1105/16128*x^11/b^4/(b*x^2+a)^6-2431/32256*x^9/b^5/(b*x^2+a)^5-2431/
28672*x^7/b^6/(b*x^2+a)^4-2431/24576*x^5/b^7/(b*x^2+a)^3-12155/98304*x^3/b^
8/(b*x^2+a)^2-12155/65536*x/b^9/(b*x^2+a)+12155/65536*arctan(x*b^(1/2)/a^(1
/2))/b^(19/2)/a^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {288, 205}

$$-\frac{17x^{15}}{288b^2\left(a+bx^2\right)^8}-\frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7}-\frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6}-\frac{2431x^9}{32256b^5\left(a+bx^2\right)^5}-\frac{2431x^7}{28672b^6\left(a+bx^2\right)^4}-\frac{}{24576}$$

Antiderivative was successfully verified.

```
[In] Int[x^18/(a + b*x^2)^10,x]
```

```
[Out] -x^17/(18*b*(a + b*x^2)^9) - (17*x^15)/(288*b^2*(a + b*x^2)^8) - (85*x^13)/
(1344*b^3*(a + b*x^2)^7) - (1105*x^11)/(16128*b^4*(a + b*x^2)^6) - (2431*x^
9)/(32256*b^5*(a + b*x^2)^5) - (2431*x^7)/(28672*b^6*(a + b*x^2)^4) - (2431
*x^5)/(24576*b^7*(a + b*x^2)^3) - (12155*x^3)/(98304*b^8*(a + b*x^2)^2) - (
12155*x)/(65536*b^9*(a + b*x^2)) + (12155*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(655
36*Sqrt[a]*b^(19/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{18}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{17}}{18b\left(a+bx^2\right)^9} + \frac{17\int \frac{x^{16}}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} + \frac{85\int \frac{x^{14}}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} + \frac{1105\int \frac{x^{12}}{\left(a+bx^2\right)^7}\,dx}{1344b^3}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} + \frac{12155\int \frac{}{(a}}{1612}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{243}{32256b^5(}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{243}{32256b^5(}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{243}{32256b^5(}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{243}{32256b^5(}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{243}{32256b^5(}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{243}{32256b^5(}$$

Mathematica [A] time = 0.08, size = 134, normalized size = 0.68

$$\frac{\frac{765765\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{b}x\left(765765a^8+6636630a^7bx^2+25423398a^6b^2x^4+56404062a^5b^3x^6+79659008a^4b^4x^8+73947042a^3b^5x^{10}+44765658a^2b^6x^{12}+\right.}{\left(a+bx^2\right)^9}}{4128768b^{19/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^18/(a + b*x^2)^10,x]
```

```
[Out] (-((Sqrt[b]*x*(765765*a^8 + 6636630*a^7*b*x^2 + 25423398*a^6*b^2*x^4 + 5640
4062*a^5*b^3*x^6 + 79659008*a^4*b^4*x^8 + 73947042*a^3*b^5*x^10 + 44765658*
a^2*b^6*x^12 + 16759722*a*b^7*x^14 + 3363003*b^8*x^16))/(a + b*x^2)^9) + (7
65765*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a])/(4128768*b^(19/2))
```

fricas [A] time = 0.88, size = 650, normalized size = 3.30

$$\left[-\frac{6726006\,ab^9x^{17}+33519444\,a^2b^8x^{15}+89531316\,a^3b^7x^{13}+147894084\,a^4b^6x^{11}+159318016\,a^5b^5x^9+11280}{8257536\left(ab^{1}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^18/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [-1/8257536*(6726006*a*b^9*x^17 + 33519444*a^2*b^8*x^15 + 89531316*a^3*b^7*
x^13 + 147894084*a^4*b^6*x^11 + 159318016*a^5*b^5*x^9 + 112808124*a^6*b^4*x
^7 + 50846796*a^7*b^3*x^5 + 13273260*a^8*b^2*x^3 + 1531530*a^9*b*x + 765765
*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5
*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a
^9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^19*x^18
+ 9*a^2*b^18*x^16 + 36*a^3*b^17*x^14 + 84*a^4*b^16*x^12 + 126*a^5*b^15*x^10
 + 126*a^6*b^14*x^8 + 84*a^7*b^13*x^6 + 36*a^8*b^12*x^4 + 9*a^9*b^11*x^2 +
a^10*b^10), -1/4128768*(3363003*a*b^9*x^17 + 16759722*a^2*b^8*x^15 + 447656
58*a^3*b^7*x^13 + 73947042*a^4*b^6*x^11 + 79659008*a^5*b^5*x^9 + 56404062*a
^6*b^4*x^7 + 25423398*a^7*b^3*x^5 + 6636630*a^8*b^2*x^3 + 765765*a^9*b*x -
765765*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a
^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x
^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^19*x^18 + 9*a^2*b^18*x^16 +
 36*a^3*b^17*x^14 + 84*a^4*b^16*x^12 + 126*a^5*b^15*x^10 + 126*a^6*b^14*x^8
 + 84*a^7*b^13*x^6 + 36*a^8*b^12*x^4 + 9*a^9*b^11*x^2 + a^10*b^10)]
```

giac [A] time = 0.62, size = 122, normalized size = 0.62

$$\frac{12155\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,b^9}-\frac{3363003\,b^8x^{17}+16759722\,ab^7x^{15}+44765658\,a^2b^6x^{13}+73947042\,a^3b^5x^{11}+7965900}{4128768\left(bx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^18/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 12155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^9) - 1/4128768*(3363003*b^8*
x^17 + 16759722*a*b^7*x^15 + 44765658*a^2*b^6*x^13 + 73947042*a^3*b^5*x^11
+ 79659008*a^4*b^4*x^9 + 56404062*a^5*b^3*x^7 + 25423398*a^6*b^2*x^5 + 6636
630*a^7*b*x^3 + 765765*a^8*x)/((b*x^2 + a)^9*b^9)
```

maple [A] time = 0.02, size = 124, normalized size = 0.63

$$\frac{12155\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\;b^9}+\frac{-\frac{53381x^{17}}{65536b}-\frac{399041a\,x^{15}}{98304b^2}-\frac{355283a^2x^{13}}{32768b^3}-\frac{4108169a^3x^{11}}{229376b^4}-\frac{2431a^4x^9}{126b^5}-\frac{3133559a^5x^7}{229376b^6}-\frac{201773a^6x^5}{32768b^7}-\frac{15}{9}}{\left(b\,x^2+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^18/(b*x^2+a)^10,x)
```

```
[Out] (-12155/65536*a^8/b^9*x-158015/98304*a^7/b^8*x^3-201773/32768*a^6/b^7*x^5-3
133559/229376*a^5/b^6*x^7-2431/126*a^4/b^5*x^9-4108169/229376*a^3/b^4*x^11-
355283/32768*a^2/b^3*x^13-399041/98304*a/b^2*x^15-53381/65536/b*x^17)/(b*x^
2+a)^9+12155/65536/b^9/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.17, size = 213, normalized size = 1.08

$$-\frac{3363003\,b^8x^{17}+16759722\,ab^7x^{15}+44765658\,a^2b^6x^{13}+73947042\,a^3b^5x^{11}+79659008\,a^4b^4x^9+56404062}{4128768\left(b^{18}x^{18}+9\,ab^{17}x^{16}+36\,a^2b^{16}x^{14}+84\,a^3b^{15}x^{12}+126\,a^4b^{14}x^{10}+126\,a^5b^{13}x^8+84\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^18/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/4128768*(3363003*b^8*x^17 + 16759722*a*b^7*x^15 + 44765658*a^2*b^6*x^13
+ 73947042*a^3*b^5*x^11 + 79659008*a^4*b^4*x^9 + 56404062*a^5*b^3*x^7 + 254
23398*a^6*b^2*x^5 + 6636630*a^7*b*x^3 + 765765*a^8*x)/(b^18*x^18 + 9*a*b^17
*x^16 + 36*a^2*b^16*x^14 + 84*a^3*b^15*x^12 + 126*a^4*b^14*x^10 + 126*a^5*b
^13*x^8 + 84*a^6*b^12*x^6 + 36*a^7*b^11*x^4 + 9*a^8*b^10*x^2 + a^9*b^9) + 1
2155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^9)
```

mupad [B] time = 4.96, size = 210, normalized size = 1.07

$$\frac{12155\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,\sqrt{a}\,b^{19/2}}-\frac{\frac{53381\,x^{17}}{65536\,b}+\frac{399041\,a\,x^{15}}{98304\,b^2}+\frac{12155\,a^8\,x}{65536\,b^9}+\frac{355283\,a^2\,x^{13}}{32768\,b^3}+\frac{4108169\,a^3\,x^{11}}{229376\,b^4}+\frac{2431\,a^4\,x^9}{126\,b^5}+\frac{3133559\,a^5\,x^7}{229376\,b^6}+\cdots}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^18/(a + b*x^2)^10,x)
```

```
[Out] (12155*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(1/2)*b^(19/2)) - ((53381*x^17)/
(65536*b) + (399041*a*x^15)/(98304*b^2) + (12155*a^8*x)/(65536*b^9) + (3552
83*a^2*x^13)/(32768*b^3) + (4108169*a^3*x^11)/(229376*b^4) + (2431*a^4*x^9)
/(126*b^5) + (3133559*a^5*x^7)/(229376*b^6) + (201773*a^6*x^5)/(32768*b^7)
+ (158015*a^7*x^3)/(98304*b^8))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^1
6 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 +
84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

sympy [A] time = 1.55, size = 277, normalized size = 1.41

$$-\frac{12155\sqrt{-\frac{1}{ab^{19}}}\log\left(-ab^9\sqrt{-\frac{1}{ab^{19}}}+x\right)}{131072}+\frac{12155\sqrt{-\frac{1}{ab^{19}}}\log\left(ab^9\sqrt{-\frac{1}{ab^{19}}}+x\right)}{131072}+\frac{-765765a^8x-\cdots}{4128768a^9b^9+37158912a^8b^{10}x^2+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**18/(b*x**2+a)**10,x)
```

```
[Out] -12155*sqrt(-1/(a*b**19))*log(-a*b**9*sqrt(-1/(a*b**19)) + x)/131072 + 1215
5*sqrt(-1/(a*b**19))*log(a*b**9*sqrt(-1/(a*b**19)) + x)/131072 + (-765765*a
**8*x - 6636630*a**7*b*x**3 - 25423398*a**6*b**2*x**5 - 56404062*a**5*b**3*
x**7 - 79659008*a**4*b**4*x**9 - 73947042*a**3*b**5*x**11 - 44765658*a**2*b
**6*x**13 - 16759722*a*b**7*x**15 - 3363003*b**8*x**17)/(4128768*a**9*b**9
+ 37158912*a**8*b**10*x**2 + 148635648*a**7*b**11*x**4 + 346816512*a**6*b**
12*x**6 + 520224768*a**5*b**13*x**8 + 520224768*a**4*b**14*x**10 + 34681651
2*a**3*b**15*x**12 + 148635648*a**2*b**16*x**14 + 37158912*a*b**17*x**16 +
4128768*b**18*x**18)
```

3.213 $\int \frac{x^{16}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=198

$$\frac{715\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{3/2}b^{17/2}}+\frac{715x}{65536ab^8\left(a+bx^2\right)}-\frac{715x}{32768b^8\left(a+bx^2\right)^2}-\frac{715x^3}{24576b^7\left(a+bx^2\right)^3}-\frac{143x^5}{4096b^6\left(a+bx^2\right)^4}-\frac{1}{3584b^5}$$

```
[Out] -1/18*x^15/b/(b*x^2+a)^9-5/96*x^13/b^2/(b*x^2+a)^8-65/1344*x^11/b^3/(b*x^2+
a)^7-715/16128*x^9/b^4/(b*x^2+a)^6-143/3584*x^7/b^5/(b*x^2+a)^5-143/4096*x^
5/b^6/(b*x^2+a)^4-715/24576*x^3/b^7/(b*x^2+a)^3-715/32768*x/b^8/(b*x^2+a)^2
+715/65536*x/a/b^8/(b*x^2+a)+715/65536*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^
(17/2)
```

Rubi [A] time = 0.12, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{715\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{3/2}b^{17/2}}-\frac{5x^{13}}{96b^2\left(a+bx^2\right)^8}-\frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7}-\frac{715x^9}{16128b^4\left(a+bx^2\right)^6}-\frac{143x^7}{3584b^5\left(a+bx^2\right)^5}-\frac{143x^5}{4096b^6\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^16/(a + b*x^2)^10,x]
```

```
[Out] -x^15/(18*b*(a + b*x^2)^9) - (5*x^13)/(96*b^2*(a + b*x^2)^8) - (65*x^11)/(1
344*b^3*(a + b*x^2)^7) - (715*x^9)/(16128*b^4*(a + b*x^2)^6) - (143*x^7)/(3
584*b^5*(a + b*x^2)^5) - (143*x^5)/(4096*b^6*(a + b*x^2)^4) - (715*x^3)/(24
576*b^7*(a + b*x^2)^3) - (715*x)/(32768*b^8*(a + b*x^2)^2) + (715*x)/(65536
*a*b^8*(a + b*x^2)) + (715*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(3/2)*b^(1
7/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{16}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{15}}{18b\left(a+bx^2\right)^9} + \frac{5\int \frac{x^{14}}{\left(a+bx^2\right)^9}\,dx}{6b}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} + \frac{65\int \frac{x^{12}}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} + \frac{715\int \frac{x^{10}}{\left(a+bx^2\right)^7}\,dx}{1344b^3}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} + \frac{715\int \frac{x^8}{(a+bx}}{1792b}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143x}{3584b^5\,(a+}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143x}{3584b^5\,(a+}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143x}{3584b^5\,(a+}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143x}{3584b^5\,(a+}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143x}{3584b^5\,(a+}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143x}{3584b^5\,(a+}$$

Mathematica [A] time = 0.07, size = 138, normalized size = 0.70

$$\frac{\frac{\sqrt{a}\,\sqrt{b}\,x\left(-45045a^8-390390a^7bx^2-1495494a^6b^2x^4-3317886a^5b^3x^6-4685824a^4b^4x^8-4349826a^3b^5x^{10}-2633274a^2b^6x^{12}-985866ab^7x^{14}+45045b^8x^{16}\right)}{\left(a+bx^2\right)^9}}{4128768a^{3/2}b^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^16/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 - 1495494*a^6*b^2*x^4 -
3317886*a^5*b^3*x^6 - 4685824*a^4*b^4*x^8 - 4349826*a^3*b^5*x^10 - 2633274*
a^2*b^6*x^12 - 985866*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(4128768*a^(3/2)*b^(17/2))
```

fricas [A] time = 0.96, size = 654, normalized size = 3.30

$$\left[\frac{90090\,ab^{9}x^{17}-1971732\,a^{2}b^{8}x^{15}-5266548\,a^{3}b^{7}x^{13}-8699652\,a^{4}b^{6}x^{11}-9371648\,a^{5}b^{5}x^{9}-6635772\,a^{6}b^{4}x^{7}-}{8257536\left(a^{2}b^{18}x^{18}+9\,a^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(90090*a*b^9*x^17 - 1971732*a^2*b^8*x^15 - 5266548*a^3*b^7*x^13
- 8699652*a^4*b^6*x^11 - 9371648*a^5*b^5*x^9 - 6635772*a^6*b^4*x^7 - 299098
8*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^18*x^18 + 9*a^3*b^17*x^
16 + 36*a^4*b^16*x^14 + 84*a^5*b^15*x^12 + 126*a^6*b^14*x^10 + 126*a^7*b^13
*x^8 + 84*a^8*b^12*x^6 + 36*a^9*b^11*x^4 + 9*a^10*b^10*x^2 + a^11*b^9), 1/4
128768*(45045*a*b^9*x^17 - 985866*a^2*b^8*x^15 - 2633274*a^3*b^7*x^13 - 434
9826*a^4*b^6*x^11 - 4685824*a^5*b^5*x^9 - 3317886*a^6*b^4*x^7 - 1495494*a^7
*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*x
^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^
8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a))/(a^2*b^18*x^18 + 9*a^3*b^17*x^16 + 36*a^4*b^16*x^14 + 84*a^5
*b^15*x^12 + 126*a^6*b^14*x^10 + 126*a^7*b^13*x^8 + 84*a^8*b^12*x^6 + 36*a^
9*b^11*x^4 + 9*a^10*b^10*x^2 + a^11*b^9)]
```

giac [A] time = 0.63, size = 128, normalized size = 0.65

$$\frac{715\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,ab^{8}}+\frac{45045\,b^{8}x^{17}-985866\,ab^{7}x^{15}-2633274\,a^{2}b^{6}x^{13}-4349826\,a^{3}b^{5}x^{11}-4685824\,a^{4}b^{4}x^{9}-3}{4128768\left(bx^{2}+a\right)^{9}ab^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 715/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^8) + 1/4128768*(45045*b^8*x^
17 - 985866*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 4349826*a^3*b^5*x^11 - 4685
824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6*b^2*x^5 - 390390*a^7*b*
x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a*b^8)
```

maple [A] time = 0.02, size = 124, normalized size = 0.63

$$\frac{715\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\;a\,b^{8}}+\frac{\frac{715x^{17}}{65536a}-\frac{23473x^{15}}{98304b}-\frac{20899a\,x^{13}}{32768b^{2}}-\frac{241657a^{2}x^{11}}{229376b^{3}}-\frac{143a^{3}x^{9}}{126b^{4}}-\frac{184327a^{4}x^{7}}{229376b^{5}}-\frac{11869a^{5}x^{5}}{32768b^{6}}-\frac{9295a^{6}x^{3}}{98304b^{7}}-\frac{715a^{7}}{65536}}{\left(b\,x^{2}+a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^16/(b*x^2+a)^10,x)
```

```
[Out] (-715/65536*a^7/b^8*x-9295/98304*a^6/b^7*x^3-11869/32768*a^5/b^6*x^5-184327
/229376*a^4/b^5*x^7-143/126*a^3/b^4*x^9-241657/229376*a^2/b^3*x^11-20899/32
768*a/b^2*x^13-23473/98304/b*x^15+715/65536/a*x^17)/(b*x^2+a)^9+715/65536/a
/b^8/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.16, size = 219, normalized size = 1.11

$$\frac{45045\,b^{8}x^{17}-985866\,ab^{7}x^{15}-2633274\,a^{2}b^{6}x^{13}-4349826\,a^{3}b^{5}x^{11}-4685824\,a^{4}b^{4}x^{9}-3317886\,a^{5}b^{3}x^{7}-14}{4128768\left(ab^{17}x^{18}+9\,a^{2}b^{16}x^{16}+36\,a^{3}b^{15}x^{14}+84\,a^{4}b^{14}x^{12}+126\,a^{5}b^{13}x^{10}+126\,a^{6}b^{12}x^{8}+84\,a^{7}b^{11}x^{6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(45045*b^8*x^17 - 985866*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 4349
826*a^3*b^5*x^11 - 4685824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6*
b^2*x^5 - 390390*a^7*b*x^3 - 45045*a^8*x)/(a*b^17*x^18 + 9*a^2*b^16*x^16 +
36*a^3*b^15*x^14 + 84*a^4*b^14*x^12 + 126*a^5*b^13*x^10 + 126*a^6*b^12*x^8
+ 84*a^7*b^11*x^6 + 36*a^8*b^10*x^4 + 9*a^9*b^9*x^2 + a^10*b^8) + 715/65536
*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^8)
```

mupad [B] time = 4.76, size = 207, normalized size = 1.05

$$\frac{715\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{3/2}\,b^{17/2}}-\frac{\frac{23473\,x^{15}}{98304\,b}-\frac{715\,x^{17}}{65536\,a}+\frac{20899\,a\,x^{13}}{32768\,b^2}+\frac{715\,a^7\,x}{65536\,b^8}+\frac{241657\,a^2\,x^{11}}{229376\,b^3}+\frac{143\,a^3\,x^9}{126\,b^4}+\frac{184327\,a^4\,x^7}{229376\,b^5}+\frac{11869\,a^5\,x}{32768\,b^6}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^16/(a + b*x^2)^10,x)
```

```
[Out] (715*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(3/2)*b^(17/2)) - ((23473*x^15)/(9
8304*b) - (715*x^17)/(65536*a) + (20899*a*x^13)/(32768*b^2) + (715*a^7*x)/(
65536*b^8) + (241657*a^2*x^11)/(229376*b^3) + (143*a^3*x^9)/(126*b^4) + (18
4327*a^4*x^7)/(229376*b^5) + (11869*a^5*x^5)/(32768*b^6) + (9295*a^6*x^3)/(
98304*b^7))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 +
 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36
*a^2*b^7*x^14)
```

sympy [A] time = 1.46, size = 289, normalized size = 1.46

$$-\frac{715\sqrt{-\frac{1}{a^3b^{17}}}\log\left(-a^2b^8\sqrt{-\frac{1}{a^3b^{17}}}+x\right)}{131072}+\frac{715\sqrt{-\frac{1}{a^3b^{17}}}\log\left(a^2b^8\sqrt{-\frac{1}{a^3b^{17}}}+x\right)}{131072}+\frac{-4504}{4128768a^{10}b^8+37158912a^9b^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**16/(b*x**2+a)**10,x)
```

```
[Out] -715*sqrt(-1/(a**3*b**17))*log(-a**2*b**8*sqrt(-1/(a**3*b**17)) + x)/131072
 + 715*sqrt(-1/(a**3*b**17))*log(a**2*b**8*sqrt(-1/(a**3*b**17)) + x)/13107
2 + (-45045*a**8*x - 390390*a**7*b*x**3 - 1495494*a**6*b**2*x**5 - 3317886*
a**5*b**3*x**7 - 4685824*a**4*b**4*x**9 - 4349826*a**3*b**5*x**11 - 2633274
*a**2*b**6*x**13 - 985866*a*b**7*x**15 + 45045*b**8*x**17)/(4128768*a**10*b
**8 + 37158912*a**9*b**9*x**2 + 148635648*a**8*b**10*x**4 + 346816512*a**7*
b**11*x**6 + 520224768*a**6*b**12*x**8 + 520224768*a**5*b**13*x**10 + 34681
6512*a**4*b**14*x**12 + 148635648*a**3*b**15*x**14 + 37158912*a**2*b**16*x*
*16 + 4128768*a*b**17*x**18)
```

3.214 $\int \frac{x^{14}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=199

$$\frac{143\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{5/2}b^{15/2}}+\frac{143x}{65536a^2b^7\left(a+bx^2\right)}+\frac{143x}{98304ab^7\left(a+bx^2\right)^2}-\frac{143x}{24576b^7\left(a+bx^2\right)^3}-\frac{143x^3}{12288b^6\left(a+bx^2\right)^4}-\frac{}{768}$$

```
[Out] -1/18*x^13/b/(b*x^2+a)^9-13/288*x^11/b^2/(b*x^2+a)^8-143/4032*x^9/b^3/(b*x^
2+a)^7-143/5376*x^7/b^4/(b*x^2+a)^6-143/7680*x^5/b^5/(b*x^2+a)^5-143/12288*
x^3/b^6/(b*x^2+a)^4-143/24576*x/b^7/(b*x^2+a)^3+143/98304*x/a/b^7/(b*x^2+a)
^2+143/65536*x/a^2/b^7/(b*x^2+a)+143/65536*arctan(x*b^(1/2)/a^(1/2))/a^(5/2
)/b^(15/2)
```

Rubi [A] time = 0.12, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {288, 199, 205}

$$\frac{143x}{65536a^2b^7\left(a+bx^2\right)}+\frac{143\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{5/2}b^{15/2}}-\frac{13x^{11}}{288b^2\left(a+bx^2\right)^8}-\frac{143x^9}{4032b^3\left(a+bx^2\right)^7}-\frac{143x^7}{5376b^4\left(a+bx^2\right)^6}-\frac{143}{7680b^5(a}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a + b*x^2)^10,x]
```

```
[Out] -x^13/(18*b*(a + b*x^2)^9) - (13*x^11)/(288*b^2*(a + b*x^2)^8) - (143*x^9)/
(4032*b^3*(a + b*x^2)^7) - (143*x^7)/(5376*b^4*(a + b*x^2)^6) - (143*x^5)/(
7680*b^5*(a + b*x^2)^5) - (143*x^3)/(12288*b^6*(a + b*x^2)^4) - (143*x)/(24
576*b^7*(a + b*x^2)^3) + (143*x)/(98304*a*b^7*(a + b*x^2)^2) + (143*x)/(655
36*a^2*b^7*(a + b*x^2)) + (143*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(5/2)*
b^(15/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{13}}{18b\left(a+bx^2\right)^9} + \frac{13\int \frac{x^{12}}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} + \frac{143\int \frac{x^{10}}{\left(a+bx^2\right)^8}\,dx}{288b^2}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} + \frac{143\int \frac{x^8}{\left(a+bx^2\right)^7}\,dx}{448b^3}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} + \frac{143\int \frac{x^6}{(a+bx}}{768b^4}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143x}{7680b^5(a -}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143x}{7680b^5(a -}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143x}{7680b^5(a -}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143x}{7680b^5(a -}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143x}{7680b^5(a -}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143x}{7680b^5(a -}$$

Mathematica [A] time = 0.06, size = 138, normalized size = 0.69

$$\frac{\frac{\sqrt{a}\sqrt{b}\,x\left(-45045a^8-390390a^7bx^2-1495494a^6b^2x^4-3317886a^5b^3x^6-4685824a^4b^4x^8-4349826a^3b^5x^{10}-2633274a^2b^6x^{12}+390390ab^7x^{14}+45045b^8x^{16}\right)}{\left(a+bx^2\right)^9}}{20643840a^{5/2}b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 - 1495494*a^6*b^2*x^4 -
3317886*a^5*b^3*x^6 - 4685824*a^4*b^4*x^8 - 4349826*a^3*b^5*x^10 - 2633274*
a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(20643840*a^(5/2)*b^(15/2))
```

fricas [A] time = 0.76, size = 654, normalized size = 3.29

$$\left[\frac{90090\,ab^9x^{17}+780780\,a^2b^8x^{15}-5266548\,a^3b^7x^{13}-8699652\,a^4b^6x^{11}-9371648\,a^5b^5x^9-6635772\,a^6b^4x^7-}{41287680\left(a^3b^{17}x^{18}+9\,a^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/41287680*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 - 5266548*a^3*b^7*x^13
- 8699652*a^4*b^6*x^11 - 9371648*a^5*b^5*x^9 - 6635772*a^6*b^4*x^7 - 299098
8*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^17*x^18 + 9*a^4*b^16*x^
16 + 36*a^5*b^15*x^14 + 84*a^6*b^14*x^12 + 126*a^7*b^13*x^10 + 126*a^8*b^12
*x^8 + 84*a^9*b^11*x^6 + 36*a^10*b^10*x^4 + 9*a^11*b^9*x^2 + a^12*b^8), 1/2
0643840*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 - 2633274*a^3*b^7*x^13 - 43
49826*a^4*b^6*x^11 - 4685824*a^5*b^5*x^9 - 3317886*a^6*b^4*x^7 - 1495494*a^
7*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*
x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x
^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(
sqrt(a*b)*x/a))/(a^3*b^17*x^18 + 9*a^4*b^16*x^16 + 36*a^5*b^15*x^14 + 84*a^
6*b^14*x^12 + 126*a^7*b^13*x^10 + 126*a^8*b^12*x^8 + 84*a^9*b^11*x^6 + 36*a
^10*b^10*x^4 + 9*a^11*b^9*x^2 + a^12*b^8)]
```

giac [A] time = 0.64, size = 128, normalized size = 0.64

$$\frac{143\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^2b^7}+\frac{45045\,b^8x^{17}+390390\,ab^7x^{15}-2633274\,a^2b^6x^{13}-4349826\,a^3b^5x^{11}-4685824\,a^4b^4x^9-3}{20643840\left(bx^2+a\right)^9a^2b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 143/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^7) + 1/20643840*(45045*b^8
*x^17 + 390390*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 4349826*a^3*b^5*x^11 - 4
685824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6*b^2*x^5 - 390390*a^7
*b*x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^2*b^7)
```

maple [A] time = 0.02, size = 122, normalized size = 0.61

$$\frac{143\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\;a^2b^7}+\frac{\frac{143b\,x^{17}}{65536a^2}+\frac{1859x^{15}}{98304a}-\frac{20899x^{13}}{163840b}-\frac{241657a\,x^{11}}{1146880b^2}-\frac{143a^2x^9}{630b^3}-\frac{184327a^3x^7}{1146880b^4}-\frac{11869a^4x^5}{163840b^5}-\frac{1859a^5x^3}{98304b^6}-\frac{143a^6x}{65536b^7}}{\left(b\,x^2+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^2+a)^10,x)
```

```
[Out] (-143/65536*a^6/b^7*x-1859/98304*a^5/b^6*x^3-11869/163840*a^4/b^5*x^5-18432
7/1146880*a^3/b^4*x^7-143/630*a^2/b^3*x^9-241657/1146880*a/b^2*x^11-20899/1
63840/b*x^13+1859/98304/a*x^15+143/65536/a^2*b*x^17)/(b*x^2+a)^9+143/65536/
a^2/b^7/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.16, size = 221, normalized size = 1.11

$$\frac{45045\,b^8x^{17}+390390\,ab^7x^{15}-2633274\,a^2b^6x^{13}-4349826\,a^3b^5x^{11}-4685824\,a^4b^4x^9-3317886\,a^5b^3x^7-14}{20643840\left(a^2b^{16}x^{18}+9\,a^3b^{15}x^{16}+36\,a^4b^{14}x^{14}+84\,a^5b^{13}x^{12}+126\,a^6b^{12}x^{10}+126\,a^7b^{11}x^8+84\,a^8b^{10}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/20643840*(45045*b^8*x^17 + 390390*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 434
9826*a^3*b^5*x^11 - 4685824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6
*b^2*x^5 - 390390*a^7*b*x^3 - 45045*a^8*x)/(a^2*b^16*x^18 + 9*a^3*b^15*x^16
 + 36*a^4*b^14*x^14 + 84*a^5*b^13*x^12 + 126*a^6*b^12*x^10 + 126*a^7*b^11*x
^8 + 84*a^8*b^10*x^6 + 36*a^9*b^9*x^4 + 9*a^10*b^8*x^2 + a^11*b^7) + 143/65
536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^7)
```

mupad [B] time = 4.75, size = 205, normalized size = 1.03

$$\frac{143\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{5/2}\,b^{15/2}}-\frac{\frac{20899\,x^{13}}{163840\,b}-\frac{1859\,x^{15}}{98304\,a}+\frac{241657\,a\,x^{11}}{1146880\,b^2}+\frac{143\,a^6\,x}{65536\,b^7}-\frac{143\,b\,x^{17}}{65536\,a^2}+\frac{143\,a^2\,x^9}{630\,b^3}+\frac{184327\,a^3\,x^7}{1146880\,b^4}+\frac{11869\,a^4\,x^5}{163840\,b^5}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a + b*x^2)^10,x)
```

```
[Out] (143*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(5/2)*b^(15/2)) - ((20899*x^13)/(1
63840*b) - (1859*x^15)/(98304*a) + (241657*a*x^11)/(1146880*b^2) + (143*a^6
*x)/(65536*b^7) - (143*b*x^17)/(65536*a^2) + (143*a^2*x^9)/(630*b^3) + (184
327*a^3*x^7)/(1146880*b^4) + (11869*a^4*x^5)/(163840*b^5) + (1859*a^5*x^3)/
(98304*b^6))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4
+ 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 3
6*a^2*b^7*x^14)
```

sympy [A] time = 1.35, size = 291, normalized size = 1.46

$$-\frac{143\sqrt{-\frac{1}{a^5b^{15}}}\,\log\left(-a^3b^7\sqrt{-\frac{1}{a^5b^{15}}}+x\right)}{131072}+\frac{143\sqrt{-\frac{1}{a^5b^{15}}}\,\log\left(a^3b^7\sqrt{-\frac{1}{a^5b^{15}}}+x\right)}{131072}+\frac{-}{20643840a^{11}b^7+185794560a^{10}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**2+a)**10,x)
```

```
[Out] -143*sqrt(-1/(a**5*b**15))*log(-a**3*b**7*sqrt(-1/(a**5*b**15)) + x)/131072
 + 143*sqrt(-1/(a**5*b**15))*log(a**3*b**7*sqrt(-1/(a**5*b**15)) + x)/13107
2 + (-45045*a**8*x - 390390*a**7*b*x**3 - 1495494*a**6*b**2*x**5 - 3317886*
a**5*b**3*x**7 - 4685824*a**4*b**4*x**9 - 4349826*a**3*b**5*x**11 - 2633274
*a**2*b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(20643840*a**11*
b**7 + 185794560*a**10*b**8*x**2 + 743178240*a**9*b**9*x**4 + 1734082560*a*
*8*b**10*x**6 + 2601123840*a**7*b**11*x**8 + 2601123840*a**6*b**12*x**10 +
1734082560*a**5*b**13*x**12 + 743178240*a**4*b**14*x**14 + 185794560*a**3*b
**15*x**16 + 20643840*a**2*b**16*x**18)
```

3.215 $\int \frac{x^{12}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=200

$$\frac{55\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{7/2}b^{13/2}}+\frac{55x}{65536a^3b^6\left(a+bx^2\right)}+\frac{55x}{98304a^2b^6\left(a+bx^2\right)^2}+\frac{11x}{24576ab^6\left(a+bx^2\right)^3}-\frac{11x}{4096b^6\left(a+bx^2\right)^4}-\frac{}{153}$$

```
[Out] -1/18*x^11/b/(b*x^2+a)^9-11/288*x^9/b^2/(b*x^2+a)^8-11/448*x^7/b^3/(b*x^2+a
)^7-11/768*x^5/b^4/(b*x^2+a)^6-11/1536*x^3/b^5/(b*x^2+a)^5-11/4096*x/b^6/(b
*x^2+a)^4+11/24576*x/a/b^6/(b*x^2+a)^3+55/98304*x/a^2/b^6/(b*x^2+a)^2+55/65
536*x/a^3/b^6/(b*x^2+a)+55/65536*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(13/2)
```

Rubi [A] time = 0.12, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{55x}{65536a^3b^6\left(a+bx^2\right)}+\frac{55x}{98304a^2b^6\left(a+bx^2\right)^2}+\frac{55\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{7/2}b^{13/2}}-\frac{11x^9}{288b^2\left(a+bx^2\right)^8}-\frac{11x^7}{448b^3\left(a+bx^2\right)^7}-\frac{11}{768b^4\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^2)^10,x]
```

```
[Out] -x^11/(18*b*(a + b*x^2)^9) - (11*x^9)/(288*b^2*(a + b*x^2)^8) - (11*x^7)/(4
48*b^3*(a + b*x^2)^7) - (11*x^5)/(768*b^4*(a + b*x^2)^6) - (11*x^3)/(1536*b
^5*(a + b*x^2)^5) - (11*x)/(4096*b^6*(a + b*x^2)^4) + (11*x)/(24576*a*b^6*(
a + b*x^2)^3) + (55*x)/(98304*a^2*b^6*(a + b*x^2)^2) + (55*x)/(65536*a^3*b^
6*(a + b*x^2)) + (55*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(7/2)*b^(13/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{12}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{11}}{18b\left(a+bx^2\right)^9} + \frac{11\int \frac{x^{10}}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} + \frac{11\int \frac{x^8}{\left(a+bx^2\right)^8}\,dx}{32b^2}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} + \frac{11\int \frac{x^6}{\left(a+bx^2\right)^7}\,dx}{64b^3}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} + \frac{55\int \frac{x^4}{\left(a+bx^2\right)^6}\,d}{768b^4}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+b\right.}$$

Mathematica [A] time = 0.07, size = 138, normalized size = 0.69

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(-3465a^8-30030a^7bx^2-115038a^6b^2x^4-255222a^5b^3x^6-360448a^4b^4x^8-334602a^3b^5x^{10}+115038a^2b^6x^{12}+30030ab^7x^{14}+3465b^8x^{16}\right)}{\left(a+bx^2\right)^9}+3465}{4128768a^{7/2}b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-3465*a^8 - 30030*a^7*b*x^2 - 115038*a^6*b^2*x^4 - 255
222*a^5*b^3*x^6 - 360448*a^4*b^4*x^8 - 334602*a^3*b^5*x^10 + 115038*a^2*b^6
*x^12 + 30030*a*b^7*x^14 + 3465*b^8*x^16))/(a + b*x^2)^9 + 3465*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(4128768*a^(7/2)*b^(13/2))
```

fricas [A] time = 0.60, size = 654, normalized size = 3.27

$$\left[\frac{6930\,ab^9x^{17}+60060\,a^2b^8x^{15}+230076\,a^3b^7x^{13}-669204\,a^4b^6x^{11}-720896\,a^5b^5x^9-510444\,a^6b^4x^7-23007}{8257536\left(a^4b^{16}x^{18}+9\,a^5b^{15}x^{16}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(6930*a*b^9*x^17 + 60060*a^2*b^8*x^15 + 230076*a^3*b^7*x^13 - 66
9204*a^4*b^6*x^11 - 720896*a^5*b^5*x^9 - 510444*a^6*b^4*x^7 - 230076*a^7*b^
3*x^5 - 60060*a^8*b^2*x^3 - 6930*a^9*b*x - 3465*(b^9*x^18 + 9*a*b^8*x^16 +
36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84
*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 -
2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^16*x^18 + 9*a^5*b^15*x^16 + 36*a^6
*b^14*x^14 + 84*a^7*b^13*x^12 + 126*a^8*b^12*x^10 + 126*a^9*b^11*x^8 + 84*a
^10*b^10*x^6 + 36*a^11*b^9*x^4 + 9*a^12*b^8*x^2 + a^13*b^7), 1/4128768*(346
5*a*b^9*x^17 + 30030*a^2*b^8*x^15 + 115038*a^3*b^7*x^13 - 334602*a^4*b^6*x^
11 - 360448*a^5*b^5*x^9 - 255222*a^6*b^4*x^7 - 115038*a^7*b^3*x^5 - 30030*a
^8*b^2*x^3 - 3465*a^9*b*x + 3465*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14
 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 +
36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b
^16*x^18 + 9*a^5*b^15*x^16 + 36*a^6*b^14*x^14 + 84*a^7*b^13*x^12 + 126*a^8*
b^12*x^10 + 126*a^9*b^11*x^8 + 84*a^10*b^10*x^6 + 36*a^11*b^9*x^4 + 9*a^12*
b^8*x^2 + a^13*b^7)]
```

giac [A] time = 0.63, size = 128, normalized size = 0.64

$$\frac{55\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^3b^6}+\frac{3465\,b^8x^{17}+30030\,ab^7x^{15}+115038\,a^2b^6x^{13}-334602\,a^3b^5x^{11}-360448\,a^4b^4x^9-255222}{4128768\left(bx^2+a\right)^9a^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^6) + 1/4128768*(3465*b^8*x^
17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 - 334602*a^3*b^5*x^11 - 360448*
a^4*b^4*x^9 - 255222*a^5*b^3*x^7 - 115038*a^6*b^2*x^5 - 30030*a^7*b*x^3 - 3
465*a^8*x)/((b*x^2 + a)^9*a^3*b^6)
```

maple [A] time = 0.02, size = 122, normalized size = 0.61

$$\frac{55\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\;a^3b^6}+\frac{\frac{55b^2x^{17}}{65536a^3}+\frac{715b\,x^{15}}{98304a^2}+\frac{913x^{13}}{32768a}-\frac{18589x^{11}}{229376b}-\frac{11a\,x^9}{126b^2}-\frac{14179a^2x^7}{229376b^3}-\frac{913a^3x^5}{32768b^4}-\frac{715a^4x^3}{98304b^5}-\frac{55a^5x}{65536b^6}}{\left(b\,x^2+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2+a)^10,x)
```

```
[Out] (-55/65536*a^5/b^6*x-715/98304*a^4/b^5*x^3-913/32768*a^3/b^4*x^5-14179/2293
76*a^2/b^3*x^7-11/126*a/b^2*x^9-18589/229376/b*x^11+913/32768/a*x^13+715/98
304/a^2*b*x^15+55/65536*b^2/a^3*x^17)/(b*x^2+a)^9+55/65536/a^3/b^6/(a*b)^(1
/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.09, size = 221, normalized size = 1.10

$$\frac{3465\,b^8x^{17}+30030\,ab^7x^{15}+115038\,a^2b^6x^{13}-334602\,a^3b^5x^{11}-360448\,a^4b^4x^9-255222\,a^5b^3x^7-115038\,a^6}{4128768\left(a^3b^{15}x^{18}+9\,a^4b^{14}x^{16}+36\,a^5b^{13}x^{14}+84\,a^6b^{12}x^{12}+126\,a^7b^{11}x^{10}+126\,a^8b^{10}x^8+84\,a^9b^9x^6+36\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(3465*b^8*x^17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 - 334602*
a^3*b^5*x^11 - 360448*a^4*b^4*x^9 - 255222*a^5*b^3*x^7 - 115038*a^6*b^2*x^5
 - 30030*a^7*b*x^3 - 3465*a^8*x)/(a^3*b^15*x^18 + 9*a^4*b^14*x^16 + 36*a^5*
b^13*x^14 + 84*a^6*b^12*x^12 + 126*a^7*b^11*x^10 + 126*a^8*b^10*x^8 + 84*a^
9*b^9*x^6 + 36*a^10*b^8*x^4 + 9*a^11*b^7*x^2 + a^12*b^6) + 55/65536*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^6)
```

mupad [B] time = 0.15, size = 205, normalized size = 1.02

$$\frac{55\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{7/2}\,b^{13/2}}-\frac{\frac{18589\,x^{11}}{229376\,b}-\frac{913\,x^{13}}{32768\,a}+\frac{11\,a\,x^9}{126\,b^2}+\frac{55\,a^5\,x}{65536\,b^6}-\frac{715\,b\,x^{15}}{98304\,a^2}+\frac{14179\,a^2\,x^7}{229376\,b^3}+\frac{913\,a^3\,x^5}{32768\,b^4}+\frac{715\,a^4\,x^3}{98304\,b^5}-\cdots}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x^2)^10,x)
```

```
[Out] (55*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(7/2)*b^(13/2)) - ((18589*x^11)/(22
9376*b) - (913*x^13)/(32768*a) + (11*a*x^9)/(126*b^2) + (55*a^5*x)/(65536*b
^6) - (715*b*x^15)/(98304*a^2) + (14179*a^2*x^7)/(229376*b^3) + (913*a^3*x^
5)/(32768*b^4) + (715*a^4*x^3)/(98304*b^5) - (55*b^2*x^17)/(65536*a^3))/(a^
9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6
 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

sympy [A] time = 1.27, size = 291, normalized size = 1.46

$$-\frac{55\sqrt{-\frac{1}{a^7b^{13}}}\log\left(-a^4b^6\sqrt{-\frac{1}{a^7b^{13}}}+x\right)}{131072}+\frac{55\sqrt{-\frac{1}{a^7b^{13}}}\log\left(a^4b^6\sqrt{-\frac{1}{a^7b^{13}}}+x\right)}{131072}+\frac{-3\cdots}{4128768a^{12}b^6+37158912a^{11}b^7x^2+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x**2+a)**10,x)
```

```
[Out] -55*sqrt(-1/(a**7*b**13))*log(-a**4*b**6*sqrt(-1/(a**7*b**13)) + x)/131072
+ 55*sqrt(-1/(a**7*b**13))*log(a**4*b**6*sqrt(-1/(a**7*b**13)) + x)/131072
+ (-3465*a**8*x - 30030*a**7*b*x**3 - 115038*a**6*b**2*x**5 - 255222*a**5*b
**3*x**7 - 360448*a**4*b**4*x**9 - 334602*a**3*b**5*x**11 + 115038*a**2*b**
6*x**13 + 30030*a*b**7*x**15 + 3465*b**8*x**17)/(4128768*a**12*b**6 + 37158
912*a**11*b**7*x**2 + 148635648*a**10*b**8*x**4 + 346816512*a**9*b**9*x**6
+ 520224768*a**8*b**10*x**8 + 520224768*a**7*b**11*x**10 + 346816512*a**6*b
**12*x**12 + 148635648*a**5*b**13*x**14 + 37158912*a**4*b**14*x**16 + 41287
68*a**3*b**15*x**18)
```

3.216 $\int \frac{x^{10}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=201

$$\frac{35\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{9/2}b^{11/2}}+\frac{35x}{65536a^4b^5\left(a+bx^2\right)}+\frac{35x}{98304a^3b^5\left(a+bx^2\right)^2}+\frac{7x}{24576a^2b^5\left(a+bx^2\right)^3}+\frac{x}{4096ab^5\left(a+bx^2\right)^4}-$$

```
[Out] -1/18*x^9/b/(b*x^2+a)^9-1/32*x^7/b^2/(b*x^2+a)^8-1/64*x^5/b^3/(b*x^2+a)^7-5
/768*x^3/b^4/(b*x^2+a)^6-1/512*x/b^5/(b*x^2+a)^5+1/4096*x/a/b^5/(b*x^2+a)^4
+7/24576*x/a^2/b^5/(b*x^2+a)^3+35/98304*x/a^3/b^5/(b*x^2+a)^2+35/65536*x/a^
4/b^5/(b*x^2+a)+35/65536*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/b^(11/2)
```

Rubi [A] time = 0.12, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {288, 199, 205}

$$\frac{35x}{65536a^4b^5\left(a+bx^2\right)}+\frac{35x}{98304a^3b^5\left(a+bx^2\right)^2}+\frac{7x}{24576a^2b^5\left(a+bx^2\right)^3}+\frac{35\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{9/2}b^{11/2}}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}-$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^10,x]
```

```
[Out] -x^9/(18*b*(a + b*x^2)^9) - x^7/(32*b^2*(a + b*x^2)^8) - x^5/(64*b^3*(a + b
*x^2)^7) - (5*x^3)/(768*b^4*(a + b*x^2)^6) - x/(512*b^5*(a + b*x^2)^5) + x/
(4096*a*b^5*(a + b*x^2)^4) + (7*x)/(24576*a^2*b^5*(a + b*x^2)^3) + (35*x)/(
98304*a^3*b^5*(a + b*x^2)^2) + (35*x)/(65536*a^4*b^5*(a + b*x^2)) + (35*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(9/2)*b^(11/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^9}{18b\left(a+bx^2\right)^9} + \frac{\int \frac{x^8}{\left(a+bx^2\right)^9}\,dx}{2b}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} + \frac{7\int \frac{x^6}{\left(a+bx^2\right)^8}\,dx}{32b^2}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} + \frac{5\int \frac{x^4}{\left(a+bx^2\right)^7}\,dx}{64b^3}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} - \frac{5x^3}{768b^4\left(a+bx^2\right)^6} + \frac{5\int \frac{x^2}{\left(a+bx^2\right)^6}\,dx}{256b^4}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} - \frac{5x^3}{768b^4\left(a+bx^2\right)^6} - \frac{x}{512b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} - \frac{5x^3}{768b^4\left(a+bx^2\right)^6} - \frac{x}{512b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} - \frac{5x^3}{768b^4\left(a+bx^2\right)^6} - \frac{x}{512b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} - \frac{5x^3}{768b^4\left(a+bx^2\right)^6} - \frac{x}{512b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} - \frac{5x^3}{768b^4\left(a+bx^2\right)^6} - \frac{x}{512b^5\left(a+bx^2\right)^5}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9} - \frac{x^7}{32b^2\left(a+bx^2\right)^8} - \frac{x^5}{64b^3\left(a+bx^2\right)^7} - \frac{5x^3}{768b^4\left(a+bx^2\right)^6} - \frac{x}{512b^5\left(a+bx^2\right)^5}$$

Mathematica [A] time = 0.07, size = 138, normalized size = 0.69

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(-315a^8-2730a^7bx^2-10458a^6b^2x^4-23202a^5b^3x^6-32768a^4b^4x^8+23202a^3b^5x^{10}+10458a^2b^6x^{12}+2730ab^7x^{14}+315b^8x^{16}\right)}{\left(a+bx^2\right)^9} + 315\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{589824a^{9/2}b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-315*a^8 - 2730*a^7*b*x^2 - 10458*a^6*b^2*x^4 - 23202*
a^5*b^3*x^6 - 32768*a^4*b^4*x^8 + 23202*a^3*b^5*x^10 + 10458*a^2*b^6*x^12 +
 2730*a*b^7*x^14 + 315*b^8*x^16))/(a + b*x^2)^9 + 315*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(589824*a^(9/2)*b^(11/2))
```

fricas [A] time = 1.16, size = 654, normalized size = 3.25

$$\left[\frac{630\,ab^9x^{17} + 5460\,a^2b^8x^{15} + 20916\,a^3b^7x^{13} + 46404\,a^4b^6x^{11} - 65536\,a^5b^5x^9 - 46404\,a^6b^4x^7 - 20916\,a^7b^3x^5 - 5}{1179648\left(a^5b^{15}x^{18} + 9\,a^6b^{14}x^{16} + 36\,a^7b^{1}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(630*a*b^9*x^17 + 5460*a^2*b^8*x^15 + 20916*a^3*b^7*x^13 + 46404
*a^4*b^6*x^11 - 65536*a^5*b^5*x^9 - 46404*a^6*b^4*x^7 - 20916*a^7*b^3*x^5 -
 5460*a^8*b^2*x^3 - 630*a^9*b*x - 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7
*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x
^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)))/(a^5*b^15*x^18 + 9*a^6*b^14*x^16 + 36*a^7*b^13*x^14
 + 84*a^8*b^12*x^12 + 126*a^9*b^11*x^10 + 126*a^10*b^10*x^8 + 84*a^11*b^9*x
^6 + 36*a^12*b^8*x^4 + 9*a^13*b^7*x^2 + a^14*b^6), 1/589824*(315*a*b^9*x^17
 + 2730*a^2*b^8*x^15 + 10458*a^3*b^7*x^13 + 23202*a^4*b^6*x^11 - 32768*a^5*
b^5*x^9 - 23202*a^6*b^4*x^7 - 10458*a^7*b^3*x^5 - 2730*a^8*b^2*x^3 - 315*a^
9*b*x + 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 +
126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^
8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^15*x^18 + 9*a^6*b^14
*x^16 + 36*a^7*b^13*x^14 + 84*a^8*b^12*x^12 + 126*a^9*b^11*x^10 + 126*a^10*
b^10*x^8 + 84*a^11*b^9*x^6 + 36*a^12*b^8*x^4 + 9*a^13*b^7*x^2 + a^14*b^6)]
```

giac [A] time = 0.63, size = 128, normalized size = 0.64

$$\frac{35\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^4b^5}+\frac{315\,b^8x^{17}+2730\,ab^7x^{15}+10458\,a^2b^6x^{13}+23202\,a^3b^5x^{11}-32768\,a^4b^4x^9-23202\,a^5b^3x^7+\cdots}{589824\left(bx^2+a\right)^9a^4b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 35/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b^5) + 1/589824*(315*b^8*x^17
 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b^5*x^11 - 32768*a^4*b^
4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*a^7*b*x^3 - 315*a^8*x)
/((b*x^2 + a)^9*a^4*b^5)
```

maple [A] time = 0.02, size = 122, normalized size = 0.61

$$\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\;a^4b^5}+\frac{\frac{35b^3x^{17}}{65536a^4}+\frac{455b^2x^{15}}{98304a^3}+\frac{581b\,x^{13}}{32768a^2}+\frac{1289x^{11}}{32768a}-\frac{x^9}{18b}-\frac{1289a\,x^7}{32768b^2}-\frac{581a^2x^5}{32768b^3}-\frac{455a^3x^3}{98304b^4}-\frac{35a^4x}{65536b^5}}{\left(b\,x^2+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^10,x)
```

```
[Out] (-35/65536*a^4/b^5*x-455/98304*a^3/b^4*x^3-581/32768*a^2/b^3*x^5-1289/32768
*a/b^2*x^7-1/18/b*x^9+1289/32768/a*x^11+581/32768/a^2*b*x^13+455/98304*b^2/
a^3*x^15+35/65536*b^3/a^4*x^17)/(b*x^2+a)^9+35/65536/a^4/b^5/(a*b)^(1/2)*ar
ctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.08, size = 221, normalized size = 1.10

$$\frac{315\,b^8x^{17}+2730\,ab^7x^{15}+10458\,a^2b^6x^{13}+23202\,a^3b^5x^{11}-32768\,a^4b^4x^9-23202\,a^5b^3x^7-10458\,a^6b^2x^5-\cdots}{589824\left(a^4b^{14}x^{18}+9\,a^5b^{13}x^{16}+36\,a^6b^{12}x^{14}+84\,a^7b^{11}x^{12}+126\,a^8b^{10}x^{10}+126\,a^9b^9x^8+84\,a^{10}b^8x^6+36\,a^{11}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/589824*(315*b^8*x^17 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b
^5*x^11 - 32768*a^4*b^4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*
```

```
a^7*b*x^3 - 315*a^8*x)/(a^4*b^14*x^18 + 9*a^5*b^13*x^16 + 36*a^6*b^12*x^14
+ 84*a^7*b^11*x^12 + 126*a^8*b^10*x^10 + 126*a^9*b^9*x^8 + 84*a^10*b^8*x^6
+ 36*a^11*b^7*x^4 + 9*a^12*b^6*x^2 + a^13*b^5) + 35/65536*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^4*b^5)
```

mupad [B] time = 4.77, size = 205, normalized size = 1.02

$$\frac{35\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{9/2}\,b^{11/2}}-\frac{\frac{x^9}{18\,b}-\frac{1289\,x^{11}}{32768\,a}+\frac{1289\,a\,x^7}{32768\,b^2}+\frac{35\,a^4\,x}{65536\,b^5}-\frac{581\,b\,x^{13}}{32768\,a^2}+\frac{581\,a^2\,x^5}{32768\,b^3}+\frac{455\,a^3\,x^3}{98304\,b^4}-\frac{455\,b^2\,x^{15}}{98304\,a^3}-\frac{35}{65}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2)^10,x)
```

```
[Out] (35*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(9/2)*b^(11/2)) - (x^9/(18*b) - (12
89*x^11)/(32768*a) + (1289*a*x^7)/(32768*b^2) + (35*a^4*x)/(65536*b^5) - (5
81*b*x^13)/(32768*a^2) + (581*a^2*x^5)/(32768*b^3) + (455*a^3*x^3)/(98304*b
^4) - (455*b^2*x^15)/(98304*a^3) - (35*b^3*x^17)/(65536*a^4))/(a^9 + b^9*x^
18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5
*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

sympy [A] time = 1.19, size = 291, normalized size = 1.45

$$-\frac{35\sqrt{-\frac{1}{a^9b^{11}}}\,\log\left(-a^5b^5\sqrt{-\frac{1}{a^9b^{11}}}+x\right)}{131072}+\frac{35\sqrt{-\frac{1}{a^9b^{11}}}\,\log\left(a^5b^5\sqrt{-\frac{1}{a^9b^{11}}}+x\right)}{131072}+\frac{-3}{589824a^{13}b^5+5308416a^{12}b^6x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**10,x)
```

```
[Out] -35*sqrt(-1/(a**9*b**11))*log(-a**5*b**5*sqrt(-1/(a**9*b**11)) + x)/131072
+ 35*sqrt(-1/(a**9*b**11))*log(a**5*b**5*sqrt(-1/(a**9*b**11)) + x)/131072
+ (-315*a**8*x - 2730*a**7*b*x**3 - 10458*a**6*b**2*x**5 - 23202*a**5*b**3*
x**7 - 32768*a**4*b**4*x**9 + 23202*a**3*b**5*x**11 + 10458*a**2*b**6*x**13
 + 2730*a*b**7*x**15 + 315*b**8*x**17)/(589824*a**13*b**5 + 5308416*a**12*b
**6*x**2 + 21233664*a**11*b**7*x**4 + 49545216*a**10*b**8*x**6 + 74317824*a
**9*b**9*x**8 + 74317824*a**8*b**10*x**10 + 49545216*a**7*b**11*x**12 + 212
33664*a**6*b**12*x**14 + 5308416*a**5*b**13*x**16 + 589824*a**4*b**14*x**18
)
```

3.217 $\int \frac{x^8}{\left(a+bx^2\right)^{10}} \, dx$

Optimal. Leaf size=202

$$\frac{35\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{11/2}b^{9/2}}+\frac{35x}{65536a^5b^4\left(a+bx^2\right)}+\frac{35x}{98304a^4b^4\left(a+bx^2\right)^2}+\frac{7x}{24576a^3b^4\left(a+bx^2\right)^3}+\frac{x}{4096a^2b^4\left(a+bx^2\right)^4}+\cdots$$

```
[Out] -1/18*x^7/b/(b*x^2+a)^9-7/288*x^5/b^2/(b*x^2+a)^8-5/576*x^3/b^3/(b*x^2+a)^7
-5/2304*x/b^4/(b*x^2+a)^6+1/4608*x/a/b^4/(b*x^2+a)^5+1/4096*x/a^2/b^4/(b*x^
2+a)^4+7/24576*x/a^3/b^4/(b*x^2+a)^3+35/98304*x/a^4/b^4/(b*x^2+a)^2+35/6553
6*x/a^5/b^4/(b*x^2+a)+35/65536*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)/b^(9/2)
```

Rubi [A] time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{35x}{65536a^5b^4\left(a+bx^2\right)}+\frac{35x}{98304a^4b^4\left(a+bx^2\right)^2}+\frac{7x}{24576a^3b^4\left(a+bx^2\right)^3}+\frac{x}{4096a^2b^4\left(a+bx^2\right)^4}+\frac{35\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{11/2}b^{9/2}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^10,x]
```

```
[Out] -x^7/(18*b*(a + b*x^2)^9) - (7*x^5)/(288*b^2*(a + b*x^2)^8) - (5*x^3)/(576*
b^3*(a + b*x^2)^7) - (5*x)/(2304*b^4*(a + b*x^2)^6) + x/(4608*a*b^4*(a + b*
x^2)^5) + x/(4096*a^2*b^4*(a + b*x^2)^4) + (7*x)/(24576*a^3*b^4*(a + b*x^2)
^3) + (35*x)/(98304*a^4*b^4*(a + b*x^2)^2) + (35*x)/(65536*a^5*b^4*(a + b*x
^2)) + (35*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(11/2)*b^(9/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^7}{18b\left(a+bx^2\right)^9} + \frac{7\int \frac{x^6}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} + \frac{35\int \frac{x^4}{\left(a+bx^2\right)^8}\,dx}{288b^2}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} + \frac{5\int \frac{x^2}{\left(a+bx^2\right)^7}\,dx}{192b^3}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} - \frac{5x}{2304b^4\left(a+bx^2\right)^6} + \frac{5\int \frac{1}{\left(a+bx^2\right)^6}\,d}{2304b^4}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} - \frac{5x}{2304b^4\left(a+bx^2\right)^6} + \frac{x}{4608ab^4\left(a\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} - \frac{5x}{2304b^4\left(a+bx^2\right)^6} + \frac{x}{4608ab^4\left(a\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} - \frac{5x}{2304b^4\left(a+bx^2\right)^6} + \frac{x}{4608ab^4\left(a\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} - \frac{5x}{2304b^4\left(a+bx^2\right)^6} + \frac{x}{4608ab^4\left(a\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} - \frac{5x}{2304b^4\left(a+bx^2\right)^6} + \frac{x}{4608ab^4\left(a\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9} - \frac{7x^5}{288b^2\left(a+bx^2\right)^8} - \frac{5x^3}{576b^3\left(a+bx^2\right)^7} - \frac{5x}{2304b^4\left(a+bx^2\right)^6} + \frac{x}{4608ab^4\left(a\right.}$$

Mathematica [A] time = 0.06, size = 138, normalized size = 0.68

$$\frac{\frac{\sqrt{a}\sqrt{b}\,x\left(-315a^8-2730a^7bx^2-10458a^6b^2x^4-23202a^5b^3x^6+32768a^4b^4x^8+23202a^3b^5x^{10}+10458a^2b^6x^{12}+2730ab^7x^{14}+315b^8x^{16}\right)}{\left(a+bx^2\right)^9}+315\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{589824a^{11/2}b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-315*a^8 - 2730*a^7*b*x^2 - 10458*a^6*b^2*x^4 - 23202*
a^5*b^3*x^6 + 32768*a^4*b^4*x^8 + 23202*a^3*b^5*x^10 + 10458*a^2*b^6*x^12 +
 2730*a*b^7*x^14 + 315*b^8*x^16))/(a + b*x^2)^9 + 315*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(589824*a^(11/2)*b^(9/2))
```

fricas [A] time = 0.86, size = 654, normalized size = 3.24

$$\left[\frac{630\,ab^9x^{17}+5460\,a^2b^8x^{15}+20916\,a^3b^7x^{13}+46404\,a^4b^6x^{11}+65536\,a^5b^5x^9-46404\,a^6b^4x^7-20916\,a^7b^3x^5-5}{1179648\left(a^6b^{14}x^{18}+9\,a^7b^{13}x^{16}+36\,a^8b^{1}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(630*a*b^9*x^17 + 5460*a^2*b^8*x^15 + 20916*a^3*b^7*x^13 + 46404
*a^4*b^6*x^11 + 65536*a^5*b^5*x^9 - 46404*a^6*b^4*x^7 - 20916*a^7*b^3*x^5 -
 5460*a^8*b^2*x^3 - 630*a^9*b*x - 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7
*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x
^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)))/(a^6*b^14*x^18 + 9*a^7*b^13*x^16 + 36*a^8*b^12*x^14
 + 84*a^9*b^11*x^12 + 126*a^10*b^10*x^10 + 126*a^11*b^9*x^8 + 84*a^12*b^8*x
^6 + 36*a^13*b^7*x^4 + 9*a^14*b^6*x^2 + a^15*b^5), 1/589824*(315*a*b^9*x^17
 + 2730*a^2*b^8*x^15 + 10458*a^3*b^7*x^13 + 23202*a^4*b^6*x^11 + 32768*a^5*
b^5*x^9 - 23202*a^6*b^4*x^7 - 10458*a^7*b^3*x^5 - 2730*a^8*b^2*x^3 - 315*a^
9*b*x + 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 +
126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^
8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^6*b^14*x^18 + 9*a^7*b^13
*x^16 + 36*a^8*b^12*x^14 + 84*a^9*b^11*x^12 + 126*a^10*b^10*x^10 + 126*a^11
*b^9*x^8 + 84*a^12*b^8*x^6 + 36*a^13*b^7*x^4 + 9*a^14*b^6*x^2 + a^15*b^5)]
```

giac [A] time = 0.60, size = 128, normalized size = 0.63

$$\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,a^5b^4}+\frac{315\,b^8x^{17}+2730\,ab^7x^{15}+10458\,a^2b^6x^{13}+23202\,a^3b^5x^{11}+32768\,a^4b^4x^9-23202\,a^5b^3x^7}{589824\left(bx^2+a\right)^9a^5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 35/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5*b^4) + 1/589824*(315*b^8*x^17
 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b^5*x^11 + 32768*a^4*b^
4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*a^7*b*x^3 - 315*a^8*x)
/((b*x^2 + a)^9*a^5*b^4)
```

maple [A] time = 0.02, size = 122, normalized size = 0.60

$$\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\;a^5b^4}+\frac{\frac{35b^4x^{17}}{65536a^5}+\frac{455b^3x^{15}}{98304a^4}+\frac{581b^2x^{13}}{32768a^3}+\frac{1289b\,x^{11}}{32768a^2}+\frac{x^9}{18a}-\frac{1289x^7}{32768b}-\frac{581a\,x^5}{32768b^2}-\frac{455a^2x^3}{98304b^3}-\frac{35a^3x}{65536b^4}}{\left(b\,x^2+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^10,x)
```

```
[Out] (-35/65536*a^3/b^4*x-455/98304*a^2/b^3*x^3-581/32768*a/b^2*x^5-1289/32768/b
*x^7+1/18/a*x^9+1289/32768/a^2*b*x^11+581/32768*b^2/a^3*x^13+455/98304*b^3/
a^4*x^15+35/65536*b^4/a^5*x^17)/(b*x^2+a)^9+35/65536/a^5/b^4/(a*b)^(1/2)*ar
ctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.08, size = 221, normalized size = 1.09

$$\frac{315\,b^8x^{17}+2730\,ab^7x^{15}+10458\,a^2b^6x^{13}+23202\,a^3b^5x^{11}+32768\,a^4b^4x^9-23202\,a^5b^3x^7-10458\,a^6b^2x^5}{589824\left(a^5b^{13}x^{18}+9\,a^6b^{12}x^{16}+36\,a^7b^{11}x^{14}+84\,a^8b^{10}x^{12}+126\,a^9b^9x^{10}+126\,a^{10}b^8x^8+84\,a^{11}b^7x^6+36\,a^{12}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/589824*(315*b^8*x^17 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b
^5*x^11 + 32768*a^4*b^4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*
```

```
a^7*b*x^3 - 315*a^8*x)/(a^5*b^13*x^18 + 9*a^6*b^12*x^16 + 36*a^7*b^11*x^14
+ 84*a^8*b^10*x^12 + 126*a^9*b^9*x^10 + 126*a^10*b^8*x^8 + 84*a^11*b^7*x^6
+ 36*a^12*b^6*x^4 + 9*a^13*b^5*x^2 + a^14*b^4) + 35/65536*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^5*b^4)
```

mupad [B] time = 4.74, size = 204, normalized size = 1.01

$$\frac{\frac{x^9}{18\,a}-\frac{1289\,x^7}{32768\,b}-\frac{581\,a\,x^5}{32768\,b^2}-\frac{35\,a^3\,x}{65536\,b^4}+\frac{1289\,b\,x^{11}}{32768\,a^2}-\frac{455\,a^2\,x^3}{98304\,b^3}+\frac{581\,b^2\,x^{13}}{32768\,a^3}+\frac{455\,b^3\,x^{15}}{98304\,a^4}+\frac{35\,b^4\,x^{17}}{65536\,a^5}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2)^10,x)
```

```
[Out] (x^9/(18*a) - (1289*x^7)/(32768*b) - (581*a*x^5)/(32768*b^2) - (35*a^3*x)/(
65536*b^4) + (1289*b*x^11)/(32768*a^2) - (455*a^2*x^3)/(98304*b^3) + (581*b
^2*x^13)/(32768*a^3) + (455*b^3*x^15)/(98304*a^4) + (35*b^4*x^17)/(65536*a^
5))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*
b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7
*x^14) + (35*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(11/2)*b^(9/2))
```

sympy [A] time = 1.15, size = 291, normalized size = 1.44

$$-\frac{35\sqrt{-\frac{1}{a^{11}b^9}}\log\left(-a^6b^4\sqrt{-\frac{1}{a^{11}b^9}}+x\right)}{131072}+\frac{35\sqrt{-\frac{1}{a^{11}b^9}}\log\left(a^6b^4\sqrt{-\frac{1}{a^{11}b^9}}+x\right)}{131072}+\frac{-3}{589824a^{14}b^4+5308416a^{13}b^5x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**10,x)
```

```
[Out] -35*sqrt(-1/(a**11*b**9))*log(-a**6*b**4*sqrt(-1/(a**11*b**9)) + x)/131072
+ 35*sqrt(-1/(a**11*b**9))*log(a**6*b**4*sqrt(-1/(a**11*b**9)) + x)/131072
+ (-315*a**8*x - 2730*a**7*b*x**3 - 10458*a**6*b**2*x**5 - 23202*a**5*b**3*
x**7 + 32768*a**4*b**4*x**9 + 23202*a**3*b**5*x**11 + 10458*a**2*b**6*x**13
 + 2730*a*b**7*x**15 + 315*b**8*x**17)/(589824*a**14*b**4 + 5308416*a**13*b
**5*x**2 + 21233664*a**12*b**6*x**4 + 49545216*a**11*b**7*x**6 + 74317824*a
**10*b**8*x**8 + 74317824*a**9*b**9*x**10 + 49545216*a**8*b**10*x**12 + 212
33664*a**7*b**11*x**14 + 5308416*a**6*b**12*x**16 + 589824*a**5*b**13*x**18
)
```

3.218 $\int \frac{x^6}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=203

$$\frac{55\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{13/2}b^{7/2}}+\frac{55x}{65536a^6b^3\left(a+bx^2\right)}+\frac{55x}{98304a^5b^3\left(a+bx^2\right)^2}+\frac{11x}{24576a^4b^3\left(a+bx^2\right)^3}+\frac{11x}{28672a^3b^3\left(a+bx^2\right)^4}+$$

```
[Out] -1/18*x^5/b/(b*x^2+a)^9-5/288*x^3/b^2/(b*x^2+a)^8-5/1344*x/b^3/(b*x^2+a)^7+
5/16128*x/a/b^3/(b*x^2+a)^6+11/32256*x/a^2/b^3/(b*x^2+a)^5+11/28672*x/a^3/b
^3/(b*x^2+a)^4+11/24576*x/a^4/b^3/(b*x^2+a)^3+55/98304*x/a^5/b^3/(b*x^2+a)^
2+55/65536*x/a^6/b^3/(b*x^2+a)+55/65536*arctan(x*b^(1/2)/a^(1/2))/a^(13/2)/
b^(7/2)
```

Rubi [A] time = 0.11, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{55x}{65536a^6b^3\left(a+bx^2\right)}+\frac{55x}{98304a^5b^3\left(a+bx^2\right)^2}+\frac{11x}{24576a^4b^3\left(a+bx^2\right)^3}+\frac{11x}{28672a^3b^3\left(a+bx^2\right)^4}+\frac{11x}{32256a^2b^3\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^10,x]
```

```
[Out] -x^5/(18*b*(a + b*x^2)^9) - (5*x^3)/(288*b^2*(a + b*x^2)^8) - (5*x)/(1344*b
^3*(a + b*x^2)^7) + (5*x)/(16128*a*b^3*(a + b*x^2)^6) + (11*x)/(32256*a^2*b
^3*(a + b*x^2)^5) + (11*x)/(28672*a^3*b^3*(a + b*x^2)^4) + (11*x)/(24576*a^
4*b^3*(a + b*x^2)^3) + (55*x)/(98304*a^5*b^3*(a + b*x^2)^2) + (55*x)/(65536
*a^6*b^3*(a + b*x^2)) + (55*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(13/2)*b^
(7/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{(a+bx^2)^{10}}\,dx = -\frac{x^5}{18b(a+bx^2)^9} + \frac{5\int \frac{x^4}{(a+bx^2)^9}\,dx}{18b}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} + \frac{5\int \frac{x^2}{(a+bx^2)^8}\,dx}{96b^2}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5\int \frac{1}{(a+bx^2)^7}\,dx}{1344b^3}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{55\int \frac{1}{(a+b}}{16128}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{}{32256a^2}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{}{32256a^2}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{}{32256a^2}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{}{32256a^2}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{}{32256a^2}$$

$$= -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{}{32256a^2}$$

Mathematica [A] time = 0.07, size = 138, normalized size = 0.68

$$\frac{\frac{\sqrt{a}\sqrt{b}x(-3465a^8-30030a^7bx^2-115038a^6b^2x^4+334602a^5b^3x^6+360448a^4b^4x^8+255222a^3b^5x^{10}+115038a^2b^6x^{12}+30030ab^7x^{14}+3465b^8x^{16})}{(a+bx^2)^9} + 3465}{4128768a^{13/2}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-3465*a^8 - 30030*a^7*b*x^2 - 115038*a^6*b^2*x^4 + 334
602*a^5*b^3*x^6 + 360448*a^4*b^4*x^8 + 255222*a^3*b^5*x^10 + 115038*a^2*b^6
*x^12 + 30030*a*b^7*x^14 + 3465*b^8*x^16))/(a + b*x^2)^9 + 3465*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(4128768*a^(13/2)*b^(7/2))
```

fricas [A] time = 0.86, size = 654, normalized size = 3.22

$$\left[\frac{6930\,ab^9x^{17} + 60060\,a^2b^8x^{15} + 230076\,a^3b^7x^{13} + 510444\,a^4b^6x^{11} + 720896\,a^5b^5x^9 + 669204\,a^6b^4x^7 - 230076\,a}{8257536\left(a^7b^{13}x^{18} + 9\,a^8b^{12}x^{16} + 3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(6930*a*b^9*x^17 + 60060*a^2*b^8*x^15 + 230076*a^3*b^7*x^13 + 51
0444*a^4*b^6*x^11 + 720896*a^5*b^5*x^9 + 669204*a^6*b^4*x^7 - 230076*a^7*b^
3*x^5 - 60060*a^8*b^2*x^3 - 6930*a^9*b*x - 3465*(b^9*x^18 + 9*a*b^8*x^16 +
36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84
*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 -
2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^7*b^13*x^18 + 9*a^8*b^12*x^16 + 36*a^9
*b^11*x^14 + 84*a^10*b^10*x^12 + 126*a^11*b^9*x^10 + 126*a^12*b^8*x^8 + 84*
a^13*b^7*x^6 + 36*a^14*b^6*x^4 + 9*a^15*b^5*x^2 + a^16*b^4), 1/4128768*(346
5*a*b^9*x^17 + 30030*a^2*b^8*x^15 + 115038*a^3*b^7*x^13 + 255222*a^4*b^6*x^
11 + 360448*a^5*b^5*x^9 + 334602*a^6*b^4*x^7 - 115038*a^7*b^3*x^5 - 30030*a
^8*b^2*x^3 - 3465*a^9*b*x + 3465*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14
 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 +
36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^7*b
^13*x^18 + 9*a^8*b^12*x^16 + 36*a^9*b^11*x^14 + 84*a^10*b^10*x^12 + 126*a^1
1*b^9*x^10 + 126*a^12*b^8*x^8 + 84*a^13*b^7*x^6 + 36*a^14*b^6*x^4 + 9*a^15*
b^5*x^2 + a^16*b^4)]
```

giac [A] time = 0.61, size = 128, normalized size = 0.63

$$\frac{55 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^6 b^3} + \frac{3465\, b^8 x^{17} + 30030\, a b^7 x^{15} + 115038\, a^2 b^6 x^{13} + 255222\, a^3 b^5 x^{11} + 360448\, a^4 b^4 x^9 + 334602}{4128768 \left(bx^2 + a\right)^9 a^6 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6*b^3) + 1/4128768*(3465*b^8*x^
17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 + 255222*a^3*b^5*x^11 + 360448*
a^4*b^4*x^9 + 334602*a^5*b^3*x^7 - 115038*a^6*b^2*x^5 - 30030*a^7*b*x^3 - 3
465*a^8*x)/((b*x^2 + a)^9*a^6*b^3)
```

maple [A] time = 0.02, size = 122, normalized size = 0.60

$$\frac{55 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\; a^6 b^3} + \frac{\frac{55 b^5 x^{17}}{65536 a^6} + \frac{715 b^4 x^{15}}{98304 a^5} + \frac{913 b^3 x^{13}}{32768 a^4} + \frac{14179 b^2 x^{11}}{229376 a^3} + \frac{11 b\, x^9}{126 a^2} + \frac{18589 x^7}{229376 a} - \frac{913 x^5}{32768 b} - \frac{715 a\, x^3}{98304 b^2} - \frac{55 a^2 x}{65536 b^3}}{\left(b\, x^2 + a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^10,x)
```

```
[Out] (-55/65536*a^2/b^3*x-715/98304*a/b^2*x^3-913/32768/b*x^5+18589/229376/a*x^7
+11/126/a^2*b*x^9+14179/229376*b^2/a^3*x^11+913/32768*b^3/a^4*x^13+715/9830
4*b^4/a^5*x^15+55/65536/a^6*b^5*x^17)/(b*x^2+a)^9+55/65536/a^6/b^3/(a*b)^(1
/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.14, size = 221, normalized size = 1.09

$$\frac{3465\, b^8 x^{17} + 30030\, a b^7 x^{15} + 115038\, a^2 b^6 x^{13} + 255222\, a^3 b^5 x^{11} + 360448\, a^4 b^4 x^9 + 334602\, a^5 b^3 x^7 - 115038\, a^6}{4128768 \left(a^6 b^{12} x^{18} + 9\, a^7 b^{11} x^{16} + 36\, a^8 b^{10} x^{14} + 84\, a^9 b^9 x^{12} + 126\, a^{10} b^8 x^{10} + 126\, a^{11} b^7 x^8 + 84\, a^{12} b^6 x^6 + 36\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(3465*b^8*x^17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 + 255222*
a^3*b^5*x^11 + 360448*a^4*b^4*x^9 + 334602*a^5*b^3*x^7 - 115038*a^6*b^2*x^5
 - 30030*a^7*b*x^3 - 3465*a^8*x)/(a^6*b^12*x^18 + 9*a^7*b^11*x^16 + 36*a^8*
b^10*x^14 + 84*a^9*b^9*x^12 + 126*a^10*b^8*x^10 + 126*a^11*b^7*x^8 + 84*a^1
2*b^6*x^6 + 36*a^13*b^5*x^4 + 9*a^14*b^4*x^2 + a^15*b^3) + 55/65536*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^6*b^3)
```

mupad [B] time = 4.79, size = 204, normalized size = 1.00

$$\frac{\frac{18589\,x^7}{229376\,a}-\frac{913\,x^5}{32768\,b}-\frac{715\,a\,x^3}{98304\,b^2}-\frac{55\,a^2\,x}{65536\,b^3}+\frac{11\,b\,x^9}{126\,a^2}+\frac{14179\,b^2\,x^{11}}{229376\,a^3}+\frac{913\,b^3\,x^{13}}{32768\,a^4}+\frac{715\,b^4\,x^{15}}{98304\,a^5}+\frac{55\,b^5\,x^{17}}{65536\,a^6}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^10,x)
```

```
[Out] ((18589*x^7)/(229376*a) - (913*x^5)/(32768*b) - (715*a*x^3)/(98304*b^2) - (
55*a^2*x)/(65536*b^3) + (11*b*x^9)/(126*a^2) + (14179*b^2*x^11)/(229376*a^3
) + (913*b^3*x^13)/(32768*a^4) + (715*b^4*x^15)/(98304*a^5) + (55*b^5*x^17)
/(65536*a^6))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4
 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 +
36*a^2*b^7*x^14) + (55*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(13/2)*b^(7/2))
```

sympy [A] time = 1.08, size = 291, normalized size = 1.43

$$-\frac{55\sqrt{-\frac{1}{a^{13}b^7}}\,\log\left(-a^7b^3\sqrt{-\frac{1}{a^{13}b^7}}+x\right)}{131072}+\frac{55\sqrt{-\frac{1}{a^{13}b^7}}\,\log\left(a^7b^3\sqrt{-\frac{1}{a^{13}b^7}}+x\right)}{131072}+\frac{-3}{4128768a^{15}b^3+37158912a^{14}b^4x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**10,x)
```

```
[Out] -55*sqrt(-1/(a**13*b**7))*log(-a**7*b**3*sqrt(-1/(a**13*b**7)) + x)/131072
+ 55*sqrt(-1/(a**13*b**7))*log(a**7*b**3*sqrt(-1/(a**13*b**7)) + x)/131072
+ (-3465*a**8*x - 30030*a**7*b*x**3 - 115038*a**6*b**2*x**5 + 334602*a**5*b
**3*x**7 + 360448*a**4*b**4*x**9 + 255222*a**3*b**5*x**11 + 115038*a**2*b**
6*x**13 + 30030*a*b**7*x**15 + 3465*b**8*x**17)/(4128768*a**15*b**3 + 37158
912*a**14*b**4*x**2 + 148635648*a**13*b**5*x**4 + 346816512*a**12*b**6*x**6
 + 520224768*a**11*b**7*x**8 + 520224768*a**10*b**8*x**10 + 346816512*a**9*
b**9*x**12 + 148635648*a**8*b**10*x**14 + 37158912*a**7*b**11*x**16 + 41287
68*a**6*b**12*x**18)
```

3.219 $\int \frac{x^4}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=204

$$\frac{143\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{15/2}b^{5/2}}+\frac{143x}{65536a^7b^2(a+bx^2)}+\frac{143x}{98304a^6b^2(a+bx^2)^2}+\frac{143x}{122880a^5b^2(a+bx^2)^3}+\frac{143x}{143360a^4b^2(a+bx^2}$$

```
[Out] -1/18*x^3/b/(b*x^2+a)^9-1/96*x/b^2/(b*x^2+a)^8+1/1344*x/a/b^2/(b*x^2+a)^7+1
3/16128*x/a^2/b^2/(b*x^2+a)^6+143/161280*x/a^3/b^2/(b*x^2+a)^5+143/143360*x
/a^4/b^2/(b*x^2+a)^4+143/122880*x/a^5/b^2/(b*x^2+a)^3+143/98304*x/a^6/b^2/(
b*x^2+a)^2+143/65536*x/a^7/b^2/(b*x^2+a)+143/65536*arctan(x*b^(1/2)/a^(1/2)
)/a^(15/2)/b^(5/2)
```

Rubi [A] time = 0.11, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{143x}{65536a^7b^2(a+bx^2)}+\frac{143x}{98304a^6b^2(a+bx^2)^2}+\frac{143x}{122880a^5b^2(a+bx^2)^3}+\frac{143x}{143360a^4b^2(a+bx^2)^4}+\frac{143x}{161280a^3b^2(a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^10,x]
```

```
[Out] -x^3/(18*b*(a + b*x^2)^9) - x/(96*b^2*(a + b*x^2)^8) + x/(1344*a*b^2*(a + b
*x^2)^7) + (13*x)/(16128*a^2*b^2*(a + b*x^2)^6) + (143*x)/(161280*a^3*b^2*(
a + b*x^2)^5) + (143*x)/(143360*a^4*b^2*(a + b*x^2)^4) + (143*x)/(122880*a^
5*b^2*(a + b*x^2)^3) + (143*x)/(98304*a^6*b^2*(a + b*x^2)^2) + (143*x)/(655
36*a^7*b^2*(a + b*x^2)) + (143*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(15/2)
*b^(5/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^3}{18b\left(a+bx^2\right)^9} + \frac{\int \frac{x^2}{\left(a+bx^2\right)^9}\,dx}{6b}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{\int \frac{1}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13\int \frac{1}{\left(a+bx^2\right)^7}\,dx}{1344ab^2}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13x}{16128a^2b^2\left(a+bx^2\right)^6} + \frac{143\int \cdots}{1612\cdots}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13x}{16128a^2b^2\left(a+bx^2\right)^6} + \frac{\cdots}{161280\cdots}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13x}{16128a^2b^2\left(a+bx^2\right)^6} + \frac{\cdots}{161280\cdots}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13x}{16128a^2b^2\left(a+bx^2\right)^6} + \frac{\cdots}{161280\cdots}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13x}{16128a^2b^2\left(a+bx^2\right)^6} + \frac{\cdots}{161280\cdots}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13x}{16128a^2b^2\left(a+bx^2\right)^6} + \frac{\cdots}{161280\cdots}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13x}{16128a^2b^2\left(a+bx^2\right)^6} + \frac{\cdots}{161280\cdots}$$

Mathematica [A] time = 0.06, size = 138, normalized size = 0.68

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(-45045a^8-390390a^7bx^2+2633274a^6b^2x^4+4349826a^5b^3x^6+4685824a^4b^4x^8+3317886a^3b^5x^{10}+1495494a^2b^6x^{12}+390390ab^7x^{14}+45045b^8x^{16}\right)}{\left(a+bx^2\right)^9}\cdots}{20643840a^{15/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 + 2633274*a^6*b^2*x^4 +
4349826*a^5*b^3*x^6 + 4685824*a^4*b^4*x^8 + 3317886*a^3*b^5*x^10 + 1495494*
a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(20643840*a^(15/2)*b^(5/2))
```

fricas [A] time = 0.93, size = 654, normalized size = 3.21

$$\left[\frac{90090\,ab^9x^{17} + 780780\,a^2b^8x^{15} + 2990988\,a^3b^7x^{13} + 6635772\,a^4b^6x^{11} + 9371648\,a^5b^5x^9 + 8699652\,a^6b^4x^7 + 5\cdots}{41287680\left(a^8b^{12}x^{18} + 9\,a^9b^{11}\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/41287680*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 + 2990988*a^3*b^7*x^13
+ 6635772*a^4*b^6*x^11 + 9371648*a^5*b^5*x^9 + 8699652*a^6*b^4*x^7 + 526654
8*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^8*b^12*x^18 + 9*a^9*b^11*x^
16 + 36*a^10*b^10*x^14 + 84*a^11*b^9*x^12 + 126*a^12*b^8*x^10 + 126*a^13*b^
7*x^8 + 84*a^14*b^6*x^6 + 36*a^15*b^5*x^4 + 9*a^16*b^4*x^2 + a^17*b^3), 1/2
0643840*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 + 1495494*a^3*b^7*x^13 + 33
17886*a^4*b^6*x^11 + 4685824*a^5*b^5*x^9 + 4349826*a^6*b^4*x^7 + 2633274*a^
7*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*
x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x
^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(
sqrt(a*b)*x/a))/(a^8*b^12*x^18 + 9*a^9*b^11*x^16 + 36*a^10*b^10*x^14 + 84*a
^11*b^9*x^12 + 126*a^12*b^8*x^10 + 126*a^13*b^7*x^8 + 84*a^14*b^6*x^6 + 36*
a^15*b^5*x^4 + 9*a^16*b^4*x^2 + a^17*b^3)]
```

giac [A] time = 0.63, size = 128, normalized size = 0.63

$$\frac{143 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^7 b^2} + \frac{45045 b^8 x^{17} + 390390 a b^7 x^{15} + 1495494 a^2 b^6 x^{13} + 3317886 a^3 b^5 x^{11} + 4685824 a^4 b^4 x^9 + \cdots}{20643840 \left(bx^2 + a\right)^9 a^7 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 143/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7*b^2) + 1/20643840*(45045*b^8
*x^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 3317886*a^3*b^5*x^11 + 4
685824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6*b^2*x^5 - 390390*a^7
*b*x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^7*b^2)
```

maple [A] time = 0.02, size = 122, normalized size = 0.60

$$\frac{143 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\; a^7 b^2} + \frac{\frac{143 b^6 x^{17}}{65536 a^7} + \frac{1859 b^5 x^{15}}{98304 a^6} + \frac{11869 b^4 x^{13}}{163840 a^5} + \frac{184327 b^3 x^{11}}{1146880 a^4} + \frac{143 b^2 x^9}{630 a^3} + \frac{241657 b\, x^7}{1146880 a^2} + \frac{20899 x^5}{163840 a} - \frac{1859 x^3}{98304 b} - \frac{143 a \cdots}{65536 \cdots}}{\left(b\, x^2 + a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^10,x)
```

```
[Out] (-143/65536*a/b^2*x-1859/98304/b*x^3+20899/163840/a*x^5+241657/1146880/a^2*
b*x^7+143/630*b^2/a^3*x^9+184327/1146880*b^3/a^4*x^11+11869/163840*b^4/a^5*
x^13+1859/98304/a^6*b^5*x^15+143/65536/a^7*b^6*x^17)/(b*x^2+a)^9+143/65536/
a^7/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.19, size = 221, normalized size = 1.08

$$\frac{45045 b^8 x^{17} + 390390 a b^7 x^{15} + 1495494 a^2 b^6 x^{13} + 3317886 a^3 b^5 x^{11} + 4685824 a^4 b^4 x^9 + 4349826 a^5 b^3 x^7 + 26\cdots}{20643840 \left(a^7 b^{11} x^{18} + 9 a^8 b^{10} x^{16} + 36 a^9 b^9 x^{14} + 84 a^{10} b^8 x^{12} + 126 a^{11} b^7 x^{10} + 126 a^{12} b^6 x^8 + 84 a^{13} b^5 x^{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/20643840*(45045*b^8*x^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 331
7886*a^3*b^5*x^11 + 4685824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6
*b^2*x^5 - 390390*a^7*b*x^3 - 45045*a^8*x)/(a^7*b^11*x^18 + 9*a^8*b^10*x^16
 + 36*a^9*b^9*x^14 + 84*a^10*b^8*x^12 + 126*a^11*b^7*x^10 + 126*a^12*b^6*x^
8 + 84*a^13*b^5*x^6 + 36*a^14*b^4*x^4 + 9*a^15*b^3*x^2 + a^16*b^2) + 143/65
536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7*b^2)
```

mupad [B] time = 4.74, size = 204, normalized size = 1.00

$$\frac{\frac{20899\,x^5}{163840\,a}-\frac{1859\,x^3}{98304\,b}+\frac{241657\,b\,x^7}{1146880\,a^2}+\frac{143\,b^2\,x^9}{630\,a^3}+\frac{184327\,b^3\,x^{11}}{1146880\,a^4}+\frac{11869\,b^4\,x^{13}}{163840\,a^5}+\frac{1859\,b^5\,x^{15}}{98304\,a^6}+\frac{143\,b^6\,x^{17}}{65536\,a^7}-\frac{143\,a\,x}{65536\,b^2}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^10,x)
```

```
[Out] ((20899*x^5)/(163840*a) - (1859*x^3)/(98304*b) + (241657*b*x^7)/(1146880*a^
2) + (143*b^2*x^9)/(630*a^3) + (184327*b^3*x^11)/(1146880*a^4) + (11869*b^4
*x^13)/(163840*a^5) + (1859*b^5*x^15)/(98304*a^6) + (143*b^6*x^17)/(65536*a
^7) - (143*a*x)/(65536*b^2))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 +
 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*
a^3*b^6*x^12 + 36*a^2*b^7*x^14) + (143*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^
(15/2)*b^(5/2))
```

sympy [A] time = 1.04, size = 291, normalized size = 1.43

$$-\frac{143\sqrt{-\frac{1}{a^{15}b^5}}\log\left(-a^8b^2\sqrt{-\frac{1}{a^{15}b^5}}+x\right)}{131072}+\frac{143\sqrt{-\frac{1}{a^{15}b^5}}\log\left(a^8b^2\sqrt{-\frac{1}{a^{15}b^5}}+x\right)}{131072}+\frac{-}{20643840a^{16}b^2+185794560a^{15}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**10,x)
```

```
[Out] -143*sqrt(-1/(a**15*b**5))*log(-a**8*b**2*sqrt(-1/(a**15*b**5)) + x)/131072
 + 143*sqrt(-1/(a**15*b**5))*log(a**8*b**2*sqrt(-1/(a**15*b**5)) + x)/13107
2 + (-45045*a**8*x - 390390*a**7*b*x**3 + 2633274*a**6*b**2*x**5 + 4349826*
a**5*b**3*x**7 + 4685824*a**4*b**4*x**9 + 3317886*a**3*b**5*x**11 + 1495494
*a**2*b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(20643840*a**16*
b**2 + 185794560*a**15*b**3*x**2 + 743178240*a**14*b**4*x**4 + 1734082560*a
**13*b**5*x**6 + 2601123840*a**12*b**6*x**8 + 2601123840*a**11*b**7*x**10 +
 1734082560*a**10*b**8*x**12 + 743178240*a**9*b**9*x**14 + 185794560*a**8*b
**10*x**16 + 20643840*a**7*b**11*x**18)
```

3.220 $\int \frac{x^2}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=205

$$\frac{715\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{17/2}b^{3/2}}+\frac{715x}{65536a^8b\left(a+bx^2\right)}+\frac{715x}{98304a^7b\left(a+bx^2\right)^2}+\frac{143x}{24576a^6b\left(a+bx^2\right)^3}+\frac{143x}{28672a^5b\left(a+bx^2\right)^4}+$$

```
[Out] -1/18*x/b/(b*x^2+a)^9+1/288*x/a/b/(b*x^2+a)^8+5/1344*x/a^2/b/(b*x^2+a)^7+65
/16128*x/a^3/b/(b*x^2+a)^6+143/32256*x/a^4/b/(b*x^2+a)^5+143/28672*x/a^5/b/
(b*x^2+a)^4+143/24576*x/a^6/b/(b*x^2+a)^3+715/98304*x/a^7/b/(b*x^2+a)^2+715
/65536*x/a^8/b/(b*x^2+a)+715/65536*arctan(x*b^(1/2)/a^(1/2))/a^(17/2)/b^(3/
2)
```

Rubi [A] time = 0.11, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{715\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{17/2}b^{3/2}}+\frac{715x}{65536a^8b\left(a+bx^2\right)}+\frac{715x}{98304a^7b\left(a+bx^2\right)^2}+\frac{143x}{24576a^6b\left(a+bx^2\right)^3}+\frac{143x}{28672a^5b\left(a+bx^2\right)^4}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^10,x]
```

```
[Out] -x/(18*b*(a + b*x^2)^9) + x/(288*a*b*(a + b*x^2)^8) + (5*x)/(1344*a^2*b*(a
+ b*x^2)^7) + (65*x)/(16128*a^3*b*(a + b*x^2)^6) + (143*x)/(32256*a^4*b*(a
+ b*x^2)^5) + (143*x)/(28672*a^5*b*(a + b*x^2)^4) + (143*x)/(24576*a^6*b*(a
 + b*x^2)^3) + (715*x)/(98304*a^7*b*(a + b*x^2)^2) + (715*x)/(65536*a^8*b*(
a + b*x^2)) + (715*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(17/2)*b^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{\int \frac{1}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5\int \frac{1}{\left(a+bx^2\right)^8}\,dx}{96ab}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65\int \frac{1}{\left(a+bx^2\right)^7}\,dx}{1344a^2b}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{715\int \ldots}{161\ldots}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{\ldots}{32256a\ldots}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{\ldots}{32256a\ldots}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{\ldots}{32256a\ldots}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{\ldots}{32256a\ldots}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{\ldots}{32256a\ldots}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{\ldots}{32256a\ldots}$$

Mathematica [A] time = 0.06, size = 138, normalized size = 0.67

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(-45045a^8+985866a^7bx^2+2633274a^6b^2x^4+4349826a^5b^3x^6+4685824a^4b^4x^8+3317886a^3b^5x^{10}+1495494a^2b^6x^{12}+390390ab^7x^{14}+45045b^8x^{16}\right)}{\left(a+bx^2\right)^9} \ldots}{4128768a^{17/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 + 985866*a^7*b*x^2 + 2633274*a^6*b^2*x^4 +
4349826*a^5*b^3*x^6 + 4685824*a^4*b^4*x^8 + 3317886*a^3*b^5*x^10 + 1495494*
a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(4128768*a^(17/2)*b^(3/2))
```

fricas [A] time = 0.84, size = 654, normalized size = 3.19

$$\left[\frac{90090\,ab^9x^{17} + 780780\,a^2b^8x^{15} + 2990988\,a^3b^7x^{13} + 6635772\,a^4b^6x^{11} + 9371648\,a^5b^5x^9 + 8699652\,a^6b^4x^7 + 5\ldots}{8257536\left(a^9b^{11}x^{18} + 9\,a^{10}b^1\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 + 2990988*a^3*b^7*x^13 +
 6635772*a^4*b^6*x^11 + 9371648*a^5*b^5*x^9 + 8699652*a^6*b^4*x^7 + 5266548
*a^7*b^3*x^5 + 1971732*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^9*b^11*x^18 + 9*a^10*b^10*x
^16 + 36*a^11*b^9*x^14 + 84*a^12*b^8*x^12 + 126*a^13*b^7*x^10 + 126*a^14*b^
6*x^8 + 84*a^15*b^5*x^6 + 36*a^16*b^4*x^4 + 9*a^17*b^3*x^2 + a^18*b^2), 1/4
128768*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 + 1495494*a^3*b^7*x^13 + 331
7886*a^4*b^6*x^11 + 4685824*a^5*b^5*x^9 + 4349826*a^6*b^4*x^7 + 2633274*a^7
*b^3*x^5 + 985866*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*x
^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^
8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a))/(a^9*b^11*x^18 + 9*a^10*b^10*x^16 + 36*a^11*b^9*x^14 + 84*a^
12*b^8*x^12 + 126*a^13*b^7*x^10 + 126*a^14*b^6*x^8 + 84*a^15*b^5*x^6 + 36*a
^16*b^4*x^4 + 9*a^17*b^3*x^2 + a^18*b^2)]
```

giac [A] time = 0.62, size = 128, normalized size = 0.62

$$\frac{715 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^8 b} + \frac{45045\, b^8 x^{17} + 390390\, a b^7 x^{15} + 1495494\, a^2 b^6 x^{13} + 3317886\, a^3 b^5 x^{11} + 4685824\, a^4 b^4 x^9 + 4}{4128768 \left(bx^2 + a\right)^9 a^8 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 715/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8*b) + 1/4128768*(45045*b^8*x^
17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 3317886*a^3*b^5*x^11 + 4685
824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6*b^2*x^5 + 985866*a^7*b*
x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^8*b)
```

maple [A] time = 0.02, size = 124, normalized size = 0.60

$$\frac{715 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\; a^8 b} + \frac{\frac{715 b^7 x^{17}}{65536 a^8} + \frac{9295 b^6 x^{15}}{98304 a^7} + \frac{11869 b^5 x^{13}}{32768 a^6} + \frac{184327 b^4 x^{11}}{229376 a^5} + \frac{143 b^3 x^9}{126 a^4} + \frac{241657 b^2 x^7}{229376 a^3} + \frac{20899 b\, x^5}{32768 a^2} + \frac{23473 x^3}{98304 a} - \frac{7}{65}}{\left(b\, x^2 + a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^10,x)
```

```
[Out] (-715/65536/b*x+23473/98304/a*x^3+20899/32768/a^2*b*x^5+241657/229376*b^2/a
^3*x^7+143/126*b^3/a^4*x^9+184327/229376*b^4/a^5*x^11+11869/32768/a^6*b^5*x
^13+9295/98304/a^7*b^6*x^15+715/65536/a^8*b^7*x^17)/(b*x^2+a)^9+715/65536/a
^8/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.09, size = 219, normalized size = 1.07

$$\frac{45045\, b^8 x^{17} + 390390\, a b^7 x^{15} + 1495494\, a^2 b^6 x^{13} + 3317886\, a^3 b^5 x^{11} + 4685824\, a^4 b^4 x^9 + 4349826\, a^5 b^3 x^7 + 26}{4128768 \left(a^8 b^{10} x^{18} + 9\, a^9 b^9 x^{16} + 36\, a^{10} b^8 x^{14} + 84\, a^{11} b^7 x^{12} + 126\, a^{12} b^6 x^{10} + 126\, a^{13} b^5 x^8 + 84\, a^{14} b^4 x^{6}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(45045*b^8*x^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 3317
886*a^3*b^5*x^11 + 4685824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6*
b^2*x^5 + 985866*a^7*b*x^3 - 45045*a^8*x)/(a^8*b^10*x^18 + 9*a^9*b^9*x^16 +
 36*a^10*b^8*x^14 + 84*a^11*b^7*x^12 + 126*a^12*b^6*x^10 + 126*a^13*b^5*x^8
 + 84*a^14*b^4*x^6 + 36*a^15*b^3*x^4 + 9*a^16*b^2*x^2 + a^17*b) + 715/65536
*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8*b)
```

mupad [B] time = 0.17, size = 206, normalized size = 1.00

$$\frac{\frac{23473\,x^3}{98304\,a}-\frac{715\,x}{65536\,b}+\frac{20899\,b\,x^5}{32768\,a^2}+\frac{241657\,b^2\,x^7}{229376\,a^3}+\frac{143\,b^3\,x^9}{126\,a^4}+\frac{184327\,b^4\,x^{11}}{229376\,a^5}+\frac{11869\,b^5\,x^{13}}{32768\,a^6}+\frac{9295\,b^6\,x^{15}}{98304\,a^7}+\frac{715\,b^7\,x^{17}}{65536\,a^8}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^10,x)
```

```
[Out] ((23473*x^3)/(98304*a) - (715*x)/(65536*b) + (20899*b*x^5)/(32768*a^2) + (2
41657*b^2*x^7)/(229376*a^3) + (143*b^3*x^9)/(126*a^4) + (184327*b^4*x^11)/(
229376*a^5) + (11869*b^5*x^13)/(32768*a^6) + (9295*b^6*x^15)/(98304*a^7) +
(715*b^7*x^17)/(65536*a^8))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 +
36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a
^3*b^6*x^12 + 36*a^2*b^7*x^14) + (715*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(
17/2)*b^(3/2))
```

sympy [A] time = 1.03, size = 286, normalized size = 1.40

$$-\frac{715\sqrt{-\frac{1}{a^{17}b^3}}\log\left(-a^9b\sqrt{-\frac{1}{a^{17}b^3}}+x\right)}{131072}+\frac{715\sqrt{-\frac{1}{a^{17}b^3}}\log\left(a^9b\sqrt{-\frac{1}{a^{17}b^3}}+x\right)}{131072}+\frac{-45045a}{4128768a^{17}b+37158912a^{16}b^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**10,x)
```

```
[Out] -715*sqrt(-1/(a**17*b**3))*log(-a**9*b*sqrt(-1/(a**17*b**3)) + x)/131072 +
715*sqrt(-1/(a**17*b**3))*log(a**9*b*sqrt(-1/(a**17*b**3)) + x)/131072 + (-
45045*a**8*x + 985866*a**7*b*x**3 + 2633274*a**6*b**2*x**5 + 4349826*a**5*b
**3*x**7 + 4685824*a**4*b**4*x**9 + 3317886*a**3*b**5*x**11 + 1495494*a**2*
b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(4128768*a**17*b + 371
58912*a**16*b**2*x**2 + 148635648*a**15*b**3*x**4 + 346816512*a**14*b**4*x*
*6 + 520224768*a**13*b**5*x**8 + 520224768*a**12*b**6*x**10 + 346816512*a**
11*b**7*x**12 + 148635648*a**10*b**8*x**14 + 37158912*a**9*b**9*x**16 + 412
8768*a**8*b**10*x**18)
```

3.221 $\int \frac{1}{\left(a+bx^2\right)^{10}} dx$

Optimal. Leaf size=181

$$\frac{12155\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{19/2}\sqrt{b}}+\frac{12155x}{65536a^9\left(a+bx^2\right)}+\frac{12155x}{98304a^8\left(a+bx^2\right)^2}+\frac{2431x}{24576a^7\left(a+bx^2\right)^3}+\frac{2431x}{28672a^6\left(a+bx^2\right)^4}+\frac{}{3225}$$

```
[Out] 1/18*x/a/(b*x^2+a)^9+17/288*x/a^2/(b*x^2+a)^8+85/1344*x/a^3/(b*x^2+a)^7+110
5/16128*x/a^4/(b*x^2+a)^6+2431/32256*x/a^5/(b*x^2+a)^5+2431/28672*x/a^6/(b*
x^2+a)^4+2431/24576*x/a^7/(b*x^2+a)^3+12155/98304*x/a^8/(b*x^2+a)^2+12155/6
5536*x/a^9/(b*x^2+a)+12155/65536*arctan(x*b^(1/2)/a^(1/2))/a^(19/2)/b^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 205}

$$\frac{12155x}{65536a^9\left(a+bx^2\right)}+\frac{12155x}{98304a^8\left(a+bx^2\right)^2}+\frac{2431x}{24576a^7\left(a+bx^2\right)^3}+\frac{2431x}{28672a^6\left(a+bx^2\right)^4}+\frac{2431x}{32256a^5\left(a+bx^2\right)^5}+\frac{}{161}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-10),x]
```

```
[Out] x/(18*a*(a + b*x^2)^9) + (17*x)/(288*a^2*(a + b*x^2)^8) + (85*x)/(1344*a^3*
(a + b*x^2)^7) + (1105*x)/(16128*a^4*(a + b*x^2)^6) + (2431*x)/(32256*a^5*(
a + b*x^2)^5) + (2431*x)/(28672*a^6*(a + b*x^2)^4) + (2431*x)/(24576*a^7*(a
 + b*x^2)^3) + (12155*x)/(98304*a^8*(a + b*x^2)^2) + (12155*x)/(65536*a^9*(
a + b*x^2)) + (12155*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(19/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{10}}\,dx = \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17\int \frac{1}{\left(a+bx^2\right)^9}\,dx}{18a}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85\int \frac{1}{\left(a+bx^2\right)^8}\,dx}{96a^2}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105\int \frac{1}{\left(a+bx^2\right)^7}\,dx}{1344a^3}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{12155\int \cdots}{16128\cdots}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431\cdots}{32256a^5(a\cdots}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431\cdots}{32256a^5(a\cdots}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431\cdots}{32256a^5(a\cdots}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431\cdots}{32256a^5(a\cdots}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431\cdots}{32256a^5(a\cdots}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431\cdots}{32256a^5(a\cdots}$$

Mathematica [**A**] time = 0.10, size = 131, normalized size = 0.72

$$\frac{\frac{765765\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{19/2}\sqrt{b}} + \frac{3363003a^8x+16759722a^7bx^3+44765658a^6b^2x^5+73947042a^5b^3x^7+79659008a^4b^4x^9+56404062a^3b^5x^{11}+25423398a^2b^6x^{13}+66\cdots}{a^9\left(a+bx^2\right)^9}}{4128768}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-10),x]
```

```
[Out] ((3363003*a^8*x + 16759722*a^7*b*x^3 + 44765658*a^6*b^2*x^5 + 73947042*a^5*
b^3*x^7 + 79659008*a^4*b^4*x^9 + 56404062*a^3*b^5*x^11 + 25423398*a^2*b^6*x
^13 + 6636630*a*b^7*x^15 + 765765*b^8*x^17)/(a^9*(a + b*x^2)^9) + (765765*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(19/2)*Sqrt[b]))/4128768
```

fricas [B] time = 0.91, size = 650, normalized size = 3.59

$$\left[\frac{1531530\,ab^{9}x^{17}+13273260\,a^{2}b^{8}x^{15}+50846796\,a^{3}b^{7}x^{13}+112808124\,a^{4}b^{6}x^{11}+159318016\,a^{5}b^{5}x^{9}+147894}{8257536\,\left(a^{10}b^{1}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(1531530*a*b^9*x^17 + 13273260*a^2*b^8*x^15 + 50846796*a^3*b^7*x
^13 + 112808124*a^4*b^6*x^11 + 159318016*a^5*b^5*x^9 + 147894084*a^6*b^4*x^
7 + 89531316*a^7*b^3*x^5 + 33519444*a^8*b^2*x^3 + 6726006*a^9*b*x - 765765*
(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*
x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^
9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^10*b^10*x^1
8 + 9*a^11*b^9*x^16 + 36*a^12*b^8*x^14 + 84*a^13*b^7*x^12 + 126*a^14*b^6*x^
10 + 126*a^15*b^5*x^8 + 84*a^16*b^4*x^6 + 36*a^17*b^3*x^4 + 9*a^18*b^2*x^2
+ a^19*b), 1/4128768*(765765*a*b^9*x^17 + 6636630*a^2*b^8*x^15 + 25423398*a
^3*b^7*x^13 + 56404062*a^4*b^6*x^11 + 79659008*a^5*b^5*x^9 + 73947042*a^6*b
^4*x^7 + 44765658*a^7*b^3*x^5 + 16759722*a^8*b^2*x^3 + 3363003*a^9*b*x + 76
5765*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4
*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2
 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^10*b^10*x^18 + 9*a^11*b^9*x^16
+ 36*a^12*b^8*x^14 + 84*a^13*b^7*x^12 + 126*a^14*b^6*x^10 + 126*a^15*b^5*x^
8 + 84*a^16*b^4*x^6 + 36*a^17*b^3*x^4 + 9*a^18*b^2*x^2 + a^19*b)]
```

giac [A] time = 0.60, size = 122, normalized size = 0.67

$$\frac{12155\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{9}}+\frac{765765\,b^{8}x^{17}+6636630\,ab^{7}x^{15}+25423398\,a^{2}b^{6}x^{13}+56404062\,a^{3}b^{5}x^{11}+79659008\,a}{4128768\,\left(bx^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 12155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^9) + 1/4128768*(765765*b^8*x
^17 + 6636630*a*b^7*x^15 + 25423398*a^2*b^6*x^13 + 56404062*a^3*b^5*x^11 +
79659008*a^4*b^4*x^9 + 73947042*a^5*b^3*x^7 + 44765658*a^6*b^2*x^5 + 167597
22*a^7*b*x^3 + 3363003*a^8*x)/((b*x^2 + a)^9*a^9)
```

maple [A] time = 0.01, size = 156, normalized size = 0.86

$$\frac{x}{18\left(b\,x^{2}+a\right)^{9}a}+\frac{17x}{288\left(b\,x^{2}+a\right)^{8}a^{2}}+\frac{85x}{1344\left(b\,x^{2}+a\right)^{7}a^{3}}+\frac{1105x}{16128\left(b\,x^{2}+a\right)^{6}a^{4}}+\frac{2431x}{32256\left(b\,x^{2}+a\right)^{5}a^{5}}+\frac{2}{28672\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^10,x)
```

```
[Out] 1/18*x/a/(b*x^2+a)^9+17/288*x/a^2/(b*x^2+a)^8+85/1344*x/a^3/(b*x^2+a)^7+110
5/16128*x/a^4/(b*x^2+a)^6+2431/32256*x/a^5/(b*x^2+a)^5+2431/28672*x/a^6/(b*
x^2+a)^4+2431/24576*x/a^7/(b*x^2+a)^3+12155/98304*x/a^8/(b*x^2+a)^2+12155/6
5536*x/a^9/(b*x^2+a)+12155/65536/a^9/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.11, size = 212, normalized size = 1.17

$$\frac{765765\,b^{8}x^{17}+6636630\,ab^{7}x^{15}+25423398\,a^{2}b^{6}x^{13}+56404062\,a^{3}b^{5}x^{11}+79659008\,a^{4}b^{4}x^{9}+73947042\,a^{5}b}{4128768\,\left(a^{9}b^{9}x^{18}+9\,a^{10}b^{8}x^{16}+36\,a^{11}b^{7}x^{14}+84\,a^{12}b^{6}x^{12}+126\,a^{13}b^{5}x^{10}+126\,a^{14}b^{4}x^{8}+8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(765765*b^8*x^17 + 6636630*a*b^7*x^15 + 25423398*a^2*b^6*x^13 + 5
6404062*a^3*b^5*x^11 + 79659008*a^4*b^4*x^9 + 73947042*a^5*b^3*x^7 + 447656
58*a^6*b^2*x^5 + 16759722*a^7*b*x^3 + 3363003*a^8*x)/(a^9*b^9*x^18 + 9*a^10
*b^8*x^16 + 36*a^11*b^7*x^14 + 84*a^12*b^6*x^12 + 126*a^13*b^5*x^10 + 126*a
^14*b^4*x^8 + 84*a^15*b^3*x^6 + 36*a^16*b^2*x^4 + 9*a^17*b*x^2 + a^18) + 12
155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^9)
```

mupad [B] time = 4.74, size = 209, normalized size = 1.15

$$\frac{\frac{53381\,x}{65536\,a}+\frac{399041\,b\,x^3}{98304\,a^2}+\frac{355283\,b^2\,x^5}{32768\,a^3}+\frac{4108169\,b^3\,x^7}{229376\,a^4}+\frac{2431\,b^4\,x^9}{126\,a^5}+\frac{3133559\,b^5\,x^{11}}{229376\,a^6}+\frac{201773\,b^6\,x^{13}}{32768\,a^7}+\frac{158015\,b^7\,x^{15}}{98304\,a^8}+\frac{12155\,b^8\,x^{1}}{65536\,a^9}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^10,x)
```

```
[Out] ((53381*x)/(65536*a) + (399041*b*x^3)/(98304*a^2) + (355283*b^2*x^5)/(32768
*a^3) + (4108169*b^3*x^7)/(229376*a^4) + (2431*b^4*x^9)/(126*a^5) + (313355
9*b^5*x^11)/(229376*a^6) + (201773*b^6*x^13)/(32768*a^7) + (158015*b^7*x^15
)/(98304*a^8) + (12155*b^8*x^17)/(65536*a^9))/(a^9 + b^9*x^18 + 9*a^8*b*x^2
 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a
^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14) + (12155*atan((b^(1/2)*x)/
a^(1/2)))/(65536*a^(19/2)*b^(1/2))
```

sympy [A] time = 1.04, size = 272, normalized size = 1.50

$$-\frac{12155\sqrt{-\frac{1}{a^{19}b}}\log\left(-a^{10}\sqrt{-\frac{1}{a^{19}b}}+x\right)}{131072}+\frac{12155\sqrt{-\frac{1}{a^{19}b}}\log\left(a^{10}\sqrt{-\frac{1}{a^{19}b}}+x\right)}{131072}+\frac{3363003a^8x+}{4128768a^{18}+37158912a^{17}bx^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**10,x)
```

```
[Out] -12155*sqrt(-1/(a**19*b))*log(-a**10*sqrt(-1/(a**19*b)) + x)/131072 + 12155
*sqrt(-1/(a**19*b))*log(a**10*sqrt(-1/(a**19*b)) + x)/131072 + (3363003*a**
8*x + 16759722*a**7*b*x**3 + 44765658*a**6*b**2*x**5 + 73947042*a**5*b**3*x
**7 + 79659008*a**4*b**4*x**9 + 56404062*a**3*b**5*x**11 + 25423398*a**2*b*
*6*x**13 + 6636630*a*b**7*x**15 + 765765*b**8*x**17)/(4128768*a**18 + 37158
912*a**17*b*x**2 + 148635648*a**16*b**2*x**4 + 346816512*a**15*b**3*x**6 +
520224768*a**14*b**4*x**8 + 520224768*a**13*b**5*x**10 + 346816512*a**12*b*
*6*x**12 + 148635648*a**11*b**7*x**14 + 37158912*a**10*b**8*x**16 + 4128768
*a**9*b**9*x**18)
```

3.222 $\int \frac{1}{x^2(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=209

$$-\frac{230945\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{21/2}}-\frac{230945}{65536a^{10}x}+\frac{230945}{196608a^9x\left(a+bx^2\right)}+\frac{46189}{98304a^8x\left(a+bx^2\right)^2}+\frac{46189}{172032a^7x\left(a+bx^2\right)^3}+$$

```
[Out] -230945/65536/a^10/x+1/18/a/x/(b*x^2+a)^9+19/288/a^2/x/(b*x^2+a)^8+323/4032
/a^3/x/(b*x^2+a)^7+1615/16128/a^4/x/(b*x^2+a)^6+4199/32256/a^5/x/(b*x^2+a)^
5+46189/258048/a^6/x/(b*x^2+a)^4+46189/172032/a^7/x/(b*x^2+a)^3+46189/98304
/a^8/x/(b*x^2+a)^2+230945/196608/a^9/x/(b*x^2+a)-230945/65536*arctan(x*b^(1
/2)/a^(1/2))*b^(1/2)/a^(21/2)
```

Rubi [A] time = 0.13, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$\frac{230945}{196608a^9x\left(a+bx^2\right)}+\frac{46189}{98304a^8x\left(a+bx^2\right)^2}+\frac{46189}{172032a^7x\left(a+bx^2\right)^3}+\frac{46189}{258048a^6x\left(a+bx^2\right)^4}+\frac{4199}{32256a^5x\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^10),x]
```

```
[Out] -230945/(65536*a^10*x) + 1/(18*a*x*(a + b*x^2)^9) + 19/(288*a^2*x*(a + b*x^
2)^8) + 323/(4032*a^3*x*(a + b*x^2)^7) + 1615/(16128*a^4*x*(a + b*x^2)^6) +
 4199/(32256*a^5*x*(a + b*x^2)^5) + 46189/(258048*a^6*x*(a + b*x^2)^4) + 46
189/(172032*a^7*x*(a + b*x^2)^3) + 46189/(98304*a^8*x*(a + b*x^2)^2) + 2309
45/(196608*a^9*x*(a + b*x^2)) - (230945*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(65536*a^(21/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^{10}}\,dx = \frac{1}{18ax\left(a+bx^2\right)^9}+\frac{19\int \frac{1}{x^2\left(a+bx^2\right)^9}\,dx}{18a}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9}+\frac{19}{288a^2x\left(a+bx^2\right)^8}+\frac{323\int \frac{1}{x^2\left(a+bx^2\right)^8}\,dx}{288a^2}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9}+\frac{19}{288a^2x\left(a+bx^2\right)^8}+\frac{323}{4032a^3x\left(a+bx^2\right)^7}+\frac{1615\int \frac{1}{x^2\left(a+bx^2\right)^7}\,dx}{1344a^3}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9}+\frac{19}{288a^2x\left(a+bx^2\right)^8}+\frac{323}{4032a^3x\left(a+bx^2\right)^7}+\frac{1615}{16128a^4x\left(a+bx^2\right)^6}+ \frac{209}{}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9}+\frac{19}{288a^2x\left(a+bx^2\right)^8}+\frac{323}{4032a^3x\left(a+bx^2\right)^7}+\frac{1615}{16128a^4x\left(a+bx^2\right)^6}+ \frac{}{322}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9}+\frac{19}{288a^2x\left(a+bx^2\right)^8}+\frac{323}{4032a^3x\left(a+bx^2\right)^7}+\frac{1615}{16128a^4x\left(a+bx^2\right)^6}+ \frac{}{322}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9}+\frac{19}{288a^2x\left(a+bx^2\right)^8}+\frac{323}{4032a^3x\left(a+bx^2\right)^7}+\frac{1615}{16128a^4x\left(a+bx^2\right)^6}+ \frac{}{322}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9}+\frac{19}{288a^2x\left(a+bx^2\right)^8}+\frac{323}{4032a^3x\left(a+bx^2\right)^7}+\frac{1615}{16128a^4x\left(a+bx^2\right)^6}+ \frac{}{322}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9}+\frac{19}{288a^2x\left(a+bx^2\right)^8}+\frac{323}{4032a^3x\left(a+bx^2\right)^7}+\frac{1615}{16128a^4x\left(a+bx^2\right)^6}+ \frac{}{322}$$

$$= -\frac{230945}{65536a^{10}x}+\frac{1}{18ax\left(a+bx^2\right)^9}+\frac{19}{288a^2x\left(a+bx^2\right)^8}+\frac{323}{4032a^3x\left(a+bx^2\right)^7}+\frac{161}{16128a^4x\,(}$$

$$= -\frac{230945}{65536a^{10}x}+\frac{1}{18ax\left(a+bx^2\right)^9}+\frac{19}{288a^2x\left(a+bx^2\right)^8}+\frac{323}{4032a^3x\left(a+bx^2\right)^7}+\frac{161}{16128a^4x\,(}$$

Mathematica [A] time = 0.10, size = 147, normalized size = 0.70

$$-\frac{\frac{\sqrt{a}\left(4128768a^9+63897057a^8bx^2+318434718a^7b^2x^4+8505475 02a^6b^3x^6+1404993798a^5b^4x^8+1513521152a^4b^5x^{10}+1071677178a^3b^6x^{12}+483044562a\right.}{x\left(a+bx^2\right)^9}}{4128768a^{21/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^10),x]
```

```
[Out] (-((Sqrt[a]*(4128768*a^9 + 63897057*a^8*b*x^2 + 318434718*a^7*b^2*x^4 + 850
547502*a^6*b^3*x^6 + 1404993798*a^5*b^4*x^8 + 1513521152*a^4*b^5*x^10 + 107
1677178*a^3*b^6*x^12 + 483044562*a^2*b^7*x^14 + 126095970*a*b^8*x^16 + 1454
9535*b^9*x^18))/(x*(a + b*x^2)^9)) - 14549535*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(4128768*a^(21/2))
```

fricas [A] time = 1.10, size = 664, normalized size = 3.18

$$\left[-\frac{29099070\,b^9x^{18}+252191940\,ab^8x^{16}+966089124\,a^2b^7x^{14}+2143354356\,a^3b^6x^{12}+3027042304\,a^4b^5x^{10}+\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [-1/8257536*(29099070*b^9*x^18 + 252191940*a*b^8*x^16 + 966089124*a^2*b^7*x
^14 + 2143354356*a^3*b^6*x^12 + 3027042304*a^4*b^5*x^10 + 2809987596*a^5*b^
4*x^8 + 1701095004*a^6*b^3*x^6 + 636869436*a^7*b^2*x^4 + 127794114*a^8*b*x^
2 + 8257536*a^9 - 14549535*(b^9*x^19 + 9*a*b^8*x^17 + 36*a^2*b^7*x^15 + 84*
a^3*b^6*x^13 + 126*a^4*b^5*x^11 + 126*a^5*b^4*x^9 + 84*a^6*b^3*x^7 + 36*a^7
*b^2*x^5 + 9*a^8*b*x^3 + a^9*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
a)/(b*x^2 + a)))/(a^10*b^9*x^19 + 9*a^11*b^8*x^17 + 36*a^12*b^7*x^15 + 84*a
^13*b^6*x^13 + 126*a^14*b^5*x^11 + 126*a^15*b^4*x^9 + 84*a^16*b^3*x^7 + 36*
a^17*b^2*x^5 + 9*a^18*b*x^3 + a^19*x), -1/4128768*(14549535*b^9*x^18 + 1260
95970*a*b^8*x^16 + 483044562*a^2*b^7*x^14 + 1071677178*a^3*b^6*x^12 + 15135
21152*a^4*b^5*x^10 + 1404993798*a^5*b^4*x^8 + 850547502*a^6*b^3*x^6 + 31843
4718*a^7*b^2*x^4 + 63897057*a^8*b*x^2 + 4128768*a^9 + 14549535*(b^9*x^19 +
9*a*b^8*x^17 + 36*a^2*b^7*x^15 + 84*a^3*b^6*x^13 + 126*a^4*b^5*x^11 + 126*a
^5*b^4*x^9 + 84*a^6*b^3*x^7 + 36*a^7*b^2*x^5 + 9*a^8*b*x^3 + a^9*x)*sqrt(b/
a)*arctan(x*sqrt(b/a)))/(a^10*b^9*x^19 + 9*a^11*b^8*x^17 + 36*a^12*b^7*x^15
 + 84*a^13*b^6*x^13 + 126*a^14*b^5*x^11 + 126*a^15*b^4*x^9 + 84*a^16*b^3*x^
7 + 36*a^17*b^2*x^5 + 9*a^18*b*x^3 + a^19*x)]
```

giac [A] time = 0.63, size = 134, normalized size = 0.64

$$-\frac{230945\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{10}}-\frac{1}{a^{10}x}-\frac{10420767\,b^9x^{17}+88937058\,ab^8x^{15}+334408914\,a^2b^7x^{13}+724860666\,a^3b^6x\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -230945/65536*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^10) - 1/(a^10*x) - 1/412
8768*(10420767*b^9*x^17 + 88937058*a*b^8*x^15 + 334408914*a^2*b^7*x^13 + 72
4860666*a^3*b^6*x^11 + 993296384*a^4*b^5*x^9 + 884769030*a^5*b^4*x^7 + 5037
30990*a^6*b^3*x^5 + 169799070*a^7*b^2*x^3 + 26738145*a^8*b*x)/((b*x^2 + a)^
9*a^10)
```

maple [A] time = 0.02, size = 206, normalized size = 0.99

$$-\frac{165409b^9x^{17}}{65536\left(b\,x^2+a\right)^9a^{10}}-\frac{2117549b^8x^{15}}{98304\left(b\,x^2+a\right)^9a^9}-\frac{2654039b^7x^{13}}{32768\left(b\,x^2+a\right)^9a^8}-\frac{40270037b^6x^{11}}{229376\left(b\,x^2+a\right)^9a^7}-\frac{30313b^5x^9}{126\left(b\,x^2+a\right)^9a^6}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^10,x)
```

```
[Out] -1/a^10/x-424415/65536/a^2*b/(b*x^2+a)^9*x-4042835/98304/a^3*b^2/(b*x^2+a)^
9*x^3-3997865/32768/a^4*b^3/(b*x^2+a)^9*x^5-49153835/229376/a^5*b^4/(b*x^2+
a)^9*x^7-30313/126/a^6*b^5/(b*x^2+a)^9*x^9-40270037/229376/a^7*b^6/(b*x^2+a
)^9*x^11-2654039/32768/a^8*b^7/(b*x^2+a)^9*x^13-2117549/98304/a^9*b^8/(b*x^
```

```
2+a)^9*x^15-165409/65536/a^10*b^9/(b*x^2+a)^9*x^17-230945/65536/a^10*b/(a*b
)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.13, size = 225, normalized size = 1.08

$$-\frac{14549535\,b^9x^{18}+126095970\,ab^8x^{16}+483044562\,a^2b^7x^{14}+1071677178\,a^3b^6x^{12}+1513521152\,a^4b^5x^{10}+1404}{4128768\left(a^{10}b^9x^{19}+9\,a^{11}b^8x^{17}+36\,a^{12}b^7x^{15}+84\,a^{13}b^6x^{13}+126\,a^{14}b^5x^{11}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/4128768*(14549535*b^9*x^18 + 126095970*a*b^8*x^16 + 483044562*a^2*b^7*x^
14 + 1071677178*a^3*b^6*x^12 + 1513521152*a^4*b^5*x^10 + 1404993798*a^5*b^4
*x^8 + 850547502*a^6*b^3*x^6 + 318434718*a^7*b^2*x^4 + 63897057*a^8*b*x^2 +
 4128768*a^9)/(a^10*b^9*x^19 + 9*a^11*b^8*x^17 + 36*a^12*b^7*x^15 + 84*a^13
*b^6*x^13 + 126*a^14*b^5*x^11 + 126*a^15*b^4*x^9 + 84*a^16*b^3*x^7 + 36*a^1
7*b^2*x^5 + 9*a^18*b*x^3 + a^19*x) - 230945/65536*b*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^10)
```

mupad [B] time = 5.09, size = 220, normalized size = 1.05

$$-\frac{\frac{1}{a}+\frac{1014239\,b\,x^2}{65536\,a^2}+\frac{7581779\,b^2\,x^4}{98304\,a^3}+\frac{6750377\,b^3\,x^6}{32768\,a^4}+\frac{78055211\,b^4\,x^8}{229376\,a^5}+\frac{46189\,b^5\,x^{10}}{126\,a^6}+\frac{59537621\,b^6\,x^{12}}{229376\,a^7}+\frac{3833687\,b^7\,x^{14}}{32768\,a^8}+\frac{3002285\,b^8\,x}{98304\,a^9}}{a^9\,x+9\,a^8\,b\,x^3+36\,a^7\,b^2\,x^5+84\,a^6\,b^3\,x^7+126\,a^5\,b^4\,x^9+126\,a^4\,b^5\,x^{11}+84\,a^3\,b^6\,x^{13}+36\,a^2\,b^7\,x^{15}+9\,a\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^10),x)
```

```
[Out] - (1/a + (1014239*b*x^2)/(65536*a^2) + (7581779*b^2*x^4)/(98304*a^3) + (675
0377*b^3*x^6)/(32768*a^4) + (78055211*b^4*x^8)/(229376*a^5) + (46189*b^5*x^
10)/(126*a^6) + (59537621*b^6*x^12)/(229376*a^7) + (3833687*b^7*x^14)/(3276
8*a^8) + (3002285*b^8*x^16)/(98304*a^9) + (230945*b^9*x^18)/(65536*a^10))/(
a^9*x + b^9*x^19 + 9*a^8*b*x^3 + 9*a*b^8*x^17 + 36*a^7*b^2*x^5 + 84*a^6*b^3
*x^7 + 126*a^5*b^4*x^9 + 126*a^4*b^5*x^11 + 84*a^3*b^6*x^13 + 36*a^2*b^7*x^
15) - (230945*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(21/2))
```

sympy [A] time = 1.35, size = 282, normalized size = 1.35

$$\frac{230945\sqrt{-\frac{b}{a^{21}}}\log\left(-\frac{a^{11}\sqrt{-\frac{b}{a^{21}}}}{b}+x\right)}{131072}-\frac{230945\sqrt{-\frac{b}{a^{21}}}\log\left(\frac{a^{11}\sqrt{-\frac{b}{a^{21}}}}{b}+x\right)}{131072}+\frac{-4128768a^9-63897057a^8bx^2-31}{4128768a^{19}x+37158912a^{18}bx^3+1486}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**10,x)
```

```
[Out] 230945*sqrt(-b/a**21)*log(-a**11*sqrt(-b/a**21)/b + x)/131072 - 230945*sqrt
(-b/a**21)*log(a**11*sqrt(-b/a**21)/b + x)/131072 + (-4128768*a**9 - 638970
57*a**8*b*x**2 - 318434718*a**7*b**2*x**4 - 850547502*a**6*b**3*x**6 - 1404
993798*a**5*b**4*x**8 - 1513521152*a**4*b**5*x**10 - 1071677178*a**3*b**6*x
**12 - 483044562*a**2*b**7*x**14 - 126095970*a*b**8*x**16 - 14549535*b**9*x
**18)/(4128768*a**19*x + 37158912*a**18*b*x**3 + 148635648*a**17*b**2*x**5
+ 346816512*a**16*b**3*x**7 + 520224768*a**15*b**4*x**9 + 520224768*a**14*b
**5*x**11 + 346816512*a**13*b**6*x**13 + 148635648*a**12*b**7*x**15 + 37158
912*a**11*b**8*x**17 + 4128768*a**10*b**9*x**19)
```

3.223 $\int \frac{1}{x^4(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=220

$$\frac{1616615b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{23/2}}+\frac{1616615b}{65536a^{11}x}-\frac{1616615}{196608a^{10}x^3}+\frac{323323}{65536a^9x^3\left(a+bx^2\right)}+\frac{46189}{32768a^8x^3\left(a+bx^2\right)^2}+\frac{4}{73728a^7x}$$

```
[Out] -1616615/196608/a^10/x^3+1616615/65536*b/a^11/x+1/18/a/x^3/(b*x^2+a)^9+7/96
/a^2/x^3/(b*x^2+a)^8+19/192/a^3/x^3/(b*x^2+a)^7+323/2304/a^4/x^3/(b*x^2+a)^
6+323/1536/a^5/x^3/(b*x^2+a)^5+4199/12288/a^6/x^3/(b*x^2+a)^4+46189/73728/a
^7/x^3/(b*x^2+a)^3+46189/32768/a^8/x^3/(b*x^2+a)^2+323323/65536/a^9/x^3/(b*
x^2+a)+1616615/65536*b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(23/2)
```

Rubi [A] time = 0.14, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$\frac{1616615b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{23/2}}+\frac{323323}{65536a^9x^3\left(a+bx^2\right)}+\frac{46189}{32768a^8x^3\left(a+bx^2\right)^2}+\frac{46189}{73728a^7x^3\left(a+bx^2\right)^3}+\frac{4199}{12288a^6x^3\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^10),x]
```

```
[Out] -1616615/(196608*a^10*x^3) + (1616615*b)/(65536*a^11*x) + 1/(18*a*x^3*(a +
b*x^2)^9) + 7/(96*a^2*x^3*(a + b*x^2)^8) + 19/(192*a^3*x^3*(a + b*x^2)^7) +
 323/(2304*a^4*x^3*(a + b*x^2)^6) + 323/(1536*a^5*x^3*(a + b*x^2)^5) + 4199
/(12288*a^6*x^3*(a + b*x^2)^4) + 46189/(73728*a^7*x^3*(a + b*x^2)^3) + 4618
9/(32768*a^8*x^3*(a + b*x^2)^2) + 323323/(65536*a^9*x^3*(a + b*x^2)) + (161
6615*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(23/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4 (a+bx^2)^{10}}\,dx = \frac{1}{18ax^3 (a+bx^2)^9} + \frac{7\int \frac{1}{x^4(a+bx^2)^9}\,dx}{6a}$$

$$= \frac{1}{18ax^3 (a+bx^2)^9} + \frac{7}{96a^2x^3 (a+bx^2)^8} + \frac{133\int \frac{1}{x^4(a+bx^2)^8}\,dx}{96a^2}$$

$$= \frac{1}{18ax^3 (a+bx^2)^9} + \frac{7}{96a^2x^3 (a+bx^2)^8} + \frac{19}{192a^3x^3 (a+bx^2)^7} + \frac{323\int \frac{1}{x^4(a+bx^2)^7}\,dx}{192a^3}$$

$$= \frac{1}{18ax^3 (a+bx^2)^9} + \frac{7}{96a^2x^3 (a+bx^2)^8} + \frac{19}{192a^3x^3 (a+bx^2)^7} + \frac{323}{2304a^4x^3 (a+bx^2)^6} + \frac{16}{}$$

$$= \frac{1}{18ax^3 (a+bx^2)^9} + \frac{7}{96a^2x^3 (a+bx^2)^8} + \frac{19}{192a^3x^3 (a+bx^2)^7} + \frac{323}{2304a^4x^3 (a+bx^2)^6} + \frac{}{15}$$

$$= \frac{1}{18ax^3 (a+bx^2)^9} + \frac{7}{96a^2x^3 (a+bx^2)^8} + \frac{19}{192a^3x^3 (a+bx^2)^7} + \frac{323}{2304a^4x^3 (a+bx^2)^6} + \frac{}{15}$$

$$= \frac{1}{18ax^3 (a+bx^2)^9} + \frac{7}{96a^2x^3 (a+bx^2)^8} + \frac{19}{192a^3x^3 (a+bx^2)^7} + \frac{323}{2304a^4x^3 (a+bx^2)^6} + \frac{}{15}$$

$$= \frac{1}{18ax^3 (a+bx^2)^9} + \frac{7}{96a^2x^3 (a+bx^2)^8} + \frac{19}{192a^3x^3 (a+bx^2)^7} + \frac{323}{2304a^4x^3 (a+bx^2)^6} + \frac{}{15}$$

$$= \frac{1}{18ax^3 (a+bx^2)^9} + \frac{7}{96a^2x^3 (a+bx^2)^8} + \frac{19}{192a^3x^3 (a+bx^2)^7} + \frac{323}{2304a^4x^3 (a+bx^2)^6} + \frac{}{15}$$

$$= -\frac{1616615}{196608a^{10}x^3} + \frac{1}{18ax^3 (a+bx^2)^9} + \frac{7}{96a^2x^3 (a+bx^2)^8} + \frac{19}{192a^3x^3 (a+bx^2)^7} + \frac{}{2304a^4x}$$

$$= -\frac{1616615}{196608a^{10}x^3} + \frac{1616615b}{65536a^{11}x} + \frac{1}{18ax^3 (a+bx^2)^9} + \frac{7}{96a^2x^3 (a+bx^2)^8} + \frac{19}{192a^3x^3 (a+bx^2}$$

$$= -\frac{1616615}{196608a^{10}x^3} + \frac{1616615b}{65536a^{11}x} + \frac{1}{18ax^3 (a+bx^2)^9} + \frac{7}{96a^2x^3 (a+bx^2)^8} + \frac{19}{192a^3x^3 (a+bx^2}$$

Mathematica [A] time = 0.09, size = 157, normalized size = 0.71

$$\frac{\frac{\sqrt{a}\left(-196608a^{10}+4128768a^9bx^2+63897057a^8b^2x^4+3184347718a^7b^3x^6+850547502a^6b^4x^8+1404993798a^5b^5x^{10}+1513521152a^4b^6x^{12}+1071677178a^3b\right.}{x^3(a+bx^2)^9}}{589824a^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^10),x]
```

```
[Out] ((Sqrt[a]*(-196608*a^10 + 4128768*a^9*b*x^2 + 63897057*a^8*b^2*x^4 + 318434
718*a^7*b^3*x^6 + 850547502*a^6*b^4*x^8 + 1404993798*a^5*b^5*x^10 + 1513521
```

```
152*a^4*b^6*x^12 + 1071677178*a^3*b^7*x^14 + 483044562*a^2*b^8*x^16 + 12609
5970*a*b^9*x^18 + 14549535*b^10*x^20))/(x^3*(a + b*x^2)^9) + 14549535*b^(3/
2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(589824*a^(23/2))
```

fricas [A] time = 0.97, size = 700, normalized size = 3.18

$$\left[\frac{29099070\,b^{10}x^{20} + 252191940\,ab^{9}x^{18} + 966089124\,a^{2}b^{8}x^{16} + 2143354356\,a^{3}b^{7}x^{14} + 3027042304\,a^{4}b^{6}x^{12} + \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(29099070*b^10*x^20 + 252191940*a*b^9*x^18 + 966089124*a^2*b^8*x
^16 + 2143354356*a^3*b^7*x^14 + 3027042304*a^4*b^6*x^12 + 2809987596*a^5*b^
5*x^10 + 1701095004*a^6*b^4*x^8 + 636869436*a^7*b^3*x^6 + 127794114*a^8*b^2
*x^4 + 8257536*a^9*b*x^2 - 393216*a^10 + 14549535*(b^10*x^21 + 9*a*b^9*x^19
 + 36*a^2*b^8*x^17 + 84*a^3*b^7*x^15 + 126*a^4*b^6*x^13 + 126*a^5*b^5*x^11
+ 84*a^6*b^4*x^9 + 36*a^7*b^3*x^7 + 9*a^8*b^2*x^5 + a^9*b*x^3)*sqrt(-b/a)*l
og((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^11*b^9*x^21 + 9*a^12*b^8
*x^19 + 36*a^13*b^7*x^17 + 84*a^14*b^6*x^15 + 126*a^15*b^5*x^13 + 126*a^16*
b^4*x^11 + 84*a^17*b^3*x^9 + 36*a^18*b^2*x^7 + 9*a^19*b*x^5 + a^20*x^3), 1/
589824*(14549535*b^10*x^20 + 126095970*a*b^9*x^18 + 483044562*a^2*b^8*x^16
+ 1071677178*a^3*b^7*x^14 + 1513521152*a^4*b^6*x^12 + 1404993798*a^5*b^5*x^
10 + 850547502*a^6*b^4*x^8 + 318434718*a^7*b^3*x^6 + 63897057*a^8*b^2*x^4 +
 4128768*a^9*b*x^2 - 196608*a^10 + 14549535*(b^10*x^21 + 9*a*b^9*x^19 + 36*
a^2*b^8*x^17 + 84*a^3*b^7*x^15 + 126*a^4*b^6*x^13 + 126*a^5*b^5*x^11 + 84*a
^6*b^4*x^9 + 36*a^7*b^3*x^7 + 9*a^8*b^2*x^5 + a^9*b*x^3)*sqrt(b/a)*arctan(x
*sqrt(b/a)))/(a^11*b^9*x^21 + 9*a^12*b^8*x^19 + 36*a^13*b^7*x^17 + 84*a^14*
b^6*x^15 + 126*a^15*b^5*x^13 + 126*a^16*b^4*x^11 + 84*a^17*b^3*x^9 + 36*a^1
8*b^2*x^7 + 9*a^19*b*x^5 + a^20*x^3)]
```

giac [A] time = 0.63, size = 148, normalized size = 0.67

$$\frac{1616615\,b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{11}} + \frac{30\,bx^{2} - a}{3\,a^{11}x^{3}} + \frac{8651295\,b^{10}x^{17} + 73208418\,ab^{9}x^{15} + 272477394\,a^{2}b^{8}x^{13} + 583302906\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1616615/65536*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^11) + 1/3*(30*b*x^2 -
a)/(a^11*x^3) + 1/589824*(8651295*b^10*x^17 + 73208418*a*b^9*x^15 + 2724773
94*a^2*b^8*x^13 + 583302906*a^3*b^7*x^11 + 786857984*a^4*b^6*x^9 + 68658816
6*a^5*b^5*x^7 + 379867950*a^6*b^4*x^5 + 122613150*a^7*b^3*x^3 + 17890785*a^
8*b^2*x)/((b*x^2 + a)^9*a^11)
```

maple [A] time = 0.02, size = 219, normalized size = 1.00

$$\frac{961255 b^{10} x^{17}}{65536 \left(b\,x^{2}+a\right)^{9} a^{11}}+\frac{12201403 b^{9} x^{15}}{98304 \left(b\,x^{2}+a\right)^{9} a^{10}}+\frac{15137633 b^{8} x^{13}}{32768 \left(b\,x^{2}+a\right)^{9} a^{9}}+\frac{32405717 b^{7} x^{11}}{32768 \left(b\,x^{2}+a\right)^{9} a^{8}}+\frac{24013 b^{6} x^{9}}{18 \left(b\,x^{2}+a\right)^{9} a^{7}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^10,x)
```

```
[Out] -1/3/a^10/x^3+10*b/a^11/x+1987865/65536/a^3*b^2/(b*x^2+a)^9*x+20435525/9830
4/a^4*b^3/(b*x^2+a)^9*x^3+21103775/32768/a^5*b^4/(b*x^2+a)^9*x^5+38143787/3
2768/a^6*b^5/(b*x^2+a)^9*x^7+24013/18/a^7*b^6/(b*x^2+a)^9*x^9+32405717/3276
8/a^8*b^7/(b*x^2+a)^9*x^11+15137633/32768/a^9*b^8/(b*x^2+a)^9*x^13+12201403
/98304/a^10*b^9/(b*x^2+a)^9*x^15+961255/65536/a^11*b^10/(b*x^2+a)^9*x^17+16
16615/65536/a^11*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.30, size = 240, normalized size = 1.09

$$\frac{14549535\,b^{10}x^{20} + 126095970\,ab^{9}x^{18} + 483044562\,a^{2}b^{8}x^{16} + 1071677178\,a^{3}b^{7}x^{14} + 1513521152\,a^{4}b^{6}x^{12} + 1404}{589824\left(a^{11}b^{9}x^{21} + 9\,a^{12}b^{8}x^{19} + 36\,a^{13}b^{7}x^{17} + 84\,a^{14}b^{6}x^{15} + 126\,a^{15}b^{5}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/589824*(14549535*b^10*x^20 + 126095970*a*b^9*x^18 + 483044562*a^2*b^8*x^1
6 + 1071677178*a^3*b^7*x^14 + 1513521152*a^4*b^6*x^12 + 1404993798*a^5*b^5*
x^10 + 850547502*a^6*b^4*x^8 + 318434718*a^7*b^3*x^6 + 63897057*a^8*b^2*x^4
 + 4128768*a^9*b*x^2 - 196608*a^10)/(a^11*b^9*x^21 + 9*a^12*b^8*x^19 + 36*a
^13*b^7*x^17 + 84*a^14*b^6*x^15 + 126*a^15*b^5*x^13 + 126*a^16*b^4*x^11 + 8
4*a^17*b^3*x^9 + 36*a^18*b^2*x^7 + 9*a^19*b*x^5 + a^20*x^3) + 1616615/65536
*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^11)
```

mupad [B] time = 5.11, size = 234, normalized size = 1.06

$$\frac{\frac{7\,b\,x^2}{a^2} - \frac{1}{3\,a} + \frac{7099673\,b^2\,x^4}{65536\,a^3} + \frac{53072453\,b^3\,x^6}{98304\,a^4} + \frac{47252639\,b^4\,x^8}{32768\,a^5} + \frac{78055211\,b^5\,x^{10}}{32768\,a^6} + \frac{46189\,b^6\,x^{12}}{18\,a^7} + \frac{59537621\,b^7\,x^{14}}{32768\,a^8} + \frac{26835809\,b^8\,x^{16}}{32768\,a^9} +}{a^9\,x^3 + 9\,a^8\,b\,x^5 + 36\,a^7\,b^2\,x^7 + 84\,a^6\,b^3\,x^9 + 126\,a^5\,b^4\,x^{11} + 126\,a^4\,b^5\,x^{13} + 84\,a^3\,b^6\,x^{15} + 36\,a^2\,b^7\,x^{17} +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^10),x)
```

```
[Out] ((7*b*x^2)/a^2 - 1/(3*a) + (7099673*b^2*x^4)/(65536*a^3) + (53072453*b^3*x^
6)/(98304*a^4) + (47252639*b^4*x^8)/(32768*a^5) + (78055211*b^5*x^10)/(3276
8*a^6) + (46189*b^6*x^12)/(18*a^7) + (59537621*b^7*x^14)/(32768*a^8) + (268
35809*b^8*x^16)/(32768*a^9) + (21015995*b^9*x^18)/(98304*a^10) + (1616615*b
^10*x^20)/(65536*a^11))/(a^9*x^3 + b^9*x^21 + 9*a^8*b*x^5 + 9*a*b^8*x^19 +
36*a^7*b^2*x^7 + 84*a^6*b^3*x^9 + 126*a^5*b^4*x^11 + 126*a^4*b^5*x^13 + 84*
a^3*b^6*x^15 + 36*a^2*b^7*x^17) + (1616615*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)
))/(65536*a^(23/2))
```

sympy [A] time = 1.45, size = 304, normalized size = 1.38

$$-\frac{1616615\sqrt{-\frac{b^3}{a^{23}}}\,\log\left(-\frac{a^{12}\sqrt{-\frac{b^3}{a^{23}}}}{b^2}+x\right)}{131072}+\frac{1616615\sqrt{-\frac{b^3}{a^{23}}}\,\log\left(\frac{a^{12}\sqrt{-\frac{b^3}{a^{23}}}}{b^2}+x\right)}{131072}+\frac{-196608a^{10}+4128768a^9bx^2+638}{589824a^{20}x^3+5308416}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**10,x)
```

```
[Out] -1616615*sqrt(-b**3/a**23)*log(-a**12*sqrt(-b**3/a**23)/b**2 + x)/131072 +
1616615*sqrt(-b**3/a**23)*log(a**12*sqrt(-b**3/a**23)/b**2 + x)/131072 + (-
196608*a**10 + 4128768*a**9*b*x**2 + 63897057*a**8*b**2*x**4 + 318434718*a*
*7*b**3*x**6 + 850547502*a**6*b**4*x**8 + 1404993798*a**5*b**5*x**10 + 1513
521152*a**4*b**6*x**12 + 1071677178*a**3*b**7*x**14 + 483044562*a**2*b**8*x
**16 + 126095970*a*b**9*x**18 + 14549535*b**10*x**20)/(589824*a**20*x**3 +
5308416*a**19*b*x**5 + 21233664*a**18*b**2*x**7 + 49545216*a**17*b**3*x**9
```

```
+ 74317824*a**16*b**4*x**11 + 74317824*a**15*b**5*x**13 + 49545216*a**14*b*
*6*x**15 + 21233664*a**13*b**7*x**17 + 5308416*a**12*b**8*x**19 + 589824*a*
*11*b**9*x**21)
```

3.224 $\int \frac{1}{x^6\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=233

$$-\frac{7436429b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{25/2}}-\frac{7436429b^2}{65536a^{12}x}+\frac{7436429b}{196608a^{11}x^3}-\frac{7436429}{327680a^{10}x^5}+\frac{1062347}{65536a^9x^5\left(a+bx^2\right)}+\frac{1062347}{294912a^8x^5\left(a+b\right.}$$

```
[Out] -7436429/327680/a^10/x^5+7436429/196608*b/a^11/x^3-7436429/65536*b^2/a^12/x
+1/18/a/x^5/(b*x^2+a)^9+23/288/a^2/x^5/(b*x^2+a)^8+23/192/a^3/x^5/(b*x^2+a)
^7+437/2304/a^4/x^5/(b*x^2+a)^6+7429/23040/a^5/x^5/(b*x^2+a)^5+7429/12288/a
^6/x^5/(b*x^2+a)^4+96577/73728/a^7/x^5/(b*x^2+a)^3+1062347/294912/a^8/x^5/(
b*x^2+a)^2+1062347/65536/a^9/x^5/(b*x^2+a)-7436429/65536*b^(5/2)*arctan(x*b
^(1/2)/a^(1/2))/a^(25/2)
```

Rubi [A] time = 0.16, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{7436429b^2}{65536a^{12}x}-\frac{7436429b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{25/2}}+\frac{7436429b}{196608a^{11}x^3}+\frac{1062347}{65536a^9x^5\left(a+bx^2\right)}+\frac{1062347}{294912a^8x^5\left(a+bx^2\right)^2}+\frac{9}{73728a^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^10),x]
```

```
[Out] -7436429/(327680*a^10*x^5) + (7436429*b)/(196608*a^11*x^3) - (7436429*b^2)/
(65536*a^12*x) + 1/(18*a*x^5*(a + b*x^2)^9) + 23/(288*a^2*x^5*(a + b*x^2)^8
) + 23/(192*a^3*x^5*(a + b*x^2)^7) + 437/(2304*a^4*x^5*(a + b*x^2)^6) + 742
9/(23040*a^5*x^5*(a + b*x^2)^5) + 7429/(12288*a^6*x^5*(a + b*x^2)^4) + 9657
7/(73728*a^7*x^5*(a + b*x^2)^3) + 1062347/(294912*a^8*x^5*(a + b*x^2)^2) +
1062347/(65536*a^9*x^5*(a + b*x^2)) - (7436429*b^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(65536*a^(25/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^2\right)^{10}}\,dx = \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23\int \frac{1}{x^6\left(a+bx^2\right)^9}\,dx}{18a}$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{161\int \frac{1}{x^6\left(a+bx^2\right)^8}\,dx}{96a^2}$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437\int \frac{1}{x^6\left(a+bx^2\right)^7}\,dx}{192a^3}$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} +$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{}{2304}$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{7436429b}{196608a^{11}x^3} + \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a\right.}$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{7436429b}{196608a^{11}x^3} - \frac{7436429b^2}{65536a^{12}x} + \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8}$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{7436429b}{196608a^{11}x^3} - \frac{7436429b^2}{65536a^{12}x} + \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8}$$

Mathematica [A] time = 0.09, size = 169, normalized size = 0.73

$$\frac{-334639305b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right) - \sqrt{a}\left(589824a^{11} - 4521984a^{10}bx^2 + 94961664a^9b^2x^4 + 1469632311a^8b^3x^6 + 7323998514a^7b^4x^8 + 195625925\ldots\right.}{2949120\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^10),x]
```

```
[Out] (-((Sqrt[a]*(589824*a^11 - 4521984*a^10*b*x^2 + 94961664*a^9*b^2*x^4 + 1469
632311*a^8*b^3*x^6 + 7323998514*a^7*b^4*x^8 + 19562592546*a^6*b^5*x^10 + 32
314857354*a^5*b^6*x^12 + 34810986496*a^4*b^7*x^14 + 24648575094*a^3*b^8*x^1
6 + 11110024926*a^2*b^9*x^18 + 2900207310*a*b^10*x^20 + 334639305*b^11*x^22
))/(x^5*(a + b*x^2)^9)) - 334639305*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2
949120*a^(25/2))
```

fricas [A] time = 1.01, size = 726, normalized size = 3.12

$$\left[-\frac{669278610\, b^{11} x^{22} + 5800414620\, a b^{10} x^{20} + 22220049852\, a^{2} b^{9} x^{18} + 49297150188\, a^{3} b^{8} x^{16} + 69621972992\, a^{4} b^{7}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [-1/5898240*(669278610*b^11*x^22 + 5800414620*a*b^10*x^20 + 22220049852*a^2
*b^9*x^18 + 49297150188*a^3*b^8*x^16 + 69621972992*a^4*b^7*x^14 + 646297147
08*a^5*b^6*x^12 + 39125185092*a^6*b^5*x^10 + 14647997028*a^7*b^4*x^8 + 2939
264622*a^8*b^3*x^6 + 189923328*a^9*b^2*x^4 - 9043968*a^10*b*x^2 + 1179648*a
^11 - 334639305*(b^11*x^23 + 9*a*b^10*x^21 + 36*a^2*b^9*x^19 + 84*a^3*b^8*x
^17 + 126*a^4*b^7*x^15 + 126*a^5*b^6*x^13 + 84*a^6*b^5*x^11 + 36*a^7*b^4*x^
9 + 9*a^8*b^3*x^7 + a^9*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)))/(a^12*b^9*x^23 + 9*a^13*b^8*x^21 + 36*a^14*b^7*x^19 + 84*
a^15*b^6*x^17 + 126*a^16*b^5*x^15 + 126*a^17*b^4*x^13 + 84*a^18*b^3*x^11 +
36*a^19*b^2*x^9 + 9*a^20*b*x^7 + a^21*x^5), -1/2949120*(334639305*b^11*x^22
 + 2900207310*a*b^10*x^20 + 11110024926*a^2*b^9*x^18 + 24648575094*a^3*b^8*
x^16 + 34810986496*a^4*b^7*x^14 + 32314857354*a^5*b^6*x^12 + 19562592546*a^
6*b^5*x^10 + 7323998514*a^7*b^4*x^8 + 1469632311*a^8*b^3*x^6 + 94961664*a^9
*b^2*x^4 - 4521984*a^10*b*x^2 + 589824*a^11 + 334639305*(b^11*x^23 + 9*a*b^
10*x^21 + 36*a^2*b^9*x^19 + 84*a^3*b^8*x^17 + 126*a^4*b^7*x^15 + 126*a^5*b^
6*x^13 + 84*a^6*b^5*x^11 + 36*a^7*b^4*x^9 + 9*a^8*b^3*x^7 + a^9*b^2*x^5)*sq
rt(b/a)*arctan(x*sqrt(b/a)))/(a^12*b^9*x^23 + 9*a^13*b^8*x^21 + 36*a^14*b^7
*x^19 + 84*a^15*b^6*x^17 + 126*a^16*b^5*x^15 + 126*a^17*b^4*x^13 + 84*a^18*
b^3*x^11 + 36*a^19*b^2*x^9 + 9*a^20*b*x^7 + a^21*x^5)]
```

giac [A] time = 0.60, size = 159, normalized size = 0.68

$$-\frac{7436429\, b^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{65536\, \sqrt{a b}\, a^{12}} - \frac{825\, b^{2} x^{4} - 50\, a b x^{2} + 3\, a^{2}}{15\, a^{12} x^{5}} - \frac{172437705\, b^{11} x^{17} + 1450223310\, a b^{10} x^{15} + 5358651102\, \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -7436429/65536*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^12) - 1/15*(825*b^2*x
^4 - 50*a*b*x^2 + 3*a^2)/(a^12*x^5) - 1/2949120*(172437705*b^11*x^17 + 1450
223310*a*b^10*x^15 + 5358651102*a^2*b^9*x^13 + 11372226678*a^3*b^8*x^11 + 1
5178104832*a^4*b^7*x^9 + 13066540938*a^5*b^6*x^7 + 7101970722*a^6*b^5*x^5 +
 2236176690*a^7*b^4*x^3 + 314167095*a^8*b^3*x)/((b*x^2 + a)^9*a^12)
```

maple [A] time = 0.03, size = 230, normalized size = 0.99

$$-\frac{3831949 b^{11} x^{17}}{65536 \left(b\, x^{2}+a\right)^{9} a^{12}}-\frac{48340777 b^{10} x^{15}}{98304 \left(b\, x^{2}+a\right)^{9} a^{11}}-\frac{297702839 b^{9} x^{13}}{163840 \left(b\, x^{2}+a\right)^{9} a^{10}}-\frac{631790371 b^{8} x^{11}}{163840 \left(b\, x^{2}+a\right)^{9} a^{9}}-\frac{463199 b^{7} x^{9}}{90 \left(b\, x^{2}+a\right)^{9} a^{8}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^10,x)
```

```
[Out] -1/5/a^10/x^5-55*b^2/a^12/x+10/3*b/a^11/x^3-6981491/65536/a^4*b^3/(b*x^2+a)
^9*x-74539223/98304/a^5*b^4/(b*x^2+a)^9*x^3-394553929/163840/a^6*b^5/(b*x^2
+a)^9*x^5-725918941/163840/a^7*b^6/(b*x^2+a)^9*x^7-463199/90/a^8*b^7/(b*x^2
+a)^9*x^9-631790371/163840/a^9*b^8/(b*x^2+a)^9*x^11-297702839/163840/a^10*b
^9/(b*x^2+a)^9*x^13-48340777/98304/a^11*b^10/(b*x^2+a)^9*x^15-3831949/65536
/a^12*b^11/(b*x^2+a)^9*x^17-7436429/65536/a^12*b^3/(a*b)^(1/2)*arctan(1/(a*
b)^(1/2)*b*x)
```

maxima [A] time = 3.35, size = 251, normalized size = 1.08

$$-\frac{334639305\,b^{11}x^{22}+2900207310\,ab^{10}x^{20}+11110024926\,a^{2}b^{9}x^{18}+24648575094\,a^{3}b^{8}x^{16}+34810986496\,a^{4}}{2949120\left(a^{12}b^{9}x^{23}+9\,a^{13}b^{8}x^{21}+36\,a^{14}b^{7}x^{19}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/2949120*(334639305*b^11*x^22 + 2900207310*a*b^10*x^20 + 11110024926*a^2*
b^9*x^18 + 24648575094*a^3*b^8*x^16 + 34810986496*a^4*b^7*x^14 + 3231485735
4*a^5*b^6*x^12 + 19562592546*a^6*b^5*x^10 + 7323998514*a^7*b^4*x^8 + 146963
2311*a^8*b^3*x^6 + 94961664*a^9*b^2*x^4 - 4521984*a^10*b*x^2 + 589824*a^11)
/(a^12*b^9*x^23 + 9*a^13*b^8*x^21 + 36*a^14*b^7*x^19 + 84*a^15*b^6*x^17 + 1
26*a^16*b^5*x^15 + 126*a^17*b^4*x^13 + 84*a^18*b^3*x^11 + 36*a^19*b^2*x^9 +
 9*a^20*b*x^7 + a^21*x^5) - 7436429/65536*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a^12)
```

mupad [B] time = 5.89, size = 246, normalized size = 1.06

$$-\frac{\frac{1}{5\,a}-\frac{23\,b\,x^2}{15\,a^2}+\frac{161\,b^2\,x^4}{5\,a^3}+\frac{163292479\,b^3\,x^6}{327680\,a^4}+\frac{1220666419\,b^4\,x^8}{491520\,a^5}+\frac{1086810697\,b^5\,x^{10}}{163840\,a^6}+\frac{1795269853\,b^6\,x^{12}}{163840\,a^7}+\frac{1062347\,b^7\,x^{14}}{90\,a^8}+\frac{1369}{1}}{a^9\,x^5+9\,a^8\,b\,x^7+36\,a^7\,b^2\,x^9+84\,a^6\,b^3\,x^{11}+126\,a^5\,b^4\,x^{13}+126\,a^4\,b^5\,x^{15}+84\,a^3\,b^6\,x^1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^10),x)
```

```
[Out] - (1/(5*a) - (23*b*x^2)/(15*a^2) + (161*b^2*x^4)/(5*a^3) + (163292479*b^3*x
^6)/(327680*a^4) + (1220666419*b^4*x^8)/(491520*a^5) + (1086810697*b^5*x^10
)/(163840*a^6) + (1795269853*b^6*x^12)/(163840*a^7) + (1062347*b^7*x^14)/(9
0*a^8) + (1369365283*b^8*x^16)/(163840*a^9) + (617223607*b^9*x^18)/(163840*
a^10) + (96673577*b^10*x^20)/(98304*a^11) + (7436429*b^11*x^22)/(65536*a^12
))/(a^9*x^5 + b^9*x^23 + 9*a^8*b*x^7 + 9*a*b^8*x^21 + 36*a^7*b^2*x^9 + 84*a
^6*b^3*x^11 + 126*a^5*b^4*x^13 + 126*a^4*b^5*x^15 + 84*a^3*b^6*x^17 + 36*a^
2*b^7*x^19) - (7436429*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(25/2))
```

sympy [A] time = 1.50, size = 316, normalized size = 1.36

$$\frac{7436429\sqrt{-\frac{b^5}{a^{25}}}\log\left(-\frac{a^{13}\sqrt{-\frac{b^5}{a^{25}}}}{b^3}+x\right)}{131072}-\frac{7436429\sqrt{-\frac{b^5}{a^{25}}}\log\left(\frac{a^{13}\sqrt{-\frac{b^5}{a^{25}}}}{b^3}+x\right)}{131072}+\frac{-589824a^{11}+4521984a^{10}bx^2-9}{2949120a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**10,x)
```

```
[Out] 7436429*sqrt(-b**5/a**25)*log(-a**13*sqrt(-b**5/a**25)/b**3 + x)/131072 - 7
436429*sqrt(-b**5/a**25)*log(a**13*sqrt(-b**5/a**25)/b**3 + x)/131072 + (-5
```

```
89824*a**11 + 4521984*a**10*b*x**2 - 94961664*a**9*b**2*x**4 - 1469632311*a
**8*b**3*x**6 - 7323998514*a**7*b**4*x**8 - 19562592546*a**6*b**5*x**10 - 3
2314857354*a**5*b**6*x**12 - 34810986496*a**4*b**7*x**14 - 24648575094*a**3
*b**8*x**16 - 11110024926*a**2*b**9*x**18 - 2900207310*a*b**10*x**20 - 3346
39305*b**11*x**22)/(2949120*a**21*x**5 + 26542080*a**20*b*x**7 + 106168320*
a**19*b**2*x**9 + 247726080*a**18*b**3*x**11 + 371589120*a**17*b**4*x**13 +
 371589120*a**16*b**5*x**15 + 247726080*a**15*b**6*x**17 + 106168320*a**14*
b**7*x**19 + 26542080*a**13*b**8*x**21 + 2949120*a**12*b**9*x**23)
```

3.225 $\int \frac{x^3}{a-bx^2} \, dx$

Optimal. Leaf size=28

$$-\frac{a \log \left(a-b x^2\right)}{2 b^2}-\frac{x^2}{2 b}$$

```
[Out] -1/2*x^2/b-1/2*a*ln(-b*x^2+a)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 43}

$$-\frac{a \log \left(a-b x^2\right)}{2 b^2}-\frac{x^2}{2 b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2),x]
```

```
[Out] -x^2/(2*b) - (a*Log[a - b*x^2])/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a-b x^2} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{x}{a-b x} \, dx,x,x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(-\frac{1}{b}-\frac{a}{b (-a+b x)}\right) \, dx,x,x^2\right)\\
&= -\frac{x^2}{2 b}-\frac{a \log \left(a-b x^2\right)}{2 b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{a \log \left(a-b x^2\right)}{2 b^2}-\frac{x^2}{2 b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2),x]
```

```
[Out] -1/2*x^2/b - (a*Log[a - b*x^2])/(2*b^2)
```

fricas [A] time = 0.75, size = 23, normalized size = 0.82

$$-\frac{b x^2 + a \log \left(b x^2 - a\right)}{2 \, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a),x, algorithm="fricas")
[Out] -1/2*(b*x^2 + a*log(b*x^2 - a))/b^2
```

giac [A] time = 0.62, size = 26, normalized size = 0.93

$$-\frac{x^2}{2\,b}-\frac{a\log\left(\left|bx^2-a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a),x, algorithm="giac")
[Out] -1/2*x^2/b - 1/2*a*log(abs(b*x^2 - a))/b^2
```

maple [A] time = 0.00, size = 26, normalized size = 0.93

$$-\frac{x^2}{2b}-\frac{a\ln\left(b\,x^2-a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a),x)
[Out] -1/2/b*x^2-1/2*a/b^2*ln(b*x^2-a)
```

maxima [A] time = 1.36, size = 25, normalized size = 0.89

$$-\frac{x^2}{2\,b}-\frac{a\log\left(bx^2-a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a),x, algorithm="maxima")
[Out] -1/2*x^2/b - 1/2*a*log(b*x^2 - a)/b^2
```

mupad [B] time = 0.05, size = 23, normalized size = 0.82

$$-\frac{b\,x^2+a\,\ln\left(b\,x^2-a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2),x)
[Out] -(b*x^2 + a*log(b*x^2 - a))/(2*b^2)
```

sympy [A] time = 0.14, size = 22, normalized size = 0.79

$$-\frac{a\log\left(-a+bx^2\right)}{2b^2}-\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a),x)
[Out] -a*log(-a + b*x**2)/(2*b**2) - x**2/(2*b)
```

3.226 $\int \frac{x^2}{a-bx^2}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{x}{b}$$

```
[Out] -x/b+arctanh(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = $\{321, 208\}$

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2),x]
[Out] -(x/b) + (Sqrt[a]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a-bx^2}\,dx&=-\frac{x}{b}+\frac{a\int \frac{1}{a-bx^2}\,dx}{b}\\&=-\frac{x}{b}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2),x]
[Out] -(x/b) + (Sqrt[a]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

fricas [A] time = 1.06, size = 80, normalized size = 2.58

$$\left[\frac{\sqrt{\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{\frac{a}{b}}+a}{bx^2-a}\right)-2\,x}{2\,b},\,-\frac{\sqrt{-\frac{a}{b}}\arctan\left(\frac{bx\sqrt{-\frac{a}{b}}}{a}\right)+x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a/b)*log((b*x^2 + 2*b*x*sqrt(a/b) + a)/(b*x^2 - a)) - 2*x)/b, -(
sqrt(-a/b)*arctan(b*x*sqrt(-a/b)/a) + x)/b]
```

giac [A] time = 0.63, size = 29, normalized size = 0.94

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b) - x/b
```

maple [A] time = 0.00, size = 27, normalized size = 0.87

$$\frac{a\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a),x)
```

```
[Out] -1/b*x+a/b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 42, normalized size = 1.35

$$-\frac{a\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*a*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*b) - x/b
```

mupad [B] time = 4.57, size = 23, normalized size = 0.74

$$\frac{\sqrt{a}\;\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2),x)
```

```
[Out] (a^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/b^(3/2) - x/b
```

sympy [A] time = 0.15, size = 49, normalized size = 1.58

$$-\frac{\sqrt{\frac{a}{b^3}}\log\left(-b\sqrt{\frac{a}{b^3}}+x\right)}{2}+\frac{\sqrt{\frac{a}{b^3}}\log\left(b\sqrt{\frac{a}{b^3}}+x\right)}{2}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a),x)
```

```
[Out] -sqrt(a/b**3)*log(-b*sqrt(a/b**3) + x)/2 + sqrt(a/b**3)*log(b*sqrt(a/b**3)
+ x)/2 - x/b
```

3.227 $\int \frac{x}{a-bx^2} dx$

Optimal. Leaf size=16

$$-\frac{\log\left(a-bx^2\right)}{2b}$$

```
[Out] -1/2*ln(-b*x^2+a)/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {260}

$$-\frac{\log\left(a-bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^2),x]
```

```
[Out] -Log[a - b*x^2]/(2*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{a-bx^2} dx = -\frac{\log\left(a-bx^2\right)}{2b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\log\left(a-bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^2),x]
```

```
[Out] -1/2*Log[a - b*x^2]/b
```

fricas [A] time = 0.76, size = 15, normalized size = 0.94

$$-\frac{\log\left(bx^2-a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*log(b*x^2 - a)/b
```

giac [A] time = 0.61, size = 16, normalized size = 1.00

$$-\frac{\log\left(\left|bx^2-a\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a),x, algorithm="giac")
[Out] -1/2*log(abs(b*x^2 - a))/b
```

maple [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\ln\left(b\,x^2-a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a),x)
[Out] -1/2/b*ln(b*x^2-a)
```

maxima [A] time = 1.29, size = 15, normalized size = 0.94

$$-\frac{\log\left(bx^2-a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 - a)/b
```

mupad [B] time = 0.03, size = 15, normalized size = 0.94

$$-\frac{\ln\left(b\,x^2-a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2),x)
[Out] -log(b*x^2 - a)/(2*b)
```

sympy [A] time = 0.12, size = 12, normalized size = 0.75

$$-\frac{\log\left(-a+bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a),x)
[Out] -log(-a + b*x**2)/(2*b)
```

3.228 $\int \frac{1}{a-bx^2} dx$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] arctanh(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{a-bx^2}\,dx = \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

fricas [A] time = 0.71, size = 68, normalized size = 2.83

$$\left[\frac{\sqrt{ab}\,\log\left(\frac{bx^2+2\,\sqrt{ab}\,x+a}{bx^2-a}\right)}{2\,ab}, -\frac{\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a))/(a*b), -sqrt(-a
*b)*arctan(sqrt(-a*b)*x/a)/(a*b)]
```

giac [A] time = 0.59, size = 18, normalized size = 0.75

$$-\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a),x, algorithm="giac")
[Out] -arctan(b*x/sqrt(-a*b))/sqrt(-a*b)
```

maple [A] time = 0.00, size = 16, normalized size = 0.67

$$\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a),x)
[Out] 1/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.00, size = 31, normalized size = 1.29

$$-\frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a),x, algorithm="maxima")
[Out] -1/2*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/sqrt(a*b)
```

mupad [B] time = 0.18, size = 16, normalized size = 0.67

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2),x)
[Out] atanh((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

sympy [B] time = 0.14, size = 46, normalized size = 1.92

$$-\frac{\sqrt{\frac{1}{ab}}\log\left(-a\sqrt{\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{\frac{1}{ab}}\log\left(a\sqrt{\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a),x)
[Out] -sqrt(1/(a*b))*log(-a*sqrt(1/(a*b)) + x)/2 + sqrt(1/(a*b))*log(a*sqrt(1/(a*
b)) + x)/2
```

3.229 $\int \frac{1}{x(a-bx^2)} dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{a} - \frac{\log\left(a - bx^2\right)}{2a}$$

```
[Out] ln(x)/a-1/2*ln(-b*x^2+a)/a
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log\left(a - bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2)),x]
```

```
[Out] Log[x]/a - Log[a - b*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a - bx^2\right)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a-bx)} dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, x^2\right)}{2a} + \frac{b\,\text{Subst}\left(\int \frac{1}{a-bx} dx, x, x^2\right)}{2a} \\
&= \frac{\log(x)}{a} - \frac{\log\left(a - bx^2\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\log(x)}{a} - \frac{\log\left(a - bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)),x]
[Out] Log[x]/a - Log[a - b*x^2]/(2*a)
```

fricas [A] time = 1.22, size = 20, normalized size = 0.87

$$-\frac{\log\left(bx^2-a\right)-2\,\log(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a),x, algorithm="fricas")
[Out] -1/2*(log(b*x^2 - a) - 2*log(x))/a
```

giac [A] time = 0.58, size = 26, normalized size = 1.13

$$\frac{\log\left(x^2\right)}{2\,a}-\frac{\log\left(\left|bx^2-a\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a),x, algorithm="giac")
[Out] 1/2*log(x^2)/a - 1/2*log(abs(b*x^2 - a))/a
```

maple [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\ln(x)}{a}-\frac{\ln\left(b\,x^2-a\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a),x)
[Out] 1/a*ln(x)-1/2/a*ln(b*x^2-a)
```

maxima [A] time = 1.32, size = 25, normalized size = 1.09

$$-\frac{\log\left(bx^2-a\right)}{2\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 - a)/a + 1/2*log(x^2)/a
```

mupad [B] time = 4.54, size = 21, normalized size = 0.91

$$\frac{\ln(x)}{a}-\frac{\ln\left(a-b\,x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2)),x)
[Out] log(x)/a - log(a - b*x^2)/(2*a)
```

sympy [A] time = 0.21, size = 15, normalized size = 0.65

$$\frac{\log\left(x\right)}{a}-\frac{\log\left(-\frac{a}{b}+x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a),x)
```

```
[Out] log(x)/a - log(-a/b + x**2)/(2*a)
```

3.230 $\int \frac{1}{x^2(a-bx^2)}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

```
[Out] -1/a/x+arctanh(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {325, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)),x]
```

```
[Out] -(1/(a*x)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a-bx^2\right)}\,dx&=-\frac{1}{ax}+\frac{b\int \frac{1}{a-bx^2}\,dx}{a}\\&=-\frac{1}{ax}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)),x]
```

```
[Out] -(1/(a*x)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 0.84, size = 82, normalized size = 2.48

$$\left[\frac{x\sqrt{\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right)-2}{2\,ax},\ -\frac{x\sqrt{-\frac{b}{a}}\arctan\left(x\sqrt{-\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)) - 2)/(a*x)
, -(x*sqrt(-b/a)*arctan(x*sqrt(-b/a)) + 1)/(a*x)]
```

giac [A] time = 0.62, size = 31, normalized size = 0.94

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/(a*x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.88

$$\frac{b\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a),x)
```

```
[Out] -1/a/x+1/a*b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.89, size = 44, normalized size = 1.33

$$-\frac{b\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*b*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a) - 1/(a*x)
```

mupad [B] time = 4.61, size = 25, normalized size = 0.76

$$\frac{\sqrt{b}\ \mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)),x)
```

```
[Out] (b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/a^(3/2) - 1/(a*x)
```

sympy [B] time = 0.19, size = 58, normalized size = 1.76

$$-\frac{\sqrt{\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{\frac{b}{a^3}}}{b}+x\right)}{2}+\frac{\sqrt{\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a),x)
```

```
[Out] -sqrt(b/a**3)*log(-a**2*sqrt(b/a**3)/b + x)/2 + sqrt(b/a**3)*log(a**2*sqrt(
b/a**3)/b + x)/2 - 1/(a*x)
```

3.231 $\int \frac{1}{x^3(a-bx^2)} dx$

Optimal. Leaf size=35

$$-\frac{b \log(a-bx^2)}{2a^2}+\frac{b \log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2+b*ln(x)/a^2-1/2*b*ln(-b*x^2+a)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$-\frac{b \log(a-bx^2)}{2a^2}+\frac{b \log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)),x]
```

```
[Out] -1/(2*a*x^2) + (b*Log[x])/a^2 - (b*Log[a - b*x^2])/(2*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a-bx^2\right)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a-bx)} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{b}{a^2x}+\frac{b^2}{a^2(a-bx)}\right) dx, x, x^2\right)\\
&= -\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$-\frac{b \log(a-bx^2)}{2a^2}+\frac{b \log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)),x]
```

```
[Out] -1/2*1/(a*x^2) + (b*Log[x])/a^2 - (b*Log[a - b*x^2])/(2*a^2)
```

fricas [A] time = 0.82, size = 33, normalized size = 0.94

$$-\frac{bx^2\log\left(bx^2-a\right)-2\,bx^2\log(x)+a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a),x, algorithm="fricas")
[Out] -1/2*(b*x^2*log(b*x^2 - a) - 2*b*x^2*log(x) + a)/(a^2*x^2)
```

giac [A] time = 0.58, size = 43, normalized size = 1.23

$$\frac{b\log\left(x^2\right)}{2\,a^2}-\frac{b\log\left(\left|bx^2-a\right|\right)}{2\,a^2}-\frac{bx^2+a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a),x, algorithm="giac")
[Out] 1/2*b*log(x^2)/a^2 - 1/2*b*log(abs(b*x^2 - a))/a^2 - 1/2*(b*x^2 + a)/(a^2*x
^2)
```

maple [A] time = 0.01, size = 33, normalized size = 0.94

$$\frac{b\ln(x)}{a^2}-\frac{b\ln\left(b\,x^2-a\right)}{2a^2}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a),x)
[Out] -1/2/a/x^2+1/a^2*b*ln(x)-1/2*b/a^2*ln(b*x^2-a)
```

maxima [A] time = 1.39, size = 35, normalized size = 1.00

$$-\frac{b\log\left(bx^2-a\right)}{2\,a^2}+\frac{b\log\left(x^2\right)}{2\,a^2}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a),x, algorithm="maxima")
[Out] -1/2*b*log(b*x^2 - a)/a^2 + 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)
```

mupad [B] time = 0.07, size = 31, normalized size = 0.89

$$\frac{b\,\ln(x)}{a^2}-\frac{b\,\ln\left(a-b\,x^2\right)}{2\,a^2}-\frac{1}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2)),x)
[Out] (b*log(x))/a^2 - (b*log(a - b*x^2))/(2*a^2) - 1/(2*a*x^2)
```

sympy [A] time = 0.27, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^2}+\frac{b\log\left(x\right)}{a^2}-\frac{b\log\left(-\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a),x)
[Out] -1/(2*a*x**2) + b*log(x)/a**2 - b*log(-a/b + x**2)/(2*a**2)
```

3.232 $\int \frac{x^3}{(a-bx^2)^2} dx$

Optimal. Leaf size=35

$$\frac{a}{2b^2(a-bx^2)}+\frac{\log(a-bx^2)}{2b^2}$$

```
[Out] 1/2*a/b^2/(-b*x^2+a)+1/2*ln(-b*x^2+a)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {266, 43}

$$\frac{a}{2b^2(a-bx^2)}+\frac{\log(a-bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2)^2,x]
```

```
[Out] a/(2*b^2*(a - b*x^2)) + Log[a - b*x^2]/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a-bx^2)^2} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a-bx)^2} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a}{b(-a+bx)^2}+\frac{1}{b(-a+bx)}\right) dx, x, x^2\right)\\
&= \frac{a}{2b^2(a-bx^2)}+\frac{\log(a-bx^2)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.83

$$\frac{\frac{a}{a-bx^2}+\log(a-bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2)^2,x]
```

```
[Out] (a/(a - b*x^2) + Log[a - b*x^2])/(2*b^2)
```

fricas [A] time = 0.82, size = 42, normalized size = 1.20

$$\frac{(bx^2-a)\log(bx^2-a)-a}{2\,(b^3x^2-ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/2*((b*x^2 - a)*log(b*x^2 - a) - a)/(b^3*x^2 - a*b^2)
```

giac [A] time = 0.63, size = 53, normalized size = 1.51

$$-\frac{\frac{\log\left(\frac{|bx^2-a|}{(bx^2-a)^2|b|}\right)}{b}+\frac{a}{(bx^2-a)b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^2,x, algorithm="giac")
[Out] -1/2*(log(abs(b*x^2 - a)/((b*x^2 - a)^2*abs(b)))/b + a/((b*x^2 - a)*b))/b
```

maple [A] time = 0.01, size = 34, normalized size = 0.97

$$-\frac{a}{2\,(b\,x^2-a)\,b^2}+\frac{\ln(b\,x^2-a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a)^2,x)
[Out] 1/2/b^2*ln(b*x^2-a)-1/2*a/b^2/(b*x^2-a)
```

maxima [A] time = 1.35, size = 35, normalized size = 1.00

$$-\frac{a}{2\,(b^3x^2-ab^2)}+\frac{\log(bx^2-a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^2,x, algorithm="maxima")
[Out] -1/2*a/(b^3*x^2 - a*b^2) + 1/2*log(b*x^2 - a)/b^2
```

mupad [B] time = 0.04, size = 32, normalized size = 0.91

$$\frac{\ln(b\,x^2-a)}{2\,b^2}+\frac{a}{2\,b^2\,(a-b\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2)^2,x)
[Out] log(b*x^2 - a)/(2*b^2) + a/(2*b^2*(a - b*x^2))
```

sympy [A] time = 0.20, size = 29, normalized size = 0.83

$$-\frac{a}{-2ab^2+2b^3x^2}+\frac{\log(-a+bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a)**2,x)
```

```
[Out] -a/(-2*a*b**2 + 2*b**3*x**2) + log(-a + b*x**2)/(2*b**2)
```

3.233 $\int \frac{x^2}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=46

$$\frac{x}{2b\left(a-bx^2\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

```
[Out] 1/2*x/b/(-b*x^2+a)-1/2*arctanh(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {288, 208}

$$\frac{x}{2b\left(a-bx^2\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^2,x]
[Out] x/(2*b*(a - b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a-bx^2\right)^2}\,dx &= \frac{x}{2b\left(a-bx^2\right)}-\frac{\int \frac{1}{a-bx^2}\,dx}{2b}\\ &= \frac{x}{2b\left(a-bx^2\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.02

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2b\left(bx^2-a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^2,x]
[Out] -1/2*x/(b*(-a + b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

fricas [A] time = 0.83, size = 127, normalized size = 2.76

$$\left[-\frac{2\,abx - \left(bx^2 - a\right)\sqrt{ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{ab}\,x + a}{bx^2 - a}\right)}{4\,\left(ab^3x^2 - a^2b^2\right)}, -\frac{abx - \left(bx^2 - a\right)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{2\,\left(ab^3x^2 - a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x - (b*x^2 - a)*sqrt(a*b)*log((b*x^2 - 2*sqrt(a*b)*x + a)/(b*x
^2 - a)))/(a*b^3*x^2 - a^2*b^2), -1/2*(a*b*x - (b*x^2 - a)*sqrt(-a*b)*arcta
n(sqrt(-a*b)*x/a))/(a*b^3*x^2 - a^2*b^2)]
```

giac [A] time = 0.61, size = 39, normalized size = 0.85

$$\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}\,b} - \frac{x}{2\,\left(bx^2 - a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b) - 1/2*x/((b*x^2 - a)*b)
```

maple [A] time = 0.01, size = 38, normalized size = 0.83

$$-\frac{x}{2\left(b\,x^2 - a\right)b} - \frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^2,x)
```

```
[Out] -1/2/b*x/(b*x^2-a)-1/2/b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.00, size = 52, normalized size = 1.13

$$-\frac{x}{2\,\left(b^2x^2 - ab\right)} + \frac{\log\left(\frac{bx - \sqrt{ab}}{bx + \sqrt{ab}}\right)}{4\,\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(b^2*x^2 - a*b) + 1/4*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt
(a*b)*b)
```

mupad [B] time = 4.68, size = 34, normalized size = 0.74

$$\frac{x}{2\,b\,\left(a - b\,x^2\right)} - \frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\;b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2)^2,x)
```

```
[Out] x/(2*b*(a - b*x^2)) - atanh((b^(1/2)*x)/a^(1/2))/(2*a^(1/2)*b^(3/2))
```

sympy [A] time = 0.21, size = 71, normalized size = 1.54

$$-\frac{x}{-2ab+2b^2x^2}+\frac{\sqrt{\frac{1}{ab^3}}\log\left(-ab\sqrt{\frac{1}{ab^3}}+x\right)}{4}-\frac{\sqrt{\frac{1}{ab^3}}\log\left(ab\sqrt{\frac{1}{ab^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**2,x)
```

```
[Out] -x/(-2*a*b + 2*b**2*x**2) + sqrt(1/(a*b**3))*log(-a*b*sqrt(1/(a*b**3)) + x)
/4 - sqrt(1/(a*b**3))*log(a*b*sqrt(1/(a*b**3)) + x)/4
```

3.234 $\int \frac{x}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2b(a-bx^2)}$$

[Out] 1/2/b/(-b*x^2+a)

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {261}

$$\frac{1}{2b(a-bx^2)}$$

Antiderivative was successfully verified.

[In] Int[x/(a - b*x^2)^2,x]

[Out] 1/(2*b*(a - b*x^2))

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a-bx^2)^2}\,dx = \frac{1}{2b(a-bx^2)}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2b(a-bx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a - b*x^2)^2,x]

[Out] 1/(2*b*(a - b*x^2))

fricas [A] time = 1.11, size = 16, normalized size = 0.94

$$-\frac{1}{2(b^2x^2-ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-b*x^2+a)^2,x, algorithm="fricas")

[Out] -1/2/(b^2*x^2 - a*b)

giac [A] time = 0.61, size = 16, normalized size = 0.94

$$-\frac{1}{2(bx^2-a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2/((b*x^2 - a)*b)
```

maple [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{1}{2\left(b\,x^{2}-a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^2,x)
```

```
[Out] -1/2/b/(b*x^2-a)
```

maxima [A] time = 1.38, size = 16, normalized size = 0.94

$$-\frac{1}{2\left(bx^{2}-a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2/((b*x^2 - a)*b)
```

mupad [B] time = 0.02, size = 15, normalized size = 0.88

$$\frac{1}{2\,b\,\left(a-b\,x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2)^2,x)
```

```
[Out] 1/(2*b*(a - b*x^2))
```

sympy [A] time = 0.16, size = 15, normalized size = 0.88

$$-\frac{1}{-2ab+2b^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**2,x)
```

```
[Out] -1/(-2*a*b + 2*b**2*x**2)
```

3.235 $\int \frac{1}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a-bx^2\right)}$$

```
[Out] 1/2*x/a/(-b*x^2+a)+1/2*arctanh(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {199, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a-bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a - b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a-bx^2\right)^2}\,dx&=\frac{x}{2a\left(a-bx^2\right)}+\frac{\int \frac{1}{a-bx^2}\,dx}{2a}\\&=\frac{x}{2a\left(a-bx^2\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.02

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}-\frac{x}{2a\left(bx^2-a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-2),x]
```

```
[Out] -1/2*x/(a*(-a + b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

fricas [**A**] time = 0.86, size = 126, normalized size = 2.74

$$\left[-\frac{2\,abx-\left(bx^2-a\right)\sqrt{ab}\,\log\left(\frac{bx^2+2\,\sqrt{ab}\,x+a}{bx^2-a}\right)}{4\,\left(a^2b^2x^2-a^3b\right)},\,-\frac{abx+\left(bx^2-a\right)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{2\,\left(a^2b^2x^2-a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x - (b*x^2 - a)*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x
^2 - a)))/(a^2*b^2*x^2 - a^3*b), -1/2*(a*b*x + (b*x^2 - a)*sqrt(-a*b)*arcta
n(sqrt(-a*b)*x/a))/(a^2*b^2*x^2 - a^3*b)]
```

giac [**A**] time = 0.58, size = 39, normalized size = 0.85

$$-\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}\,a}-\frac{x}{2\,\left(bx^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/2*x/((b*x^2 - a)*a)
```

maple [**A**] time = 0.00, size = 38, normalized size = 0.83

$$-\frac{x}{2\left(b\,x^2-a\right)a}+\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^2,x)
```

```
[Out] -1/2*x/a/(b*x^2-a)+1/2/a/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x)
```

maxima [**A**] time = 2.96, size = 52, normalized size = 1.13

$$-\frac{x}{2\,\left(abx^2-a^2\right)}-\frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(a*b*x^2 - a^2) - 1/4*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt
(a*b)*a)
```

mupad [**B**] time = 4.51, size = 34, normalized size = 0.74

$$\frac{x}{2\,a\,\left(a-b\,x^2\right)}+\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^2,x)
```

```
[Out] x/(2*a*(a - b*x^2)) + atanh((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

sympy [A] time = 0.22, size = 71, normalized size = 1.54

$$-\frac{x}{-2a^2+2abx^2}-\frac{\sqrt{\frac{1}{a^3b}}\log\left(-a^2\sqrt{\frac{1}{a^3b}}+x\right)}{4}+\frac{\sqrt{\frac{1}{a^3b}}\log\left(a^2\sqrt{\frac{1}{a^3b}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**2,x)
```

```
[Out] -x/(-2*a**2 + 2*a*b*x**2) - sqrt(1/(a**3*b))*log(-a**2*sqrt(1/(a**3*b)) + x
)/4 + sqrt(1/(a**3*b))*log(a**2*sqrt(1/(a**3*b)) + x)/4
```

3.236 $\int \frac{1}{x(a-bx^2)^2} dx$

Optimal. Leaf size=40

$$-\frac{\log(a-bx^2)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a(a-bx^2)}$$

[Out] 1/2/a/(-b*x^2+a)+ln(x)/a^2-1/2*ln(-b*x^2+a)/a^2

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$-\frac{\log(a-bx^2)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a(a-bx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a - b*x^2)^2),x]

[Out] 1/(2*a*(a - b*x^2)) + Log[x]/a^2 - Log[a - b*x^2]/(2*a^2)

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a-bx^2)^2} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a-bx)^2} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x}+\frac{b}{a(a-bx)^2}+\frac{b}{a^2(a-bx)}\right) dx, x, x^2\right)\\
&= \frac{1}{2a(a-bx^2)}+\frac{\log(x)}{a^2}-\frac{\log(a-bx^2)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.88

$$\frac{\frac{a}{a-bx^2}-\log(a-bx^2)+2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a - b*x^2)^2),x]

[Out] (a/(a - b*x^2) + 2*Log[x] - Log[a - b*x^2])/(2*a^2)

fricas [A] time = 0.96, size = 53, normalized size = 1.32

$$-\frac{\left(bx^2-a\right)\log\left(bx^2-a\right)-2\left(bx^2-a\right)\log(x)+a}{2\left(a^2bx^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 - a)*log(b*x^2 - a) - 2*(b*x^2 - a)*log(x) + a)/(a^2*b*x^2 - a
^3)
```

giac [A] time = 0.63, size = 51, normalized size = 1.28

$$\frac{\log\left(x^2\right)}{2\,a^2}-\frac{\log\left(\left|bx^2-a\right|\right)}{2\,a^2}+\frac{bx^2-2\,a}{2\left(bx^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 - a))/a^2 + 1/2*(b*x^2 - 2*a)/((b*x^2
- a)*a^2)
```

maple [A] time = 0.01, size = 39, normalized size = 0.98

$$-\frac{1}{2\left(b\,x^2-a\right)a}+\frac{\ln(x)}{a^2}-\frac{\ln\left(b\,x^2-a\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a)^2,x)
```

```
[Out] 1/a^2*ln(x)-1/2/a^2*ln(b*x^2-a)-1/2/a/(b*x^2-a)
```

maxima [A] time = 1.36, size = 41, normalized size = 1.02

$$-\frac{1}{2\left(abx^2-a^2\right)}-\frac{\log\left(bx^2-a\right)}{2\,a^2}+\frac{\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(a*b*x^2 - a^2) - 1/2*log(b*x^2 - a)/a^2 + 1/2*log(x^2)/a^2
```

mupad [B] time = 0.06, size = 36, normalized size = 0.90

$$\frac{\ln(x)}{a^2}+\frac{1}{2\,a\,\left(a-b\,x^2\right)}-\frac{\ln\left(a-b\,x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2)^2),x)
```

```
[Out] log(x)/a^2 + 1/(2*a*(a - b*x^2)) - log(a - b*x^2)/(2*a^2)
```

sympy [A] time = 0.30, size = 34, normalized size = 0.85

$$-\frac{1}{-2a^2+2abx^2}+\frac{\log(x)}{a^2}-\frac{\log\left(-\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a)**2,x)
```

```
[Out] -1/(-2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(-a/b + x**2)/(2*a**2)
```

3.237 $\int \frac{1}{x^2(a-bx^2)^2}\,dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a-bx^2\right)}$$

```
[Out] -3/2/a^2/x+1/2/a/x/(-b*x^2+a)+3/2*arctanh(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2
)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {290, 325, 208}

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a-bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^2),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a - b*x^2)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2(a-bx^2)^2}\,dx = \frac{1}{2ax(a-bx^2)} + \frac{3\int \frac{1}{x^2(a-bx^2)}\,dx}{2a}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax(a-bx^2)} + \frac{(3b)\int \frac{1}{a-bx^2}\,dx}{2a^2}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax(a-bx^2)} + \frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.97

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{bx}{2a^2(bx^2-a)} - \frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^2),x]
```

```
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(-a + b*x^2)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/
Sqrt[a]])/(2*a^(5/2))
```

fricas [A] time = 0.87, size = 140, normalized size = 2.41

$$\left[-\frac{6\,bx^2 - 3\left(bx^3 - ax\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right) - 4\,a}{4\left(a^2bx^3 - a^3x\right)}, -\frac{3\,bx^2 + 3\left(bx^3 - ax\right)\sqrt{-\frac{b}{a}}\arctan\left(x\sqrt{-\frac{b}{a}}\right) - 2\,a}{2\left(a^2bx^3 - a^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 - a*x)*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a
)/(b*x^2 - a)) - 4*a)/(a^2*b*x^3 - a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 - a*x)*
sqrt(-b/a)*arctan(x*sqrt(-b/a)) - 2*a)/(a^2*b*x^3 - a^3*x)]
```

giac [A] time = 0.58, size = 50, normalized size = 0.86

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,a^2} - \frac{3\,bx^2 - 2\,a}{2\left(bx^3 - ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^2) - 1/2*(3*b*x^2 - 2*a)/((b*x^
3 - a*x)*a^2)
```

maple [A] time = 0.01, size = 47, normalized size = 0.81

$$-\frac{\left(\frac{x}{2b\,x^2-2a} - \frac{3\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)b}{a^2} - \frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^2,x)
```

```
[Out] -1/a^2/x-1/a^2*b*(1/2*x/(b*x^2-a)-3/2/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x
))
```

maxima [A] time = 2.98, size = 65, normalized size = 1.12

$$-\frac{3\,bx^2-2\,a}{2\left(a^2bx^3-a^3x\right)}-\frac{3\,b\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x^2 - 2*a)/(a^2*b*x^3 - a^3*x) - 3/4*b*log((b*x - sqrt(a*b))/(b*x
 + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

mupad [B] time = 4.63, size = 45, normalized size = 0.78

$$\frac{3\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}-\frac{\frac{1}{a}-\frac{3\,b\,x^2}{2\,a^2}}{a\,x-b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^2),x)
```

```
[Out] (3*b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/(2*a^(5/2)) - (1/a - (3*b*x^2)/(2*a^
2))/(a*x - b*x^3)
```

sympy [A] time = 0.30, size = 83, normalized size = 1.43

$$-\frac{3\sqrt{\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{3\sqrt{\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{2a-3bx^2}{-2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**2,x)
```

```
[Out] -3*sqrt(b/a**5)*log(-a**3*sqrt(b/a**5)/b + x)/4 + 3*sqrt(b/a**5)*log(a**3*s
qrt(b/a**5)/b + x)/4 + (2*a - 3*b*x**2)/(-2*a**3*x + 2*a**2*b*x**3)
```

3.238 $\int \frac{1}{x^3\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=52

$$-\frac{b\log\left(a-bx^2\right)}{a^3}+\frac{2b\log(x)}{a^3}+\frac{b}{2a^2\left(a-bx^2\right)}-\frac{1}{2a^2x^2}$$

```
[Out] -1/2/a^2/x^2+1/2*b/a^2/(-b*x^2+a)+2*b*ln(x)/a^3-b*ln(-b*x^2+a)/a^3
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{b}{2a^2\left(a-bx^2\right)}-\frac{b\log\left(a-bx^2\right)}{a^3}+\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)^2),x]
```

```
[Out] -1/(2*a^2*x^2) + b/(2*a^2*(a - b*x^2)) + (2*b*Log[x])/a^3 - (b*Log[a - b*x^
2])/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a-bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a-bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^2(a-bx)^2}+\frac{2b^2}{a^3(a-bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{2a^2x^2}+\frac{b}{2a^2\left(a-bx^2\right)}+\frac{2b\log(x)}{a^3}-\frac{b\log\left(a-bx^2\right)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.85

$$\frac{\frac{ab}{a-bx^2}-2b\log\left(a-bx^2\right)-\frac{a}{x^2}+4b\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)^2),x]
```

```
[Out] (-(a/x^2) + (a*b)/(a - b*x^2) + 4*b*Log[x] - 2*b*Log[a - b*x^2])/(2*a^3)
```

fricas [A] time = 0.84, size = 80, normalized size = 1.54

$$-\frac{2\,abx^2 - a^2 + 2\left(b^2x^4 - abx^2\right)\log\left(bx^2 - a\right) - 4\left(b^2x^4 - abx^2\right)\log(x)}{2\left(a^3bx^4 - a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 - a^2 + 2*(b^2*x^4 - a*b*x^2)*log(b*x^2 - a) - 4*(b^2*x^4 -
 a*b*x^2)*log(x))/(a^3*b*x^4 - a^4*x^2)
```

giac [A] time = 0.60, size = 56, normalized size = 1.08

$$\frac{b\log\left(x^2\right)}{a^3} - \frac{b\log\left(\left|bx^2 - a\right|\right)}{a^3} - \frac{2\,bx^2 - a}{2\left(bx^4 - ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] b*log(x^2)/a^3 - b*log(abs(b*x^2 - a))/a^3 - 1/2*(2*b*x^2 - a)/((b*x^4 - a*
x^2)*a^2)
```

maple [A] time = 0.01, size = 51, normalized size = 0.98

$$-\frac{b}{2\left(b\,x^2 - a\right)a^2} + \frac{2b\ln(x)}{a^3} - \frac{b\ln\left(b\,x^2 - a\right)}{a^3} - \frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2/x^2+2/a^3*b*ln(x)-b/a^3*ln(b*x^2-a)-1/2*b/a^2/(b*x^2-a)
```

maxima [A] time = 1.31, size = 57, normalized size = 1.10

$$-\frac{2\,bx^2 - a}{2\left(a^2bx^4 - a^3x^2\right)} - \frac{b\log\left(bx^2 - a\right)}{a^3} + \frac{b\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 - a)/(a^2*b*x^4 - a^3*x^2) - b*log(b*x^2 - a)/a^3 + b*log(x^2
)/a^3
```

mupad [B] time = 4.59, size = 55, normalized size = 1.06

$$\frac{2\,b\,\ln(x)}{a^3} - \frac{b\,\ln\left(a - b\,x^2\right)}{a^3} - \frac{\frac{1}{2\,a} - \frac{b\,x^2}{a^2}}{a\,x^2 - b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2)^2),x)
```

```
[Out] (2*b*log(x))/a^3 - (b*log(a - b*x^2))/a^3 - (1/(2*a) - (b*x^2)/a^2)/(a*x^2
- b*x^4)
```

sympy [**A**] time = 0.37, size = 49, normalized size = 0.94

$$\frac{a-2bx^2}{-2a^3x^2+2a^2bx^4}+\frac{2b\log(x)}{a^3}-\frac{b\log\left(-\frac{a}{b}+x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a)**2,x)
```

```
[Out] (a - 2*b*x**2)/(-2*a**3*x**2 + 2*a**2*b*x**4) + 2*b*log(x)/a**3 - b*log(-a/
b + x**2)/a**3
```

3.239 $\int \frac{x^3}{(a-bx^2)^3} dx$

Optimal. Leaf size=20

$$\frac{x^4}{4a(a-bx^2)^2}$$

```
[Out] 1/4*x^4/a/(-b*x^2+a)^2
```

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {264}

$$\frac{x^4}{4a(a-bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2)^3,x]
```

```
[Out] x^4/(4*a*(a - b*x^2)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{(a-bx^2)^3} dx = \frac{x^4}{4a(a-bx^2)^2}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.25

$$-\frac{a-2bx^2}{4b^2(a-bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2)^3,x]
```

```
[Out] -1/4*(a - 2*b*x^2)/(b^2*(a - b*x^2)^2)
```

fricas [A] time = 0.96, size = 38, normalized size = 1.90

$$\frac{2\,bx^2-a}{4\,(b^4x^4-2\,ab^3x^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x^2 - a)/(b^4*x^4 - 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 0.62, size = 26, normalized size = 1.30

$$\frac{2\,bx^2 - a}{4\,\left(bx^2 - a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*b*x^2 - a)/((b*x^2 - a)^2*b^2)
```

maple [A] time = 0.01, size = 35, normalized size = 1.75

$$\frac{a}{4\left(b\,x^2 - a\right)^2 b^2} + \frac{1}{2\left(b\,x^2 - a\right) b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a)^3,x)
```

```
[Out] 1/4*a/b^2/(b*x^2-a)^2+1/2/b^2/(b*x^2-a)
```

maxima [A] time = 1.31, size = 38, normalized size = 1.90

$$\frac{2\,bx^2 - a}{4\left(b^4x^4 - 2\,ab^3x^2 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x^2 - a)/(b^4*x^4 - 2*a*b^3*x^2 + a^2*b^2)
```

mupad [B] time = 0.04, size = 37, normalized size = 1.85

$$-\frac{\frac{a}{4\,b^2} - \frac{x^2}{2\,b}}{a^2 - 2\,a\,b\,x^2 + b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2)^3,x)
```

```
[Out] -(a/(4*b^2) - x^2/(2*b))/(a^2 + b^2*x^4 - 2*a*b*x^2)
```

sympy [B] time = 0.27, size = 36, normalized size = 1.80

$$-\frac{a - 2bx^2}{4a^2b^2 - 8ab^3x^2 + 4b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a)**3,x)
```

```
[Out] -(a - 2*b*x**2)/(4*a**2*b**2 - 8*a*b**3*x**2 + 4*b**4*x**4)
```

3.240 $\int \frac{x^2}{\left(a-bx^2\right)^3}\,dx$

Optimal. Leaf size=67

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}-\frac{x}{8ab\left(a-bx^2\right)}+\frac{x}{4b\left(a-bx^2\right)^2}$$

```
[Out] 1/4*x/b/(-b*x^2+a)^2-1/8*x/a/b/(-b*x^2+a)-1/8*arctanh(x*b^(1/2)/a^(1/2))/a^
(3/2)/b^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {288, 199, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}-\frac{x}{8ab\left(a-bx^2\right)}+\frac{x}{4b\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^3,x]
```

```
[Out] x/(4*b*(a - b*x^2)^2) - x/(8*a*b*(a - b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]
]/(8*a^(3/2)*b^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a-bx^2\right)^3}\,dx = \frac{x}{4b\left(a-bx^2\right)^2} - \frac{\int \frac{1}{\left(a-bx^2\right)^2}\,dx}{4b}$$
$$= \frac{x}{4b\left(a-bx^2\right)^2} - \frac{x}{8ab\left(a-bx^2\right)} - \frac{\int \frac{1}{a-bx^2}\,dx}{8ab}$$
$$= \frac{x}{4b\left(a-bx^2\right)^2} - \frac{x}{8ab\left(a-bx^2\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.84

$$\frac{x\left(a+bx^2\right)}{8ab\left(a-bx^2\right)^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^3,x]
```

```
[Out] (x*(a + b*x^2))/(8*a*b*(a - b*x^2)^2) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(8*a^(
3/2)*b^(3/2))
```

fricas [A] time = 0.89, size = 188, normalized size = 2.81

$$\left[\frac{2\,ab^2x^3 + 2\,a^2bx + \left(b^2x^4 - 2\,abx^2 + a^2\right)\sqrt{ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{ab}\,x + a}{bx^2 - a}\right)}{16\left(a^2b^4x^4 - 2\,a^3b^3x^2 + a^4b^2\right)}, \frac{ab^2x^3 + a^2bx + \left(b^2x^4 - 2\,abx^2 + a^2\right)\sqrt{-ab}\,\text{ar}}{8\left(a^2b^4x^4 - 2\,a^3b^3x^2 + a^4b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*a*b^2*x^3 + 2*a^2*b*x + (b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(a*b)*log(
(b*x^2 - 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^2*b^4*x^4 - 2*a^3*b^3*x^2 + a^
4*b^2), 1/8*(a*b^2*x^3 + a^2*b*x + (b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(-a*b)*a
rctan(sqrt(-a*b)*x/a))/(a^2*b^4*x^4 - 2*a^3*b^3*x^2 + a^4*b^2)]
```

giac [A] time = 0.63, size = 53, normalized size = 0.79

$$\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\sqrt{-ab}\,ab} + \frac{bx^3 + ax}{8\left(bx^2 - a\right)^2 ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a*b) + 1/8*(b*x^3 + a*x)/((b*x^2 - a
)^2*a*b)
```

maple [A] time = 0.01, size = 52, normalized size = 0.78

$$-\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;ab} - \frac{-\frac{x^3}{8a} - \frac{x}{8b}}{\left(b\,x^2 - a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^3,x)
```

```
[Out] -(-1/8/a*x^3-1/8/b*x)/(b*x^2-a)^2-1/8/b/a/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)
*b*x)
```

maxima [A] time = 2.89, size = 76, normalized size = 1.13

$$\frac{bx^3 + ax}{8\left(ab^3x^4 - 2\,a^2b^2x^2 + a^3b\right)} + \frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{16\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^3 + a*x)/(a*b^3*x^4 - 2*a^2*b^2*x^2 + a^3*b) + 1/16*log((b*x - sqr
t(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a*b)
```

mupad [B] time = 4.62, size = 54, normalized size = 0.81

$$\frac{\frac{x}{8\,b}+\frac{x^3}{8\,a}}{a^2-2\,a\,b\,x^2+b^2\,x^4}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2)^3,x)
```

```
[Out] (x/(8*b) + x^3/(8*a))/(a^2 + b^2*x^4 - 2*a*b*x^2) - atanh((b^(1/2)*x)/a^(1/
2))/(8*a^(3/2)*b^(3/2))
```

sympy [B] time = 0.32, size = 105, normalized size = 1.57

$$\frac{\sqrt{\frac{1}{a^3b^3}}\log\left(-a^2b\sqrt{\frac{1}{a^3b^3}}+x\right)}{16}-\frac{\sqrt{\frac{1}{a^3b^3}}\log\left(a^2b\sqrt{\frac{1}{a^3b^3}}+x\right)}{16}-\frac{-ax-bx^3}{8a^3b-16a^2b^2x^2+8ab^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**3,x)
```

```
[Out] sqrt(1/(a**3*b**3))*log(-a**2*b*sqrt(1/(a**3*b**3)) + x)/16 - sqrt(1/(a**3*
b**3))*log(a**2*b*sqrt(1/(a**3*b**3)) + x)/16 - (-a*x - b*x**3)/(8*a**3*b -
 16*a**2*b**2*x**2 + 8*a*b**3*x**4)
```

3.241 $\int \frac{x}{(a-bx^2)^3} \, dx$

Optimal. Leaf size=17

$$\frac{1}{4b\left(a-bx^2\right)^2}$$

```
[Out] 1/4/b/(-b*x^2+a)^2
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {261}

$$\frac{1}{4b\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^2)^3,x]
```

```
[Out] 1/(4*b*(a - b*x^2)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a-bx^2\right)^3} \, dx = \frac{1}{4b\left(a-bx^2\right)^2}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{4b\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^2)^3,x]
```

```
[Out] 1/(4*b*(a - b*x^2)^2)
```

fricas [A] time = 0.98, size = 26, normalized size = 1.53

$$\frac{1}{4\left(b^3x^4-2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4/(b^3*x^4 - 2*a*b^2*x^2 + a^2*b)
```

giac [A] time = 0.61, size = 16, normalized size = 0.94

$$\frac{1}{4\left(bx^2-a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/4/((b*x^2 - a)^2*b)
```

maple [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{4\left(b\,x^{2}-a\right)^{2} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^3,x)
```

```
[Out] 1/4/b/(b*x^2-a)^2
```

maxima [A] time = 1.34, size = 16, normalized size = 0.94

$$\frac{1}{4\left(bx^{2}-a\right)^{2} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4/((b*x^2 - a)^2*b)
```

mupad [B] time = 0.03, size = 26, normalized size = 1.53

$$\frac{1}{4\,a^{2}\,b-8\,a\,b^{2}\,x^{2}+4\,b^{3}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2)^3,x)
```

```
[Out] 1/(4*a^2*b + 4*b^3*x^4 - 8*a*b^2*x^2)
```

sympy [B] time = 0.24, size = 26, normalized size = 1.53

$$\frac{1}{4a^{2}b-8ab^{2}x^{2}+4b^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**3,x)
```

```
[Out] 1/(4*a**2*b - 8*a*b**2*x**2 + 4*b**3*x**4)
```

3.242 $\int \frac{1}{\left(a-bx^2\right)^3} \, dx$

Optimal. Leaf size=64

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{3x}{8a^2\left(a-bx^2\right)}+\frac{x}{4a\left(a-bx^2\right)^2}$$

```
[Out] 1/4*x/a/(-b*x^2+a)^2+3/8*x/a^2/(-b*x^2+a)+3/8*arctanh(x*b^(1/2)/a^(1/2))/a^
(5/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {199, 208}

$$\frac{3x}{8a^2\left(a-bx^2\right)}+\frac{3 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{x}{4a\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-3),x]
```

```
[Out] x/(4*a*(a - b*x^2)^2) + (3*x)/(8*a^2*(a - b*x^2)) + (3*ArcTanh[(Sqrt[b]*x)/
Sqrt[a]])/(8*a^(5/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a-bx^2\right)^3} \, dx &= \frac{x}{4a\left(a-bx^2\right)^2}+\frac{3\int \frac{1}{\left(a-bx^2\right)^2} \, dx}{4a}\\
&= \frac{x}{4a\left(a-bx^2\right)^2}+\frac{3x}{8a^2\left(a-bx^2\right)}+\frac{3\int \frac{1}{a-bx^2} \, dx}{8a^2}\\
&= \frac{x}{4a\left(a-bx^2\right)^2}+\frac{3x}{8a^2\left(a-bx^2\right)}+\frac{3 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.88

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{5ax-3bx^3}{8a^2\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-3),x]
```

```
[Out] (5*a*x - 3*b*x^3)/(8*a^2*(a - b*x^2)^2) + (3*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/
(8*a^(5/2)*Sqrt[b])
```

fricas [A] time = 0.90, size = 188, normalized size = 2.94

$$\left[-\frac{6\,ab^2x^3 - 10\,a^2bx - 3\left(b^2x^4 - 2\,abx^2 + a^2\right)\sqrt{ab}\,\log\left(\frac{bx^2+2\,\sqrt{ab}\,x+a}{bx^2-a}\right)}{16\left(a^3b^3x^4 - 2\,a^4b^2x^2 + a^5b\right)}, -\frac{3\,ab^2x^3 - 5\,a^2bx + 3\left(b^2x^4 - 2\,abx^2 + a^2\right)\sqrt{}}{8\left(a^3b^3x^4 - 2\,a^4b^2x^2 + a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(6*a*b^2*x^3 - 10*a^2*b*x - 3*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(a*b)*
log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^3*b^3*x^4 - 2*a^4*b^2*x^2
+ a^5*b), -1/8*(3*a*b^2*x^3 - 5*a^2*b*x + 3*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqr
t(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^3*b^3*x^4 - 2*a^4*b^2*x^2 + a^5*b)]
```

giac [A] time = 0.62, size = 49, normalized size = 0.77

$$-\frac{3\,\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\,\sqrt{-ab}\,a^2} - \frac{3\,bx^3 - 5\,ax}{8\left(bx^2 - a\right)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/8*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^2) - 1/8*(3*b*x^3 - 5*a*x)/((b*x^
2 - a)^2*a^2)
```

maple [A] time = 0.00, size = 61, normalized size = 0.95

$$\frac{x}{4\left(b\,x^2 - a\right)^2 a} + \frac{-\frac{3x}{8\left(b\,x^2-a\right)a} + \frac{3\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^3,x)
```

```
[Out] 1/4*x/a/(b*x^2-a)^2+3/4/a*(-1/2/(b*x^2-a)/a*x+1/2/a/(a*b)^(1/2)*arctanh(1/(
a*b)^(1/2)*b*x))
```

maxima [A] time = 2.99, size = 73, normalized size = 1.14

$$-\frac{3\,bx^3 - 5\,ax}{8\left(a^2b^2x^4 - 2\,a^3bx^2 + a^4\right)} - \frac{3\,\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b*x^3 - 5*a*x)/(a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4) - 3/16*log((b*x -
sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

mupad [B] time = 4.60, size = 55, normalized size = 0.86

$$\frac{\frac{5\,x}{8\,a}-\frac{3\,b\,x^3}{8\,a^2}}{a^2-2\,a\,b\,x^2+b^2\,x^4}+\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^3,x)
```

```
[Out] ((5*x)/(8*a) - (3*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 - 2*a*b*x^2) + (3*atanh((b
^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2))
```

sympy [A] time = 0.33, size = 99, normalized size = 1.55

$$-\frac{3\sqrt{\frac{1}{a^5b}}\log\left(-a^3\sqrt{\frac{1}{a^5b}}+x\right)}{16}+\frac{3\sqrt{\frac{1}{a^5b}}\log\left(a^3\sqrt{\frac{1}{a^5b}}+x\right)}{16}-\frac{-5ax+3bx^3}{8a^4-16a^3bx^2+8a^2b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(1/(a**5*b))*log(-a**3*sqrt(1/(a**5*b)) + x)/16 + 3*sqrt(1/(a**5*b))
*log(a**3*sqrt(1/(a**5*b)) + x)/16 - (-5*a*x + 3*b*x**3)/(8*a**4 - 16*a**3*
b*x**2 + 8*a**2*b**2*x**4)
```

3.243 $\int \frac{1}{x\left(a-bx^2\right)^3}\,dx$

Optimal. Leaf size=57

$$-\frac{\log\left(a-bx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{1}{2a^2\left(a-bx^2\right)}+\frac{1}{4a\left(a-bx^2\right)^2}$$

[Out] 1/4/a/(-b*x^2+a)^2+1/2/a^2/(-b*x^2+a)+ln(x)/a^3-1/2*ln(-b*x^2+a)/a^3

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{1}{2a^2\left(a-bx^2\right)}-\frac{\log\left(a-bx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{1}{4a\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a - b*x^2)^3),x]

[Out] 1/(4*a*(a - b*x^2)^2) + 1/(2*a^2*(a - b*x^2)) + Log[x]/a^3 - Log[a - b*x^2]/(2*a^3)

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a-bx^2\right)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a-bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x}+\frac{b}{a(a-bx)^3}+\frac{b}{a^2(a-bx)^2}+\frac{b}{a^3(a-bx)}\right)dx,x,x^2\right)\\
&= \frac{1}{4a\left(a-bx^2\right)^2}+\frac{1}{2a^2\left(a-bx^2\right)}+\frac{\log(x)}{a^3}-\frac{\log\left(a-bx^2\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.79

$$\frac{\frac{a\left(3a-2bx^2\right)}{\left(a-bx^2\right)^2}-2\log\left(a-bx^2\right)+4\log(x)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)^3),x]
```

```
[Out] ((a*(3*a - 2*b*x^2))/(a - b*x^2)^2 + 4*Log[x] - 2*Log[a - b*x^2])/(4*a^3)
```

fricas [A] time = 0.73, size = 92, normalized size = 1.61

$$-\frac{2\,abx^2-3\,a^2+2\,\left(b^2x^4-2\,abx^2+a^2\right)\log\left(bx^2-a\right)-4\,\left(b^2x^4-2\,abx^2+a^2\right)\log(x)}{4\,\left(a^3b^2x^4-2\,a^4bx^2+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b*x^2 - 3*a^2 + 2*(b^2*x^4 - 2*a*b*x^2 + a^2)*log(b*x^2 - a) - 4*
(b^2*x^4 - 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 - 2*a^4*b*x^2 + a^5)
```

giac [A] time = 0.63, size = 63, normalized size = 1.11

$$\frac{\log\left(x^2\right)}{2\,a^3}-\frac{\log\left(\left|bx^2-a\right|\right)}{2\,a^3}+\frac{3\,b^2x^4-8\,abx^2+6\,a^2}{4\,\left(bx^2-a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^3 - 1/2*log(abs(b*x^2 - a))/a^3 + 1/4*(3*b^2*x^4 - 8*a*b*x^2
 + 6*a^2)/((b*x^2 - a)^2*a^3)
```

maple [A] time = 0.01, size = 55, normalized size = 0.96

$$\frac{1}{4\left(b\,x^2-a\right)^2a}-\frac{1}{2\left(b\,x^2-a\right)a^2}+\frac{\ln(x)}{a^3}-\frac{\ln\left(b\,x^2-a\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a)^3,x)
```

```
[Out] 1/a^3*ln(x)-1/2/a^3*ln(b*x^2-a)+1/4/a/(b*x^2-a)^2-1/2/a^2/(b*x^2-a)
```

maxima [A] time = 1.37, size = 62, normalized size = 1.09

$$-\frac{2\,bx^2-3\,a}{4\,\left(a^2b^2x^4-2\,a^3bx^2+a^4\right)}-\frac{\log\left(bx^2-a\right)}{2\,a^3}+\frac{\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x^2 - 3*a)/(a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4) - 1/2*log(b*x^2 - a)
/a^3 + 1/2*log(x^2)/a^3
```

mupad [B] time = 0.06, size = 57, normalized size = 1.00

$$\frac{\ln(x)}{a^3}+\frac{\frac{3}{4\,a}-\frac{b\,x^2}{2\,a^2}}{a^2-2\,a\,b\,x^2+b^2\,x^4}-\frac{\ln\left(a-b\,x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2)^3),x)
```

```
[Out] log(x)/a^3 + (3/(4*a) - (b*x^2)/(2*a^2))/(a^2 + b^2*x^4 - 2*a*b*x^2) - log(
a - b*x^2)/(2*a^3)
```

sympy [A] time = 0.42, size = 56, normalized size = 0.98

$$-\frac{-3a+2bx^2}{4a^4-8a^3bx^2+4a^2b^2x^4}+\frac{\log(x)}{a^3}-\frac{\log\left(-\frac{a}{b}+x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a)**3,x)
```

```
[Out] -(-3*a + 2*b*x**2)/(4*a**4 - 8*a**3*b*x**2 + 4*a**2*b**2*x**4) + log(x)/a**
3 - log(-a/b + x**2)/(2*a**3)
```

3.244 $\int \frac{1}{x^2(a-bx^2)^3}\,dx$

Optimal. Leaf size=78

$$\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15}{8a^3x}+\frac{5}{8a^2x\left(a-bx^2\right)}+\frac{1}{4ax\left(a-bx^2\right)^2}$$

```
[Out] -15/8/a^3/x+1/4/a/x/(-b*x^2+a)^2+5/8/a^2/x/(-b*x^2+a)+15/8*arctanh(x*b^(1/2
)/a^(1/2))*b^(1/2)/a^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {290, 325, 208}

$$\frac{5}{8a^2x\left(a-bx^2\right)}+\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15}{8a^3x}+\frac{1}{4ax\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^3),x]
```

```
[Out] -15/(8*a^3*x) + 1/(4*a*x*(a - b*x^2)^2) + 5/(8*a^2*x*(a - b*x^2)) + (15*Sqr
t[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a-bx^2\right)^3}\,dx &= \frac{1}{4ax\left(a-bx^2\right)^2}+\frac{5\int \frac{1}{x^2\left(a-bx^2\right)^2}\,dx}{4a} \\
&= \frac{1}{4ax\left(a-bx^2\right)^2}+\frac{5}{8a^2x\left(a-bx^2\right)}+\frac{15\int \frac{1}{x^2\left(a-bx^2\right)}\,dx}{8a^2} \\
&= -\frac{15}{8a^3x}+\frac{1}{4ax\left(a-bx^2\right)^2}+\frac{5}{8a^2x\left(a-bx^2\right)}+\frac{(15b)\int \frac{1}{a-bx^2}\,dx}{8a^3} \\
&= -\frac{15}{8a^3x}+\frac{1}{4ax\left(a-bx^2\right)^2}+\frac{5}{8a^2x\left(a-bx^2\right)}+\frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.88

$$
\frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}+\frac{-8a^2+25abx^2-15b^2x^4}{8a^3x\left(a-bx^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^3),x]
```

```
[Out] (-8*a^2 + 25*a*b*x^2 - 15*b^2*x^4)/(8*a^3*x*(a - b*x^2)^2) + (15*Sqrt[b]*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

fricas [A] time = 0.87, size = 202, normalized size = 2.59

$$
\left[-\frac{30\,b^2x^4-50\,abx^2-15\left(b^2x^5-2\,abx^3+a^2x\right)\sqrt{\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right)+16\,a^2}{16\left(a^3b^2x^5-2\,a^4bx^3+a^5x\right)},\,-\frac{15\,b^2x^4-25\,abx^2+15\left(b^2x^5-2\right.}{8\left(a^3b^2x^5\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(30*b^2*x^4 - 50*a*b*x^2 - 15*(b^2*x^5 - 2*a*b*x^3 + a^2*x)*sqrt(b/a
)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)) + 16*a^2)/(a^3*b^2*x^5 - 2
*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 - 25*a*b*x^2 + 15*(b^2*x^5 - 2*a*b*x^
3 + a^2*x)*sqrt(-b/a)*arctan(x*sqrt(-b/a)) + 8*a^2)/(a^3*b^2*x^5 - 2*a^4*b*
x^3 + a^5*x)]
```

giac [A] time = 0.63, size = 61, normalized size = 0.78

$$
-\frac{15\,b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\sqrt{-ab}\,a^3}-\frac{7\,b^2x^3-9\,abx}{8\left(bx^2-a\right)^2a^3}-\frac{1}{a^3x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^3) - 1/8*(7*b^2*x^3 - 9*a*b*x)
/((b*x^2 - a)^2*a^3) - 1/(a^3*x)
```

maple [A] time = 0.01, size = 56, normalized size = 0.72

$$-\frac{\left(-\frac{15\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}+\frac{\frac{7}{8}b\,x^{3}-\frac{9}{8}ax}{\left(b\,x^{2}-a\right)^{2}}\right)b}{a^{3}}-\frac{1}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^3,x)
```

```
[Out] -1/a^3/x-1/a^3*b*((7/8*b*x^3-9/8*a*x)/(b*x^2-a)^2-15/8/(a*b)^(1/2)*arctanh(
1/(a*b)^(1/2)*b*x))
```

maxima [A] time = 2.91, size = 86, normalized size = 1.10

$$-\frac{15\,b^{2}x^{4}-25\,abx^{2}+8\,a^{2}}{8\left(a^{3}b^{2}x^{5}-2\,a^{4}bx^{3}+a^{5}x\right)}-\frac{15\,b\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(15*b^2*x^4 - 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 - 2*a^4*b*x^3 + a^5*x)
- 15/16*b*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^3)
```

mupad [B] time = 4.60, size = 66, normalized size = 0.85

$$\frac{15\,\sqrt{b}\ \mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}-\frac{\frac{1}{a}-\frac{25\,b\,x^{2}}{8\,a^{2}}+\frac{15\,b^{2}\,x^{4}}{8\,a^{3}}}{a^{2}\,x-2\,a\,b\,x^{3}+b^{2}\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^3),x)
```

```
[Out] (15*b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2)) - (1/a - (25*b*x^2)/(8*
a^2) + (15*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 - 2*a*b*x^3)
```

sympy [A] time = 0.43, size = 107, normalized size = 1.37

$$-\frac{15\sqrt{\frac{b}{a^{7}}}\log\left(-\frac{a^{4}\sqrt{\frac{b}{a^{7}}}}{b}+x\right)}{16}+\frac{15\sqrt{\frac{b}{a^{7}}}\log\left(\frac{a^{4}\sqrt{\frac{b}{a^{7}}}}{b}+x\right)}{16}-\frac{8a^{2}-25abx^{2}+15b^{2}x^{4}}{8a^{5}x-16a^{4}bx^{3}+8a^{3}b^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**3,x)
```

```
[Out] -15*sqrt(b/a**7)*log(-a**4*sqrt(b/a**7)/b + x)/16 + 15*sqrt(b/a**7)*log(a**
4*sqrt(b/a**7)/b + x)/16 - (8*a**2 - 25*a*b*x**2 + 15*b**2*x**4)/(8*a**5*x
- 16*a**4*b*x**3 + 8*a**3*b**2*x**5)
```

3.245 $\int \frac{1}{x^3(a-bx^2)^3} \, dx$

Optimal. Leaf size=69

$$-\frac{3b\log\left(a-bx^2\right)}{2a^4}+\frac{3b\log(x)}{a^4}+\frac{b}{a^3\left(a-bx^2\right)}-\frac{1}{2a^3x^2}+\frac{b}{4a^2\left(a-bx^2\right)^2}$$

```
[Out] -1/2/a^3/x^2+1/4*b/a^2/(-b*x^2+a)^2+b/a^3/(-b*x^2+a)+3*b*ln(x)/a^4-3/2*b*ln
(-b*x^2+a)/a^4
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{b}{a^3\left(a-bx^2\right)}+\frac{b}{4a^2\left(a-bx^2\right)^2}-\frac{3b\log\left(a-bx^2\right)}{2a^4}+\frac{3b\log(x)}{a^4}-\frac{1}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)^3),x]
```

```
[Out] -1/(2*a^3*x^2) + b/(4*a^2*(a - b*x^2)^2) + b/(a^3*(a - b*x^2)) + (3*b*Log[x
])/a^4 - (3*b*Log[a - b*x^2])/(2*a^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a-bx^2\right)^3} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a-bx)^3} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^3x^2}+\frac{3b}{a^4x}+\frac{b^2}{a^2(a-bx)^3}+\frac{2b^2}{a^3(a-bx)^2}+\frac{3b^2}{a^4(a-bx)}\right) dx, x, x^2\right)\\
&= -\frac{1}{2a^3x^2}+\frac{b}{4a^2\left(a-bx^2\right)^2}+\frac{b}{a^3\left(a-bx^2\right)}+\frac{3b\log(x)}{a^4}-\frac{3b\log\left(a-bx^2\right)}{2a^4}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 60, normalized size = 0.87

$$\frac{\frac{a\left(-2a^2+9abx^2-6b^2x^4\right)}{\left(ax-bx^3\right)^2}-6b\log\left(a-bx^2\right)+12b\log(x)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)^3),x]
```

```
[Out] ((a*(-2*a^2 + 9*a*b*x^2 - 6*b^2*x^4))/(a*x - b*x^3)^2 + 12*b*Log[x] - 6*b*L
og[a - b*x^2])/(4*a^4)
```

fricas [A] time = 1.15, size = 121, normalized size = 1.75

$$-\frac{6\,ab^2x^4 - 9\,a^2bx^2 + 2\,a^3 + 6\left(b^3x^6 - 2\,ab^2x^4 + a^2bx^2\right)\log\left(bx^2 - a\right) - 12\left(b^3x^6 - 2\,ab^2x^4 + a^2bx^2\right)\log(x)}{4\left(a^4b^2x^6 - 2\,a^5bx^4 + a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3 + 6*(b^3*x^6 - 2*a*b^2*x^4 + a^2*b*
x^2)*log(b*x^2 - a) - 12*(b^3*x^6 - 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4*b
^2*x^6 - 2*a^5*b*x^4 + a^6*x^2)
```

giac [A] time = 0.63, size = 84, normalized size = 1.22

$$\frac{3\,b\log\left(x^2\right)}{2\,a^4} - \frac{3\,b\log\left(\left|bx^2 - a\right|\right)}{2\,a^4} + \frac{9\,b^3x^4 - 22\,ab^2x^2 + 14\,a^2b}{4\left(bx^2 - a\right)^2a^4} - \frac{3\,bx^2 + a}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/2*b*log(x^2)/a^4 - 3/2*b*log(abs(b*x^2 - a))/a^4 + 1/4*(9*b^3*x^4 - 22*a*
b^2*x^2 + 14*a^2*b)/((b*x^2 - a)^2*a^4) - 1/2*(3*b*x^2 + a)/(a^4*x^2)
```

maple [A] time = 0.01, size = 68, normalized size = 0.99

$$\frac{b}{4\left(b\,x^2 - a\right)^2a^2} - \frac{b}{\left(b\,x^2 - a\right)a^3} + \frac{3b\ln(x)}{a^4} - \frac{3b\ln\left(b\,x^2 - a\right)}{2a^4} - \frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a)^3,x)
```

```
[Out] -1/2/a^3/x^2+3/a^4*b*ln(x)-3/2/a^4*b*ln(b*x^2-a)+1/4/a^2*b/(b*x^2-a)^2-1/a^
3*b/(b*x^2-a)
```

maxima [A] time = 1.31, size = 79, normalized size = 1.14

$$-\frac{6\,b^2x^4 - 9\,abx^2 + 2\,a^2}{4\left(a^3b^2x^6 - 2\,a^4bx^4 + a^5x^2\right)} - \frac{3\,b\log\left(bx^2 - a\right)}{2\,a^4} + \frac{3\,b\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*b^2*x^4 - 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 - 2*a^4*b*x^4 + a^5*x^2)
- 3/2*b*log(b*x^2 - a)/a^4 + 3/2*b*log(x^2)/a^4
```

mupad [B] time = 4.61, size = 76, normalized size = 1.10

$$\frac{3\,b\,\ln(x)}{a^4} - \frac{3\,b\,\ln\left(a - b\,x^2\right)}{2\,a^4} - \frac{\frac{1}{2\,a} - \frac{9\,b\,x^2}{4\,a^2} + \frac{3\,b^2\,x^4}{2\,a^3}}{a^2\,x^2 - 2\,a\,b\,x^4 + b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2)^3),x)
```

```
[Out] (3*b*log(x))/a^4 - (3*b*log(a - b*x^2))/(2*a^4) - (1/(2*a) - (9*b*x^2)/(4*a
^2) + (3*b^2*x^4)/(2*a^3))/(a^2*x^2 + b^2*x^6 - 2*a*b*x^4)
```

sympy [A] time = 0.51, size = 78, normalized size = 1.13

$$-\frac{2a^2 - 9abx^2 + 6b^2x^4}{4a^5x^2 - 8a^4bx^4 + 4a^3b^2x^6} + \frac{3b\log(x)}{a^4} - \frac{3b\log\left(-\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a)**3,x)
```

```
[Out] -(2*a**2 - 9*a*b*x**2 + 6*b**2*x**4)/(4*a**5*x**2 - 8*a**4*b*x**4 + 4*a**3*
b**2*x**6) + 3*b*log(x)/a**4 - 3*b*log(-a/b + x**2)/(2*a**4)
```

3.246 $\int \frac{x^3}{\left(a-bx^2\right)^5}\,dx$

Optimal. Leaf size=36

$$\frac{a}{8b^2\left(a-bx^2\right)^4}-\frac{1}{6b^2\left(a-bx^2\right)^3}$$

```
[Out] 1/8*a/b^2/(-b*x^2+a)^4-1/6/b^2/(-b*x^2+a)^3
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 43}

$$\frac{a}{8b^2\left(a-bx^2\right)^4}-\frac{1}{6b^2\left(a-bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2)^5,x]
```

```
[Out] a/(8*b^2*(a - b*x^2)^4) - 1/(6*b^2*(a - b*x^2)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a-bx^2\right)^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a-bx)^5}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a}{b(a-bx)^5}-\frac{1}{b(a-bx)^4}\right)dx,x,x^2\right)\\
&= \frac{a}{8b^2\left(a-bx^2\right)^4}-\frac{1}{6b^2\left(a-bx^2\right)^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.69

$$-\frac{a-4bx^2}{24b^2\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2)^5,x]
```

```
[Out] -1/24*(a - 4*b*x^2)/(b^2*(a - b*x^2)^4)
```

fricas [A] time = 0.99, size = 60, normalized size = 1.67

$$\frac{4\,bx^2 - a}{24\left(b^6x^8 - 4\,ab^5x^6 + 6\,a^2b^4x^4 - 4\,a^3b^3x^2 + a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*(4*b*x^2 - a)/(b^6*x^8 - 4*a*b^5*x^6 + 6*a^2*b^4*x^4 - 4*a^3*b^3*x^2 +
 a^4*b^2)
```

giac [A] time = 0.59, size = 39, normalized size = 1.08

$$\frac{\frac{4}{\left(bx^2-a\right)^3b} + \frac{3\,a}{\left(bx^2-a\right)^4b}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/24*(4/((b*x^2 - a)^3*b) + 3*a/((b*x^2 - a)^4*b))/b
```

maple [A] time = 0.01, size = 35, normalized size = 0.97

$$\frac{a}{8\left(b\,x^2 - a\right)^4 b^2} + \frac{1}{6\left(b\,x^2 - a\right)^3 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a)^5,x)
```

```
[Out] 1/6/b^2/(b*x^2-a)^3+1/8*a/b^2/(b*x^2-a)^4
```

maxima [A] time = 1.36, size = 60, normalized size = 1.67

$$\frac{4\,bx^2 - a}{24\left(b^6x^8 - 4\,ab^5x^6 + 6\,a^2b^4x^4 - 4\,a^3b^3x^2 + a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*(4*b*x^2 - a)/(b^6*x^8 - 4*a*b^5*x^6 + 6*a^2*b^4*x^4 - 4*a^3*b^3*x^2 +
 a^4*b^2)
```

mupad [B] time = 4.60, size = 59, normalized size = 1.64

$$-\frac{\frac{a}{24\,b^2} - \frac{x^2}{6\,b}}{a^4 - 4\,a^3\,b\,x^2 + 6\,a^2\,b^2\,x^4 - 4\,a\,b^3\,x^6 + b^4\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2)^5,x)
```

```
[Out] -(a/(24*b^2) - x^2/(6*b))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^
2*b^2*x^4)
```

sympy [B] time = 0.44, size = 60, normalized size = 1.67

$$-\frac{a - 4bx^2}{24a^4b^2 - 96a^3b^3x^2 + 144a^2b^4x^4 - 96ab^5x^6 + 24b^6x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a)**5,x)
```

```
[Out] -(a - 4*b*x**2)/(24*a**4*b**2 - 96*a**3*b**3*x**2 + 144*a**2*b**4*x**4 - 96
*a*b**5*x**6 + 24*b**6*x**8)
```

3.247 $\int \frac{x^2}{\left(a-bx^2\right)^5}\,dx$

Optimal. Leaf size=109

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}-\frac{5x}{128a^3b\left(a-bx^2\right)}-\frac{5x}{192a^2b\left(a-bx^2\right)^2}-\frac{x}{48ab\left(a-bx^2\right)^3}+\frac{x}{8b\left(a-bx^2\right)^4}$$

```
[Out] 1/8*x/b/(-b*x^2+a)^4-1/48*x/a/b/(-b*x^2+a)^3-5/192*x/a^2/b/(-b*x^2+a)^2-5/1
28*x/a^3/b/(-b*x^2+a)-5/128*arctanh(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {288, 199, 208}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}-\frac{5x}{128a^3b\left(a-bx^2\right)}-\frac{5x}{192a^2b\left(a-bx^2\right)^2}-\frac{x}{48ab\left(a-bx^2\right)^3}+\frac{x}{8b\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^5,x]
```

```
[Out] x/(8*b*(a - b*x^2)^4) - x/(48*a*b*(a - b*x^2)^3) - (5*x)/(192*a^2*b*(a - b*
x^2)^2) - (5*x)/(128*a^3*b*(a - b*x^2)) - (5*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/
(128*a^(7/2)*b^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a-bx^2\right)^5}\,dx = \frac{x}{8b\left(a-bx^2\right)^4} - \frac{\int \frac{1}{\left(a-bx^2\right)^4}\,dx}{8b}$$
$$= \frac{x}{8b\left(a-bx^2\right)^4} - \frac{x}{48ab\left(a-bx^2\right)^3} - \frac{5\int \frac{1}{\left(a-bx^2\right)^3}\,dx}{48ab}$$
$$= \frac{x}{8b\left(a-bx^2\right)^4} - \frac{x}{48ab\left(a-bx^2\right)^3} - \frac{5x}{192a^2b\left(a-bx^2\right)^2} - \frac{5\int \frac{1}{\left(a-bx^2\right)^2}\,dx}{64a^2b}$$
$$= \frac{x}{8b\left(a-bx^2\right)^4} - \frac{x}{48ab\left(a-bx^2\right)^3} - \frac{5x}{192a^2b\left(a-bx^2\right)^2} - \frac{5x}{128a^3b\left(a-bx^2\right)} - \frac{5\int \frac{1}{a-bx^2}\,dx}{128a^3b}$$
$$= \frac{x}{8b\left(a-bx^2\right)^4} - \frac{x}{48ab\left(a-bx^2\right)^3} - \frac{5x}{192a^2b\left(a-bx^2\right)^2} - \frac{5x}{128a^3b\left(a-bx^2\right)} - \frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 0.74

$$\frac{15a^3x+73a^2bx^3-55ab^2x^5+15b^3x^7}{384a^3b\left(a-bx^2\right)^4} - \frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^5,x]
```

```
[Out] (15*a^3*x + 73*a^2*b*x^3 - 55*a*b^2*x^5 + 15*b^3*x^7)/(384*a^3*b*(a - b*x^2
)^4) - (5*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2))
```

fricas [A] time = 0.96, size = 324, normalized size = 2.97

$$\left[\frac{30\,ab^4x^7-110\,a^2b^3x^5+146\,a^3b^2x^3+30\,a^4bx+15\left(b^4x^8-4\,ab^3x^6+6\,a^2b^2x^4-4\,a^3bx^2+a^4\right)\sqrt{ab}\,\log\left(\frac{bx^2-2}{bx}\right.}{768\left(a^4b^6x^8-4\,a^5b^5x^6+6\,a^6b^4x^4-4\,a^7b^3x^2+a^8b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] [1/768*(30*a*b^4*x^7 - 110*a^2*b^3*x^5 + 146*a^3*b^2*x^3 + 30*a^4*b*x + 15*
(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(a*b)*log((
b*x^2 - 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^4*b^6*x^8 - 4*a^5*b^5*x^6 + 6*a
^6*b^4*x^4 - 4*a^7*b^3*x^2 + a^8*b^2), 1/384*(15*a*b^4*x^7 - 55*a^2*b^3*x^5
 + 73*a^3*b^2*x^3 + 15*a^4*b*x + 15*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4
- 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^4*b^6*x^8 - 4*a^
5*b^5*x^6 + 6*a^6*b^4*x^4 - 4*a^7*b^3*x^2 + a^8*b^2)]
```

giac [A] time = 0.63, size = 77, normalized size = 0.71

$$\frac{5\,\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128\,\sqrt{-ab}\,a^3b} + \frac{15\,b^3x^7-55\,ab^2x^5+73\,a^2bx^3+15\,a^3x}{384\left(bx^2-a\right)^4a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 5/128*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^3*b) + 1/384*(15*b^3*x^7 - 55*a*
b^2*x^5 + 73*a^2*b*x^3 + 15*a^3*x)/((b*x^2 - a)^4*a^3*b)
```

maple [A] time = 0.01, size = 72, normalized size = 0.66

$$-\frac{5\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\sqrt{ab}\;a^3b}-\frac{-\frac{5b^2x^7}{128a^3}+\frac{55b\,x^5}{384a^2}-\frac{73x^3}{384a}-\frac{5x}{128b}}{\left(b\,x^2-a\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^5,x)
```

```
[Out] -(-5/128/a^3*b^2*x^7+55/384/a^2*b*x^5-73/384/a*x^3-5/128/b*x)/(b*x^2-a)^4-5
/128/a^3/b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.98, size = 124, normalized size = 1.14

$$\frac{15\,b^3x^7-55\,ab^2x^5+73\,a^2bx^3+15\,a^3x}{384\left(a^3b^5x^8-4\,a^4b^4x^6+6\,a^5b^3x^4-4\,a^6b^2x^2+a^7b\right)}+\frac{5\,\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/384*(15*b^3*x^7 - 55*a*b^2*x^5 + 73*a^2*b*x^3 + 15*a^3*x)/(a^3*b^5*x^8 -
4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 - 4*a^6*b^2*x^2 + a^7*b) + 5/256*log((b*x - s
qrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^3*b)
```

mupad [B] time = 4.76, size = 96, normalized size = 0.88

$$\frac{\frac{5\,x}{128\,b}+\frac{73\,x^3}{384\,a}-\frac{55\,b\,x^5}{384\,a^2}+\frac{5\,b^2\,x^7}{128\,a^3}}{a^4-4\,a^3\,b\,x^2+6\,a^2\,b^2\,x^4-4\,a\,b^3\,x^6+b^4\,x^8}-\frac{5\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128\,a^{7/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2)^5,x)
```

```
[Out] ((5*x)/(128*b) + (73*x^3)/(384*a) - (55*b*x^5)/(384*a^2) + (5*b^2*x^7)/(128
*a^3))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4) - (5*ata
nh((b^(1/2)*x)/a^(1/2)))/(128*a^(7/2)*b^(3/2))
```

sympy [A] time = 0.50, size = 160, normalized size = 1.47

$$\frac{5\sqrt{\frac{1}{a^7b^3}}\,\log\left(-a^4b\sqrt{\frac{1}{a^7b^3}}+x\right)}{256}-\frac{5\sqrt{\frac{1}{a^7b^3}}\,\log\left(a^4b\sqrt{\frac{1}{a^7b^3}}+x\right)}{256}-\frac{-15a^3x-73a^2bx^3+55ab^2x^5-15b^3x^7}{384a^7b-1536a^6b^2x^2+2304a^5b^3x^4-1536a^4b^4x^6+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**5,x)
```

```
[Out] 5*sqrt(1/(a**7*b**3))*log(-a**4*b*sqrt(1/(a**7*b**3)) + x)/256 - 5*sqrt(1/(
a**7*b**3))*log(a**4*b*sqrt(1/(a**7*b**3)) + x)/256 - (-15*a**3*x - 73*a**2
*b*x**3 + 55*a*b**2*x**5 - 15*b**3*x**7)/(384*a**7*b - 1536*a**6*b**2*x**2
+ 2304*a**5*b**3*x**4 - 1536*a**4*b**4*x**6 + 384*a**3*b**5*x**8)
```

3.248 $\int \frac{x}{(a-bx^2)^5}\,dx$

Optimal. Leaf size=17

$$\frac{1}{8b(a-bx^2)^4}$$

```
[Out] 1/8/b/(-b*x^2+a)^4
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {261}

$$\frac{1}{8b(a-bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^2)^5,x]
```

```
[Out] 1/(8*b*(a - b*x^2)^4)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a-bx^2)^5}\,dx = \frac{1}{8b(a-bx^2)^4}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{8b(a-bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^2)^5,x]
```

```
[Out] 1/(8*b*(a - b*x^2)^4)
```

fricas [B] time = 0.81, size = 48, normalized size = 2.82

$$\frac{1}{8\left(b^5x^8 - 4\,ab^4x^6 + 6\,a^2b^3x^4 - 4\,a^3b^2x^2 + a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/8/(b^5*x^8 - 4*a*b^4*x^6 + 6*a^2*b^3*x^4 - 4*a^3*b^2*x^2 + a^4*b)
```

giac [A] time = 0.62, size = 16, normalized size = 0.94

$$\frac{1}{8(bx^2-a)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/8/((b*x^2 - a)^4*b)
```

maple [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{8\left(b\,x^{2}-a\right)^{4} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^5,x)
```

```
[Out] 1/8/b/(b*x^2-a)^4
```

maxima [A] time = 1.31, size = 16, normalized size = 0.94

$$\frac{1}{8\left(bx^{2}-a\right)^{4} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/8/((b*x^2 - a)^4*b)
```

mupad [B] time = 4.69, size = 48, normalized size = 2.82

$$\frac{1}{8\,a^{4}\,b-32\,a^{3}\,b^{2}\,x^{2}+48\,a^{2}\,b^{3}\,x^{4}-32\,a\,b^{4}\,x^{6}+8\,b^{5}\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2)^5,x)
```

```
[Out] 1/(8*a^4*b + 8*b^5*x^8 - 32*a*b^4*x^6 - 32*a^3*b^2*x^2 + 48*a^2*b^3*x^4)
```

sympy [B] time = 0.40, size = 49, normalized size = 2.88

$$\frac{1}{8a^{4}b-32a^{3}b^{2}x^{2}+48a^{2}b^{3}x^{4}-32ab^{4}x^{6}+8b^{5}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**5,x)
```

```
[Out] 1/(8*a**4*b - 32*a**3*b**2*x**2 + 48*a**2*b**3*x**4 - 32*a*b**4*x**6 + 8*b*
*5*x**8)
```

3.249 $\int \frac{1}{\left(a-bx^2\right)^5} dx$

Optimal. Leaf size=100

$$\frac{35 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}}+\frac{35x}{128a^4\left(a-bx^2\right)}+\frac{35x}{192a^3\left(a-bx^2\right)^2}+\frac{7x}{48a^2\left(a-bx^2\right)^3}+\frac{x}{8a\left(a-bx^2\right)^4}$$

```
[Out] 1/8*x/a/(-b*x^2+a)^4+7/48*x/a^2/(-b*x^2+a)^3+35/192*x/a^3/(-b*x^2+a)^2+35/1
28*x/a^4/(-b*x^2+a)+35/128*arctanh(x*b^(1/2)/a^(1/2))/a^(9/2)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {199, 208}

$$\frac{35x}{128a^4\left(a-bx^2\right)}+\frac{35x}{192a^3\left(a-bx^2\right)^2}+\frac{7x}{48a^2\left(a-bx^2\right)^3}+\frac{35 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}}+\frac{x}{8a\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-5),x]
```

```
[Out] x/(8*a*(a - b*x^2)^4) + (7*x)/(48*a^2*(a - b*x^2)^3) + (35*x)/(192*a^3*(a -
 b*x^2)^2) + (35*x)/(128*a^4*(a - b*x^2)) + (35*ArcTanh[(Sqrt[b]*x)/Sqrt[a]
])/(128*a^(9/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^5}\,dx = \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7\int \frac{1}{\left(a-bx^2\right)^4}\,dx}{8a}$$
$$= \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7x}{48a^2\left(a-bx^2\right)^3} + \frac{35\int \frac{1}{\left(a-bx^2\right)^3}\,dx}{48a^2}$$
$$= \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7x}{48a^2\left(a-bx^2\right)^3} + \frac{35x}{192a^3\left(a-bx^2\right)^2} + \frac{35\int \frac{1}{\left(a-bx^2\right)^2}\,dx}{64a^3}$$
$$= \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7x}{48a^2\left(a-bx^2\right)^3} + \frac{35x}{192a^3\left(a-bx^2\right)^2} + \frac{35x}{128a^4\left(a-bx^2\right)} + \frac{35\int \frac{1}{a-bx^2}\,dx}{128a^4}$$
$$= \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7x}{48a^2\left(a-bx^2\right)^3} + \frac{35x}{192a^3\left(a-bx^2\right)^2} + \frac{35x}{128a^4\left(a-bx^2\right)} + \frac{35\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.79

$$\frac{\frac{\sqrt{a}\,x\left(279a^3-511a^2bx^2+385ab^2x^4-105b^3x^6\right)}{\left(a-bx^2\right)^4}+\frac{105\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}}{384a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-5),x]
```

```
[Out] ((Sqrt[a]*x*(279*a^3 - 511*a^2*b*x^2 + 385*a*b^2*x^4 - 105*b^3*x^6))/(a - b
*x^2)^4 + (105*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(384*a^(9/2))
```

fricas [A] time = 0.92, size = 320, normalized size = 3.20

$$\left[-\frac{210\,ab^4x^7 - 770\,a^2b^3x^5 + 1022\,a^3b^2x^3 - 558\,a^4bx - 105\left(b^4x^8 - 4\,ab^3x^6 + 6\,a^2b^2x^4 - 4\,a^3bx^2 + a^4\right)\sqrt{ab}\,\log\left(\frac{bx}{}\right.}{768\left(a^5b^5x^8 - 4\,a^6b^4x^6 + 6\,a^7b^3x^4 - 4\,a^8b^2x^2 + a^9b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/768*(210*a*b^4*x^7 - 770*a^2*b^3*x^5 + 1022*a^3*b^2*x^3 - 558*a^4*b*x -
 105*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(a*b)*
log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^5*b^5*x^8 - 4*a^6*b^4*x^6
+ 6*a^7*b^3*x^4 - 4*a^8*b^2*x^2 + a^9*b), -1/384*(105*a*b^4*x^7 - 385*a^2*b
^3*x^5 + 511*a^3*b^2*x^3 - 279*a^4*b*x + 105*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2
*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^5*b^5*x
^8 - 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 - 4*a^8*b^2*x^2 + a^9*b)]
```

giac [A] time = 0.62, size = 71, normalized size = 0.71

$$-\frac{35\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128\sqrt{-ab}\,a^4} - \frac{105\,b^3x^7 - 385\,ab^2x^5 + 511\,a^2bx^3 - 279\,a^3x}{384\left(bx^2-a\right)^4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] -35/128*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^4) - 1/384*(105*b^3*x^7 - 385*
a*b^2*x^5 + 511*a^2*b*x^3 - 279*a^3*x)/((b*x^2 - a)^4*a^4)
```

maple [A] time = 0.00, size = 107, normalized size = 1.07

$$\frac{x}{8\left(b x^2-a\right)^4 a}+\frac{-\frac{7x}{48\left(bx^2-a\right)^3 a}-\frac{35\left(-\frac{x}{4\left(bx^2-a\right)^2 a}-\frac{3\left(-\frac{x}{2\left(bx^2-a\right)a}+\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}\right)}{4a}\right)}{48a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^5,x)
```

```
[Out] 1/8*x/a/(b*x^2-a)^4+7/8/a*(-1/6*x/a/(b*x^2-a)^3-5/6/a*(-1/4/(b*x^2-a)^2/a*x
-3/4/a*(-1/2/(b*x^2-a)/a*x+1/2/a/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x))))
```

maxima [A] time = 2.90, size = 117, normalized size = 1.17

$$-\frac{105\,b^3x^7-385\,ab^2x^5+511\,a^2bx^3-279\,a^3x}{384\left(a^4b^4x^8-4\,a^5b^3x^6+6\,a^6b^2x^4-4\,a^7bx^2+a^8\right)}-\frac{35\,\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] -1/384*(105*b^3*x^7 - 385*a*b^2*x^5 + 511*a^2*b*x^3 - 279*a^3*x)/(a^4*b^4*x
^8 - 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 - 4*a^7*b*x^2 + a^8) - 35/256*log((b*x -
 sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^4)
```

mupad [B] time = 4.79, size = 99, normalized size = 0.99

$$\frac{\frac{93\,x}{128\,a}-\frac{511\,b\,x^3}{384\,a^2}+\frac{385\,b^2\,x^5}{384\,a^3}-\frac{35\,b^3\,x^7}{128\,a^4}}{a^4-4\,a^3\,b\,x^2+6\,a^2\,b^2\,x^4-4\,a\,b^3\,x^6+b^4\,x^8}+\frac{35\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128\,a^{9/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^5,x)
```

```
[Out] ((93*x)/(128*a) - (511*b*x^3)/(384*a^2) + (385*b^2*x^5)/(384*a^3) - (35*b^3
*x^7)/(128*a^4))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4
) + (35*atanh((b^(1/2)*x)/a^(1/2)))/(128*a^(9/2)*b^(1/2))
```

sympy [A] time = 0.52, size = 146, normalized size = 1.46

$$-\frac{35\sqrt{\frac{1}{a^9b}}\,\log\left(-a^5\sqrt{\frac{1}{a^9b}}+x\right)}{256}+\frac{35\sqrt{\frac{1}{a^9b}}\,\log\left(a^5\sqrt{\frac{1}{a^9b}}+x\right)}{256}-\frac{-279a^3x+511a^2bx^3-385ab^2x^5+105b^3}{384a^8-1536a^7bx^2+2304a^6b^2x^4-1536a^5b^3x^6+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**5,x)
```

```
[Out] -35*sqrt(1/(a**9*b))*log(-a**5*sqrt(1/(a**9*b)) + x)/256 + 35*sqrt(1/(a**9*
b))*log(a**5*sqrt(1/(a**9*b)) + x)/256 - (-279*a**3*x + 511*a**2*b*x**3 - 3
85*a*b**2*x**5 + 105*b**3*x**7)/(384*a**8 - 1536*a**7*b*x**2 + 2304*a**6*b*
*2*x**4 - 1536*a**5*b**3*x**6 + 384*a**4*b**4*x**8)
```

3.250 $\int \frac{1}{x\left(a-bx^2\right)^5} dx$

Optimal. Leaf size=91

$$-\frac{\log\left(a-bx^2\right)}{2a^5}+\frac{\log(x)}{a^5}+\frac{1}{2a^4\left(a-bx^2\right)}+\frac{1}{4a^3\left(a-bx^2\right)^2}+\frac{1}{6a^2\left(a-bx^2\right)^3}+\frac{1}{8a\left(a-bx^2\right)^4}$$

```
[Out] 1/8/a/(-b*x^2+a)^4+1/6/a^2/(-b*x^2+a)^3+1/4/a^3/(-b*x^2+a)^2+1/2/a^4/(-b*x^
2+a)+ln(x)/a^5-1/2*ln(-b*x^2+a)/a^5
```

Rubi [A] time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{1}{2a^4\left(a-bx^2\right)}+\frac{1}{4a^3\left(a-bx^2\right)^2}+\frac{1}{6a^2\left(a-bx^2\right)^3}-\frac{\log\left(a-bx^2\right)}{2a^5}+\frac{\log(x)}{a^5}+\frac{1}{8a\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2)^5),x]
```

```
[Out] 1/(8*a*(a - b*x^2)^4) + 1/(6*a^2*(a - b*x^2)^3) + 1/(4*a^3*(a - b*x^2)^2) +
 1/(2*a^4*(a - b*x^2)) + Log[x]/a^5 - Log[a - b*x^2]/(2*a^5)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a-bx^2\right)^5} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a-bx)^5} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^5x}+\frac{b}{a(a-bx)^5}+\frac{b}{a^2(a-bx)^4}+\frac{b}{a^3(a-bx)^3}+\frac{b}{a^4(a-bx)^2}+\frac{b}{a^5(a-bx)}\right) dx\right.\\
&= \frac{1}{8a\left(a-bx^2\right)^4}+\frac{1}{6a^2\left(a-bx^2\right)^3}+\frac{1}{4a^3\left(a-bx^2\right)^2}+\frac{1}{2a^4\left(a-bx^2\right)}+\frac{\log(x)}{a^5}-\frac{\log\left(a-bx^2\right)}{2a^5}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.74

$$\frac{\frac{a\left(25a^3-52a^2bx^2+42ab^2x^4-12b^3x^6\right)}{\left(a-bx^2\right)^4}-12\log\left(a-bx^2\right)+24\log(x)}{24a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)^5),x]
```

```
[Out] ((a*(25*a^3 - 52*a^2*b*x^2 + 42*a*b^2*x^4 - 12*b^3*x^6))/(a - b*x^2)^4 + 24
*Log[x] - 12*Log[a - b*x^2])/(24*a^5)
```

fricas [B] time = 0.84, size = 180, normalized size = 1.98

$$-\frac{12\,ab^3x^6-42\,a^2b^2x^4+52\,a^3bx^2-25\,a^4+12\left(b^4x^8-4\,ab^3x^6+6\,a^2b^2x^4-4\,a^3bx^2+a^4\right)\log\left(bx^2-a\right)-24\,(b}{24\left(a^5b^4x^8-4\,a^6b^3x^6+6\,a^7b^2x^4-4\,a^8bx^2+a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(12*a*b^3*x^6 - 42*a^2*b^2*x^4 + 52*a^3*b*x^2 - 25*a^4 + 12*(b^4*x^8
- 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*log(b*x^2 - a) - 24*(b^4
*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*log(x))/(a^5*b^4*x^
8 - 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 - 4*a^8*b*x^2 + a^9)
```

giac [A] time = 0.61, size = 85, normalized size = 0.93

$$\frac{\log\left(x^2\right)}{2\,a^5}-\frac{\log\left(\left|bx^2-a\right|\right)}{2\,a^5}+\frac{25\,b^4x^8-112\,ab^3x^6+192\,a^2b^2x^4-152\,a^3bx^2+50\,a^4}{24\left(bx^2-a\right)^4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^5 - 1/2*log(abs(b*x^2 - a))/a^5 + 1/24*(25*b^4*x^8 - 112*a*b
^3*x^6 + 192*a^2*b^2*x^4 - 152*a^3*b*x^2 + 50*a^4)/((b*x^2 - a)^4*a^5)
```

maple [A] time = 0.01, size = 87, normalized size = 0.96

$$\frac{1}{8\left(b\,x^2-a\right)^4a}-\frac{1}{6\left(b\,x^2-a\right)^3a^2}+\frac{1}{4\left(b\,x^2-a\right)^2a^3}-\frac{1}{2\left(b\,x^2-a\right)a^4}+\frac{\ln(x)}{a^5}-\frac{\ln\left(b\,x^2-a\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a)^5,x)
```

```
[Out] ln(x)/a^5-1/2/a^5*ln(b*x^2-a)-1/6/a^2/(b*x^2-a)^3+1/4/a^3/(b*x^2-a)^2+1/8/a
/(b*x^2-a)^4-1/2/a^4/(b*x^2-a)
```

maxima [A] time = 1.39, size = 106, normalized size = 1.16

$$-\frac{12\,b^3x^6-42\,ab^2x^4+52\,a^2bx^2-25\,a^3}{24\left(a^4b^4x^8-4\,a^5b^3x^6+6\,a^6b^2x^4-4\,a^7bx^2+a^8\right)}-\frac{\log\left(bx^2-a\right)}{2\,a^5}+\frac{\log\left(x^2\right)}{2\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(12*b^3*x^6 - 42*a*b^2*x^4 + 52*a^2*b*x^2 - 25*a^3)/(a^4*b^4*x^8 - 4*
a^5*b^3*x^6 + 6*a^6*b^2*x^4 - 4*a^7*b*x^2 + a^8) - 1/2*log(b*x^2 - a)/a^5 +
 1/2*log(x^2)/a^5
```

mupad [B] time = 5.24, size = 101, normalized size = 1.11

$$\frac{\ln(x)}{a^5}+\frac{\frac{25}{24\,a}-\frac{13\,b\,x^2}{6\,a^2}+\frac{7\,b^2\,x^4}{4\,a^3}-\frac{b^3\,x^6}{2\,a^4}}{a^4-4\,a^3\,b\,x^2+6\,a^2\,b^2\,x^4-4\,a\,b^3\,x^6+b^4\,x^8}-\frac{\ln\left(a-b\,x^2\right)}{2\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2)^5),x)
```

```
[Out] log(x)/a^5 + (25/(24*a) - (13*b*x^2)/(6*a^2) + (7*b^2*x^4)/(4*a^3) - (b^3*x
^6)/(2*a^4))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4) -
log(a - b*x^2)/(2*a^5)
```

sympy [A] time = 0.64, size = 104, normalized size = 1.14

$$-\frac{-25a^3 + 52a^2bx^2 - 42ab^2x^4 + 12b^3x^6}{24a^8 - 96a^7bx^2 + 144a^6b^2x^4 - 96a^5b^3x^6 + 24a^4b^4x^8} + \frac{\log(x)}{a^5} - \frac{\log\left(-\frac{a}{b} + x^2\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a)**5,x)
```

```
[Out] -(-25*a**3 + 52*a**2*b*x**2 - 42*a*b**2*x**4 + 12*b**3*x**6)/(24*a**8 - 96*
a**7*b*x**2 + 144*a**6*b**2*x**4 - 96*a**5*b**3*x**6 + 24*a**4*b**4*x**8) +
 log(x)/a**5 - log(-a/b + x**2)/(2*a**5)
```

3.251 $\int \frac{1}{x^2(a-bx^2)^5}\,dx$

Optimal. Leaf size=118

$$\frac{315\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}}-\frac{315}{128a^5x}+\frac{105}{128a^4x\left(a-bx^2\right)}+\frac{21}{64a^3x\left(a-bx^2\right)^2}+\frac{3}{16a^2x\left(a-bx^2\right)^3}+\frac{1}{8ax\left(a-bx^2\right)^4}$$

```
[Out] -315/128/a^5/x+1/8/a/x/(-b*x^2+a)^4+3/16/a^2/x/(-b*x^2+a)^3+21/64/a^3/x/(-b
*x^2+a)^2+105/128/a^4/x/(-b*x^2+a)+315/128*arctanh(x*b^(1/2)/a^(1/2))*b^(1/
2)/a^(11/2)
```

Rubi [A] time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {290, 325, 208}

$$\frac{105}{128a^4x\left(a-bx^2\right)}+\frac{21}{64a^3x\left(a-bx^2\right)^2}+\frac{3}{16a^2x\left(a-bx^2\right)^3}+\frac{315\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}}-\frac{315}{128a^5x}+\frac{1}{8ax\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^5),x]
```

```
[Out] -315/(128*a^5*x) + 1/(8*a*x*(a - b*x^2)^4) + 3/(16*a^2*x*(a - b*x^2)^3) + 2
1/(64*a^3*x*(a - b*x^2)^2) + 105/(128*a^4*x*(a - b*x^2)) + (315*Sqrt[b]*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(11/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a-bx^2\right)^5}\,dx = \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{9\int \frac{1}{x^2\left(a-bx^2\right)^4}\,dx}{8a}$$

$$= \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21\int \frac{1}{x^2\left(a-bx^2\right)^3}\,dx}{16a^2}$$

$$= \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21}{64a^3x\left(a-bx^2\right)^2} + \frac{105\int \frac{1}{x^2\left(a-bx^2\right)^2}\,dx}{64a^3}$$

$$= \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21}{64a^3x\left(a-bx^2\right)^2} + \frac{105}{128a^4x\left(a-bx^2\right)} + \frac{315\int \frac{1}{x^2(a-bx^2}}{128a^4}$$

$$= -\frac{315}{128a^5x} + \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21}{64a^3x\left(a-bx^2\right)^2} + \frac{105}{128a^4x\left(a-bx^2\right)} + (3$$

$$= -\frac{315}{128a^5x} + \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21}{64a^3x\left(a-bx^2\right)^2} + \frac{105}{128a^4x\left(a-bx^2\right)} + 3$$

Mathematica [A] time = 0.06, size = 92, normalized size = 0.78

$$\frac{\frac{\sqrt{a}\left(-128a^4+837a^3bx^2-1533a^2b^2x^4+1155ab^3x^6-315b^4x^8\right)}{x\left(a-bx^2\right)^4}+315\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^5),x]
```

```
[Out] ((Sqrt[a]*(-128*a^4 + 837*a^3*b*x^2 - 1533*a^2*b^2*x^4 + 1155*a*b^3*x^6 - 3
15*b^4*x^8))/(x*(a - b*x^2)^4) + 315*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/
(128*a^(11/2))
```

fricas [A] time = 0.82, size = 334, normalized size = 2.83

$$\left[-\frac{630\,b^4x^8 - 2310\,ab^3x^6 + 3066\,a^2b^2x^4 - 1674\,a^3bx^2 + 256\,a^4 - 315\left(b^4x^9 - 4\,ab^3x^7 + 6\,a^2b^2x^5 - 4\,a^3bx^3 + a^4x\right)}{256\left(a^5b^4x^9 - 4\,a^6b^3x^7 + 6\,a^7b^2x^5 - 4\,a^8bx^3 + a^9x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/256*(630*b^4*x^8 - 2310*a*b^3*x^6 + 3066*a^2*b^2*x^4 - 1674*a^3*b*x^2 +
 256*a^4 - 315*(b^4*x^9 - 4*a*b^3*x^7 + 6*a^2*b^2*x^5 - 4*a^3*b*x^3 + a^4*x
)*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)))/(a^5*b^4*x^9 -
4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 - 4*a^8*b*x^3 + a^9*x), -1/128*(315*b^4*x^8 -
 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 - 837*a^3*b*x^2 + 128*a^4 + 315*(b^4*x^9
 - 4*a*b^3*x^7 + 6*a^2*b^2*x^5 - 4*a^3*b*x^3 + a^4*x)*sqrt(-b/a)*arctan(x*s
qrt(-b/a)))/(a^5*b^4*x^9 - 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 - 4*a^8*b*x^3 + a^
9*x)]
```

giac [A] time = 0.63, size = 83, normalized size = 0.70

$$-\frac{315\,b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128\,\sqrt{-ab}\,a^5}-\frac{1}{a^5x}-\frac{187\,b^4x^7-643\,ab^3x^5+765\,a^2b^2x^3-325\,a^3bx}{128\,\left(bx^2-a\right)^4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] -315/128*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^5) - 1/(a^5*x) - 1/128*(187
*b^4*x^7 - 643*a*b^3*x^5 + 765*a^2*b^2*x^3 - 325*a^3*b*x)/((b*x^2 - a)^4*a^
5)
```

maple [A] time = 0.02, size = 78, normalized size = 0.66

$$-\frac{\left(-\frac{315\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}}+\frac{\frac{187}{128}b^3x^7-\frac{643}{128}a\,b^2x^5+\frac{765}{128}a^2b\,x^3-\frac{325}{128}a^3x}{\left(b\,x^2-a\right)^4}\right)b}{a^5}-\frac{1}{a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^5,x)
```

```
[Out] -1/a^5/x-1/a^5*b*((187/128*b^3*x^7-643/128*a*b^2*x^5+765/128*a^2*b*x^3-325/
128*a^3*x)/(b*x^2-a)^4-315/128/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x))
```

maxima [A] time = 2.90, size = 130, normalized size = 1.10

$$-\frac{315\,b^4x^8-1155\,ab^3x^6+1533\,a^2b^2x^4-837\,a^3bx^2+128\,a^4}{128\,\left(a^5b^4x^9-4\,a^6b^3x^7+6\,a^7b^2x^5-4\,a^8bx^3+a^9x\right)}-\frac{315\,b\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] -1/128*(315*b^4*x^8 - 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 - 837*a^3*b*x^2 + 1
28*a^4)/(a^5*b^4*x^9 - 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 - 4*a^8*b*x^3 + a^9*x)
 - 315/256*b*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^5)
```

mupad [B] time = 5.17, size = 110, normalized size = 0.93

$$\frac{315\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128\,a^{11/2}}-\frac{\frac{1}{a}-\frac{837\,b\,x^2}{128\,a^2}+\frac{1533\,b^2\,x^4}{128\,a^3}-\frac{1155\,b^3\,x^6}{128\,a^4}+\frac{315\,b^4\,x^8}{128\,a^5}}{a^4\,x-4\,a^3\,b\,x^3+6\,a^2\,b^2\,x^5-4\,a\,b^3\,x^7+b^4\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^5),x)
```

```
[Out] (315*b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/(128*a^(11/2)) - (1/a - (837*b*x^2
)/(128*a^2) + (1533*b^2*x^4)/(128*a^3) - (1155*b^3*x^6)/(128*a^4) + (315*b^
4*x^8)/(128*a^5))/(a^4*x + b^4*x^9 - 4*a^3*b*x^3 - 4*a*b^3*x^7 + 6*a^2*b^2*
x^5)
```

sympy [A] time = 0.66, size = 155, normalized size = 1.31

$$-\frac{315\sqrt{\frac{b}{a^{11}}}\log\left(-\frac{a^6\sqrt{\frac{b}{a^{11}}}}{b}+x\right)}{256}+\frac{315\sqrt{\frac{b}{a^{11}}}\log\left(\frac{a^6\sqrt{\frac{b}{a^{11}}}}{b}+x\right)}{256}-\frac{128a^4-837a^3bx^2+1533a^2b^2x^4-1155ab^3x^6+}{128a^9x-512a^8bx^3+768a^7b^2x^5-512a^6b^3x^7+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**5,x)
```

```
[Out] -315*sqrt(b/a**11)*log(-a**6*sqrt(b/a**11)/b + x)/256 + 315*sqrt(b/a**11)*l
og(a**6*sqrt(b/a**11)/b + x)/256 - (128*a**4 - 837*a**3*b*x**2 + 1533*a**2*
b**2*x**4 - 1155*a*b**3*x**6 + 315*b**4*x**8)/(128*a**9*x - 512*a**8*b*x**3
 + 768*a**7*b**2*x**5 - 512*a**6*b**3*x**7 + 128*a**5*b**4*x**9)
```

3.252 $\int \frac{1}{x^3 \left(a-bx^2\right)^5} \, dx$

Optimal. Leaf size=106

$$-\frac{5b \log \left(a-bx^2\right)}{2a^6}+\frac{5b \log(x)}{a^6}+\frac{2b}{a^5 \left(a-bx^2\right)}-\frac{1}{2a^5x^2}+\frac{3b}{4a^4 \left(a-bx^2\right)^2}+\frac{b}{3a^3 \left(a-bx^2\right)^3}+\frac{b}{8a^2 \left(a-bx^2\right)^4}$$

```
[Out] -1/2/a^5/x^2+1/8*b/a^2/(-b*x^2+a)^4+1/3*b/a^3/(-b*x^2+a)^3+3/4*b/a^4/(-b*x^
2+a)^2+2*b/a^5/(-b*x^2+a)+5*b*ln(x)/a^6-5/2*b*ln(-b*x^2+a)/a^6
```

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{2b}{a^5 \left(a-bx^2\right)}+\frac{3b}{4a^4 \left(a-bx^2\right)^2}+\frac{b}{3a^3 \left(a-bx^2\right)^3}+\frac{b}{8a^2 \left(a-bx^2\right)^4}-\frac{5b \log \left(a-bx^2\right)}{2a^6}+\frac{5b \log(x)}{a^6}-\frac{1}{2a^5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)^5),x]
```

```
[Out] -1/(2*a^5*x^2) + b/(8*a^2*(a - b*x^2)^4) + b/(3*a^3*(a - b*x^2)^3) + (3*b)/
(4*a^4*(a - b*x^2)^2) + (2*b)/(a^5*(a - b*x^2)) + (5*b*Log[x])/a^6 - (5*b*L
og[a - b*x^2])/(2*a^6)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3 \left(a-bx^2\right)^5} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{x^2(a-bx)^5} \, dx, x, x^2\right)\\ &= \frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{a^5x^2}+\frac{5b}{a^6x}+\frac{b^2}{a^2(a-bx)^5}+\frac{2b^2}{a^3(a-bx)^4}+\frac{3b^2}{a^4(a-bx)^3}+\frac{4b^2}{a^5(a-bx)^2}+\right.\right.\\ &= -\frac{1}{2a^5x^2}+\frac{b}{8a^2 \left(a-bx^2\right)^4}+\frac{b}{3a^3 \left(a-bx^2\right)^3}+\frac{3b}{4a^4 \left(a-bx^2\right)^2}+\frac{2b}{a^5 \left(a-bx^2\right)}+\frac{5b \log(x)}{a^6}\end{aligned}$$

Mathematica [A] time = 0.07, size = 83, normalized size = 0.78

$$\frac{\frac{a\left(-12a^4+125a^3bx^2-260a^2b^2x^4+210ab^3x^6-60b^4x^8\right)}{x^2\left(a-bx^2\right)^4}-60b \log \left(a-bx^2\right)+120b \log(x)}{24a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)^5),x]
```

```
[Out] ((a*(-12*a^4 + 125*a^3*b*x^2 - 260*a^2*b^2*x^4 + 210*a*b^3*x^6 - 60*b^4*x^8
))/(x^2*(a - b*x^2)^4) + 120*b*Log[x] - 60*b*Log[a - b*x^2])/(24*a^6)
```

fricas [B] time = 0.72, size = 209, normalized size = 1.97

$$-\frac{60\,ab^4x^8 - 210\,a^2b^3x^6 + 260\,a^3b^2x^4 - 125\,a^4bx^2 + 12\,a^5 + 60\left(b^5x^{10} - 4\,ab^4x^8 + 6\,a^2b^3x^6 - 4\,a^3b^2x^4 + a^4bx^2\right)\mathrm{lo}}{24\left(a^6b^4x^{10} - 4\,a^7b^3x^8 + 6\,a^8b^2x^6 - 4\,a^9bx^4 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(60*a*b^4*x^8 - 210*a^2*b^3*x^6 + 260*a^3*b^2*x^4 - 125*a^4*b*x^2 + 1
2*a^5 + 60*(b^5*x^10 - 4*a*b^4*x^8 + 6*a^2*b^3*x^6 - 4*a^3*b^2*x^4 + a^4*b*
x^2)*log(b*x^2 - a) - 120*(b^5*x^10 - 4*a*b^4*x^8 + 6*a^2*b^3*x^6 - 4*a^3*b
^2*x^4 + a^4*b*x^2)*log(x))/(a^6*b^4*x^10 - 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 -
 4*a^9*b*x^4 + a^10*x^2)
```

giac [A] time = 0.64, size = 106, normalized size = 1.00

$$\frac{5\,b\log\left(x^2\right)}{2\,a^6} - \frac{5\,b\log\left(\left|bx^2 - a\right|\right)}{2\,a^6} - \frac{5\,bx^2 + a}{2\,a^6x^2} + \frac{125\,b^5x^8 - 548\,ab^4x^6 + 912\,a^2b^3x^4 - 688\,a^3b^2x^2 + 202\,a^4b}{24\left(bx^2 - a\right)^4a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 5/2*b*log(x^2)/a^6 - 5/2*b*log(abs(b*x^2 - a))/a^6 - 1/2*(5*b*x^2 + a)/(a^6
*x^2) + 1/24*(125*b^5*x^8 - 548*a*b^4*x^6 + 912*a^2*b^3*x^4 - 688*a^3*b^2*x
^2 + 202*a^4*b)/((b*x^2 - a)^4*a^6)
```

maple [A] time = 0.02, size = 102, normalized size = 0.96

$$\frac{b}{8\left(b\,x^2 - a\right)^4a^2} - \frac{b}{3\left(b\,x^2 - a\right)^3a^3} + \frac{3b}{4\left(b\,x^2 - a\right)^2a^4} - \frac{2b}{\left(b\,x^2 - a\right)a^5} + \frac{5b\ln(x)}{a^6} - \frac{5b\ln\left(b\,x^2 - a\right)}{2a^6} - \frac{1}{2a^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a)^5,x)
```

```
[Out] -1/2/a^5/x^2+5*b*ln(x)/a^6-1/3/a^3*b/(b*x^2-a)^3-5/2/a^6*b*ln(b*x^2-a)+3/4/
a^4*b/(b*x^2-a)^2+1/8/a^2*b/(b*x^2-a)^4-2/a^5*b/(b*x^2-a)
```

maxima [A] time = 1.35, size = 123, normalized size = 1.16

$$-\frac{60\,b^4x^8 - 210\,ab^3x^6 + 260\,a^2b^2x^4 - 125\,a^3bx^2 + 12\,a^4}{24\left(a^5b^4x^{10} - 4\,a^6b^3x^8 + 6\,a^7b^2x^6 - 4\,a^8bx^4 + a^9x^2\right)} - \frac{5\,b\log\left(bx^2 - a\right)}{2\,a^6} + \frac{5\,b\log\left(x^2\right)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(60*b^4*x^8 - 210*a*b^3*x^6 + 260*a^2*b^2*x^4 - 125*a^3*b*x^2 + 12*a^
4)/(a^5*b^4*x^10 - 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6 - 4*a^8*b*x^4 + a^9*x^2) -
 5/2*b*log(b*x^2 - a)/a^6 + 5/2*b*log(x^2)/a^6
```

mupad [B] time = 5.35, size = 120, normalized size = 1.13

$$\frac{5\,b\,\ln(x)}{a^6}-\frac{5\,b\,\ln\left(a-b\,x^2\right)}{2\,a^6}-\frac{\frac{1}{2\,a}-\frac{125\,b\,x^2}{24\,a^2}+\frac{65\,b^2\,x^4}{6\,a^3}-\frac{35\,b^3\,x^6}{4\,a^4}+\frac{5\,b^4\,x^8}{2\,a^5}}{a^4\,x^2-4\,a^3\,b\,x^4+6\,a^2\,b^2\,x^6-4\,a\,b^3\,x^8+b^4\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2)^5),x)
```

```
[Out] (5*b*log(x))/a^6 - (5*b*log(a - b*x^2))/(2*a^6) - (1/(2*a) - (125*b*x^2)/(2
4*a^2) + (65*b^2*x^4)/(6*a^3) - (35*b^3*x^6)/(4*a^4) + (5*b^4*x^8)/(2*a^5))
/(a^4*x^2 + b^4*x^10 - 4*a^3*b*x^4 - 4*a*b^3*x^8 + 6*a^2*b^2*x^6)
```

sympy [A] time = 0.76, size = 126, normalized size = 1.19

$$-\frac{12a^4-125a^3bx^2+260a^2b^2x^4-210ab^3x^6+60b^4x^8}{24a^9x^2-96a^8bx^4+144a^7b^2x^6-96a^6b^3x^8+24a^5b^4x^{10}}+\frac{5b\log{(x)}}{a^6}-\frac{5b\log{\left(-\frac{a}{b}+x^2\right)}}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a)**5,x)
```

```
[Out] -(12*a**4 - 125*a**3*b*x**2 + 260*a**2*b**2*x**4 - 210*a*b**3*x**6 + 60*b**
4*x**8)/(24*a**9*x**2 - 96*a**8*b*x**4 + 144*a**7*b**2*x**6 - 96*a**6*b**3*
x**8 + 24*a**5*b**4*x**10) + 5*b*log(x)/a**6 - 5*b*log(-a/b + x**2)/(2*a**6
)
```

3.253 $\int \frac{1}{x(1+bx^2)}\,dx$

Optimal. Leaf size=15

$$\log(x)-\frac{1}{2}\log\left(bx^2+1\right)$$

```
[Out] ln(x)-1/2*ln(b*x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 36, 29, 31}

$$\log(x)-\frac{1}{2}\log\left(bx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + b*x^2)),x]
```

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(1+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)-\frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{1+bx}\,dx,x,x^2\right)\\
&= \log(x)-\frac{1}{2}\log\left(1+bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\log(x)-\frac{1}{2}\log\left(bx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + b*x^2)),x]
```

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

fricas [A] time = 1.30, size = 13, normalized size = 0.87

$$-\frac{1}{2}\log\left(bx^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*log(b*x^2 + 1) + log(x)
```

giac [A] time = 0.62, size = 18, normalized size = 1.20

$$\frac{1}{2}\log\left(x^2\right)-\frac{1}{2}\log\left(\left|bx^2+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\ln(x)-\frac{\ln\left(b\,x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+1),x)
```

```
[Out] ln(x)-1/2*ln(b*x^2+1)
```

maxima [A] time = 1.35, size = 17, normalized size = 1.13

$$-\frac{1}{2}\log\left(bx^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x^2 + 1) + 1/2*log(x^2)
```

mupad [B] time = 5.11, size = 14, normalized size = 0.93

$$\ln(x)-\frac{\ln\left(\frac{3\,b\,x^2}{2}+\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + 1)),x)
```

```
[Out] log(x) - log((3*b*x^2)/2 + 3/2)/2
```

sympy [A] time = 0.13, size = 12, normalized size = 0.80

$$\log{\left(x \right)}-\frac{\log{\left(x^{2}+\frac{1}{b} \right)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+1),x)
```

```
[Out] log(x) - log(x**2 + 1/b)/2
```

3.254 $\int \frac{1}{x(-1+bx^2)}\,dx$

Optimal. Leaf size=18

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

[Out] -ln(x)+1/2*ln(-b*x^2+1)

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 36, 29, 31}

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(-1 + b*x^2)),x]

[Out] -Log[x] + Log[1 - b*x^2]/2

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(-1+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(-1+bx)}\,dx,x,x^2\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)\right)+\frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{-1+bx}\,dx,x,x^2\right)\\ &= -\log(x)+\frac{1}{2}\log\left(1-bx^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + b*x^2)),x]
```

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

fricas [A] time = 0.96, size = 15, normalized size = 0.83

$$\frac{1}{2}\log\left(bx^2-1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 - 1) - log(x)
```

giac [A] time = 0.62, size = 18, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*log(x^2) + 1/2*log(abs(b*x^2 - 1))
```

maple [A] time = 0.00, size = 16, normalized size = 0.89

$$-\ln(x)+\frac{\ln\left(b\,x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2-1),x)
```

```
[Out] -ln(x)+1/2*ln(b*x^2-1)
```

maxima [A] time = 1.37, size = 17, normalized size = 0.94

$$\frac{1}{2}\log\left(bx^2-1\right)-\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 - 1) - 1/2*log(x^2)
```

mupad [B] time = 0.06, size = 16, normalized size = 0.89

$$\frac{\ln\left(\frac{3}{2}-\frac{3\,b\,x^2}{2}\right)}{2}-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 - 1)),x)
```

```
[Out] log(3/2 - (3*b*x^2)/2)/2 - log(x)
```

sympy [A] time = 0.14, size = 12, normalized size = 0.67

$$-\log\left(x\right)+\frac{\log\left(x^2-\frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2-1),x)
```

```
[Out] -log(x) + log(x**2 - 1/b)/2
```

3.255 $\int \frac{1}{x^3(1+bx^2)} \, dx$

Optimal. Leaf size=26

$$\frac{1}{2} b \log \left(b x^2+1\right)-b \log (x)-\frac{1}{2 x^2}$$

```
[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2} b \log \left(b x^2+1\right)-b \log (x)-\frac{1}{2 x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + b*x^2)),x]
```

```
[Out] -1/(2*x^2) - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(1+b x^2\right)} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{x^2(1+b x)} \, dx,x,x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+b x}\right) \, dx,x,x^2\right)\\
&= -\frac{1}{2 x^2}-b \log (x)+\frac{1}{2} b \log \left(1+b x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{1}{2} b \log \left(b x^2+1\right)-b \log (x)-\frac{1}{2 x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + b*x^2)),x]
```

```
[Out] -1/2*1/x^2 - b*Log[x] + (b*Log[1 + b*x^2])/2
```

fricas [A] time = 0.92, size = 28, normalized size = 1.08

$$\frac{b x^2 \log \left(b x^2+1\right)-2 \, b x^2 \log (x)-1}{2 \, x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + 1) - 2*b*x^2*log(x) - 1)/x^2
```

giac [A] time = 0.63, size = 32, normalized size = 1.23

$$-\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2+1\right|\right)+\frac{bx^2-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 + 1)) + 1/2*(b*x^2 - 1)/x^2
```

maple [A] time = 0.01, size = 23, normalized size = 0.88

$$-b\ln(x)+\frac{b\ln\left(b\,x^2+1\right)}{2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+1),x)
```

```
[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

maxima [A] time = 1.34, size = 24, normalized size = 0.92

$$\frac{1}{2}\,b\log\left(bx^2+1\right)-\frac{1}{2}\,b\log\left(x^2\right)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + 1) - 1/2*b*log(x^2) - 1/2/x^2
```

mupad [B] time = 0.06, size = 22, normalized size = 0.85

$$\frac{b\,\ln\left(b\,x^2+1\right)}{2}-b\,\ln(x)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + 1)),x)
```

```
[Out] (b*log(b*x^2 + 1))/2 - b*log(x) - 1/(2*x^2)
```

sympy [A] time = 0.20, size = 22, normalized size = 0.85

$$-b\log\left(x\right)+\frac{b\log\left(x^2+\frac{1}{b}\right)}{2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+1),x)
```

```
[Out] -b*log(x) + b*log(x**2 + 1/b)/2 - 1/(2*x**2)
```

3.256 $\int \frac{1}{x^3(-1+bx^2)}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

```
[Out] 1/2/x^2-b*ln(x)+1/2*b*ln(-b*x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-1 + b*x^2)),x]
```

```
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(-1+bx^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(-1+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{-1+bx}\right)dx,x,x^2\right)\\
&= \frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1-bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-1 + b*x^2)),x]
```

```
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

fricas [A] time = 0.69, size = 28, normalized size = 1.04

$$\frac{bx^2\log\left(bx^2-1\right)-2\,bx^2\log(x)+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2-1),x, algorithm="fricas")
[Out] 1/2*(b*x^2*log(b*x^2 - 1) - 2*b*x^2*log(x) + 1)/x^2
```

giac [A] time = 0.59, size = 32, normalized size = 1.19

$$-\frac{1}{2}\,b\log\left(x^{2}\right)+\frac{1}{2}\,b\log\left(\left|bx^{2}-1\right|\right)+\frac{bx^{2}+1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2-1),x, algorithm="giac")
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 - 1)) + 1/2*(b*x^2 + 1)/x^2
```

maple [A] time = 0.00, size = 23, normalized size = 0.85

$$-b\ln(x)+\frac{b\ln\left(b\,x^{2}-1\right)}{2}+\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2-1),x)
[Out] 1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2-1)
```

maxima [A] time = 1.34, size = 24, normalized size = 0.89

$$\frac{1}{2}\,b\log\left(bx^{2}-1\right)-\frac{1}{2}\,b\log\left(x^{2}\right)+\frac{1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2-1),x, algorithm="maxima")
[Out] 1/2*b*log(b*x^2 - 1) - 1/2*b*log(x^2) + 1/2/x^2
```

mupad [B] time = 0.06, size = 22, normalized size = 0.81

$$\frac{b\,\ln\left(b\,x^{2}-1\right)}{2}-b\,\ln(x)+\frac{1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 - 1)),x)
[Out] (b*log(b*x^2 - 1))/2 - b*log(x) + 1/(2*x^2)
```

sympy [A] time = 0.21, size = 22, normalized size = 0.81

$$-b\log\left(x\right)+\frac{b\log\left(x^{2}-\frac{1}{b}\right)}{2}+\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2-1),x)
[Out] -b*log(x) + b*log(x**2 - 1/b)/2 + 1/(2*x**2)
```

3.257 $\int \frac{1}{-1+a+ax^2} dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

```
[Out] -arctanh(x*a^(1/2)/(1-a)^(1/2))/((1-a)*a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + a + a*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*x)/Sqrt[1 - a]]/Sqrt[(1 - a)*a])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{-1+a+ax^2}\,dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.93

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-1}}\right)}{\sqrt{a-1}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + a + a*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*x)/Sqrt[-1 + a]]/(Sqrt[-1 + a]*Sqrt[a])
```

fricas [A] time = 0.83, size = 82, normalized size = 2.73

$$\left[-\frac{\sqrt{-a^2+a}\,\log\left(\frac{ax^2-2\sqrt{-a^2+a}\,x-a+1}{ax^2+a-1}\right)}{2\left(a^2-a\right)}, \frac{\arctan\left(\frac{\sqrt{a^2-a}\,x}{a-1}\right)}{\sqrt{a^2-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a-1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + a)*log((a*x^2 - 2*sqrt(-a^2 + a)*x - a + 1)/(a*x^2 + a -
1))/(a^2 - a), arctan(sqrt(a^2 - a)*x/(a - 1))/sqrt(a^2 - a)]
```

giac [A] time = 0.58, size = 23, normalized size = 0.77

$$\frac{\arctan\left(\frac{ax}{\sqrt{a^2-a}}\right)}{\sqrt{a^2-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a-1),x, algorithm="giac")
[Out] arctan(a*x/sqrt(a^2 - a))/sqrt(a^2 - a)
```

maple [A] time = 0.01, size = 20, normalized size = 0.67

$$\frac{\arctan\left(\frac{ax}{\sqrt{(a-1)a}}\right)}{\sqrt{(a-1)\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+a-1),x)
[Out] 1/((a-1)*a)^(1/2)*arctan(x*a/((a-1)*a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a-1),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 positive or negative?
```

mupad [B] time = 0.17, size = 23, normalized size = 0.77

$$\frac{\operatorname{atan}\left(\frac{a\,x}{\sqrt{a^2-a}}\right)}{\sqrt{a^2-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*x^2 - 1),x)
[Out] atan((a*x)/(a^2 - a)^(1/2))/(a^2 - a)^(1/2)
```

sympy [B] time = 0.17, size = 83, normalized size = 2.77

$$-\frac{\sqrt{-\frac{1}{a(a-1)}}\log\left(-a\sqrt{-\frac{1}{a(a-1)}}+x+\sqrt{-\frac{1}{a(a-1)}}\right)}{2}+\frac{\sqrt{-\frac{1}{a(a-1)}}\log\left(a\sqrt{-\frac{1}{a(a-1)}}+x-\sqrt{-\frac{1}{a(a-1)}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+a-1),x)
[Out] -sqrt(-1/(a*(a - 1)))*log(-a*sqrt(-1/(a*(a - 1))) + x + sqrt(-1/(a*(a - 1))
))/2 + sqrt(-1/(a*(a - 1)))*log(a*sqrt(-1/(a*(a - 1))) + x - sqrt(-1/(a*(a
- 1))))/2
```

3.258 $\int \frac{1}{-c-d+(c-d)x^2} dx$

Optimal. Leaf size=37

$$-\frac{\tanh^{-1}\left(\frac{x\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}$$

```
[Out] -arctanh(x*(c-d)^(1/2)/(c+d)^(1/2))/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {208}

$$-\frac{\tanh^{-1}\left(\frac{x\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(-c - d + (c - d)*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(Sqrt[c - d]*x)/Sqrt[c + d]]/(Sqrt[c - d]*Sqrt[c + d]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{-c-d+(c-d)x^2} dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{c-d}\,x}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.19

$$\frac{\tan^{-1}\left(\frac{x\sqrt{c-d}}{\sqrt{-c-d}}\right)}{\sqrt{-c-d}\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-c - d + (c - d)*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[c - d]*x)/Sqrt[-c - d]]/(Sqrt[-c - d]*Sqrt[c - d])
```

fricas [A] time = 0.94, size = 102, normalized size = 2.76

$$\left[\frac{\log\left(\frac{(c-d)x^2-2\sqrt{c^2-d^2}\,x+c+d}{(c-d)x^2-c-d}\right)}{2\sqrt{c^2-d^2}}, \frac{\sqrt{-c^2+d^2}\,\arctan\left(\frac{\sqrt{-c^2+d^2}\,x}{c+d}\right)}{c^2-d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c-d+(c-d)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(((c - d)*x^2 - 2*sqrt(c^2 - d^2)*x + c + d)/((c - d)*x^2 - c - d))
/sqrt(c^2 - d^2), sqrt(-c^2 + d^2)*arctan(sqrt(-c^2 + d^2)*x/(c + d))/(c^2
- d^2)]
```

giac [**A**] time = 0.59, size = 33, normalized size = 0.89

$$\frac{\arctan\left(\frac{cx-dx}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c-d+(c-d)*x^2),x, algorithm="giac")
```

```
[Out] arctan((c*x - d*x)/sqrt(-c^2 + d^2))/sqrt(-c^2 + d^2)
```

maple [**A**] time = 0.01, size = 33, normalized size = 0.89

$$-\frac{\operatorname{arctanh}\left(\frac{(c-d)x}{\sqrt{(c+d)(c-d)}}\right)}{\sqrt{(c+d)(c-d)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c-d+(c-d)*x^2),x)
```

```
[Out] -1/((c+d)*(c-d))^(1/2)*arctanh((c-d)*x/((c+d)*(c-d))^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c-d+(c-d)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?
```

mupad [**B**] time = 0.29, size = 29, normalized size = 0.78

$$-\frac{\operatorname{atanh}\left(\frac{x\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c+d}\,\sqrt{c-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(c + d - x^2*(c - d)),x)
```

```
[Out] -atanh((x*(c - d)^(1/2))/(c + d)^(1/2))/((c + d)^(1/2)*(c - d)^(1/2))
```

sympy [**B**] time = 0.23, size = 87, normalized size = 2.35

$$\frac{\sqrt{\frac{1}{(c-d)(c+d)}}\log\left(-c\sqrt{\frac{1}{(c-d)(c+d)}}-d\sqrt{\frac{1}{(c-d)(c+d)}}+x\right)}{2}-\frac{\sqrt{\frac{1}{(c-d)(c+d)}}\log\left(c\sqrt{\frac{1}{(c-d)(c+d)}}+d\sqrt{\frac{1}{(c-d)(c+d)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c-d+(c-d)*x**2),x)
```

```
[Out] sqrt(1/((c - d)*(c + d)))*log(-c*sqrt(1/((c - d)*(c + d))) - d*sqrt(1/((c -
 d)*(c + d))) + x)/2 - sqrt(1/((c - d)*(c + d)))*log(c*sqrt(1/((c - d)*(c +
 d))) + d*sqrt(1/((c - d)*(c + d))) + x)/2
```

3.259 $\int \frac{1}{x(1+bx^2)^2}\,dx$

Optimal. Leaf size=28

$$\frac{1}{2\left(bx^2+1\right)}-\frac{1}{2}\log\left(bx^2+1\right)+\log(x)$$

```
[Out] 1/2/(b*x^2+1)+ln(x)-1/2*ln(b*x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2\left(bx^2+1\right)}-\frac{1}{2}\log\left(bx^2+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + b*x^2)^2),x]
```

```
[Out] 1/(2*(1 + b*x^2)) + Log[x] - Log[1 + b*x^2]/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+bx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}-\frac{b}{(1+bx)^2}-\frac{b}{1+bx}\right)dx, x, x^2\right)\\
&= \frac{1}{2\left(1+bx^2\right)}+\log(x)-\frac{1}{2}\log\left(1+bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.89

$$\frac{1}{2bx^2+2}-\frac{1}{2}\log\left(bx^2+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + b*x^2)^2),x]
```

```
[Out] (2 + 2*b*x^2)^(-1) + Log[x] - Log[1 + b*x^2]/2
```

fricas [A] time = 0.72, size = 40, normalized size = 1.43

$$-\frac{\left(bx^2+1\right)\log\left(bx^2+1\right)-2\left(bx^2+1\right)\log(x)-1}{2\left(bx^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*((b*x^2 + 1)*log(b*x^2 + 1) - 2*(b*x^2 + 1)*log(x) - 1)/(b*x^2 + 1)
```

giac [A] time = 0.63, size = 36, normalized size = 1.29

$$\frac{bx^2+2}{2\left(bx^2+1\right)}+\frac{1}{2}\log\left(x^2\right)-\frac{1}{2}\log\left(\left|bx^2+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1)^2,x, algorithm="giac")
[Out] 1/2*(b*x^2 + 2)/(b*x^2 + 1) + 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

maple [A] time = 0.01, size = 25, normalized size = 0.89

$$\ln(x)-\frac{\ln\left(b\,x^2+1\right)}{2}+\frac{1}{2b\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+1)^2,x)
[Out] 1/2/(b*x^2+1)+ln(x)-1/2*ln(b*x^2+1)
```

maxima [A] time = 1.35, size = 28, normalized size = 1.00

$$\frac{1}{2\left(bx^2+1\right)}-\frac{1}{2}\log\left(bx^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1)^2,x, algorithm="maxima")
[Out] 1/2/(b*x^2 + 1) - 1/2*log(b*x^2 + 1) + 1/2*log(x^2)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.86

$$\ln(x)-\frac{\ln\left(\frac{3\,b\,x^2}{2}+\frac{3}{2}\right)}{2}+\frac{1}{2\,\left(b\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + 1)^2),x)
[Out] log(x) - log((3*b*x^2)/2 + 3/2)/2 + 1/(2*(b*x^2 + 1))
```

sympy [A] time = 0.20, size = 22, normalized size = 0.79

$$\log\left(x\right)-\frac{\log\left(x^2+\frac{1}{b}\right)}{2}+\frac{1}{2bx^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+1)**2,x)
[Out] log(x) - log(x**2 + 1/b)/2 + 1/(2*b*x**2 + 2)
```

3.260 $\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx$

Optimal. Leaf size=30

$$\frac{1}{2\left(1-bx^2\right)}-\frac{1}{2}\log\left(1-bx^2\right)+\log(x)$$

[Out] 1/2/(-b*x^2+1)+ln(x)-1/2*ln(-b*x^2+1)

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2\left(1-bx^2\right)}-\frac{1}{2}\log\left(1-bx^2\right)+\log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(-1 + b*x^2)^2),x]

[Out] 1/(2*(1 - b*x^2)) + Log[x] - Log[1 - b*x^2]/2

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-1+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{b}{(-1+bx)^2}-\frac{b}{-1+bx}\right)dx,x,x^2\right)\\ &= \frac{1}{2\left(1-bx^2\right)}+\log(x)-\frac{1}{2}\log\left(1-bx^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.87

$$\frac{1}{2-2bx^2}-\frac{1}{2}\log\left(1-bx^2\right)+\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(-1 + b*x^2)^2),x]

[Out] (2 - 2*b*x^2)^(-1) + Log[x] - Log[1 - b*x^2]/2

fricas [A] time = 0.56, size = 40, normalized size = 1.33

$$-\frac{\left(bx^2-1\right)\log\left(bx^2-1\right)-2\left(bx^2-1\right)\log(x)+1}{2\left(bx^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1)^2,x, algorithm="fricas")
[Out] -1/2*((b*x^2 - 1)*log(b*x^2 - 1) - 2*(b*x^2 - 1)*log(x) + 1)/(b*x^2 - 1)
```

giac [A] time = 0.63, size = 36, normalized size = 1.20

$$\frac{bx^2-2}{2\left(bx^2-1\right)}+\frac{1}{2}\log\left(x^2\right)-\frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1)^2,x, algorithm="giac")
[Out] 1/2*(b*x^2 - 2)/(b*x^2 - 1) + 1/2*log(x^2) - 1/2*log(abs(b*x^2 - 1))
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$\ln(x)-\frac{\ln\left(b\,x^2-1\right)}{2}-\frac{1}{2\left(b\,x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2-1)^2,x)
[Out] ln(x)-1/2/(b*x^2-1)-1/2*ln(b*x^2-1)
```

maxima [A] time = 1.35, size = 28, normalized size = 0.93

$$-\frac{1}{2\left(bx^2-1\right)}-\frac{1}{2}\log\left(bx^2-1\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1)^2,x, algorithm="maxima")
[Out] -1/2/(b*x^2 - 1) - 1/2*log(b*x^2 - 1) + 1/2*log(x^2)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.87

$$\ln(x)-\frac{\ln\left(\frac{3\,b\,x^2}{2}-\frac{3}{2}\right)}{2}-\frac{1}{2\,\left(b\,x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 - 1)^2),x)
[Out] log(x) - log((3*b*x^2)/2 - 3/2)/2 - 1/(2*(b*x^2 - 1))
```

sympy [A] time = 0.21, size = 22, normalized size = 0.73

$$\log\left(x\right)-\frac{\log\left(x^2-\frac{1}{b}\right)}{2}-\frac{1}{2bx^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2-1)**2,x)
[Out] log(x) - log(x**2 - 1/b)/2 - 1/(2*b*x**2 - 2)
```

3.261 $\int \frac{1}{a+(b-ac)x^2}\,dx$

Optimal. Leaf size=34

$$\frac{\tan^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

[Out] arctan(x*(-a*c+b)^(1/2)/a^(1/2))/a^(1/2)/(-a*c+b)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

Antiderivative was successfully verified.

[In] Int[(a + (b - a*c)*x^2)^(-1),x]

[Out] ArcTan[(Sqrt[b - a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b - a*c])

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+(b-ac)x^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b-ac}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.06

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{ac-b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{ac-b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + (b - a*c)*x^2)^(-1),x]

[Out] ArcTanh[(Sqrt[-b + a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[-b + a*c])

fricas [A] time = 0.87, size = 106, normalized size = 3.12

$$\left[\frac{\log\left(\frac{(ac-b)x^2+2\sqrt{a^2c-ab}\,x+a}{(ac-b)x^2-a}\right)}{2\sqrt{a^2c-ab}}, -\frac{\sqrt{-a^2c+ab}\arctan\left(\frac{\sqrt{-a^2c+ab}\,x}{a}\right)}{a^2c-ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+(-a*c+b)*x^2),x, algorithm="fricas")

```
[Out] [1/2*log(((a*c - b)*x^2 + 2*sqrt(a^2*c - a*b)*x + a)/((a*c - b)*x^2 - a))/s
qrt(a^2*c - a*b), -sqrt(-a^2*c + a*b)*arctan(sqrt(-a^2*c + a*b)*x/a)/(a^2*c
 - a*b)]
```

giac [A] time = 0.63, size = 37, normalized size = 1.09

$$-\frac{\arctan\left(\frac{acx-bx}{\sqrt{-a^2c+ab}}\right)}{\sqrt{-a^2c+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x^2),x, algorithm="giac")
```

```
[Out] -arctan((a*c*x - b*x)/sqrt(-a^2*c + a*b))/sqrt(-a^2*c + a*b)
```

maple [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{\operatorname{arctanh}\left(\frac{(ac-b)x}{\sqrt{(ac-b)a}}\right)}{\sqrt{(ac-b)\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(-a*c+b)*x^2),x)
```

```
[Out] 1/(a*(a*c-b))^(1/2)*arctanh((a*c-b)*x/(a*(a*c-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*c-b>0)', see `assume?` for more d
etails)Is a*c-b positive or negative?
```

mupad [B] time = 5.14, size = 38, normalized size = 1.12

$$-\frac{\operatorname{atanh}\left(\frac{x\,(2\,b-2\,a\,c)}{2\,\sqrt{a}\,\sqrt{a\,c-b}}\right)}{\sqrt{a}\,\sqrt{a\,c-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + x^2*(b - a*c)),x)
```

```
[Out] -atanh((x*(2*b - 2*a*c))/(2*a^(1/2)*(a*c - b)^(1/2)))/(a^(1/2)*(a*c - b)^(1
/2))
```

sympy [B] time = 0.25, size = 60, normalized size = 1.76

$$-\frac{\sqrt{\frac{1}{a(ac-b)}}\log\left(-a\sqrt{\frac{1}{a(ac-b)}}+x\right)}{2}+\frac{\sqrt{\frac{1}{a(ac-b)}}\log\left(a\sqrt{\frac{1}{a(ac-b)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x**2),x)
```

```
[Out] -sqrt(1/(a*(a*c - b)))*log(-a*sqrt(1/(a*(a*c - b))) + x)/2 + sqrt(1/(a*(a*c
 - b)))*log(a*sqrt(1/(a*(a*c - b))) + x)/2
```

3.262 $\int \frac{1}{a-(b-ac)x^2}\,dx$

Optimal. Leaf size=34

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b-ac}}$$

[Out] arctanh(x*(-a*c+b)^(1/2)/a^(1/2))/a^(1/2)/(-a*c+b)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b-ac}}$$

Antiderivative was successfully verified.

[In] Int[(a - (b - a*c)*x^2)^(-1),x]

[Out] ArcTanh[(Sqrt[b - a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b - a*c])

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{a-(b-ac)x^2}\,dx = \frac{\tanh^{-1}\left(\frac{\sqrt{b-ac}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b-ac}}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.06

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ac-b}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{ac-b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - (b - a*c)*x^2)^(-1),x]

[Out] ArcTan[(Sqrt[-b + a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[-b + a*c])

fricas [A] time = 0.83, size = 105, normalized size = 3.09

$$\left[-\frac{\sqrt{-a^2c+ab}\,\log\left(\frac{(ac-b)x^2-2\sqrt{-a^2c+ab}\,x-a}{(ac-b)x^2+a}\right)}{2\left(a^2c-ab\right)},\ \frac{\arctan\left(\frac{\sqrt{a^2c-ab}\,x}{a}\right)}{\sqrt{a^2c-ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-(-a*c+b)*x^2),x, algorithm="fricas")

```
[Out] [-1/2*sqrt(-a^2*c + a*b)*log(((a*c - b)*x^2 - 2*sqrt(-a^2*c + a*b)*x - a)/(
(a*c - b)*x^2 + a))/(a^2*c - a*b), arctan(sqrt(a^2*c - a*b)*x/a)/sqrt(a^2*c
 - a*b)]
```

giac [**A**] time = 0.62, size = 36, normalized size = 1.06

$$\frac{\arctan\left(\frac{acx-bx}{\sqrt{a^2c-ab}}\right)}{\sqrt{a^2c-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(-a*c+b)*x^2),x, algorithm="giac")
```

```
[Out] arctan((a*c*x - b*x)/sqrt(a^2*c - a*b))/sqrt(a^2*c - a*b)
```

maple [**A**] time = 0.00, size = 34, normalized size = 1.00

$$\frac{\arctan\left(\frac{(ac-b)x}{\sqrt{(ac-b)a}}\right)}{\sqrt{(ac-b)\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-(-a*c+b)*x^2),x)
```

```
[Out] 1/((a*c-b)*a)^(1/2)*arctan((a*c-b)/((a*c-b)*a)^(1/2)*x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(-a*c+b)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*c-b>0)', see `assume?` for more d
etails)Is a*c-b positive or negative?
```

mupad [**B**] time = 4.64, size = 38, normalized size = 1.12

$$-\frac{\mathrm{atan}\left(\frac{x\,\left(2\,b-2\,a\,c\right)}{2\,\sqrt{a^2\,c-a\,b}}\right)}{\sqrt{a^2\,c-a\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - x^2*(b - a*c)),x)
```

```
[Out] -atan((x*(2*b - 2*a*c))/(2*(a^2*c - a*b)^(1/2)))/(a^2*c - a*b)^(1/2)
```

sympy [**B**] time = 0.23, size = 66, normalized size = 1.94

$$-\frac{\sqrt{-\frac{1}{a(ac-b)}}\log\left(-a\sqrt{-\frac{1}{a(ac-b)}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a(ac-b)}}\log\left(a\sqrt{-\frac{1}{a(ac-b)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(-a*c+b)*x**2),x)
```

```
[Out] -sqrt(-1/(a*(a*c - b)))*log(-a*sqrt(-1/(a*(a*c - b))) + x)/2 + sqrt(-1/(a*(
a*c - b)))*log(a*sqrt(-1/(a*(a*c - b))) + x)/2
```

3.263 $\int \frac{1}{c(a-d)-(b-c)x^2}\,dx$

Optimal. Leaf size=50

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-c}}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{c}\sqrt{a-d}\sqrt{b-c}}$$

```
[Out] arctanh(x*(b-c)^(1/2)/c^(1/2)/(a-d)^(1/2))/(b-c)^(1/2)/c^(1/2)/(a-d)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-c}}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{c}\sqrt{a-d}\sqrt{b-c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a - d) - (b - c)*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[b - c]*x)/(Sqrt[c]*Sqrt[a - d])]/(Sqrt[b - c]*Sqrt[c]*Sqrt[a
- d])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{c(a-d)-(b-c)x^2}\,dx = \frac{\tanh^{-1}\left(\frac{\sqrt{b-c}\,x}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{b-c}\sqrt{c}\sqrt{a-d}}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x\sqrt{c-b}}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{c}\sqrt{a-d}\sqrt{c-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a - d) - (b - c)*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[-b + c]*x)/(Sqrt[c]*Sqrt[a - d])]/(Sqrt[c]*Sqrt[-b + c]*Sqrt[a
 - d])
```

fricas [B] time = 0.83, size = 182, normalized size = 3.64

$$\left[\frac{\log\left(\frac{(b-c)x^2+ac-cd+2\sqrt{abc-ac^2-(bc-c^2)d}\,x}{(b-c)x^2-ac+cd}\right)}{2\sqrt{abc-ac^2-(bc-c^2)d}}, \frac{\sqrt{-abc+ac^2+(bc-c^2)d}\arctan\left(-\frac{\sqrt{-abc+ac^2+(bc-c^2)d}\,x}{ac-cd}\right)}{abc-ac^2-(bc-c^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(((b - c)*x^2 + a*c - c*d + 2*sqrt(a*b*c - a*c^2 - (b*c - c^2)*d)*x
)/((b - c)*x^2 - a*c + c*d))/sqrt(a*b*c - a*c^2 - (b*c - c^2)*d), sqrt(-a*b
*c + a*c^2 + (b*c - c^2)*d)*arctan(-sqrt(-a*b*c + a*c^2 + (b*c - c^2)*d)*x/
(a*c - c*d))/(a*b*c - a*c^2 - (b*c - c^2)*d)]
```

giac [A] time = 0.64, size = 58, normalized size = 1.16

$$-\frac{\arctan\left(\frac{bx-cx}{\sqrt{-abc+ac^2+bcd-c^2d}}\right)}{\sqrt{-abc+ac^2+bcd-c^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="giac")
```

```
[Out] -arctan((b*x - c*x)/sqrt(-a*b*c + a*c^2 + b*c*d - c^2*d))/sqrt(-a*b*c + a*c
^2 + b*c*d - c^2*d)
```

maple [A] time = 0.01, size = 38, normalized size = 0.76

$$\frac{\operatorname{arctanh}\left(\frac{(b-c)x}{\sqrt{(a-d)(b-c)c}}\right)}{\sqrt{(a-d)\,(b-c)\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a-d)-(b-c)*x^2),x)
```

```
[Out] 1/(c*(a-d)*(b-c))^(1/2)*arctanh((b-c)*x/(c*(a-d)*(b-c))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((c-b)*(d-a)>0)', see `assume?` for
more details)Is (c-b)*(d-a) positive or negative?
```

mupad [B] time = 4.88, size = 46, normalized size = 0.92

$$\frac{\operatorname{atanh}\left(\frac{x\,(2\,b-2\,c)}{2\,\sqrt{c}\,\sqrt{a-d}\,\sqrt{b-c}}\right)}{\sqrt{c}\,\sqrt{a-d}\,\sqrt{b-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a - d) - x^2*(b - c)),x)
```

```
[Out] atanh((x*(2*b - 2*c))/(2*c^(1/2)*(a - d)^(1/2)*(b - c)^(1/2)))/(c^(1/2)*(a
- d)^(1/2)*(b - c)^(1/2))
```

sympy [B] time = 0.32, size = 104, normalized size = 2.08

$$-\frac{\sqrt{\frac{1}{c(a-d)(b-c)}}\log\left(-ac\sqrt{\frac{1}{c(a-d)(b-c)}}+cd\sqrt{\frac{1}{c(a-d)(b-c)}}+x\right)}{2}+\frac{\sqrt{\frac{1}{c(a-d)(b-c)}}\log\left(ac\sqrt{\frac{1}{c(a-d)(b-c)}}-cd\sqrt{\frac{1}{c(a-d)(b-c)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x**2),x)
```

```
[Out] -sqrt(1/(c*(a - d)*(b - c)))*log(-a*c*sqrt(1/(c*(a - d)*(b - c))) + c*d*sqr
t(1/(c*(a - d)*(b - c))) + x)/2 + sqrt(1/(c*(a - d)*(b - c)))*log(a*c*sqrt(
1/(c*(a - d)*(b - c))) - c*d*sqrt(1/(c*(a - d)*(b - c))) + x)/2
```

3.264 $\int x^{7/2} \left(a + bx^2\right) \, dx$

Optimal. Leaf size=21

$$\frac{2}{9}ax^{9/2} + \frac{2}{13}bx^{13/2}$$

```
[Out] 2/9*a*x^(9/2)+2/13*b*x^(13/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{9}ax^{9/2} + \frac{2}{13}bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(9/2))/9 + (2*b*x^(13/2))/13
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int x^{7/2} \left(a + bx^2\right) \, dx &= \int \left(ax^{7/2} + bx^{11/2}\right) \, dx \\
&= \frac{2}{9}ax^{9/2} + \frac{2}{13}bx^{13/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{9}ax^{9/2} + \frac{2}{13}bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(9/2))/9 + (2*b*x^(13/2))/13
```

fricas [A] time = 0.72, size = 18, normalized size = 0.86

$$\frac{2}{117} \left(9 \, bx^6 + 13 \, ax^4\right) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/117*(9*b*x^6 + 13*a*x^4)*sqrt(x)
```

giac [A] time = 0.63, size = 13, normalized size = 0.62

$$\frac{2}{13} \, bx^{\frac{13}{2}} + \frac{2}{9} \, ax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a),x, algorithm="giac")
[Out] 2/13*b*x^(13/2) + 2/9*a*x^(9/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(9b\,x^{2}+13a\right)x^{\frac{9}{2}}}{117}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a),x)
[Out] 2/117*x^(9/2)*(9*b*x^2+13*a)
```

maxima [A] time = 1.33, size = 13, normalized size = 0.62

$$\frac{2}{13}\,bx^{\frac{13}{2}} + \frac{2}{9}\,ax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a),x, algorithm="maxima")
[Out] 2/13*b*x^(13/2) + 2/9*a*x^(9/2)
```

mupad [B] time = 4.50, size = 15, normalized size = 0.71

$$\frac{2\,x^{9/2}\,\left(9\,b\,x^2+13\,a\right)}{117}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2),x)
[Out] (2*x^(9/2)*(13*a + 9*b*x^2))/117
```

sympy [A] time = 5.45, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{9}{2}}}{9} + \frac{2bx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a),x)
[Out] 2*a*x**(9/2)/9 + 2*b*x**(13/2)/13
```

3.265 $\int x^{5/2}\left(a+bx^2\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}$$

```
[Out] 2/7*a*x^(7/2)+2/11*b*x^(11/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)\,dx&=\int\left(ax^{5/2}+bx^{9/2}\right)\,dx\\&=\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11
```

fricas [A] time = 0.68, size = 18, normalized size = 0.86

$$\frac{2}{77}\left(7\,bx^5+11\,ax^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b*x^5 + 11*a*x^3)*sqrt(x)
```

giac [A] time = 0.60, size = 13, normalized size = 0.62

$$\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(7b\,x^{2}+11a\right)x^{\frac{7}{2}}}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a),x)
```

```
[Out] 2/77*x^(7/2)*(7*b*x^2+11*a)
```

maxima [A] time = 1.29, size = 13, normalized size = 0.62

$$\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{7/2}\,\left(7\,b\,x^2+11\,a\right)}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2),x)
```

```
[Out] (2*x^(7/2)*(11*a + 7*b*x^2))/77
```

sympy [A] time = 2.41, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a),x)
```

```
[Out] 2*a*x**(7/2)/7 + 2*b*x**(11/2)/11
```

3.266 $\int x^{3/2}\left(a+bx^2\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}$$

```
[Out] 2/5*a*x^(5/2)+2/9*b*x^(9/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2),x]
[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)\,dx &= \int \left(ax^{3/2}+bx^{7/2}\right)\,dx\\ &= \frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2),x]
[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9
```

fricas [A] time = 1.29, size = 18, normalized size = 0.86

$$\frac{2}{45}\left(5\,bx^4+9\,ax^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a),x, algorithm="fricas")
[Out] 2/45*(5*b*x^4 + 9*a*x^2)*sqrt(x)
```

giac [A] time = 0.63, size = 13, normalized size = 0.62

$$\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(5b\,x^{2}+9a\right)x^{\frac{5}{2}}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a),x)
```

```
[Out] 2/45*x^(5/2)*(5*b*x^2+9*a)
```

maxima [A] time = 1.33, size = 13, normalized size = 0.62

$$\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

mupad [B] time = 0.02, size = 15, normalized size = 0.71

$$\frac{2\,x^{5/2}\,\left(5\,b\,x^2+9\,a\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2),x)
```

```
[Out] (2*x^(5/2)*(9*a + 5*b*x^2))/45
```

sympy [A] time = 0.95, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{5}{2}}}{5}+\frac{2bx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a),x)
```

```
[Out] 2*a*x**(5/2)/5 + 2*b*x**(9/2)/9
```

3.267 $\int \sqrt{x}\left(a+bx^2\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}$$

[Out] 2/3*a*x^(3/2)+2/7*b*x^(7/2)

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]*(a + b*x^2),x]

[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)\,dx &= \int \left(a\sqrt{x}+bx^{5/2}\right)\,dx\\ &= \frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]*(a + b*x^2),x]

[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7

fricas [A] time = 0.72, size = 16, normalized size = 0.76

$$\frac{2}{21}\left(3\,bx^3+7\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*x^(1/2),x, algorithm="fricas")

[Out] 2/21*(3*b*x^3 + 7*a*x)*sqrt(x)

giac [A] time = 0.61, size = 13, normalized size = 0.62

$$\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(3b\,x^{2}+7a\right)x^{\frac{3}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*x^(1/2),x)
```

```
[Out] 2/21*x^(3/2)*(3*b*x^2+7*a)
```

maxima [A] time = 1.31, size = 13, normalized size = 0.62

$$\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{3/2}\,\left(3\,b\,x^2+7\,a\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2),x)
```

```
[Out] (2*x^(3/2)*(7*a + 3*b*x^2))/21
```

sympy [A] time = 1.30, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{3}{2}}}{3}+\frac{2bx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*x**(1/2),x)
```

```
[Out] 2*a*x**(3/2)/3 + 2*b*x**(7/2)/7
```

3.268 $\int \frac{a+bx^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=19

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}$$

[Out] 2/5*b*x^(5/2)+2*a*x^(1/2)

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)/Sqrt[x],x]

[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{\sqrt{x}}\,dx &= \int \left(\frac{a}{\sqrt{x}}+bx^{3/2}\right)dx\\ &= 2a\sqrt{x}+\frac{2}{5}bx^{5/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)/Sqrt[x],x]

[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5

fricas [A] time = 0.63, size = 14, normalized size = 0.74

$$\frac{2}{5}\left(bx^2+5\,a\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/x^(1/2),x, algorithm="fricas")

[Out] 2/5*(b*x^2 + 5*a)*sqrt(x)

giac [A] time = 0.63, size = 13, normalized size = 0.68

$$\frac{2}{5}\,bx^{\frac{5}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

maple [A] time = 0.00, size = 15, normalized size = 0.79

$$\frac{2\left(b\,x^{2}+5a\right)\sqrt{x}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(1/2),x)
```

```
[Out] 2/5*x^(1/2)*(b*x^2+5*a)
```

maxima [A] time = 1.30, size = 13, normalized size = 0.68

$$\frac{2}{5}\,bx^{\frac{5}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.74

$$\frac{2\,\sqrt{x}\,\left(b\,x^{2}+5\,a\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(5*a + b*x^2))/5
```

sympy [A] time = 0.23, size = 17, normalized size = 0.89

$$2a\sqrt{x}+\frac{2bx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(1/2),x)
```

```
[Out] 2*a*sqrt(x) + 2*b*x**(5/2)/5
```

3.269 $\int \frac{a+bx^2}{x^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{2}{3}bx^{3/2}-\frac{2a}{\sqrt{x}}$$

```
[Out] 2/3*b*x^(3/2)-2*a/x^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{3}bx^{3/2}-\frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^{3/2}}\,dx &= \int \left(\frac{a}{x^{3/2}}+b\sqrt{x}\right)dx\\ &= -\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2}{3}bx^{3/2}-\frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3
```

fricas [A] time = 0.81, size = 14, normalized size = 0.74

$$\frac{2\left(bx^2-3\,a\right)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x^2 - 3*a)/sqrt(x)
```

giac [A] time = 0.59, size = 13, normalized size = 0.68

$$\frac{2}{3}\,bx^{\frac{3}{2}}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{2\left(-b\,x^{2}+3a\right)}{3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(3/2),x)
```

```
[Out] -2/3*(-b*x^2+3*a)/x^(1/2)
```

maxima [A] time = 1.35, size = 13, normalized size = 0.68

$$\frac{2}{3}\,bx^{\frac{3}{2}}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$-\frac{6\,a-2\,b\,x^{2}}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^(3/2),x)
```

```
[Out] -(6*a - 2*b*x^2)/(3*x^(1/2))
```

sympy [A] time = 0.38, size = 17, normalized size = 0.89

$$-\frac{2a}{\sqrt{x}}+\frac{2bx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(3/2),x)
```

```
[Out] -2*a/sqrt(x) + 2*b*x**(3/2)/3
```

3.270 $\int \frac{a+bx^2}{x^{5/2}}\,dx$

Optimal. Leaf size=19

$$2b\sqrt{x}-\frac{2a}{3x^{3/2}}$$

```
[Out] -2/3*a/x^(3/2)+2*b*x^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$2b\sqrt{x}-\frac{2a}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^(5/2),x]
[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^{5/2}}\,dx &= \int \left(\frac{a}{x^{5/2}}+\frac{b}{\sqrt{x}}\right)dx\\ &= -\frac{2a}{3x^{3/2}}+2b\sqrt{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$2b\sqrt{x}-\frac{2a}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^(5/2),x]
[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x]
```

fricas [A] time = 0.95, size = 15, normalized size = 0.79

$$\frac{2\left(3\,bx^2-a\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(5/2),x, algorithm="fricas")
[Out] 2/3*(3*b*x^2 - a)/x^(3/2)
```

giac [A] time = 0.60, size = 13, normalized size = 0.68

$$2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{2\left(-3b\,x^{2}+a\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(5/2),x)
```

```
[Out] -2/3*(-3*b*x^2+a)/x^(3/2)
```

maxima [A] time = 1.32, size = 13, normalized size = 0.68

$$2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$-\frac{2\,a-6\,b\,x^2}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^(5/2),x)
```

```
[Out] -(2*a - 6*b*x^2)/(3*x^(3/2))
```

sympy [A] time = 0.56, size = 17, normalized size = 0.89

$$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(5/2),x)
```

```
[Out] -2*a/(3*x**(3/2)) + 2*b*sqrt(x)
```

3.271 $\int \frac{a+bx^2}{x^{7/2}} \, dx$

Optimal. Leaf size=19

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}$$

```
[Out] -2/5*a/x^(5/2)-2*b/x^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^(7/2),x]
[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^{7/2}} \, dx &= \int \left(\frac{a}{x^{7/2}}+\frac{b}{x^{3/2}}\right) dx\\ &= -\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^(7/2),x]
[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x]
```

fricas [A] time = 0.90, size = 13, normalized size = 0.68

$$-\frac{2\left(5\,bx^2+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(7/2),x, algorithm="fricas")
[Out] -2/5*(5*b*x^2 + a)/x^(5/2)
```

giac [A] time = 0.59, size = 13, normalized size = 0.68

$$-\frac{2\left(5\,bx^2+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(7/2),x, algorithm="giac")
[Out] -2/5*(5*b*x^2 + a)/x^(5/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{2\left(5b\,x^2+a\right)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(7/2),x)
[Out] -2/5*(5*b*x^2+a)/x^(5/2)
```

maxima [A] time = 1.31, size = 13, normalized size = 0.68

$$-\frac{2\left(5\,bx^2+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(7/2),x, algorithm="maxima")
[Out] -2/5*(5*b*x^2 + a)/x^(5/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$-\frac{10\,b\,x^2+2\,a}{5\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^(7/2),x)
[Out] -(2*a + 10*b*x^2)/(5*x^(5/2))
```

sympy [A] time = 1.24, size = 19, normalized size = 1.00

$$-\frac{2a}{5x^{\frac{5}{2}}}-\frac{2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(7/2),x)
[Out] -2*a/(5*x**(5/2)) - 2*b/sqrt(x)
```

3.272 $\int x^{7/2} \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{9}a^2x^{9/2}+\frac{4}{13}abx^{13/2}+\frac{2}{17}b^2x^{17/2}$$

```
[Out] 2/9*a^2*x^(9/2)+4/13*a*b*x^(13/2)+2/17*b^2*x^(17/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{2}{9}a^2x^{9/2}+\frac{4}{13}abx^{13/2}+\frac{2}{17}b^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*a^2*x^(9/2))/9 + (4*a*b*x^(13/2))/13 + (2*b^2*x^(17/2))/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^2\,dx &= \int \left(a^2x^{7/2}+2abx^{11/2}+b^2x^{15/2}\right)dx\\ &= \frac{2}{9}a^2x^{9/2}+\frac{4}{13}abx^{13/2}+\frac{2}{17}b^2x^{17/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{9/2}\left(221a^2+306abx^2+117b^2x^4\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*x^(9/2)*(221*a^2 + 306*a*b*x^2 + 117*b^2*x^4))/1989
```

fricas [A] time = 0.84, size = 29, normalized size = 0.81

$$\frac{2}{1989}\left(117\,b^2x^8+306\,abx^6+221\,a^2x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/1989*(117*b^2*x^8 + 306*a*b*x^6 + 221*a^2*x^4)*sqrt(x)
```

giac [A] time = 0.62, size = 24, normalized size = 0.67

$$\frac{2}{17}\,b^2x^{\frac{17}{2}}+\frac{4}{13}\,abx^{\frac{13}{2}}+\frac{2}{9}\,a^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/17*b^2*x^(17/2) + 4/13*a*b*x^(13/2) + 2/9*a^2*x^(9/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.75

$$\frac{2\left(117b^2x^4+306ab\,x^2+221a^2\right)x^{\frac{9}{2}}}{1989}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2,x)
```

```
[Out] 2/1989*x^(9/2)*(117*b^2*x^4+306*a*b*x^2+221*a^2)
```

maxima [A] time = 1.36, size = 24, normalized size = 0.67

$$\frac{2}{17}\,b^2x^{\frac{17}{2}}+\frac{4}{13}\,abx^{\frac{13}{2}}+\frac{2}{9}\,a^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/17*b^2*x^(17/2) + 4/13*a*b*x^(13/2) + 2/9*a^2*x^(9/2)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.69

$$x^{9/2}\left(\frac{2\,a^2}{9}+\frac{4\,a\,b\,x^2}{13}+\frac{2\,b^2\,x^4}{17}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2)^2,x)
```

```
[Out] x^(9/2)*((2*a^2)/9 + (2*b^2*x^4)/17 + (4*a*b*x^2)/13)
```

sympy [A] time = 10.60, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{9}{2}}}{9}+\frac{4abx^{\frac{13}{2}}}{13}+\frac{2b^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(9/2)/9 + 4*a*b*x**(13/2)/13 + 2*b**2*x**(17/2)/17
```

3.273 $\int x^{5/2}\left(a+bx^2\right)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}b^2x^{15/2}$$

```
[Out] 2/7*a^2*x^(7/2)+4/11*a*b*x^(11/2)+2/15*b^2*x^(15/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}b^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2,x]
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*b^2*x^(15/2))/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\,dx&=\int\left(a^2x^{5/2}+2abx^{9/2}+b^2x^{13/2}\right)dx\\&=\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}b^2x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{7/2}\left(165a^2+210abx^2+77b^2x^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2,x]
[Out] (2*x^(7/2)*(165*a^2 + 210*a*b*x^2 + 77*b^2*x^4))/1155
```

fricas [A] time = 1.02, size = 29, normalized size = 0.81

$$\frac{2}{1155}\left(77\,b^2x^7+210\,abx^5+165\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="fricas")
[Out] 2/1155*(77*b^2*x^7 + 210*a*b*x^5 + 165*a^2*x^3)*sqrt(x)
```

giac [A] time = 0.63, size = 24, normalized size = 0.67

$$\frac{2}{15}\,b^2x^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/15*b^2*x^(15/2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.75

$$\frac{2\left(77b^2x^4+210ab\,x^2+165a^2\right)x^{\frac{7}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2,x)
```

```
[Out] 2/1155*x^(7/2)*(77*b^2*x^4+210*a*b*x^2+165*a^2)
```

maxima [A] time = 1.26, size = 24, normalized size = 0.67

$$\frac{2}{15}\,b^2x^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/15*b^2*x^(15/2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.72

$$\frac{2\,x^{7/2}\,\left(165\,a^2+210\,a\,b\,x^2+77\,b^2\,x^4\right)}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2)^2,x)
```

```
[Out] (2*x^(7/2)*(165*a^2 + 77*b^2*x^4 + 210*a*b*x^2))/1155
```

sympy [A] time = 5.51, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{11}{2}}}{11}+\frac{2b^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(11/2)/11 + 2*b**2*x**(15/2)/15
```

3.274 $\int x^{3/2} \left(a + bx^2\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{5}a^2x^{5/2} + \frac{4}{9}abx^{9/2} + \frac{2}{13}b^2x^{13/2}$$

```
[Out] 2/5*a^2*x^(5/2)+4/9*a*b*x^(9/2)+2/13*b^2*x^(13/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{270\}$

$$\frac{2}{5}a^2x^{5/2} + \frac{4}{9}abx^{9/2} + \frac{2}{13}b^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*b^2*x^(13/2))/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2} \left(a + bx^2\right)^2 \, dx &= \int \left(a^2x^{3/2} + 2abx^{7/2} + b^2x^{11/2}\right) dx\\ &= \frac{2}{5}a^2x^{5/2} + \frac{4}{9}abx^{9/2} + \frac{2}{13}b^2x^{13/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2}{585}x^{5/2}\left(117a^2 + 130abx^2 + 45b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*x^(5/2)*(117*a^2 + 130*a*b*x^2 + 45*b^2*x^4))/585
```

fricas [A] time = 0.95, size = 29, normalized size = 0.81

$$\frac{2}{585}\left(45\,b^2x^6 + 130\,abx^4 + 117\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*x^6 + 130*a*b*x^4 + 117*a^2*x^2)*sqrt(x)
```

giac [A] time = 0.63, size = 24, normalized size = 0.67

$$\frac{2}{13}\,b^2x^{\frac{13}{2}} + \frac{4}{9}\,abx^{\frac{9}{2}} + \frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/13*b^2*x^(13/2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(45b^2x^4+130ab\,x^2+117a^2\right)x^{\frac{5}{2}}}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2,x)
```

```
[Out] 2/585*x^(5/2)*(45*b^2*x^4+130*a*b*x^2+117*a^2)
```

maxima [A] time = 1.36, size = 24, normalized size = 0.67

$$\frac{2}{13}\,b^2x^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/13*b^2*x^(13/2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.72

$$\frac{2\,x^{5/2}\,\left(117\,a^2+130\,a\,b\,x^2+45\,b^2\,x^4\right)}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2)^2,x)
```

```
[Out] (2*x^(5/2)*(117*a^2 + 45*b^2*x^4 + 130*a*b*x^2))/585
```

sympy [A] time = 2.55, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(13/2)/13
```

3.275 $\int \sqrt{x}\left(a+bx^2\right)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}b^2x^{11/2}$$

```
[Out] 2/3*a^2*x^(3/2)+4/7*a*b*x^(7/2)+2/11*b^2*x^(11/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}b^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2,x]
```

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(7/2))/7 + (2*b^2*x^(11/2))/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^2\,dx &= \int \left(a^2\sqrt{x}+2abx^{5/2}+b^2x^{9/2}\right)dx\\ &= \frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}b^2x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2}{231}x^{3/2}\left(77a^2+66abx^2+21b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2,x]
```

```
[Out] (2*x^(3/2)*(77*a^2 + 66*a*b*x^2 + 21*b^2*x^4))/231
```

fricas [A] time = 0.82, size = 27, normalized size = 0.75

$$\frac{2}{231}\left(21\,b^2x^5+66\,abx^3+77\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*b^2*x^5 + 66*a*b*x^3 + 77*a^2*x)*sqrt(x)
```

giac [A] time = 0.58, size = 24, normalized size = 0.67

$$\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.75

$$\frac{2\left(21b^{2}x^{4}+66ab\,x^{2}+77a^{2}\right)x^{\frac{3}{2}}}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*x^(1/2),x)
```

```
[Out] 2/231*x^(3/2)*(21*b^2*x^4+66*a*b*x^2+77*a^2)
```

maxima [A] time = 1.34, size = 24, normalized size = 0.67

$$\frac{2}{11}\,b^{2}x^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^{2}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.72

$$\frac{2\,x^{3/2}\,\left(77\,a^2+66\,a\,b\,x^2+21\,b^2\,x^4\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2)^2,x)
```

```
[Out] (2*x^(3/2)*(77*a^2 + 21*b^2*x^4 + 66*a*b*x^2))/231
```

sympy [A] time = 1.70, size = 34, normalized size = 0.94

$$\frac{2a^{2}x^{\frac{3}{2}}}{3}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{2b^{2}x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*x**(1/2),x)
```

```
[Out] 2*a**2*x**(3/2)/3 + 4*a*b*x**(7/2)/7 + 2*b**2*x**(11/2)/11
```

3.276 $\int \frac{(a+bx^2)^2}{\sqrt{x}} \, dx$

Optimal. Leaf size=34

$$2a^2\sqrt{x} + \frac{4}{5}abx^{5/2} + \frac{2}{9}b^2x^{9/2}$$

```
[Out] 4/5*a*b*x^(5/2)+2/9*b^2*x^(9/2)+2*a^2*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$2a^2\sqrt{x} + \frac{4}{5}abx^{5/2} + \frac{2}{9}b^2x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/Sqrt[x],x]
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*b^2*x^(9/2))/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{\sqrt{x}} \, dx &= \int \left(\frac{a^2}{\sqrt{x}} + 2abx^{3/2} + b^2x^{7/2}\right) dx \\ &= 2a^2\sqrt{x} + \frac{4}{5}abx^{5/2} + \frac{2}{9}b^2x^{9/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.88

$$\frac{2}{45}\sqrt{x}\left(45a^2 + 18abx^2 + 5b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/Sqrt[x],x]
[Out] (2*Sqrt[x]*(45*a^2 + 18*a*b*x^2 + 5*b^2*x^4))/45
```

fricas [A] time = 1.03, size = 26, normalized size = 0.76

$$\frac{2}{45}\left(5\,b^2x^4 + 18\,abx^2 + 45\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
[Out] 2/45*(5*b^2*x^4 + 18*a*b*x^2 + 45*a^2)*sqrt(x)
```

giac [A] time = 0.63, size = 24, normalized size = 0.71

$$\frac{2}{9}\,b^2x^{\frac{9}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + 2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(1/2),x, algorithm="giac")
[Out] 2/9*b^2*x^(9/2) + 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.79

$$\frac{2\left(5b^2x^4 + 18ab\,x^2 + 45a^2\right)\sqrt{x}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(1/2),x)
[Out] 2/45*x^(1/2)*(5*b^2*x^4+18*a*b*x^2+45*a^2)
```

maxima [A] time = 1.30, size = 24, normalized size = 0.71

$$\frac{2}{9}\,b^2x^{\frac{9}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + 2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
[Out] 2/9*b^2*x^(9/2) + 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

mupad [B] time = 0.03, size = 26, normalized size = 0.76

$$\frac{2\,\sqrt{x}\,\left(45\,a^2 + 18\,a\,b\,x^2 + 5\,b^2\,x^4\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^(1/2),x)
[Out] (2*x^(1/2)*(45*a^2 + 5*b^2*x^4 + 18*a*b*x^2))/45
```

sympy [A] time = 0.75, size = 32, normalized size = 0.94

$$2a^2\sqrt{x} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(1/2),x)
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/9
```

3.277 $\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}b^2x^{7/2}$$

```
[Out] 4/3*a*b*x^(3/2)+2/7*b^2*x^(7/2)-2*a^2/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$-\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}b^2x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*b^2*x^(7/2))/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx &= \int \left(\frac{a^2}{x^{3/2}}+2ab\sqrt{x}+b^2x^{5/2}\right)dx\\ &= -\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}b^2x^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.88

$$\frac{2\left(-21a^2+14abx^2+3b^2x^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^(3/2),x]
```

```
[Out] (2*(-21*a^2 + 14*a*b*x^2 + 3*b^2*x^4))/(21*Sqrt[x])
```

fricas [A] time = 0.83, size = 26, normalized size = 0.76

$$\frac{2\left(3\,b^2x^4+14\,abx^2-21\,a^2\right)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b^2*x^4 + 14*a*b*x^2 - 21*a^2)/sqrt(x)
```

giac [A] time = 0.63, size = 24, normalized size = 0.71

$$\frac{2}{7}\,b^2x^{\frac{7}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2),x, algorithm="giac")
[Out] 2/7*b^2*x^(7/2) + 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.79

$$-\frac{2\left(-3b^2x^4-14ab\,x^2+21a^2\right)}{21\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(3/2),x)
[Out] -2/21*(-3*b^2*x^4-14*a*b*x^2+21*a^2)/x^(1/2)
```

maxima [A] time = 1.33, size = 24, normalized size = 0.71

$$\frac{2}{7}\,b^2x^{\frac{7}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2),x, algorithm="maxima")
[Out] 2/7*b^2*x^(7/2) + 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.76

$$\frac{-42\,a^2+28\,a\,b\,x^2+6\,b^2\,x^4}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^(3/2),x)
[Out] (6*b^2*x^4 - 42*a^2 + 28*a*b*x^2)/(21*x^(1/2))
```

sympy [A] time = 0.94, size = 32, normalized size = 0.94

$$-\frac{2a^2}{\sqrt{x}}+\frac{4abx^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(3/2),x)
[Out] -2*a**2/sqrt(x) + 4*a*b*x**(3/2)/3 + 2*b**2*x**(7/2)/7
```

3.278 $\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}b^2x^{5/2}$$

```
[Out] -2/3*a^2/x^(3/2)+2/5*b^2*x^(5/2)+4*a*b*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$-\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}b^2x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^(5/2),x]
[Out] (-2*a^2)/(3*x^(3/2)) + 4*a*b*Sqrt[x] + (2*b^2*x^(5/2))/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx &= \int \left(\frac{a^2}{x^{5/2}}+\frac{2ab}{\sqrt{x}}+b^2x^{3/2}\right)dx\\ &= -\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}b^2x^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.88

$$\frac{2\left(-5a^2+30abx^2+3b^2x^4\right)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^(5/2),x]
[Out] (2*(-5*a^2 + 30*a*b*x^2 + 3*b^2*x^4))/(15*x^(3/2))
```

fricas [A] time = 0.68, size = 26, normalized size = 0.76

$$\frac{2\left(3\,b^2x^4+30\,abx^2-5\,a^2\right)}{15\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^4 + 30*a*b*x^2 - 5*a^2)/x^(3/2)
```

giac [A] time = 0.64, size = 24, normalized size = 0.71

$$\frac{2}{5}\,b^2x^{\frac{5}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2),x, algorithm="giac")
[Out] 2/5*b^2*x^(5/2) + 4*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.79

$$-\frac{2\left(-3b^2x^4 - 30ab\,x^2 + 5a^2\right)}{15x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(5/2),x)
[Out] -2/15*(-3*b^2*x^4-30*a*b*x^2+5*a^2)/x^(3/2)
```

maxima [A] time = 1.35, size = 24, normalized size = 0.71

$$\frac{2}{5}\,b^2x^{\frac{5}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2),x, algorithm="maxima")
[Out] 2/5*b^2*x^(5/2) + 4*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.76

$$\frac{-10\,a^2 + 60\,a\,b\,x^2 + 6\,b^2\,x^4}{15\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^(5/2),x)
[Out] (6*b^2*x^4 - 10*a^2 + 60*a*b*x^2)/(15*x^(3/2))
```

sympy [A] time = 1.11, size = 32, normalized size = 0.94

$$-\frac{2a^2}{3x^{\frac{3}{2}}} + 4ab\sqrt{x} + \frac{2b^2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(5/2),x)
[Out] -2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5/2)/5
```

3.279 $\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}b^2x^{3/2}$$

```
[Out] -2/5*a^2/x^(5/2)+2/3*b^2*x^(3/2)-4*a*b/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$-\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}b^2x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^(7/2),x]
```

```
[Out] (-2*a^2)/(5*x^(5/2)) - (4*a*b)/Sqrt[x] + (2*b^2*x^(3/2))/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx &= \int \left(\frac{a^2}{x^{7/2}}+\frac{2ab}{x^{3/2}}+b^2\sqrt{x}\right)dx\\ &= -\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}b^2x^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.88

$$\frac{2\left(-3a^2-30abx^2+5b^2x^4\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^(7/2),x]
```

```
[Out] (2*(-3*a^2 - 30*a*b*x^2 + 5*b^2*x^4))/(15*x^(5/2))
```

fricas [A] time = 0.65, size = 26, normalized size = 0.76

$$\frac{2\left(5\,b^2x^4-30\,abx^2-3\,a^2\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*b^2*x^4 - 30*a*b*x^2 - 3*a^2)/x^(5/2)
```

giac [A] time = 0.60, size = 25, normalized size = 0.74

$$\frac{2}{3}\,b^{2}x^{\frac{3}{2}}-\frac{2\left(10\,abx^{2}+a^{2}\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2),x, algorithm="giac")
[Out] 2/3*b^2*x^(3/2) - 2/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.79

$$-\frac{2\left(-5b^{2}x^{4}+30ab\,x^{2}+3a^{2}\right)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(7/2),x)
[Out] -2/15*(-5*b^2*x^4+30*a*b*x^2+3*a^2)/x^(5/2)
```

maxima [A] time = 1.31, size = 25, normalized size = 0.74

$$\frac{2}{3}\,b^{2}x^{\frac{3}{2}}-\frac{2\left(10\,abx^{2}+a^{2}\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2),x, algorithm="maxima")
[Out] 2/3*b^2*x^(3/2) - 2/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

mupad [B] time = 0.03, size = 26, normalized size = 0.76

$$-\frac{6\,a^{2}+60\,a\,b\,x^{2}-10\,b^{2}\,x^{4}}{15\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^(7/2),x)
[Out] -(6*a^2 - 10*b^2*x^4 + 60*a*b*x^2)/(15*x^(5/2))
```

sympy [A] time = 1.71, size = 32, normalized size = 0.94

$$-\frac{2a^{2}}{5x^{\frac{5}{2}}}-\frac{4ab}{\sqrt{x}}+\frac{2b^{2}x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(7/2),x)
[Out] -2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 2*b**2*x**(3/2)/3
```

3.280 $\int x^{7/2}\left(a+bx^2\right)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{9}a^3x^{9/2}+\frac{6}{13}a^2bx^{13/2}+\frac{6}{17}ab^2x^{17/2}+\frac{2}{21}b^3x^{21/2}$$

```
[Out] 2/9*a^3*x^(9/2)+6/13*a^2*b*x^(13/2)+6/17*a*b^2*x^(17/2)+2/21*b^3*x^(21/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{6}{13}a^2bx^{13/2}+\frac{2}{9}a^3x^{9/2}+\frac{6}{17}ab^2x^{17/2}+\frac{2}{21}b^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(9/2))/9 + (6*a^2*b*x^(13/2))/13 + (6*a*b^2*x^(17/2))/17 + (2*b^3*
x^(21/2))/21
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^3\,dx &= \int \left(a^3x^{7/2}+3a^2bx^{11/2}+3ab^2x^{15/2}+b^3x^{19/2}\right)dx\\ &= \frac{2}{9}a^3x^{9/2}+\frac{6}{13}a^2bx^{13/2}+\frac{6}{17}ab^2x^{17/2}+\frac{2}{21}b^3x^{21/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{9/2}\left(1547a^3+3213a^2bx^2+2457ab^2x^4+663b^3x^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*x^(9/2)*(1547*a^3 + 3213*a^2*b*x^2 + 2457*a*b^2*x^4 + 663*b^3*x^6))/1392
3
```

fricas [A] time = 0.64, size = 40, normalized size = 0.78

$$\frac{2}{13923}\left(663\,b^3x^{10}+2457\,ab^2x^8+3213\,a^2bx^6+1547\,a^3x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^3*x^10 + 2457*a*b^2*x^8 + 3213*a^2*b*x^6 + 1547*a^3*x^4)*sqr
t(x)
```

giac [A] time = 0.61, size = 35, normalized size = 0.69

$$\frac{2}{21}\,b^3x^{\frac{21}{2}} + \frac{6}{17}\,ab^2x^{\frac{17}{2}} + \frac{6}{13}\,a^2bx^{\frac{13}{2}} + \frac{2}{9}\,a^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/21*b^3*x^(21/2) + 6/17*a*b^2*x^(17/2) + 6/13*a^2*b*x^(13/2) + 2/9*a^3*x^(
9/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(663b^3x^6+2457a\,b^2x^4+3213a^2b\,x^2+1547a^3\right)x^{\frac{9}{2}}}{13923}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^3,x)
```

```
[Out] 2/13923*x^(9/2)*(663*b^3*x^6+2457*a*b^2*x^4+3213*a^2*b*x^2+1547*a^3)
```

maxima [A] time = 1.34, size = 35, normalized size = 0.69

$$\frac{2}{21}\,b^3x^{\frac{21}{2}} + \frac{6}{17}\,ab^2x^{\frac{17}{2}} + \frac{6}{13}\,a^2bx^{\frac{13}{2}} + \frac{2}{9}\,a^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/21*b^3*x^(21/2) + 6/17*a*b^2*x^(17/2) + 6/13*a^2*b*x^(13/2) + 2/9*a^3*x^(
9/2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{9/2}}{9}+\frac{2\,b^3\,x^{21/2}}{21}+\frac{6\,a^2\,b\,x^{13/2}}{13}+\frac{6\,a\,b^2\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2)^3,x)
```

```
[Out] (2*a^3*x^(9/2))/9 + (2*b^3*x^(21/2))/21 + (6*a^2*b*x^(13/2))/13 + (6*a*b^2*
x^(17/2))/17
```

sympy [A] time = 19.69, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{9}{2}}}{9}+\frac{6a^2bx^{\frac{13}{2}}}{13}+\frac{6ab^2x^{\frac{17}{2}}}{17}+\frac{2b^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(9/2)/9 + 6*a**2*b*x**(13/2)/13 + 6*a*b**2*x**(17/2)/17 + 2*b**3*
x**(21/2)/21
```

3.281 $\int x^{5/2}\left(a+bx^2\right)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{2}{5}ab^2x^{15/2}+\frac{2}{19}b^3x^{19/2}$$

```
[Out] 2/7*a^3*x^(7/2)+6/11*a^2*b*x^(11/2)+2/5*a*b^2*x^(15/2)+2/19*b^3*x^(19/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{6}{11}a^2bx^{11/2}+\frac{2}{7}a^3x^{7/2}+\frac{2}{5}ab^2x^{15/2}+\frac{2}{19}b^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*b^2*x^(15/2))/5 + (2*b^3*x
^(19/2))/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^3\,dx&=\int\left(a^3x^{5/2}+3a^2bx^{9/2}+3ab^2x^{13/2}+b^3x^{17/2}\right)\,dx\\&=\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{2}{5}ab^2x^{15/2}+\frac{2}{19}b^3x^{19/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{7/2}\left(1045a^3+1995a^2bx^2+1463ab^2x^4+385b^3x^6\right)}{7315}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*x^(7/2)*(1045*a^3 + 1995*a^2*b*x^2 + 1463*a*b^2*x^4 + 385*b^3*x^6))/7315
```

fricas [A] time = 0.67, size = 40, normalized size = 0.78

$$\frac{2}{7315}\left(385\,b^3x^9+1463\,ab^2x^7+1995\,a^2bx^5+1045\,a^3x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^3*x^9 + 1463*a*b^2*x^7 + 1995*a^2*b*x^5 + 1045*a^3*x^3)*sqrt(
x)
```

giac [**A**] time = 0.63, size = 35, normalized size = 0.69

$$\frac{2}{19}\,b^3x^{\frac{19}{2}}+\frac{2}{5}\,ab^2x^{\frac{15}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/19*b^3*x^(19/2) + 2/5*a*b^2*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(7
/2)
```

maple [**A**] time = 0.00, size = 38, normalized size = 0.75

$$\frac{2\left(385b^3x^6+1463a\,b^2x^4+1995a^2b\,x^2+1045a^3\right)x^{\frac{7}{2}}}{7315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^3,x)
```

```
[Out] 2/7315*x^(7/2)*(385*b^3*x^6+1463*a*b^2*x^4+1995*a^2*b*x^2+1045*a^3)
```

maxima [**A**] time = 1.34, size = 35, normalized size = 0.69

$$\frac{2}{19}\,b^3x^{\frac{19}{2}}+\frac{2}{5}\,ab^2x^{\frac{15}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/19*b^3*x^(19/2) + 2/5*a*b^2*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(7
/2)
```

mupad [**B**] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{7/2}}{7}+\frac{2\,b^3\,x^{19/2}}{19}+\frac{6\,a^2\,b\,x^{11/2}}{11}+\frac{2\,a\,b^2\,x^{15/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2)^3,x)
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*b^3*x^(19/2))/19 + (6*a^2*b*x^(11/2))/11 + (2*a*b^2*
x^(15/2))/5
```

sympy [**A**] time = 10.62, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{7}{2}}}{7}+\frac{6a^2bx^{\frac{11}{2}}}{11}+\frac{2ab^2x^{\frac{15}{2}}}{5}+\frac{2b^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 6*a**2*b*x**(11/2)/11 + 2*a*b**2*x**(15/2)/5 + 2*b**3*x
**(19/2)/19
```

3.282 $\int x^{3/2} \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{13}ab^2x^{13/2}+\frac{2}{17}b^3x^{17/2}$$

```
[Out] 2/5*a^3*x^(5/2)+2/3*a^2*b*x^(9/2)+6/13*a*b^2*x^(13/2)+2/17*b^3*x^(17/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{2}{3}a^2bx^{9/2}+\frac{2}{5}a^3x^{5/2}+\frac{6}{13}ab^2x^{13/2}+\frac{2}{17}b^3x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^(13/2))/13 + (2*b^3*x^
(17/2))/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^3\,dx&=\int\left(a^3x^{3/2}+3a^2bx^{7/2}+3ab^2x^{11/2}+b^3x^{15/2}\right)\,dx\\&=\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{13}ab^2x^{13/2}+\frac{2}{17}b^3x^{17/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{5/2}\left(663a^3+1105a^2bx^2+765ab^2x^4+195b^3x^6\right)}{3315}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*x^(5/2)*(663*a^3 + 1105*a^2*b*x^2 + 765*a*b^2*x^4 + 195*b^3*x^6))/3315
```

fricas [A] time = 0.85, size = 40, normalized size = 0.78

$$\frac{2}{3315}\left(195\,b^3x^8+765\,ab^2x^6+1105\,a^2bx^4+663\,a^3x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/3315*(195*b^3*x^8 + 765*a*b^2*x^6 + 1105*a^2*b*x^4 + 663*a^3*x^2)*sqrt(x)
```

giac [A] time = 0.63, size = 35, normalized size = 0.69

$$\frac{2}{17}\,b^3x^{\frac{17}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/17*b^3*x^(17/2) + 6/13*a*b^2*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5/
2)
```

maple [A] time = 0.00, size = 38, normalized size = 0.75

$$\frac{2\left(195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3\right)x^{\frac{5}{2}}}{3315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^3,x)
```

```
[Out] 2/3315*x^(5/2)*(195*b^3*x^6+765*a*b^2*x^4+1105*a^2*b*x^2+663*a^3)
```

maxima [A] time = 1.37, size = 35, normalized size = 0.69

$$\frac{2}{17}\,b^3x^{\frac{17}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/17*b^3*x^(17/2) + 6/13*a*b^2*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5/
2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{5/2}}{5}+\frac{2\,b^3\,x^{17/2}}{17}+\frac{2\,a^2\,b\,x^{9/2}}{3}+\frac{6\,a\,b^2\,x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2)^3,x)
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*b^3*x^(17/2))/17 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^
(13/2))/13
```

sympy [A] time = 5.73, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{2a^2bx^{\frac{9}{2}}}{3}+\frac{6ab^2x^{\frac{13}{2}}}{13}+\frac{2b^3x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 2*a**2*b*x**(9/2)/3 + 6*a*b**2*x**(13/2)/13 + 2*b**3*x*
*(17/2)/17
```

3.283 $\int \sqrt{x}\left(a+bx^2\right)^3 dx$

Optimal. Leaf size=51

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{15}b^3x^{15/2}$$

```
[Out] 2/3*a^3*x^(3/2)+6/7*a^2*b*x^(7/2)+6/11*a*b^2*x^(11/2)+2/15*b^3*x^(15/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}a^3x^{3/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{15}b^3x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*b^2*x^(11/2))/11 + (2*b^3*x^
(15/2))/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^3 dx &= \int \left(a^3\sqrt{x}+3a^2bx^{5/2}+3ab^2x^{9/2}+b^3x^{13/2}\right) dx\\ &= \frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{15}b^3x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{3/2}\left(385a^3+495a^2bx^2+315ab^2x^4+77b^3x^6\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^3,x]
```

```
[Out] (2*x^(3/2)*(385*a^3 + 495*a^2*b*x^2 + 315*a*b^2*x^4 + 77*b^3*x^6))/1155
```

fricas [A] time = 0.92, size = 38, normalized size = 0.75

$$\frac{2}{1155}\left(77\,b^3x^7+315\,ab^2x^5+495\,a^2bx^3+385\,a^3x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^3*x^7 + 315*a*b^2*x^5 + 495*a^2*b*x^3 + 385*a^3*x)*sqrt(x)
```

giac [A] time = 0.58, size = 35, normalized size = 0.69

$$\frac{2}{15}\,b^3x^{\frac{15}{2}}+\frac{6}{11}\,ab^2x^{\frac{11}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*b^3*x^(15/2) + 6/11*a*b^2*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3/
2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(77b^3x^6+315a\,b^2x^4+495a^2b\,x^2+385a^3\right)x^{\frac{3}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*x^(1/2),x)
```

```
[Out] 2/1155*x^(3/2)*(77*b^3*x^6+315*a*b^2*x^4+495*a^2*b*x^2+385*a^3)
```

maxima [A] time = 1.34, size = 35, normalized size = 0.69

$$\frac{2}{15}\,b^3x^{\frac{15}{2}}+\frac{6}{11}\,ab^2x^{\frac{11}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*b^3*x^(15/2) + 6/11*a*b^2*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3/
2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{3/2}}{3}+\frac{2\,b^3\,x^{15/2}}{15}+\frac{6\,a^2\,b\,x^{7/2}}{7}+\frac{6\,a\,b^2\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2)^3,x)
```

```
[Out] (2*a^3*x^(3/2))/3 + (2*b^3*x^(15/2))/15 + (6*a^2*b*x^(7/2))/7 + (6*a*b^2*x^
(11/2))/11
```

sympy [A] time = 2.17, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{3}{2}}}{3}+\frac{6a^2bx^{\frac{7}{2}}}{7}+\frac{6ab^2x^{\frac{11}{2}}}{11}+\frac{2b^3x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*x**(1/2),x)
```

```
[Out] 2*a**3*x**(3/2)/3 + 6*a**2*b*x**(7/2)/7 + 6*a*b**2*x**(11/2)/11 + 2*b**3*x*
*(15/2)/15
```

3.284 $\int \frac{\left(a+bx^2\right)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=49

$$2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{13}b^3x^{13/2}$$

```
[Out] 6/5*a^2*b*x^(5/2)+2/3*a*b^2*x^(9/2)+2/13*b^3*x^(13/2)+2*a^3*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{6}{5}a^2bx^{5/2}+2a^3\sqrt{x}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{13}b^3x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*b^2*x^(9/2))/3 + (2*b^3*x^(13/2)
)/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^3}{\sqrt{x}}\,dx &= \int \left(\frac{a^3}{\sqrt{x}}+3a^2bx^{3/2}+3ab^2x^{7/2}+b^3x^{11/2}\right)dx\\ &= 2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{13}b^3x^{13/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.84

$$\frac{2}{195}\sqrt{x}\left(195a^3+117a^2bx^2+65ab^2x^4+15b^3x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(195*a^3 + 117*a^2*b*x^2 + 65*a*b^2*x^4 + 15*b^3*x^6))/195
```

fricas [A] time = 0.85, size = 37, normalized size = 0.76

$$\frac{2}{195}\left(15\,b^3x^6+65\,ab^2x^4+117\,a^2bx^2+195\,a^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/195*(15*b^3*x^6 + 65*a*b^2*x^4 + 117*a^2*b*x^2 + 195*a^3)*sqrt(x)
```

giac [A] time = 0.59, size = 35, normalized size = 0.71

$$\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(1/2),x, algorithm="giac")
[Out] 2/13*b^3*x^(13/2) + 2/3*a*b^2*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)
```

maple [A] time = 0.01, size = 38, normalized size = 0.78

$$\frac{2\left(15b^3x^6+65a\,b^2x^4+117a^2b\,x^2+195a^3\right)\sqrt{x}}{195}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(1/2),x)
[Out] 2/195*x^(1/2)*(15*b^3*x^6+65*a*b^2*x^4+117*a^2*b*x^2+195*a^3)
```

maxima [A] time = 1.34, size = 35, normalized size = 0.71

$$\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
[Out] 2/13*b^3*x^(13/2) + 2/3*a*b^2*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.71

$$2\,a^3\,\sqrt{x}+\frac{2\,b^3\,x^{13/2}}{13}+\frac{6\,a^2\,b\,x^{5/2}}{5}+\frac{2\,a\,b^2\,x^{9/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^(1/2),x)
[Out] 2*a^3*x^(1/2) + (2*b^3*x^(13/2))/13 + (6*a^2*b*x^(5/2))/5 + (2*a*b^2*x^(9/2
))/3
```

sympy [A] time = 2.10, size = 48, normalized size = 0.98

$$2a^3\sqrt{x}+\frac{6a^2bx^{\frac{5}{2}}}{5}+\frac{2ab^2x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(1/2),x)
[Out] 2*a**3*sqrt(x) + 6*a**2*b*x**(5/2)/5 + 2*a*b**2*x**(9/2)/3 + 2*b**3*x**(13/
2)/13
```

3.285 $\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{11}b^3x^{11/2}$$

```
[Out] 2*a^2*b*x^(3/2)+6/7*a*b^2*x^(7/2)+2/11*b^3*x^(11/2)-2*a^3/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$2a^2bx^{3/2}-\frac{2a^3}{\sqrt{x}}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{11}b^3x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(7/2))/7 + (2*b^3*x^(11/2))
/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx &= \int \left(\frac{a^3}{x^{3/2}}+3a^2b\sqrt{x}+3ab^2x^{5/2}+b^3x^{9/2}\right)dx\\ &= -\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{11}b^3x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.87

$$\frac{2\left(-77a^3+77a^2bx^2+33ab^2x^4+7b^3x^6\right)}{77\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^(3/2),x]
```

```
[Out] (2*(-77*a^3 + 77*a^2*b*x^2 + 33*a*b^2*x^4 + 7*b^3*x^6))/(77*Sqrt[x])
```

fricas [A] time = 0.87, size = 37, normalized size = 0.79

$$\frac{2\left(7\,b^3x^6+33\,ab^2x^4+77\,a^2bx^2-77\,a^3\right)}{77\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b^3*x^6 + 33*a*b^2*x^4 + 77*a^2*b*x^2 - 77*a^3)/sqrt(x)
```

giac [A] time = 0.58, size = 35, normalized size = 0.74

$$\frac{2}{11}\,b^{3}x^{\frac{11}{2}}+\frac{6}{7}\,ab^{2}x^{\frac{7}{2}}+2\,a^{2}bx^{\frac{3}{2}}-\frac{2\,a^{3}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(3/2),x, algorithm="giac")
[Out] 2/11*b^3*x^(11/2) + 6/7*a*b^2*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

maple [A] time = 0.01, size = 38, normalized size = 0.81

$$-\frac{2\left(-7b^{3}x^{6}-33a\,b^{2}x^{4}-77a^{2}b\,x^{2}+77a^{3}\right)}{77\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(3/2),x)
[Out] -2/77*(-7*b^3*x^6-33*a*b^2*x^4-77*a^2*b*x^2+77*a^3)/x^(1/2)
```

maxima [A] time = 1.39, size = 35, normalized size = 0.74

$$\frac{2}{11}\,b^{3}x^{\frac{11}{2}}+\frac{6}{7}\,ab^{2}x^{\frac{7}{2}}+2\,a^{2}bx^{\frac{3}{2}}-\frac{2\,a^{3}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(3/2),x, algorithm="maxima")
[Out] 2/11*b^3*x^(11/2) + 6/7*a*b^2*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.74

$$\frac{2\,b^{3}\,x^{11/2}}{11}-\frac{2\,a^{3}}{\sqrt{x}}+2\,a^{2}\,b\,x^{3/2}+\frac{6\,a\,b^{2}\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^(3/2),x)
[Out] (2*b^3*x^(11/2))/11 - (2*a^3)/x^(1/2) + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(7/2))
/7
```

sympy [A] time = 2.31, size = 46, normalized size = 0.98

$$-\frac{2a^{3}}{\sqrt{x}}+2a^{2}bx^{\frac{3}{2}}+\frac{6ab^{2}x^{\frac{7}{2}}}{7}+\frac{2b^{3}x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(3/2),x)
[Out] -2*a**3/sqrt(x) + 2*a**2*b*x**(3/2) + 6*a*b**2*x**(7/2)/7 + 2*b**3*x**(11/2
)/11
```

3.286 $\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{9}b^3x^{9/2}$$

```
[Out] -2/3*a^3/x^(3/2)+6/5*a*b^2*x^(5/2)+2/9*b^3*x^(9/2)+6*a^2*b*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$6a^2b\sqrt{x}-\frac{2a^3}{3x^{3/2}}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{9}b^3x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(9/
2))/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx &= \int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{\sqrt{x}}+3ab^2x^{3/2}+b^3x^{7/2}\right)dx\\ &= -\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{9}b^3x^{9/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.84

$$\frac{2\left(-15a^3+135a^2bx^2+27ab^2x^4+5b^3x^6\right)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^(5/2),x]
```

```
[Out] (2*(-15*a^3 + 135*a^2*b*x^2 + 27*a*b^2*x^4 + 5*b^3*x^6))/(45*x^(3/2))
```

fricas [A] time = 0.94, size = 37, normalized size = 0.76

$$\frac{2\left(5\,b^3x^6+27\,ab^2x^4+135\,a^2bx^2-15\,a^3\right)}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^3*x^6 + 27*a*b^2*x^4 + 135*a^2*b*x^2 - 15*a^3)/x^(3/2)
```

giac [A] time = 0.58, size = 35, normalized size = 0.71

$$\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{6}{5}\,ab^2x^{\frac{5}{2}}+6\,a^2b\sqrt{x}-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*b^3*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.78

$$-\frac{2\left(-5b^3x^6-27a\,b^2x^4-135a^2b\,x^2+15a^3\right)}{45x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(5/2),x)
```

```
[Out] -2/45*(-5*b^3*x^6-27*a*b^2*x^4-135*a^2*b*x^2+15*a^3)/x^(3/2)
```

maxima [A] time = 1.35, size = 35, normalized size = 0.71

$$\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{6}{5}\,ab^2x^{\frac{5}{2}}+6\,a^2b\sqrt{x}-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*b^3*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.71

$$\frac{2\,b^3\,x^{9/2}}{9}-\frac{2\,a^3}{3\,x^{3/2}}+6\,a^2\,b\,\sqrt{x}+\frac{6\,a\,b^2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^(5/2),x)
```

```
[Out] (2*b^3*x^(9/2))/9 - (2*a^3)/(3*x^(3/2)) + 6*a^2*b*x^(1/2) + (6*a*b^2*x^(5/2
))/5
```

sympy [A] time = 2.80, size = 48, normalized size = 0.98

$$-\frac{2a^3}{3x^{\frac{3}{2}}}+6a^2b\sqrt{x}+\frac{6ab^2x^{\frac{5}{2}}}{5}+\frac{2b^3x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) + 6*a**2*b*sqrt(x) + 6*a*b**2*x**(5/2)/5 + 2*b**3*x**(
9/2)/9
```

3.287 $\int \frac{(a+bx^2)^3}{x^{7/2}} dx$

Optimal. Leaf size=47

$$-\frac{2a^3}{5x^{5/2}} - \frac{6a^2b}{\sqrt{x}} + 2ab^2x^{3/2} + \frac{2}{7}b^3x^{7/2}$$

```
[Out] -2/5*a^3/x^(5/2)+2*a*b^2*x^(3/2)+2/7*b^3*x^(7/2)-6*a^2*b/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$-\frac{6a^2b}{\sqrt{x}} - \frac{2a^3}{5x^{5/2}} + 2ab^2x^{3/2} + \frac{2}{7}b^3x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^(7/2),x]
```

```
[Out] (-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(7/2)
)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^{7/2}} dx &= \int \left(\frac{a^3}{x^{7/2}} + \frac{3a^2b}{x^{3/2}} + 3ab^2\sqrt{x} + b^3x^{5/2}\right) dx \\
&= -\frac{2a^3}{5x^{5/2}} - \frac{6a^2b}{\sqrt{x}} + 2ab^2x^{3/2} + \frac{2}{7}b^3x^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.87

$$\frac{2\left(-7a^3 - 105a^2bx^2 + 35ab^2x^4 + 5b^3x^6\right)}{35x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^(7/2),x]
```

```
[Out] (2*(-7*a^3 - 105*a^2*b*x^2 + 35*a*b^2*x^4 + 5*b^3*x^6))/(35*x^(5/2))
```

fricas [A] time = 0.75, size = 37, normalized size = 0.79

$$\frac{2\left(5\,b^3x^6 + 35\,ab^2x^4 - 105\,a^2bx^2 - 7\,a^3\right)}{35\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^6 + 35*a*b^2*x^4 - 105*a^2*b*x^2 - 7*a^3)/x^(5/2)
```

giac [**A**] time = 0.61, size = 36, normalized size = 0.77

$$\frac{2}{7} b^3 x^{\frac{7}{2}} + 2 ab^2 x^{\frac{3}{2}} - \frac{2 \left(15 a^2 b x^2 + a^3\right)}{5 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(7/2),x, algorithm="giac")
[Out] 2/7*b^3*x^(7/2) + 2*a*b^2*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

maple [**A**] time = 0.00, size = 38, normalized size = 0.81

$$-\frac{2 \left(-5 b^3 x^6 - 35 a b^2 x^4 + 105 a^2 b x^2 + 7 a^3\right)}{35 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(7/2),x)
[Out] -2/35*(-5*b^3*x^6-35*a*b^2*x^4+105*a^2*b*x^2+7*a^3)/x^(5/2)
```

maxima [**A**] time = 1.35, size = 36, normalized size = 0.77

$$\frac{2}{7} b^3 x^{\frac{7}{2}} + 2 ab^2 x^{\frac{3}{2}} - \frac{2 \left(15 a^2 b x^2 + a^3\right)}{5 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(7/2),x, algorithm="maxima")
[Out] 2/7*b^3*x^(7/2) + 2*a*b^2*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

mupad [**B**] time = 0.04, size = 37, normalized size = 0.79

$$-\frac{14 a^3 + 210 a^2 b x^2 - 70 a b^2 x^4 - 10 b^3 x^6}{35 x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^(7/2),x)
[Out] -(14*a^3 - 10*b^3*x^6 + 210*a^2*b*x^2 - 70*a*b^2*x^4)/(35*x^(5/2))
```

sympy [**A**] time = 3.86, size = 46, normalized size = 0.98

$$-\frac{2a^3}{5x^{\frac{5}{2}}} - \frac{6a^2 b}{\sqrt{x}} + 2ab^2 x^{\frac{3}{2}} + \frac{2b^3 x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(7/2),x)
[Out] -2*a**3/(5*x**(5/2)) - 6*a**2*b/sqrt(x) + 2*a*b**2*x**(3/2) + 2*b**3*x**(7/
2)/7
```

3.288 $\int \frac{x^{7/2}}{a+bx^2}\,dx$

Optimal. Leaf size=215

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}}+$$

```
[Out] 2/5*x^(5/2)/b-1/2*a^(5/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(9/4)
*2^(1/2)+1/2*a^(5/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(9/4)*2^(1
/2)-1/4*a^(5/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(9/
4)*2^(1/2)+1/4*a^(5/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2)
)/b^(9/4)*2^(1/2)-2*a*x^(1/2)/b^2
```

Rubi [A] time = 0.20, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2),x]
```

```
[Out] (-2*a*Sqrt[x])/b^2 + (2*x^(5/2))/(5*b) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4)) - (a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4)) + (a^(5/4)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}}{a+bx^2}\,dx &= \frac{2x^{5/2}}{5b}-\frac{a\int \frac{x^{3/2}}{a+bx^2}\,dx}{b}\\
&= -\frac{2a\sqrt{x}}{b^2}+\frac{2x^{5/2}}{5b}+\frac{a^2\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{b^2}\\
&= -\frac{2a\sqrt{x}}{b^2}+\frac{2x^{5/2}}{5b}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{2a\sqrt{x}}{b^2}+\frac{2x^{5/2}}{5b}+\frac{a^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b^2}+\frac{a^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{2a\sqrt{x}}{b^2}+\frac{2x^{5/2}}{5b}+\frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2b^{5/2}}+\frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2a\sqrt{x}}{b^2}+\frac{2x^{5/2}}{5b}-\frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}}+\frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\right.}{2\sqrt{2}\,b^{9/4}}\\
&= -\frac{2a\sqrt{x}}{b^2}+\frac{2x^{5/2}}{5b}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}}+\frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}}-\frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{\right.}{2\sqrt{2}\,b^{
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 203, normalized size = 0.94

$$\frac{-5\sqrt{2}\,a^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+5\sqrt{2}\,a^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-10\sqrt{2}\,a^{5/4}\tan^{-1}}{20b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2),x]
```

```
[Out] (-40*a*b^(1/4)*Sqrt[x] + 8*b^(5/4)*x^(5/2) - 10*Sqrt[2]*a^(5/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 10*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)] - 5*Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 5*Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(20*b^(9/4))
```

fricas [A] time = 0.98, size = 170, normalized size = 0.79

$$\frac{20\,b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{ab^7\sqrt{x}\left(-\frac{a^5}{b^9}\right)^{\frac{3}{4}}-\sqrt{b^4\sqrt{-\frac{a^5}{b^9}}+a^2x}\,b^7\left(-\frac{a^5}{b^9}\right)^{\frac{3}{4}}}{a^5}\right)+5\,b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}}+a\sqrt{x}\right)-5\,b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/10*(20*b^2*(-a^5/b^9)^(1/4)*arctan(-(a*b^7*sqrt(x)*(-a^5/b^9)^(3/4) - sqr
t(b^4*sqrt(-a^5/b^9) + a^2*x)*b^7*(-a^5/b^9)^(3/4))/a^5) + 5*b^2*(-a^5/b^9)
^(1/4)*log(b^2*(-a^5/b^9)^(1/4) + a*sqrt(x)) - 5*b^2*(-a^5/b^9)^(1/4)*log(-
b^2*(-a^5/b^9)^(1/4) + a*sqrt(x)) + 4*(b*x^2 - 5*a)*sqrt(x))/b^2
```

giac [A] time = 0.62, size = 196, normalized size = 0.91

$$\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^3}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^3}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}a\log\left(\sqrt{2}\sqrt{x}\,\left(\right.\right.}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/b^3 + 1/2*sqrt(2)*(a*b^3)^(1/4)*a*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^3 + 1/4*sqrt(2)*(a*b^3)^(1/4
)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 - 1/4*sqrt(2)*(a*b
^3)^(1/4)*a*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 2/5*(b^
4*x^(5/2) - 5*a*b^3*sqrt(x))/b^5
```

maple [A] time = 0.01, size = 152, normalized size = 0.71

$$\frac{2x^{\frac{5}{2}}}{5b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b^2}-\frac{2a\sqrt{}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a),x)
```

```
[Out] 2/5/b*x^(5/2)-2*a/b^2*x^(1/2)+1/4*a/b^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/
4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
)+1/2*a/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*a
/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.08, size = 194, normalized size = 0.90

$$\frac{2\left(bx^{\frac{5}{2}}-5a\sqrt{x}\right)}{5b^2}+\frac{\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{}\right.}{b^{\frac{1}{4}}}}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x^(5/2) - 5*a*sqrt(x))/b^2 + 1/4*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(
2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqr
t(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)
*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b))
+ sqrt(2)*a^(5/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a)
)/b^(1/4) - sqrt(2)*a^(5/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*
x + sqrt(a))/b^(1/4))/b^2
```

mupad [B] time = 4.49, size = 67, normalized size = 0.31

$$\frac{2\,x^{5/2}}{5\,b}-\frac{2\,a\,\sqrt{x}}{b^2}-\frac{(-a)^{5/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{b^{9/4}}+\frac{(-a)^{5/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/4}}\right)\,1\mathrm{i}}{b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2),x)
```

```
[Out] (2*x^(5/2))/(5*b) - (2*a*x^(1/2))/b^2 - ((-a)^(5/4)*atan((b^(1/4)*x^(1/2))/
(-a)^(1/4)))/b^(9/4) + ((-a)^(5/4)*atan((b^(1/4)*x^(1/2)*1i)/(-a)^(1/4))*1i
)/b^(9/4)
```

sympy [A] time = 50.31, size = 192, normalized size = 0.89

$$\begin{cases} \tilde{\infty}x^{\frac{5}{2}} & \text{for } a = \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for } a = \\ \frac{2x^{\frac{9}{2}}}{9a} & \text{for } b = \\ -\frac{\sqrt[4]{-1}\,a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^2}+\frac{\sqrt[4]{-1}\,a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^2}-\frac{\sqrt[4]{-1}\,a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{b^2}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{\frac{5}{2}}}{5b} & \text{otherv} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*b), Eq(a, 0))
, (2*x**(9/2)/(9*a), Eq(b, 0)), (-(-1)**(1/4)*a**(5/4)*(1/b)**(1/4)*log(-(-
1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2) + (-1)**(1/4)*a**(5/4)*
(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2) - (-
1)**(1/4)*a**(5/4)*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(
1/4)))/b**2 - 2*a*sqrt(x)/b**2 + 2*x**(5/2)/(5*b), True))
```

3.289 $\int \frac{x^{5/2}}{a+bx^2} dx$

Optimal. Leaf size=204

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}}-$$

```
[Out] 2/3*x^(3/2)/b+1/2*a^(3/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(7/4)
*2^(1/2)-1/2*a^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(7/4)*2^(1
/2)-1/4*a^(3/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(7/
4)*2^(1/2)+1/4*a^(3/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2)
)/b^(7/4)*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2))/(3*b) + (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*b^(7/4)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/(Sqrt[2]*b^(7/4)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4)) + (a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{a+bx^2}\,dx &= \frac{2x^{3/2}}{3b} - \frac{a\int \frac{\sqrt{x}}{a+bx^2}\,dx}{b}\\
&= \frac{2x^{3/2}}{3b} - \frac{(2a)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b}\\
&= \frac{2x^{3/2}}{3b} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}}\\
&= \frac{2x^{3/2}}{3b} - \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} - \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} - \frac{a^{3/4}\,\text{Subst}}{}\\
&= \frac{2x^{3/2}}{3b} - \frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{7/4}} + \frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{7/4}} - \frac{a^{3/4}\,\text{S}}{}\\
&= \frac{2x^{3/2}}{3b} + \frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}} - \frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}} - \frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{2\sqrt{2}\,b^{7/4}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.38

$$\frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{b^{7/4}} - \frac{(-a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{b^{7/4}} + \frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2))/(3*b) + ((-a)^(3/4)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/b^(7/
4) - ((-a)^(3/4)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/b^(7/4)
```

fricas [A] time = 0.96, size = 165, normalized size = 0.81

$$\frac{12\,b\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^2b^2\sqrt{x}\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}-\sqrt{-a^3b^3\sqrt{-\frac{a^3}{b^7}}+a^4x}\,b^2\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}}{a^3}\right)-3\,b\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}\log\left(b^5\left(-\frac{a^3}{b^7}\right)^{\frac{3}{4}}+a^2\sqrt{x}\right)+3\,b\left(-\frac{a^3}{b^7}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(12*b*(-a^3/b^7)^(1/4)*arctan(-(a^2*b^2*sqrt(x)*(-a^3/b^7)^(1/4) - sqrt
(-a^3*b^3*sqrt(-a^3/b^7) + a^4*x)*b^2*(-a^3/b^7)^(1/4))/a^3) - 3*b*(-a^3/b^
7)^(1/4)*log(b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) + 3*b*(-a^3/b^7)^(1/4)*log
(-b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) + 4*x^(3/2))/b
```

giac [A] time = 0.65, size = 178, normalized size = 0.87

$$\frac{2\,x^{\frac{3}{2}}}{3\,b}-\frac{\sqrt{2}\,\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^4}-\frac{\sqrt{2}\,\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^4}+\frac{\sqrt{2}\,\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\right.}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)/b - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/4*sqrt(2)*
(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 - 1/4*sq
rt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4
```

maple [A] time = 0.00, size = 143, normalized size = 0.70

$$\frac{2x^{\frac{3}{2}}}{3b}-\frac{\sqrt{2}\;a\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}-\frac{\sqrt{2}\;a\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}-\frac{\sqrt{2}\;a\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a),x)
```

```
[Out] 2/3/b*x^(3/2)-1/4*a/b^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/
2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2*a/b^2/(a/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*a/b^2/(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.04, size = 186, normalized size = 0.91

$$-\frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b +
 2/3*x^(3/2)/b
```

mupad [B] time = 0.09, size = 54, normalized size = 0.26

$$\frac{2\,x^{3/2}}{3\,b}+\frac{{\left(-a\right)}^{3/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{b^{7/4}}-\frac{{\left(-a\right)}^{3/4}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2),x)
```

```
[Out] (2*x^(3/2))/(3*b) + ((-a)^(3/4)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(7/4)
 - ((-a)^(3/4)*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(7/4)
```

sympy [A] time = 15.29, size = 180, normalized size = 0.88

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for}\: a = 0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for}\: b = 0 \\ \frac{(-1)^{\frac{3}{4}}a^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^{2}\sqrt[4]{\frac{1}{b}}}-\frac{(-1)^{\frac{3}{4}}a^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^{2}\sqrt[4]{\frac{1}{b}}}-\frac{(-1)^{\frac{3}{4}}a^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{b^{2}\sqrt[4]{\frac{1}{b}}}+\frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*b), Eq(a, 0))
, (2*x**(7/2)/(7*a), Eq(b, 0)), ((-1)**(3/4)*a**(3/4)*log(-(-1)**(1/4)*a**(
1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2*(1/b)**(1/4)) - (-1)**(3/4)*a**(3/4)*l
og((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2*(1/b)**(1/4)) - (-1
)**(3/4)*a**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(b**2*(
1/b)**(1/4)) + 2*x**(3/2)/(3*b), True))
```

3.290 $\int \frac{x^{3/2}}{a+bx^2} dx$

Optimal. Leaf size=202

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}}+\frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}}-$$

```
[Out] 1/2*a^(1/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(5/4)*2^(1/2)-1/2*a
^(1/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(5/4)*2^(1/2)+1/4*a^(1/4
)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)*2^(1/2)-1/4
*a^(1/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)*2^(1
/2)+2*x^(1/2)/b
```

Rubi [A] time = 0.16, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}}+\frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[x])/b + (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*b^(5/4)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*b^(5/4)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)) - (a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{a+bx^2}\,dx &= \frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{b}\\
&= \frac{2\sqrt{x}}{b}-\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b}\\
&= \frac{2\sqrt{x}}{b}-\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b}-\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b}\\
&= \frac{2\sqrt{x}}{b}-\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2b^{3/2}}-\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2b^{3/2}}+\frac{\sqrt[4]{a}\,\text{Su}}{}\\
&= \frac{2\sqrt{x}}{b}+\frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\,\text{Su}}{}\\
&= \frac{2\sqrt{x}}{b}+\frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{2\sqrt{2}\,b^{5/4}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 189, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+2\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{4b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2),x]
```

```
[Out] (8*b^(1/4)*Sqrt[x] + 2*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)] - 2*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
 + Sqrt[2]*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x] - Sqrt[2]*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(4*b^(5/4))
```

fricas [A] time = 0.96, size = 124, normalized size = 0.61

$$-\frac{4\,b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^2\sqrt{-\frac{a}{b^5}}+x}\,b^4\left(-\frac{a}{b^5}\right)^{\frac{3}{4}}-b^4\sqrt{x}\left(-\frac{a}{b^5}\right)^{\frac{3}{4}}}{a}\right)+b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{a}{b^5}\right.\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*(-a/b^5)^(1/4)*arctan((sqrt(b^2*sqrt(-a/b^5) + x)*b^4*(-a/b^5)^(3
/4) - b^4*sqrt(x)*(-a/b^5)^(3/4))/a) + b*(-a/b^5)^(1/4)*log(b*(-a/b^5)^(1/4
) + sqrt(x)) - b*(-a/b^5)^(1/4)*log(-b*(-a/b^5)^(1/4) + sqrt(x)) - 4*sqrt(x
))/b
```

giac [A] time = 0.65, size = 178, normalized size = 0.88

$$-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right.}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/b^2 - 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^2 - 1/4*sqrt(2)*(a*b^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^2 + 1/4*sqrt(2)*(a*b^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^2 + 2*sqrt(x)/b
```

maple [A] time = 0.01, size = 140, normalized size = 0.69

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b}+\frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a),x)
```

```
[Out] 2*x^(1/2)/b-1/4/b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/
b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2/b*(a/b)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/b*(a/b)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.90, size = 185, normalized size = 0.92

$$-\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(-\sqrt{\right.}}{}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt
(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))/b + 2*sqrt(x)
/b
```

mupad [B] time = 0.09, size = 55, normalized size = 0.27

$$\frac{2\sqrt{x}}{b}-\frac{(-a)^{1/4}\operatorname{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{b^{5/4}}-\frac{(-a)^{1/4}\operatorname{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2),x)
```

```
[Out] (2*x^(1/2))/b - ((-a)^(1/4)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(5/4) - (
(-a)^(1/4)*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(5/4)
```

sympy [A] time = 5.89, size = 172, normalized size = 0.85

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\ a=0\wedge b \\ \frac{2\sqrt{x}}{b} & \text{for}\ a=0 \\ \frac{2x^{\frac{5}{2}}}{5a} & \text{for}\ b=0 \\ \frac{\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b}-\frac{\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b}+\frac{\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{b}+\frac{2\sqrt{x}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b, Eq(a, 0)), (2*x
**(5/2)/(5*a), Eq(b, 0)), ((-1)**(1/4)*a**(1/4)*(1/b)**(1/4)*log(-(-1)**(1/
4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b) - (-1)**(1/4)*a**(1/4)*(1/b)**(1/
4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b) + (-1)**(1/4)*a**
(1/4)*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/b + 2*
sqrt(x)/b, True))
```

3.291 $\int \frac{\sqrt{x}}{a+bx^2}\,dx$

Optimal. Leaf size=192

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{\sqrt{2}\sqrt[4]{a}\,b}$$

```
[Out] -1/2*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/b^(3/4)*2^(1/2)+1/2*
arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/b^(3/4)*2^(1/2)+1/4*ln(a^
(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/b^(3/4)*2^(1/2)-1/
4*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/b^(3/4)*2^(
1/2)
```

Rubi [A] time = 0.14, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{\sqrt{2}\sqrt[4]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)) +
 Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1
/4)*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(
2*Sqrt[2]*a^(1/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{a+bx^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}}\\
&= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\cdots}\right.}{2\sqrt{2}\sqrt[4]{a}b^{3\cdots}}\\
&= \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} - \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\cdots\right.}{\sqrt{2}\sqrt[4]{a}b^{3/\cdots}}\\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} - \frac{\log\left(\sqrt{a}\cdots\right.}{}\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.28

$$\frac{a\left(\tan^{-1}\left(\frac{a\sqrt[4]{b}\sqrt{x}}{(-a)^{5/4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)\right)}{(-a)^{5/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2),x]
```

```
[Out] (a*(ArcTan[(a*b^(1/4)*Sqrt[x])/(-a)^(5/4)] + ArcTanh[(b^(1/4)*Sqrt[x])/(-a)
^(1/4)]))/((-a)^(5/4)*b^(3/4))
```

fricas [A] time = 0.66, size = 126, normalized size = 0.66

$$-2\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-ab\sqrt{-\frac{1}{ab^3}}+x\,b}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}-b\sqrt{x}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\right)+\frac{1}{2}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*(-1/(a*b^3))^(1/4)*arctan(sqrt(-a*b*sqrt(-1/(a*b^3)) + x)*b*(-1/(a*b^3))
^(1/4) - b*sqrt(x)*(-1/(a*b^3))^(1/4)) + 1/2*(-1/(a*b^3))^(1/4)*log(a*b^2*(
-1/(a*b^3))^(3/4) + sqrt(x)) - 1/2*(-1/(a*b^3))^(1/4)*log(-a*b^2*(-1/(a*b^3
))^(3/4) + sqrt(x))
```

giac [A] time = 0.60, size = 182, normalized size = 0.95

$$\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^{3}}+\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^{3}}-\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+\right.}{4\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b^3) + 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(a*b^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) + 1/4*sqrt(2
)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3)
```

maple [A] time = 0.01, size = 132, normalized size = 0.69

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/4/b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x
+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2/b/(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.98, size = 172, normalized size = 0.90

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}\,\sqrt{b}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{4\,a^{\frac{1}{4}}b^{\frac{3}{4}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 1/2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + 1/4*sqrt(2)*log
(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
```

mupad [B] time = 0.07, size = 38, normalized size = 0.20

$$\frac{\operatorname{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)-\operatorname{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{(-a)^{1/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2),x)
```

```
[Out] (atan((b^(1/4)*x^(1/2))/(-a)^(1/4)) - atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/
((-a)^(1/4)*b^(3/4))
```

sympy [A] time = 3.17, size = 165, normalized size = 0.86

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\ a = 0 \wedge b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for}\ a = 0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for}\ b = 0 \\ -\frac{(-1)^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{a}\,b\sqrt[4]{\frac{1}{b}}}+\frac{(-1)^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{a}\,b\sqrt[4]{\frac{1}{b}}}+\frac{(-1)^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{\sqrt[4]{a}\,b\sqrt[4]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b*sqrt(x)), Eq(a, 0)), (
2*x**(3/2)/(3*a), Eq(b, 0)), (-(-1)**(3/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)*
*(1/4) + sqrt(x))/(2*a**(1/4)*b*(1/b)**(1/4)) + (-1)**(3/4)*log((-1)**(1/4)
*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(1/4)*b*(1/b)**(1/4)) + (-1)**(3/4)
*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(1/4)*b*(1/b)**(1/4)
), True))
```

3.292 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=192

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right)}{\sqrt{2}\,a^{3/4}\cdots}$$

```
[Out] -1/2*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)*2^(1/2)+1/2*
arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)*2^(1/2)-1/4*ln(a^
(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)+1/
4*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(1/4)*2^(
1/2)
```

Rubi [A] time = 0.15, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right)}{\sqrt{2}\,a^{3/4}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/4)) -
 Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(3
/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(
2*Sqrt[2]*a^(3/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\text{Subst}\left(\int \cdots\right)}{2\sqrt{2}\cdots}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\cdots\right)}{\sqrt{2}\cdots}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\log \cdots}{\cdots}$$

Mathematica [A] time = 0.03, size = 146, normalized size = 0.76

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(\cdots\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2
*Sqrt[2]*a^(3/4)*b^(1/4))
```

fricas [A] time = 0.96, size = 126, normalized size = 0.66

$$2\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2\sqrt{-\frac{1}{a^3b}}+x}\,a^2b\left(-\frac{1}{a^3b}\right)^{\frac{3}{4}}-a^2b\sqrt{x}\left(-\frac{1}{a^3b}\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\sqrt{x}\right)-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(-1/(a^3*b))^(1/4)*arctan(sqrt(a^2*sqrt(-1/(a^3*b)) + x)*a^2*b*(-1/(a^3*b
))^(3/4) - a^2*b*sqrt(x)*(-1/(a^3*b))^(3/4)) + 1/2*(-1/(a^3*b))^(1/4)*log(a
*(-1/(a^3*b))^(1/4) + sqrt(x)) - 1/2*(-1/(a^3*b))^(1/4)*log(-a*(-1/(a^3*b))
^(1/4) + sqrt(x))
```

giac [A] time = 0.63, size = 182, normalized size = 0.95

$$\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}-\right.}{4\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b) + 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b) + 1/4*sqrt(2)*(a*b^3)^(1/4
)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b) - 1/4*sqrt(2)*(a*b
^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b)
```

maple [A] time = 0.00, size = 132, normalized size = 0.69

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/x^(1/2),x)
```

```
[Out] 1/4*(a/b)^(1/4)/a*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x
-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2*(a/b)^(1/4)/a*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.04, size = 172, normalized size = 0.90

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{4\,a^{\frac{3}{4}}b^{\frac{1}{4}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 1/2*sqrt(2)*arct
```

```
an(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 1/4*sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - 1/4*sqrt(2)*log
(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4))
```

mupad [B] time = 0.08, size = 37, normalized size = 0.19

$$-\frac{\operatorname{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{(-a)^{3/4}\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)),x)
```

```
[Out] -(atan((b^(1/4)*x^(1/2))/(-a)^(1/4)) + atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))
/((-a)^(3/4)*b^(1/4))
```

sympy [A] time = 5.94, size = 160, normalized size = 0.83

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for}\: a = 0 \\ \frac{2\sqrt{x}}{a} & \text{for}\: b = 0 \\ -\frac{\sqrt[4]{-1}\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{3}{4}}}+\frac{\sqrt[4]{-1}\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{3}{4}}}-\frac{\sqrt[4]{-1}\sqrt[4]{\frac{1}{b}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(a, 0)
), (2*sqrt(x)/a, Eq(b, 0)), (-(-1)**(1/4)*(1/b)**(1/4)*log(-(-1)**(1/4)*a**
(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)) + (-1)**(1/4)*(1/b)**(1/4)*log((
-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)) - (-1)**(1/4)*(1/b
)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/a**(3/4), True))
```

3.293 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)} dx$

Optimal. Leaf size=202

$$-\frac{\sqrt[4]{b} \log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x}+\sqrt{a}+\sqrt{b} x\right)}{2 \sqrt{2} a^{5/4}}+\frac{\sqrt[4]{b} \log \left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x}+\sqrt{a}+\sqrt{b} x\right)}{2 \sqrt{2} a^{5/4}}+\frac{\sqrt[4]{b} \tan^{-1}\left(1-\frac{\sqrt{2} \sqrt[4]{b} \sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2} a^{5/4}}-$$

```
[Out] 1/2*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)*2^(1/2)-1/2*b
^(1/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)*2^(1/2)-1/4*b^(1/4
)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)*2^(1/2)+1/4
*b^(1/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)*2^(1
/2)-2/a/x^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b} \log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x}+\sqrt{a}+\sqrt{b} x\right)}{2 \sqrt{2} a^{5/4}}+\frac{\sqrt[4]{b} \log \left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x}+\sqrt{a}+\sqrt{b} x\right)}{2 \sqrt{2} a^{5/4}}+\frac{\sqrt[4]{b} \tan^{-1}\left(1-\frac{\sqrt{2} \sqrt[4]{b} \sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2} a^{5/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] -2/(a*Sqrt[x]) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*a^(5/4)) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(5/4)) - (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx &= -\frac{2}{a\sqrt{x}}-\frac{b\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a}\\
&= -\frac{2}{a\sqrt{x}}-\frac{(2b)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{a}\\
&= -\frac{2}{a\sqrt{x}}+\frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{a}-\frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{a}\\
&= -\frac{2}{a\sqrt{x}}-\frac{\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2a}-\frac{\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2a}-\sqrt[4]{b}\,\mathrm{Su}\\
&= -\frac{2}{a\sqrt{x}}-\frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}}+\frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}}-\\
&= -\frac{2}{a\sqrt{x}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}}-\frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}}-\frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{2\sqrt{2}\,a^{5/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{bx^2}{a}\right)}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -((b*x^2)/a)])/(a*Sqrt[x])
```

fricas [A] time = 0.85, size = 142, normalized size = 0.70

$$\frac{4\,ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{ab\sqrt{x}\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}-\sqrt{-a^3b\sqrt{-\frac{b}{a^5}}+b^2x}\,a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}}{b}\right)-ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}+b\sqrt{x}\right)+ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log}{2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x*(-b/a^5)^(1/4)*arctan(-(a*b*sqrt(x)*(-b/a^5)^(1/4) - sqrt(-a^3*b
*sqrt(-b/a^5) + b^2*x)*a*(-b/a^5)^(1/4))/b) - a*x*(-b/a^5)^(1/4)*log(a^4*(-
b/a^5)^(3/4) + b*sqrt(x)) + a*x*(-b/a^5)^(1/4)*log(-a^4*(-b/a^5)^(3/4) + b*
sqrt(x)) - 4*sqrt(x))/(a*x)
```

giac [A] time = 0.62, size = 190, normalized size = 0.94

$$-\frac{2}{a\sqrt{x}}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^2}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{\right.}{4\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -2/(a*sqrt(x)) - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) - 1/2*sqrt(2)*(a*b^3)^(3/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) +
 1/4*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(a^2*b^2) - 1/4*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(a^2*b^2)
```

maple [A] time = 0.01, size = 140, normalized size = 0.69

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a),x)
```

```
[Out] -2/a/x^(1/2)-1/4/a/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a
/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2/a/(a/b)^(1/4)*2
```

```
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/a/(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.04, size = 186, normalized size = 0.92

$$-\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}\cdots\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\cdots\right.}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a -
 2/(a*sqrt(x))
```

mupad [B] time = 4.50, size = 54, normalized size = 0.27

$$\frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{5/4}}-\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{5/4}}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(5/4) - ((-b)^(1/4)*atan
(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(5/4) - 2/(a*x^(1/2))
```

sympy [A] time = 12.16, size = 170, normalized size = 0.84

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for}\: a = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for}\: b = 0 \\ -\frac{2}{a\sqrt{x}}+\frac{(-1)^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}}-\frac{(-1)^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}}-\frac{(-1)^{\frac{3}{4}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)
), (-2/(a*sqrt(x)), Eq(b, 0)), (-2/(a*sqrt(x)) + (-1)**(3/4)*log(-(-1)**(1/
4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)*(1/b)**(1/4)) - (-1)**(3/4)
*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)*(1/b)**(1/4))
 - (-1)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(5/4)*
(1/b)**(1/4)), True))
```

3.294 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)} dx$

Optimal. Leaf size=204

$$\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}}-\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}}-\frac{b}{}$$

```
[Out] -2/3/a/x^(3/2)+1/2*b^(3/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4
)*2^(1/2)-1/2*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)*2^(
1/2)+1/4*b^(3/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7
/4)*2^(1/2)-1/4*b^(3/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2
))/a^(7/4)*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}}-\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}}-\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*a^(7/4)) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*a^(7/4)) + (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)) - (b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx &= -\frac{2}{3ax^{3/2}}-\frac{b\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{a}\\
&= -\frac{2}{3ax^{3/2}}-\frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{a}\\
&= -\frac{2}{3ax^{3/2}}-\frac{b\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{a^{3/2}}-\frac{b\,\mathrm{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{a^{3/2}}\\
&= -\frac{2}{3ax^{3/2}}-\frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2a^{3/2}}-\frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2a^{3/2}}\\
&= -\frac{2}{3ax^{3/2}}+\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}}-\frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}}\\
&= -\frac{2}{3ax^{3/2}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}}-\frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}}+\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}\,a^{7/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.14

$$
-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{bx^2}{a}\right)}{3ax^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, -((b*x^2)/a)])/(3*a*x^(3/2))
```

fricas [A] time = 0.68, size = 167, normalized size = 0.82

$$-\frac{12\,ax^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^5b\sqrt{x}\left(-\frac{b^3}{a^7}\right)^{\frac{3}{4}}-\sqrt{a^4\sqrt{-\frac{b^3}{a^7}}+b^2x}\,a^5\left(-\frac{b^3}{a^7}\right)^{\frac{3}{4}}}{b^3}\right)+3\,ax^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}+b\sqrt{x}\right)-3\,ax^2\left(-\right.}{6\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*x^2*(-b^3/a^7)^(1/4)*arctan(-(a^5*b*sqrt(x)*(-b^3/a^7)^(3/4) - s
qrt(a^4*sqrt(-b^3/a^7) + b^2*x)*a^5*(-b^3/a^7)^(3/4))/b^3) + 3*a*x^2*(-b^3/
a^7)^(1/4)*log(a^2*(-b^3/a^7)^(1/4) + b*sqrt(x)) - 3*a*x^2*(-b^3/a^7)^(1/4)
*log(-a^2*(-b^3/a^7)^(1/4) + b*sqrt(x)) + 4*sqrt(x))/(a*x^2)
```

giac [A] time = 0.64, size = 178, normalized size = 0.87

$$-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right.}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/a^2 - 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^2 - 1/4*sqrt(2)*(a*b^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^2 + 1/4*sqrt(2)*(a*b^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^2 - 2/3/(a*x^(3/2)
)
```

maple [A] time = 0.01, size = 143, normalized size = 0.70

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a^2}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^2}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a^2}-\frac{2}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a),x)
```

```
[Out] -1/4/a^2*b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2
))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2/a^2*b*(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/a^2*b*(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-2/3/a/x^(3/2)
```

maxima [A] time = 3.13, size = 187, normalized size = 0.92

$$-\frac{\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right.}{a^{\frac{3}{4}}}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*
b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(
2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)
*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/a - 2
/3/(a*x^(3/2))
```

mupad [B] time = 0.09, size = 53, normalized size = 0.26

$$\frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{a^{7/4}}-\frac{2}{3\,a\,x^{3/2}}+\frac{(-b)^{3/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{a^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(3/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(7/4) - 2/(3*a*x^(3/2)) +
 ((-b)^(3/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(7/4)
```

sympy [A] time = 30.40, size = 178, normalized size = 0.87

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\ a = 0 \wedge b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for}\ a = 0 \\ -\frac{2}{3ax^{\frac{3}{2}}} & \text{for}\ b = 0 \\ -\frac{2}{3ax^{\frac{3}{2}}}+\frac{\sqrt[4]{-1}\,b\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{4}}}-\frac{\sqrt[4]{-1}\,b\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{4}}}+\frac{\sqrt[4]{-1}\,b\sqrt[4]{\frac{1}{b}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{7}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0)
), (-2/(3*a*x**(3/2)), Eq(b, 0)), (-2/(3*a*x**(3/2)) + (-1)**(1/4)*b*(1/b)*
*(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(7/4)) - (-1
)**(1/4)*b*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2
*a**(7/4)) + (-1)**(1/4)*b*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*
(1/b)**(1/4)))/a**(7/4), True))
```

3.295 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=215

$$\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}}-\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}}+$$

[Out] -2/5/a/x^(5/2)-1/2*b^(5/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4
)*2^(1/2)+1/2*b^(5/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)*2^(
1/2)+1/4*b^(5/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9
/4)*2^(1/2)-1/4*b^(5/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2
))/a^(9/4)*2^(1/2)+2*b/a^2/x^(1/2)

Rubi [A] time = 0.18, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}}-\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}}+$$

Antiderivative was successfully verified.

[In] Int[1/(x^(7/2)*(a + b*x^2)),x]

[Out] -2/(5*a*x^(5/2)) + (2*b)/(a^2*Sqrt[x]) - (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)) - (b^(5/4)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4))

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx &= -\frac{2}{5ax^{5/2}}-\frac{b\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{a}\\
&= -\frac{2}{5ax^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{b^2\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a^2}\\
&= -\frac{2}{5ax^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{\left(2b^2\right)\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{a^2}\\
&= -\frac{2}{5ax^{5/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{b^{3/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{a^2}+\frac{b^{3/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{a^2}\\
&= -\frac{2}{5ax^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2a^2}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\right.}{2a^2}\\
&= -\frac{2}{5ax^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}}-\frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}\,a^{9/4}}\\
&= -\frac{2}{5ax^{5/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}}+\frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}}+\frac{b^{5/4}\log\left(\sqrt{a}-\right.}{}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, -((b*x^2)/a)])/(5*a*x^(5/2))
```

fricas [A] time = 0.89, size = 193, normalized size = 0.90

$$-\frac{20\,a^2x^3\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^2b^4\sqrt{x}\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}}-\sqrt{-a^5b^5\sqrt{-\frac{b^5}{a^9}}+b^8x}\,a^2\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}}}{b^5}\right)-5\,a^2x^3\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}}\log\left(a^7\left(-\frac{b^5}{a^9}\right)^{\frac{3}{4}}+b^4\sqrt{x}\right)+5\cdots}{10\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/10*(20*a^2*x^3*(-b^5/a^9)^(1/4)*arctan(-(a^2*b^4*sqrt(x)*(-b^5/a^9)^(1/4
) - sqrt(-a^5*b^5*sqrt(-b^5/a^9) + b^8*x)*a^2*(-b^5/a^9)^(1/4))/b^5) - 5*a^
2*x^3*(-b^5/a^9)^(1/4)*log(a^7*(-b^5/a^9)^(3/4) + b^4*sqrt(x)) + 5*a^2*x^3*
(-b^5/a^9)^(1/4)*log(-a^7*(-b^5/a^9)^(3/4) + b^4*sqrt(x)) - 4*(5*b*x^2 - a)
*sqrt(x))/(a^2*x^3)
```

giac [A] time = 0.62, size = 200, normalized size = 0.93

$$\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+\cdots\right.}{4\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a^3*b) + 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) - 1/4*sqrt(2)*(a*b^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) + 1/4*sqrt(2
)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) +
 2/5*(5*b*x^2 - a)/(a^2*x^(5/2))
```

maple [A] time = 0.01, size = 152, normalized size = 0.71

$$\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2}+\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2}+\frac{\sqrt{2}\,b\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{5a\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a),x)
```

```
[Out] -2/5/a/x^(5/2)+2*b/a^2/x^(1/2)+1/4*b/a^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1
/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)
```

```
))+1/2*b/a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*
b/a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.04, size = 198, normalized size = 0.92

$$\frac{b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^
2 + 2/5*(5*b*x^2 - a)/(a^2*x^(5/2))
```

mupad [B] time = 4.48, size = 66, normalized size = 0.31

$$\frac{(-b)^{5/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{9/4}}-\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{9/4}}-\frac{\frac{2}{5a}-\frac{2bx^2}{a^2}}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(5/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(9/4) - ((-b)^(5/4)*atan
(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(9/4) - (2/(5*a) - (2*b*x^2)/a^2)/x^(5/2)
```

sympy [A] time = 109.16, size = 190, normalized size = 0.88

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\ a = 0 \wedge b = 0 \\ -\frac{2}{9bx^{\frac{9}{2}}} & \text{for}\ a = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for}\ b = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}}+\frac{2b}{a^2\sqrt{x}}-\frac{(-1)^{\frac{3}{4}}b\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}}+\frac{(-1)^{\frac{3}{4}}b\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}}+\frac{(-1)^{\frac{3}{4}}b\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b*x**(9/2)), Eq(a, 0)
), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + 2*b/(a**2*sqrt(x)) -
 (-1)**(3/4)*b*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(9/4
)*(1/b)**(1/4)) + (-1)**(3/4)*b*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqr
t(x))/(2*a**(9/4)*(1/b)**(1/4)) + (-1)**(3/4)*b*atan((-1)**(3/4)*sqrt(x)/(a
**(1/4)*(1/b)**(1/4)))/(a**(9/4)*(1/b)**(1/4)), True))
```

3.296 $\int \frac{x^{7/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=230

$$\frac{5\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{9/4}}$$

```
[Out] -1/2*x^(5/2)/b/(b*x^2+a)+5/8*a^(1/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/
4))/b^(9/4)*2^(1/2)-5/8*a^(1/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b
^(9/4)*2^(1/2)+5/16*a^(1/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^
(1/2))/b^(9/4)*2^(1/2)-5/16*a^(1/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^
(1/2)*x^(1/2))/b^(9/4)*2^(1/2)+5/2*x^(1/2)/b^2
```

Rubi [A] time = 0.17, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{5\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2)^2,x]
```

```
[Out] (5*Sqrt[x])/(2*b^2) - x^(5/2)/(2*b*(a + b*x^2)) + (5*a^(1/4)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*a^(1/4)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(9/4)) + (5*a^(1/4)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(9/4))
 - (5*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
8*Sqrt[2]*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)^2}\,dx = -\frac{x^{5/2}}{2b\left(a+bx^2\right)} + \frac{5\int \frac{x^{3/2}}{a+bx^2}\,dx}{4b}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b\left(a+bx^2\right)} - \frac{(5a)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4b^2}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b\left(a+bx^2\right)} - \frac{(5a)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b\left(a+bx^2\right)} - \frac{\left(5\sqrt{a}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4b^2} - \frac{\left(5\sqrt{a}\right)\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,d\right.}{4b^2}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b\left(a+bx^2\right)} - \frac{\left(5\sqrt{a}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{5/2}} - \frac{\left(5\sqrt{a}\right)\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}+\right.}{8b^{5/2}}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b\left(a+bx^2\right)} + \frac{5\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}} - \frac{5\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\right.}{8\sqrt{2}b^{9/4}}$$

$$= \frac{5\sqrt{x}}{2b^2} - \frac{x^{5/2}}{2b\left(a+bx^2\right)} + \frac{5\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{9/4}} - \frac{5\sqrt[4]{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{9/4}} + \frac{5\sqrt[4]{a}\log}{}$$

Mathematica [A] time = 0.10, size = 221, normalized size = 0.96

$$\frac{\frac{32b^{5/4}x^{5/2}}{a+bx^2} + \frac{40a\sqrt[4]{b}\sqrt{x}}{a+bx^2} + 5\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) - 5\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\right.}{16b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2)^2,x]
```

```
[Out] ((40*a*b^(1/4)*Sqrt[x])/(a + b*x^2) + (32*b^(5/4)*x^(5/2))/(a + b*x^2) + 10
*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 10*Sqrt[2]
*a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 5*Sqrt[2]*a^(1/4)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 5*Sqrt[2]*a^(1
/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(16*b^(9/4)
)
```

fricas [A] time = 1.13, size = 192, normalized size = 0.83

$$-\frac{20\left(b^3x^2+ab^2\right)\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^4\sqrt{-\frac{a}{b^9}}+x}\,b^7\left(-\frac{a}{b^9}\right)^{\frac{3}{4}} - b^7\sqrt{x}\left(-\frac{a}{b^9}\right)^{\frac{3}{4}}}{a}\right) + 5\left(b^3x^2+ab^2\right)\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}\log\left(5b^2\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}+5\right.}{8\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(20*(b^3*x^2 + a*b^2)*(-a/b^9)^(1/4)*arctan((sqrt(b^4*sqrt(-a/b^9) + x
)*b^7*(-a/b^9)^(3/4) - b^7*sqrt(x)*(-a/b^9)^(3/4))/a) + 5*(b^3*x^2 + a*b^2)
```

```
*(-a/b^9)^(1/4)*log(5*b^2*(-a/b^9)^(1/4) + 5*sqrt(x)) - 5*(b^3*x^2 + a*b^2)
*(-a/b^9)^(1/4)*log(-5*b^2*(-a/b^9)^(1/4) + 5*sqrt(x)) - 4*(4*b*x^2 + 5*a)*
sqrt(x))/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.65, size = 196, normalized size = 0.85

$$-\frac{5\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8b^{3}}-\frac{5\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8b^{3}}-\frac{5\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right.\right.}{16b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/b^3 - 5/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^3 - 5/16*sqrt(2)*(a*b^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 5/16*sqrt(2)*(a*b^3)
^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 1/2*a*sqrt(x
)/((b*x^2 + a)*b^2) + 2*sqrt(x)/b^2
```

maple [A] time = 0.01, size = 158, normalized size = 0.69

$$\frac{a\sqrt{x}}{2\left(bx^{2}+a\right)b^{2}}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8b^{2}}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^{2}}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right.}{16b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)^2,x)
```

```
[Out] 2*x^(1/2)/b^2+1/2*a/b^2*x^(1/2)/(b*x^2+a)-5/16/b^2*(a/b)^(1/4)*2^(1/2)*ln((
x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(
a/b)^(1/2)))-5/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
+1)-5/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.02, size = 206, normalized size = 0.90

$$\frac{a\sqrt{x}}{2\left(b^{3}x^{2}+ab^{2}\right)}-\frac{5\left(\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\right.}{b^{\frac{1}{4}}}\right.}{16b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*sqrt(x)/(b^3*x^2 + a*b^2) - 5/16*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2
)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt
(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*
b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) +
 sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))
/b^(1/4) - sqrt(2)*a^(1/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/b^(1/4))/b^2 + 2*sqrt(x)/b^2
```

mupad [B] time = 4.51, size = 80, normalized size = 0.35

$$\frac{2\sqrt{x}}{b^2}-\frac{5(-a)^{1/4}\operatorname{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{4b^{9/4}}+\frac{a\sqrt{x}}{2\left(b^3x^2+ab^2\right)}+\frac{(-a)^{1/4}\operatorname{atan}\left(\frac{b^{1/4}\sqrt{x}\,1\mathrm{i}}{(-a)^{1/4}}\right)5\mathrm{i}}{4b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2)^2,x)
```

```
[Out] (2*x^(1/2))/b^2 - (5*(-a)^(1/4)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*b^(9
/4)) + ((-a)^(1/4)*atan((b^(1/4)*x^(1/2)*1i)/(-a)^(1/4))*5i)/(4*b^(9/4)) +
(a*x^(1/2))/(2*(a*b^2 + b^3*x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.297 $\int \frac{x^{5/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=218

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{3\tan^{-1}\left(\sqrt{2}\right.}{4\sqrt{2}\sqrt[4]{}}$$

```
[Out] -1/2*x^(3/2)/b/(b*x^2+a)-3/8*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1
/4)/b^(7/4)*2^(1/2)+3/8*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/b
^(7/4)*2^(1/2)+3/16*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a
^(1/4)/b^(7/4)*2^(1/2)-3/16*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^
(1/2))/a^(1/4)/b^(7/4)*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {288, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{3\tan^{-1}\left(\sqrt{2}\right.}{4\sqrt{2}\sqrt[4]{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] -x^(3/2)/(2*b*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2\right)^2}\,dx = -\frac{x^{3/2}}{2b\left(a+bx^2\right)}+\frac{3\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4b}$$

$$= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2b}$$

$$= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}-\frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4b^{3/2}}+\frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4b^{3/2}}$$

$$= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right.}{8b^2}$$

$$= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}+\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right.}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}$$

$$= -\frac{x^{3/2}}{2b\left(a+bx^2\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}}+\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{8\sqrt{2}\sqrt[4]{a}\,b}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.20

$$\frac{2x^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{a}-\frac{1}{a+bx^2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] (2*x^(3/2)*(-(a + b*x^2)^(-1) + Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)
]/a))/b
```

fricas [A] time = 1.00, size = 185, normalized size = 0.85

$$-\frac{12\left(b^2x^2+ab\right)\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-ab^3\sqrt{-\frac{1}{ab^7}}+x}\,b^2\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}-b^2\sqrt{x}\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\right)-3\left(b^2x^2+ab\right)\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\log\left(ab\right.}{8\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(12*(b^2*x^2 + a*b)*(-1/(a*b^7))^(1/4)*arctan(sqrt(-a*b^3*sqrt(-1/(a*b
^7)) + x)*b^2*(-1/(a*b^7))^(1/4) - b^2*sqrt(x)*(-1/(a*b^7))^(1/4)) - 3*(b^2
*x^2 + a*b)*(-1/(a*b^7))^(1/4)*log(a*b^5*(-1/(a*b^7))^(3/4) + sqrt(x)) + 3*
(b^2*x^2 + a*b)*(-1/(a*b^7))^(1/4)*log(-a*b^5*(-1/(a*b^7))^(3/4) + sqrt(x))
 + 4*x^(3/2))/(b^2*x^2 + a*b)
```

giac [A] time = 0.62, size = 199, normalized size = 0.91

$$-\frac{x^{\frac{3}{2}}}{2\left(bx^2+a\right)b}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab^4}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab^4}-\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^(3/2)/((b*x^2 + a)*b) + 3/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) + 3/8*sqrt(2)*(a*b^
3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))
/(a*b^4) - 3/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
 sqrt(a/b))/(a*b^4) + 3/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)
^(1/4) + x + sqrt(a/b))/(a*b^4)
```

maple [A] time = 0.01, size = 149, normalized size = 0.68

$$-\frac{x^{\frac{3}{2}}}{2\left(b\,x^2+a\right)b}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}+\frac{3\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/2*x^(3/2)/b/(b*x^2+a)+3/16/b^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(
1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+3/8/
b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+3/8/b^2/(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.99, size = 195, normalized size = 0.89

$$-\frac{x^{\frac{3}{2}}}{2\left(b^{2}x^{2}+ab\right)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x^(3/2)/(b^2*x^2 + a*b) + 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*
a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*s
qrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b))
- sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/
4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sq
rt(a))/(a^(1/4)*b^(3/4)))/b
```

mupad [B] time = 0.08, size = 64, normalized size = 0.29

$$\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{4\,{\left(-a\right)}^{1/4}\,b^{7/4}}-\frac{x^{3/2}}{2\,b\,\left(b\,x^2+a\right)}-\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{4\,{\left(-a\right)}^{1/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2)^2,x)
```

```
[Out] (3*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(1/4)*b^(7/4)) - x^(3/2)/(2*
b*(a + b*x^2)) - (3*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(1/4)*b^(7
/4))
```

sympy [A] time = 153.06, size = 595, normalized size = 2.73

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} \\ -\frac{2}{b^{2}\sqrt{x}} \\ \frac{2x^{\frac{7}{2}}}{7a^{2}} \\ -\frac{4\sqrt[4]{-1}\sqrt[4]{a}bx^{\frac{3}{2}}\sqrt[4]{\frac{1}{b}}}{8\sqrt[4]{-1}a^{\frac{5}{4}}b^{2}\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}\sqrt[4]{a}b^{3}x^{2}\sqrt[4]{\frac{1}{b}}}+\frac{3a\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{8\sqrt[4]{-1}a^{\frac{5}{4}}b^{2}\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}\sqrt[4]{a}b^{3}x^{2}\sqrt[4]{\frac{1}{b}}}-\frac{3a\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{8\sqrt[4]{-1}a^{\frac{5}{4}}b^{2}\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}\sqrt[4]{a}b^{3}x^{2}\sqrt[4]{\frac{1}{b}}}-\frac{6a\,\mathrm{atan}\left(\cdots\right.}{8\sqrt[4]{-1}a^{\frac{5}{4}}b^{2}\sqrt[4]{\frac{1}{b}}+8\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b**2*sqrt(x)), Eq(a, 0))
, (2*x**(7/2)/(7*a**2), Eq(b, 0)), (-4*(-1)**(1/4)*a**(1/4)*b*x**(3/2)*(1/b
)**(1/4)/(8*(-1)**(1/4)*a**(5/4)*b**2*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(1/4)
*b**3*x**2*(1/b)**(1/4)) + 3*a*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqr
t(x))/(8*(-1)**(1/4)*a**(5/4)*b**2*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(1/4)*b*
*3*x**2*(1/b)**(1/4)) - 3*a*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x)
)/(8*(-1)**(1/4)*a**(5/4)*b**2*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(1/4)*b**3*x
**2*(1/b)**(1/4)) - 6*a*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(
8*(-1)**(1/4)*a**(5/4)*b**2*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(1/4)*b**3*x**2
*(1/b)**(1/4)) + 3*b*x**2*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))
/(8*(-1)**(1/4)*a**(5/4)*b**2*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(1/4)*b**3*x*
*2*(1/b)**(1/4)) - 3*b*x**2*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x)
)/(8*(-1)**(1/4)*a**(5/4)*b**2*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(1/4)*b**3*x
**2*(1/b)**(1/4)) - 6*b*x**2*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4
)))/(8*(-1)**(1/4)*a**(5/4)*b**2*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(1/4)*b**3
*x**2*(1/b)**(1/4)), True))
```

3.298 $\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{4\sqrt{2}a^{3/}}$$

```
[Out] -1/8*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(5/4)*2^(1/2)+1/8*
arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(5/4)*2^(1/2)-1/16*ln(a
^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1
/16*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(5/4)*2
^(1/2)-1/2*x^(1/2)/b/(b*x^2+a)
```

Rubi [A] time = 0.15, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{4\sqrt{2}a^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] -Sqrt[x]/(2*b*(a + b*x^2)) - ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/
(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
/(4*Sqrt[2]*a^(3/4)*b^(5/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x]/(8*Sqrt[2]*a^(3/4)*b^(5/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(8*Sqrt[2]*a^(3/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{\left(a+bx^2\right)^2}\,dx &= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)}+\frac{\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4b}\\
&= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2b}\\
&= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4\sqrt{a}\,b}+\frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4\sqrt{a}\,b}\\
&= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{a}\,b^{3/2}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{a}\,b^{3/2}}-\cdots\\
&= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)}-\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}+\cdots\\
&= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\cdots\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 198, normalized size = 0.91

$$\frac{-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}-\frac{2\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}}-\frac{8\cdots}{a\cdots}}{16b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] ((-8*b^(1/4)*Sqrt[x])/(a + b*x^2) - (2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/a^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/a^(3/4) - (Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/a^(3/4) + (Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x] + Sqrt[b]*x])/a^(3/4))/(16*b^(5/4))
```

fricas [A] time = 0.95, size = 187, normalized size = 0.86

$$\frac{4\left(b^2x^2+ab\right)\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x}\,a^2b^4\left(-\frac{1}{a^3b^5}\right)^{\frac{3}{4}}-a^2b^4\sqrt{x}\left(-\frac{1}{a^3b^5}\right)^{\frac{3}{4}}\right)+\left(b^2x^2+ab\right)\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}\cdots}{8\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*arctan(sqrt(a^2*b^2*sqrt(-1/(a^
3*b^5)) + x)*a^2*b^4*(-1/(a^3*b^5))^(3/4) - a^2*b^4*sqrt(x)*(-1/(a^3*b^5))^
(3/4)) + (b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*log(a*b*(-1/(a^3*b^5))^(1/4)
+ sqrt(x)) - (b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*log(-a*b*(-1/(a^3*b^5))^(
1/4) + sqrt(x)) - 4*sqrt(x))/(b^2*x^2 + a*b)
```

giac [A] time = 0.65, size = 199, normalized size = 0.91

$$\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^2}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^2}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots\right)}{16\,ab^2}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^2) + 1/16*sqrt(2)*(a*b^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2) - 1/16*sqrt
(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2)
 - 1/2*sqrt(x)/((b*x^2 + a)*b)
```

maple [A] time = 0.01, size = 158, normalized size = 0.72

$$-\frac{\sqrt{x}}{2\left(bx^2+a\right)b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/2*x^(1/2)/b/(b*x^2+a)+1/16/b*(a/b)^(1/4)/a*2^(1/2)*ln((x+(a/b)^(1/4)*2^(
1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/8/
b*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/8/b*(a/b)^(
1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.88, size = 195, normalized size = 0.89

$$\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqr
t(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b - 1
/2*sqrt(x)/(b^2*x^2 + a*b)
```

mupad [B] time = 4.65, size = 64, normalized size = 0.29

$$-\frac{\sqrt{x}}{2\,b\,\left(b\,x^2+a\right)}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{4\,{(-a)}^{3/4}\,b^{5/4}}-\frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{4\,{(-a)}^{3/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2)^2,x)
```

```
[Out] - x^(1/2)/(2*b*(a + b*x^2)) - atan((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(3
/4)*b^(5/4)) - atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(3/4)*b^(5/4))
```

sympy [A] time = 78.16, size = 440, normalized size = 2.02

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} \\ -\frac{2}{3b^2x^{\frac{3}{2}}} \\ \frac{2x^{\frac{5}{2}}}{5a^2} \\ -\frac{\sqrt[4]{-1}\,a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{8a^2b+8ab^2x^2}+\frac{\sqrt[4]{-1}\,a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{8a^2b+8ab^2x^2}-\frac{2\sqrt[4]{-1}\,a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{8a^2b+8ab^2x^2}-\frac{\sqrt[4]{-1}\sqrt[4]{a}\,bx^2\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{\right.}}{8a^2b+8ab^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b**2*x**(3/2)), Eq(a,
 0)), (2*x**(5/2)/(5*a**2), Eq(b, 0)), (-(-1)**(1/4)*a**(5/4)*(1/b)**(1/4)*
```

```
log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**2*b + 8*a*b**2*x**2
) + (-1)**(1/4)*a**(5/4)*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4)
 + sqrt(x))/(8*a**2*b + 8*a*b**2*x**2) - 2*(-1)**(1/4)*a**(5/4)*(1/b)**(1/4
)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*a**2*b + 8*a*b**2*x*
*2) - (-1)**(1/4)*a**(1/4)*b*x**2*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1
/b)**(1/4) + sqrt(x))/(8*a**2*b + 8*a*b**2*x**2) + (-1)**(1/4)*a**(1/4)*b*x
**2*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**2*b
 + 8*a*b**2*x**2) - 2*(-1)**(1/4)*a**(1/4)*b*x**2*(1/b)**(1/4)*atan((-1)**(
3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*a**2*b + 8*a*b**2*x**2) - 4*a*sqrt
(x)/(8*a**2*b + 8*a*b**2*x**2), True))
```

3.299 $\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=218

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{4\sqrt{2}\,a^{5/4}b^{3/4}}$$

```
[Out] 1/2*x^(3/2)/a/(b*x^2+a)-1/8*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/
4)/b^(3/4)*2^(1/2)+1/8*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^
(3/4)*2^(1/2)+1/16*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^
(5/4)/b^(3/4)*2^(1/2)-1/16*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(
1/2))/a^(5/4)/b^(3/4)*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{4\sqrt{2}\,a^{5/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2)^2,x]
```

```
[Out] x^(3/2)/(2*a*(a + b*x^2)) - ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(
4*Sqrt[2]*a^(5/4)*b^(3/4)) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/
(4*Sqrt[2]*a^(5/4)*b^(3/4)) + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x]/(8*Sqrt[2]*a^(5/4)*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x]/(8*Sqrt[2]*a^(5/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2}\,dx &= \frac{x^{3/2}}{2a\left(a+bx^2\right)}+\frac{\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a}\\
&= \frac{x^{3/2}}{2a\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2a}\\
&= \frac{x^{3/2}}{2a\left(a+bx^2\right)}-\frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a\sqrt{b}}+\frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a\sqrt{b}}\\
&= \frac{x^{3/2}}{2a\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8ab}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8ab}+\\
&= \frac{x^{3/2}}{2a\left(a+bx^2\right)}+\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}+\\
&= \frac{x^{3/2}}{2a\left(a+bx^2\right)}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{8\sqrt{2}\,a^{5/4}b^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2)^2,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)])/(3*a^2)
```

fricas [A] time = 0.69, size = 182, normalized size = 0.83

$$-\frac{4\left(abx^2+a^2\right)\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-a^3b\sqrt{-\frac{1}{a^5b^3}}+x}\,ab\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}-ab\sqrt{x}\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\right)-\left(abx^2+a^2\right)\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\log\left(a\right.}{8\left(abx^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(a*b*x^2 + a^2)*(-1/(a^5*b^3))^(1/4)*arctan(sqrt(-a^3*b*sqrt(-1/(a^
5*b^3)) + x)*a*b*(-1/(a^5*b^3))^(1/4) - a*b*sqrt(x)*(-1/(a^5*b^3))^(1/4)) -
 (a*b*x^2 + a^2)*(-1/(a^5*b^3))^(1/4)*log(a^4*b^2*(-1/(a^5*b^3))^(3/4) + sq
rt(x)) + (a*b*x^2 + a^2)*(-1/(a^5*b^3))^(1/4)*log(-a^4*b^2*(-1/(a^5*b^3))^(
3/4) + sqrt(x)) - 4*x^(3/2))/(a*b*x^2 + a^2)
```

giac [A] time = 0.62, size = 199, normalized size = 0.91

$$\frac{x^{\frac{3}{2}}}{2\left(bx^2+a\right)a}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b^3}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b^3}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^(3/2)/((b*x^2 + a)*a) + 1/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1/8*sqrt(2)*(a*b
^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(a^2*b^3) - 1/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
x + sqrt(a/b))/(a^2*b^3) + 1/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3)
```

maple [A] time = 0.01, size = 158, normalized size = 0.72

$$\frac{x^{\frac{3}{2}}}{2\left(bx^2+a\right)a}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{\sqrt{2}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*x^(3/2)/a/(b*x^2+a)+1/16/a/b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1
/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/8/a
```

```
/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/8/a/b/(a/b)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.02, size = 194, normalized size = 0.89

$$\frac{x^{\frac{3}{2}}}{2\left(abx^2+a^2\right)}+\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^(3/2)/(a*b*x^2 + a^2) + 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a
^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sq
rt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) -
 sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4
)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqr
t(a))/(a^(1/4)*b^(3/4)))/a
```

mupad [B] time = 0.09, size = 64, normalized size = 0.29

$$\frac{x^{3/2}}{2\,a\,\left(b\,x^2+a\right)}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{4\,{\left(-a\right)}^{5/4}\,b^{3/4}}+\frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{4\,{\left(-a\right)}^{5/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2)^2,x)
```

```
[Out] x^(3/2)/(2*a*(a + b*x^2)) - atan((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(5/4
)*b^(3/4)) + atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(5/4)*b^(3/4))
```

sympy [A] time = 50.14, size = 578, normalized size = 2.65

$$\begin{cases}\frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ -\frac{2}{5b^2x^{\frac{5}{2}}} \\ \frac{2x^{\frac{3}{2}}}{3a^2} \\ \frac{4\sqrt[4]{-1}\sqrt[4]{a}bx^{\frac{3}{2}}\sqrt[4]{\frac{1}{b}}}{8\sqrt[4]{-1}a^{\frac{9}{4}}b\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}a^{\frac{5}{4}}b^2x^2\sqrt[4]{\frac{1}{b}}}+\frac{a\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{8\sqrt[4]{-1}a^{\frac{9}{4}}b\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}a^{\frac{5}{4}}b^2x^2\sqrt[4]{\frac{1}{b}}}-\frac{a\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{8\sqrt[4]{-1}a^{\frac{9}{4}}b\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}a^{\frac{5}{4}}b^2x^2\sqrt[4]{\frac{1}{b}}}-\frac{2a\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{8\sqrt[4]{-1}a^{\frac{9}{4}}b\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}a^{\frac{5}{4}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b**2*x**(5/2)), Eq(a,
 0)), (2*x**(3/2)/(3*a**2), Eq(b, 0)), (4*(-1)**(1/4)*a**(1/4)*b*x**(3/2)*(
1/b)**(1/4)/(8*(-1)**(1/4)*a**(9/4)*b*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(5/4)
*b**2*x**2*(1/b)**(1/4)) + a*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(
x))/(8*(-1)**(1/4)*a**(9/4)*b*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(5/4)*b**2*x*
```

```
*2*(1/b)**(1/4)) - a*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*(-
1)**(1/4)*a**(9/4)*b*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(5/4)*b**2*x**2*(1/b)*
*(1/4)) - 2*a*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*(-1)**(1
/4)*a**(9/4)*b*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(5/4)*b**2*x**2*(1/b)**(1/4)
) + b*x**2*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*(-1)**(1/4)
*a**(9/4)*b*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(5/4)*b**2*x**2*(1/b)**(1/4)) -
 b*x**2*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*(-1)**(1/4)*a**
(9/4)*b*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(5/4)*b**2*x**2*(1/b)**(1/4)) - 2*b
*x**2*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*(-1)**(1/4)*a**(
9/4)*b*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(5/4)*b**2*x**2*(1/b)**(1/4)), True)
)
```

3.300 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\tan^{-1}}{4}$$

```
[Out] -3/8*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(1/4)*2^(1/2)+3/8*
arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(1/4)*2^(1/2)-3/16*ln(a
^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(1/4)*2^(1/2)+3
/16*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(1/4)*2
^(1/2)+1/2*x^(1/2)/a/(b*x^2+a)
```

Rubi [A] time = 0.15, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\tan^{-1}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[x]/(2*a*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(4*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx &= \frac{\sqrt{x}}{2a\left(a+bx^2\right)}+\frac{3\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4a}\\
&= \frac{\sqrt{x}}{2a\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2a}\\
&= \frac{\sqrt{x}}{2a\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a^{3/2}}+\frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a^{3/2}}\\
&= \frac{\sqrt{x}}{2a\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8a^{3/2}\sqrt{b}}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right.}{8a^{3/2}\sqrt{b}}\\
&= \frac{\sqrt{x}}{2a\left(a+bx^2\right)}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right.}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}\\
&= \frac{\sqrt{x}}{2a\left(a+bx^2\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 199, normalized size = 0.91

$$\frac{\frac{8a^{3/4}\sqrt{x}}{a+bx^2}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{\sqrt[4]{b}}}{16a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] ((8*a^(3/4)*Sqrt[x])/(a + b*x^2) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/b^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/b^(1/4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/b^(1/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/b^(1/4))/(16*a^(7/4))
```

fricas [A] time = 0.91, size = 179, normalized size = 0.82

$$\frac{12\left(abx^2+a^2\right)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^4\sqrt{-\frac{1}{a^7b}}+x}\,a^5b\left(-\frac{1}{a^7b}\right)^{\frac{3}{4}}-a^5b\sqrt{x}\left(-\frac{1}{a^7b}\right)^{\frac{3}{4}}\right)+3\left(abx^2+a^2\right)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\log\left(a\right.}{8\left(abx^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(12*(a*b*x^2 + a^2)*(-1/(a^7*b))^(1/4)*arctan(sqrt(a^4*sqrt(-1/(a^7*b))
 + x)*a^5*b*(-1/(a^7*b))^(3/4) - a^5*b*sqrt(x)*(-1/(a^7*b))^(3/4)) + 3*(a*b
*x^2 + a^2)*(-1/(a^7*b))^(1/4)*log(a^2*(-1/(a^7*b))^(1/4) + sqrt(x)) - 3*(a
*b*x^2 + a^2)*(-1/(a^7*b))^(1/4)*log(-a^2*(-1/(a^7*b))^(1/4) + sqrt(x)) + 4
*sqrt(x))/(a*b*x^2 + a^2)
```

giac [A] time = 0.65, size = 199, normalized size = 0.91

$$\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\right.\right.}{16a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a^2*b) + 3/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b) + 3/16*sqrt(2)*(a*b^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b) - 3/16*sqrt
(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b)
 + 1/2*sqrt(x)/((b*x^2 + a)*a)
```

maple [A] time = 0.01, size = 149, normalized size = 0.68

$$\frac{\sqrt{x}}{2\left(bx^2+a\right)a}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^2}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^2}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+}\right.}{16a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/x^(1/2),x)
```

```
[Out] 1/2*x^(1/2)/a/(b*x^2+a)+3/16/a^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1
/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+3/8/a
^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+3/8/a^2*(a/b)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.05, size = 194, normalized size = 0.89

$$\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}\cdots\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqr
t(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a + 1
/2*sqrt(x)/(a*b*x^2 + a^2)
```

mupad [B] time = 0.09, size = 64, normalized size = 0.29

$$\frac{\sqrt{x}}{2a\left(bx^2+a\right)}+\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{4(-a)^{7/4}b^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{4(-a)^{7/4}b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)^2),x)
```

```
[Out] x^(1/2)/(2*a*(a + b*x^2)) + (3*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^
(7/4)*b^(1/4)) + (3*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(7/4)*b^(1
/4))
```

sympy [A] time = 75.79, size = 434, normalized size = 1.99

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} \\ -\frac{2}{7b^2x^{\frac{7}{2}}} \\ \frac{2\sqrt{x}}{a^2} \\ -\frac{3\sqrt[4]{-1}a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{8a^3+8a^2bx^2}+\frac{3\sqrt[4]{-1}a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{8a^3+8a^2bx^2}-\frac{6\sqrt[4]{-1}a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{8a^3+8a^2bx^2}-\frac{3\sqrt[4]{-1}\sqrt[4]{a}bx^2\sqrt[4]{\frac{1}{b}}\log\left(\cdots\right.}{8a^3+8a\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b**2*x**(7/2)), Eq(a,
 0)), (2*sqrt(x)/a**2, Eq(b, 0)), (-3*(-1)**(1/4)*a**(5/4)*(1/b)**(1/4)*log
(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**3 + 8*a**2*b*x**2) + 3
*(-1)**(1/4)*a**(5/4)*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) +
sqrt(x))/(8*a**3 + 8*a**2*b*x**2) - 6*(-1)**(1/4)*a**(5/4)*(1/b)**(1/4)*ata
n((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*a**3 + 8*a**2*b*x**2) - 3
*(-1)**(1/4)*a**(1/4)*b*x**2*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**
(1/4) + sqrt(x))/(8*a**3 + 8*a**2*b*x**2) + 3*(-1)**(1/4)*a**(1/4)*b*x**2*(
1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**3 + 8*a*
*2*b*x**2) - 6*(-1)**(1/4)*a**(1/4)*b*x**2*(1/b)**(1/4)*atan((-1)**(3/4)*sq
rt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*a**3 + 8*a**2*b*x**2) + 4*a*sqrt(x)/(8*a*
*3 + 8*a**2*b*x**2), True))
```

3.301 $\int \frac{1}{x^{3/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}}-$$

```
[Out] 5/8*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)*2^(1/2)-5/8*b
^(1/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)*2^(1/2)-5/16*b^(1/
4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)*2^(1/2)+5/
16*b^(1/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)*2^
(1/2)-5/2/a^2/x^(1/2)+1/2/a/(b*x^2+a)/x^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] -5/(2*a^2*Sqrt[x]) + 1/(2*a*Sqrt[x]*(a + b*x^2)) + (5*b^(1/4)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)) - (5*b^(1/4)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)) - (5*b^(1/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)
) + (5*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*a^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx = \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} + \frac{5\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{4a}$$

$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} - \frac{(5b)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a^2}$$

$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} - \frac{(5b)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a^2}$$

$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} + \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^2} - \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{}}{}\right.}{4a}$$

$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} - \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8a^2} - \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}}\right.}{8a^2}$$

$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} - \frac{5\sqrt[4]{b}\,\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}} + \frac{5\sqrt[4]{b}\,\log\left(\sqrt{a}+\right.}{8}$$

$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} + \frac{5\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}} - \frac{5\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}} -$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 2, 3/4, -((b*x^2)/a)])/(a^2*Sqrt[x])
```

fricas [A] time = 0.95, size = 208, normalized size = 0.90

$$\frac{20\left(a^2bx^3+a^3x\right)\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\,a^2b\sqrt{x}\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}-\sqrt{-15625\,a^5b\sqrt{-\frac{b}{a^9}}+15625\,b^2x}\,a^2\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}}{125\,b}\right)-5\left(a^2bx^3+a^3x\right)\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}\log}{8\left(a^2bx^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(20*(a^2*b*x^3 + a^3*x)*(-b/a^9)^(1/4)*arctan(-1/125*(125*a^2*b*sqrt(x)
*(-b/a^9)^(1/4) - sqrt(-15625*a^5*b*sqrt(-b/a^9) + 15625*b^2*x)*a^2*(-b/a^9
)^(1/4))/b) - 5*(a^2*b*x^3 + a^3*x)*(-b/a^9)^(1/4)*log(125*a^7*(-b/a^9)^(3/
4) + 125*b*sqrt(x)) + 5*(a^2*b*x^3 + a^3*x)*(-b/a^9)^(1/4)*log(-125*a^7*(-b
/a^9)^(3/4) + 125*b*sqrt(x)) - 4*(5*b*x^2 + 4*a)*sqrt(x))/(a^2*b*x^3 + a^3*
x)
```

giac [A] time = 0.62, size = 210, normalized size = 0.91

$$-\frac{5\,bx^{2}+4\,a}{2\left(bx^{\frac{5}{2}}+a\sqrt{x}\right)a^{2}}-\frac{5\,\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{3}b^{2}}-\frac{5\,\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{3}b^{2}}+\frac{5\,\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*b*x^2 + 4*a)/((b*x^(5/2) + a*sqrt(x))*a^2) - 5/8*sqrt(2)*(a*b^3)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*
b^2) - 5/8*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 5/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) - 5/16*sqrt(2)*(a*b^3)^(3/4
)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2)
```

maple [A] time = 0.02, size = 158, normalized size = 0.69

$$-\frac{b\,x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}-\frac{5\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}-\frac{2}{a^{2}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -2/a^2/x^(1/2)-1/2/a^2*b*x^(3/2)/(b*x^2+a)-5/16/a^2/(a/b)^(1/4)*2^(1/2)*ln(
(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+
(a/b)^(1/2)))-5/8/a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)-5/8/a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.98, size = 208, normalized size = 0.90

$$-\frac{5\,bx^{2}+4\,a}{2\left(a^{2}bx^{\frac{5}{2}}+a^{3}\sqrt{x}\right)}-\frac{5\,b\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{16\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(5*b*x^2 + 4*a)/(a^2*b*x^(5/2) + a^3*sqrt(x)) - 5/16*b*(2*sqrt(2)*arct
an(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sq
rt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqr
t(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*
sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^2
```

mupad [B] time = 0.08, size = 77, normalized size = 0.33

$$\frac{5\,{\left(-b\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{9/4}}-\frac{5\,{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{9/4}}-\frac{\frac{2}{a}+\frac{5\,b\,x^2}{2\,a^2}}{a\,\sqrt{x}+b\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)^2),x)
```

```
[Out] (5*(-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(9/4)) - (5*(-b)^(1
/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(9/4)) - (2/a + (5*b*x^2)/(2*a
^2))/(a*x^(1/2) + b*x^(5/2))
```

sympy [A] time = 150.18, size = 700, normalized size = 3.04

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} \\ -\frac{2}{9b^2x^{\frac{9}{2}}} \\ -\frac{2}{a^2\sqrt{x}} \\ -\frac{16\sqrt[4]{-1}\,a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}}{8\sqrt[4]{-1}\,a^{\frac{13}{4}}\sqrt{x}\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}\,a^{\frac{9}{4}}bx^{\frac{5}{2}}\sqrt[4]{\frac{1}{b}}}-\frac{20\sqrt[4]{-1}\sqrt[4]{a}\,bx^2\sqrt[4]{\frac{1}{b}}}{8\sqrt[4]{-1}\,a^{\frac{13}{4}}\sqrt{x}\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}\,a^{\frac{9}{4}}bx^{\frac{5}{2}}\sqrt[4]{\frac{1}{b}}}-\frac{5a\sqrt{x}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{8\sqrt[4]{-1}\,a^{\frac{13}{4}}\sqrt{x}\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}\,a^{\frac{9}{4}}bx^{\frac{5}{2}}\sqrt[4]{\frac{1}{b}}}+\frac{5a\sqrt{x}\log\left(\sqrt[4]{-1}\right.}{8\sqrt[4]{-1}\,a^{\frac{13}{4}}\sqrt{x}\sqrt[4]{\frac{1}{b}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b**2*x**(9/2)), Eq(a,
 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)), (-16*(-1)**(1/4)*a**(5/4)*(1/b)**(1/4)
/(8*(-1)**(1/4)*a**(13/4)*sqrt(x)*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(9/4)*b*x
**(5/2)*(1/b)**(1/4)) - 20*(-1)**(1/4)*a**(1/4)*b*x**2*(1/b)**(1/4)/(8*(-1)
**(1/4)*a**(13/4)*sqrt(x)*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(9/4)*b*x**(5/2)*
(1/b)**(1/4)) - 5*a*sqrt(x)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x
))/(8*(-1)**(1/4)*a**(13/4)*sqrt(x)*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(9/4)*b
*x**(5/2)*(1/b)**(1/4)) + 5*a*sqrt(x)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4)
 + sqrt(x))/(8*(-1)**(1/4)*a**(13/4)*sqrt(x)*(1/b)**(1/4) + 8*(-1)**(1/4)*a
**(9/4)*b*x**(5/2)*(1/b)**(1/4)) + 10*a*sqrt(x)*atan((-1)**(3/4)*sqrt(x)/(a
**(1/4)*(1/b)**(1/4)))/(8*(-1)**(1/4)*a**(13/4)*sqrt(x)*(1/b)**(1/4) + 8*(-
1)**(1/4)*a**(9/4)*b*x**(5/2)*(1/b)**(1/4)) - 5*b*x**(5/2)*log(-(-1)**(1/4)
*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*(-1)**(1/4)*a**(13/4)*sqrt(x)*(1/b)**(
1/4) + 8*(-1)**(1/4)*a**(9/4)*b*x**(5/2)*(1/b)**(1/4)) + 5*b*x**(5/2)*log((
-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*(-1)**(1/4)*a**(13/4)*sqrt(x
)*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(9/4)*b*x**(5/2)*(1/b)**(1/4)) + 10*b*x**
(5/2)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*(-1)**(1/4)*a**(
13/4)*sqrt(x)*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(9/4)*b*x**(5/2)*(1/b)**(1/4)
), True))
```

3.302 $\int \frac{1}{x^{5/2}(a+bx^2)^2}\,dx$

Optimal. Leaf size=230

$$\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}}$$

```
[Out] -7/6/a^2/x^(3/2)+1/2/a/x^(3/2)/(b*x^2+a)+7/8*b^(3/4)*arctan(1-b^(1/4)*2^(1/
2)*x^(1/2)/a^(1/4))/a^(11/4)*2^(1/2)-7/8*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x
^(1/2)/a^(1/4))/a^(11/4)*2^(1/2)+7/16*b^(3/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*
b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)*2^(1/2)-7/16*b^(3/4)*ln(a^(1/2)+x*b^(1/2)
+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] -7/(6*a^2*x^(3/2)) + 1/(2*a*x^(3/2)*(a + b*x^2)) + (7*b^(3/4)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)) - (7*b^(3/4)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)) + (7*b^(3/4)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11
/4)) - (7*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx = \frac{1}{2ax^{3/2}\left(a+bx^2\right)}+\frac{7\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx}{4a}$$

$$= -\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}-\frac{(7b)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4a^2}$$

$$= -\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}-\frac{(7b)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2a^2}$$

$$= -\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}-\frac{(7b)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a^{5/2}}-\frac{(7b)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}+\cdots}{a+\cdots}\right.}{4a^{5\cdots}}$$

$$= -\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}-\frac{\left(7\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8a^{5/2}}-\frac{\left(7\sqrt{b}\right)\mathrm{Sub}\cdots}{\cdots}$$

$$= -\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}+\frac{7b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\,a^{11/4}}-\frac{7b^{3/4}\log\left(\sqrt{a}\cdots\right.}{\cdots}$$

$$= -\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{11/4}}-\frac{7b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{11/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},2;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 2, 1/4, -((b*x^2)/a)])/(3*a^2*x^(3/2))
```

fricas [A] time = 1.07, size = 228, normalized size = 0.99

$$-\frac{84\left(a^2bx^4+a^3x^2\right)\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^8b\sqrt{x}\left(-\frac{b^3}{a^{11}}\right)^{\frac{3}{4}}-\sqrt{a^6\sqrt{-\frac{b^3}{a^{11}}}+b^2x}\,a^8\left(-\frac{b^3}{a^{11}}\right)^{\frac{3}{4}}}{b^3}\right)+21\left(a^2bx^4+a^3x^2\right)\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\log\left(7\cdots\right.}{24\left(a^2bx^4+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(84*(a^2*b*x^4 + a^3*x^2)*(-b^3/a^11)^(1/4)*arctan(-(a^8*b*sqrt(x)*(-
b^3/a^11)^(3/4) - sqrt(a^6*sqrt(-b^3/a^11) + b^2*x)*a^8*(-b^3/a^11)^(3/4))/
b^3) + 21*(a^2*b*x^4 + a^3*x^2)*(-b^3/a^11)^(1/4)*log(7*a^3*(-b^3/a^11)^(1/
4) + 7*b*sqrt(x)) - 21*(a^2*b*x^4 + a^3*x^2)*(-b^3/a^11)^(1/4)*log(-7*a^3*(
-b^3/a^11)^(1/4) + 7*b*sqrt(x)) + 4*(7*b*x^2 + 4*a)*sqrt(x))/(a^2*b*x^4 + a
^3*x^2)
```

giac [A] time = 0.64, size = 196, normalized size = 0.85

$$-\frac{7\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{3}}-\frac{7\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{3}}-\frac{7\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right.\right.}{16a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -7/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/a^3 - 7/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^3 - 7/16*sqrt(2)*(a*b^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^3 + 7/16*sqrt(2)*(a*b^3)
^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^3 - 1/2*b*sqrt(x
)/((b*x^2 + a)*a^2) - 2/3/(a^2*x^(3/2))
```

maple [A] time = 0.02, size = 161, normalized size = 0.70

$$-\frac{b\sqrt{x}}{2\left(bx^{2}+a\right)a^{2}}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^{3}}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^{3}}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}}\right.}{16a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] -2/3/a^2/x^(3/2)-1/2/a^2*b*x^(1/2)/(b*x^2+a)-7/16/a^3*b*(a/b)^(1/4)*2^(1/2)
*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1
/2)+(a/b)^(1/2)))-7/8/a^3*b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)+1)-7/8/a^3*b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
-1)
```

maxima [A] time = 3.06, size = 209, normalized size = 0.91

$$-\frac{7bx^{2}+4a}{6\left(a^{2}bx^{\frac{7}{2}}+a^{3}x^{\frac{3}{2}}\right)}-\frac{7\left(\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\right.}{a^{\frac{3}{4}}}\right.}{16a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(7*b*x^2 + 4*a)/(a^2*b*x^(7/2) + a^3*x^(3/2)) - 7/16*(2*sqrt(2)*b*arct
an(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*
(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqr
t(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/a^2
```

mupad [B] time = 4.68, size = 77, normalized size = 0.33

$$\frac{7\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{11/4}}-\frac{\frac{2}{3\,a}+\frac{7\,b\,x^2}{6\,a^2}}{a\,x^{3/2}+b\,x^{7/2}}+\frac{7\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)^2),x)
```

```
[Out] (7*(-b)^(3/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(11/4)) - (2/(3*a) +
 (7*b*x^2)/(6*a^2))/(a*x^(3/2) + b*x^(7/2)) + (7*(-b)^(3/4)*atanh(((-b)^(1/
4)*x^(1/2))/a^(1/4)))/(4*a^(11/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.303 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=243

$$\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}-\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{13/4}}+\cdots$$

```
[Out] -9/10/a^2/x^(5/2)+1/2/a/x^(5/2)/(b*x^2+a)-9/8*b^(5/4)*arctan(1-b^(1/4)*2^(1
/2)*x^(1/2)/a^(1/4))/a^(13/4)*2^(1/2)+9/8*b^(5/4)*arctan(1+b^(1/4)*2^(1/2)*
x^(1/2)/a^(1/4))/a^(13/4)*2^(1/2)+9/16*b^(5/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)
*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)*2^(1/2)-9/16*b^(5/4)*ln(a^(1/2)+x*b^(1/2
)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)*2^(1/2)+9/2*b/a^3/x^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}-\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{13/4}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] -9/(10*a^2*x^(5/2)) + (9*b)/(2*a^3*Sqrt[x]) + 1/(2*a*x^(5/2)*(a + b*x^2)) -
 (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13
/4)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]
*a^(13/4)) + (9*b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(8*Sqrt[2]*a^(13/4)) - (9*b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx = \frac{1}{2ax^{5/2}\left(a+bx^2\right)} + \frac{9\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx}{4a}$$

$$= -\frac{9}{10a^2x^{5/2}} + \frac{1}{2ax^{5/2}\left(a+bx^2\right)} - \frac{(9b)\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{4a^2}$$

$$= -\frac{9}{10a^2x^{5/2}} + \frac{9b}{2a^3\sqrt{x}} + \frac{1}{2ax^{5/2}\left(a+bx^2\right)} + \frac{\left(9b^2\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a^3}$$

$$= -\frac{9}{10a^2x^{5/2}} + \frac{9b}{2a^3\sqrt{x}} + \frac{1}{2ax^{5/2}\left(a+bx^2\right)} + \frac{\left(9b^2\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a^3}$$

$$= -\frac{9}{10a^2x^{5/2}} + \frac{9b}{2a^3\sqrt{x}} + \frac{1}{2ax^{5/2}\left(a+bx^2\right)} - \frac{\left(9b^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^3} + \frac{\left(9b^{3/2}\right)}{}$$

$$= -\frac{9}{10a^2x^{5/2}} + \frac{9b}{2a^3\sqrt{x}} + \frac{1}{2ax^{5/2}\left(a+bx^2\right)} + \frac{(9b)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8a^3} + \frac{(9b)}{}$$

$$= -\frac{9}{10a^2x^{5/2}} + \frac{9b}{2a^3\sqrt{x}} + \frac{1}{2ax^{5/2}\left(a+bx^2\right)} + \frac{9b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}} - \frac{9b^{5/4}}{}$$

$$= -\frac{9}{10a^2x^{5/2}} + \frac{9b}{2a^3\sqrt{x}} + \frac{1}{2ax^{5/2}\left(a+bx^2\right)} - \frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{13/4}} + \frac{9b^{5/4}\tan^{-1}\left(1+\right.}{4\sqrt{2}\,a^{13/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},2;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 2, -1/4, -((b*x^2)/a)])/(5*a^2*x^(5/2))
```

fricas [A] time = 0.49, size = 251, normalized size = 1.03

$$-\frac{180\left(a^3bx^5+a^4x^3\right)\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{729\,a^3b^4\sqrt{x}\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}}-\sqrt{-531441\,a^7b^5\sqrt{-\frac{b^5}{a^{13}}}+531441\,b^8x}\,a^3\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}}}{729\,b^5}\right)-45\left(a^3bx^5+a^4x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(180*(a^3*b*x^5 + a^4*x^3)*(-b^5/a^13)^(1/4)*arctan(-1/729*(729*a^3*b
^4*sqrt(x)*(-b^5/a^13)^(1/4) - sqrt(-531441*a^7*b^5*sqrt(-b^5/a^13) + 53144
1*b^8*x)*a^3*(-b^5/a^13)^(1/4))/b^5) - 45*(a^3*b*x^5 + a^4*x^3)*(-b^5/a^13)
```

```
^(1/4)*log(729*a^10*(-b^5/a^13)^(3/4) + 729*b^4*sqrt(x)) + 45*(a^3*b*x^5 +
a^4*x^3)*(-b^5/a^13)^(1/4)*log(-729*a^10*(-b^5/a^13)^(3/4) + 729*b^4*sqrt(x
)) - 4*(45*b^2*x^4 + 36*a*b*x^2 - 4*a^2)*sqrt(x))/(a^3*b*x^5 + a^4*x^3)
```

giac [A] time = 0.66, size = 220, normalized size = 0.91

$$\frac{b^2x^{\frac{3}{2}}}{2\left(bx^2+a\right)a^3}+\frac{9\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^4b}+\frac{9\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^4b}-\frac{9\sqrt{2}\left(ab^3\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^(3/2)/((b*x^2 + a)*a^3) + 9/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) + 9/8*sqrt(2)*
(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(
1/4))/(a^4*b) - 9/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
+ x + sqrt(a/b))/(a^4*b) + 9/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) + 2/5*(10*b*x^2 - a)/(a^3*x^(5/2))
```

maple [A] time = 0.02, size = 172, normalized size = 0.71

$$\frac{b^2x^{\frac{3}{2}}}{2\left(bx^2+a\right)a^3}+\frac{9\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3}+\frac{9\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3}+\frac{9\sqrt{2}\,b\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}a^3}+\frac{4b}{a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)^2,x)
```

```
[Out] -2/5/a^2/x^(5/2)+4/a^3*b/x^(1/2)+1/2*b^2/a^3*x^(3/2)/(b*x^2+a)+9/16*b/a^3/(
a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^
(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+9/8*b/a^3/(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)+9/8*b/a^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.04, size = 221, normalized size = 0.91

$$\frac{45\,b^2x^4+36\,abx^2-4\,a^2}{10\left(a^3bx^{\frac{9}{2}}+a^4x^{\frac{5}{2}}\right)}+\frac{9\,b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{}\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{16\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*(45*b^2*x^4 + 36*a*b*x^2 - 4*a^2)/(a^3*b*x^(9/2) + a^4*x^(5/2)) + 9/16
*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqr
t(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b
```

```
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3
```

mupad [B] time = 4.69, size = 87, normalized size = 0.36

$$\frac{\frac{18\,b\,x^2}{5\,a^2}-\frac{2}{5\,a}+\frac{9\,b^2\,x^4}{2\,a^3}}{a\,x^{5/2}+b\,x^{9/2}}-\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{13/4}}+\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)^2),x)
```

```
[Out] ((18*b*x^2)/(5*a^2) - 2/(5*a) + (9*b^2*x^4)/(2*a^3))/(a*x^(5/2) + b*x^(9/2)
) - (9*(-b)^(5/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(13/4)) + (9*(-b
)^(5/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.304 $\int \frac{x^{7/2}}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}}+\frac{5\tan^{-1}}{32}$$

```
[Out] -1/4*x^(5/2)/b/(b*x^2+a)^2-5/64*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a
^(3/4)/b^(9/4)*2^(1/2)+5/64*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/
4)/b^(9/4)*2^(1/2)-5/128*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/
2))/a^(3/4)/b^(9/4)*2^(1/2)+5/128*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1
/2)*x^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)-5/16*x^(1/2)/b^2/(b*x^2+a)
```

Rubi [A] time = 0.18, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}}+\frac{5\tan^{-1}}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2)^3,x]
```

```
[Out] -x^(5/2)/(4*b*(a + b*x^2)^2) - (5*Sqrt[x])/(16*b^2*(a + b*x^2)) - (5*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4)) + (5*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4))
 - (5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[
2]*a^(3/4)*b^(9/4)) + (5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)^3}\,dx = -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2} + \frac{5\int \frac{x^{3/2}}{\left(a+bx^2\right)^2}\,dx}{8b}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2} - \frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)} + \frac{5\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32b^2}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2} - \frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16b^2}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2} - \frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32\sqrt{a}\,b^2} + \frac{5\,\mathrm{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\right.}{32\sqrt{a}\,b^2}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2} - \frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{a}\,b^{5/2}} + \frac{5\,\mathrm{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}+\cdots}\right.}{64}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2} - \frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)} - \frac{5\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{64\sqrt{2}\,a^{3/4}b^{9/4}} + \frac{5\log\left(\sqrt{a} + \sqrt{2}\right.}{64\sqrt{}}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2} - \frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)} - \frac{5\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}} + \frac{5\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}} - \frac{5\,\mathrm{lo}}{}$$

Mathematica [A] time = 0.11, size = 242, normalized size = 1.01

$$\frac{-\frac{15\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}} + \frac{15\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}} - \frac{30\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{30\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right.}{a^{3/4}}}{384b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2)^3,x]
```

```
[Out] ((-160*a*b^(1/4)*Sqrt[x])/(a + b*x^2)^2 - (256*b^(5/4)*x^(5/2))/(a + b*x^2)
^2 + (40*b^(1/4)*Sqrt[x])/(a + b*x^2) - (30*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (30*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/a^(3/4) - (15*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + (15*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4))/(384*b^(9/4))
```

fricas [A] time = 1.22, size = 254, normalized size = 1.06

$$\frac{20\left(b^4x^4 + 2\,ab^3x^2 + a^2b^2\right)\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2b^4\sqrt{-\frac{1}{a^3b^9}} + x}\,a^2b^7\left(-\frac{1}{a^3b^9}\right)^{\frac{3}{4}} - a^2b^7\sqrt{x}\left(-\frac{1}{a^3b^9}\right)^{\frac{3}{4}}\right) + 5\left(b^4x^4 + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-1/(a^3*b^9))^(1/4)*arctan(sqrt
(a^2*b^4*sqrt(-1/(a^3*b^9)) + x)*a^2*b^7*(-1/(a^3*b^9))^(3/4) - a^2*b^7*sqr
t(x)*(-1/(a^3*b^9))^(3/4)) + 5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-1/(a^3*b
^9))^(1/4)*log(a*b^2*(-1/(a^3*b^9))^(1/4) + sqrt(x)) - 5*(b^4*x^4 + 2*a*b^3
*x^2 + a^2*b^2)*(-1/(a^3*b^9))^(1/4)*log(-a*b^2*(-1/(a^3*b^9))^(1/4) + sqrt
(x)) - 4*(9*b*x^2 + 5*a)*sqrt(x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 0.65, size = 209, normalized size = 0.87

$$\frac{5\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^3}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^3}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right.}{128\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a*b^3) + 5/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) + 5/128*sqrt(2)*(a*b
^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) - 5/128*
sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*
b^3) - 1/16*(9*b*x^(5/2) + 5*a*sqrt(x))/((b*x^2 + a)^2*b^2)
```

maple [A] time = 0.01, size = 170, normalized size = 0.71

$$\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a\,b^2}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a\,b^2}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128a\,b^2}+\frac{-\frac{9x^{\frac{5}{2}}}{16b}-\frac{5a\sqrt{x}}{16b^2}}{\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)^3,x)
```

```
[Out] 2*(-9/32/b*x^(5/2)-5/32*a/b^2*x^(1/2))/(b*x^2+a)^2+5/128/b^2*(a/b)^(1/4)/a*
2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/
2)*x^(1/2)+(a/b)^(1/2)))+5/64/b^2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)+1)+5/64/b^2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)-1)
```

maxima [A] time = 3.09, size = 218, normalized size = 0.91

$$-\frac{9\,bx^{\frac{5}{2}}+5\,a\sqrt{x}}{16\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}+\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(9*b*x^(5/2) + 5*a*sqrt(x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) + 5/128
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
```

```
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sq
rt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt(2)*
a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^2
```

mupad [B] time = 4.69, size = 87, normalized size = 0.36

$$-\frac{\frac{9\,x^{5/2}}{16\,b}+\frac{5\,a\,\sqrt{x}}{16\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{5\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{3/4}\,b^{9/4}}-\frac{5\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{3/4}\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2)^3,x)
```

```
[Out] - ((9*x^(5/2))/(16*b) + (5*a*x^(1/2))/(16*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)
 - (5*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(3/4)*b^(9/4)) - (5*atan
h((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(3/4)*b^(9/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.305 $\int \frac{x^{5/2}}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=242

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}}+\frac{3\tan^{-1}\left(\sqrt{2}\right.}{32\sqrt{2}}$$

```
[Out] -1/4*x^(3/2)/b/(b*x^2+a)^2+3/16*x^(3/2)/a/b/(b*x^2+a)-3/64*arctan(1-b^(1/4)
*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(7/4)*2^(1/2)+3/64*arctan(1+b^(1/4)*2^(
1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(7/4)*2^(1/2)+3/128*ln(a^(1/2)+x*b^(1/2)-a^
(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/b^(7/4)*2^(1/2)-3/128*ln(a^(1/2)+x*b
^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/b^(7/4)*2^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}}+\frac{3\tan^{-1}\left(\sqrt{2}\right.}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2)^3,x]
```

```
[Out] -x^(3/2)/(4*b*(a + b*x^2)^2) + (3*x^(3/2))/(16*a*b*(a + b*x^2)) - (3*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(7/4)) + (3*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(7/4))
 + (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[
2]*a^(5/4)*b^(7/4)) - (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2\right)^3}\,dx = -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2} + \frac{3\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2}\,dx}{8b}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2} + \frac{3x^{3/2}}{16ab\left(a+bx^2\right)} + \frac{3\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32ab}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2} + \frac{3x^{3/2}}{16ab\left(a+bx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16ab}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2} + \frac{3x^{3/2}}{16ab\left(a+bx^2\right)} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32ab^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx\right.}{32ab^{3/2}}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2} + \frac{3x^{3/2}}{16ab\left(a+bx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64ab^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}}{}\right.}{64a}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2} + \frac{3x^{3/2}}{16ab\left(a+bx^2\right)} + \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{5/4}b^{7/4}} - \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{\right.}{64\sqrt{2}\,a}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2} + \frac{3x^{3/2}}{16ab\left(a+bx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}} + \frac{3\log}{}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.19

$$\frac{2x^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)}{a^2} - \frac{1}{\left(a+bx^2\right)^2}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2)^3,x]
```

```
[Out] (2*x^(3/2)*(-(a + b*x^2)^(-2) + Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)
]/a^2))/(5*b)
```

fricas [A] time = 0.96, size = 260, normalized size = 1.07

$$-\frac{12\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x}\,ab^2\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}} - ab^2\sqrt{x}\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}\right) - 3\left(ab^3x^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^5*b^7))^(1/4)*arctan(s
qrt(-a^3*b^3*sqrt(-1/(a^5*b^7)) + x)*a*b^2*(-1/(a^5*b^7))^(1/4) - a*b^2*sqr
t(x)*(-1/(a^5*b^7))^(1/4)) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^5
*b^7))^(1/4)*log(a^4*b^5*(-1/(a^5*b^7))^(3/4) + sqrt(x)) + 3*(a*b^3*x^4 + 2
```

```
*a^2*b^2*x^2 + a^3*b)*(-1/(a^5*b^7))^(1/4)*log(-a^4*b^5*(-1/(a^5*b^7))^(3/4
) + sqrt(x)) - 4*(3*b*x^3 - a*x)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*
b)
```

giac [A] time = 0.66, size = 212, normalized size = 0.88

$$\frac{3\,bx^{\frac{7}{2}}-ax^{\frac{3}{2}}}{16\left(bx^2+a\right)^2ab}+\frac{3\,\sqrt{2}\,\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^4}+\frac{3\,\sqrt{2}\,\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^4}-\frac{3\,\sqrt{2}\,\left(ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*b*x^(7/2) - a*x^(3/2))/((b*x^2 + a)^2*a*b) + 3/64*sqrt(2)*(a*b^3)^(
3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2
*b^4) + 3/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) - 3/128*sqrt(2)*(a*b^3)^(3/4)*log(sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) + 3/128*sqrt(2)*(a*b^3)^
(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4)
```

maple [A] time = 0.02, size = 169, normalized size = 0.70

$$\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a\,b^2}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a\,b^2}+\frac{3\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a\,b^2}+\frac{\frac{3x^{\frac{7}{2}}}{16a}-\frac{x^{\frac{3}{2}}}{16b}}{\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)^3,x)
```

```
[Out] 2*(3/32/a*x^(7/2)-1/32/b*x^(3/2))/(b*x^2+a)^2+3/128/b^2/a/(a/b)^(1/4)*2^(1/
2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^
(1/2)+(a/b)^(1/2)))+3/64/b^2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)+1)+3/64/b^2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)-1)
```

maxima [A] time = 2.97, size = 222, normalized size = 0.92

$$\frac{3\,bx^{\frac{7}{2}}-ax^{\frac{3}{2}}}{16\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)}+\frac{3\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right.}{a^{\frac{1}{4}}b}\right.}{128\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*b*x^(7/2) - a*x^(3/2))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 3/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4
```

```
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b)
```

mupad [B] time = 0.08, size = 85, normalized size = 0.35

$$\frac{\frac{3\,x^{7/2}}{16\,a}-\frac{x^{3/2}}{16\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{5/4}\,b^{7/4}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{5/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2)^3,x)
```

```
[Out] ((3*x^(7/2))/(16*a) - x^(3/2)/(16*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (3*atan
((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(5/4)*b^(7/4)) + (3*atanh((b^(1/4)
*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(5/4)*b^(7/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.306 $\int \frac{x^{3/2}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}}+\frac{3\tan^{-1}}{32}$$

```
[Out] -3/64*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(5/4)*2^(1/2)+3/6
4*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(5/4)*2^(1/2)-3/128*l
n(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(5/4)*2^(1/2
)+3/128*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(5/
4)*2^(1/2)-1/4*x^(1/2)/b/(b*x^2+a)^2+1/16*x^(1/2)/a/b/(b*x^2+a)
```

Rubi [A] time = 0.16, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}}+\frac{3\tan^{-1}}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2)^3,x]
```

```
[Out] -Sqrt[x]/(4*b*(a + b*x^2)^2) + Sqrt[x]/(16*a*b*(a + b*x^2)) - (3*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(5/4)) + (3*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(5/4)) - (
3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a
^(7/4)*b^(5/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(64*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)^3}\,dx = -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2} + \frac{\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx}{8b}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2} + \frac{\sqrt{x}}{16ab\left(a+bx^2\right)} + \frac{3\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32ab}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2} + \frac{\sqrt{x}}{16ab\left(a+bx^2\right)} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16ab}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2} + \frac{\sqrt{x}}{16ab\left(a+bx^2\right)} + \frac{3\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^{3/2}b} + \frac{3\,\mathrm{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\right.}{32a^{3/2}b}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2} + \frac{\sqrt{x}}{16ab\left(a+bx^2\right)} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^{3/2}b^{3/2}} + \frac{3\,\mathrm{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}+}\right.}{64}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2} + \frac{\sqrt{x}}{16ab\left(a+bx^2\right)} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{5/4}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\right.}{64\sqrt{}}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2} + \frac{\sqrt{x}}{16ab\left(a+bx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}} - \frac{3\,\mathrm{l}}{}$$

Mathematica [A] time = 0.11, size = 223, normalized size = 0.92

$$\frac{-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{7/4}} + \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{7/4}} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}} + }{128b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2)^3,x]
```

```
[Out] ((-32*b^(1/4)*Sqrt[x])/(a + b*x^2)^2 + (8*b^(1/4)*Sqrt[x])/(a^2 + a*b*x^2)
- (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(7/4) + (6*Sq
rt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(7/4) - (3*Sqrt[2]*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (3*Sqr
t[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4))/(
128*b^(5/4))
```

fricas [A] time = 0.95, size = 257, normalized size = 1.06

$$\frac{12\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{1}{a^7b^5}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x}\,a^5b^4\left(-\frac{1}{a^7b^5}\right)^{\frac{3}{4}} - a^5b^4\sqrt{x}\left(-\frac{1}{a^7b^5}\right)^{\frac{3}{4}}\right) + 3\left(ab^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^7*b^5))^(1/4)*arctan(sq
rt(a^4*b^2*sqrt(-1/(a^7*b^5)) + x)*a^5*b^4*(-1/(a^7*b^5))^(3/4) - a^5*b^4*s
qrt(x)*(-1/(a^7*b^5))^(3/4)) + 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a
^7*b^5))^(1/4)*log(a^2*b*(-1/(a^7*b^5))^(1/4) + sqrt(x)) - 3*(a*b^3*x^4 + 2
*a^2*b^2*x^2 + a^3*b)*(-1/(a^7*b^5))^(1/4)*log(-a^2*b*(-1/(a^7*b^5))^(1/4)
+ sqrt(x)) + 4*(b*x^2 - 3*a)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

giac [A] time = 0.66, size = 211, normalized size = 0.87

$$\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^2}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^2}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right.}{128\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a^2*b^2) + 3/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 3/128*sqrt(2)*
(a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^2) -
3/128*sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b
))/(a^2*b^2) + 1/16*(b*x^(5/2) - 3*a*sqrt(x))/((b*x^2 + a)^2*a*b)
```

maple [A] time = 0.02, size = 169, normalized size = 0.70

$$\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a^2b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a^2b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128a^2b}+\frac{\frac{x^{\frac{5}{2}}}{16a}-\frac{3\sqrt{x}}{16b}}{\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)^3,x)
```

```
[Out] 2*(1/32/a*x^(5/2)-3/32/b*x^(1/2))/(b*x^2+a)^2+3/128/b/a^2*(a/b)^(1/4)*2^(1/
2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^
(1/2)+(a/b)^(1/2)))+3/64/b/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)+1)+3/64/b/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)-1)
```

maxima [A] time = 2.89, size = 221, normalized size = 0.91

$$\frac{bx^{\frac{5}{2}}-3\,a\sqrt{x}}{16\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{128\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(b*x^(5/2) - 3*a*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 3/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))
```

```
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b)
```

mupad [B] time = 4.67, size = 85, normalized size = 0.35

$$\frac{\frac{x^{5/2}}{16\,a}-\frac{3\,\sqrt{x}}{16\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{32\,{(-a)}^{7/4}\,b^{5/4}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{32\,{(-a)}^{7/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2)^3,x)
```

```
[Out] (x^(5/2)/(16*a) - (3*x^(1/2))/(16*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (3*atan
((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(7/4)*b^(5/4)) + (3*atanh((b^(1/4)
*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(7/4)*b^(5/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.307 $\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=239

$$\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}+\frac{5\tan^{-1}\left(\sqrt{2}\right.}{32\sqrt{2}a}$$

```
[Out] 1/4*x^(3/2)/a/(b*x^2+a)^2+5/16*x^(3/2)/a^2/(b*x^2+a)-5/64*arctan(1-b^(1/4)*
2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^(3/4)*2^(1/2)+5/64*arctan(1+b^(1/4)*2^(1
/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^(3/4)*2^(1/2)+5/128*ln(a^(1/2)+x*b^(1/2)-a^(
1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(3/4)*2^(1/2)-5/128*ln(a^(1/2)+x*b^
(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(3/4)*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}+\frac{5\tan^{-1}\left(\sqrt{2}\right.}{32\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2)^3,x]
```

```
[Out] x^(3/2)/(4*a*(a + b*x^2)^2) + (5*x^(3/2))/(16*a^2*(a + b*x^2)) - (5*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4)) + (5*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4))
+ (5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2
]*a^(9/4)*b^(3/4)) - (5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2\right)^3}\,dx = \frac{x^{3/2}}{4a\left(a+bx^2\right)^2} + \frac{5\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2}\,dx}{8a}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2} + \frac{5x^{3/2}}{16a^2\left(a+bx^2\right)} + \frac{5\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^2}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2} + \frac{5x^{3/2}}{16a^2\left(a+bx^2\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16a^2}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2} + \frac{5x^{3/2}}{16a^2\left(a+bx^2\right)} - \frac{5\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^2\sqrt{b}} + \frac{5\,\mathrm{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x\right.}{32a^2\sqrt{b}}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2} + \frac{5x^{3/2}}{16a^2\left(a+bx^2\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^2b} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{b}}}\right.}{64a^2}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2} + \frac{5x^{3/2}}{16a^2\left(a+bx^2\right)} + \frac{5\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{3/4}} - \frac{5\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\right.}{64\sqrt{2}\,a^{9/}}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2} + \frac{5x^{3/2}}{16a^2\left(a+bx^2\right)} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{9/4}b^{3/4}} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{9/4}b^{3/4}} + \frac{5\log\left(\right.}{}$$

Mathematica [**C**] time = 0.01, size = 29, normalized size = 0.12

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2)^3,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)])/(3*a^3)
```

fricas [**A**] time = 1.05, size = 250, normalized size = 1.05

$$-\frac{20\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-a^5b\sqrt{-\frac{1}{a^9b^3}}+x}\,a^2b\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}-a^2b\sqrt{x}\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\right)-5\left(a^2b^2x^4+\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(20*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^9*b^3))^(1/4)*arctan(sqr
t(-a^5*b*sqrt(-1/(a^9*b^3)) + x)*a^2*b*(-1/(a^9*b^3))^(1/4) - a^2*b*sqrt(x)
*(-1/(a^9*b^3))^(1/4)) - 5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^9*b^3))
^(1/4)*log(a^7*b^2*(-1/(a^9*b^3))^(3/4) + sqrt(x)) + 5*(a^2*b^2*x^4 + 2*a^3
*b*x^2 + a^4)*(-1/(a^9*b^3))^(1/4)*log(-a^7*b^2*(-1/(a^9*b^3))^(3/4) + sqrt
(x)) - 4*(5*b*x^3 + 9*a*x)*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

giac [A] time = 0.65, size = 209, normalized size = 0.87

$$\frac{5\,bx^{\frac{7}{2}}+9\,ax^{\frac{3}{2}}}{16\left(bx^2+a\right)^2a^2}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^3}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^3}-\frac{5\sqrt{2}\left(ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*b*x^(7/2) + 9*a*x^(3/2))/((b*x^2 + a)^2*a^2) + 5/64*sqrt(2)*(a*b^3)
^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a
^3*b^3) + 5/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/
4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) - 5/128*sqrt(2)*(a*b^3)^(3/4)*log(sq
rt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) + 5/128*sqrt(2)*(a*b^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3)
```

maple [A] time = 0.01, size = 175, normalized size = 0.73

$$\frac{x^{\frac{3}{2}}}{4\left(b\,x^2+a\right)^2a}+\frac{5x^{\frac{3}{2}}}{16\left(b\,x^2+a\right)a^2}+\frac{5\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b}+\frac{5\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b}+\frac{5\sqrt{2}\;\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}}\right.}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)^3,x)
```

```
[Out] 1/4*x^(3/2)/a/(b*x^2+a)^2+5/16*x^(3/2)/a^2/(b*x^2+a)+5/128/a^2/b/(a/b)^(1/4
)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(
1/2)*x^(1/2)+(a/b)^(1/2)))+5/64/a^2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)+1)+5/64/a^2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)-1)
```

maxima [A] time = 2.99, size = 217, normalized size = 0.91

$$\frac{5\,bx^{\frac{7}{2}}+9\,ax^{\frac{3}{2}}}{16\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}+\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{\right.}}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{128\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(5*b*x^(7/2) + 9*a*x^(3/2))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 5/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^2
```

<u>**mupad**</u> **[B]** time = 0.08, size = 86, normalized size = 0.36

$$\frac{\frac{9\,x^{3/2}}{16\,a}+\frac{5\,b\,x^{7/2}}{16\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{5\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{9/4}\,b^{3/4}}-\frac{5\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{9/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2)^3,x)
```

```
[Out] ((9*x^(3/2))/(16*a) + (5*b*x^(7/2))/(16*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) +
 (5*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(9/4)*b^(3/4)) - (5*atanh(
(b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(9/4)*b^(3/4))
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.308 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21\,\mathrm{t}}{}$$

```
[Out] -21/64*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^(1/4)*2^(1/2)+2
1/64*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^(1/4)*2^(1/2)-21/
128*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/b^(1/4)*
2^(1/2)+21/128*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/
4)/b^(1/4)*2^(1/2)+1/4*x^(1/2)/a/(b*x^2+a)^2+7/16*x^(1/2)/a^2/(b*x^2+a)
```

Rubi [A] time = 0.17, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)}-\frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{21\tan^{-1}\left(1-\right.}{32\sqrt{2}\,a^{11/}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] Sqrt[x]/(4*a*(a + b*x^2)^2) + (7*Sqrt[x])/(16*a^2*(a + b*x^2)) - (21*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(1/4)) + (2
1*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(1/
4)) - (21*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*S
qrt[2]*a^(11/4)*b^(1/4)) + (21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(64*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^3}\,dx = \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx}{8a}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} + \frac{21\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32a^2}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16a^2}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^{5/2}} + \frac{21\,\text{Subst}\left(\int \frac{\sqrt{a}+}{a+}\right.}{32a}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^{5/2}\sqrt{b}} + \frac{21\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} - \frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}} + \frac{21\log\left(\sqrt{a}\right.}{6}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}} -$$

Mathematica [A] time = 0.08, size = 220, normalized size = 0.92

$$\frac{\frac{32a^{7/4}\sqrt{x}}{\left(a+bx^2\right)^2} + \frac{56a^{3/4}\sqrt{x}}{a+bx^2} - \frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt[4]{b}} + \frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt[4]{b}} - \frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} +}{128a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] ((32*a^(7/4)*Sqrt[x])/(a + b*x^2)^2 + (56*a^(3/4)*Sqrt[x])/(a + b*x^2) - (4
2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (42*Sqrt
[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (21*Sqrt[2]*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (21*Sqr
t[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4))/(
128*a^(11/4))
```

fricas [A] time = 0.91, size = 241, normalized size = 1.01

$$\frac{84\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^6\sqrt{-\frac{1}{a^{11}b}}+x\,a^8b\left(-\frac{1}{a^{11}b}\right)^{\frac{3}{4}}}-a^8b\sqrt{x}\left(-\frac{1}{a^{11}b}\right)^{\frac{3}{4}}\right)+21\left(a^2b^2x^4+\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

[Out] 1/64*(84*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^11*b))^(1/4)*arctan(sqrt(
a^6*sqrt(-1/(a^11*b)) + x)*a^8*b*(-1/(a^11*b))^(3/4) - a^8*b*sqrt(x)*(-1/(a
^11*b))^(3/4)) + 21*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^11*b))^(1/4)*l
og(a^3*(-1/(a^11*b))^(1/4) + sqrt(x)) - 21*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4
)*(-1/(a^11*b))^(1/4)*log(-a^3*(-1/(a^11*b))^(1/4) + sqrt(x)) + 4*(7*b*x^2
+ 11*a)*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)

giac [A] time = 0.65, size = 209, normalized size = 0.87

$$\frac{21\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b}+\frac{21\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b}+\frac{21\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\right.\right.}{128\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)^3/x^(1/2),x, algorithm="giac")

[Out] 21/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/(a^3*b) + 21/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) + 21/128*sqrt(2)*(
a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) - 21/
128*sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(a^3*b) + 1/16*(7*b*x^(5/2) + 11*a*sqrt(x))/((b*x^2 + a)^2*a^2)

maple [A] time = 0.01, size = 166, normalized size = 0.69

$$\frac{\sqrt{x}}{4\left(b\,x^2+a\right)^2 a}+\frac{7\sqrt{x}}{16\left(b\,x^2+a\right)a^2}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a^3}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a^3}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+a)^3/x^(1/2),x)

[Out] 1/4*x^(1/2)/a/(b*x^2+a)^2+7/16*x^(1/2)/a^2/(b*x^2+a)+21/128/a^3*(a/b)^(1/4)
*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1
/2)*x^(1/2)+(a/b)^(1/2)))+21/64/a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)+1)+21/64/a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)-1)

maxima [A] time = 3.02, size = 217, normalized size = 0.91

$$\frac{7\,bx^{\frac{5}{2}}+11\,a\sqrt{x}}{16\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}+\frac{21\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{128\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)^3/x^(1/2),x, algorithm="maxima")

[Out] 1/16*(7*b*x^(5/2) + 11*a*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 21/12
8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x

```
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1
/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt(2)
*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a^2
```

mupad [B] time = 4.67, size = 86, normalized size = 0.36

$$\frac{\frac{11\sqrt{x}}{16\,a}+\frac{7\,b\,x^{5/2}}{16\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{21\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{32\,{(-a)}^{11/4}\,b^{1/4}}-\frac{21\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{32\,{(-a)}^{11/4}\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)^3),x)
```

```
[Out] ((11*x^(1/2))/(16*a) + (7*b*x^(5/2))/(16*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)
- (21*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(11/4)*b^(1/4)) - (21*at
anh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(11/4)*b^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3/x**(1/2),x)
```

```
[Out] Timed out
```

3.309 $\int \frac{1}{x^{3/2}(a+bx^2)^3}\,dx$

Optimal. Leaf size=251

$$-\frac{45\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}}+\frac{45\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}}+\frac{45\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{}}{\sqrt[4]{a}}\right.}{32\sqrt{2}a^{13/4}}$$

```
[Out] 45/64*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)*2^(1/2)-45
/64*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)*2^(1/2)-45/1
28*b^(1/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)*2
^(1/2)+45/128*b^(1/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))
/a^(13/4)*2^(1/2)-45/16/a^3/x^(1/2)+1/4/a/(b*x^2+a)^2/x^(1/2)+9/16/a^2/(b*x
^2+a)/x^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}-\frac{45\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}}+\frac{45\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}}+\frac{45\sqrt[4]{b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] -45/(16*a^3*Sqrt[x]) + 1/(4*a*Sqrt[x]*(a + b*x^2)^2) + 9/(16*a^2*Sqrt[x]*(a
 + b*x^2)) + (45*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32
*Sqrt[2]*a^(13/4)) - (45*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)])/(32*Sqrt[2]*a^(13/4)) - (45*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(13/4)) + (45*b^(1/4)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^3}\,dx = \frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx}{8a}$$

$$= \frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}+\frac{45\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{32a^2}$$

$$= -\frac{45}{16a^3\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}-\frac{(45b)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^3}$$

$$= -\frac{45}{16a^3\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}-\frac{(45b)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16a^3}$$

$$= -\frac{45}{16a^3\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}+\frac{\left(45\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{\right.}}{32a^3}$$

$$= -\frac{45}{16a^3\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}-\frac{45\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{\right.}}{64a^3}$$

$$= -\frac{45}{16a^3\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}-\frac{45\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{64\sqrt{2}\,a^{13/4}}$$

$$= -\frac{45}{16a^3\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}+\frac{45\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{13/4}}-\frac{45\sqrt[4]{}}{}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},3;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 3, 3/4, -((b*x^2)/a)])/(a^3*Sqrt[x])
```

fricas [A] time = 0.90, size = 263, normalized size = 1.05

$$\frac{180\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125\,a^3b\sqrt{x}\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}-\sqrt{-8303765625\,a^7b\sqrt{-\frac{b}{a^{13}}}+8303765625\,b^2x}\,a^3\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}}{91125\,b}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(180*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-b/a^13)^(1/4)*arctan(-1/911
25*(91125*a^3*b*sqrt(x)*(-b/a^13)^(1/4) - sqrt(-8303765625*a^7*b*sqrt(-b/a^
```

```
13) + 8303765625*b^2*x)*a^3*(-b/a^13)^(1/4))/b) - 45*(a^3*b^2*x^5 + 2*a^4*b
*x^3 + a^5*x)*(-b/a^13)^(1/4)*log(91125*a^10*(-b/a^13)^(3/4) + 91125*b*sqrt
(x)) + 45*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-b/a^13)^(1/4)*log(-91125*a^
10*(-b/a^13)^(3/4) + 91125*b*sqrt(x)) - 4*(45*b^2*x^4 + 81*a*b*x^2 + 32*a^2
)*sqrt(x))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
```

giac [A] time = 0.65, size = 220, normalized size = 0.88

$$-\frac{2}{a^{3}\sqrt{x}}-\frac{45\,\sqrt{2}\,\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{4}b^{2}}-\frac{45\,\sqrt{2}\,\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{4}b^{2}}+\frac{45\,\sqrt{2}\,\left(ab^{3}\right)^{\frac{3}{4}}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -2/(a^3*sqrt(x)) - 45/64*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) - 45/64*sqrt(2)*(a*b^3)^(3/
4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*
b^2) + 45/128*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + s
qrt(a/b))/(a^4*b^2) - 45/128*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 1/16*(13*b^2*x^(7/2) + 17*a*b*x^(3/2)
)/((b*x^2 + a)^2*a^3)
```

maple [A] time = 0.02, size = 178, normalized size = 0.71

$$-\frac{13b^{2}x^{\frac{7}{2}}}{16\left(b\,x^{2}+a\right)^{2}a^{3}}-\frac{17b\,x^{\frac{3}{2}}}{16\left(b\,x^{2}+a\right)^{2}a^{2}}-\frac{45\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}}-\frac{45\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}}-\frac{45\sqrt{2}\;\ln\left(\frac{x-\left(\frac{a}{b}\right.}{x+\left(\frac{a}{b}\right.}\right.}{128\left(\frac{a}{b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)^3,x)
```

```
[Out] -2/a^3/x^(1/2)-13/16/a^3*b^2/(b*x^2+a)^2*x^(7/2)-17/16/a^2*b/(b*x^2+a)^2*x^
(3/2)-45/128/a^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b
)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-45/64/a^3/(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-45/64/a^3/(a/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.00, size = 230, normalized size = 0.92

$$-\frac{45\,b^{2}x^{4}+81\,abx^{2}+32\,a^{2}}{16\left(a^{3}b^{2}x^{\frac{9}{2}}+2\,a^{4}bx^{\frac{5}{2}}+a^{5}\sqrt{x}\right)}-\frac{45\,b\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\right.}{}\right.}{128\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(45*b^2*x^4 + 81*a*b*x^2 + 32*a^2)/(a^3*b^2*x^(9/2) + 2*a^4*b*x^(5/2)
 + a^5*sqrt(x)) - 45/128*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b
```

```
^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)
*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a
^(1/4)*b^(3/4)))/a^3
```

mupad [B] time = 0.10, size = 99, normalized size = 0.39

$$\frac{45\,{\left(-b\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{13/4}}-\frac{45\,{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{13/4}}-\frac{\frac{2}{a}+\frac{81\,b\,x^2}{16\,a^2}+\frac{45\,b^2\,x^4}{16\,a^3}}{a^2\,\sqrt{x}+b^2\,x^{9/2}+2\,a\,b\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)^3),x)
```

```
[Out] (45*(-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(13/4)) - (45*(-b
)^(1/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(13/4)) - (2/a + (81*b*x^
2)/(16*a^2) + (45*b^2*x^4)/(16*a^3))/(a^2*x^(1/2) + b^2*x^(9/2) + 2*a*b*x^(
5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.310 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=251

$$\frac{77b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}}-\frac{77b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}}+\frac{77b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{32\sqrt{2}a^{15/4}}$$

```
[Out] -77/48/a^3/x^(3/2)+1/4/a/x^(3/2)/(b*x^2+a)^2+11/16/a^2/x^(3/2)/(b*x^2+a)+77
/64*b^(3/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(15/4)*2^(1/2)-77/6
4*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(15/4)*2^(1/2)+77/128
*b^(3/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(15/4)*2^(
1/2)-77/128*b^(3/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a
^(15/4)*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}}-\frac{77b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}}+\frac{77b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{32\sqrt{2}a^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)^3),x]
```

```
[Out] -77/(48*a^3*x^(3/2)) + 1/(4*a*x^(3/2)*(a + b*x^2)^2) + 11/(16*a^2*x^(3/2)*(
a + b*x^2)) + (77*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(3
2*Sqrt[2]*a^(15/4)) - (77*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(32*Sqrt[2]*a^(15/4)) + (77*b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4)) - (77*b^(3/4)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx}{8a}$$

$$= \frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}+\frac{77\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx}{32a^2}$$

$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}-\frac{(77b)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32a^3}$$

$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}-\frac{(77b)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16a^3}$$

$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}-\frac{(77b)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx,x,\sqrt{}\right.}{32a^{7/2}}$$

$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}-\frac{\left(77\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\right.}{64a^{7/2}}$$

$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}+\frac{77b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{64\sqrt{2}\,a^{15/4}}$$

$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}+\frac{77b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{15/4}}-$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},3;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)^3),x]
[Out] (-2*Hypergeometric2F1[-3/4, 3, 1/4, -((b*x^2)/a)])/(3*a^3*x^(3/2))
```

fricas [A] time = 1.02, size = 283, normalized size = 1.13

$$-\frac{924\left(a^3b^2x^6+2a^4bx^4+a^5x^2\right)\left(-\frac{b^3}{a^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^{11}b\sqrt{x}\left(-\frac{b^3}{a^{15}}\right)^{\frac{3}{4}}-\sqrt{a^8\sqrt{-\frac{b^3}{a^{15}}}+b^2x}\,a^{11}\left(-\frac{b^3}{a^{15}}\right)^{\frac{3}{4}}}{b^3}\right)+231\left(a^3b^2x^6+2a^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(924*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-b^3/a^15)^(1/4)*arctan(
-(a^11*b*sqrt(x)*(-b^3/a^15)^(3/4) - sqrt(a^8*sqrt(-b^3/a^15) + b^2*x)*a^11
*(-b^3/a^15)^(3/4))/b^3) + 231*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-b^3/
a^15)^(1/4)*log(77*a^4*(-b^3/a^15)^(1/4) + 77*b*sqrt(x)) - 231*(a^3*b^2*x^6
 + 2*a^4*b*x^4 + a^5*x^2)*(-b^3/a^15)^(1/4)*log(-77*a^4*(-b^3/a^15)^(1/4) +
 77*b*sqrt(x)) + 4*(77*b^2*x^4 + 121*a*b*x^2 + 32*a^2)*sqrt(x))/(a^3*b^2*x^
6 + 2*a^4*b*x^4 + a^5*x^2)
```

giac [A]   time = 0.65, size = 208, normalized size = 0.83

$$-\frac{77\,\sqrt{2}\,\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{4}}-\frac{77\,\sqrt{2}\,\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{4}}-\frac{77\,\sqrt{2}\,\left(ab^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\,\sqrt{x}\right.}{128\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -77/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sq
rt(x))/(a/b)^(1/4))/a^4 - 77/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^4 - 77/128*sqrt(2)*(a*b^3)^
(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^4 + 77/128*sqrt(2)
*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^4 - 1/16
*(15*b^2*x^(5/2) + 19*a*b*sqrt(x))/((b*x^2 + a)^2*a^3) - 2/3/(a^3*x^(3/2))
```

maple [A]   time = 0.02, size = 181, normalized size = 0.72

$$-\frac{15b^{2}x^{\frac{5}{2}}}{16\left(b\,x^{2}+a\right)^{2}a^{3}}-\frac{19b\sqrt{x}}{16\left(b\,x^{2}+a\right)^{2}a^{2}}-\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;b\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a^{4}}-\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;b\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a^{4}}-\frac{77\left(\frac{a}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)^3,x)
```

```
[Out] -2/3/a^3/x^(3/2)-15/16/a^3*b^2/(b*x^2+a)^2*x^(5/2)-19/16/a^2*b/(b*x^2+a)^2*
x^(1/2)-77/128/a^4*b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+
(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-77/64/a^4*b*(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-77/64/a^4*b*(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A]   time = 2.97, size = 231, normalized size = 0.92

$$-\frac{77\,b^{2}x^{4}+121\,abx^{2}+32\,a^{2}}{48\left(a^{3}b^{2}x^{\frac{11}{2}}+2\,a^{4}bx^{\frac{7}{2}}+a^{5}x^{\frac{3}{2}}\right)}-\frac{77\left(\frac{2\,\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\right.}{}\right.}{128\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(77*b^2*x^4 + 121*a*b*x^2 + 32*a^2)/(a^3*b^2*x^(11/2) + 2*a^4*b*x^(7/
2) + a^5*x^(3/2)) - 77/128*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)
*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(b))) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sq
rt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqr
t(2)*b^(3/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(
3/4) - sqrt(2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + s
qrt(a))/a^(3/4))/a^3
```

mupad [B] time = 4.66, size = 99, normalized size = 0.39

$$\frac{77\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{15/4}}-\frac{\frac{2}{3\,a}+\frac{121\,b\,x^2}{48\,a^2}+\frac{77\,b^2\,x^4}{48\,a^3}}{a^2\,x^{3/2}+b^2\,x^{11/2}+2\,a\,b\,x^{7/2}}+\frac{77\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)^3),x)
```

```
[Out] (77*(-b)^(3/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(15/4)) - (2/(3*a)
 + (121*b*x^2)/(48*a^2) + (77*b^2*x^4)/(48*a^3))/(a^2*x^(3/2) + b^2*x^(11/2
) + 2*a*b*x^(7/2)) + (77*(-b)^(3/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(3
2*a^(15/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.311 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=264

$$\frac{117b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{17/4}}-\frac{117b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{17/4}}-\frac{117b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{32\sqrt{2}\,a^{17/4}}$$

```
[Out] -117/80/a^3/x^(5/2)+1/4/a/x^(5/2)/(b*x^2+a)^2+13/16/a^2/x^(5/2)/(b*x^2+a)-1
17/64*b^(5/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(17/4)*2^(1/2)+11
7/64*b^(5/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(17/4)*2^(1/2)+117
/128*b^(5/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(17/4)
*2^(1/2)-117/128*b^(5/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/
2))/a^(17/4)*2^(1/2)+117/16*b/a^4/x^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{117b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{17/4}}-\frac{117b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{17/4}}-\frac{117b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{32\sqrt{2}\,a^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] -117/(80*a^3*x^(5/2)) + (117*b)/(16*a^4*Sqrt[x]) + 1/(4*a*x^(5/2)*(a + b*x^
2)^2) + 13/(16*a^2*x^(5/2)*(a + b*x^2)) - (117*b^(5/4)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (117*b^(5/4)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (117*b^(5/4)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(17
/4)) - (117*b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(64*Sqrt[2]*a^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx}{8a}$$

$$= \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{117\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx}{32a^2}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} - \frac{(117b)\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{32a^3}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{\left(117b^2\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^4}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{\left(117b^2\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\right.}{16a^4}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} - \frac{\left(117b^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{a}-}{a+}\right.}{32a^4}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{(117b)\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}}{}}\right.}{64a^4}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{117b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\right.}{64\sqrt{2}\,a}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} - \frac{117b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{32\sqrt{2}\,a^{17/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},3;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 3, -1/4, -((b*x^2)/a)])/(5*a^3*x^(5/2))
```

fricas [A] time = 0.76, size = 306, normalized size = 1.16

$$-\frac{2340\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)\left(-\frac{b^5}{a^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{1601613\,a^4b^4\sqrt{x}\left(-\frac{b^5}{a^{17}}\right)^{\frac{1}{4}}-\sqrt{-2565164201769\,a^9b^5\sqrt{-\frac{b^5}{a^{17}}}+2565164201769\,b^8x}}{1601613\,b^5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(2340*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-b^5/a^17)^(1/4)*arctan
(-1/1601613*(1601613*a^4*b^4*sqrt(x)*(-b^5/a^17)^(1/4) - sqrt(-256516420176
9*a^9*b^5*sqrt(-b^5/a^17) + 2565164201769*b^8*x)*a^4*(-b^5/a^17)^(1/4))/b^5
) - 585*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-b^5/a^17)^(1/4)*log(1601613
*a^13*(-b^5/a^17)^(3/4) + 1601613*b^4*sqrt(x)) + 585*(a^4*b^2*x^7 + 2*a^5*b
*x^5 + a^6*x^3)*(-b^5/a^17)^(1/4)*log(-1601613*a^13*(-b^5/a^17)^(3/4) + 160
1613*b^4*sqrt(x)) - 4*(585*b^3*x^6 + 1053*a*b^2*x^4 + 416*a^2*b*x^2 - 32*a^
3)*sqrt(x))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)
```

giac [A] time = 0.63, size = 232, normalized size = 0.88

$$\frac{117\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5b}+\frac{117\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5b}-\frac{117\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{\right.}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 117/64*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sq
rt(x))/(a/b)^(1/4))/(a^5*b) + 117/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^5*b) - 117/128*sqrt(2
)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b) +
117/128*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a
/b))/(a^5*b) + 1/16*(21*b^3*x^(7/2) + 25*a*b^2*x^(3/2))/((b*x^2 + a)^2*a^4)
 + 2/5*(15*b*x^2 - a)/(a^4*x^(5/2))
```

maple [A] time = 0.02, size = 192, normalized size = 0.73

$$\frac{21b^3x^{\frac{7}{2}}}{16\left(b\,x^2+a\right)^2a^4}+\frac{25b^2x^{\frac{3}{2}}}{16\left(b\,x^2+a\right)^2a^3}+\frac{117\sqrt{2}\;b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^4}+\frac{117\sqrt{2}\;b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^4}+\frac{117\sqrt{2}\;b\ln}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)^3,x)
```

```
[Out] -2/5/a^3/x^(5/2)+6*b/a^4/x^(1/2)+21/16/a^4*b^3/(b*x^2+a)^2*x^(7/2)+25/16/a^
3*b^2/(b*x^2+a)^2*x^(3/2)+117/128/a^4*b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/
4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))
)+117/64/a^4*b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+11
7/64/a^4*b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.05, size = 243, normalized size = 0.92

$$\frac{585\,b^3x^6+1053\,ab^2x^4+416\,a^2bx^2-32\,a^3}{80\left(a^4b^2x^{\frac{13}{2}}+2\,a^5bx^{\frac{9}{2}}+a^6x^{\frac{5}{2}}\right)}+\frac{117\,b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right.}{2\sqrt{\sqrt{a}\sqrt{b}}}\right.}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right.}{128\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/80*(585*b^3*x^6 + 1053*a*b^2*x^4 + 416*a^2*b*x^2 - 32*a^3)/(a^4*b^2*x^(13
/2) + 2*a^5*b*x^(9/2) + a^6*x^(5/2)) + 117/128*b^2*(2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^
(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqr
t(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
 sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^4
```

mupad [B] time = 4.65, size = 109, normalized size = 0.41

$$\frac{\frac{26\,b\,x^2}{5\,a^2}-\frac{2}{5\,a}+\frac{1053\,b^2\,x^4}{80\,a^3}+\frac{117\,b^3\,x^6}{16\,a^4}}{a^2\,x^{5/2}+b^2\,x^{13/2}+2\,a\,b\,x^{9/2}}-\frac{117\,{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{17/4}}+\frac{117\,{\left(-b\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)^3),x)
```

```
[Out] ((26*b*x^2)/(5*a^2) - 2/(5*a) + (1053*b^2*x^4)/(80*a^3) + (117*b^3*x^6)/(16
*a^4))/(a^2*x^(5/2) + b^2*x^(13/2) + 2*a*b*x^(9/2)) - (117*(-b)^(5/4)*atan(
((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(17/4)) + (117*(-b)^(5/4)*atanh(((-b)^
(1/4)*x^(1/2))/a^(1/4)))/(32*a^(17/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.312 $\int \frac{\sqrt{x}}{a-bx^2}\,dx$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

```
[Out] -arctan(b^(1/4)*x^(1/2)/a^(1/4))/a^(1/4)/b^(3/4)+arctanh(b^(1/4)*x^(1/2)/a^
(1/4))/a^(1/4)/b^(3/4)
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {329, 298, 205, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a - b*x^2),x]
```

```
[Out] -(ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)]/(a^(1/4)*b^(3/4))) + ArcTanh[(b^(1/4)*S
qrt[x])/a^(1/4)]/(a^(1/4)*b^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{x}}{a-bx^2}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x^2}{a-bx^4}\,dx,x,\sqrt{x}\right)$$
$$= \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{a}-\sqrt{b}x^2}\,dx,x,\sqrt{x}\right)}{\sqrt{b}} - \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{a}+\sqrt{b}x^2}\,dx,x,\sqrt{x}\right)}{\sqrt{b}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a - b*x^2),x]
```

```
[Out] (-ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)] + ArcTanh[(b^(1/4)*Sqrt[x])/a^(1/4)])/(
a^(1/4)*b^(3/4))
```

fricas [B] time = 0.94, size = 117, normalized size = 2.02

$$2\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{ab\sqrt{\frac{1}{ab^3}}+x}\,b\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}-b\sqrt{x}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\right)+\frac{1}{2}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2*(1/(a*b^3))^(1/4)*arctan(sqrt(a*b*sqrt(1/(a*b^3)) + x)*b*(1/(a*b^3))^(1/4
) - b*sqrt(x)*(1/(a*b^3))^(1/4)) + 1/2*(1/(a*b^3))^(1/4)*log(a*b^2*(1/(a*b^
3))^(3/4) + sqrt(x)) - 1/2*(1/(a*b^3))^(1/4)*log(-a*b^2*(1/(a*b^3))^(3/4) +
 sqrt(x))
```

giac [B] time = 0.63, size = 194, normalized size = 3.34

$$\frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^3}+\frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^3}-\frac{\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(-\right.\right.}{4\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*sqr
t(x))/(-a/b)^(1/4))/(a*b^3) + 1/2*sqrt(2)*(-a*b^3)^(3/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(-a/b)^(1/4) - 2*sqrt(x))/(-a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(-
a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(-a/b)^(1/4) + x + sqrt(-a/b))/(a*b^3) + 1
/4*sqrt(2)*(-a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(-a/b)^(1/4) + x + sqrt(-a/b
))/(a*b^3)
```

maple [A] time = 0.01, size = 66, normalized size = 1.14

$$-\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{4}} b}+\frac{\ln\left(\frac{\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x^2+a),x)
```

```
[Out] -1/b/(a/b)^(1/4)*arctan(x^(1/2)/(a/b)^(1/4))+1/2/b/(a/b)^(1/4)*ln((x^(1/2)+
(a/b)^(1/4))/(x^(1/2)-(a/b)^(1/4)))
```

maxima [B] time = 2.97, size = 86, normalized size = 1.48

$$-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\log\left(\frac{\sqrt{b}\sqrt{x}-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sqrt{x}+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(b)*sqrt(x)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(
b)) - 1/2*log((sqrt(b)*sqrt(x) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*sqrt(x) +
sqrt(sqrt(a)*sqrt(b))))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b))
```

mupad [B] time = 0.08, size = 33, normalized size = 0.57

$$-\frac{\operatorname{atan}\left(\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right)-\operatorname{atanh}\left(\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{1/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x^2),x)
```

```
[Out] -(atan((b^(1/4)*x^(1/2))/a^(1/4)) - atanh((b^(1/4)*x^(1/2))/a^(1/4)))/(a^(1
/4)*b^(3/4))
```

sympy [A] time = 3.10, size = 122, normalized size = 2.10

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2}{b\sqrt{x}} & \text{for}\: a = 0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for}\: b = 0 \\ -\frac{\log\left(-\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{a}\, b\sqrt[4]{\frac{1}{b}}}+\frac{\log\left(\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{a}\, b\sqrt[4]{\frac{1}{b}}}-\frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{\sqrt[4]{a}\, b\sqrt[4]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x**2+a),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2/(b*sqrt(x)), Eq(a, 0)), (2
*x**(3/2)/(3*a), Eq(b, 0)), (-log(-a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(
1/4)*b*(1/b)**(1/4)) + log(a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(1/4)*b*(
1/b)**(1/4)) - atan(sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(1/4)*b*(1/b)**(1/
4)), True))
```

3.313 $\int \frac{x^{7/2}}{1+x^2}\,dx$

Optimal. Leaf size=108

$$\frac{2x^{5/2}}{5}-2\sqrt{x}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] 2/5*x^(5/2)+1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*x^(
1/2))*2^(1/2)-1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+1/4*ln(1+x+2^(1/2)*x^(1/2
))*2^(1/2)-2*x^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2x^{5/2}}{5}-2\sqrt{x}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(1 + x^2),x]
```

```
[Out] -2*Sqrt[x] + (2*x^(5/2))/5 - ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Lo
g[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{1+x^2}\,dx &= \frac{2x^{5/2}}{5} - \int \frac{x^{3/2}}{1+x^2}\,dx\\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} + \int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} + 2\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)\\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} + \mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)\\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) -\\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{x}\right.}{\sqrt{2}}\\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\right.}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 108, normalized size = 1.00

$$\frac{2x^{5/2}}{5} - 2\sqrt{x} - \frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}} + \frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(1 + x^2),x]
```

```
[Out] -2*Sqrt[x] + (2*x^(5/2))/5 - ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Lo
g[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2])
```

fricas [A] time = 0.96, size = 117, normalized size = 1.08

$$\frac{2}{5}\left(x^2-5\right)\sqrt{x}-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 2/5*(x^2 - 5)*sqrt(x) - sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1
) - sqrt(2)*sqrt(x) - 1) - sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(
x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) +
4*x + 4) - 1/4*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

giac [A] time = 0.63, size = 84, normalized size = 0.78

$$\frac{2}{5}x^{\frac{5}{2}}+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2*sqrt(x)
```

maple [A] time = 0.01, size = 72, normalized size = 0.67

$$\frac{2x^{\frac{5}{2}}}{5}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{2}+\frac{\sqrt{2}\,\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)}{4}-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2+1),x)
```

```
[Out] 2/5*x^(5/2)-2*x^(1/2)+1/4*2^(1/2)*ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^(
1/2)))+1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*x^(1/2))
*2^(1/2)
```

maxima [A] time = 3.05, size = 84, normalized size = 0.78

$$\frac{2}{5}x^{\frac{5}{2}}+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2*sqrt(x)
```

mupad [B] time = 0.08, size = 47, normalized size = 0.44

$$\frac{2\,x^{5/2}}{5}-2\,\sqrt{x}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2) - 2*x^(1/2) + (2*x^(5/2))/5
```

sympy [A] time = 4.15, size = 105, normalized size = 0.97

$$\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} - \frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4} + \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{\right.}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(x**2+1),x)
```

```
[Out] 2*x**(5/2)/5 - 2*sqrt(x) - sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sq
rt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1
)/2 + sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/2
```

3.314 $\int \frac{x^{5/2}}{1+x^2} \, dx$

Optimal. Leaf size=101

$$\frac{2x^{3/2}}{3}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] 2/3*x^(3/2)-1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctan(1+2^(1/2)*x^(
1/2))*2^(1/2)-1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+1/4*ln(1+x+2^(1/2)*x^(1/2
))*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2x^{3/2}}{3}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(1 + x^2),x]
```

```
[Out] (2*x^(3/2))/3 + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sq
rt[x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]
*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{1+x^2}\,dx &= \frac{2x^{3/2}}{3} - \int \frac{\sqrt{x}}{1+x^2}\,dx\\
&= \frac{2x^{3/2}}{3} - 2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)\\
&= \frac{2x^{3/2}}{3} + \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) - \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)\\
&= \frac{2x^{3/2}}{3} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right) - \frac{\text{Subst}\left(\int\right.}{}\\
&= \frac{2x^{3/2}}{3} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\text{Subs}}{}\\
&= \frac{2x^{3/2}}{3} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.24

$$-\frac{2}{3}x^{3/2}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-x^2\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(1 + x^2),x]
```

```
[Out] (-2*x^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -x^2]))/3
```

fricas [A] time = 1.08, size = 110, normalized size = 1.09

$$\frac{2}{3}x^{\frac{3}{2}}+\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2) + sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2
)*sqrt(x) - 1) + sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x +
 4) - sqrt(2)*sqrt(x) + 1) + 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) -
 1/4*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

giac [A] time = 0.63, size = 79, normalized size = 0.78

$$\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

maple [A] time = 0.01, size = 67, normalized size = 0.66

$$\frac{2x^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{2}-\frac{\sqrt{2}\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2+1),x)
```

```
[Out] 2/3*x^(3/2)-1/4*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))-1/2
*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)-1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)
```

maxima [A] time = 2.97, size = 79, normalized size = 0.78

$$\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

mupad [B] time = 0.04, size = 42, normalized size = 0.42

$$\frac{2\,x^{3/2}}{3}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2 + 1),x)
```

```
[Out] (2*x^(3/2))/3 - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2) - 2
^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2)
```

sympy [A] time = 1.75, size = 99, normalized size = 0.98

$$\frac{2x^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(x**2+1),x)
```

```
[Out] 2*x**(3/2)/3 - sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*
sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2
)*atan(sqrt(2)*sqrt(x) + 1)/2
```

3.315 $\int \frac{x^{3/2}}{1+x^2} \, dx$

Optimal. Leaf size=99

$$2\sqrt{x}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/
2)+1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-1/4*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)+
2*x^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {321, 329, 211, 1165, 628, 1162, 617, 204}

$$2\sqrt{x}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(1 + x^2),x]
```

```
[Out] 2*Sqrt[x] + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[x
]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqr
t[x] + x]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{1+x^2}\,dx &= 2\sqrt{x}-\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= 2\sqrt{x}-2\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= 2\sqrt{x}-\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= 2\sqrt{x}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+\frac{\text{Subst}\left(\int\right.}{}\\
&= 2\sqrt{x}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\text{Subs}}{}\\
&= 2\sqrt{x}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 99, normalized size = 1.00

$$2\sqrt{x}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(1 + x^2),x]
```

```
[Out] 2*Sqrt[x] + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[x
]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqr
t[x] + x]/(2*Sqrt[2])
```

fricas [A] time = 0.94, size = 110, normalized size = 1.11

$$\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-\frac{1}{4}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqrt(x) - 1)
 + sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*
sqrt(x) + 1) - 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 1/4*sqrt(2)*l
og(-4*sqrt(2)*sqrt(x) + 4*x + 4) + 2*sqrt(x)
```

giac [A] time = 0.60, size = 79, normalized size = 0.80

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x)
```

maple [A] time = 0.01, size = 67, normalized size = 0.68

$$-\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{2}-\frac{\sqrt{2}\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)}{4}+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2+1),x)
```

```
[Out] 2*x^(1/2)-1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)-1/4*2^(1/2)*ln((x+2^(1/2)*x
^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))-1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)
```

maxima [A] time = 2.96, size = 79, normalized size = 0.80

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x)
```

mupad [B] time = 0.04, size = 42, normalized size = 0.42

$$2\sqrt{x}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2 + 1),x)
```

```
[Out] 2*x^(1/2) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2) - 2^(1/
2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2)
```

sympy [A] time = 0.84, size = 97, normalized size = 0.98

$$2\sqrt{x}+\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(x**2+1),x)
```

```
[Out] 2*sqrt(x) + sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqr
t(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*a
tan(sqrt(2)*sqrt(x) + 1)/2
```

3.316 $\int \frac{\sqrt{x}}{1+x^2} \, dx$

Optimal. Leaf size=92

$$\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/2
)+1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-1/4*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^2),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2
] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[x] + x]
/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{1+x^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2}{-1-\sqrt{2}}\right.}{2\sqrt{}}$$
$$= \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-}\right.}{}$$
$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.24

$$\frac{2}{3}x^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^2),x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -x^2])/3
```

fricas [A] time = 0.85, size = 107, normalized size = 1.16

$$-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-\frac{1}{4}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqrt(x) - 1
) - sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)
*sqrt(x) + 1) - 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 1/4*sqrt(2)*
log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

giac [A] time = 0.63, size = 74, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

maple [A] time = 0.01, size = 62, normalized size = 0.67

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ -1\right)}{2}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ +1\right)}{2}+\frac{\sqrt{2}\ \ln\left(\frac{x-\sqrt{2}\ \sqrt{x}+1}{x+\sqrt{2}\ \sqrt{x}+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1),x)
```

```
[Out] 1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)+1/4*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/
(x+2^(1/2)*x^(1/2)+1))+1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)
```

maxima [A] time = 2.87, size = 74, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

mupad [B] time = 0.04, size = 37, normalized size = 0.40

$$\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2)
```

sympy [A] time = 0.55, size = 90, normalized size = 0.98

$$\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1),x)
```

```
[Out] sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(2)*
sqrt(x) + 1)/2
```

3.317 $\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx$

Optimal. Leaf size=92

$$-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/2
)-1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+1/4*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2
] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[x] + x]
/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)\\
&= \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right)-\frac{\text{Subst}\left(\right.}{}\\
&= -\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\text{Su}}{}\\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+\right.}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 76, normalized size = 0.83

$$\frac{-\log\left(x-\sqrt{2}\sqrt{x}+1\right)+\log\left(x+\sqrt{2}\sqrt{x}+1\right)-2\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+2\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[x]] - Log[1 - S
qrt[2]*Sqrt[x] + x] + Log[1 + Sqrt[2]*Sqrt[x] + x])/(2*Sqrt[2])
```

fricas [A] time = 0.66, size = 107, normalized size = 1.16

$$-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqrt(x) - 1
) - sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)
*sqrt(x) + 1) + 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 1/4*sqrt(2)*
log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

giac [A] time = 0.59, size = 74, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

maple [A] time = 0.00, size = 62, normalized size = 0.67

$$\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{2}+\frac{\sqrt{2}\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/x^(1/2),x)
```

```
[Out] 1/4*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))+1/2*2^(1/2)*arc
tan(2^(1/2)*x^(1/2)-1)+1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)
```

maxima [A] time = 2.89, size = 74, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

mupad [B] time = 0.03, size = 37, normalized size = 0.40

$$\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^2 + 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2)
```

sympy [A] time = 0.73, size = 90, normalized size = 0.98

$$-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/x**(1/2),x)
```

```
[Out] -sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*sqrt(2)*sqrt(x
) + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(2)
*sqrt(x) + 1)/2
```

3.318 $\int \frac{1}{x^{3/2}\left(1+x^2\right)} dx$

Optimal. Leaf size=99

$$-\frac{2}{\sqrt{x}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/
2)-1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+1/4*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)-
2/x^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2}{\sqrt{x}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(1 + x^2)),x]
```

```
[Out] -2/Sqrt[x] + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[
x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sq
rt[x] + x]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx &= -\frac{2}{\sqrt{x}}-\int \frac{\sqrt{x}}{1+x^2}\,dx \\
&= -\frac{2}{\sqrt{x}}-2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= -\frac{2}{\sqrt{x}}+\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= -\frac{2}{\sqrt{x}}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{\text{Su}}{}
\\
&= -\frac{2}{\sqrt{x}}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} \\
&= -\frac{2}{\sqrt{x}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{}\right.}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 20, normalized size = 0.20

$$
-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-x^2\right)}{\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(1 + x^2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -x^2])/Sqrt[x]
```

fricas [A] time = 0.85, size = 120, normalized size = 1.21

$$
\frac{4\sqrt{2}\,x\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+4\sqrt{2}\,x\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)}{4x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*x*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqr
t(x) - 1) + 4*sqrt(2)*x*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x +
4) - sqrt(2)*sqrt(x) + 1) + sqrt(2)*x*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - sq
rt(2)*x*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*sqrt(x))/x
```

giac [A] time = 0.63, size = 79, normalized size = 0.80

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x)
```

maple [A] time = 0.01, size = 67, normalized size = 0.68

$$-\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{2}-\frac{\sqrt{2}\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)}{4}-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(x^2+1),x)
```

```
[Out] -1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)-1/4*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)
/(x+2^(1/2)*x^(1/2)+1))-1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)-2/x^(1/2)
```

maxima [A] time = 2.99, size = 79, normalized size = 0.80

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x)
```

mupad [B] time = 0.04, size = 42, normalized size = 0.42

$$-\frac{2}{\sqrt{x}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(x^2 + 1)),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2) - 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2) - 2/x^(1/2)
```

sympy [A] time = 1.35, size = 97, normalized size = 0.98

$$-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(x**2+1),x)
```

```
[Out] -sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*sqrt(2)*sqrt(x
) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*atan(sqrt(2)
*sqrt(x) + 1)/2 - 2/sqrt(x)
```

3.319 $\int \frac{1}{x^{5/2}\left(1+x^2\right)}\,dx$

Optimal. Leaf size=101

$$-\frac{2}{3x^{3/2}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] -2/3/x^(3/2)-1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctan(1+2^(1/2)*x^
(1/2))*2^(1/2)+1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-1/4*ln(1+x+2^(1/2)*x^(1/
2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2}{3x^{3/2}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(1 + x^2)),x]
```

```
[Out] -2/(3*x^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*S
qrt[x]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2
]*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/2}\left(1+x^2\right)}\,dx &= -\frac{2}{3x^{3/2}}-\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx \\
&= -\frac{2}{3x^{3/2}}-2\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= -\frac{2}{3x^{3/2}}-\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= -\frac{2}{3x^{3/2}}-\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+ \\
&= -\frac{2}{3x^{3/2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} \\
&= -\frac{2}{3x^{3/2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\right.}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.22

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-x^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(1 + x^2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, -x^2])/(3*x^(3/2))
```

fricas [A] time = 1.23, size = 129, normalized size = 1.28

$$\frac{12\sqrt{2}\,x^2\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\,x^2\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+\right.}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*x^2*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)
*sqrt(x) - 1) + 12*sqrt(2)*x^2*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) +
 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 3*sqrt(2)*x^2*log(4*sqrt(2)*sqrt(x) + 4*
x + 4) + 3*sqrt(2)*x^2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*sqrt(x))/x^2
```

giac [A] time = 0.63, size = 79, normalized size = 0.78

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/3/x^(3/2)
```

maple [A] time = 0.01, size = 67, normalized size = 0.66

$$-\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{2}-\frac{\sqrt{2}\,\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)}{4}-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(x^2+1),x)
```

```
[Out] -1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)-1/4*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)
/(x-2^(1/2)*x^(1/2)+1))-1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)-2/3/x^(3/2)
```

maxima [A] time = 2.93, size = 79, normalized size = 0.78

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/3/x^(3/2)
```

mupad [B] time = 0.04, size = 42, normalized size = 0.42

$$-\frac{2}{3\,x^{3/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(x^2 + 1)),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2) - 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2) - 2/(3*x^(3/2))
```

sympy [A] time = 2.44, size = 99, normalized size = 0.98

$$\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(x**2+1),x)
```

```
[Out] sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*atan(sqrt(2)*
sqrt(x) + 1)/2 - 2/(3*x**(3/2))
```

3.320 $\int \frac{1}{x^{7/2}(1+x^2)}\,dx$

Optimal. Leaf size=108

$$-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

```
[Out] -2/5/x^(5/2)+1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*x^
(1/2))*2^(1/2)+1/4*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-1/4*ln(1+x+2^(1/2)*x^(1/
2))*2^(1/2)+2/x^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(1 + x^2)),x]
```

```
[Out] -2/(5*x^(5/2)) + 2/Sqrt[x] - ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Lo
g[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}\left(1+x^2\right)}\,dx &= -\frac{2}{5x^{5/2}}-\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx\\
&= -\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+\int \frac{\sqrt{x}}{1+x^2}\,dx\\
&= -\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}-\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\sqrt{}\right.\\
&= -\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+\frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{}\right.}{\sqrt{2}}\\
&= -\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log(1}{}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.20

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-x^2\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(1 + x^2)),x]
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, -x^2])/(5*x^(5/2))
```

fricas [A] time = 0.64, size = 136, normalized size = 1.26

$$-\frac{20\sqrt{2}\,x^3\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}\,x^3\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\cdots\right.}{20\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/20*(20*sqrt(2)*x^3*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2
)*sqrt(x) - 1) + 20*sqrt(2)*x^3*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x)
+ 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 5*sqrt(2)*x^3*log(4*sqrt(2)*sqrt(x) + 4
*x + 4) - 5*sqrt(2)*x^3*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(5*x^2 - 1)*s
qrt(x))/x^3
```

giac [A] time = 0.63, size = 86, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2/5*(5*x^2 - 1)/x^(5/2)
```

maple [A] time = 0.01, size = 72, normalized size = 0.67

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ -1\right)}{2}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ +1\right)}{2}+\frac{\sqrt{2}\ \ln\left(\frac{x-\sqrt{2}\ \sqrt{x}+1}{x+\sqrt{2}\ \sqrt{x}+1}\right)}{4}+\frac{2}{\sqrt{x}}-\frac{2}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^2+1),x)
```

```
[Out] 1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)+1/4*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/
(x+2^(1/2)*x^(1/2)+1))+1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)-2/5/x^(5/2)+2/
x^(1/2)
```

maxima [A] time = 3.05, size = 86, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2/5*(5*x^2 - 1)/x^(5/2)
```

mupad [B] time = 0.04, size = 48, normalized size = 0.44

$$\frac{2\,x^2-\frac{2}{5}}{x^{5/2}}+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(x^2 + 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2) + (2*x^2 - 2/5)/x^(5/2)
```

sympy [A] time = 5.46, size = 105, normalized size = 0.97

$$\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}+\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(x**2+1),x)
```

```
[Out] sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(2)*
sqrt(x) + 1)/2 + 2/sqrt(x) - 2/(5*x**(5/2))
```

3.321 $\int \frac{x^{7/2}}{\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=122

$$-\frac{x^{5/2}}{2\left(x^2+1\right)}+\frac{5\sqrt{x}}{2}+\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}+\right.}{4\sqrt{2}}$$

```
[Out] -1/2*x^(5/2)/(x^2+1)-5/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-5/8*arctan(1+2^
(1/2)*x^(1/2))*2^(1/2)+5/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-5/16*ln(1+x+2^(
1/2)*x^(1/2))*2^(1/2)+5/2*x^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^{5/2}}{2\left(x^2+1\right)}+\frac{5\sqrt{x}}{2}+\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}+\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(1 + x^2)^2,x]
```

```
[Out] (5*Sqrt[x])/2 - x^(5/2)/(2*(1 + x^2)) + (5*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*
Sqrt[2]) - (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (5*Log[1 - Sqrt[2]
*Sqrt[x] + x])/(8*Sqrt[2]) - (5*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{\left(1+x^2\right)^2}\,dx &= -\frac{x^{5/2}}{2\left(1+x^2\right)}+\frac{5}{4}\int \frac{x^{3/2}}{1+x^2}\,dx\\
&= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}-\frac{5}{4}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}-\frac{5}{2}\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}-\frac{5}{4}\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\frac{5}{4}\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}-\frac{5}{8}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{5}{8}\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\right.\\
&= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}+\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{5\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x\right.}{4\sqrt{2}}\\
&= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-
\end{aligned}$$

Mathematica [A] time = 0.05, size = 121, normalized size = 0.99

$$\frac{1}{16}\left(\frac{40\sqrt{x}}{x^2+1}+\frac{32x^{5/2}}{x^2+1}+5\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)-5\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)+10\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-1\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(1 + x^2)^2,x]
```

```
[Out] ((40*Sqrt[x])/(1 + x^2) + (32*x^(5/2))/(1 + x^2) + 10*Sqrt[2]*ArcTan[1 - Sq
rt[2]*Sqrt[x]] - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x]] + 5*Sqrt[2]*Log[1 -
 Sqrt[2]*Sqrt[x] + x] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/16
```

fricas [A] time = 0.58, size = 148, normalized size = 1.21

$$\frac{20\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x}\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(20*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(
2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 5*sqrt(2)*(x^2 + 1)*log(4*sq
rt(2)*sqrt(x) + 4*x + 4) + 5*sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x
 + 4) + 8*(4*x^2 + 5)*sqrt(x))/(x^2 + 1)
```

giac [A] time = 0.62, size = 91, normalized size = 0.75

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{5}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x) + 1/2*sqrt(x
)/(x^2 + 1)
```

maple [A] time = 0.01, size = 79, normalized size = 0.65

$$-\frac{5\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)}{8}-\frac{5\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{8}-\frac{5\sqrt{2}\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)}{16}+2\sqrt{x}+\frac{\sqrt{x}}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2+1)^2,x)
```

```
[Out] 2*x^(1/2)+1/2*x^(1/2)/(x^2+1)-5/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)-5/16*2^
(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))-5/8*2^(1/2)*arctan(2^
(1/2)*x^(1/2)+1)
```

maxima [A] time = 3.00, size = 91, normalized size = 0.75

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{5}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x) + 1/2*sqrt(x
)/(x^2 + 1)
```

mupad [B] time = 4.62, size = 55, normalized size = 0.45

$$\frac{\sqrt{x}}{2\left(x^2+1\right)}+2\sqrt{x}+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{8}-\frac{5}{8}\mathrm{i}\right)+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{8}+\frac{5}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2 + 1)^2,x)
```

```
[Out] x^(1/2)/(2*(x^2 + 1)) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/8 - 5
i/8) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(5/8 + 5i/8) + 2*x^(1/2)
```

sympy [B] time = 8.34, size = 277, normalized size = 2.27

$$\frac{32x^{\frac{5}{2}}}{16x^2+16}+\frac{40\sqrt{x}}{16x^2+16}+\frac{5\sqrt{2}\,x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{5\sqrt{2}\,x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{10\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}\right.}{16x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(x**2+1)**2,x)
```

```
[Out] 32*x**(5/2)/(16*x**2 + 16) + 40*sqrt(x)/(16*x**2 + 16) + 5*sqrt(2)*x**2*log
(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 5*sqrt(2)*x**2*log(4*sqrt(2
)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 10*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x)
- 1)/(16*x**2 + 16) - 10*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 +
16) + 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 5*sqrt(2
)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 10*sqrt(2)*atan(sqrt(2)
*sqrt(x) - 1)/(16*x**2 + 16) - 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**
2 + 16)
```

3.322 $\int \frac{x^{5/2}}{\left(1+x^2\right)^2} dx$

Optimal. Leaf size=113

$$-\frac{x^{3/2}}{2\left(x^2+1\right)}+\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

```
[Out] -1/2*x^(3/2)/(x^2+1)+3/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+3/8*arctan(1+2^
(1/2)*x^(1/2))*2^(1/2)+3/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-3/16*ln(1+x+2^(
1/2)*x^(1/2))*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^{3/2}}{2\left(x^2+1\right)}+\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(1 + x^2)^2,x]
```

```
[Out] -x^(3/2)/(2*(1 + x^2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*A
rcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*Log[1 - Sqrt[2]*Sqrt[x] + x])/
(8*Sqrt[2]) - (3*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\left(1+x^2\right)^2}\,dx &= -\frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{3}{4}\int \frac{\sqrt{x}}{1+x^2}\,dx \\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{3}{2}\operatorname{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{3}{4}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+ \\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{3\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\right.}{4\sqrt{2}} \\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{3\log\left(\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.27

$$2x^{3/2}\left({}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-x^2\right)-\frac{1}{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(1 + x^2)^2,x]
```

```
[Out] 2*x^(3/2)*(-(1 + x^2)^(-1) + Hypergeometric2F1[3/4, 2, 7/4, -x^2])
```

fricas [A] time = 0.94, size = 141, normalized size = 1.25

$$-\frac{12\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x}\right.}{16\left(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(12*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) -
sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt
(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 3*sqrt(2)*(x^2 + 1)*log(4*s
qrt(2)*sqrt(x) + 4*x + 4) - 3*sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*
x + 4) + 8*x^(3/2))/(x^2 + 1)
```

giac [A] time = 0.60, size = 86, normalized size = 0.76

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{3}{16}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*x^(3/2)/(x^2 + 1)
```

maple [A] time = 0.01, size = 74, normalized size = 0.65

$$-\frac{x^{\frac{3}{2}}}{2\left(x^2+1\right)}+\frac{3\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}-1\right)}{8}+\frac{3\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}+1\right)}{8}+\frac{3\sqrt{2}\ \ln\left(\frac{x-\sqrt{2}\ \sqrt{x}+1}{x+\sqrt{2}\ \sqrt{x}+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2+1)^2,x)
```

```
[Out] -1/2*x^(3/2)/(x^2+1)+3/16*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/
2)+1))+3/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)+3/8*2^(1/2)*arctan(2^(1/2)*x^(
1/2)-1)
```

maxima [A] time = 3.00, size = 86, normalized size = 0.76

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{3}{16}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*x^(3/2)/(x^2 + 1)
```

mupad [B] time = 4.62, size = 51, normalized size = 0.45

$$-\frac{x^{3/2}}{2\left(x^2+1\right)}+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{8}-\frac{3}{8}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{8}+\frac{3}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2 + 1)^2,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/8 - 3i/8) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(3/8 + 3i/8) - x^(3/2)/(2*(x^2 + 1))
```

sympy [B] time = 5.31, size = 264, normalized size = 2.34

$$-\frac{8x^{\frac{3}{2}}}{16x^2+16}+\frac{3\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,\sqrt{x}\,+4x+4\right)}{16x^2+16}-\frac{3\sqrt{2}\,x^2\log\left(4\sqrt{2}\,\sqrt{x}\,+4x+4\right)}{16x^2+16}+\frac{6\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}\,-1\right)}{16x^2+16}+\frac{6\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(x**2+1)**2,x)
```

```
[Out] -8*x**(3/2)/(16*x**2 + 16) + 3*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x +
4)/(16*x**2 + 16) - 3*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**
2 + 16) + 6*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 6*sqrt(
2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) + 3*sqrt(2)*log(-4*sqrt(2)
*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x
+ 4)/(16*x**2 + 16) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) +
6*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

3.323 $\int \frac{x^{3/2}}{(1+x^2)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{x}}{2\left(x^2+1\right)}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

```
[Out] 1/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2
)-1/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+1/16*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
-1/2*x^(1/2)/(x^2+1)
```

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt{x}}{2\left(x^2+1\right)}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(1 + x^2)^2,x]
```

```
[Out] -Sqrt[x]/(2*(1 + x^2)) - ArcTan[1 - Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) - Log[1 - Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
+ Log[1 + Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{\left(1+x^2\right)^2}\,dx &= -\frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{1}{4}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= -\frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\\
&= -\frac{\sqrt{x}}{2\left(1+x^2\right)}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right.}{4\sqrt{2}}\\
&= -\frac{\sqrt{x}}{2\left(1+x^2\right)}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\right.}{8\sqrt{}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 106, normalized size = 0.94

$$\frac{1}{16}\left(-\frac{8\sqrt{x}}{x^2+1}-\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)+\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)-2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+2\sqrt{2}\tan^{-1}\left(\sqrt{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(1 + x^2)^2,x]
```

```
[Out] ((-8*Sqrt[x])/(1 + x^2) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 2*Sqrt[2]
*ArcTan[1 + Sqrt[2]*Sqrt[x]] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x] + x] + Sqrt[
2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/16
```

fricas [A] time = 1.10, size = 140, normalized size = 1.24

$$-\frac{4\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+4\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+}\right.}{16\left(x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 4*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2
)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - sqrt(2)*(x^2 + 1)*log(4*sqrt(
2)*sqrt(x) + 4*x + 4) + sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
 + 8*sqrt(x))/(x^2 + 1)
```

giac [A] time = 0.63, size = 86, normalized size = 0.76

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{16}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 + 1)
```

maple [A] time = 0.01, size = 74, normalized size = 0.65

$$\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)}{8}+\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{8}+\frac{\sqrt{2}\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)}{16}-\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2+1)^2,x)
```

```
[Out] -1/2/(x^2+1)*x^(1/2)+1/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)+1/8*2^(1/2)*arct
an(2^(1/2)*x^(1/2)-1)+1/16*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1
/2)+1))
```

maxima [A] time = 2.96, size = 86, normalized size = 0.76

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{16}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 + 1)
```

mupad [B] time = 4.68, size = 51, normalized size = 0.45

$$-\frac{\sqrt{x}}{2\left(x^2+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2 + 1)^2,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/8 + 1i/8) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/8 - 1i/8) - x^(1/2)/(2*(x^2 + 1))
```

sympy [B] time = 3.39, size = 257, normalized size = 2.27

$$-\frac{8\sqrt{x}}{16x^2+16}-\frac{\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{\sqrt{2}\,x^2\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{2\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{16x^2+16}+\frac{2\sqrt{2}\,x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(x**2+1)**2,x)
```

```
[Out] -8*sqrt(x)/(16*x**2 + 16) - sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/
(16*x**2 + 16) + sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 1
6) + 2*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)*x*
*2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) - sqrt(2)*log(-4*sqrt(2)*sqrt(x
) + 4*x + 4)/(16*x**2 + 16) + sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*
x**2 + 16) + 2*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)
*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

3.324 $\int \frac{\sqrt{x}}{\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=113

$$\frac{x^{3/2}}{2\left(x^2+1\right)}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

```
[Out] 1/2*x^(3/2)/(x^2+1)+1/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/8*arctan(1+2^(
1/2)*x^(1/2))*2^(1/2)+1/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-1/16*ln(1+x+2^(1
/2)*x^(1/2))*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^{3/2}}{2\left(x^2+1\right)}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^2)^2,x]
```

```
[Out] x^(3/2)/(2*(1 + x^2)) - ArcTan[1 - Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + ArcTan[1
+ Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + Log[1 - Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2]) -
 Log[1 + Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\left(1+x^2\right)^2}\,dx &= \frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{1}{4}\int \frac{\sqrt{x}}{1+x^2}\,dx \\
&= \frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= \frac{x^{3/2}}{2\left(1+x^2\right)}-\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= \frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{1}{8}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+\frac{1}{8}\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+ \\
&= \frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\,\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\,\sqrt{x}\right)}{4\sqrt{2}} \\
&= \frac{x^{3/2}}{2\left(1+x^2\right)}-\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{x}\right)}{4\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\right.}{8\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.19

$$\frac{2}{3}x^{3/2}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^2)^2,x]
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -x^2])/3
```

fricas [A] time = 1.18, size = 140, normalized size = 1.24

$$-\frac{4\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+4\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+}\right.}{16\left(x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 4*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2
)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + sqrt(2)*(x^2 + 1)*log(4*sqrt(
2)*sqrt(x) + 4*x + 4) - sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
 - 8*x^(3/2))/(x^2 + 1)
```

giac [A] time = 0.63, size = 86, normalized size = 0.76

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{16}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1)
```

maple [A] time = 0.01, size = 74, normalized size = 0.65

$$\frac{x^{\frac{3}{2}}}{2x^2+2}+\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)}{8}+\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{8}+\frac{\sqrt{2}\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1)^2,x)
```

```
[Out] 1/2/(x^2+1)*x^(3/2)+1/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)+1/16*2^(1/2)*ln((
x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+1/8*2^(1/2)*arctan(2^(1/2)*x^(1
/2)+1)
```

maxima [A] time = 2.94, size = 86, normalized size = 0.76

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{16}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1)
```

mupad [B] time = 0.04, size = 50, normalized size = 0.44

$$\frac{x^{3/2}}{2\left(x^2+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2 + 1)^2,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/8 - 1i/8) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/8 + 1i/8) + x^(3/2)/(2*(x^2 + 1))
```

sympy [B] time = 2.17, size = 257, normalized size = 2.27

$$\frac{8x^{\frac{3}{2}}}{16x^2+16}+\frac{\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,\sqrt{x}\,+4x+4\right)}{16x^2+16}-\frac{\sqrt{2}\,x^2\log\left(4\sqrt{2}\,\sqrt{x}\,+4x+4\right)}{16x^2+16}+\frac{2\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}\,-1\right)}{16x^2+16}+\frac{2\sqrt{2}\,x^2\,\mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1)**2,x)
```

```
[Out] 8*x**(3/2)/(16*x**2 + 16) + sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/
(16*x**2 + 16) - sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 1
6) + 2*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)*x*
*2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) + sqrt(2)*log(-4*sqrt(2)*sqrt(x
) + 4*x + 4)/(16*x**2 + 16) - sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*
x**2 + 16) + 2*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)
*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

3.325 $\int \frac{1}{\sqrt{x}\left(1+x^2\right)^2} dx$

Optimal. Leaf size=113

$$\frac{\sqrt{x}}{2\left(x^2+1\right)}-\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

```
[Out] 3/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+3/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2
)-3/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+3/16*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
+1/2*x^(1/2)/(x^2+1)
```

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt{x}}{2\left(x^2+1\right)}-\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + x^2)^2),x]
```

```
[Out] Sqrt[x]/(2*(1 + x^2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*Ar
cTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) - (3*Log[1 - Sqrt[2]*Sqrt[x] + x])/(
8*Sqrt[2]) + (3*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\left(1+x^2\right)^2}\,dx &= \frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{3}{4}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx \\
&= \frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= \frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{3}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{3}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= \frac{\sqrt{x}}{2\left(1+x^2\right)}+\frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+\frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right. \\
&= \frac{\sqrt{x}}{2\left(1+x^2\right)}-\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{3\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\right.}{4\sqrt{2}} \\
&= \frac{\sqrt{x}}{2\left(1+x^2\right)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{3\,\text{lo}}{}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 107, normalized size = 0.95

$$\frac{1}{16}\left(\frac{8\sqrt{x}}{x^2+1}-3\sqrt{2}\,\log\left(x-\sqrt{2}\sqrt{x}+1\right)+3\sqrt{2}\,\log\left(x+\sqrt{2}\sqrt{x}+1\right)-6\sqrt{2}\,\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+6\sqrt{2}\,\tan^{-1}\left(\sqrt{\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + x^2)^2),x]
```

```
[Out] ((8*Sqrt[x])/(1 + x^2) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 6*Sqrt[2]*
ArcTan[1 + Sqrt[2]*Sqrt[x]] - 3*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x] + x] + 3*Sq
rt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/16
```

fricas [A] time = 1.13, size = 141, normalized size = 1.25

$$-\frac{12\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x}\right.}{16\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(12*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) -
sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt
(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 3*sqrt(2)*(x^2 + 1)*log(4*s
qrt(2)*sqrt(x) + 4*x + 4) + 3*sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*
x + 4) - 8*sqrt(x))/(x^2 + 1)
```

giac [A] time = 0.60, size = 86, normalized size = 0.76

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{16}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*sqrt(x)/(x^2 + 1)
```

maple [A] time = 0.01, size = 74, normalized size = 0.65

$$\frac{3\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)}{8}+\frac{3\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{8}+\frac{3\sqrt{2}\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)}{16}+\frac{\sqrt{x}}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/x^(1/2),x)
```

```
[Out] 1/2/(x^2+1)*x^(1/2)+3/16*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2
)+1))+3/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)+3/8*2^(1/2)*arctan(2^(1/2)*x^(1
/2)-1)
```

maxima [A] time = 2.84, size = 86, normalized size = 0.76

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{16}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*sqrt(x)/(x^2 + 1)
```

mupad [B] time = 4.66, size = 50, normalized size = 0.44

$$\frac{\sqrt{x}}{2\left(x^2+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{8}+\frac{3}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{8}-\frac{3}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^2 + 1)^2),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/8 + 3i/8) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(3/8 - 3i/8) + x^(1/2)/(2*(x^2 + 1))
```

sympy [**B**]   time = 2.99, size = 264, normalized size = 2.34

$$\frac{8\sqrt{x}}{16x^2+16}-\frac{3\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,\sqrt{x}\,+4x+4\right)}{16x^2+16}+\frac{3\sqrt{2}\,x^2\log\left(4\sqrt{2}\,\sqrt{x}\,+4x+4\right)}{16x^2+16}+\frac{6\sqrt{2}\,x^2\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}\,-1\right)}{16x^2+16}+\frac{6\sqrt{2}\,x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/x**(1/2),x)
```

```
[Out] 8*sqrt(x)/(16*x**2 + 16) - 3*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
/(16*x**2 + 16) + 3*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2
+ 16) + 6*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 6*sqrt(2)
*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) - 3*sqrt(2)*log(-4*sqrt(2)*s
qrt(x) + 4*x + 4)/(16*x**2 + 16) + 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x +
4)/(16*x**2 + 16) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 6*
sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

3.326 $\int \frac{1}{x^{3/2}\left(1+x^2\right)^2} dx$

Optimal. Leaf size=122

$$\frac{1}{2\sqrt{x}\left(x^2+1\right)}-\frac{5}{2\sqrt{x}}-\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{4\sqrt{2}}$$

```
[Out] -5/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-5/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/
2)-5/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+5/16*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2
)-5/2/x^(1/2)+1/2/(x^2+1)/x^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{2\sqrt{x}\left(x^2+1\right)}-\frac{5}{2\sqrt{x}}-\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(1 + x^2)^2),x]
```

```
[Out] -5/(2*Sqrt[x]) + 1/(2*Sqrt[x]*(1 + x^2)) + (5*ArcTan[1 - Sqrt[2]*Sqrt[x]])/
(4*Sqrt[2]) - (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) - (5*Log[1 - Sqrt
[2]*Sqrt[x] + x])/(8*Sqrt[2]) + (5*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^2}\,dx = \frac{1}{2\sqrt{x}\left(1+x^2\right)}+\frac{5}{4}\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx$$
$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}-\frac{5}{4}\int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}-\frac{5}{2}\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}+\frac{5}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\frac{5}{4}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}-\frac{5}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{5}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+$$
$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}-\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{5\,\text{Subst}\left(\int \right.}{}$$
$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}\sqrt{\right.}{8\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 20, normalized size = 0.16

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};-x^2\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(1 + x^2)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 2, 3/4, -x^2])/Sqrt[x]
```

fricas [A] time = 0.90, size = 148, normalized size = 1.21

$$20\sqrt{2}\left(x^3+x\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}\left(x^3+x\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(20*sqrt(2)*(x^3 + x)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^3 + x)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(
2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 5*sqrt(2)*(x^3 + x)*log(4*sq
rt(2)*sqrt(x) + 4*x + 4) - 5*sqrt(2)*(x^3 + x)*log(-4*sqrt(2)*sqrt(x) + 4*x
 + 4) - 8*(5*x^2 + 4)*sqrt(x))/(x^3 + x)
```

giac [A] time = 0.64, size = 92, normalized size = 0.75

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{5}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) - 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*(5*x^2 + 4)/(x^(5/
2) + sqrt(x))
```

maple [A] time = 0.01, size = 79, normalized size = 0.65

$$-\frac{x^{\frac{3}{2}}}{2\left(x^2+1\right)}-\frac{5\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{8}-\frac{5\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{8}-\frac{5\sqrt{2}\,\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)}{16}-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(x^2+1)^2,x)
```

```
[Out] -1/2/(x^2+1)*x^(3/2)-5/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)-5/8*2^(1/2)*arct
an(2^(1/2)*x^(1/2)-1)-5/16*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1
/2)+1))-2/x^(1/2)
```

maxima [A] time = 3.04, size = 92, normalized size = 0.75

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{5}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) - 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*(5*x^2 + 4)/(x^(5/
2) + sqrt(x))
```

mupad [B] time = 4.68, size = 55, normalized size = 0.45

$$-\frac{\frac{5x^2}{2}+2}{\sqrt{x}+x^{5/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{8}+\frac{5}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{8}-\frac{5}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(x^2 + 1)^2),x)
```

```
[Out] - ((5*x^2)/2 + 2)/(x^(1/2) + x^(5/2)) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 -
 1i/2))*(5/8 - 5i/8) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/8 + 5i
/8)
```

sympy [B] time = 4.56, size = 366, normalized size = 3.00

$$-\frac{5\sqrt{2}\,x^{\frac{5}{2}}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}+\frac{5\sqrt{2}\,x^{\frac{5}{2}}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{10\sqrt{2}\,x^{\frac{5}{2}}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{10\sqrt{2}\,x^{\frac{5}{2}}\,\mathrm{atan}}{16x^{\frac{5}{2}}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(x**2+1)**2,x)
```

```
[Out] -5*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**(5/2) + 16*sqr
t(x)) + 5*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**(5/2) +
16*sqrt(x)) - 10*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**(5/2) +
16*sqrt(x)) - 10*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**(5/2) +
16*sqrt(x)) - 5*sqrt(2)*sqrt(x)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**(5
/2) + 16*sqrt(x)) + 5*sqrt(2)*sqrt(x)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*
x**(5/2) + 16*sqrt(x)) - 10*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) - 1)/(16*x
**(5/2) + 16*sqrt(x)) - 10*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) + 1)/(16*x*
*(5/2) + 16*sqrt(x)) - 40*x**2/(16*x**(5/2) + 16*sqrt(x)) - 32/(16*x**(5/2)
 + 16*sqrt(x))
```

3.327 $\int \frac{1}{x^{5/2}\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=122

$$-\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}\left(x^2+1\right)}+\frac{7\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{7\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{7\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{7\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{4\sqrt{2}}$$

```
[Out] -7/6/x^(3/2)+1/2/x^(3/2)/(x^2+1)-7/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-7/8
*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+7/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-7/1
6*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{2x^{3/2}\left(x^2+1\right)}-\frac{7}{6x^{3/2}}+\frac{7\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{7\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{7\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{7\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(1 + x^2)^2),x]
```

```
[Out] -7/(6*x^(3/2)) + 1/(2*x^(3/2)*(1 + x^2)) + (7*ArcTan[1 - Sqrt[2]*Sqrt[x]])/
(4*Sqrt[2]) - (7*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (7*Log[1 - Sqrt
[2]*Sqrt[x] + x])/(8*Sqrt[2]) - (7*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/2}\left(1+x^2\right)^2}\,dx &= \frac{1}{2x^{3/2}\left(1+x^2\right)}+\frac{7}{4}\int \frac{1}{x^{5/2}\left(1+x^2\right)}\,dx\\
&= -\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}\left(1+x^2\right)}-\frac{7}{4}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= -\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}\left(1+x^2\right)}-\frac{7}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}\left(1+x^2\right)}-\frac{7}{4}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\frac{7}{4}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}\left(1+x^2\right)}-\frac{7}{8}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{7}{8}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+\cdots}\right.\\
&= -\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}\left(1+x^2\right)}+\frac{7\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{7\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{7\operatorname{Subst}\left(\int \cdots\right.}{\cdots}\\
&= -\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}\left(1+x^2\right)}+\frac{7\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{7\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{7\log\left(1-\sqrt{2}\sqrt{\cdots}\right.}{8\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.18

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},2;\frac{1}{4};-x^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(1 + x^2)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 2, 1/4, -x^2])/(3*x^(3/2))
```

fricas [A] time = 0.90, size = 158, normalized size = 1.30

$$\frac{84\sqrt{2}\left(x^4+x^2\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+84\sqrt{2}\left(x^4+x^2\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(84*sqrt(2)*(x^4 + x^2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) -
 sqrt(2)*sqrt(x) - 1) + 84*sqrt(2)*(x^4 + x^2)*arctan(1/2*sqrt(2)*sqrt(-4*s
qrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 21*sqrt(2)*(x^4 + x^2)*l
og(4*sqrt(2)*sqrt(x) + 4*x + 4) + 21*sqrt(2)*(x^4 + x^2)*log(-4*sqrt(2)*sqr
t(x) + 4*x + 4) - 8*(7*x^2 + 4)*sqrt(x))/(x^4 + x^2)
```

giac [A] time = 0.63, size = 91, normalized size = 0.75

$$-\frac{7}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{7}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{7}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{7}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -7/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 7/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 7/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 7/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 + 1)
- 2/3/x^(3/2)
```

maple [A] time = 0.01, size = 79, normalized size = 0.65

$$-\frac{7\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{8}-\frac{7\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{8}-\frac{7\sqrt{2}\,\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)}{16}-\frac{\sqrt{x}}{2\left(x^2+1\right)}-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(x^2+1)^2,x)
```

```
[Out] -1/2/(x^2+1)*x^(1/2)-7/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)-7/16*2^(1/2)*ln(
(x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))-7/8*2^(1/2)*arctan(2^(1/2)*x^(
1/2)+1)-2/3/x^(3/2)
```

maxima [A] time = 2.94, size = 92, normalized size = 0.75

$$-\frac{7}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{7}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{7}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{7}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -7/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 7/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 7/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 7/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/6*(7*x^2 + 4)/(x^(7/
2) + x^(3/2))
```

mupad [B] time = 0.08, size = 55, normalized size = 0.45

$$-\frac{\frac{7x^2}{6}+\frac{2}{3}}{x^{3/2}+x^{7/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{7}{8}-\frac{7}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{7}{8}+\frac{7}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(x^2 + 1)^2),x)
```

```
[Out] - ((7*x^2)/6 + 2/3)/(x^(3/2) + x^(7/2)) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2
 - 1i/2))*(7/8 + 7i/8) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(7/8 -
7i/8)
```

sympy [B] time = 8.33, size = 366, normalized size = 3.00

$$\frac{21\sqrt{2}\,x^{\frac{7}{2}}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{21\sqrt{2}\,x^{\frac{7}{2}}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{42\sqrt{2}\,x^{\frac{7}{2}}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{42\sqrt{2}\,x^{\frac{7}{2}}\,\mathrm{atar}}{48x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(x**2+1)**2,x)
```

```
[Out] 21*sqrt(2)*x**(7/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(48*x**(7/2) + 48*x**
(3/2)) - 21*sqrt(2)*x**(7/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(48*x**(7/2)
+ 48*x**(3/2)) - 42*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) - 1)/(48*x**(7/2)
 + 48*x**(3/2)) - 42*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) + 1)/(48*x**(7/2
) + 48*x**(3/2)) + 21*sqrt(2)*x**(3/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(4
8*x**(7/2) + 48*x**(3/2)) - 21*sqrt(2)*x**(3/2)*log(4*sqrt(2)*sqrt(x) + 4*x
 + 4)/(48*x**(7/2) + 48*x**(3/2)) - 42*sqrt(2)*x**(3/2)*atan(sqrt(2)*sqrt(x
) - 1)/(48*x**(7/2) + 48*x**(3/2)) - 42*sqrt(2)*x**(3/2)*atan(sqrt(2)*sqrt(
x) + 1)/(48*x**(7/2) + 48*x**(3/2)) - 56*x**2/(48*x**(7/2) + 48*x**(3/2)) -
 32/(48*x**(7/2) + 48*x**(3/2))
```

3.328 $\int \frac{1}{x^{7/2}\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=131

$$-\frac{9}{10x^{5/2}}+\frac{1}{2x^{5/2}\left(x^2+1\right)}+\frac{9}{2\sqrt{x}}+\frac{9\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{9\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{9\tan^{-1}}{}$$

```
[Out] -9/10/x^(5/2)+1/2/x^(5/2)/(x^2+1)+9/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+9/
8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+9/16*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-9/
16*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)+9/2/x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{2x^{5/2}\left(x^2+1\right)}-\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{9\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{9\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{9\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(1 + x^2)^2),x]
```

```
[Out] -9/(10*x^(5/2)) + 9/(2*Sqrt[x]) + 1/(2*x^(5/2)*(1 + x^2)) - (9*ArcTan[1 - S
qrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (9*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2])
+ (9*Log[1 - Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]) - (9*Log[1 + Sqrt[2]*Sqrt[x]
 + x])/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}\left(1+x^2\right)^2}\,dx &= \frac{1}{2x^{5/2}\left(1+x^2\right)}+\frac{9}{4}\int \frac{1}{x^{7/2}\left(1+x^2\right)}\,dx\\
&= -\frac{9}{10x^{5/2}}+\frac{1}{2x^{5/2}\left(1+x^2\right)}-\frac{9}{4}\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx\\
&= -\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{1}{2x^{5/2}\left(1+x^2\right)}+\frac{9}{4}\int \frac{\sqrt{x}}{1+x^2}\,dx\\
&= -\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{1}{2x^{5/2}\left(1+x^2\right)}+\frac{9}{2}\operatorname{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{1}{2x^{5/2}\left(1+x^2\right)}-\frac{9}{4}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{9}{4}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx\right.\\
&= -\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{1}{2x^{5/2}\left(1+x^2\right)}+\frac{9}{8}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+\frac{9}{8}\operatorname{Subst}\left(\int \right.\\
&= -\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{1}{2x^{5/2}\left(1+x^2\right)}+\frac{9\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{9\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\\
&= -\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{1}{2x^{5/2}\left(1+x^2\right)}-\frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{9\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+
\end{aligned}$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.17

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},2;-\frac{1}{4};-x^2\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(1 + x^2)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 2, -1/4, -x^2])/(5*x^(5/2))
```

fricas [A] time = 1.03, size = 163, normalized size = 1.24

$$-\frac{180\sqrt{2}\left(x^5+x^3\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+180\sqrt{2}\left(x^5+x^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/80*(180*sqrt(2)*(x^5 + x^3)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1)
 - sqrt(2)*sqrt(x) - 1) + 180*sqrt(2)*(x^5 + x^3)*arctan(1/2*sqrt(2)*sqrt(-
4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 45*sqrt(2)*(x^5 + x^3
)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 45*sqrt(2)*(x^5 + x^3)*log(-4*sqrt(2)*
sqrt(x) + 4*x + 4) - 8*(45*x^4 + 36*x^2 - 4)*sqrt(x))/(x^5 + x^3)
```

giac [A] time = 0.64, size = 98, normalized size = 0.75

$$\frac{9}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{9}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{9}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{9}{16}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 9/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 9/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 9/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 9/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1) +
 2/5*(10*x^2 - 1)/x^(5/2)
```

maple [A] time = 0.02, size = 84, normalized size = 0.64

$$\frac{x^{\frac{3}{2}}}{2x^2+2}+\frac{9\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{8}+\frac{9\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{8}+\frac{9\sqrt{2}\,\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)}{16}+\frac{4}{\sqrt{x}}-\frac{2}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^2+1)^2,x)
```

```
[Out] 1/2/(x^2+1)*x^(3/2)+9/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)+9/16*2^(1/2)*ln((
x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+9/8*2^(1/2)*arctan(2^(1/2)*x^(1
/2)+1)-2/5/x^(5/2)+4/x^(1/2)
```

maxima [A] time = 3.04, size = 97, normalized size = 0.74

$$\frac{9}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{9}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{9}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{9}{16}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 9/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 9/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 9/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/10*(45*x^4 + 36*x^2 -
 4)/(x^(9/2) + x^(5/2))
```

mupad [B] time = 0.07, size = 59, normalized size = 0.45

$$\frac{\frac{9x^4}{2}+\frac{18x^2}{5}-\frac{2}{5}}{x^{5/2}+x^{9/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{9}{8}-\frac{9}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{9}{8}+\frac{9}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(x^2 + 1)^2),x)
```

```
[Out] ((18*x^2)/5 + (9*x^4)/2 - 2/5)/(x^(5/2) + x^(9/2)) + 2^(1/2)*atan(2^(1/2)*x
^(1/2)*(1/2 - 1i/2))*(9/8 - 9i/8) + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/
2))*(9/8 + 9i/8)
```

sympy [B] time = 18.73, size = 384, normalized size = 2.93

$$\frac{45\sqrt{2}\,x^{\frac{9}{2}}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}-\frac{45\sqrt{2}\,x^{\frac{9}{2}}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{90\sqrt{2}\,x^{\frac{9}{2}}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{90\sqrt{2}\,x^{\frac{9}{2}}\,\mathrm{ata}}{80x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(x**2+1)**2,x)
```

```
[Out] 45*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(80*x**(9/2) + 80*x**
(5/2)) - 45*sqrt(2)*x**(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(80*x**(9/2)
+ 80*x**(5/2)) + 90*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) - 1)/(80*x**(9/2)
 + 80*x**(5/2)) + 90*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) + 1)/(80*x**(9/2
) + 80*x**(5/2)) + 45*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(8
0*x**(9/2) + 80*x**(5/2)) - 45*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x) + 4*x
 + 4)/(80*x**(9/2) + 80*x**(5/2)) + 90*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x
) - 1)/(80*x**(9/2) + 80*x**(5/2)) + 90*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(
x) + 1)/(80*x**(9/2) + 80*x**(5/2)) + 360*x**4/(80*x**(9/2) + 80*x**(5/2))
+ 288*x**2/(80*x**(9/2) + 80*x**(5/2)) - 32/(80*x**(9/2) + 80*x**(5/2))
```

3.329 $\int \frac{x^{7/2}}{(1+x^2)^3}\,dx$

Optimal. Leaf size=129

$$-\frac{5\sqrt{x}}{16\left(x^2+1\right)}-\frac{x^{5/2}}{4\left(x^2+1\right)^2}-\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{\right.}}{32}$$

```
[Out] -1/4*x^(5/2)/(x^2+1)^2+5/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+5/64*arctan(
1+2^(1/2)*x^(1/2))*2^(1/2)-5/128*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+5/128*ln(1
+x+2^(1/2)*x^(1/2))*2^(1/2)-5/16*x^(1/2)/(x^2+1)
```

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^{5/2}}{4\left(x^2+1\right)^2}-\frac{5\sqrt{x}}{16\left(x^2+1\right)}-\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{\right.}}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(1 + x^2)^3,x]
```

```
[Out] -x^(5/2)/(4*(1 + x^2)^2) - (5*Sqrt[x])/(16*(1 + x^2)) - (5*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) -
(5*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (5*Log[1 + Sqrt[2]*Sqrt[x]
+ x])/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{\left(1+x^2\right)^3}\,dx &= -\frac{x^{5/2}}{4\left(1+x^2\right)^2}+\frac{5}{8}\int \frac{x^{3/2}}{\left(1+x^2\right)^2}\,dx\\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2}-\frac{5\sqrt{x}}{16\left(1+x^2\right)}+\frac{5}{32}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2}-\frac{5\sqrt{x}}{16\left(1+x^2\right)}+\frac{5}{16}\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2}-\frac{5\sqrt{x}}{16\left(1+x^2\right)}+\frac{5}{32}\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{5}{32}\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right.\\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2}-\frac{5\sqrt{x}}{16\left(1+x^2\right)}+\frac{5}{64}\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{5}{64}\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x\right.\\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2}-\frac{5\sqrt{x}}{16\left(1+x^2\right)}-\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{5\,\mathrm{Subst}\left(\int\right.}{}\\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2}-\frac{5\sqrt{x}}{16\left(1+x^2\right)}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}\sqrt{x}\right.}{64\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 135, normalized size = 1.05

$$\frac{1}{384}\left(\frac{40\sqrt{x}}{x^2+1}-\frac{160\sqrt{x}}{\left(x^2+1\right)^2}-\frac{256x^{5/2}}{\left(x^2+1\right)^2}-15\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)+15\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)-30\sqrt{2}\tan^{-1}\right($$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(1 + x^2)^3,x]
```

```
[Out] ((-160*Sqrt[x])/(1 + x^2)^2 - (256*x^(5/2))/(1 + x^2)^2 + (40*Sqrt[x])/(1 +
 x^2) - 30*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 30*Sqrt[2]*ArcTan[1 + Sqrt
[2]*Sqrt[x]] - 15*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x] + x] + 15*Sqrt[2]*Log[1 +
 Sqrt[2]*Sqrt[x] + x])/384
```

fricas [A] time = 0.93, size = 173, normalized size = 1.34

$$-\frac{20\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 5*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 5*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) + 8*(9*x^2 + 5)*sqrt(x))/(x^4 + 2*x
^2 + 1)
```

giac [A] time = 0.69, size = 94, normalized size = 0.73

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(9*x^(5/2) + 5
*sqrt(x))/(x^2 + 1)^2
```

maple [A] time = 0.01, size = 82, normalized size = 0.64

$$\frac{5\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{64}+\frac{5\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{64}+\frac{5\sqrt{2}\,\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)}{128}+\frac{-\frac{9x^{\frac{5}{2}}}{16}-\frac{5\sqrt{x}}{16}}{\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2+1)^3,x)
```

```
[Out] 2*(-9/32*x^(5/2)-5/32*x^(1/2))/(x^2+1)^2+5/64*2^(1/2)*arctan(2^(1/2)*x^(1/2
)-1)+5/128*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))+5/64*2^(
1/2)*arctan(2^(1/2)*x^(1/2)+1)
```

maxima [A] time = 3.01, size = 99, normalized size = 0.77

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(9*x^(5/2) + 5
*sqrt(x))/(x^4 + 2*x^2 + 1)
```

mupad [B] time = 4.73, size = 62, normalized size = 0.48

$$-\frac{\frac{5\,\sqrt{x}}{16}+\frac{9\,x^{5/2}}{16}}{x^4+2\,x^2+1}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{5}{64}+\frac{5}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{5}{64}-\frac{5}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2 + 1)^3,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(5/64 + 5i/64) + 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/64 - 5i/64) - ((5*x^(1/2))/16 + (9*x^(5/2))/
16)/(2*x^2 + x^4 + 1)
```

sympy [B] time = 23.35, size = 481, normalized size = 3.73

$$-\frac{72x^{\frac{5}{2}}}{128x^4+256x^2+128}-\frac{40\sqrt{x}}{128x^4+256x^2+128}-\frac{5\sqrt{2}\,x^4\log\left(-4\sqrt{2}\,\sqrt{x}\,+4x+4\right)}{128x^4+256x^2+128}+\frac{5\sqrt{2}\,x^4\log\left(4\sqrt{2}\,\sqrt{x}\,+4x+4\right)}{128x^4+256x^2+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(x**2+1)**3,x)
```

```
[Out] -72*x**(5/2)/(128*x**4 + 256*x**2 + 128) - 40*sqrt(x)/(128*x**4 + 256*x**2
+ 128) - 5*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 5*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x
**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2
 + 128) - 10*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) + 10*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 +
 256*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 25
6*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x
**2 + 128) - 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x*
*2 + 128) + 5*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2
 + 128) + 10*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128)
+ 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

3.330 $\int \frac{x^{5/2}}{\left(1+x^2\right)^3} \, dx$

Optimal. Leaf size=129

$$\frac{3x^{3/2}}{16\left(x^2+1\right)}-\frac{x^{3/2}}{4\left(x^2+1\right)^2}+\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\right.}{32\sqrt{}}$$

```
[Out] -1/4*x^(3/2)/(x^2+1)^2+3/16*x^(3/2)/(x^2+1)+3/64*arctan(-1+2^(1/2)*x^(1/2))
*2^(1/2)+3/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+3/128*ln(1+x-2^(1/2)*x^(1/2
))*2^(1/2)-3/128*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3x^{3/2}}{16\left(x^2+1\right)}-\frac{x^{3/2}}{4\left(x^2+1\right)^2}+\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\right.}{32\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(1 + x^2)^3,x]
```

```
[Out] -x^(3/2)/(4*(1 + x^2)^2) + (3*x^(3/2))/(16*(1 + x^2)) - (3*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) +
(3*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (3*Log[1 + Sqrt[2]*Sqrt[x]
+ x])/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(1+x^2\right)^3} dx = -\frac{x^{3/2}}{4\left(1+x^2\right)^2} + \frac{3}{8}\int \frac{\sqrt{x}}{\left(1+x^2\right)^2} dx$$

$$= -\frac{x^{3/2}}{4\left(1+x^2\right)^2} + \frac{3x^{3/2}}{16\left(1+x^2\right)} + \frac{3}{32}\int \frac{\sqrt{x}}{1+x^2} dx$$

$$= -\frac{x^{3/2}}{4\left(1+x^2\right)^2} + \frac{3x^{3/2}}{16\left(1+x^2\right)} + \frac{3}{16}\text{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}}{4\left(1+x^2\right)^2} + \frac{3x^{3/2}}{16\left(1+x^2\right)} - \frac{3}{32}\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{x}\right) + \frac{3}{32}\text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x,\right.$$

$$= -\frac{x^{3/2}}{4\left(1+x^2\right)^2} + \frac{3x^{3/2}}{16\left(1+x^2\right)} + \frac{3}{64}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{x}\right) + \frac{3}{64}\text{Subst}\left(\int \frac{}{1+\sqrt{}}\right.$$

$$= -\frac{x^{3/2}}{4\left(1+x^2\right)^2} + \frac{3x^{3/2}}{16\left(1+x^2\right)} + \frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} - \frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} + \frac{3\,\text{Subst}\left(\right.}{}$$

$$= -\frac{x^{3/2}}{4\left(1+x^2\right)^2} + \frac{3x^{3/2}}{16\left(1+x^2\right)} - \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{3\log\left(1-\sqrt{2}\right.}{64\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.25

$$\frac{2}{5}x^{3/2}\left({}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-x^2\right) - \frac{1}{\left(x^2+1\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(1 + x^2)^3,x]
[Out] (2*x^(3/2)*(-(1 + x^2)^(-2) + Hypergeometric2F1[3/4, 3, 7/4, -x^2]))/5
```

fricas [A] time = 0.66, size = 175, normalized size = 1.36

$$-\frac{12\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^3,x, algorithm="fricas")
[Out] -1/128*(12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 3*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 3*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(3*x^3 - x)*sqrt(x))/(x^4 + 2*x
^2 + 1)
```

giac [A] time = 0.65, size = 94, normalized size = 0.73

$$\frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(3*x^(7/2) - x
^(3/2))/(x^2 + 1)^2
```

maple [A] time = 0.01, size = 82, normalized size = 0.64

$$\frac{3\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ -1\right)}{64}+\frac{3\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ +1\right)}{64}+\frac{3\sqrt{2}\ \ln\left(\frac{x-\sqrt{2}\ \sqrt{x}+1}{x+\sqrt{2}\ \sqrt{x}+1}\right)}{128}+\frac{\frac{3x^{\frac{7}{2}}}{16}-\frac{x^{\frac{3}{2}}}{16}}{\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2+1)^3,x)
```

```
[Out] 2*(3/32*x^(7/2)-1/32*x^(3/2))/(x^2+1)^2+3/64*2^(1/2)*arctan(2^(1/2)*x^(1/2)
-1)+3/128*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+3/64*2^(1
/2)*arctan(2^(1/2)*x^(1/2)+1)
```

maxima [A] time = 2.91, size = 99, normalized size = 0.77

$$\frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{3}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(3*x^(7/2) - x
^(3/2))/(x^4 + 2*x^2 + 1)
```

mupad [B] time = 0.07, size = 62, normalized size = 0.48

$$-\frac{\frac{x^{3/2}}{16}-\frac{3\,x^{7/2}}{16}}{x^4+2\,x^2+1}+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{64}-\frac{3}{64}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{64}+\frac{3}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2 + 1)^3,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/64 - 3i/64) + 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(3/64 + 3i/64) - (x^(3/2)/16 - (3*x^(7/2))/16)/
(2*x^2 + x^4 + 1)
```

sympy [B] time = 15.49, size = 481, normalized size = 3.73

$$\frac{24x^{\frac{7}{2}}}{128x^4+256x^2+128}-\frac{8x^{\frac{3}{2}}}{128x^4+256x^2+128}+\frac{3\sqrt{2}\,x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{3\sqrt{2}\,x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(x**2+1)**3,x)
```

```
[Out] 24*x**(7/2)/(128*x**4 + 256*x**2 + 128) - 8*x**(3/2)/(128*x**4 + 256*x**2 +
 128) + 3*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x*
```

```
*2 + 128) - 3*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**
2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 +
128) + 6*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**
2 + 128) - 6*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*
x**2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**
2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 +
 128) + 3*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 +
128) - 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 + 12
8) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128) + 6*sq
rt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

3.331 $\int \frac{x^{3/2}}{(1+x^2)^3}\,dx$

Optimal. Leaf size=129

$$\frac{\sqrt{x}}{16(x^2+1)}-\frac{\sqrt{x}}{4(x^2+1)^2}-\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{32\sqrt{2}}$$

```
[Out] 3/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+3/64*arctan(1+2^(1/2)*x^(1/2))*2^(1
/2)-3/128*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+3/128*ln(1+x+2^(1/2)*x^(1/2))*2^(
1/2)-1/4*x^(1/2)/(x^2+1)^2+1/16*x^(1/2)/(x^2+1)
```

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt{x}}{16(x^2+1)}-\frac{\sqrt{x}}{4(x^2+1)^2}-\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(1 + x^2)^3,x]
```

```
[Out] -Sqrt[x]/(4*(1 + x^2)^2) + Sqrt[x]/(16*(1 + x^2)) - (3*ArcTan[1 - Sqrt[2]*S
qrt[x]])/(32*Sqrt[2]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (3*L
og[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (3*Log[1 + Sqrt[2]*Sqrt[x] + x]
)/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(1+x^2\right)^3}\,dx = -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{1}{8}\int \frac{1}{\sqrt{x}\left(1+x^2\right)^2}\,dx$$
$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}+\frac{3}{32}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx$$
$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}+\frac{3}{16}\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}+\frac{3}{32}\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{3}{32}\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right.$$
$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}+\frac{3}{64}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{3}{64}\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x\right.$$
$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}-\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{3\,\mathrm{Subst}\left(\int \right.}{}$$
$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{3\log\left(1-\sqrt{2}\sqrt{x}\right.}{64\sqrt{2}}$$

Mathematica [A] time = 0.03, size = 121, normalized size = 0.94

$$\frac{1}{128}\left(\frac{8\sqrt{x}}{x^2+1}-\frac{32\sqrt{x}}{\left(x^2+1\right)^2}-3\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)+3\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)-6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+6\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(1 + x^2)^3,x]
```

```
[Out] ((-32*Sqrt[x])/(1 + x^2)^2 + (8*Sqrt[x])/(1 + x^2) - 6*Sqrt[2]*ArcTan[1 - S
qrt[2]*Sqrt[x]] + 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x]] - 3*Sqrt[2]*Log[1 -
 Sqrt[2]*Sqrt[x] + x] + 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/128
```

fricas [A] time = 0.66, size = 171, normalized size = 1.33

$$-\frac{12\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 3*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 3*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(x^2 - 3)*sqrt(x))/(x^4 + 2*x^2
 + 1)
```

giac [A] time = 0.64, size = 92, normalized size = 0.71

$$\frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(x^(5/2) - 3*s
qrt(x))/(x^2 + 1)^2
```

maple [A] time = 0.01, size = 82, normalized size = 0.64

$$\frac{3\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ -1\right)}{64}+\frac{3\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ +1\right)}{64}+\frac{3\sqrt{2}\ \ln\left(\frac{x+\sqrt{2}\ \sqrt{x}+1}{x-\sqrt{2}\ \sqrt{x}+1}\right)}{128}+\frac{\frac{x^{\frac{5}{2}}}{16}-\frac{3\sqrt{x}}{16}}{\left(x^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2+1)^3,x)
```

```
[Out] 2*(1/32*x^(5/2)-3/32*x^(1/2))/(x^2+1)^2+3/128*2^(1/2)*ln((x+2^(1/2)*x^(1/2)
+1)/(x-2^(1/2)*x^(1/2)+1))+3/64*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)+3/64*2^(1
/2)*arctan(2^(1/2)*x^(1/2)-1)
```

maxima [A] time = 2.97, size = 97, normalized size = 0.75

$$\frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(x^(5/2) - 3*s
qrt(x))/(x^4 + 2*x^2 + 1)
```

mupad [B] time = 4.69, size = 62, normalized size = 0.48

$$-\frac{\frac{3\sqrt{x}}{16}-\frac{x^{5/2}}{16}}{x^4+2x^2+1}+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{64}+\frac{3}{64}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{64}-\frac{3}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2 + 1)^3,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/64 + 3i/64) + 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(3/64 - 3i/64) - ((3*x^(1/2))/16 - x^(5/2)/16)/
(2*x^2 + x^4 + 1)
```

sympy [B] time = 9.81, size = 481, normalized size = 3.73

$$\frac{8x^{\frac{5}{2}}}{128x^4+256x^2+128}-\frac{24\sqrt{x}}{128x^4+256x^2+128}-\frac{3\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{3\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+\right.}{128x^4+256x^2+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(x**2+1)**3,x)
```

```
[Out] 8*x**(5/2)/(128*x**4 + 256*x**2 + 128) - 24*sqrt(x)/(128*x**4 + 256*x**2 +
128) - 3*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**
```

```
2 + 128) + 3*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*
x**2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2
 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 1
28) - 6*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2
 + 128) + 6*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2
 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 +
128) - 3*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 + 1
28) + 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 + 128
) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128) + 6*sqr
t(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

3.332 $\int \frac{\sqrt{x}}{\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=129

$$\frac{5x^{3/2}}{16\left(x^2+1\right)}+\frac{x^{3/2}}{4\left(x^2+1\right)^2}+\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}\right.}{32\sqrt{}}$$

```
[Out] 1/4*x^(3/2)/(x^2+1)^2+5/16*x^(3/2)/(x^2+1)+5/64*arctan(-1+2^(1/2)*x^(1/2))*
2^(1/2)+5/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+5/128*ln(1+x-2^(1/2)*x^(1/2)
)*2^(1/2)-5/128*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5x^{3/2}}{16\left(x^2+1\right)}+\frac{x^{3/2}}{4\left(x^2+1\right)^2}+\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}\right.}{32\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^2)^3,x]
```

```
[Out] x^(3/2)/(4*(1 + x^2)^2) + (5*x^(3/2))/(16*(1 + x^2)) - (5*ArcTan[1 - Sqrt[2
]*Sqrt[x]])/(32*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (
5*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (5*Log[1 + Sqrt[2]*Sqrt[x] +
 x])/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\left(1+x^2\right)^3}\,dx &= \frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5}{8}\int \frac{\sqrt{x}}{\left(1+x^2\right)^2}\,dx\\
&= \frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5x^{3/2}}{16\left(1+x^2\right)}+\frac{5}{32}\int \frac{\sqrt{x}}{1+x^2}\,dx\\
&= \frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5x^{3/2}}{16\left(1+x^2\right)}+\frac{5}{16}\operatorname{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= \frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5x^{3/2}}{16\left(1+x^2\right)}-\frac{5}{32}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{5}{32}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= \frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5x^{3/2}}{16\left(1+x^2\right)}+\frac{5}{64}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{5}{64}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+
\right.\\
&= \frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5x^{3/2}}{16\left(1+x^2\right)}+\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}-\frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{5\operatorname{Subst}\left(\int \frac{1}{-1-
\right.}{}\\
&= \frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{5x^{3/2}}{16\left(1+x^2\right)}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}\sqrt{x}\right.}{64\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.17

$$\frac{2}{3}x^{3/2}\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^2)^3,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -x^2])/3
```

fricas [A] time = 0.68, size = 175, normalized size = 1.36

$$-\frac{20\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 5*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 5*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(5*x^3 + 9*x)*sqrt(x))/(x^4 + 2
*x^2 + 1)
```

giac [A] time = 0.59, size = 94, normalized size = 0.73

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(5*x^(7/2) + 9
*x^(3/2))/(x^2 + 1)^2
```

maple [A] time = 0.01, size = 86, normalized size = 0.67

$$\frac{x^{\frac{3}{2}}}{4\left(x^2+1\right)^2}+\frac{5x^{\frac{3}{2}}}{16\left(x^2+1\right)}+\frac{5\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)}{64}+\frac{5\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{64}+\frac{5\sqrt{2}\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1)^3,x)
```

```
[Out] 1/4*x^(3/2)/(x^2+1)^2+5/16/(x^2+1)*x^(3/2)+5/128*2^(1/2)*ln((x-2^(1/2)*x^(1
/2)+1)/(x+2^(1/2)*x^(1/2)+1))+5/64*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)+5/64*2
^(1/2)*arctan(2^(1/2)*x^(1/2)+1)
```

maxima [A] time = 3.02, size = 99, normalized size = 0.77

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
```

```
 x + 1) + 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(5*x^(7/2) + 9
*x^(3/2))/(x^4 + 2*x^2 + 1)
```

mupad [B] time = 0.04, size = 61, normalized size = 0.47

$$\frac{\frac{9\,x^{3/2}}{16}+\frac{5\,x^{7/2}}{16}}{x^4+2\,x^2+1}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{5}{64}-\frac{5}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{5}{64}+\frac{5}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2 + 1)^3,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(5/64 - 5i/64) + 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/64 + 5i/64) + ((9*x^(3/2))/16 + (5*x^(7/2))/
16)/(2*x^2 + x^4 + 1)
```

sympy [B] time = 6.38, size = 481, normalized size = 3.73

$$\frac{40x^{\frac{7}{2}}}{128x^4+256x^2+128}+\frac{72x^{\frac{3}{2}}}{128x^4+256x^2+128}+\frac{5\sqrt{2}\,x^4\log\left(-4\sqrt{2}\,\sqrt{x}\,+4x+4\right)}{128x^4+256x^2+128}-\frac{5\sqrt{2}\,x^4\log\left(4\sqrt{2}\,\sqrt{x}\,+4x+4\right)}{128x^4+256x^2+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1)**3,x)
```

```
[Out] 40*x**(7/2)/(128*x**4 + 256*x**2 + 128) + 72*x**(3/2)/(128*x**4 + 256*x**2
+ 128) + 5*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) - 5*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x
**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2
 + 128) + 10*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) - 10*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 +
 256*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 25
6*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x
**2 + 128) + 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x*
*2 + 128) - 5*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2
 + 128) + 10*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128)
+ 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

3.333 $\int \frac{1}{\sqrt{x}\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=129

$$\frac{7\sqrt{x}}{16\left(x^2+1\right)}+\frac{\sqrt{x}}{4\left(x^2+1\right)^2}-\frac{21\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{21\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{21\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{21\tan^{-1}}{}$$

```
[Out] 21/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+21/64*arctan(1+2^(1/2)*x^(1/2))*2^
(1/2)-21/128*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+21/128*ln(1+x+2^(1/2)*x^(1/2))
*2^(1/2)+1/4*x^(1/2)/(x^2+1)^2+7/16*x^(1/2)/(x^2+1)
```

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7\sqrt{x}}{16\left(x^2+1\right)}+\frac{\sqrt{x}}{4\left(x^2+1\right)^2}-\frac{21\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{21\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{21\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{21\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + x^2)^3),x]
```

```
[Out] Sqrt[x]/(4*(1 + x^2)^2) + (7*Sqrt[x])/(16*(1 + x^2)) - (21*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (21*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) -
 (21*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (21*Log[1 + Sqrt[2]*Sqrt[
x] + x])/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\left(1+x^2\right)^3}\,dx &= \frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{7}{8}\int \frac{1}{\sqrt{x}\left(1+x^2\right)^2}\,dx\\
&= \frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{7\sqrt{x}}{16\left(1+x^2\right)}+\frac{21}{32}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\
&= \frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{7\sqrt{x}}{16\left(1+x^2\right)}+\frac{21}{16}\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= \frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{7\sqrt{x}}{16\left(1+x^2\right)}+\frac{21}{32}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{21}{32}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\right.\\
&= \frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{7\sqrt{x}}{16\left(1+x^2\right)}+\frac{21}{64}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)+\frac{21}{64}\operatorname{Subst}\left(\int \frac{}{1+\sqrt{2}}\right.\\
&= \frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{7\sqrt{x}}{16\left(1+x^2\right)}-\frac{21\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{21\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{21\operatorname{Subs}}{}\\
&= \frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{7\sqrt{x}}{16\left(1+x^2\right)}-\frac{21\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{21\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{21\log\left(1-\right.}{64}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 121, normalized size = 0.94

$$\frac{1}{128}\left(\frac{56\sqrt{x}}{x^2+1}+\frac{32\sqrt{x}}{\left(x^2+1\right)^2}-21\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)+21\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)-42\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + x^2)^3),x]
```

```
[Out] ((32*Sqrt[x])/(1 + x^2)^2 + (56*Sqrt[x])/(1 + x^2) - 42*Sqrt[2]*ArcTan[1 -
Sqrt[2]*Sqrt[x]] + 42*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x]] - 21*Sqrt[2]*Log[
1 - Sqrt[2]*Sqrt[x] + x] + 21*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/128
```

fricas [A] time = 0.79, size = 173, normalized size = 1.34

$$-\frac{84\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+84\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/128*(84*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 84*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 21*sqrt(2)
*(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 21*sqrt(2)*(x^4 + 2*x
^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(7*x^2 + 11)*sqrt(x))/(x^4 +
2*x^2 + 1)
```

giac [A] time = 0.61, size = 94, normalized size = 0.73

$$\frac{21}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{21}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{21}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 21/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 21/64*sqrt(2)*arc
tan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 21/128*sqrt(2)*log(sqrt(2)*sqrt(x
) + x + 1) - 21/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(7*x^(5/2)
 + 11*sqrt(x))/(x^2 + 1)^2
```

maple [A] time = 0.01, size = 86, normalized size = 0.67

$$\frac{21\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{64}+\frac{21\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{64}+\frac{21\sqrt{2}\,\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)}{128}+\frac{\sqrt{x}}{4\left(x^2+1\right)^2}+\frac{7\sqrt{x}}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^3/x^(1/2),x)
```

```
[Out] 1/4*x^(1/2)/(x^2+1)^2+7/16/(x^2+1)*x^(1/2)+21/128*2^(1/2)*ln((x+2^(1/2)*x^(
1/2)+1)/(x-2^(1/2)*x^(1/2)+1))+21/64*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)+21/6
4*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)
```

maxima [A] time = 2.90, size = 99, normalized size = 0.77

$$\frac{21}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{21}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{21}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 21/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 21/64*sqrt(2)*arc
tan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 21/128*sqrt(2)*log(sqrt(2)*sqrt(x
) + x + 1) - 21/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(7*x^(5/2)
 + 11*sqrt(x))/(x^4 + 2*x^2 + 1)
```

mupad [B] time = 4.75, size = 61, normalized size = 0.47

$$\frac{\frac{11\,\sqrt{x}}{16}+\frac{7\,x^{5/2}}{16}}{x^4+2\,x^2+1}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{21}{64}+\frac{21}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{21}{64}-\frac{21}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^2 + 1)^3),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(21/64 + 21i/64) + 2^(1/2)*atan(
2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(21/64 - 21i/64) + ((11*x^(1/2))/16 + (7*x^(5
/2))/16)/(2*x^2 + x^4 + 1)
```

sympy [B] time = 8.46, size = 481, normalized size = 3.73

$$\frac{56x^{\frac{5}{2}}}{128x^4+256x^2+128}+\frac{88\sqrt{x}}{128x^4+256x^2+128}-\frac{21\sqrt{2}\,x^4\log\left(-4\sqrt{2}\,\sqrt{x}\,+4x+4\right)}{128x^4+256x^2+128}+\frac{21\sqrt{2}\,x^4\log\left(4\sqrt{2}\,\sqrt{x}\,+4x+4\right.}{128x^4+256x^2+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**3/x**(1/2),x)
```

```
[Out] 56*x**(5/2)/(128*x**4 + 256*x**2 + 128) + 88*sqrt(x)/(128*x**4 + 256*x**2 +
 128) - 21*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 21*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 2
56*x**2 + 128) + 42*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*
x**2 + 128) + 42*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**
2 + 128) - 42*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 42*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4
+ 256*x**2 + 128) + 84*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 2
56*x**2 + 128) + 84*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*
x**2 + 128) - 21*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*
x**2 + 128) + 21*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 42*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 12
8) + 42*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

3.334 $\int \frac{1}{x^{3/2}\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=138

$$\frac{9}{16\sqrt{x}\left(x^2+1\right)}+\frac{1}{4\sqrt{x}\left(x^2+1\right)^2}-\frac{45}{16\sqrt{x}}-\frac{45\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{45\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{45\tan^{-1}\left(1-\sqrt{2}\right.}{32\sqrt{2}}$$

```
[Out] -45/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-45/64*arctan(1+2^(1/2)*x^(1/2))*2
^(1/2)-45/128*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+45/128*ln(1+x+2^(1/2)*x^(1/2)
)*2^(1/2)-45/16/x^(1/2)+1/4/(x^2+1)^2/x^(1/2)+9/16/(x^2+1)/x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9}{16\sqrt{x}\left(x^2+1\right)}+\frac{1}{4\sqrt{x}\left(x^2+1\right)^2}-\frac{45}{16\sqrt{x}}-\frac{45\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{45\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{45\tan^{-1}\left(1-\sqrt{2}\right.}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(1 + x^2)^3),x]
```

```
[Out] -45/(16*Sqrt[x]) + 1/(4*Sqrt[x]*(1 + x^2)^2) + 9/(16*Sqrt[x]*(1 + x^2)) + (
45*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (45*ArcTan[1 + Sqrt[2]*Sqrt[
x]])/(32*Sqrt[2]) - (45*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (45*Lo
g[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^3}\,dx = \frac{1}{4\sqrt{x}\left(1+x^2\right)^2} + \frac{9}{8}\int \frac{1}{x^{3/2}\left(1+x^2\right)^2}\,dx$$
$$= \frac{1}{4\sqrt{x}\left(1+x^2\right)^2} + \frac{9}{16\sqrt{x}\left(1+x^2\right)} + \frac{45}{32}\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx$$
$$= -\frac{45}{16\sqrt{x}} + \frac{1}{4\sqrt{x}\left(1+x^2\right)^2} + \frac{9}{16\sqrt{x}\left(1+x^2\right)} - \frac{45}{32}\int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= -\frac{45}{16\sqrt{x}} + \frac{1}{4\sqrt{x}\left(1+x^2\right)^2} + \frac{9}{16\sqrt{x}\left(1+x^2\right)} - \frac{45}{16}\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{45}{16\sqrt{x}} + \frac{1}{4\sqrt{x}\left(1+x^2\right)^2} + \frac{9}{16\sqrt{x}\left(1+x^2\right)} + \frac{45}{32}\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) - \frac{45}{32}\,\mathrm{Sub}$$
$$= -\frac{45}{16\sqrt{x}} + \frac{1}{4\sqrt{x}\left(1+x^2\right)^2} + \frac{9}{16\sqrt{x}\left(1+x^2\right)} - \frac{45}{64}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right) -$$
$$= -\frac{45}{16\sqrt{x}} + \frac{1}{4\sqrt{x}\left(1+x^2\right)^2} + \frac{9}{16\sqrt{x}\left(1+x^2\right)} - \frac{45\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} + \frac{45\log\left(1+\sqrt{2}\right.}{64\sqrt{2}}$$
$$= -\frac{45}{16\sqrt{x}} + \frac{1}{4\sqrt{x}\left(1+x^2\right)^2} + \frac{9}{16\sqrt{x}\left(1+x^2\right)} + \frac{45\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} - \frac{45\tan^{-1}\left(1+\sqrt{}\right.}{32\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 20, normalized size = 0.14

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},3;\frac{3}{4};-x^2\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(1 + x^2)^3),x]
[Out] (-2*Hypergeometric2F1[-1/4, 3, 3/4, -x^2])/Sqrt[x]
```

fricas [A] time = 0.99, size = 178, normalized size = 1.29

$$\frac{180\sqrt{2}\left(x^5+2x^3+x\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+180\sqrt{2}\left(x^5+2x^3+x\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="fricas")
[Out] 1/128*(180*sqrt(2)*(x^5 + 2*x^3 + x)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 180*sqrt(2)*(x^5 + 2*x^3 + x)*arctan(1/2*sq
rt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 45*sqrt(2
)*(x^5 + 2*x^3 + x)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 45*sqrt(2)*(x^5 + 2*
x^3 + x)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(45*x^4 + 81*x^2 + 32)*sqrt(
x))/(x^5 + 2*x^3 + x)
```

giac [A] time = 0.63, size = 99, normalized size = 0.72

$$-\frac{45}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{45}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{45}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -45/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 45/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 45/128*sqrt(2)*log(sqrt(2)*sqrt(
x) + x + 1) - 45/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x) - 1/
16*(13*x^(7/2) + 17*x^(3/2))/(x^2 + 1)^2
```

maple [A] time = 0.01, size = 87, normalized size = 0.63

$$-\frac{45\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ -1\right)}{64}-\frac{45\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ +1\right)}{64}-\frac{45\sqrt{2}\ \ln\left(\frac{x-\sqrt{2}\ \sqrt{x}+1}{x+\sqrt{2}\ \sqrt{x}+1}\right)}{128}-\frac{2}{\sqrt{x}}-\frac{2\left(\frac{13x^{\frac{7}{2}}}{32}+\frac{17x^{\frac{3}{2}}}{32}\right)}{\left(x^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(x^2+1)^3,x)
```

```
[Out] -2*(13/32*x^(7/2)+17/32*x^(3/2))/(x^2+1)^2-45/64*2^(1/2)*arctan(2^(1/2)*x^(
1/2)-1)-45/128*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))-45/6
4*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)-2/x^(1/2)
```

maxima [A] time = 2.98, size = 102, normalized size = 0.74

$$-\frac{45}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{45}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{45}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{45}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -45/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 45/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 45/128*sqrt(2)*log(sqrt(2)*sqrt(
x) + x + 1) - 45/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(45*x^4 +
 81*x^2 + 32)/(x^(9/2) + 2*x^(5/2) + sqrt(x))
```

mupad [B] time = 4.72, size = 65, normalized size = 0.47

$$-\frac{\frac{45\,x^4}{16}+\frac{81\,x^2}{16}+2}{\sqrt{x}+2\,x^{5/2}+x^{9/2}}+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\ \left(-\frac{45}{64}+\frac{45}{64}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\ \left(-\frac{45}{64}-\frac{45}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(x^2 + 1)^3),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(45/64 - 45i/64) - 2^(1/2)*ata
n(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(45/64 + 45i/64) - ((81*x^2)/16 + (45*x^4)/
16 + 2)/(x^(1/2) + 2*x^(5/2) + x^(9/2))
```

sympy [B] time = 12.63, size = 653, normalized size = 4.73

$$-\frac{45\sqrt{2}\,x^{\frac{9}{2}}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}+\frac{45\sqrt{2}\,x^{\frac{9}{2}}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{90\sqrt{2}\,x^{\frac{9}{2}}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{90\sqrt{2}\,x^{\frac{9}{2}}\,\mathrm{at}}{128x^{\frac{9}{2}}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(x**2+1)**3,x)
```

```
[Out] -45*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**(9/2) + 256*
x**(5/2) + 128*sqrt(x)) + 45*sqrt(2)*x**(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x +
 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 90*sqrt(2)*x**(9/2)*atan(
sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 90*sqrt(
2)*x**(9/2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sq
rt(x)) - 90*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**(9/2
) + 256*x**(5/2) + 128*sqrt(x)) + 90*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 180*sqrt(2)*x**(5
/2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) -
 180*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**(9/2) + 256*x**(5/2
) + 128*sqrt(x)) - 45*sqrt(2)*sqrt(x)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(12
8*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) + 45*sqrt(2)*sqrt(x)*log(4*sqrt(2)
*sqrt(x) + 4*x + 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 90*sqrt(2
)*sqrt(x)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt
(x)) - 90*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**(9/2) + 256*x**
(5/2) + 128*sqrt(x)) - 360*x**4/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x))
 - 648*x**2/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 256/(128*x**(9/2)
 + 256*x**(5/2) + 128*sqrt(x))
```

3.335 $\int \frac{1}{x^{5/2}\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{77}{48x^{3/2}}+\frac{11}{16x^{3/2}\left(x^2+1\right)}+\frac{1}{4x^{3/2}\left(x^2+1\right)^2}+\frac{77\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{77\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{77\tan^{-1}\left(1-\sqrt{2}\right.}{32\sqrt{2}}$$

```
[Out] -77/48/x^(3/2)+1/4/x^(3/2)/(x^2+1)^2+11/16/x^(3/2)/(x^2+1)-77/64*arctan(-1+
2^(1/2)*x^(1/2))*2^(1/2)-77/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+77/128*ln(
1+x-2^(1/2)*x^(1/2))*2^(1/2)-77/128*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{11}{16x^{3/2}\left(x^2+1\right)}-\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(x^2+1\right)^2}+\frac{77\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{77\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{77\tan^{-1}\left(1-\sqrt{2}\sqrt{\right.}}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(1 + x^2)^3),x]
```

```
[Out] -77/(48*x^(3/2)) + 1/(4*x^(3/2)*(1 + x^2)^2) + 11/(16*x^(3/2)*(1 + x^2)) +
(77*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (77*ArcTan[1 + Sqrt[2]*Sqrt
[x]])/(32*Sqrt[2]) + (77*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (77*L
og[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/2}\left(1+x^2\right)^3}\,dx &= \frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{8}\int \frac{1}{x^{5/2}\left(1+x^2\right)^2}\,dx \\
&= \frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)}+\frac{77}{32}\int \frac{1}{x^{5/2}\left(1+x^2\right)}\,dx \\
&= -\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)}-\frac{77}{32}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx \\
&= -\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)}-\frac{77}{16}\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right) \\
&= -\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)}-\frac{77}{32}\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\frac{77}{32}\mathrm{Subst} \\
&= -\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)}-\frac{77}{64}\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)-\frac{77}{64} \\
&= -\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)}+\frac{77\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}-\frac{77\log\left(1+\sqrt{2}\right.}{64\sqrt{2}} \\
&= -\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)}+\frac{77\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{77\tan^{-1}\left(1+\sqrt{2}\right.}{32\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.16

$$
-\frac{2\,{}_2F_1\left(-\frac{3}{4},3;\frac{1}{4};-x^2\right)}{3x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(1 + x^2)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 3, 1/4, -x^2])/(3*x^(3/2))
```

fricas [B] time = 0.94, size = 188, normalized size = 1.36

$$
\frac{924\sqrt{2}\left(x^6+2x^4+x^2\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+924\sqrt{2}\left(x^6+2x^4+x^2\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(924*sqrt(2)*(x^6 + 2*x^4 + x^2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x)
+ x + 1) - sqrt(2)*sqrt(x) - 1) + 924*sqrt(2)*(x^6 + 2*x^4 + x^2)*arctan(1/
2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 231*s
qrt(2)*(x^6 + 2*x^4 + x^2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 231*sqrt(2)*(
x^6 + 2*x^4 + x^2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(77*x^4 + 121*x^2
+ 32)*sqrt(x))/(x^6 + 2*x^4 + x^2)
```

giac [A] time = 0.62, size = 99, normalized size = 0.72

$$
-\frac{77}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{77}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{77}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{77}{128}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -77/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 77/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 77/128*sqrt(2)*log(sqrt(2)*sqrt(
x) + x + 1) + 77/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(15*x^(5/
2) + 19*sqrt(x))/(x^2 + 1)^2 - 2/3/x^(3/2)
```

maple [A] time = 0.02, size = 87, normalized size = 0.63

$$-\frac{77\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ -1\right)}{64}-\frac{77\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ +1\right)}{64}-\frac{77\sqrt{2}\ \ln\left(\frac{x+\sqrt{2}\ \sqrt{x}+1}{x-\sqrt{2}\ \sqrt{x}+1}\right)}{128}-\frac{2}{3x^{\frac{3}{2}}}-\frac{2\left(\frac{15x^{\frac{5}{2}}}{32}+\frac{19\sqrt{x}}{32}\right)}{\left(x^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(x^2+1)^3,x)
```

```
[Out] -2*(15/32*x^(5/2)+19/32*x^(1/2))/(x^2+1)^2-77/64*2^(1/2)*arctan(2^(1/2)*x^(
1/2)-1)-77/128*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))-77/6
4*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)-2/3/x^(3/2)
```

maxima [A] time = 2.85, size = 102, normalized size = 0.74

$$-\frac{77}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{77}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{77}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -77/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 77/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 77/128*sqrt(2)*log(sqrt(2)*sqrt(
x) + x + 1) + 77/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/48*(77*x^4 +
 121*x^2 + 32)/(x^(11/2) + 2*x^(7/2) + x^(3/2))
```

mupad [B] time = 4.72, size = 65, normalized size = 0.47

$$-\frac{\frac{77\,x^4}{48}+\frac{121\,x^2}{48}+\frac{2}{3}}{x^{3/2}+2\,x^{7/2}+x^{11/2}}+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{77}{64}-\frac{77}{64}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{77}{64}+\frac{77}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(x^2 + 1)^3),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(77/64 + 77i/64) - 2^(1/2)*ata
n(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(77/64 - 77i/64) - ((121*x^2)/48 + (77*x^4)
/48 + 2/3)/(x^(3/2) + 2*x^(7/2) + x^(11/2))
```

sympy [B] time = 22.42, size = 653, normalized size = 4.73

$$\frac{231\sqrt{2}\,x^{\frac{11}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{231\sqrt{2}\,x^{\frac{11}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{462\sqrt{2}\,x^{\frac{11}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{462}{38}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(x**2+1)**3,x)
```

```
[Out] 231*sqrt(2)*x**(11/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 76
8*x**(7/2) + 384*x**(3/2)) - 231*sqrt(2)*x**(11/2)*log(4*sqrt(2)*sqrt(x) +
4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(11
/2)*atan(sqrt(2)*sqrt(x) - 1)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2))
 - 462*sqrt(2)*x**(11/2)*atan(sqrt(2)*sqrt(x) + 1)/(384*x**(11/2) + 768*x**
(7/2) + 384*x**(3/2)) + 462*sqrt(2)*x**(7/2)*log(-4*sqrt(2)*sqrt(x) + 4*x +
 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(7/2)*lo
g(4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)
) - 924*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) - 1)/(384*x**(11/2) + 768*x**
(7/2) + 384*x**(3/2)) - 924*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) + 1)/(384
*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) + 231*sqrt(2)*x**(3/2)*log(-4*sqr
t(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 231
*sqrt(2)*x**(3/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 768*x**
(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(3/2)*atan(sqrt(2)*sqrt(x) - 1)/(384
*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(3/2)*atan(sqrt(
2)*sqrt(x) + 1)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 616*x**4/(3
84*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 968*x**2/(384*x**(11/2) + 768
*x**(7/2) + 384*x**(3/2)) - 256/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2
))
```

3.336 $\int \frac{1}{x^{7/2}(1+x^2)^3}\,dx$

Optimal. Leaf size=147

$$-\frac{117}{80x^{5/2}}+\frac{13}{16x^{5/2}\left(x^2+1\right)}+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}+\frac{117}{16\sqrt{x}}+\frac{117\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{117\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{117\mathrm{t}}{}$$

```
[Out] -117/80/x^(5/2)+1/4/x^(5/2)/(x^2+1)^2+13/16/x^(5/2)/(x^2+1)+117/64*arctan(-
1+2^(1/2)*x^(1/2))*2^(1/2)+117/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+117/128
*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)-117/128*ln(1+x+2^(1/2)*x^(1/2))*2^(1/2)+11
7/16/x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{13}{16x^{5/2}\left(x^2+1\right)}+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}-\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{117\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{117\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{117\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(1 + x^2)^3),x]
```

```
[Out] -117/(80*x^(5/2)) + 117/(16*Sqrt[x]) + 1/(4*x^(5/2)*(1 + x^2)^2) + 13/(16*x
^(5/2)*(1 + x^2)) - (117*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (117*A
rcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (117*Log[1 - Sqrt[2]*Sqrt[x] + x
])/(64*Sqrt[2]) - (117*Log[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}\left(1+x^2\right)^3}\,dx &= \frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{8}\int \frac{1}{x^{7/2}\left(1+x^2\right)^2}\,dx\\
&= \frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117}{32}\int \frac{1}{x^{7/2}\left(1+x^2\right)}\,dx\\
&= -\frac{117}{80x^{5/2}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}-\frac{117}{32}\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117}{32}\int \frac{\sqrt{x}}{1+x^2}\,dx\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117}{16}\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right.\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}-\frac{117}{32}\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right.\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117}{64}\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx\right.\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}-\frac{117}{}\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}-\frac{117\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{117\,\mathrm{ta}}{}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.15

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},3;-\frac{1}{4};-x^2\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(1 + x^2)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 3, -1/4, -x^2])/(5*x^(5/2))
```

fricas [A] time = 0.72, size = 193, normalized size = 1.31

$$-\frac{2340\sqrt{2}\left(x^7+2x^5+x^3\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+2340\sqrt{2}\left(x^7+2x^5+x^3\right)\arctan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/640*(2340*sqrt(2)*(x^7 + 2*x^5 + x^3)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x
) + x + 1) - sqrt(2)*sqrt(x) - 1) + 2340*sqrt(2)*(x^7 + 2*x^5 + x^3)*arctan
(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 58
5*sqrt(2)*(x^7 + 2*x^5 + x^3)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 585*sqrt(2
)*(x^7 + 2*x^5 + x^3)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(585*x^6 + 1053
*x^4 + 416*x^2 - 32)*sqrt(x))/(x^7 + 2*x^5 + x^3)
```

giac [A] time = 0.64, size = 106, normalized size = 0.72

$$\frac{117}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{117}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{117}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{11}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 117/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 117/64*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 117/128*sqrt(2)*log(sqrt(2)*sqr
t(x) + x + 1) + 117/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(21*x^
(7/2) + 25*x^(3/2))/(x^2 + 1)^2 + 2/5*(15*x^2 - 1)/x^(5/2)
```

maple [A] time = 0.01, size = 92, normalized size = 0.63

$$\frac{117\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{64}+\frac{117\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{64}+\frac{117\sqrt{2}\,\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)}{128}+\frac{6}{\sqrt{x}}-\frac{2}{5x^{\frac{5}{2}}}+\frac{\frac{21x^{\frac{7}{2}}}{16}+\frac{25x^{\frac{3}{2}}}{16}}{\left(x^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^2+1)^3,x)
```

```
[Out] 2*(21/32*x^(7/2)+25/32*x^(3/2))/(x^2+1)^2+117/128*2^(1/2)*ln((x-2^(1/2)*x^(
1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+117/64*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)+117
/64*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)-2/5/x^(5/2)+6/x^(1/2)
```

maxima [A] time = 2.95, size = 107, normalized size = 0.73

$$\frac{117}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{117}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{117}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{11}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 117/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 117/64*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 117/128*sqrt(2)*log(sqrt(2)*sqr
t(x) + x + 1) + 117/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/80*(585*x
^6 + 1053*x^4 + 416*x^2 - 32)/(x^(13/2) + 2*x^(9/2) + x^(5/2))
```

mupad [B] time = 0.07, size = 69, normalized size = 0.47

$$\frac{\frac{117\,x^6}{16}+\frac{1053\,x^4}{80}+\frac{26\,x^2}{5}-\frac{2}{5}}{x^{5/2}+2\,x^{9/2}+x^{13/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{117}{64}-\frac{117}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{117}{64}+\frac{117}{64}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(x^2 + 1)^3),x)
```

```
[Out] ((26*x^2)/5 + (1053*x^4)/80 + (117*x^6)/16 - 2/5)/(x^(5/2) + 2*x^(9/2) + x^
(13/2)) + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(117/64 - 117i/64) + 2
^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(117/64 + 117i/64)
```

sympy [B] time = 51.16, size = 678, normalized size = 4.61

$$\frac{585\sqrt{2}\,x^{\frac{13}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}-\frac{585\sqrt{2}\,x^{\frac{13}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{1170\sqrt{2}\,x^{\frac{13}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{1170}{640}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(x**2+1)**3,x)
```

```
[Out] 585*sqrt(2)*x**(13/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 12
80*x**(9/2) + 640*x**(5/2)) - 585*sqrt(2)*x**(13/2)*log(4*sqrt(2)*sqrt(x) +
 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*x**
(13/2)*atan(sqrt(2)*sqrt(x) - 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5
/2)) + 1170*sqrt(2)*x**(13/2)*atan(sqrt(2)*sqrt(x) + 1)/(640*x**(13/2) + 12
80*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sqrt(x)
+ 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) - 1170*sqrt(2)*x*
*(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 64
0*x**(5/2)) + 2340*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) - 1)/(640*x**(13/2
) + 1280*x**(9/2) + 640*x**(5/2)) + 2340*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt
(x) + 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 585*sqrt(2)*x**(5
/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*
x**(5/2)) - 585*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(
13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*x**(5/2)*atan(sqrt(2)*
sqrt(x) - 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*
x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**
(5/2)) + 4680*x**6/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 8424*x*
*4/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 3328*x**2/(640*x**(13/2
) + 1280*x**(9/2) + 640*x**(5/2)) - 256/(640*x**(13/2) + 1280*x**(9/2) + 64
0*x**(5/2))
```

3.337 $\int \frac{\sqrt{x}}{1-x^2} \, dx$

Optimal. Leaf size=15

$$\tanh^{-1}\left(\sqrt{x}\right)-\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -arctan(x^(1/2))+arctanh(x^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {329, 298, 203, 206}

$$\tanh^{-1}\left(\sqrt{x}\right)-\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 - x^2),x]
```

```
[Out] -ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{1-x^2} \, dx &= 2 \operatorname{Subst}\left(\int \frac{x^2}{1-x^4} \, dx, x, \sqrt{x}\right)\\
&= \operatorname{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, \sqrt{x}\right)-\operatorname{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \sqrt{x}\right)\\
&= -\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\tanh^{-1}\left(\sqrt{x}\right)-\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 - x^2),x]
[Out] -ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

fricas [B] time = 0.94, size = 23, normalized size = 1.53

$$-\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^2+1),x, algorithm="fricas")
[Out] -arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

giac [B] time = 0.62, size = 24, normalized size = 1.60

$$-\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^2+1),x, algorithm="giac")
[Out] -arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

maple [B] time = 0.01, size = 24, normalized size = 1.60

$$-\arctan\left(\sqrt{x}\right)-\frac{\ln\left(\sqrt{x}-1\right)}{2}+\frac{\ln\left(\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-x^2+1),x)
[Out] -1/2*ln(-1+x^(1/2))+1/2*ln(1+x^(1/2))-arctan(x^(1/2))
```

maxima [B] time = 2.97, size = 23, normalized size = 1.53

$$-\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^2+1),x, algorithm="maxima")
[Out] -arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

mupad [B] time = 0.03, size = 11, normalized size = 0.73

$$\operatorname{atanh}\left(\sqrt{x}\right)-\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(1/2)/(x^2 - 1),x)
[Out] atanh(x^(1/2)) - atan(x^(1/2))
```

sympy [B] time = 0.29, size = 26, normalized size = 1.73

$$-\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}-\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-x**2+1),x)
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 - atan(sqrt(x))
```

3.338 $\int \frac{x^{2/3}}{1+x^2} dx$

Optimal. Leaf size=73

$$-\frac{1}{2}\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x}}{x^{2/3}+1}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{x}\right)+\frac{1}{2}\tan^{-1}\left(2\sqrt[3]{x}+\sqrt{3}\right)+\tan^{-1}\left(\sqrt[3]{x}\right)$$

```
[Out] arctan(x^(1/3))+1/2*arctan(2*x^(1/3)-3^(1/2))+1/2*arctan(2*x^(1/3)+3^(1/2))
-1/2*arctanh(x^(1/3)*3^(1/2)/(1+x^(2/3)))*3^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 100, normalized size of antiderivative = 1.37, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {329, 295, 634, 618, 204, 628, 203}

$$\frac{1}{4}\sqrt{3}\log\left(x^{2/3}-\sqrt{3}\sqrt[3]{x}+1\right)-\frac{1}{4}\sqrt{3}\log\left(x^{2/3}+\sqrt{3}\sqrt[3]{x}+1\right)-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{x}\right)+\frac{1}{2}\tan^{-1}\left(2\sqrt[3]{x}+\sqrt{3}\right)+\tan^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(1 + x^2),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x^(1/3)]/2 + ArcTan[Sqrt[3] + 2*x^(1/3)]/2 + ArcTan[x^(
1/3)] + (Sqrt[3]*Log[1 - Sqrt[3]*x^(1/3) + x^(2/3)])/4 - (Sqrt[3]*Log[1 + S
qrt[3]*x^(1/3) + x^(2/3)])/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{2/3}}{1+x^2}\,dx &= 3\,\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \sqrt[3]{x}\right)\\ &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[3]{x}\right) + \text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \sqrt[3]{x}\right) + \text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\right.\\ &= \tan^{-1}\left(\sqrt[3]{x}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \sqrt[3]{x}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, \sqrt[3]{x}\right) +\\ &= \tan^{-1}\left(\sqrt[3]{x}\right) + \frac{1}{4}\sqrt{3}\log\left(1-\sqrt{3}\sqrt[3]{x}+x^{2/3}\right) - \frac{1}{4}\sqrt{3}\log\left(1+\sqrt{3}\sqrt[3]{x}+x^{2/3}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1}\right.\\ &= -\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{x}\right) + \frac{1}{2}\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{x}\right) + \tan^{-1}\left(\sqrt[3]{x}\right) + \frac{1}{4}\sqrt{3}\log\left(1-\sqrt{3}\sqrt[3]{x}+x^{2/3}\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.30

$$\frac{3}{5}x^{5/3}\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(1 + x^2),x]
```

```
[Out] (3*x^(5/3)*Hypergeometric2F1[5/6, 1, 11/6, -x^2])/5
```

fricas [B] time = 0.92, size = 108, normalized size = 1.48

$$-\frac{1}{4}\sqrt{3}\log\left(16\sqrt{3}x^{\frac{1}{3}}+16x^{\frac{2}{3}}+16\right)+\frac{1}{4}\sqrt{3}\log\left(-16\sqrt{3}x^{\frac{1}{3}}+16x^{\frac{2}{3}}+16\right)-\arctan\left(\sqrt{3}+\frac{1}{2}\sqrt{-16\sqrt{3}x^{\frac{1}{3}}+16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*log(16*sqrt(3)*x^(1/3) + 16*x^(2/3) + 16) + 1/4*sqrt(3)*log(-1
6*sqrt(3)*x^(1/3) + 16*x^(2/3) + 16) - arctan(sqrt(3) + 1/2*sqrt(-16*sqrt(3
)*x^(1/3) + 16*x^(2/3) + 16) - 2*x^(1/3)) - arctan(-sqrt(3) + 2*sqrt(sqrt(3
)*x^(1/3) + x^(2/3) + 1) - 2*x^(1/3)) + arctan(x^(1/3))
```

giac [A] time = 0.63, size = 68, normalized size = 0.93

$$-\frac{1}{4}\sqrt{3}\log\left(\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{4}\sqrt{3}\log\left(-\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{2}\arctan\left(\sqrt{3}+2x^{\frac{1}{3}}\right)+\frac{1}{2}\arctan\left(-\sqrt{3}+2x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*log(sqrt(3)*x^(1/3) + x^(2/3) + 1) + 1/4*sqrt(3)*log(-sqrt(3)*
x^(1/3) + x^(2/3) + 1) + 1/2*arctan(sqrt(3) + 2*x^(1/3)) + 1/2*arctan(-sqrt
(3) + 2*x^(1/3)) + arctan(x^(1/3))
```

maple [A] time = 0.05, size = 69, normalized size = 0.95

$$\arctan\left(x^{\frac{1}{3}}\right)+\frac{\arctan\left(2x^{\frac{1}{3}}-\sqrt{3}\right)}{2}+\frac{\arctan\left(2x^{\frac{1}{3}}+\sqrt{3}\right)}{2}+\frac{\sqrt{3}\,\ln\left(x^{\frac{2}{3}}-\sqrt{3}\,x^{\frac{1}{3}}+1\right)}{4}-\frac{\sqrt{3}\,\ln\left(x^{\frac{2}{3}}+\sqrt{3}\,x^{\frac{1}{3}}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(x^2+1),x)
```

```
[Out] arctan(x^(1/3))+1/2*arctan(2*x^(1/3)-3^(1/2))+1/2*arctan(2*x^(1/3)+3^(1/2))
+1/4*ln(1+x^(2/3)-x^(1/3)*3^(1/2))*3^(1/2)-1/4*ln(1+x^(2/3)+x^(1/3)*3^(1/2)
)*3^(1/2)
```

maxima [A] time = 2.97, size = 68, normalized size = 0.93

$$-\frac{1}{4}\sqrt{3}\log\left(\sqrt{3}\,x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{4}\sqrt{3}\log\left(-\sqrt{3}\,x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{2}\arctan\left(\sqrt{3}+2\,x^{\frac{1}{3}}\right)+\frac{1}{2}\arctan\left(-\sqrt{3}+2\,x^{\frac{1}{3}}\right)+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(3)*log(sqrt(3)*x^(1/3) + x^(2/3) + 1) + 1/4*sqrt(3)*log(-sqrt(3)*
x^(1/3) + x^(2/3) + 1) + 1/2*arctan(sqrt(3) + 2*x^(1/3)) + 1/2*arctan(-sqrt
(3) + 2*x^(1/3)) + arctan(x^(1/3))
```

mupad [B] time = 4.77, size = 57, normalized size = 0.78

$$\mathrm{atan}\left(x^{1/3}\right)-\mathrm{atan}\left(\frac{486\,x^{1/3}}{-243+\sqrt{3}\,243\mathrm{i}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\mathrm{atan}\left(\frac{486\,x^{1/3}}{243+\sqrt{3}\,243\mathrm{i}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(x^2 + 1),x)
```

```
[Out] atan(x^(1/3)) - atan((486*x^(1/3))/(3^(1/2)*243i - 243))*((3^(1/2)*1i)/2 +
1/2) - atan((486*x^(1/3))/(3^(1/2)*243i + 243))*((3^(1/2)*1i)/2 - 1/2)
```

sympy [A] time = 1.78, size = 94, normalized size = 1.29

$$\frac{\sqrt{3}\log\left(4x^{\frac{2}{3}}-4\sqrt{3}\sqrt[3]{x}+4\right)}{4}-\frac{\sqrt{3}\log\left(4x^{\frac{2}{3}}+4\sqrt{3}\sqrt[3]{x}+4\right)}{4}+\mathrm{atan}\left(\sqrt[3]{x}\right)+\frac{\mathrm{atan}\left(2\sqrt[3]{x}-\sqrt{3}\right)}{2}+\frac{\mathrm{atan}\left(2\sqrt[3]{x}+\sqrt{3}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(x**2+1),x)
```

```
[Out] sqrt(3)*log(4*x**(2/3) - 4*sqrt(3)*x**(1/3) + 4)/4 - sqrt(3)*log(4*x**(2/3)
 + 4*sqrt(3)*x**(1/3) + 4)/4 + atan(x**(1/3)) + atan(2*x**(1/3) - sqrt(3))/
2 + atan(2*x**(1/3) + sqrt(3))/2
```

3.339 $\int x^m \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=97

$$\frac{a^5x^{m+1}}{m+1}+\frac{5a^4bx^{m+3}}{m+3}+\frac{10a^3b^2x^{m+5}}{m+5}+\frac{10a^2b^3x^{m+7}}{m+7}+\frac{5ab^4x^{m+9}}{m+9}+\frac{b^5x^{m+11}}{m+11}$$

```
[Out] a^5*x^(1+m)/(1+m)+5*a^4*b*x^(3+m)/(3+m)+10*a^3*b^2*x^(5+m)/(5+m)+10*a^2*b^3
*x^(7+m)/(7+m)+5*a*b^4*x^(9+m)/(9+m)+b^5*x^(11+m)/(11+m)
```

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10a^3b^2x^{m+5}}{m+5}+\frac{10a^2b^3x^{m+7}}{m+7}+\frac{5a^4bx^{m+3}}{m+3}+\frac{a^5x^{m+1}}{m+1}+\frac{5ab^4x^{m+9}}{m+9}+\frac{b^5x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^(1 + m))/(1 + m) + (5*a^4*b*x^(3 + m))/(3 + m) + (10*a^3*b^2*x^(5 +
m))/(5 + m) + (10*a^2*b^3*x^(7 + m))/(7 + m) + (5*a*b^4*x^(9 + m))/(9 + m)
+ (b^5*x^(11 + m))/(11 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^5 \, dx &= \int \left(a^5x^m+5a^4bx^{2+m}+10a^3b^2x^{4+m}+10a^2b^3x^{6+m}+5ab^4x^{8+m}+b^5x^{10+m}\right) dx\\ &= \frac{a^5x^{1+m}}{1+m}+\frac{5a^4bx^{3+m}}{3+m}+\frac{10a^3b^2x^{5+m}}{5+m}+\frac{10a^2b^3x^{7+m}}{7+m}+\frac{5ab^4x^{9+m}}{9+m}+\frac{b^5x^{11+m}}{11+m}\end{aligned}$$

Mathematica [A] time = 0.06, size = 88, normalized size = 0.91

$$x^{m+1}\left(\frac{a^5}{m+1}+\frac{5a^4bx^2}{m+3}+\frac{10a^3b^2x^4}{m+5}+\frac{10a^2b^3x^6}{m+7}+\frac{5ab^4x^8}{m+9}+\frac{b^5x^{10}}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^5,x]
```

```
[Out] x^(1 + m)*(a^5/(1 + m) + (5*a^4*b*x^2)/(3 + m) + (10*a^3*b^2*x^4)/(5 + m) +
 (10*a^2*b^3*x^6)/(7 + m) + (5*a*b^4*x^8)/(9 + m) + (b^5*x^10)/(11 + m))
```

fricas [B] time = 0.94, size = 367, normalized size = 3.78

$$\frac{\left(\left(b^5m^5+25\,b^5m^4+230\,b^5m^3+950\,b^5m^2+1689\,b^5m+945\,b^5\right)x^{11}+5\left(ab^4m^5+27\,ab^4m^4+262\,ab^4m^3+11\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 25*b^5*m^4 + 230*b^5*m^3 + 950*b^5*m^2 + 1689*b^5*m + 945*b^5)*
x^11 + 5*(a*b^4*m^5 + 27*a*b^4*m^4 + 262*a*b^4*m^3 + 1122*a*b^4*m^2 + 2041*
a*b^4*m + 1155*a*b^4)*x^9 + 10*(a^2*b^3*m^5 + 29*a^2*b^3*m^4 + 302*a^2*b^3*
m^3 + 1366*a^2*b^3*m^2 + 2577*a^2*b^3*m + 1485*a^2*b^3)*x^7 + 10*(a^3*b^2*m
^5 + 31*a^3*b^2*m^4 + 350*a^3*b^2*m^3 + 1730*a^3*b^2*m^2 + 3489*a^3*b^2*m +
 2079*a^3*b^2)*x^5 + 5*(a^4*b*m^5 + 33*a^4*b*m^4 + 406*a^4*b*m^3 + 2262*a^4
*b*m^2 + 5353*a^4*b*m + 3465*a^4*b)*x^3 + (a^5*m^5 + 35*a^5*m^4 + 470*a^5*m
^3 + 3010*a^5*m^2 + 9129*a^5*m + 10395*a^5)*x)*x^m/(m^6 + 36*m^5 + 505*m^4
+ 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

giac [B] time = 0.69, size = 540, normalized size = 5.57

$$\frac{b^5m^5x^{11}x^m + 25\,b^5m^4x^{11}x^m + 5\,ab^4m^5x^9x^m + 230\,b^5m^3x^{11}x^m + 135\,ab^4m^4x^9x^m + 950\,b^5m^2x^{11}x^m + 10\,a^2b^3m^5x^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] (b^5*m^5*x^11*x^m + 25*b^5*m^4*x^11*x^m + 5*a*b^4*m^5*x^9*x^m + 230*b^5*m^3
*x^11*x^m + 135*a*b^4*m^4*x^9*x^m + 950*b^5*m^2*x^11*x^m + 10*a^2*b^3*m^5*x
^7*x^m + 1310*a*b^4*m^3*x^9*x^m + 1689*b^5*m*x^11*x^m + 290*a^2*b^3*m^4*x^7
*x^m + 5610*a*b^4*m^2*x^9*x^m + 945*b^5*x^11*x^m + 10*a^3*b^2*m^5*x^5*x^m +
 3020*a^2*b^3*m^3*x^7*x^m + 10205*a*b^4*m*x^9*x^m + 310*a^3*b^2*m^4*x^5*x^m
 + 13660*a^2*b^3*m^2*x^7*x^m + 5775*a*b^4*x^9*x^m + 5*a^4*b*m^5*x^3*x^m + 3
500*a^3*b^2*m^3*x^5*x^m + 25770*a^2*b^3*m*x^7*x^m + 165*a^4*b*m^4*x^3*x^m +
 17300*a^3*b^2*m^2*x^5*x^m + 14850*a^2*b^3*x^7*x^m + a^5*m^5*x*x^m + 2030*a
^4*b*m^3*x^3*x^m + 34890*a^3*b^2*m*x^5*x^m + 35*a^5*m^4*x*x^m + 11310*a^4*b
*m^2*x^3*x^m + 20790*a^3*b^2*x^5*x^m + 470*a^5*m^3*x*x^m + 26765*a^4*b*m*x^
3*x^m + 3010*a^5*m^2*x*x^m + 17325*a^4*b*x^3*x^m + 9129*a^5*m*x*x^m + 10395
*a^5*x*x^m)/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 1039
5)
```

maple [B] time = 0.01, size = 432, normalized size = 4.45

$$\frac{\left(b^5m^5x^{10} + 25b^5m^4x^{10} + 5a\,b^4m^5x^8 + 230b^5m^3x^{10} + 135a\,b^4m^4x^8 + 950b^5m^2x^{10} + 10a^2b^3m^5x^6 + 1310a\,b^4m^3x^8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^5,x)
```

```
[Out] x^(m+1)*(b^5*m^5*x^10+25*b^5*m^4*x^10+5*a*b^4*m^5*x^8+230*b^5*m^3*x^10+135*
a*b^4*m^4*x^8+950*b^5*m^2*x^10+10*a^2*b^3*m^5*x^6+1310*a*b^4*m^3*x^8+1689*b
^5*m*x^10+290*a^2*b^3*m^4*x^6+5610*a*b^4*m^2*x^8+945*b^5*x^10+10*a^3*b^2*m^
5*x^4+3020*a^2*b^3*m^3*x^6+10205*a*b^4*m*x^8+310*a^3*b^2*m^4*x^4+13660*a^2*
b^3*m^2*x^6+5775*a*b^4*x^8+5*a^4*b*m^5*x^2+3500*a^3*b^2*m^3*x^4+25770*a^2*b
^3*m*x^6+165*a^4*b*m^4*x^2+17300*a^3*b^2*m^2*x^4+14850*a^2*b^3*x^6+a^5*m^5+
2030*a^4*b*m^3*x^2+34890*a^3*b^2*m*x^4+35*a^5*m^4+11310*a^4*b*m^2*x^2+20790
*a^3*b^2*x^4+470*a^5*m^3+26765*a^4*b*m*x^2+3010*a^5*m^2+17325*a^4*b*x^2+912
9*a^5*m+10395*a^5)/(11+m)/(9+m)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.33, size = 97, normalized size = 1.00

$$\frac{b^5x^{m+11}}{m+11} + \frac{5\,ab^4x^{m+9}}{m+9} + \frac{10\,a^2b^3x^{m+7}}{m+7} + \frac{10\,a^3b^2x^{m+5}}{m+5} + \frac{5\,a^4bx^{m+3}}{m+3} + \frac{a^5x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] b^5*x^(m + 11)/(m + 11) + 5*a*b^4*x^(m + 9)/(m + 9) + 10*a^2*b^3*x^(m + 7)/
(m + 7) + 10*a^3*b^2*x^(m + 5)/(m + 5) + 5*a^4*b*x^(m + 3)/(m + 3) + a^5*x^
(m + 1)/(m + 1)
```

mupad [B] time = 5.11, size = 389, normalized size = 4.01

$$\frac{a^5\,x\,x^m\,\left(m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{b^5\,x^m\,x^{11}\,\left(m^5+25\,m^4+230\,m^3+950\,m\right.}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x*x^m*(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395))/(19524*m +
 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (b^5*x^m*x^11*(16
89*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945))/(19524*m + 12139*m^2 + 3480
*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (5*a*b^4*x^m*x^9*(2041*m + 1122*m^
2 + 262*m^3 + 27*m^4 + m^5 + 1155))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m
^4 + 36*m^5 + m^6 + 10395) + (5*a^4*b*x^m*x^3*(5353*m + 2262*m^2 + 406*m^3
+ 33*m^4 + m^5 + 3465))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5
+ m^6 + 10395) + (10*a^2*b^3*x^m*x^7*(2577*m + 1366*m^2 + 302*m^3 + 29*m^4
+ m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 1
0395) + (10*a^3*b^2*x^m*x^5*(3489*m + 1730*m^2 + 350*m^3 + 31*m^4 + m^5 + 2
079))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395)
```

sympy [A] time = 4.50, size = 1999, normalized size = 20.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**5,x)
```

```
[Out] Piecewise((-a**5/(10*x**10) - 5*a**4*b/(8*x**8) - 5*a**3*b**2/(3*x**6) - 5*
a**2*b**3/(2*x**4) - 5*a*b**4/(2*x**2) + b**5*log(x), Eq(m, -11)), (-a**5/(
8*x**8) - 5*a**4*b/(6*x**6) - 5*a**3*b**2/(2*x**4) - 5*a**2*b**3/x**2 + 5*a
*b**4*log(x) + b**5*x**2/2, Eq(m, -9)), (-a**5/(6*x**6) - 5*a**4*b/(4*x**4)
 - 5*a**3*b**2/x**2 + 10*a**2*b**3*log(x) + 5*a*b**4*x**2/2 + b**5*x**4/4,
Eq(m, -7)), (-a**5/(4*x**4) - 5*a**4*b/(2*x**2) + 10*a**3*b**2*log(x) + 5*a
**2*b**3*x**2 + 5*a*b**4*x**4/4 + b**5*x**6/6, Eq(m, -5)), (-a**5/(2*x**2)
+ 5*a**4*b*log(x) + 5*a**3*b**2*x**2 + 5*a**2*b**3*x**4/2 + 5*a*b**4*x**6/6
 + b**5*x**8/8, Eq(m, -3)), (a**5*log(x) + 5*a**4*b*x**2/2 + 5*a**3*b**2*x*
*4/2 + 5*a**2*b**3*x**6/3 + 5*a*b**4*x**8/8 + b**5*x**10/10, Eq(m, -1)), (a
**5*m**5*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 35*a**5*m**4*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 470*a**5*m**3*x*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3010*a**5*m**2*x*x**m/(m
**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 9129
*a**5*m*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*
m + 10395) + 10395*a**5*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 5*a**4*b*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 165*a**4*b*m**4*x**3*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
2030*a**4*b*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
**2 + 19524*m + 10395) + 11310*a**4*b*m**2*x**3*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 26765*a**4*b*m*x**3*x**m
/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1
7325*a**4*b*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
 19524*m + 10395) + 10*a**3*b**2*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4
+ 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 310*a**3*b**2*m**4*x**5*x**m/
(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 35
```

```
00*a**3*b**2*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + 17300*a**3*b**2*m**2*x**5*x**m/(m**6 + 36*m**5 +
505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 34890*a**3*b**2*m*x*
*5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 103
95) + 20790*a**3*b**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12
139*m**2 + 19524*m + 10395) + 10*a**2*b**3*m**5*x**7*x**m/(m**6 + 36*m**5 +
 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 290*a**2*b**3*m**4*
x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 3020*a**2*b**3*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**
3 + 12139*m**2 + 19524*m + 10395) + 13660*a**2*b**3*m**2*x**7*x**m/(m**6 +
36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 25770*a**2
*b**3*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 195
24*m + 10395) + 14850*a**2*b**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 5*a*b**4*m**5*x**9*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 135*a*b**4*m*
*4*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
+ 10395) + 1310*a*b**4*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**
3 + 12139*m**2 + 19524*m + 10395) + 5610*a*b**4*m**2*x**9*x**m/(m**6 + 36*m
**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10205*a*b**4*m
*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
10395) + 5775*a*b**4*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + b**5*m**5*x**11*x**m/(m**6 + 36*m**5 + 505*m**
4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 25*b**5*m**4*x**11*x**m/(m*
*6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 230*b
**5*m**3*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 950*b**5*m**2*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 348
0*m**3 + 12139*m**2 + 19524*m + 10395) + 1689*b**5*m*x**11*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 945*b**5*x**1
1*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1039
5), True))
```

3.340 $\int x^m \left(a+bx^2\right)^4 dx$

Optimal. Leaf size=79

$$\frac{a^4x^{m+1}}{m+1}+\frac{4a^3bx^{m+3}}{m+3}+\frac{6a^2b^2x^{m+5}}{m+5}+\frac{4ab^3x^{m+7}}{m+7}+\frac{b^4x^{m+9}}{m+9}$$

```
[Out] a^4*x^(1+m)/(1+m)+4*a^3*b*x^(3+m)/(3+m)+6*a^2*b^2*x^(5+m)/(5+m)+4*a*b^3*x^(
7+m)/(7+m)+b^4*x^(9+m)/(9+m)
```

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{6a^2b^2x^{m+5}}{m+5}+\frac{4a^3bx^{m+3}}{m+3}+\frac{a^4x^{m+1}}{m+1}+\frac{4ab^3x^{m+7}}{m+7}+\frac{b^4x^{m+9}}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^4,x]
```

```
[Out] (a^4*x^(1 + m))/(1 + m) + (4*a^3*b*x^(3 + m))/(3 + m) + (6*a^2*b^2*x^(5 + m
))/(5 + m) + (4*a*b^3*x^(7 + m))/(7 + m) + (b^4*x^(9 + m))/(9 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^4 dx &= \int \left(a^4x^m+4a^3bx^{2+m}+6a^2b^2x^{4+m}+4ab^3x^{6+m}+b^4x^{8+m}\right) dx\\ &= \frac{a^4x^{1+m}}{1+m}+\frac{4a^3bx^{3+m}}{3+m}+\frac{6a^2b^2x^{5+m}}{5+m}+\frac{4ab^3x^{7+m}}{7+m}+\frac{b^4x^{9+m}}{9+m}\end{aligned}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 0.91

$$x^{m+1}\left(\frac{a^4}{m+1}+\frac{4a^3bx^2}{m+3}+\frac{6a^2b^2x^4}{m+5}+\frac{4ab^3x^6}{m+7}+\frac{b^4x^8}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^4,x]
```

```
[Out] x^(1 + m)*(a^4/(1 + m) + (4*a^3*b*x^2)/(3 + m) + (6*a^2*b^2*x^4)/(5 + m) +
(4*a*b^3*x^6)/(7 + m) + (b^4*x^8)/(9 + m))
```

fricas [B] time = 0.81, size = 251, normalized size = 3.18

$$\frac{\left(\left(b^4m^4+16\,b^4m^3+86\,b^4m^2+176\,b^4m+105\,b^4\right)x^9+4\left(ab^3m^4+18\,ab^3m^3+104\,ab^3m^2+222\,ab^3m+135\,ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] ((b^4*m^4 + 16*b^4*m^3 + 86*b^4*m^2 + 176*b^4*m + 105*b^4)*x^9 + 4*(a*b^3*m
^4 + 18*a*b^3*m^3 + 104*a*b^3*m^2 + 222*a*b^3*m + 135*a*b^3)*x^7 + 6*(a^2*b
^2*m^4 + 20*a^2*b^2*m^3 + 130*a^2*b^2*m^2 + 300*a^2*b^2*m + 189*a^2*b^2)*x^
5 + 4*(a^3*b*m^4 + 22*a^3*b*m^3 + 164*a^3*b*m^2 + 458*a^3*b*m + 315*a^3*b)*
x^3 + (a^4*m^4 + 24*a^4*m^3 + 206*a^4*m^2 + 744*a^4*m + 945*a^4)*x)*x^m/(m^
5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

giac [B] time = 0.61, size = 365, normalized size = 4.62

$$b^4m^4x^9x^m + 16\,b^4m^3x^9x^m + 4\,ab^3m^4x^7x^m + 86\,b^4m^2x^9x^m + 72\,ab^3m^3x^7x^m + 176\,b^4mx^9x^m + 6\,a^2b^2m^4x^5x^m + 41$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^4,x, algorithm="giac")
```

```
[Out] (b^4*m^4*x^9*x^m + 16*b^4*m^3*x^9*x^m + 4*a*b^3*m^4*x^7*x^m + 86*b^4*m^2*x^
9*x^m + 72*a*b^3*m^3*x^7*x^m + 176*b^4*m*x^9*x^m + 6*a^2*b^2*m^4*x^5*x^m +
416*a*b^3*m^2*x^7*x^m + 105*b^4*x^9*x^m + 120*a^2*b^2*m^3*x^5*x^m + 888*a*b
^3*m*x^7*x^m + 4*a^3*b*m^4*x^3*x^m + 780*a^2*b^2*m^2*x^5*x^m + 540*a*b^3*x^
7*x^m + 88*a^3*b*m^3*x^3*x^m + 1800*a^2*b^2*m*x^5*x^m + a^4*m^4*x*x^m + 656
*a^3*b*m^2*x^3*x^m + 1134*a^2*b^2*x^5*x^m + 24*a^4*m^3*x*x^m + 1832*a^3*b*m
*x^3*x^m + 206*a^4*m^2*x*x^m + 1260*a^3*b*x^3*x^m + 744*a^4*m*x*x^m + 945*a
^4*x*x^m)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

maple [B] time = 0.01, size = 291, normalized size = 3.68

$$\left(b^4m^4x^8 + 16b^4m^3x^8 + 4a\,b^3m^4x^6 + 86b^4m^2x^8 + 72a\,b^3m^3x^6 + 176b^4m\,x^8 + 6a^2b^2m^4x^4 + 416a\,b^3m^2x^6 + 105b^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^4,x)
```

```
[Out] x^(m+1)*(b^4*m^4*x^8+16*b^4*m^3*x^8+4*a*b^3*m^4*x^6+86*b^4*m^2*x^8+72*a*b^3
*m^3*x^6+176*b^4*m*x^8+6*a^2*b^2*m^4*x^4+416*a*b^3*m^2*x^6+105*b^4*x^8+120*
a^2*b^2*m^3*x^4+888*a*b^3*m*x^6+4*a^3*b*m^4*x^2+780*a^2*b^2*m^2*x^4+540*a*b
^3*x^6+88*a^3*b*m^3*x^2+1800*a^2*b^2*m*x^4+a^4*m^4+656*a^3*b*m^2*x^2+1134*a
^2*b^2*x^4+24*a^4*m^3+1832*a^3*b*m*x^2+206*a^4*m^2+1260*a^3*b*x^2+744*a^4*m
+945*a^4)/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.34, size = 79, normalized size = 1.00

$$\frac{b^4x^{m+9}}{m+9} + \frac{4\,ab^3x^{m+7}}{m+7} + \frac{6\,a^2b^2x^{m+5}}{m+5} + \frac{4\,a^3bx^{m+3}}{m+3} + \frac{a^4x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] b^4*x^(m + 9)/(m + 9) + 4*a*b^3*x^(m + 7)/(m + 7) + 6*a^2*b^2*x^(m + 5)/(m
+ 5) + 4*a^3*b*x^(m + 3)/(m + 3) + a^4*x^(m + 1)/(m + 1)
```

mupad [B] time = 4.99, size = 272, normalized size = 3.44

$$\frac{a^4\,x\,x^m\,\left(m^4+24\,m^3+206\,m^2+744\,m+945\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{b^4\,x^m\,x^9\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{6\,a^2\,b^2\,x^m\,x^5}{m^5+25\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^4,x)
```

```
[Out] (a^4*x*x^m*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/(1689*m + 950*m^2 + 230*
m^3 + 25*m^4 + m^5 + 945) + (b^4*x^m*x^9*(176*m + 86*m^2 + 16*m^3 + m^4 + 1
05))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (6*a^2*b^2*x^m*x^5
*(300*m + 130*m^2 + 20*m^3 + m^4 + 189))/(1689*m + 950*m^2 + 230*m^3 + 25*m
^4 + m^5 + 945) + (4*a*b^3*x^m*x^7*(222*m + 104*m^2 + 18*m^3 + m^4 + 135))/
(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (4*a^3*b*x^m*x^3*(458*m
 + 164*m^2 + 22*m^3 + m^4 + 315))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^
5 + 945)
```

sympy [A] time = 2.85, size = 1221, normalized size = 15.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**4,x)
```

```
[Out] Piecewise((-a**4/(8*x**8) - 2*a**3*b/(3*x**6) - 3*a**2*b**2/(2*x**4) - 2*a*
b**3/x**2 + b**4*log(x), Eq(m, -9)), (-a**4/(6*x**6) - a**3*b/x**4 - 3*a**2
*b**2/x**2 + 4*a*b**3*log(x) + b**4*x**2/2, Eq(m, -7)), (-a**4/(4*x**4) - 2
*a**3*b/x**2 + 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**4/4, Eq(m, -5))
, (-a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**2 + a*b**3*x**4 + b**4
*x**6/6, Eq(m, -3)), (a**4*log(x) + 2*a**3*b*x**2 + 3*a**2*b**2*x**4/2 + 2*
a*b**3*x**6/3 + b**4*x**8/8, Eq(m, -1)), (a**4*m**4*x*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + 24*a**4*m**3*x*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 206*a**4*m**2*x*x**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 744*a**4*m*x*x**m/(m**5 + 25*m**
4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 945*a**4*x*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 4*a**3*b*m**4*x**3*x**m/(m**5 + 25*m
**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 88*a**3*b*m**3*x**3*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 656*a**3*b*m**2*x**3*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1832*a**3*b*m*x**3
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1260*a**3*b*x
**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 6*a**2*b**
2*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 12
0*a**2*b**2*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
 945) + 780*a**2*b**2*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
+ 1689*m + 945) + 1800*a**2*b**2*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 9
50*m**2 + 1689*m + 945) + 1134*a**2*b**2*x**5*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 4*a*b**3*m**4*x**7*x**m/(m**5 + 25*m**4 + 2
30*m**3 + 950*m**2 + 1689*m + 945) + 72*a*b**3*m**3*x**7*x**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 416*a*b**3*m**2*x**7*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 888*a*b**3*m*x**7*x**m/(m
**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 540*a*b**3*x**7*x**m/
(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + b**4*m**4*x**9*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 16*b**4*m**3*x**9*
x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*b**4*m**2*x
**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 176*b**4*m
*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*b**4
*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), True))
```

3.341 $\int x^m \left(a+bx^2\right)^3 dx$

Optimal. Leaf size=61

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+3}}{m+3}+\frac{3ab^2x^{m+5}}{m+5}+\frac{b^3x^{m+7}}{m+7}$$

```
[Out] a^3*x^(1+m)/(1+m)+3*a^2*b*x^(3+m)/(3+m)+3*a*b^2*x^(5+m)/(5+m)+b^3*x^(7+m)/(
7+m)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{3a^2bx^{m+3}}{m+3}+\frac{a^3x^{m+1}}{m+1}+\frac{3ab^2x^{m+5}}{m+5}+\frac{b^3x^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(3 + m))/(3 + m) + (3*a*b^2*x^(5 + m))
/(5 + m) + (b^3*x^(7 + m))/(7 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)^3\,dx&=\int\left(a^3x^m+3a^2bx^{2+m}+3ab^2x^{4+m}+b^3x^{6+m}\right)dx\\&=\frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{3+m}}{3+m}+\frac{3ab^2x^{5+m}}{5+m}+\frac{b^3x^{7+m}}{7+m}\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.92

$$x^{m+1}\left(\frac{a^3}{m+1}+\frac{3a^2bx^2}{m+3}+\frac{3ab^2x^4}{m+5}+\frac{b^3x^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x^2)/(3 + m) + (3*a*b^2*x^4)/(5 + m) + (b
^3*x^6)/(7 + m))
```

fricas [B] time = 0.86, size = 157, normalized size = 2.57

$$\frac{\left(\left(b^3m^3+9\,b^3m^2+23\,b^3m+15\,b^3\right)x^7+3\left(ab^2m^3+11\,ab^2m^2+31\,ab^2m+21\,ab^2\right)x^5+3\left(a^2bm^3+13\,a^2bm^2+47\right.\right.}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 9*b^3*m^2 + 23*b^3*m + 15*b^3)*x^7 + 3*(a*b^2*m^3 + 11*a*b^2*m^
2 + 31*a*b^2*m + 21*a*b^2)*x^5 + 3*(a^2*b*m^3 + 13*a^2*b*m^2 + 47*a^2*b*m +
 35*a^2*b)*x^3 + (a^3*m^3 + 15*a^3*m^2 + 71*a^3*m + 105*a^3)*x)*x^m/(m^4 +
16*m^3 + 86*m^2 + 176*m + 105)
```

giac [B] time = 0.66, size = 224, normalized size = 3.67

$$\frac{b^3m^3x^7x^m + 9\,b^3m^2x^7x^m + 3\,ab^2m^3x^5x^m + 23\,b^3mx^7x^m + 33\,ab^2m^2x^5x^m + 15\,b^3x^7x^m + 3\,a^2bm^3x^3x^m + 93\,ab^2}{m^4 + 16\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^7*x^m + 9*b^3*m^2*x^7*x^m + 3*a*b^2*m^3*x^5*x^m + 23*b^3*m*x^7*x
^m + 33*a*b^2*m^2*x^5*x^m + 15*b^3*x^7*x^m + 3*a^2*b*m^3*x^3*x^m + 93*a*b^2
*m*x^5*x^m + 39*a^2*b*m^2*x^3*x^m + 63*a*b^2*x^5*x^m + a^3*m^3*x*x^m + 141*
a^2*b*m*x^3*x^m + 15*a^3*m^2*x*x^m + 105*a^2*b*x^3*x^m + 71*a^3*m*x*x^m + 1
05*a^3*x*x^m)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

maple [B] time = 0.01, size = 178, normalized size = 2.92

$$\frac{\left(b^3m^3x^6 + 9b^3m^2x^6 + 3a\,b^2m^3x^4 + 23b^3m\,x^6 + 33a\,b^2m^2x^4 + 15b^3x^6 + 3a^2b\,m^3x^2 + 93a\,b^2m\,x^4 + 39a^2b\,m^2x^2}{(m+7)\,(m+5)\,(m+3)\,(m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^3,x)
```

```
[Out] x^(m+1)*(b^3*m^3*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^
2*x^4+15*b^3*x^6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x
^4+a^3*m^3+141*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)/(m+7)
/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.34, size = 61, normalized size = 1.00

$$\frac{b^3x^{m+7}}{m+7} + \frac{3\,ab^2x^{m+5}}{m+5} + \frac{3\,a^2bx^{m+3}}{m+3} + \frac{a^3x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] b^3*x^(m + 7)/(m + 7) + 3*a*b^2*x^(m + 5)/(m + 5) + 3*a^2*b*x^(m + 3)/(m +
3) + a^3*x^(m + 1)/(m + 1)
```

mupad [B] time = 4.79, size = 167, normalized size = 2.74

$$x^m\left(\frac{a^3\,x\,\left(m^3+15\,m^2+71\,m+105\right)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{b^3\,x^7\,\left(m^3+9\,m^2+23\,m+15\right)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{3\,a\,b^2\,x^5\,\left(m^3+11\,m^2+31\,n\right.}{m^4+16\,m^3+86\,m^2+176\,n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^3,x)
```

```
[Out] x^m*((a^3*x*(71*m + 15*m^2 + m^3 + 105))/(176*m + 86*m^2 + 16*m^3 + m^4 + 1
05) + (b^3*x^7*(23*m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4 +
105) + (3*a*b^2*x^5*(31*m + 11*m^2 + m^3 + 21))/(176*m + 86*m^2 + 16*m^3 +
m^4 + 105) + (3*a^2*b*x^3*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2 + 16*
m^3 + m^4 + 105))
```

sympy [A] time = 1.66, size = 683, normalized size = 11.20

$$\begin{cases} -\frac{a^3}{6x^6} - \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} + b^3\log(x) \\ -\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + 3ab^2\log(x) + \frac{b^3x^2}{2} \\ -\frac{a^3}{2x^2} + 3a^2b\log(x) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4} \\ a^3\log(x) + \frac{3a^2bx^2}{2} + \frac{3ab^2x^4}{4} + \frac{b^3x^6}{6} \\ \frac{a^3m^3xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{15a^3m^2xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{71a^3mxx^m}{m^4+16m^3+86m^2+176m+105} + \frac{105a^3xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{3a^2b}{m^4+16m^3+8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**3,x)
```

```
[Out] Piecewise((-a**3/(6*x**6) - 3*a**2*b/(4*x**4) - 3*a*b**2/(2*x**2) + b**3*lo
g(x), Eq(m, -7)), (-a**3/(4*x**4) - 3*a**2*b/(2*x**2) + 3*a*b**2*log(x) + b
**3*x**2/2, Eq(m, -5)), (-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2
 + b**3*x**4/4, Eq(m, -3)), (a**3*log(x) + 3*a**2*b*x**2/2 + 3*a*b**2*x**4/
4 + b**3*x**6/6, Eq(m, -1)), (a**3*m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 +
176*m + 105) + 15*a**3*m**2*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
 + 71*a**3*m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 105*a**3*x*x
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 3*a**2*b*m**3*x**3*x**m/(m**
4 + 16*m**3 + 86*m**2 + 176*m + 105) + 39*a**2*b*m**2*x**3*x**m/(m**4 + 16*
m**3 + 86*m**2 + 176*m + 105) + 141*a**2*b*m*x**3*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 105*a**2*b*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176
*m + 105) + 3*a*b**2*m**3*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105
) + 33*a*b**2*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 93*
a*b**2*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 63*a*b**2*x**
5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b**3*m**3*x**7*x**m/(m**4
 + 16*m**3 + 86*m**2 + 176*m + 105) + 9*b**3*m**2*x**7*x**m/(m**4 + 16*m**3
 + 86*m**2 + 176*m + 105) + 23*b**3*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 +
 176*m + 105) + 15*b**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105),
 True))
```

3.342 $\int x^m \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=43

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+3}}{m+3}+\frac{b^2x^{m+5}}{m+5}$$

```
[Out] a^2*x^(1+m)/(1+m)+2*a*b*x^(3+m)/(3+m)+b^2*x^(5+m)/(5+m)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+3}}{m+3}+\frac{b^2x^{m+5}}{m+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(3 + m))/(3 + m) + (b^2*x^(5 + m))/(5 +
m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^2 \, dx &= \int \left(a^2x^m+2abx^{2+m}+b^2x^{4+m}\right) dx\\ &= \frac{a^2x^{1+m}}{1+m}+\frac{2abx^{3+m}}{3+m}+\frac{b^2x^{5+m}}{5+m}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.93

$$x^{m+1}\left(\frac{a^2}{m+1}+\frac{2abx^2}{m+3}+\frac{b^2x^4}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + (b^2*x^4)/(5 + m))
```

fricas [A] time = 1.06, size = 85, normalized size = 1.98

$$\frac{\left(\left(b^2m^2+4\,b^2m+3\,b^2\right)x^5+2\left(abm^2+6\,abm+5\,ab\right)x^3+\left(a^2m^2+8\,a^2m+15\,a^2\right)x\right)x^m}{m^3+9\,m^2+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 4*b^2*m + 3*b^2)*x^5 + 2*(a*b*m^2 + 6*a*b*m + 5*a*b)*x^3 + (a^2
*m^2 + 8*a^2*m + 15*a^2)*x)*x^m/(m^3 + 9*m^2 + 23*m + 15)
```

giac [B] time = 0.65, size = 117, normalized size = 2.72

$$\frac{b^2m^2x^5x^m + 4\,b^2mx^5x^m + 2\,abm^2x^3x^m + 3\,b^2x^5x^m + 12\,abmx^3x^m + a^2m^2xx^m + 10\,abx^3x^m + 8\,a^2mxx^m + 15\,a^2xx}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^5*x^m + 4*b^2*m*x^5*x^m + 2*a*b*m^2*x^3*x^m + 3*b^2*x^5*x^m + 12
*a*b*m*x^3*x^m + a^2*m^2*x*x^m + 10*a*b*x^3*x^m + 8*a^2*m*x*x^m + 15*a^2*x*
x^m)/(m^3 + 9*m^2 + 23*m + 15)
```

maple [B] time = 0.01, size = 93, normalized size = 2.16

$$\frac{\left(b^2m^2x^4 + 4b^2m\,x^4 + 2ab\,m^2x^2 + 3b^2x^4 + 12abm\,x^2 + a^2m^2 + 10ab\,x^2 + 8a^2m + 15a^2\right)x^{m+1}}{(m+5)\,(m+3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2,x)
```

```
[Out] x^(m+1)*(b^2*m^2*x^4+4*b^2*m*x^4+2*a*b*m^2*x^2+3*b^2*x^4+12*a*b*m*x^2+a^2*m
^2+10*a*b*x^2+8*a^2*m+15*a^2)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.31, size = 43, normalized size = 1.00

$$\frac{b^2x^{m+5}}{m+5} + \frac{2\,abx^{m+3}}{m+3} + \frac{a^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 5)/(m + 5) + 2*a*b*x^(m + 3)/(m + 3) + a^2*x^(m + 1)/(m + 1)
```

mupad [B] time = 4.75, size = 93, normalized size = 2.16

$$x^m\left(\frac{a^2\,x\,\left(m^2+8\,m+15\right)}{m^3+9\,m^2+23\,m+15} + \frac{b^2\,x^5\,\left(m^2+4\,m+3\right)}{m^3+9\,m^2+23\,m+15} + \frac{2\,a\,b\,x^3\,\left(m^2+6\,m+5\right)}{m^3+9\,m^2+23\,m+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^2,x)
```

```
[Out] x^m*((a^2*x*(8*m + m^2 + 15))/(23*m + 9*m^2 + m^3 + 15) + (b^2*x^5*(4*m + m
^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (2*a*b*x^3*(6*m + m^2 + 5))/(23*m + 9*
m^2 + m^3 + 15))
```

sympy [A] time = 0.94, size = 306, normalized size = 7.12

$$\begin{cases} -\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2\log(x) \\ -\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2} \\ a^2\log(x) + abx^2 + \frac{b^2x^4}{4} \\ \frac{a^2m^2xx^m}{m^3+9m^2+23m+15} + \frac{8a^2mxx^m}{m^3+9m^2+23m+15} + \frac{15a^2xx^m}{m^3+9m^2+23m+15} + \frac{2abm^2x^3x^m}{m^3+9m^2+23m+15} + \frac{12abmx^3x^m}{m^3+9m^2+23m+15} + \frac{10abx^3x^m}{m^3+9m^2+23m+15} + \frac{b^2m^2}{m^3+9m^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) - a*b/x**2 + b**2*log(x), Eq(m, -5)), (-a**2/(2*x
**2) + 2*a*b*log(x) + b**2*x**2/2, Eq(m, -3)), (a**2*log(x) + a*b*x**2 + b*
*2*x**4/4, Eq(m, -1)), (a**2*m**2*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 8*a*
*2*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a**2*x*x**m/(m**3 + 9*m**2 + 2
3*m + 15) + 2*a*b*m**2*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 12*a*b*m*x**
3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 10*a*b*x**3*x**m/(m**3 + 9*m**2 + 23*m
 + 15) + b**2*m**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 4*b**2*m*x**5*x*
*m/(m**3 + 9*m**2 + 23*m + 15) + 3*b**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 1
5), True))
```

3.343 $\int x^m \left(a+bx^2\right) dx$

Optimal. Leaf size=25

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+3}}{m+3}$$

```
[Out] a*x^(1+m)/(1+m)+b*x^(3+m)/(3+m)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(3 + m))/(3 + m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right) dx &= \int \left(ax^m+bx^{2+m}\right) dx\\ &= \frac{ax^{1+m}}{1+m}+\frac{bx^{3+m}}{3+m}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(3 + m))/(3 + m)
```

fricas [A] time = 0.94, size = 33, normalized size = 1.32

$$\frac{\left((bm+b)x^3+(am+3\,a)x\right)x^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^3 + (a*m + 3*a)*x)*x^m/(m^2 + 4*m + 3)
```

giac [A] time = 0.69, size = 43, normalized size = 1.72

$$\frac{bmx^3x^m+bx^3x^m+amxx^m+3\,axx^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b*m*x^3*x^m + b*x^3*x^m + a*m*x*x^m + 3*a*x*x^m)/(m^2 + 4*m + 3)
```

maple [A] time = 0.00, size = 35, normalized size = 1.40

$$\frac{\left(bm\,x^{2}+b\,x^{2}+am+3a\right)x^{m+1}}{\left(m+3\right)\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a),x)
```

```
[Out] x^(m+1)*(b*m*x^2+b*x^2+a*m+3*a)/(m+3)/(m+1)
```

maxima [A] time = 1.30, size = 25, normalized size = 1.00

$$\frac{bx^{m+3}}{m+3}+\frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] b*x^(m + 3)/(m + 3) + a*x^(m + 1)/(m + 1)
```

mupad [B] time = 4.80, size = 34, normalized size = 1.36

$$\frac{x^{m+1}\,\left(3\,a+a\,m+b\,x^2+b\,m\,x^2\right)}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2),x)
```

```
[Out] (x^(m + 1)*(3*a + a*m + b*x^2 + b*m*x^2))/(4*m + m^2 + 3)
```

sympy [A] time = 0.41, size = 94, normalized size = 3.76

$$\begin{cases} -\frac{a}{2x^{2}}+b\log\left(x\right) & \text{for}\: m=-3 \\ a\log\left(x\right)+\frac{bx^{2}}{2} & \text{for}\: m=-1 \\ \frac{amxx^{m}}{m^{2}+4m+3}+\frac{3axx^{m}}{m^{2}+4m+3}+\frac{bmx^{3}x^{m}}{m^{2}+4m+3}+\frac{bx^{3}x^{m}}{m^{2}+4m+3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a),x)
```

```
[Out] Piecewise((-a/(2*x**2) + b*log(x), Eq(m, -3)), (a*log(x) + b*x**2/2, Eq(m,
-1)), (a*m*x*x**m/(m**2 + 4*m + 3) + 3*a*x*x**m/(m**2 + 4*m + 3) + b*m*x**3
*x**m/(m**2 + 4*m + 3) + b*x**3*x**m/(m**2 + 4*m + 3), True))
```

3.344 $\int \frac{x^m}{a+bx^2} dx$

Optimal. Leaf size=39

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(1
+ m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{a+bx^2}\,dx = \frac{x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.05

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
*(1 + m))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^2 + a), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^m}{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a),x)
```

```
[Out] int(x^m/(b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^2),x)
```

```
[Out] int(x^m/(a + b*x^2), x)
```

sympy [C] time = 1.25, size = 88, normalized size = 2.26

$$\frac{mxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{xx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a),x)
```

```
[Out] m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/
(4*a*gamma(m/2 + 3/2)) + x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 +
 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2))
```

3.345 $\int \frac{x^m}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=39

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^2(m+1)}$$

```
[Out] x^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^2*(
1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\left(a+bx^2\right)^2} \, dx = \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^2(1+m)}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.05

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^2*(1 + m))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^2 + a)^2, x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(b\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^2,x)
```

```
[Out] int(x^m/(b*x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{(b\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^2)^2,x)
```

```
[Out] int(x^m/(a + b*x^2)^2, x)
```

sympy [C] time = 6.22, size = 374, normalized size = 9.59

$$-\frac{am^2xx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{2amxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{axx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**2,x)
```

```
[Out] -a*m**2*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 +
 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*m*x*
x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 +
3/2)) + a*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*x*
```

```
x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 +
3/2)) - b*m**2*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*g
amma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2))
+ b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2))
```

3.346 $\int \frac{x^m}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=39

$$\frac{x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3(m+1)}$$

```
[Out] x^(1+m)*hypergeom([3, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^3/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^2)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^3*(
1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\left(a+bx^2\right)^3}\,dx=\frac{x^{1+m}\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^3(1+m)}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.05

$$\frac{x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^3*(1 + m))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^2 + a)^3, x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(b\,x^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^3,x)
```

```
[Out] int(x^m/(b*x^2+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^2 + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\left(b\,x^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^2)^3,x)
```

```
[Out] int(x^m/(a + b*x^2)^3, x)
```

sympy [C] time = 20.44, size = 1556, normalized size = 39.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**3,x)
```

```
[Out] a**2*m**3*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a*
*3*b**2*x**4*gamma(m/2 + 3/2)) - 3*a**2*m**2*x*x**m*lerchphi(b*x**2*exp_pol
ar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a
**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 2*a**2*
m**2*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gam
ma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - a**2*m*x*x**m*lerchph
i(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m
```

```
/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 +
 3/2)) + 8*a**2*m*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a*
*4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 3*a**2*x
*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32
*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**
4*gamma(m/2 + 3/2)) + 10*a**2*x*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 +
3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)
) + 2*a*b*m**3*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*g
amma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2)
 + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 6*a*b*m**2*x**3*x**m*lerchphi(b*x*
*2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3
/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2))
 - 2*a*b*m**2*x**3*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**
4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 2*a*b*m*x
**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/
(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*
x**4*gamma(m/2 + 3/2)) + 4*a*b*m*x**3*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(
m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2
+ 3/2)) + 6*a*b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*
gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2
) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 6*a*b*x**3*x**m*gamma(m/2 + 1/2)/
(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*
x**4*gamma(m/2 + 3/2)) + b**2*m**3*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi
)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x
**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 3*b**2*m**2*x*
*5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(
32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x
**4*gamma(m/2 + 3/2)) - b**2*m*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a,
 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*
gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 3*b**2*x**5*x**m*l
erchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*g
amma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma
(m/2 + 3/2))
```

3.347 $\int \frac{(cx)^{1+m}}{a+bx^2} dx$

Optimal. Leaf size=44

$$\frac{(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac(m+2)}$$

```
[Out] (c*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-b*x^2/a)/a/c/(2+m)
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {364}

$$\frac{(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1 + m)/(a + b*x^2),x]
```

```
[Out] ((c*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a
*c*(2 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{1+m}}{a+bx^2}\,dx = \frac{(cx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{ac(2+m)}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 1.02

$$\frac{cx^2(cx)^m\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+2}{2}+1;-\frac{bx^2}{a}\right)}{a(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1 + m)/(a + b*x^2),x]
```

```
[Out] (c*x^2*(c*x)^m*Hypergeometric2F1[1, (2 + m)/2, 1 + (2 + m)/2, -((b*x^2)/a)]
)/(a*(2 + m))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^{m+1}}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1+m)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(m + 1)/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m+1}}{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1+m)/(b*x^2+a),x, algorithm="giac")
[Out] integrate((c*x)^(m + 1)/(b*x^2 + a), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m+1}}{b\,x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(m+1)/(b*x^2+a),x)
[Out] int((c*x)^(m+1)/(b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m+1}}{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1+m)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((c*x)^(m + 1)/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{m+1}}{b\,x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(m + 1)/(a + b*x^2),x)
[Out] int((c*x)^(m + 1)/(a + b*x^2), x)
```

sympy [C] time = 2.95, size = 92, normalized size = 2.09

$$\frac{cc^m m x^2 x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + 1\right)\Gamma\left(\frac{m}{2} + 1\right)}{4a\Gamma\left(\frac{m}{2} + 2\right)} + \frac{cc^m x^2 x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + 1\right)\Gamma\left(\frac{m}{2} + 1\right)}{2a\Gamma\left(\frac{m}{2} + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1+m)/(b*x**2+a),x)
[Out] c*c**m*m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2
 + 1)/(4*a*gamma(m/2 + 2)) + c*c**m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2))
```

3.348 $\int \frac{(cx)^m}{a+bx^2}\,dx$

Optimal. Leaf size=44

$$\frac{(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}$$

```
[Out] (c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/c/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {364}

$$\frac{(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(a + b*x^2),x]
```

```
[Out] ((c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
*c*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^m}{a+bx^2}\,dx=\frac{(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ac(1+m)}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.95

$$\frac{x(cx)^m\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
*(1 + m))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(cx)^m}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^m/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2+a),x, algorithm="giac")
[Out] integrate((c*x)^m/(b*x^2 + a), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^2+a),x)
[Out] int((c*x)^m/(b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((c*x)^m/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^m}{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a + b*x^2),x)
[Out] int((c*x)^m/(a + b*x^2), x)
```

sympy [C] time = 1.25, size = 95, normalized size = 2.16

$$\frac{c^m m x x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{c^m x x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(b*x**2+a),x)
[Out] c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(4*a*gamma(m/2 + 3/2)) + c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2))
```

3.349 $\int \frac{(cx)^{-1+m}}{a+bx^2} dx$

Optimal. Leaf size=38

$$\frac{(cx)^m {}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)}{acm}$$

```
[Out] (c*x)^m*hypergeom([1, 1/2*m],[1+1/2*m],-b*x^2/a)/a/c/m
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {364}

$$\frac{(cx)^m {}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)}{acm}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + m)/(a + b*x^2),x]
```

```
[Out] ((c*x)^m*Hypergeometric2F1[1, m/2, (2 + m)/2, -((b*x^2)/a)])/(a*c*m)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{-1+m}}{a+bx^2} dx = \frac{(cx)^m {}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};-\frac{bx^2}{a}\right)}{acm}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$\frac{x(cx)^{m-1} {}_2F_1\left(1,\frac{m}{2};\frac{m}{2}+1;-\frac{bx^2}{a}\right)}{am}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + m)/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^(-1 + m)*Hypergeometric2F1[1, m/2, 1 + m/2, -((b*x^2)/a)])/(a*m)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^{m-1}}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+m)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(m - 1)/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m-1}}{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+m)/(b*x^2+a),x, algorithm="giac")
[Out] integrate((c*x)^(m - 1)/(b*x^2 + a), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m-1}}{b\,x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(m-1)/(b*x^2+a),x)
[Out] int((c*x)^(m-1)/(b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m-1}}{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+m)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((c*x)^(m - 1)/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c\,x)^{m-1}}{b\,x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(m - 1)/(a + b*x^2),x)
[Out] int((c*x)^(m - 1)/(a + b*x^2), x)
```

sympy [C] time = 7.15, size = 39, normalized size = 1.03

$$\frac{c^m m x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2}\right) \Gamma\left(\frac{m}{2}\right)}{4ac\Gamma\left(\frac{m}{2} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+m)/(b*x**2+a),x)
[Out] c**m*m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2)*gamma(m/2)/(4*a*c*ga
mma(m/2 + 1))
```

3.350 $\int \frac{(cx)^{-2+m}}{a+bx^2}\,dx$

Optimal. Leaf size=47

$$-\frac{(cx)^{m-1}\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{bx^2}{a}\right)}{ac(1-m)}$$

```
[Out] -(c*x)^(-1+m)*hypergeom([1, -1/2+1/2*m],[1/2+1/2*m],-b*x^2/a)/a/c/(1-m)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {364}

$$-\frac{(cx)^{m-1}\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{bx^2}{a}\right)}{ac(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-2 + m)/(a + b*x^2),x]
```

```
[Out] -(((c*x)^(-1 + m)*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((b*x^2)/a)]
)/(a*c*(1 - m)))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{-2+m}}{a+bx^2}\,dx = -\frac{(cx)^{-1+m}\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{bx^2}{a}\right)}{ac(1-m)}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.94

$$\frac{x(cx)^{m-2}\,{}_2F_1\left(1,\frac{m-1}{2};\frac{m-1}{2}+1;-\frac{bx^2}{a}\right)}{a(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-2 + m)/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^(-2 + m)*Hypergeometric2F1[1, (-1 + m)/2, 1 + (-1 + m)/2, -((b*x^2
)/a)])/(a*(-1 + m))
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(cx)^{m-2}}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-2+m)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(m - 2)/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m-2}}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-2+m)/(b*x^2+a),x, algorithm="giac")
[Out] integrate((c*x)^(m - 2)/(b*x^2 + a), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m-2}}{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(m-2)/(b*x^2+a),x)
[Out] int((c*x)^(m-2)/(b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m-2}}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-2+m)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((c*x)^(m - 2)/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{m-2}}{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(m - 2)/(a + b*x^2),x)
[Out] int((c*x)^(m - 2)/(a + b*x^2), x)
```

sympy [C] time = 11.57, size = 102, normalized size = 2.17

$$\frac{c^m m x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} - \frac{1}{2}\right)\Gamma\left(\frac{m}{2} - \frac{1}{2}\right)}{4ac^2 x\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)} - \frac{c^m x^m \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} - \frac{1}{2}\right)\Gamma\left(\frac{m}{2} - \frac{1}{2}\right)}{4ac^2 x\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-2+m)/(b*x**2+a),x)
[Out] c**m*m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 - 1/2)*gamma(m/2 - 1/
2)/(4*a*c**2*x*gamma(m/2 + 1/2)) - c**m*x**m*lerchphi(b*x**2*exp_polar(I*pi
)/a, 1, m/2 - 1/2)*gamma(m/2 - 1/2)/(4*a*c**2*x*gamma(m/2 + 1/2))
```

3.351 $\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx$

Optimal. Leaf size=45

$$-\frac{(cx)^{m-2}\,{}_2F_1\left(1,\frac{m-2}{2};\frac{m}{2};-\frac{bx^2}{a}\right)}{ac(2-m)}$$

```
[Out] -(c*x)^(-2+m)*hypergeom([1, -1+1/2*m],[1/2*m],-b*x^2/a)/a/c/(2-m)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {364}

$$-\frac{(cx)^{m-2}\,{}_2F_1\left(1,\frac{m-2}{2};\frac{m}{2};-\frac{bx^2}{a}\right)}{ac(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-3 + m)/(a + b*x^2),x]
```

```
[Out] -(((c*x)^(-2 + m)*Hypergeometric2F1[1, (-2 + m)/2, m/2, -((b*x^2)/a)])/(a*c
*(2 - m)))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx=-\frac{(cx)^{-2+m}\,{}_2F_1\left(1,\frac{1}{2}(-2+m);\frac{m}{2};-\frac{bx^2}{a}\right)}{ac(2-m)}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.98

$$\frac{x(cx)^{m-3}\,{}_2F_1\left(1,\frac{m-2}{2};\frac{m-2}{2}+1;-\frac{bx^2}{a}\right)}{a(m-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-3 + m)/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^(-3 + m)*Hypergeometric2F1[1, (-2 + m)/2, 1 + (-2 + m)/2, -((b*x^2
)/a)])/(a*(-2 + m))
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^{m-3}}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-3+m)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(m - 3)/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m-3}}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-3+m)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(m - 3)/(b*x^2 + a), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m-3}}{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(m-3)/(b*x^2+a),x)
```

```
[Out] int((c*x)^(m-3)/(b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{m-3}}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-3+m)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(m - 3)/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{m-3}}{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(m - 3)/(a + b*x^2),x)
```

```
[Out] int((c*x)^(m - 3)/(a + b*x^2), x)
```

sympy [C] time = 21.42, size = 92, normalized size = 2.04

$$\frac{c^m m x^m \Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}-1\right)\Gamma\left(\frac{m}{2}-1\right)}{4ac^3x^2\Gamma\left(\frac{m}{2}\right)} - \frac{c^m x^m \Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}-1\right)\Gamma\left(\frac{m}{2}-1\right)}{2ac^3x^2\Gamma\left(\frac{m}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-3+m)/(b*x**2+a),x)
```

```
[Out] c**m*m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 - 1)*gamma(m/2 - 1)/(
4*a*c**3*x**2*gamma(m/2)) - c**m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1,
 m/2 - 1)*gamma(m/2 - 1)/(2*a*c**3*x**2*gamma(m/2))
```

3.352 $\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2} dx$

Optimal. Leaf size=36

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{ax^2}{b}\right)}{m+1}$$

```
[Out] x^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-a*x^2/b)/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{ax^2}{b}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 + (a*x^2)/b)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((a*x^2)/b)])/(1 + m
)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2} dx = \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{ax^2}{b}\right)}{1+m}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.06

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{ax^2}{b}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(1 + (a*x^2)/b)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((a*x^2)/b)])/(1
 + m)
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^m}{a^2x^4+2\,abx^2+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+a*x^2/b)^2,x, algorithm="fricas")
[Out] integral(b^2*x^m/(a^2*x^4 + 2*a*b*x^2 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax^2}{b}+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+a*x^2/b)^2,x, algorithm="giac")
[Out] integrate(x^m/(a*x^2/b + 1)^2, x)
```

maple [C] time = 0.28, size = 92, normalized size = 2.56

$$\frac{\left(\frac{2\left(-\frac{m^2}{4}+\frac{1}{4}\right)x^{m+1}\left(\frac{a}{b}\right)^{\frac{m}{2}+\frac{1}{2}}\Phi\left(-\frac{a\,x^2}{b},1,\frac{m}{2}+\frac{1}{2}\right)}{m+1}+\frac{2x^{m+1}\left(\frac{a}{b}\right)^{\frac{m}{2}+\frac{1}{2}}}{\frac{2a\,x^2}{b}+2}\right)\left(\frac{a}{b}\right)^{-\frac{m}{2}-\frac{1}{2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(1+a/b*x^2)^2,x)
[Out] 1/2*(a/b)^(-1/2-1/2*m)*(2*x^(m+1)*(a/b)^(1/2*m+1/2)/(2+2*a/b*x^2)+2/(m+1)*x
^(m+1)*(a/b)^(1/2*m+1/2)*(-1/4*m^2+1/4)*LerchPhi(-a/b*x^2,1,1/2*m+1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax^2}{b}+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(1+a*x^2/b)^2,x, algorithm="maxima")
[Out] integrate(x^m/(a*x^2/b + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\left(\frac{a\,x^2}{b}+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x^2)/b + 1)^2,x)
[Out] int(x^m/((a*x^2)/b + 1)^2, x)
```

sympy [C] time = 6.16, size = 343, normalized size = 9.53

$$-\frac{am^2x^3x^m\Phi\left(\frac{ax^2e^{i\pi}}{b},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{ax^3x^m\Phi\left(\frac{ax^2e^{i\pi}}{b},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}-\frac{bm^2xx^m\Phi\left(\frac{ax^2e^{i\pi}}{b},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma(}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(1+a*x**2/b)**2,x)
```

```
[Out] -a*m**2*x**3*x**m*lerchphi(a*x**2*exp_polar(I*pi)/b, 1, m/2 + 1/2)*gamma(m/
2 + 1/2)/(8*a*x**2*gamma(m/2 + 3/2) + 8*b*gamma(m/2 + 3/2)) + a*x**3*x**m*l
erchphi(a*x**2*exp_polar(I*pi)/b, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a*x**2*
gamma(m/2 + 3/2) + 8*b*gamma(m/2 + 3/2)) - b*m**2*x*x**m*lerchphi(a*x**2*ex
p_polar(I*pi)/b, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a*x**2*gamma(m/2 + 3/2)
+ 8*b*gamma(m/2 + 3/2)) + 2*b*m*x*x**m*gamma(m/2 + 1/2)/(8*a*x**2*gamma(m/2
 + 3/2) + 8*b*gamma(m/2 + 3/2)) + b*x*x**m*lerchphi(a*x**2*exp_polar(I*pi)/
b, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a*x**2*gamma(m/2 + 3/2) + 8*b*gamma(m/
2 + 3/2)) + 2*b*x*x**m*gamma(m/2 + 1/2)/(8*a*x**2*gamma(m/2 + 3/2) + 8*b*ga
mma(m/2 + 3/2))
```

3.353 $\int x^7 \sqrt{a+bx^2}\,dx$

Optimal. Leaf size=80

$$-\frac{a^3\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{3a^2\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{\left(a+bx^2\right)^{9/2}}{9b^4}-\frac{3a\left(a+bx^2\right)^{7/2}}{7b^4}$$

```
[Out] -1/3*a^3*(b*x^2+a)^(3/2)/b^4+3/5*a^2*(b*x^2+a)^(5/2)/b^4-3/7*a*(b*x^2+a)^(7
/2)/b^4+1/9*(b*x^2+a)^(9/2)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{5/2}}{5b^4}-\frac{a^3\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{\left(a+bx^2\right)^{9/2}}{9b^4}-\frac{3a\left(a+bx^2\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Sqrt[a + b*x^2],x]
```

```
[Out] -(a^3*(a + b*x^2)^(3/2))/(3*b^4) + (3*a^2*(a + b*x^2)^(5/2))/(5*b^4) - (3*a
*(a + b*x^2)^(7/2))/(7*b^4) + (a + b*x^2)^(9/2)/(9*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7\sqrt{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3\sqrt{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx,x,x^2\right)\\
&= -\frac{a^3\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{3a^2\left(a+bx^2\right)^{5/2}}{5b^4}-\frac{3a\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{\left(a+bx^2\right)^{9/2}}{9b^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^{3/2}\left(-16a^3+24a^2bx^2-30ab^2x^4+35b^3x^6\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Sqrt[a + b*x^2],x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-16*a^3 + 24*a^2*b*x^2 - 30*a*b^2*x^4 + 35*b^3*x^6))/(3
15*b^4)
```

fricas [A] time = 0.87, size = 57, normalized size = 0.71

$$\frac{\left(35\, b^{4} x^{8} + 5\, a b^{3} x^{6} - 6\, a^{2} b^{2} x^{4} + 8\, a^{3} b x^{2} - 16\, a^{4}\right) \sqrt{b x^{2} + a}}{315\, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*x^8 + 5*a*b^3*x^6 - 6*a^2*b^2*x^4 + 8*a^3*b*x^2 - 16*a^4)*sqr
t(b*x^2 + a)/b^4
```

giac [A] time = 0.60, size = 57, normalized size = 0.71

$$\frac{35 \left(b x^{2} + a\right)^{\frac{9}{2}} - 135 \left(b x^{2} + a\right)^{\frac{7}{2}} a + 189 \left(b x^{2} + a\right)^{\frac{5}{2}} a^{2} - 105 \left(b x^{2} + a\right)^{\frac{3}{2}} a^{3}}{315\, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/
2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^{2} + a\right)^{\frac{3}{2}} \left(-35 b^{3} x^{6} + 30 a\, b^{2} x^{4} - 24 a^{2} b\, x^{2} + 16 a^{3}\right)}{315 b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(1/2),x)
```

```
[Out] -1/315*(b*x^2+a)^(3/2)*(-35*b^3*x^6+30*a*b^2*x^4-24*a^2*b*x^2+16*a^3)/b^4
```

maxima [A] time = 1.36, size = 73, normalized size = 0.91

$$\frac{\left(b x^{2} + a\right)^{\frac{3}{2}} x^{6}}{9\, b} - \frac{2 \left(b x^{2} + a\right)^{\frac{3}{2}} a x^{4}}{21\, b^{2}} + \frac{8 \left(b x^{2} + a\right)^{\frac{3}{2}} a^{2} x^{2}}{105\, b^{3}} - \frac{16 \left(b x^{2} + a\right)^{\frac{3}{2}} a^{3}}{315\, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(3/2)*x^6/b - 2/21*(b*x^2 + a)^(3/2)*a*x^4/b^2 + 8/105*(b*x
^2 + a)^(3/2)*a^2*x^2/b^3 - 16/315*(b*x^2 + a)^(3/2)*a^3/b^4
```

mupad [B] time = 4.59, size = 55, normalized size = 0.69

$$\sqrt{b\,x^2+a}\,\left(\frac{x^8}{9}-\frac{16\,a^4}{315\,b^4}+\frac{a\,x^6}{63\,b}-\frac{2\,a^2\,x^4}{105\,b^2}+\frac{8\,a^3\,x^2}{315\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*(x^8/9 - (16*a^4)/(315*b^4) + (a*x^6)/(63*b) - (2*a^2*x^4
)/(105*b^2) + (8*a^3*x^2)/(315*b^3))
```

sympy [A] time = 1.42, size = 110, normalized size = 1.38

$$\begin{cases} -\frac{16a^4\sqrt{a+bx^2}}{315b^4} + \frac{8a^3x^2\sqrt{a+bx^2}}{315b^3} - \frac{2a^2x^4\sqrt{a+bx^2}}{105b^2} + \frac{ax^6\sqrt{a+bx^2}}{63b} + \frac{x^8\sqrt{a+bx^2}}{9} & \text{for } b \neq 0 \\ \frac{\sqrt{a}\,x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**4*sqrt(a + b*x**2)/(315*b**4) + 8*a**3*x**2*sqrt(a + b*x*
*2)/(315*b**3) - 2*a**2*x**4*sqrt(a + b*x**2)/(105*b**2) + a*x**6*sqrt(a +
b*x**2)/(63*b) + x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*x**8/8, True)
)
```

3.354 $\int x^5\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{\left(a+bx^2\right)^{7/2}}{7b^3}-\frac{2a\left(a+bx^2\right)^{5/2}}{5b^3}$$

```
[Out] 1/3*a^2*(b*x^2+a)^(3/2)/b^3-2/5*a*(b*x^2+a)^(5/2)/b^3+1/7*(b*x^2+a)^(7/2)/b
^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{\left(a+bx^2\right)^{7/2}}{7b^3}-\frac{2a\left(a+bx^2\right)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(a + b*x^2)^(3/2))/(3*b^3) - (2*a*(a + b*x^2)^(5/2))/(5*b^3) + (a + b*
x^2)^(7/2)/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{a+bx^2}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int x^2\sqrt{a+bx}\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx,x,x^2\right)\\&=\frac{a^2\left(a+bx^2\right)^{3/2}}{3b^3}-\frac{2a\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{\left(a+bx^2\right)^{7/2}}{7b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^2\right)^{3/2}\left(8a^2-12abx^2+15b^2x^4\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^2],x]
```

```
[Out] ((a + b*x^2)^(3/2)*(8*a^2 - 12*a*b*x^2 + 15*b^2*x^4))/(105*b^3)
```

fricas [A] time = 0.55, size = 46, normalized size = 0.78

$$\frac{\left(15\,b^3x^6+3\,ab^2x^4-4\,a^2bx^2+8\,a^3\right)\sqrt{bx^2+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*x^6 + 3*a*b^2*x^4 - 4*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a)/b^3
```

giac [A] time = 0.59, size = 43, normalized size = 0.73

$$\frac{15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)
*a^2)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\left(15b^2x^4-12ab\,x^2+8a^2\right)}{105b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/105*(b*x^2+a)^(3/2)*(15*b^2*x^4-12*a*b*x^2+8*a^2)/b^3
```

maxima [A] time = 1.32, size = 53, normalized size = 0.90

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}x^4}{7\,b}-\frac{4\left(bx^2+a\right)^{\frac{3}{2}}ax^2}{35\,b^2}+\frac{8\left(bx^2+a\right)^{\frac{3}{2}}a^2}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(3/2)*x^4/b - 4/35*(b*x^2 + a)^(3/2)*a*x^2/b^2 + 8/105*(b*x
^2 + a)^(3/2)*a^2/b^3
```

mupad [B] time = 4.66, size = 44, normalized size = 0.75

$$\sqrt{b\,x^2+a}\,\left(\frac{x^6}{7}+\frac{8\,a^3}{105\,b^3}+\frac{a\,x^4}{35\,b}-\frac{4\,a^2\,x^2}{105\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*(x^6/7 + (8*a^3)/(105*b^3) + (a*x^4)/(35*b) - (4*a^2*x^2)
/(105*b^2))
```

sympy [A] time = 0.70, size = 87, normalized size = 1.47

$$\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^6}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2
)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne
(b, 0)), (sqrt(a)*x**6/6, True))
```

3.355 $\int x^3\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^2\right)^{5/2}}{5b^2}-\frac{a\left(a+bx^2\right)^{3/2}}{3b^2}$$

```
[Out] -1/3*a*(b*x^2+a)^(3/2)/b^2+1/5*(b*x^2+a)^(5/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{5/2}}{5b^2}-\frac{a\left(a+bx^2\right)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^2],x]
```

```
[Out] -(a*(a + b*x^2)^(3/2))/(3*b^2) + (a + b*x^2)^(5/2)/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\sqrt{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx,x,x^2\right)\\ &= -\frac{a\left(a+bx^2\right)^{3/2}}{3b^2}+\frac{\left(a+bx^2\right)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{3/2}\left(3bx^2-2a\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^2],x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-2*a + 3*b*x^2))/(15*b^2)
```

fricas [A] time = 0.61, size = 34, normalized size = 0.89

$$\frac{\left(3\,b^2x^4+abx^2-2\,a^2\right)\sqrt{bx^2+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*b^2*x^4 + a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a)/b^2
```

giac [A] time = 0.60, size = 29, normalized size = 0.76

$$\frac{3 \left(bx^{2}+a\right)^{\frac{5}{2}}-5 \left(bx^{2}+a\right)^{\frac{3}{2}} a}{15 \, b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/15*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)/b^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{\left(b \,x^{2}+a \right)^{\frac{3}{2}} \left(-3 b \,x^{2}+2 a \right)}{15 b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(1/2),x)
[Out] -1/15*(b*x^2+a)^(3/2)*(-3*b*x^2+2*a)/b^2
```

maxima [A] time = 1.34, size = 33, normalized size = 0.87

$$\frac{\left(bx^{2}+a\right)^{\frac{3}{2}} x^{2}}{5 \, b}-\frac{2 \left(bx^{2}+a\right)^{\frac{3}{2}} a}{15 \, b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 1/5*(b*x^2 + a)^(3/2)*x^2/b - 2/15*(b*x^2 + a)^(3/2)*a/b^2
```

mupad [B] time = 4.65, size = 33, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{x^4}{5}-\frac{2\,a^2}{15\,b^2}+\frac{a\,x^2}{15\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(1/2),x)
[Out] (a + b*x^2)^(1/2)*(x^4/5 - (2*a^2)/(15*b^2) + (a*x^2)/(15*b))
```

sympy [A] time = 0.32, size = 63, normalized size = 1.66

$$\begin{cases} - \frac{2 a^{2} \sqrt{a + b x^{2}}}{15 b^{2}} + \frac{a x^{2} \sqrt{a + b x^{2}}}{15 b} + \frac{x^{4} \sqrt{a + b x^{2}}}{5} & \text{for}\: b \neq 0 \\\frac{\sqrt{a} x^{4}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(1/2),x)
[Out] Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15
*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True))
```

3.356 $\int x\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^2\right)^{3/2}}{3b}$$

```
[Out] 1/3*(b*x^2+a)^(3/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^2],x]
[Out] (a + b*x^2)^(3/2)/(3*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{a+bx^2}\,dx=\frac{\left(a+bx^2\right)^{3/2}}{3b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^2],x]
[Out] (a + b*x^2)^(3/2)/(3*b)
```

fricas [A] time = 0.73, size = 14, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/3*(b*x^2 + a)^(3/2)/b
```

giac [A] time = 0.60, size = 14, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*x^2 + a)^(3/2)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(3/2)/b
```

maxima [A] time = 1.33, size = 14, normalized size = 0.78

$$\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(b*x^2 + a)^(3/2)/b
```

mupad [B] time = 4.63, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^{2}+a\right)^{3/2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(3/2)/(3*b)
```

sympy [A] time = 0.17, size = 39, normalized size = 2.17

$$\begin{cases} \frac{a\sqrt{a+bx^{2}}}{3b}+\frac{x^{2}\sqrt{a+bx^{2}}}{3} & \text{for}\; b\neq 0 \\ \frac{\sqrt{a}\,x^{2}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (
sqrt(a)*x**2/2, True))
```

3.357 $\int \frac{\sqrt{a+bx^2}}{x}\,dx$

Optimal. Leaf size=37

$$\sqrt{a+bx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 208}

$$\sqrt{a+bx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x,x]
```

```
[Out] Sqrt[a + b*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\
&= \sqrt{a+bx^2} + \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \sqrt{a+bx^2} + \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\
&= \sqrt{a+bx^2} - \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\sqrt{a+bx^2} - \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x,x]
```

```
[Out] Sqrt[a + b*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

fricas [A] time = 0.99, size = 77, normalized size = 2.08

$$\left[\frac{1}{2}\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+\sqrt{bx^2+a},\ \sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqrt(b*x
^2 + a), sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + sqrt(b*x^2 + a)]
```

giac [A] time = 0.59, size = 33, normalized size = 0.89

$$\frac{a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x^2 + a)
```

maple [A] time = 0.00, size = 39, normalized size = 1.05

$$-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\sqrt{b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x,x)
```

```
[Out] (b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

maxima [A] time = 1.36, size = 27, normalized size = 0.73

$$-\sqrt{a}\ \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)
```

mupad [B] time = 4.66, size = 29, normalized size = 0.78

$$\sqrt{b\,x^2+a}-\sqrt{a}\ \operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x,x)
```

```
[Out] (a + b*x^2)^(1/2) - a^(1/2)*atanh((a + b*x^2)^(1/2)/a^(1/2))
```

sympy [A] time = 1.41, size = 56, normalized size = 1.51

$$-\sqrt{a}\ \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{a}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x,x)
```

```
[Out] -sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) +
sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

3.358 $\int \frac{\sqrt{a+bx^2}}{x^3} \, dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{a+bx^2}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

[Out] -1/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)-1/2*(b*x^2+a)^(1/2)/x^2

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 208}

$$-\frac{\sqrt{a+bx^2}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x^2]/x^3,x]

[Out] -Sqrt[a + b*x^2]/(2*x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{2x^2} + \frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{2x^2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)\\
&= -\frac{\sqrt{a+bx^2}}{2x^2} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 1.26

$$-\frac{bx^2\sqrt{\frac{bx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)+a+bx^2}{2x^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^3,x]
```

```
[Out] -1/2*(a + b*x^2 + b*x^2*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a]])/(
x^2*Sqrt[a + b*x^2])
```

fricas [A] time = 0.49, size = 106, normalized size = 2.26

$$\left[\frac{\sqrt{a}\,bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,\sqrt{bx^2+a}\,a}{4\,ax^2}, \frac{\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\,a}{2\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*sqrt(b*x^2 + a)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2
 + a)) - sqrt(b*x^2 + a)*a)/(a*x^2)]
```

giac [A] time = 0.61, size = 46, normalized size = 0.98

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx^2+a}\,b}{x^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x^2 + a)*b/x^2)
/b
```

maple [A] time = 0.00, size = 63, normalized size = 1.34

$$-\frac{b\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2\sqrt{a}}+\frac{\sqrt{b\,x^2+a}\,b}{2a}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(3/2)-1/2/a^(1/2)*b*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))
/x)+1/2/a*b*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.37, size = 51, normalized size = 1.09

$$-\frac{b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,\sqrt{a}}+\frac{\sqrt{bx^2+a}\,b}{2\,a}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*sqrt(b*x^2 + a)*b/a - 1/
2*(b*x^2 + a)^(3/2)/(a*x^2)
```

mupad [B] time = 4.80, size = 35, normalized size = 0.74

$$-\frac{\sqrt{b\,x^2+a}}{2\,x^2}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^3,x)
```

```
[Out] - (a + b*x^2)^(1/2)/(2*x^2) - (b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/
2))
```

sympy [A] time = 1.93, size = 42, normalized size = 0.89

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - b*asinh(sqrt(a)/(sqrt(b)*x))/(2*sqrt(
a))
```

3.359 $\int \frac{\sqrt{a+bx^2}}{x^5}\,dx$

Optimal. Leaf size=71

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}-\frac{b\sqrt{a+bx^2}}{8ax^2}-\frac{\sqrt{a+bx^2}}{4x^4}$$

```
[Out] 1/8*b^2*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-1/4*(b*x^2+a)^(1/2)/x^4-1/
8*b*(b*x^2+a)^(1/2)/a/x^2
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}-\frac{b\sqrt{a+bx^2}}{8ax^2}-\frac{\sqrt{a+bx^2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^5,x]
```

```
[Out] -Sqrt[a + b*x^2]/(4*x^4) - (b*Sqrt[a + b*x^2])/(8*a*x^2) + (b^2*ArcTanh[Sqr
t[a + b*x^2]/Sqrt[a]])/(8*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{4x^4} + \frac{1}{8}b\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{4x^4} - \frac{b\sqrt{a+bx^2}}{8ax^2} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{16a}\\
&= -\frac{\sqrt{a+bx^2}}{4x^4} - \frac{b\sqrt{a+bx^2}}{8ax^2} - \frac{b\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{8a}\\
&= -\frac{\sqrt{a+bx^2}}{4x^4} - \frac{b\sqrt{a+bx^2}}{8ax^2} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.55

$$-\frac{b^2\left(a+bx^2\right)^{3/2}{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx^2}{a}+1\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^5,x]
```

```
[Out] -1/3*(b^2*(a + b*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^2)/a])/
a^3
```

fricas [A] time = 1.02, size = 131, normalized size = 1.85

$$\left[\frac{\sqrt{a}\,b^2x^4\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(abx^2+2a^2\right)\sqrt{bx^2+a}}{16a^2x^4}, -\frac{\sqrt{-a}\,b^2x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(abx^2+2a^2\right)\sqrt{bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(a)*b^2*x^4*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
- 2*(a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a^2*x^4), -1/8*(sqrt(-a)*b^2*x^4*ar
ctan(sqrt(-a)/sqrt(b*x^2 + a)) + (a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a^2*x^
4)]
```

giac [A] time = 0.60, size = 72, normalized size = 1.01

$$-\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}b^3+\sqrt{bx^2+a}\,ab^3}{ab^2x^4}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/8*(b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x^2 + a)^(3/2
)*b^3 + sqrt(b*x^2 + a)*a*b^3)/(a*b^2*x^4))/b
```

maple [A] time = 0.01, size = 85, normalized size = 1.20

$$\frac{b^2 \ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^2+a}\,b^2}{8a^2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}} b}{8a^2x^2}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(3/2)+1/8/a^2*b/x^2*(b*x^2+a)^(3/2)+1/8/a^(3/2)*b^2*ln
((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/8/a^2*b^2*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.32, size = 73, normalized size = 1.03

$$\frac{b^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{3}{2}}}-\frac{\sqrt{bx^2+a}\,b^2}{8\,a^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}} b}{8\,a^2x^2}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/8*sqrt(b*x^2 + a)*b^2/a^2
 + 1/8*(b*x^2 + a)^(3/2)*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(3/2)/(a*x^4)
```

mupad [B] time = 4.91, size = 54, normalized size = 0.76

$$\frac{b^2 \operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{3/2}}-\frac{\sqrt{b\,x^2+a}}{8\,x^4}-\frac{\left(b\,x^2+a\right)^{3/2}}{8\,a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^5,x)
```

```
[Out] (b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(3/2)) - (a + b*x^2)^(1/2)/(8*x
^4) - (a + b*x^2)^(3/2)/(8*a*x^4)
```

sympy [A] time = 3.73, size = 92, normalized size = 1.30

$$-\frac{a}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{b^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}}+\frac{b^2 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] -a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(8*x**3*sqrt(a/(b*x**2
) + 1)) - b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + b**2*asinh(sqrt(a)/(sqrt(
b)*x))/(8*a**(3/2))
```

3.360 $\int \frac{\sqrt{a+bx^2}}{x^7}\,dx$

Optimal. Leaf size=95

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}+\frac{b^2\sqrt{a+bx^2}}{16a^2x^2}-\frac{\sqrt{a+bx^2}}{6x^6}-\frac{b\sqrt{a+bx^2}}{24ax^4}$$

```
[Out] -1/16*b^3*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)-1/6*(b*x^2+a)^(1/2)/x^6-
1/24*b*(b*x^2+a)^(1/2)/a/x^4+1/16*b^2*(b*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{b^2\sqrt{a+bx^2}}{16a^2x^2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}-\frac{b\sqrt{a+bx^2}}{24ax^4}-\frac{\sqrt{a+bx^2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^7,x]
```

```
[Out] -Sqrt[a + b*x^2]/(6*x^6) - (b*Sqrt[a + b*x^2])/(24*a*x^4) + (b^2*Sqrt[a + b
*x^2])/(16*a^2*x^2) - (b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} + \frac{1}{12}b\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{16a}\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4} + \frac{b^2\sqrt{a+bx^2}}{16a^2x^2} + \frac{b^3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{32a^2}\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4} + \frac{b^2\sqrt{a+bx^2}}{16a^2x^2} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{16a^2}\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4} + \frac{b^2\sqrt{a+bx^2}}{16a^2x^2} - \frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.41

$$\frac{b^3\left(a+bx^2\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{bx^2}{a}+1\right)}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^7,x]
```

```
[Out] (b^3*(a + b*x^2)^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (b*x^2)/a])/(3*a^
4)
```

fricas [A] time = 0.97, size = 157, normalized size = 1.65

$$\left[\frac{3\sqrt{a}\,b^3x^6\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(3ab^2x^4-2a^2bx^2-8a^3\right)\sqrt{bx^2+a}}{96a^3x^6}, \frac{3\sqrt{-a}\,b^3x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(\ldots}{48\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(a)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + 2*(3*a*b^2*x^4 - 2*a^2*b*x^2 - 8*a^3)*sqrt(b*x^2 + a))/(a^3*x^6), 1/48*
(3*sqrt(-a)*b^3*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*a*b^2*x^4 - 2*a^2
*b*x^2 - 8*a^3)*sqrt(b*x^2 + a))/(a^3*x^6)]
```

giac [A] time = 0.68, size = 92, normalized size = 0.97

$$\frac{\frac{3b^4\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}b^4-8\left(bx^2+a\right)^{\frac{3}{2}}ab^4-3\sqrt{bx^2+a}\,a^2b^4}{a^2b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*b^4*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a
)^(5/2)*b^4 - 8*(b*x^2 + a)^(3/2)*a*b^4 - 3*sqrt(b*x^2 + a)*a^2*b^4)/(a^2*b
^3*x^6))/b
```

maple [A] time = 0.01, size = 105, normalized size = 1.11

$$-\frac{b^{3} \ln \left(\frac{2 a+2 \sqrt{b \,x^{2}+a}\, \sqrt{a}}{x}\right)}{16 a^{\frac{5}{2}}}+\frac{\sqrt{b \,x^{2}+a}\, b^{3}}{16 a^{3}}-\frac{\left(b \,x^{2}+a \right)^{\frac{3}{2}} b^{2}}{16 a^{3} x^{2}}+\frac{\left(b \,x^{2}+a \right)^{\frac{3}{2}} b}{8 a^{2} x^{4}}-\frac{\left(b \,x^{2}+a \right)^{\frac{3}{2}}}{6 a \,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^7,x)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(3/2)+1/8/a^2*b/x^4*(b*x^2+a)^(3/2)-1/16/a^3*b^2/x^2*(
b*x^2+a)^(3/2)-1/16/a^(5/2)*b^3*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+1/16/
a^3*b^3*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.34, size = 93, normalized size = 0.98

$$-\frac{b^{3} \operatorname{arsinh}\left(\frac{a}{\sqrt{a b}\, |x|}\right)}{16 \, a^{\frac{5}{2}}} + \frac{\sqrt{b x^{2} + a} b^{3}}{16 \, a^{3}} - \frac{{\left(b x^{2} + a\right)}^{\frac{3}{2}} b^{2}}{16 \, a^{3} x^{2}} + \frac{{\left(b x^{2} + a\right)}^{\frac{3}{2}} b}{8 \, a^{2} x^{4}} - \frac{{\left(b x^{2} + a\right)}^{\frac{3}{2}}}{6 \, a x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/16*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/16*sqrt(b*x^2 + a)*b^3/
a^3 - 1/16*(b*x^2 + a)^(3/2)*b^2/(a^3*x^2) + 1/8*(b*x^2 + a)^(3/2)*b/(a^2*x
^4) - 1/6*(b*x^2 + a)^(3/2)/(a*x^6)
```

mupad [B] time = 4.94, size = 74, normalized size = 0.78

$$\frac{{\left(b\,x^2+a\right)}^{5/2}}{16\,a^2\,x^6}-\frac{{\left(b\,x^2+a\right)}^{3/2}}{6\,a\,x^6}-\frac{\sqrt{b\,x^2+a}}{16\,x^6}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}}{16\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^7,x)
```

```
[Out] (b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(5/2)) - (a + b*x^2)^(1
/2)/(16*x^6) - (a + b*x^2)^(3/2)/(6*a*x^6) + (a + b*x^2)^(5/2)/(16*a^2*x^6)
```

sympy [A] time = 5.91, size = 117, normalized size = 1.23

$$- \frac{a}{6 \sqrt{b} x^{7} \sqrt{\frac{a}{b x^{2}} + 1}} - \frac{5 \sqrt{b}}{24 x^{5} \sqrt{\frac{a}{b x^{2}} + 1}} + \frac{b^{\frac{3}{2}}}{48 a x^{3} \sqrt{\frac{a}{b x^{2}} + 1}} + \frac{b^{\frac{5}{2}}}{16 a^{2} x \sqrt{\frac{a}{b x^{2}} + 1}} - \frac{b^{3} \operatorname{asinh}{\left(\frac{\sqrt{a}}{\sqrt{b} x} \right)}}{16 a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] -a/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*sqrt(b)/(24*x**5*sqrt(a/(b*x**
2) + 1)) + b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + b**(5/2)/(16*a**2*x*
sqrt(a/(b*x**2) + 1)) - b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5/2))
```

3.361 $\int x^4\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=94

$$\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}-\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}$$

```
[Out] 1/16*a^3*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-1/16*a^2*x*(b*x^2+a)^(1
/2)/b^2+1/24*a*x^3*(b*x^2+a)^(1/2)/b+1/6*x^5*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*x^2],x]
```

```
[Out] -(a^2*x*Sqrt[a + b*x^2])/(16*b^2) + (a*x^3*Sqrt[a + b*x^2])/(24*b) + (x^5*S
qrt[a + b*x^2])/6 + (a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\sqrt{a+bx^2}\,dx = \frac{1}{6}x^5\sqrt{a+bx^2} + \frac{1}{6}a\int \frac{x^4}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{ax^3\sqrt{a+bx^2}}{24b} + \frac{1}{6}x^5\sqrt{a+bx^2} - \frac{a^2\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{8b}$$
$$= -\frac{a^2x\sqrt{a+bx^2}}{16b^2} + \frac{ax^3\sqrt{a+bx^2}}{24b} + \frac{1}{6}x^5\sqrt{a+bx^2} + \frac{a^3\int \frac{1}{\sqrt{a+bx^2}}\,dx}{16b^2}$$
$$= -\frac{a^2x\sqrt{a+bx^2}}{16b^2} + \frac{ax^3\sqrt{a+bx^2}}{24b} + \frac{1}{6}x^5\sqrt{a+bx^2} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{16b^2}$$
$$= -\frac{a^2x\sqrt{a+bx^2}}{16b^2} + \frac{ax^3\sqrt{a+bx^2}}{24b} + \frac{1}{6}x^5\sqrt{a+bx^2} + \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.82

$$\frac{a^3\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{16b^{5/2}} + \sqrt{a+bx^2}\left(-\frac{a^2x}{16b^2} + \frac{ax^3}{24b} + \frac{x^5}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*x^2],x]
```

```
[Out] Sqrt[a + b*x^2]*(-1/16*(a^2*x)/b^2 + (a*x^3)/(24*b) + x^5/6) + (a^3*Log[b*x
 + Sqrt[b]*Sqrt[a + b*x^2]])/(16*b^(5/2))
```

fricas [A] time = 1.11, size = 146, normalized size = 1.55

$$\left[\frac{3\,a^3\sqrt{b}\,\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{b}\,x - a\right) + 2\left(8\,b^3x^5 + 2\,ab^2x^3 - 3\,a^2bx\right)\sqrt{bx^2+a}}{96\,b^3}, -\frac{3\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8
*b^3*x^5 + 2*a*b^2*x^3 - 3*a^2*b*x)*sqrt(b*x^2 + a))/b^3, -1/48*(3*a^3*sqrt
(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 2*a*b^2*x^3 - 3*a^2*
b*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.69, size = 64, normalized size = 0.68

$$\frac{1}{48}\left(2\left(4\,x^2 + \frac{a}{b}\right)x^2 - \frac{3\,a^2}{b^2}\right)\sqrt{bx^2+a}\,x - \frac{a^3\log\left(\left|-\sqrt{b}\,x + \sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2 + a/b)*x^2 - 3*a^2/b^2)*sqrt(b*x^2 + a)*x - 1/16*a^3*log(abs
(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 77, normalized size = 0.82

$$\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}x^3}{6b} + \frac{a^3\ln\left(\sqrt{b}\,x + \sqrt{b\,x^2+a}\right)}{16b^{\frac{5}{2}}} + \frac{\sqrt{b\,x^2+a}\,a^2x}{16b^2} - \frac{\left(b\,x^2+a\right)^{\frac{3}{2}}ax}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*x^3*(b*x^2+a)^(3/2)/b-1/8*a/b^2*x*(b*x^2+a)^(3/2)+1/16*a^2*x*(b*x^2+a)^
(1/2)/b^2+1/16*a^3/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.31, size = 69, normalized size = 0.73

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}x^3}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}ax}{8\,b^2}+\frac{\sqrt{bx^2+a}\,a^2x}{16\,b^2}+\frac{a^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(3/2)*x^3/b - 1/8*(b*x^2 + a)^(3/2)*a*x/b^2 + 1/16*sqrt(b*x
^2 + a)*a^2*x/b^2 + 1/16*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^4*(a + b*x^2)^(1/2), x)
```

sympy [A] time = 5.77, size = 117, normalized size = 1.24

$$-\frac{a^{\frac{5}{2}}x}{16b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{3}{2}}x^3}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{5\sqrt{a}\,x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}+\frac{bx^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/2),x)
```

```
[Out] -a**(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - a**(3/2)*x**3/(48*b*sqrt(1 + b*x
**2/a)) + 5*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a)) + a**3*asinh(sqrt(b)*x/sqr
t(a))/(16*b**(5/2)) + b*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.362 $\int x^2\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=70

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}$$

```
[Out] -1/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/8*a*x*(b*x^2+a)^(1/2)
/b+1/4*x^3*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{1}{4}x^3\sqrt{a+bx^2}+\frac{ax\sqrt{a+bx^2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^2],x]
```

```
[Out] (a*x*Sqrt[a + b*x^2])/(8*b) + (x^3*Sqrt[a + b*x^2])/4 - (a^2*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx^2}\,dx &= \frac{1}{4}x^3\sqrt{a+bx^2}+\frac{1}{4}a\int\frac{x^2}{\sqrt{a+bx^2}}\,dx\\ &= \frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}-\frac{a^2\int\frac{1}{\sqrt{a+bx^2}}\,dx}{8b}\\ &= \frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}-\frac{a^2\,\mathrm{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8b}\\ &= \frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.91

$$\sqrt{a+bx^2}\left(\frac{ax}{8b}+\frac{x^3}{4}\right)-\frac{a^2\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^2],x]
```

```
[Out] Sqrt[a + b*x^2]*((a*x)/(8*b) + x^3/4) - (a^2*Log[b*x + Sqrt[b]*Sqrt[a + b*x
^2]])/(8*b^(3/2))
```

fricas [A] time = 0.91, size = 119, normalized size = 1.70

$$\left[\frac{a^2\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(2\,b^2x^3+abx\right)\sqrt{bx^2+a}}{16\,b^2},\ \frac{a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\bigl(2\,b^2x^3+a}{8\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*b
^2*x^3 + a*b*x)*sqrt(b*x^2 + a))/b^2, 1/8*(a^2*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) + (2*b^2*x^3 + a*b*x)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.71, size = 50, normalized size = 0.71

$$\frac{1}{8}\sqrt{bx^2+a}\left(2\,x^2+\frac{a}{b}\right)x+\frac{a^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*x^2 + a/b)*x + 1/8*a^2*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

maple [A] time = 0.00, size = 57, normalized size = 0.81

$$-\frac{a^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}-\frac{\sqrt{b\,x^2+a}\;ax}{8b}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}x}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*x*(b*x^2+a)^(3/2)/b-1/8*a*x*(b*x^2+a)^(1/2)/b-1/8*a^2/b^(3/2)*ln(b^(1/2
)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.36, size = 49, normalized size = 0.70

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}x}{4\,b}-\frac{\sqrt{bx^2+a}\,ax}{8\,b}-\frac{a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*x/b - 1/8*sqrt(b*x^2 + a)*a*x/b - 1/8*a^2*arcsinh(b*x
/sqrt(a*b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{b\,x^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^(1/2), x)
```

sympy [A] time = 3.53, size = 92, normalized size = 1.31

$$\frac{a^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3\sqrt{a}\,x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{a^2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{bx^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/2),x)
```

```
[Out] a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*sqrt(a)*x**3/(8*sqrt(1 + b*x**2/a))
 - a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + b*x**5/(4*sqrt(a)*sqrt(1 +
b*x**2/a))
```

3.363 $\int \sqrt{a+bx^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] 1/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+1/2*x*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]
)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx^2}\,dx &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{1}{2}a\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.07

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

fricas [A] time = 0.90, size = 94, normalized size = 2.04

$$\left[\frac{2\sqrt{bx^2+a}\,bx + a\sqrt{b}\,\log\left(-2\,bx^2 - 2\sqrt{bx^2+a}\,\sqrt{b}\,x - a\right)}{4\,b}, \frac{\sqrt{bx^2+a}\,bx - a\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b, 1/2*(sqrt(b*x^2 + a)*b*x - a*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)))/b]
```

giac [A] time = 0.66, size = 37, normalized size = 0.80

$$\frac{1}{2}\sqrt{bx^2+a}\,x - \frac{a\log\left(\left|-\sqrt{b}\,x + \sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b
)
```

maple [A] time = 0.00, size = 36, normalized size = 0.78

$$\frac{a\ln\left(\sqrt{b}\,x + \sqrt{b\,x^2+a}\right)}{2\sqrt{b}} + \frac{\sqrt{b\,x^2+a}\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.33, size = 28, normalized size = 0.61

$$\frac{1}{2}\sqrt{bx^2+a}\,x + \frac{a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x + 1/2*a*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 4.67, size = 35, normalized size = 0.76

$$\frac{x\sqrt{b\,x^2+a}}{2} + \frac{a\,\ln\left(\sqrt{b}\,x + \sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2),x)
```

```
[Out] (x*(a + b*x^2)^(1/2))/2 + (a*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/(2*b^(1/2)
)
```

sympy **[A]** time = 1.83, size = 41, normalized size = 0.89

$$\frac{\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

3.364 $\int \frac{\sqrt{a+bx^2}}{x^2} dx$

Optimal. Leaf size=42

$$\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)-(b*x^2+a)^(1/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 217, 206}

$$\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x^2]/x) + Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^2}\,dx &= -\frac{\sqrt{a+bx^2}}{x}+b\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= -\frac{\sqrt{a+bx^2}}{x}+b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= -\frac{\sqrt{a+bx^2}}{x}+\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 1.50

$$-\frac{-\sqrt{a}\sqrt{b}\,x\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+a+bx^2}{x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^2,x]
```

```
[Out] -((a + b*x^2 - Sqrt[a]*Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sq
rt[a]])/(x*Sqrt[a + b*x^2]))
```

fricas [A] time = 0.75, size = 88, normalized size = 2.10

$$\left[\frac{\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,\sqrt{bx^2+a}}{2\,x},\,-\frac{\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*sqrt(b*
x^2 + a))/x, -(sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + sqrt(b*x^2 +
 a))/x]
```

giac [A] time = 0.65, size = 57, normalized size = 1.36

$$-\frac{1}{2}\,\sqrt{b}\,\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,a\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a*sqrt(b)/((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)
```

maple [A] time = 0.00, size = 54, normalized size = 1.29

$$\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)+\frac{\sqrt{b\,x^2+a}\,bx}{a}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^2,x)
```

```
[Out] -1/a/x*(b*x^2+a)^(3/2)+1/a*b*x*(b*x^2+a)^(1/2)+b^(1/2)*ln(b^(1/2)*x+(b*x^2+
a)^(1/2))
```

maxima [A] time = 1.37, size = 28, normalized size = 0.67

$$\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\sqrt{bx^2+a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] sqrt(b)*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)/x
```

mupad [B] time = 4.82, size = 56, normalized size = 1.33

$$-\frac{\sqrt{b\,x^2+a}}{x}-\frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^2,x)
```

```
[Out] - (a + b*x^2)^(1/2)/x - (b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^(
1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

sympy [A] time = 1.42, size = 56, normalized size = 1.33

$$-\frac{\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{bx}{\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -sqrt(a)/(x*sqrt(1 + b*x**2/a)) + sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - b*x/(s
qrt(a)*sqrt(1 + b*x**2/a))
```

3.365 $\int \frac{\sqrt{a+bx^2}}{x^4}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+bx^2\right)^{3/2}}{3ax^3}$$

```
[Out] -1/3*(b*x^2+a)^(3/2)/a/x^3
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\left(a+bx^2\right)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^4,x]
```

```
[Out] -(a + b*x^2)^(3/2)/(3*a*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{x^4}\,dx=-\frac{\left(a+bx^2\right)^{3/2}}{3ax^3}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{\left(a+bx^2\right)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^4,x]
```

```
[Out] -1/3*(a + b*x^2)^(3/2)/(a*x^3)
```

fricas [A] time = 0.81, size = 17, normalized size = 0.81

$$-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(b*x^2 + a)^(3/2)/(a*x^3)
```

giac [B] time = 0.68, size = 59, normalized size = 2.81

$$\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 b^{\frac{3}{2}}+a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) + a^2*b^(3/2))/((sqrt(b)*x -
 sqrt(b*x^2 + a))^2 - a)^3
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^4,x)
```

```
[Out] -1/3*(b*x^2+a)^(3/2)/a/x^3
```

maxima [A] time = 1.32, size = 17, normalized size = 0.81

$$-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(b*x^2 + a)^(3/2)/(a*x^3)
```

mupad [B] time = 4.57, size = 17, normalized size = 0.81

$$-\frac{\left(b\,x^2+a\right)^{3/2}}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^4,x)
```

```
[Out] -(a + b*x^2)^(3/2)/(3*a*x^3)
```

sympy [B] time = 0.71, size = 42, normalized size = 2.00

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**4,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - b**(3/2)*sqrt(a/(b*x**2) + 1)/(3*a
)
```

3.366 $\int \frac{\sqrt{a+bx^2}}{x^6} dx$

Optimal. Leaf size=44

$$\frac{2b\left(a+bx^2\right)^{3/2}}{15a^2x^3}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}$$

```
[Out] -1/5*(b*x^2+a)^(3/2)/a/x^5+2/15*b*(b*x^2+a)^(3/2)/a^2/x^3
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2b\left(a+bx^2\right)^{3/2}}{15a^2x^3}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^6,x]
```

```
[Out] -(a + b*x^2)^(3/2)/(5*a*x^5) + (2*b*(a + b*x^2)^(3/2))/(15*a^2*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{x^6}\,dx &= -\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}-\frac{(2b)\int \frac{\sqrt{a+bx^2}}{x^4}\,dx}{5a}\\ &= -\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}+\frac{2b\left(a+bx^2\right)^{3/2}}{15a^2x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{\left(a+bx^2\right)^{3/2}\left(2bx^2-3a\right)}{15a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^6,x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-3*a + 2*b*x^2))/(15*a^2*x^5)
```

fricas [A] time = 0.83, size = 38, normalized size = 0.86

$$\frac{\left(2\,b^2x^4-abx^2-3\,a^2\right)\sqrt{bx^2+a}}{15\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(2*b^2*x^4 - a*b*x^2 - 3*a^2)*sqrt(b*x^2 + a)/(a^2*x^5)
```

giac [B] time = 0.68, size = 112, normalized size = 2.55

$$\frac{4\left(15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6 b^{\frac{5}{2}}+5\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4 ab^{\frac{5}{2}}+5\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2 a^2b^{\frac{5}{2}}-a^3b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 4/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2) + 5*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*a*b^(5/2) + 5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2) - a^3*b
^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

maple [A] time = 0.00, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\left(-2b\,x^2+3a\right)}{15a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^6,x)
```

```
[Out] -1/15*(b*x^2+a)^(3/2)*(-2*b*x^2+3*a)/a^2/x^5
```

maxima [A] time = 1.28, size = 36, normalized size = 0.82

$$\frac{2\left(bx^2+a\right)^{\frac{3}{2}}b}{15\,a^2x^3}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^2 + a)^(3/2)*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(3/2)/(a*x^5)
```

mupad [B] time = 4.77, size = 37, normalized size = 0.84

$$-\frac{\sqrt{b\,x^2+a}\,\left(3\,a^2+a\,b\,x^2-2\,b^2\,x^4\right)}{15\,a^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^6,x)
```

```
[Out] -((a + b*x^2)^(1/2)*(3*a^2 - 2*b^2*x^4 + a*b*x^2))/(15*a^2*x^5)
```

sympy [A] time = 0.91, size = 68, normalized size = 1.55

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2}+\frac{2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**6,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*
a*x**2) + 2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2)
```

3.367 $\int \frac{\sqrt{a+bx^2}}{x^8}\,dx$

Optimal. Leaf size=68

$$-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^3}+\frac{4b\left(a+bx^2\right)^{3/2}}{35a^2x^5}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}$$

```
[Out] -1/7*(b*x^2+a)^(3/2)/a/x^7+4/35*b*(b*x^2+a)^(3/2)/a^2/x^5-8/105*b^2*(b*x^2+
a)^(3/2)/a^3/x^3
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^3}+\frac{4b\left(a+bx^2\right)^{3/2}}{35a^2x^5}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^8,x]
```

```
[Out] -(a + b*x^2)^(3/2)/(7*a*x^7) + (4*b*(a + b*x^2)^(3/2))/(35*a^2*x^5) - (8*b^
2*(a + b*x^2)^(3/2))/(105*a^3*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^8}\,dx &= -\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}-\frac{(4b)\int \frac{\sqrt{a+bx^2}}{x^6}\,dx}{7a}\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}+\frac{4b\left(a+bx^2\right)^{3/2}}{35a^2x^5}+\frac{\left(8b^2\right)\int \frac{\sqrt{a+bx^2}}{x^4}\,dx}{35a^2}\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}+\frac{4b\left(a+bx^2\right)^{3/2}}{35a^2x^5}-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^2\right)^{3/2}\left(15a^2-12abx^2+8b^2x^4\right)}{105a^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^8,x]
```

```
[Out] -1/105*((a + b*x^2)^(3/2)*(15*a^2 - 12*a*b*x^2 + 8*b^2*x^4))/(a^3*x^7)
```

fricas [A] time = 0.98, size = 49, normalized size = 0.72

$$-\frac{\left(8\,b^3x^6 - 4\,ab^2x^4 + 3\,a^2bx^2 + 15\,a^3\right)\sqrt{bx^2 + a}}{105\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(8*b^3*x^6 - 4*a*b^2*x^4 + 3*a^2*b*x^2 + 15*a^3)*sqrt(b*x^2 + a)/(a^
3*x^7)
```

giac [B] time = 0.65, size = 138, normalized size = 2.03

$$\frac{16\left(70\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^8 b^{\frac{7}{2}} + 35\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^6 ab^{\frac{7}{2}} + 21\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^4 a^2b^{\frac{7}{2}} - 7\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)\right.}{105\left(\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^2 - a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(7/2) + 35*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*a*b^(7/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(7/2) - 7
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(7/2) + a^4*b^(7/2))/((sqrt(b)*x - s
qrt(b*x^2 + a))^2 - a)^7
```

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^2 + a\right)^{\frac{3}{2}}\left(8b^2x^4 - 12ab\,x^2 + 15a^2\right)}{105a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^8,x)
```

```
[Out] -1/105*(b*x^2+a)^(3/2)*(8*b^2*x^4-12*a*b*x^2+15*a^2)/a^3/x^7
```

maxima [A] time = 1.32, size = 56, normalized size = 0.82

$$-\frac{8\left(bx^2 + a\right)^{\frac{3}{2}}b^2}{105\,a^3x^3} + \frac{4\left(bx^2 + a\right)^{\frac{3}{2}}b}{35\,a^2x^5} - \frac{\left(bx^2 + a\right)^{\frac{3}{2}}}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] -8/105*(b*x^2 + a)^(3/2)*b^2/(a^3*x^3) + 4/35*(b*x^2 + a)^(3/2)*b/(a^2*x^5)
 - 1/7*(b*x^2 + a)^(3/2)/(a*x^7)
```

mupad [B] time = 4.71, size = 73, normalized size = 1.07

$$\frac{4\,b^2\,\sqrt{b\,x^2 + a}}{105\,a^2\,x^3} - \frac{b\,\sqrt{b\,x^2 + a}}{35\,a\,x^5} - \frac{\sqrt{b\,x^2 + a}}{7\,x^7} - \frac{8\,b^3\,\sqrt{b\,x^2 + a}}{105\,a^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^8,x)
```

```
[Out] (4*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^3) - (b*(a + b*x^2)^(1/2))/(35*a*x^5)
- (a + b*x^2)^(1/2)/(7*x^7) - (8*b^3*(a + b*x^2)^(1/2))/(105*a^3*x)
```

sympy [B] time = 1.30, size = 359, normalized size = 5.28

$$-\frac{15a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{33a^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{17a^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**8,x)
```

```
[Out] -15*a**5*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*
x**8 + 105*a**3*b**6*x**10) - 33*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*a**3*b*
*(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8
+ 105*a**3*b**6*x**10) - 3*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a*
*5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*a*b**(17/2)*x
**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**
3*b**6*x**10) - 8*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6
+ 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10)
```

3.368 $\int \frac{\sqrt{a+bx^2}}{x^{10}} \, dx$

Optimal. Leaf size=92

$$\frac{16b^3\left(a+bx^2\right)^{3/2}}{315a^4x^3}-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^5}+\frac{2b\left(a+bx^2\right)^{3/2}}{21a^2x^7}-\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}$$

```
[Out] -1/9*(b*x^2+a)^(3/2)/a/x^9+2/21*b*(b*x^2+a)^(3/2)/a^2/x^7-8/105*b^2*(b*x^2+
a)^(3/2)/a^3/x^5+16/315*b^3*(b*x^2+a)^(3/2)/a^4/x^3
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{16b^3\left(a+bx^2\right)^{3/2}}{315a^4x^3}-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^5}+\frac{2b\left(a+bx^2\right)^{3/2}}{21a^2x^7}-\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^10,x]
```

```
[Out] -(a + b*x^2)^(3/2)/(9*a*x^9) + (2*b*(a + b*x^2)^(3/2))/(21*a^2*x^7) - (8*b^
2*(a + b*x^2)^(3/2))/(105*a^3*x^5) + (16*b^3*(a + b*x^2)^(3/2))/(315*a^4*x^
3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^{10}} \, dx &= -\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}-\frac{(2b)\int \frac{\sqrt{a+bx^2}}{x^8} \, dx}{3a}\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}+\frac{2b\left(a+bx^2\right)^{3/2}}{21a^2x^7}+\frac{\left(8b^2\right)\int \frac{\sqrt{a+bx^2}}{x^6} \, dx}{21a^2}\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}+\frac{2b\left(a+bx^2\right)^{3/2}}{21a^2x^7}-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^5}-\frac{\left(16b^3\right)\int \frac{\sqrt{a+bx^2}}{x^4} \, dx}{105a^3}\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}+\frac{2b\left(a+bx^2\right)^{3/2}}{21a^2x^7}-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^5}+\frac{16b^3\left(a+bx^2\right)^{3/2}}{315a^4x^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.58

$$\frac{\left(a+bx^2\right)^{3/2}\left(-35a^3+30a^2bx^2-24ab^2x^4+16b^3x^6\right)}{315a^4x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^10,x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-35*a^3 + 30*a^2*b*x^2 - 24*a*b^2*x^4 + 16*b^3*x^6))/(3
15*a^4*x^9)
```

fricas [A] time = 0.86, size = 60, normalized size = 0.65

$$\frac{\left(16\, b^{4} x^{8} - 8\, a b^{3} x^{6} + 6\, a^{2} b^{2} x^{4} - 5\, a^{3} b x^{2} - 35\, a^{4}\right)\sqrt{b x^{2} + a}}{315\, a^{4} x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(16*b^4*x^8 - 8*a*b^3*x^6 + 6*a^2*b^2*x^4 - 5*a^3*b*x^2 - 35*a^4)*sqr
t(b*x^2 + a)/(a^4*x^9)
```

giac [B] time = 0.70, size = 166, normalized size = 1.80

$$\frac{32\left(315\left(\sqrt{b}\,x - \sqrt{b x^{2} + a}\right)^{10} b^{\frac{9}{2}} + 189\left(\sqrt{b}\,x - \sqrt{b x^{2} + a}\right)^{8} a b^{\frac{9}{2}} + 84\left(\sqrt{b}\,x - \sqrt{b x^{2} + a}\right)^{6} a^{2} b^{\frac{9}{2}} - 36\left(\sqrt{b}\,x - \sqrt{b\right.\right.}{315\left(\left(\sqrt{b}\,x - \sqrt{b x^{2} + a}\right)^{2} - a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] 32/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(9/2) + 189*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*a*b^(9/2) + 84*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(9/2)
- 36*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(9/2) + 9*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a^4*b^(9/2) - a^5*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^
9
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-16 b^{3} x^{6}+24 a\,b^{2} x^{4}-30 a^{2} b\,x^{2}+35 a^{3}\right)}{315 a^{4} x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^10,x)
```

```
[Out] -1/315*(b*x^2+a)^(3/2)*(-16*b^3*x^6+24*a*b^2*x^4-30*a^2*b*x^2+35*a^3)/x^9/a
^4
```

maxima [A] time = 1.34, size = 76, normalized size = 0.83

$$\frac{16\left(b x^{2} + a\right)^{\frac{3}{2}} b^{3}}{315\, a^{4} x^{3}} - \frac{8\left(b x^{2} + a\right)^{\frac{3}{2}} b^{2}}{105\, a^{3} x^{5}} + \frac{2\left(b x^{2} + a\right)^{\frac{3}{2}} b}{21\, a^{2} x^{7}} - \frac{\left(b x^{2} + a\right)^{\frac{3}{2}}}{9\, a x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] 16/315*(b*x^2 + a)^(3/2)*b^3/(a^4*x^3) - 8/105*(b*x^2 + a)^(3/2)*b^2/(a^3*x
^5) + 2/21*(b*x^2 + a)^(3/2)*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(3/2)/(a*x^9)
```

mupad [B] time = 4.95, size = 93, normalized size = 1.01

$$\frac{2\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^5}-\frac{b\,\sqrt{b\,x^2+a}}{63\,a\,x^7}-\frac{\sqrt{b\,x^2+a}}{9\,x^9}-\frac{8\,b^3\,\sqrt{b\,x^2+a}}{315\,a^3\,x^3}+\frac{16\,b^4\,\sqrt{b\,x^2+a}}{315\,a^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^10,x)
```

```
[Out] (2*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^5) - (b*(a + b*x^2)^(1/2))/(63*a*x^7)
- (a + b*x^2)^(1/2)/(9*x^9) - (8*b^3*(a + b*x^2)^(1/2))/(315*a^3*x^3) + (16
*b^4*(a + b*x^2)^(1/2))/(315*a^4*x)
```

sympy [B] time = 1.74, size = 575, normalized size = 6.25

$$-\frac{35a^7b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}-\frac{110a^6b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**10,x)
```

```
[Out] -35*a**7*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**1
0*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*a**6*b**(21/2)
*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945
*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*a**5*b**(23/2)*x**4*sqrt(a/
(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x
**12 + 315*a**4*b**12*x**14) - 40*a**4*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/
(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**
4*b**12*x**14) + 5*a**3*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*
x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14)
+ 30*a**2*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a*
*6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) + 40*a*b**(31
/2)*x**12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) + 16*b**(33/2)*x**14*sqrt(a/(
b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x*
*12 + 315*a**4*b**12*x**14)
```

3.369 $\int x^7 \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=80

$$-\frac{a^3 \left(a+bx^2\right)^{5/2}}{5b^4}+\frac{3a^2 \left(a+bx^2\right)^{7/2}}{7b^4}+\frac{\left(a+bx^2\right)^{11/2}}{11b^4}-\frac{a \left(a+bx^2\right)^{9/2}}{3b^4}$$

```
[Out] -1/5*a^3*(b*x^2+a)^(5/2)/b^4+3/7*a^2*(b*x^2+a)^(7/2)/b^4-1/3*a*(b*x^2+a)^(9
/2)/b^4+1/11*(b*x^2+a)^(11/2)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2 \left(a+bx^2\right)^{7/2}}{7b^4}-\frac{a^3 \left(a+bx^2\right)^{5/2}}{5b^4}+\frac{\left(a+bx^2\right)^{11/2}}{11b^4}-\frac{a \left(a+bx^2\right)^{9/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(3/2),x]
```

```
[Out] -(a^3*(a + b*x^2)^(5/2))/(5*b^4) + (3*a^2*(a + b*x^2)^(7/2))/(7*b^4) - (a*(
a + b*x^2)^(9/2))/(3*b^4) + (a + b*x^2)^(11/2)/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7 \left(a+bx^2\right)^{3/2} dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{3/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3(a+bx)^{3/2}}{b^3}+\frac{3a^2(a+bx)^{5/2}}{b^3}-\frac{3a(a+bx)^{7/2}}{b^3}+\frac{(a+bx)^{9/2}}{b^3}\right)dx,x,x^2\right)\\ &= -\frac{a^3 \left(a+bx^2\right)^{5/2}}{5b^4}+\frac{3a^2 \left(a+bx^2\right)^{7/2}}{7b^4}-\frac{a \left(a+bx^2\right)^{9/2}}{3b^4}+\frac{\left(a+bx^2\right)^{11/2}}{11b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^{5/2} \left(-16a^3+40a^2bx^2-70ab^2x^4+105b^3x^6\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(3/2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-16*a^3 + 40*a^2*b*x^2 - 70*a*b^2*x^4 + 105*b^3*x^6))/(
1155*b^4)
```

fricas [A] time = 0.99, size = 68, normalized size = 0.85

$$\frac{\left(105\,b^5x^{10}+140\,ab^4x^8+5\,a^2b^3x^6-6\,a^3b^2x^4+8\,a^4bx^2-16\,a^5\right)\sqrt{bx^2+a}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(105*b^5*x^10 + 140*a*b^4*x^8 + 5*a^2*b^3*x^6 - 6*a^3*b^2*x^4 + 8*a^
4*b*x^2 - 16*a^5)*sqrt(b*x^2 + a)/b^4
```

giac [A] time = 0.68, size = 57, normalized size = 0.71

$$\frac{105\left(bx^2+a\right)^{\frac{11}{2}}-385\left(bx^2+a\right)^{\frac{9}{2}}a+495\left(bx^2+a\right)^{\frac{7}{2}}a^2-231\left(bx^2+a\right)^{\frac{5}{2}}a^3}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1155*(105*(b*x^2 + a)^(11/2) - 385*(b*x^2 + a)^(9/2)*a + 495*(b*x^2 + a)^
(7/2)*a^2 - 231*(b*x^2 + a)^(5/2)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\left(-105b^3x^6+70a\,b^2x^4-40a^2b\,x^2+16a^3\right)}{1155b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(3/2),x)
```

```
[Out] -1/1155*(b*x^2+a)^(5/2)*(-105*b^3*x^6+70*a*b^2*x^4-40*a^2*b*x^2+16*a^3)/b^4
```

maxima [A] time = 1.33, size = 73, normalized size = 0.91

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}x^6}{11\,b}-\frac{2\left(bx^2+a\right)^{\frac{5}{2}}ax^4}{33\,b^2}+\frac{8\left(bx^2+a\right)^{\frac{5}{2}}a^2x^2}{231\,b^3}-\frac{16\left(bx^2+a\right)^{\frac{5}{2}}a^3}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^2 + a)^(5/2)*x^6/b - 2/33*(b*x^2 + a)^(5/2)*a*x^4/b^2 + 8/231*(b*
x^2 + a)^(5/2)*a^2*x^2/b^3 - 16/1155*(b*x^2 + a)^(5/2)*a^3/b^4
```

mupad [B] time = 4.60, size = 64, normalized size = 0.80

$$\sqrt{b\,x^2+a}\,\left(\frac{4\,a\,x^8}{33}+\frac{b\,x^{10}}{11}-\frac{16\,a^5}{1155\,b^4}+\frac{a^2\,x^6}{231\,b}-\frac{2\,a^3\,x^4}{385\,b^2}+\frac{8\,a^4\,x^2}{1155\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((4*a*x^8)/33 + (b*x^10)/11 - (16*a^5)/(1155*b^4) + (a^2*
x^6)/(231*b) - (2*a^3*x^4)/(385*b^2) + (8*a^4*x^2)/(1155*b^3))
```

sympy [A] time = 3.81, size = 133, normalized size = 1.66

$$\begin{cases} -\frac{16a^5\sqrt{a+bx^2}}{1155b^4} + \frac{8a^4x^2\sqrt{a+bx^2}}{1155b^3} - \frac{2a^3x^4\sqrt{a+bx^2}}{385b^2} + \frac{a^2x^6\sqrt{a+bx^2}}{231b} + \frac{4ax^8\sqrt{a+bx^2}}{33} + \frac{bx^{10}\sqrt{a+bx^2}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-16*a**5*sqrt(a + b*x**2)/(1155*b**4) + 8*a**4*x**2*sqrt(a + b*x
**2)/(1155*b**3) - 2*a**3*x**4*sqrt(a + b*x**2)/(385*b**2) + a**2*x**6*sqrt
(a + b*x**2)/(231*b) + 4*a*x**8*sqrt(a + b*x**2)/33 + b*x**10*sqrt(a + b*x*
*2)/11, Ne(b, 0)), (a**(3/2)*x**8/8, True))
```

3.370 $\int x^5 \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{\left(a+bx^2\right)^{9/2}}{9b^3}-\frac{2a\left(a+bx^2\right)^{7/2}}{7b^3}$$

```
[Out] 1/5*a^2*(b*x^2+a)^(5/2)/b^3-2/7*a*(b*x^2+a)^(7/2)/b^3+1/9*(b*x^2+a)^(9/2)/b
^3
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{\left(a+bx^2\right)^{9/2}}{9b^3}-\frac{2a\left(a+bx^2\right)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(3/2),x]
```

```
[Out] (a^2*(a + b*x^2)^(5/2))/(5*b^3) - (2*a*(a + b*x^2)^(7/2))/(7*b^3) + (a + b*
x^2)^(9/2)/(9*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5\left(a+bx^2\right)^{3/2} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^{3/2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{a^2(a+bx)^{3/2}}{b^2}-\frac{2a(a+bx)^{5/2}}{b^2}+\frac{(a+bx)^{7/2}}{b^2}\right)dx,x,x^2\right)\\
&= \frac{a^2\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{2a\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{\left(a+bx^2\right)^{9/2}}{9b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^2\right)^{5/2}\left(8a^2-20abx^2+35b^2x^4\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(3/2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(8*a^2 - 20*a*b*x^2 + 35*b^2*x^4))/(315*b^3)
```

fricas [A] time = 0.85, size = 57, normalized size = 0.97

$$\frac{\left(35\,b^4x^8+50\,ab^3x^6+3\,a^2b^2x^4-4\,a^3bx^2+8\,a^4\right)\sqrt{bx^2+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*x^8 + 50*a*b^3*x^6 + 3*a^2*b^2*x^4 - 4*a^3*b*x^2 + 8*a^4)*sqr
t(b*x^2 + a)/b^3
```

giac [A] time = 0.64, size = 43, normalized size = 0.73

$$\frac{35\left(bx^2+a\right)^{\frac{9}{2}}-90\left(bx^2+a\right)^{\frac{7}{2}}a+63\left(bx^2+a\right)^{\frac{5}{2}}a^2}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*(b*x^2 + a)^(9/2) - 90*(b*x^2 + a)^(7/2)*a + 63*(b*x^2 + a)^(5/2)
*a^2)/b^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\left(35b^2x^4-20ab\,x^2+8a^2\right)}{315b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(5/2)*(35*b^2*x^4-20*a*b*x^2+8*a^2)/b^3
```

maxima [A] time = 1.32, size = 53, normalized size = 0.90

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}x^4}{9\,b}-\frac{4\left(bx^2+a\right)^{\frac{5}{2}}ax^2}{63\,b^2}+\frac{8\left(bx^2+a\right)^{\frac{5}{2}}a^2}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(5/2)*x^4/b - 4/63*(b*x^2 + a)^(5/2)*a*x^2/b^2 + 8/315*(b*x
^2 + a)^(5/2)*a^2/b^3
```

mupad [B] time = 4.66, size = 53, normalized size = 0.90

$$\sqrt{b\,x^2+a}\,\left(\frac{10\,a\,x^6}{63}+\frac{b\,x^8}{9}+\frac{8\,a^4}{315\,b^3}+\frac{a^2\,x^4}{105\,b}-\frac{4\,a^3\,x^2}{315\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((10*a*x^6)/63 + (b*x^8)/9 + (8*a^4)/(315*b^3) + (a^2*x^4
)/(105*b) - (4*a^3*x^2)/(315*b^2))
```

sympy [A] time = 2.24, size = 109, normalized size = 1.85

$$\begin{cases} \frac{8a^4\sqrt{a+bx^2}}{315b^3} - \frac{4a^3x^2\sqrt{a+bx^2}}{315b^2} + \frac{a^2x^4\sqrt{a+bx^2}}{105b} + \frac{10ax^6\sqrt{a+bx^2}}{63} + \frac{bx^8\sqrt{a+bx^2}}{9} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((8*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*a**3*x**2*sqrt(a + b*x**2
)/(315*b**2) + a**2*x**4*sqrt(a + b*x**2)/(105*b) + 10*a*x**6*sqrt(a + b*x*
*2)/63 + b*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(3/2)*x**6/6, True))
```

3.371 $\int x^3 \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^2\right)^{7/2}}{7b^2}-\frac{a\left(a+bx^2\right)^{5/2}}{5b^2}$$

```
[Out] -1/5*a*(b*x^2+a)^(5/2)/b^2+1/7*(b*x^2+a)^(7/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{7/2}}{7b^2}-\frac{a\left(a+bx^2\right)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(3/2),x]
```

```
[Out] -(a*(a + b*x^2)^(5/2))/(5*b^2) + (a + b*x^2)^(7/2)/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^2\right)^{3/2} dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^{3/2} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a(a+bx)^{3/2}}{b}+\frac{(a+bx)^{5/2}}{b}\right) dx, x, x^2\right)\\
&= -\frac{a\left(a+bx^2\right)^{5/2}}{5b^2}+\frac{\left(a+bx^2\right)^{7/2}}{7b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{5/2}\left(5bx^2-2a\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(3/2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-2*a + 5*b*x^2))/(35*b^2)
```

fricas [A] time = 0.97, size = 45, normalized size = 1.18

$$\frac{\left(5\,b^3x^6+8\,ab^2x^4+a^2bx^2-2\,a^3\right)\sqrt{bx^2+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^3*x^6 + 8*a*b^2*x^4 + a^2*b*x^2 - 2*a^3)*sqrt(b*x^2 + a)/b^2
```

giac [A] time = 0.65, size = 29, normalized size = 0.76

$$\frac{5\left(bx^2+a\right)^{\frac{7}{2}}-7\left(bx^2+a\right)^{\frac{5}{2}}a}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/35*(5*(b*x^2 + a)^(7/2) - 7*(b*x^2 + a)^(5/2)*a)/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\left(-5b\,x^2+2a\right)}{35b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(3/2),x)
```

```
[Out] -1/35*(b*x^2+a)^(5/2)*(-5*b*x^2+2*a)/b^2
```

maxima [A] time = 1.33, size = 33, normalized size = 0.87

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}x^2}{7\,b}-\frac{2\left(bx^2+a\right)^{\frac{5}{2}}a}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(5/2)*x^2/b - 2/35*(b*x^2 + a)^(5/2)*a/b^2
```

mupad [B] time = 4.64, size = 42, normalized size = 1.11

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,a\,x^4}{35}+\frac{b\,x^6}{7}-\frac{2\,a^3}{35\,b^2}+\frac{a^2\,x^2}{35\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a*x^4)/35 + (b*x^6)/7 - (2*a^3)/(35*b^2) + (a^2*x^2)/
(35*b))
```

sympy [A] time = 1.13, size = 85, normalized size = 2.24

$$\begin{cases} -\frac{2a^3\sqrt{a+bx^2}}{35b^2}+\frac{a^2x^2\sqrt{a+bx^2}}{35b}+\frac{8ax^4\sqrt{a+bx^2}}{35}+\frac{bx^6\sqrt{a+bx^2}}{7} & \text{for } b\neq 0 \\ \frac{a^{\frac{3}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-2*a**3*sqrt(a + b*x**2)/(35*b**2) + a**2*x**2*sqrt(a + b*x**2)/
(35*b) + 8*a*x**4*sqrt(a + b*x**2)/35 + b*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)
), (a**(3/2)*x**4/4, True))
```

3.372 $\int x\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

```
[Out] 1/5*(b*x^2+a)^(5/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(3/2),x]
[Out] (a + b*x^2)^(5/2)/(5*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{3/2}\,dx=\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(3/2),x]
[Out] (a + b*x^2)^(5/2)/(5*b)
```

fricas [B] time = 0.94, size = 32, normalized size = 1.78

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{bx^2+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] 1/5*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/b
```

giac [A] time = 0.67, size = 14, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/5*(b*x^2 + a)^(5/2)/b
```

maple [A]   time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/5*(b*x^2+a)^(5/2)/b
```

maxima [A]   time = 1.33, size = 14, normalized size = 0.78

$$\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^2 + a)^(5/2)/b
```

mupad [B]   time = 4.57, size = 14, normalized size = 0.78

$$\frac{{\left(b\,x^2+a\right)}^{5/2}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(5/2)/(5*b)
```

sympy [A]   time = 0.59, size = 61, normalized size = 3.39

$$\begin{cases} \frac{a^{2}\sqrt{a+bx^{2}}}{5b}+\frac{2ax^{2}\sqrt{a+bx^{2}}}{5}+\frac{bx^{4}\sqrt{a+bx^{2}}}{5} & \text{for}\: b\neq 0 \\ \frac{a^{\frac{3}{2}}x^{2}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((a**2*sqrt(a + b*x**2)/(5*b) + 2*a*x**2*sqrt(a + b*x**2)/5 + b*x*
*4*sqrt(a + b*x**2)/5, Ne(b, 0)), (a**(3/2)*x**2/2, True))
```

3.373 $\int \frac{\left(a+bx^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=54

$$a^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+a\sqrt{a+bx^2}+\frac{1}{3}\left(a+bx^2\right)^{3/2}$$

```
[Out] 1/3*(b*x^2+a)^(3/2)-a^(3/2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+a*(b*x^2+a)^(1
/2)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 208}

$$a^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+a\sqrt{a+bx^2}+\frac{1}{3}\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x,x]
```

```
[Out] a*Sqrt[a + b*x^2] + (a + b*x^2)^(3/2)/3 - a^(3/2)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,x^2\right)\\ &= \frac{1}{3}\left(a+bx^2\right)^{3/2}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^2\right)\\ &= a\sqrt{a+bx^2}+\frac{1}{3}\left(a+bx^2\right)^{3/2}+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= a\sqrt{a+bx^2}+\frac{1}{3}\left(a+bx^2\right)^{3/2}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\ &= a\sqrt{a+bx^2}+\frac{1}{3}\left(a+bx^2\right)^{3/2}-a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.93

$$\frac{1}{3}\sqrt{a+bx^2}\left(4a+bx^2\right)-a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x,x]
[Out] (Sqrt[a + b*x^2]*(4*a + b*x^2))/3 - a^(3/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]
```

fricas [A] time = 0.85, size = 100, normalized size = 1.85

$$\left[\frac{1}{2}\,a^{\frac{3}{2}}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+\frac{1}{3}\left(bx^2+4\,a\right)\sqrt{bx^2+a}\,,\,\sqrt{-a}\,a\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\frac{1}{3}\left(bx^2+4\,a\right)\sqrt{b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x,x, algorithm="fricas")
[Out] [1/2*a^(3/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/3*(b*x
^2 + 4*a)*sqrt(b*x^2 + a), sqrt(-a)*a*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 1/
3*(b*x^2 + 4*a)*sqrt(b*x^2 + a)]
```

giac [A] time = 0.63, size = 48, normalized size = 0.89

$$\frac{a^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}+\sqrt{bx^2+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x,x, algorithm="giac")
[Out] a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/3*(b*x^2 + a)^(3/2) + sqr
t(b*x^2 + a)*a
```

maple [A] time = 0.01, size = 52, normalized size = 0.96

$$-a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\sqrt{b\,x^2+a}\,a+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x,x)
```

```
[Out] 1/3*(b*x^2+a)^(3/2)-ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)*a^(3/2)+a*(b*x^2+
a)^(1/2)
```

maxima [A] time = 1.28, size = 40, normalized size = 0.74

$$-a^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right) + \frac{1}{3}\left(bx^2 + a\right)^{\frac{3}{2}} + \sqrt{bx^2 + a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2) + sqrt(b*x^2
 + a)*a
```

mupad [B] time = 4.70, size = 42, normalized size = 0.78

$$a\sqrt{b\,x^2 + a} - a^{3/2}\operatorname{atanh}\left(\frac{\sqrt{b\,x^2 + a}}{\sqrt{a}}\right) + \frac{\left(b\,x^2 + a\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x,x)
```

```
[Out] a*(a + b*x^2)^(1/2) - a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + (a + b*x^2
)^(3/2)/3
```

sympy [A] time = 1.92, size = 78, normalized size = 1.44

$$\frac{4a^{\frac{3}{2}}\sqrt{1 + \frac{bx^2}{a}}}{3} + \frac{a^{\frac{3}{2}}\log\left(\frac{bx^2}{a}\right)}{2} - a^{\frac{3}{2}}\log\left(\sqrt{1 + \frac{bx^2}{a}} + 1\right) + \frac{\sqrt{a}\,bx^2\sqrt{1 + \frac{bx^2}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x,x)
```

```
[Out] 4*a**(3/2)*sqrt(1 + b*x**2/a)/3 + a**(3/2)*log(b*x**2/a)/2 - a**(3/2)*log(s
qrt(1 + b*x**2/a) + 1) + sqrt(a)*b*x**2*sqrt(1 + b*x**2/a)/3
```

3.374 $\int \frac{(a+bx^2)^{3/2}}{x^3} \, dx$

Optimal. Leaf size=63

$$-\frac{(a+bx^2)^{3/2}}{2x^2}+\frac{3}{2}b\sqrt{a+bx^2}-\frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -1/2*(b*x^2+a)^(3/2)/x^2-3/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+3/2
*b*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$-\frac{(a+bx^2)^{3/2}}{2x^2}+\frac{3}{2}b\sqrt{a+bx^2}-\frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^3,x]
```

```
[Out] (3*b*Sqrt[a + b*x^2])/2 - (a + b*x^2)^(3/2)/(2*x^2) - (3*Sqrt[a]*b*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{1}{4}(3b)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{2}b\sqrt{a+bx^2} - \frac{\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{1}{4}(3ab)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \frac{3}{2}b\sqrt{a+bx^2} - \frac{\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{1}{2}(3a)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)\\
&= \frac{3}{2}b\sqrt{a+bx^2} - \frac{\left(a+bx^2\right)^{3/2}}{2x^2} - \frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.59

$$\frac{b\left(a+bx^2\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^2}{a}+1\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^3,x]
[Out] (b*(a + b*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^2)/a])/(5*a^2)
```

fricas [A] time = 1.01, size = 119, normalized size = 1.89

$$\left[\frac{3\sqrt{a}\,bx^2\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(2bx^2-a\right)\sqrt{bx^2+a}}{4x^2}, \frac{3\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(2bx^2-a\right)\sqrt{bx^2+\ldots}}{2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
[Out] [1/4*(3*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
 2*(2*b*x^2 - a)*sqrt(b*x^2 + a))/x^2, 1/2*(3*sqrt(-a)*b*x^2*arctan(sqrt(-a
)/sqrt(b*x^2 + a)) + (2*b*x^2 - a)*sqrt(b*x^2 + a))/x^2]
```

giac [A] time = 0.68, size = 63, normalized size = 1.00

$$\frac{\frac{3ab^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{bx^2+a}\,b^2-\frac{\sqrt{bx^2+a}\,ab}{x^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x^2 + a)*
b^2 - sqrt(b*x^2 + a)*a*b/x^2)/b
```

maple [A] time = 0.01, size = 75, normalized size = 1.19

$$-\frac{3\sqrt{a}\,b\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2}+\frac{3\sqrt{b\,x^2+a}\,b}{2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}b}{2a}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(5/2)+1/2/a*b*(b*x^2+a)^(3/2)-3/2*a^(1/2)*b*ln((2*a+2*
(b*x^2+a)^(1/2)*a^(1/2))/x)+3/2*b*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.34, size = 63, normalized size = 1.00

$$-\frac{3}{2}\sqrt{a}\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{3}{2}\sqrt{bx^2+a}\,b+\frac{\left(bx^2+a\right)^{\frac{3}{2}}b}{2\,a}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 3/2*sqrt(b*x^2 + a)*b + 1/2*
(b*x^2 + a)^(3/2)*b/a - 1/2*(b*x^2 + a)^(5/2)/(a*x^2)
```

mupad [B] time = 4.82, size = 47, normalized size = 0.75

$$b\sqrt{b\,x^2+a}-\frac{a\sqrt{b\,x^2+a}}{2\,x^2}-\frac{3\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^3,x)
```

```
[Out] b*(a + b*x^2)^(1/2) - (a*(a + b*x^2)^(1/2))/(2*x^2) - (3*a^(1/2)*b*atanh((a
 + b*x^2)^(1/2)/a^(1/2)))/2
```

sympy [A] time = 2.33, size = 88, normalized size = 1.40

$$-\frac{3\sqrt{a}\,b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2}-\frac{a^2}{2\sqrt{b}\,x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{a\sqrt{b}}{2x\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] -3*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - a**2/(2*sqrt(b)*x**3*sqrt(a/(b*
x**2) + 1)) + a*sqrt(b)/(2*x*sqrt(a/(b*x**2) + 1)) + b**(3/2)*x/sqrt(a/(b*x
**2) + 1)
```

3.375 $\int \frac{\left(a+bx^2\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=68

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{\left(a+bx^2\right)^{3/2}}{4x^4}$$

```
[Out] -1/4*(b*x^2+a)^(3/2)/x^4-3/8*b^2*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)-3
/8*b*(b*x^2+a)^(1/2)/x^2
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 208}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{\left(a+bx^2\right)^{3/2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^5,x]
```

```
[Out] (-3*b*Sqrt[a + b*x^2])/(8*x^2) - (a + b*x^2)^(3/2)/(4*x^4) - (3*b^2*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{4x^4}+\frac{1}{8}(3b)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{\left(a+bx^2\right)^{3/2}}{4x^4}+\frac{1}{16}\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{\left(a+bx^2\right)^{3/2}}{4x^4}+\frac{1}{8}(3b)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)\\
&= -\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{\left(a+bx^2\right)^{3/2}}{4x^4}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 76, normalized size = 1.12

$$
-\frac{2a^2+3b^2x^4\sqrt{\frac{bx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)+7abx^2+5b^2x^4}{8x^4\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^5,x]
```

```
[Out] -1/8*(2*a^2 + 7*a*b*x^2 + 5*b^2*x^4 + 3*b^2*x^4*Sqrt[1 + (b*x^2)/a]*ArcTanh
[Sqrt[1 + (b*x^2)/a]])/(x^4*Sqrt[a + b*x^2])
```

fricas [A] time = 1.28, size = 136, normalized size = 2.00

$$
\left[\frac{3\sqrt{a}\,b^2x^4\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(5abx^2+2a^2\right)\sqrt{bx^2+a}}{16ax^4}, \frac{3\sqrt{-a}\,b^2x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(5abx^2+2a^2\right)\sqrt{bx^2+a}}{8ax^4}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) - 2*(5*a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a*x^4), 1/8*(3*sqrt(-a)*b^2*x^4
*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (5*a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a
*x^4)]
```

giac [A] time = 0.65, size = 70, normalized size = 1.03

$$
\frac{\frac{3b^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}b^3-3\sqrt{bx^2+a}\,ab^3}{b^2x^4}}{8b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (5*(b*x^2 + a)^(3/2)
*b^3 - 3*sqrt(b*x^2 + a)*a*b^3)/(b^2*x^4))/b
```

maple [A] time = 0.01, size = 102, normalized size = 1.50

$$-\frac{3b^{2}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{8\sqrt{a}}+\frac{3\sqrt{b\,x^{2}+a}\,b^{2}}{8a}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{2}}{8a^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}b}{8a^{2}x^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{4a\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(5/2)-1/8/a^2*b/x^2*(b*x^2+a)^(5/2)+1/8/a^2*b^2*(b*x^2
+a)^(3/2)-3/8/a^(1/2)*b^2*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/8/a*b^2*(
b*x^2+a)^(1/2)
```

maxima [A] time = 1.37, size = 90, normalized size = 1.32

$$-\frac{3\,b^{2}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,\sqrt{a}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}{8\,a^{2}}+\frac{3\,\sqrt{bx^{2}+a}\,b^{2}}{8\,a}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}b}{8\,a^{2}x^{2}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -3/8*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/8*(b*x^2 + a)^(3/2)*b^2/
a^2 + 3/8*sqrt(b*x^2 + a)*b^2/a - 1/8*(b*x^2 + a)^(5/2)*b/(a^2*x^2) - 1/4*(
b*x^2 + a)^(5/2)/(a*x^4)
```

mupad [B] time = 4.91, size = 52, normalized size = 0.76

$$\frac{3\,a\,\sqrt{b\,x^{2}+a}}{8\,x^{4}}-\frac{3\,b^{2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{8\,\sqrt{a}}-\frac{5\,\left(b\,x^{2}+a\right)^{3/2}}{8\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^5,x)
```

```
[Out] (3*a*(a + b*x^2)^(1/2))/(8*x^4) - (3*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/
(8*a^(1/2)) - (5*(a + b*x^2)^(3/2))/(8*x^4)
```

sympy [A] time = 2.98, size = 71, normalized size = 1.04

$$-\frac{a\sqrt{b}\,\sqrt{\frac{a}{bx^{2}}+1}}{4x^{3}}-\frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{8x}-\frac{3b^{2}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**5,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(4*x**3) - 5*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(8*x) - 3*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a))
```

3.376 $\int \frac{(a+bx^2)^{3/2}}{x^7} \, dx$

Optimal. Leaf size=92

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}} - \frac{b^2\sqrt{a+bx^2}}{16ax^2} - \frac{(a+bx^2)^{3/2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{8x^4}$$

```
[Out] -1/6*(b*x^2+a)^(3/2)/x^6+1/16*b^3*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-
1/8*b*(b*x^2+a)^(1/2)/x^4-1/16*b^2*(b*x^2+a)^(1/2)/a/x^2
```

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}} - \frac{b^2\sqrt{a+bx^2}}{16ax^2} - \frac{b\sqrt{a+bx^2}}{8x^4} - \frac{(a+bx^2)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^7,x]
```

```
[Out] -(b*Sqrt[a + b*x^2])/(8*x^4) - (b^2*Sqrt[a + b*x^2])/(16*a*x^2) - (a + b*x^
2)^(3/2)/(6*x^6) + (b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{6x^6}+\frac{1}{4}b\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4}-\frac{\left(a+bx^2\right)^{3/2}}{6x^6}+\frac{1}{16}b^2\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4}-\frac{b^2\sqrt{a+bx^2}}{16ax^2}-\frac{\left(a+bx^2\right)^{3/2}}{6x^6}-\frac{b^3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{32a}\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4}-\frac{b^2\sqrt{a+bx^2}}{16ax^2}-\frac{\left(a+bx^2\right)^{3/2}}{6x^6}-\frac{b^2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{16a}\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4}-\frac{b^2\sqrt{a+bx^2}}{16ax^2}-\frac{\left(a+bx^2\right)^{3/2}}{6x^6}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.42

$$\frac{b^3\left(a+bx^2\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{bx^2}{a}+1\right)}{5a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^7,x]
```

```
[Out] (b^3*(a + b*x^2)^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x^2)/a])/(5*a^
4)
```

fricas [A] time = 0.91, size = 157, normalized size = 1.71

$$\left[\frac{3\sqrt{a}\,b^3x^6\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(3ab^2x^4+14a^2bx^2+8a^3\right)\sqrt{bx^2+a}}{96a^2x^6},-\frac{3\sqrt{-a}\,b^3x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(a)*b^3*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) - 2*(3*a*b^2*x^4 + 14*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a^2*x^6), -1/4
8*(3*sqrt(-a)*b^3*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*a*b^2*x^4 + 14*
a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a^2*x^6)]
```

giac [A] time = 0.64, size = 92, normalized size = 1.00

$$-\frac{\frac{3b^4\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}b^4+8\left(bx^2+a\right)^{\frac{3}{2}}ab^4-3\sqrt{bx^2+a}\,a^2b^4}{ab^3x^6}}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/48*(3*b^4*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (3*(b*x^2 + a)
^(5/2)*b^4 + 8*(b*x^2 + a)^(3/2)*a*b^4 - 3*sqrt(b*x^2 + a)*a^2*b^4)/(a*b^3*
x^6))/b
```

maple [A] time = 0.01, size = 122, normalized size = 1.33

$$\frac{b^{3}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{16a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{2}+a}\,b^{3}}{16a^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{3}}{48a^{3}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}b^{2}}{48a^{3}x^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}b}{24a^{2}x^{4}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6a\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^7,x)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(5/2)+1/24/a^2*b/x^4*(b*x^2+a)^(5/2)+1/48/a^3*b^2/x^2*
(b*x^2+a)^(5/2)-1/48/a^3*b^3*(b*x^2+a)^(3/2)+1/16/a^(3/2)*b^3*ln((2*a+2*(b*
x^2+a)^(1/2)*a^(1/2))/x)-1/16/a^2*b^3*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.33, size = 110, normalized size = 1.20

$$\frac{b^{3}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{3}{2}}}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}b^{3}}{48\,a^{3}}-\frac{\sqrt{bx^{2}+a}\,b^{3}}{16\,a^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}b^{2}}{48\,a^{3}x^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}b}{24\,a^{2}x^{4}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}}{6\,ax^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/48*(b*x^2 + a)^(3/2)*b^3
/a^3 - 1/16*sqrt(b*x^2 + a)*b^3/a^2 + 1/48*(b*x^2 + a)^(5/2)*b^2/(a^3*x^2)
+ 1/24*(b*x^2 + a)^(5/2)*b/(a^2*x^4) - 1/6*(b*x^2 + a)^(5/2)/(a*x^6)
```

mupad [B] time = 4.94, size = 72, normalized size = 0.78

$$\frac{a\,\sqrt{b\,x^{2}+a}}{16\,x^{6}}-\frac{\left(b\,x^{2}+a\right)^{3/2}}{6\,x^{6}}-\frac{\left(b\,x^{2}+a\right)^{5/2}}{16\,a\,x^{6}}-\frac{b^{3}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^{2}+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{16\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^7,x)
```

```
[Out] (a*(a + b*x^2)^(1/2))/(16*x^6) - (b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*
1i)/(16*a^(3/2)) - (a + b*x^2)^(3/2)/(6*x^6) - (a + b*x^2)^(5/2)/(16*a*x^6)
```

sympy [A] time = 5.33, size = 119, normalized size = 1.29

$$-\frac{a^{2}}{6\sqrt{b}\,x^{7}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{11a\sqrt{b}}{24x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{17b^{\frac{3}{2}}}{48x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{b^{\frac{5}{2}}}{16ax\sqrt{\frac{a}{bx^{2}}+1}}+\frac{b^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**7,x)
```

```
[Out] -a**2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 11*a*sqrt(b)/(24*x**5*sqrt(a/
(b*x**2) + 1)) - 17*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - b**(5/2)/(16*
a*x*sqrt(a/(b*x**2) + 1)) + b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(3/2))
```

3.377 $\int \frac{\left(a+bx^2\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=116

$$-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}+\frac{3b^3\sqrt{a+bx^2}}{128a^2x^2}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}-\frac{\left(a+bx^2\right)^{3/2}}{8x^8}-\frac{b\sqrt{a+bx^2}}{16x^6}$$

```
[Out] -1/8*(b*x^2+a)^(3/2)/x^8-3/128*b^4*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
-1/16*b*(b*x^2+a)^(1/2)/x^6-1/64*b^2*(b*x^2+a)^(1/2)/a/x^4+3/128*b^3*(b*x^2
+a)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{3b^3\sqrt{a+bx^2}}{128a^2x^2}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}-\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{\left(a+bx^2\right)^{3/2}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^9,x]
```

```
[Out] -(b*Sqrt[a + b*x^2])/(16*x^6) - (b^2*Sqrt[a + b*x^2])/(64*a*x^4) + (3*b^3*S
qrt[a + b*x^2])/(128*a^2*x^2) - (a + b*x^2)^(3/2)/(8*x^8) - (3*b^4*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^9}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{8x^8}+\frac{1}{16}(3b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{\left(a+bx^2\right)^{3/2}}{8x^8}+\frac{1}{32}b^2\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}-\frac{\left(a+bx^2\right)^{3/2}}{8x^8}-\frac{\left(3b^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{128a}\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}+\frac{3b^3\sqrt{a+bx^2}}{128a^2x^2}-\frac{\left(a+bx^2\right)^{3/2}}{8x^8}+\frac{\left(3b^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x\right.}{256a^2}\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}+\frac{3b^3\sqrt{a+bx^2}}{128a^2x^2}-\frac{\left(a+bx^2\right)^{3/2}}{8x^8}+\frac{\left(3b^3\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{}\right.}{128a^2}\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}+\frac{3b^3\sqrt{a+bx^2}}{128a^2x^2}-\frac{\left(a+bx^2\right)^{3/2}}{8x^8}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.34

$$-\frac{b^4\left(a+bx^2\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{bx^2}{a}+1\right)}{5a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^9,x]
```

```
[Out] -1/5*(b^4*(a + b*x^2)^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, 1 + (b*x^2)/a])/
a^5
```

fricas [A] time = 0.64, size = 179, normalized size = 1.54

$$\left[\frac{3\sqrt{a}\,b^4x^8\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\,\sqrt{a}+2a}{x^2}\right)+2\left(3ab^3x^6-2a^2b^2x^4-24a^3bx^2-16a^4\right)\sqrt{bx^2+a}}{256a^3x^8},\ \frac{3\sqrt{-a}\,b^4x^8\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/256*(3*sqrt(a)*b^4*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*(3*a*b^3*x^6 - 2*a^2*b^2*x^4 - 24*a^3*b*x^2 - 16*a^4)*sqrt(b*x^2 + a
))/(a^3*x^8), 1/128*(3*sqrt(-a)*b^4*x^8*arctan(sqrt(-a)/sqrt(b*x^2 + a)) +
```

```
(3*a*b^3*x^6 - 2*a^2*b^2*x^4 - 24*a^3*b*x^2 - 16*a^4)*sqrt(b*x^2 + a))/(a^3
*x^8)]
```

giac [A] time = 0.65, size = 109, normalized size = 0.94

$$\frac{\frac{3\,b^5 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2} + \frac{3\left(bx^2+a\right)^{\frac{7}{2}} b^5 - 11\left(bx^2+a\right)^{\frac{5}{2}} ab^5 - 11\left(bx^2+a\right)^{\frac{3}{2}} a^2 b^5 + 3\sqrt{bx^2+a}\,a^3 b^5}{a^2 b^4 x^8}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/128*(3*b^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 +
a)^(7/2)*b^5 - 11*(b*x^2 + a)^(5/2)*a*b^5 - 11*(b*x^2 + a)^(3/2)*a^2*b^5 +
3*sqrt(b*x^2 + a)*a^3*b^5)/(a^2*b^4*x^8))/b
```

maple [A] time = 0.02, size = 142, normalized size = 1.22

$$-\frac{3b^4 \ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{128a^{\frac{5}{2}}}+\frac{3\sqrt{b\,x^2+a}\;b^4}{128a^3}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}} b^4}{128a^4}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}} b^3}{128a^4x^2}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}} b^2}{64a^3x^4}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}} b}{16a^2x^6}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{8ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^9,x)
```

```
[Out] -1/8/a/x^8*(b*x^2+a)^(5/2)+1/16/a^2*b/x^6*(b*x^2+a)^(5/2)-1/64/a^3*b^2/x^4*
(b*x^2+a)^(5/2)-1/128/a^4*b^3/x^2*(b*x^2+a)^(5/2)+1/128/a^4*b^4*(b*x^2+a)^(
3/2)-3/128/a^(5/2)*b^4*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/128/a^3*b^4*
(b*x^2+a)^(1/2)
```

maxima [A] time = 1.38, size = 130, normalized size = 1.12

$$-\frac{3\,b^4 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{5}{2}}}+\frac{\left(bx^2+a\right)^{\frac{3}{2}} b^4}{128\,a^4}+\frac{3\,\sqrt{bx^2+a}\,b^4}{128\,a^3}-\frac{\left(bx^2+a\right)^{\frac{5}{2}} b^3}{128\,a^4x^2}-\frac{\left(bx^2+a\right)^{\frac{5}{2}} b^2}{64\,a^3x^4}+\frac{\left(bx^2+a\right)^{\frac{5}{2}} b}{16\,a^2x^6}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{8\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -3/128*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/128*(b*x^2 + a)^(3/2)*
b^4/a^4 + 3/128*sqrt(b*x^2 + a)*b^4/a^3 - 1/128*(b*x^2 + a)^(5/2)*b^3/(a^4*
x^2) - 1/64*(b*x^2 + a)^(5/2)*b^2/(a^3*x^4) + 1/16*(b*x^2 + a)^(5/2)*b/(a^2
*x^6) - 1/8*(b*x^2 + a)^(5/2)/(a*x^8)
```

mupad [B] time = 5.21, size = 89, normalized size = 0.77

$$\frac{3\,a\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{11\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^8}-\frac{11\,{\left(b\,x^2+a\right)}^{5/2}}{128\,a\,x^8}+\frac{3\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a^2\,x^8}+\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,3{}\mathrm{i}}{128\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^9,x)
```

```
[Out] (b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(128*a^(5/2)) - (11*(a + b*x^
2)^(3/2))/(128*x^8) + (3*a*(a + b*x^2)^(1/2))/(128*x^8) - (11*(a + b*x^2)^(
5/2))/(128*a*x^8) + (3*(a + b*x^2)^(7/2))/(128*a^2*x^8)
```

sympy [**A**]   time = 8.26, size = 148, normalized size = 1.28

$$-\frac{a^2}{8\sqrt{b}\,x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{5a\sqrt{b}}{16x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{13b^{\frac{3}{2}}}{64x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{5}{2}}}{128ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3b^{\frac{7}{2}}}{128a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{3b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{128a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**9,x)
```

```
[Out] -a**2/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 5*a*sqrt(b)/(16*x**7*sqrt(a/(
b*x**2) + 1)) - 13*b**(3/2)/(64*x**5*sqrt(a/(b*x**2) + 1)) + b**(5/2)/(128*
a*x**3*sqrt(a/(b*x**2) + 1)) + 3*b**(7/2)/(128*a**2*x*sqrt(a/(b*x**2) + 1))
 - 3*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(5/2))
```

3.378 $\int x^4 \left(a+bx^2\right)^{3/2} \, dx$

Optimal. Leaf size=115

$$\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}-\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{1}{16}ax^5\sqrt{a+bx^2}$$

```
[Out] 1/8*x^5*(b*x^2+a)^(3/2)+3/128*a^4*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2
)-3/128*a^3*x*(b*x^2+a)^(1/2)/b^2+1/64*a^2*x^3*(b*x^2+a)^(1/2)/b+1/16*a*x^5
*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {279, 321, 217, 206}

$$-\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{1}{16}ax^5\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(3/2),x]
```

```
[Out] (-3*a^3*x*Sqrt[a + b*x^2])/(128*b^2) + (a^2*x^3*Sqrt[a + b*x^2])/(64*b) + (
a*x^5*Sqrt[a + b*x^2])/16 + (x^5*(a + b*x^2)^(3/2))/8 + (3*a^4*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+bx^2\right)^{3/2}dx &= \frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{1}{8}(3a)\int x^4\sqrt{a+bx^2}\,dx\\
&= \frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{1}{16}a^2\int\frac{x^4}{\sqrt{a+bx^2}}dx\\
&= \frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}-\frac{\left(3a^3\right)\int\frac{x^2}{\sqrt{a+bx^2}}dx}{64b}\\
&= -\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{\left(3a^4\right)\int\frac{1}{\sqrt{a+bx^2}}}{128b^2}\\
&= -\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{\left(3a^4\right)\mathrm{Subst}\left(\int\right.}{}\\
&= -\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{3a^4\tanh^{-1}\left(\right.}{128b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 94, normalized size = 0.82

$$\frac{\sqrt{a+bx^2}\left(\frac{3a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\sqrt{b}x\left(-3a^3+2a^2bx^2+24ab^2x^4+16b^3x^6\right)\right)}{128b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-3*a^3 + 2*a^2*b*x^2 + 24*a*b^2*x^4 + 16*b^3*x
^6) + (3*a^(7/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(128*b
^(5/2))
```

fricas [A] time = 0.91, size = 168, normalized size = 1.46

$$\left[\frac{3\,a^4\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(16\,b^4x^7+24\,ab^3x^5+2\,a^2b^2x^3-3\,a^3bx\right)\sqrt{bx^2+a}}{256\,b^3},-\frac{3\,a^4\sqrt{-b}\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*a^4*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
16*b^4*x^7 + 24*a*b^3*x^5 + 2*a^2*b^2*x^3 - 3*a^3*b*x)*sqrt(b*x^2 + a))/b^3
, -1/128*(3*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (16*b^4*x^7 +
 24*a*b^3*x^5 + 2*a^2*b^2*x^3 - 3*a^3*b*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.66, size = 76, normalized size = 0.66

$$\frac{1}{128}\left(2\left(4\left(2\,bx^2+3\,a\right)x^2+\frac{a^2}{b}\right)x^2-\frac{3\,a^3}{b^2}\right)\sqrt{bx^2+a}\,x-\frac{3\,a^4\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*(2*(4*(2*b*x^2 + 3*a)*x^2 + a^2/b)*x^2 - 3*a^3/b^2)*sqrt(b*x^2 + a)*x
 - 3/128*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 95, normalized size = 0.83

$$\frac{3a^{4}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{5}{2}}}+\frac{3\sqrt{b\,x^{2}+a}\,a^{3}x}{128b^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}x^{3}}{8b}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}x}{64b^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}ax}{16b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/8*x^3*(b*x^2+a)^(5/2)/b-1/16*a/b^2*x*(b*x^2+a)^(5/2)+1/64*a^2/b^2*x*(b*x^
2+a)^(3/2)+3/128*a^3*x*(b*x^2+a)^(1/2)/b^2+3/128*a^4/b^(5/2)*ln(b^(1/2)*x+(
b*x^2+a)^(1/2))
```

maxima [A] time = 1.36, size = 87, normalized size = 0.76

$$\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}x^{3}}{8\,b}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}ax}{16\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}x}{64\,b^{2}}+\frac{3\,\sqrt{bx^{2}+a}\,a^{3}x}{128\,b^{2}}+\frac{3\,a^{4}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(5/2)*x^3/b - 1/16*(b*x^2 + a)^(5/2)*a*x/b^2 + 1/64*(b*x^2
+ a)^(3/2)*a^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*a^3*x/b^2 + 3/128*a^4*arcsinh(
b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{4}\left(b\,x^{2}+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(3/2),x)
```

```
[Out] int(x^4*(a + b*x^2)^(3/2), x)
```

sympy [A] time = 8.20, size = 148, normalized size = 1.29

$$-\frac{3a^{\frac{7}{2}}x}{128b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{a^{\frac{5}{2}}x^{3}}{128b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{13a^{\frac{3}{2}}x^{5}}{64\sqrt{1+\frac{bx^{2}}{a}}}+\frac{5\sqrt{a}\,bx^{7}}{16\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3a^{4}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{b^{2}x^{9}}{8\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(3/2),x)
```

```
[Out] -3*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**2/a)) - a**(5/2)*x**3/(128*b*sqrt(1 +
 b*x**2/a)) + 13*a**(3/2)*x**5/(64*sqrt(1 + b*x**2/a)) + 5*sqrt(a)*b*x**7/(
16*sqrt(1 + b*x**2/a)) + 3*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) + b
**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.379 $\int x^2 \left(a+bx^2\right)^{3/2} \, dx$

Optimal. Leaf size=91

$$-\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}$$

```
[Out] 1/6*x^3*(b*x^2+a)^(3/2)-1/16*a^3*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)
+1/16*a^2*x*(b*x^2+a)^(1/2)/b+1/8*a*x^3*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(3/2),x]
```

```
[Out] (a^2*x*Sqrt[a + b*x^2])/(16*b) + (a*x^3*Sqrt[a + b*x^2])/8 + (x^3*(a + b*x^
2)^(3/2))/6 - (a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^{3/2}dx=\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}+\frac{1}{2}a\int x^2\sqrt{a+bx^2}\,dx$$
$$=\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}a^2\int\frac{x^2}{\sqrt{a+bx^2}}dx$$
$$=\frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}-\frac{a^3\int\frac{1}{\sqrt{a+bx^2}}dx}{16b}$$
$$=\frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}-\frac{a^3\,\text{Subst}\left(\int\frac{1}{1-bx^2}dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{16b}$$
$$=\frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

Mathematica [A] time = 0.12, size = 83, normalized size = 0.91

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}x\left(3a^2+14abx^2+8b^2x^4\right)-\frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{48b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(3*a^2 + 14*a*b*x^2 + 8*b^2*x^4) - (3*a^(5/2)*A
rcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(48*b^(3/2))
```

fricas [A] time = 0.96, size = 145, normalized size = 1.59

$$\left[\frac{3\,a^3\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(8\,b^3x^5+14\,ab^2x^3+3\,a^2bx\right)\sqrt{bx^2+a}}{96\,b^2},\ \frac{3\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}}{\sqrt{bx^2}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8
*b^3*x^5 + 14*a*b^2*x^3 + 3*a^2*b*x)*sqrt(b*x^2 + a))/b^2, 1/48*(3*a^3*sqrt
(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*b^3*x^5 + 14*a*b^2*x^3 + 3*a^2
*b*x)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.64, size = 63, normalized size = 0.69

$$\frac{1}{48}\left(2\left(4\,bx^2+7\,a\right)x^2+\frac{3\,a^2}{b}\right)\sqrt{bx^2+a}\,x+\frac{a^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*x^2 + 7*a)*x^2 + 3*a^2/b)*sqrt(b*x^2 + a)*x + 1/16*a^3*log(abs
(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

maple [A] time = 0.00, size = 75, normalized size = 0.82

$$-\frac{a^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16b^{\frac{3}{2}}}-\frac{\sqrt{b\,x^2+a}\,a^2x}{16b}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}ax}{24b}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}x}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/6*x*(b*x^2+a)^(5/2)/b-1/24*a/b*x*(b*x^2+a)^(3/2)-1/16*a^2*x*(b*x^2+a)^(1/
2)/b-1/16*a^3/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.33, size = 67, normalized size = 0.74

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}x}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}ax}{24\,b}-\frac{\sqrt{bx^2+a}\,a^2x}{16\,b}-\frac{a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*x/b - 1/24*(b*x^2 + a)^(3/2)*a*x/b - 1/16*sqrt(b*x^2
+ a)*a^2*x/b - 1/16*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^(3/2), x)
```

sympy [A] time = 5.21, size = 119, normalized size = 1.31

$$\frac{a^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17a^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11\sqrt{a}\,bx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{a^3\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{b^2x^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*a**(3/2)*x**3/(48*sqrt(1 + b*x**2
/a)) + 11*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - a**3*asinh(sqrt(b)*x/sqr
t(a))/(16*b**(3/2)) + b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.380 $\int \left(a+bx^2\right)^{3/2} \, dx$

Optimal. Leaf size=65

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

```
[Out] 1/4*x*(b*x^2+a)^(3/2)+3/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+3/
8*a*x*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*x*Sqrt[a + b*x^2])/8 + (x*(a + b*x^2)^(3/2))/4 + (3*a^2*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^{3/2} \, dx &= \frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{4}(3a)\int \sqrt{a+bx^2} \, dx\\ &= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{a+bx^2}} \, dx\\ &= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 65, normalized size = 1.00

$$\frac{1}{8}\sqrt{a+bx^2}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}+5ax+2bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(5*a*x + 2*b*x^3 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/8
```

fricas [A] time = 1.06, size = 124, normalized size = 1.91

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(2\,b^2x^3+5\,abx\right)\sqrt{bx^2+a}}{16\,b},\,-\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(2\,b^2x^3\right.}{8\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.63, size = 49, normalized size = 0.75

$$\frac{1}{8}\left(2\,bx^2+5\,a\right)\sqrt{bx^2+a}\,x-\frac{3\,a^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2 + 5*a)*sqrt(b*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/sqrt(b)
```

maple [A] time = 0.00, size = 51, normalized size = 0.78

$$\frac{3a^2\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{8\sqrt{b}}+\frac{3\sqrt{b\,x^2+a}\;ax}{8}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*x*(b*x^2+a)^(3/2)+3/8*a*x*(b*x^2+a)^(1/2)+3/8*a^2/b^(1/2)*ln(b^(1/2)*x+
(b*x^2+a)^(1/2))
```

maxima [A] time = 1.31, size = 43, normalized size = 0.66

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{bx^2+a}\,ax+\frac{3\,a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*x + 3/8*sqrt(b*x^2 + a)*a*x + 3/8*a^2*arcsinh(b*x/sqr
t(a*b))/sqrt(b)
```

mupad [B] time = 4.61, size = 37, normalized size = 0.57

$$\frac{x\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2),x)
```

```
[Out] (x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(3/2)
```

sympy [A] time = 2.90, size = 70, normalized size = 1.08

$$\frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{8}+\frac{\sqrt{a}\,bx^3\sqrt{1+\frac{bx^2}{a}}}{4}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2),x)
```

```
[Out] 5*a**(3/2)*x*sqrt(1 + b*x**2/a)/8 + sqrt(a)*b*x**3*sqrt(1 + b*x**2/a)/4 + 3
*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b))
```

3.381 $\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=63

$$-\frac{\left(a+bx^2\right)^{3/2}}{x}+\frac{3}{2}bx\sqrt{a+bx^2}+\frac{3}{2}a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

```
[Out] -(b*x^2+a)^(3/2)/x+3/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)+3/2*b*x
*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = $\{277, 195, 217, 206\}$

$$-\frac{\left(a+bx^2\right)^{3/2}}{x}+\frac{3}{2}bx\sqrt{a+bx^2}+\frac{3}{2}a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^2,x]
```

```
[Out] (3*b*x*Sqrt[a + b*x^2])/2 - (a + b*x^2)^(3/2)/x + (3*a*Sqrt[b]*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx &= -\frac{\left(a+bx^2\right)^{3/2}}{x}+(3b)\int \sqrt{a+bx^2}\,dx\\ &= \frac{3}{2}bx\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{3/2}}{x}+\frac{1}{2}(3ab)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{3}{2}bx\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{3/2}}{x}+\frac{1}{2}(3ab)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{3}{2}bx\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{3/2}}{x}+\frac{3}{2}a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.79

$$-\frac{a\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^2)/a)])/(x*S
qrt[1 + (b*x^2)/a]))
```

fricas [A] time = 0.83, size = 112, normalized size = 1.78

$$\left[\frac{3\,a\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,\sqrt{bx^2+a}\left(bx^2-2\,a\right)}{4\,x},-\frac{3\,a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\left(bx^2-2\,a\right)}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqr
t(b*x^2 + a)*(b*x^2 - 2*a))/x, -1/2*(3*a*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) - sqrt(b*x^2 + a)*(b*x^2 - 2*a))/x]
```

giac [A] time = 0.67, size = 73, normalized size = 1.16

$$\frac{1}{2}\,\sqrt{bx^2+a}\,bx-\frac{3}{4}\,a\sqrt{b}\,\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,a^2\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b*x - 3/4*a*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2
) + 2*a^2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

maple [A] time = 0.00, size = 69, normalized size = 1.10

$$\frac{3a\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2}+\frac{3\sqrt{b\,x^2+a}\,bx}{2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}bx}{a}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^2,x)
```

```
[Out] -1/a/x*(b*x^2+a)^(5/2)+1/a*b*x*(b*x^2+a)^(3/2)+3/2*b*x*(b*x^2+a)^(1/2)+3/2*
a*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.34, size = 43, normalized size = 0.68

$$\frac{3}{2}\sqrt{bx^2+a}\,bx+\frac{3}{2}\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b*x^2 + a)*b*x + 3/2*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - (b*x^2 + a
)^(3/2)/x
```

mupad [B] time = 5.15, size = 40, normalized size = 0.63

$$-\frac{\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^2,x)
```

```
[Out] -((a + b*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a
 + 1)^(3/2))
```

sympy [A] time = 2.37, size = 88, normalized size = 1.40

$$-\frac{a^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,bx}{2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2}+\frac{b^2x^3}{2\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] -a**(3/2)/(x*sqrt(1 + b*x**2/a)) - sqrt(a)*b*x/(2*sqrt(1 + b*x**2/a)) + 3*a
*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2 + b**2*x**3/(2*sqrt(a)*sqrt(1 + b*x**2/
a))
```

3.382 $\int \frac{(a+bx^2)^{3/2}}{x^4} dx$

Optimal. Leaf size=61

$$b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b\sqrt{a+bx^2}}{x}-\frac{(a+bx^2)^{3/2}}{3x^3}$$

```
[Out] -1/3*(b*x^2+a)^(3/2)/x^3+b^(3/2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))-b*(b*x^
2+a)^(1/2)/x
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 217, 206}

$$b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b\sqrt{a+bx^2}}{x}-\frac{(a+bx^2)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^4,x]
```

```
[Out] -((b*Sqrt[a + b*x^2])/x) - (a + b*x^2)^(3/2)/(3*x^3) + b^(3/2)*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx &= -\frac{(a+bx^2)^{3/2}}{3x^3}+b\int \frac{\sqrt{a+bx^2}}{x^2}\,dx\\
&= -\frac{b\sqrt{a+bx^2}}{x}-\frac{(a+bx^2)^{3/2}}{3x^3}+b^2\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= -\frac{b\sqrt{a+bx^2}}{x}-\frac{(a+bx^2)^{3/2}}{3x^3}+b^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= -\frac{b\sqrt{a+bx^2}}{x}-\frac{(a+bx^2)^{3/2}}{3x^3}+b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.85

$$-\frac{a\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^4,x]
```

```
[Out] -1/3*(a*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^2)/a)])/
(x^3*Sqrt[1 + (b*x^2)/a])
```

fricas [A] time = 0.91, size = 112, normalized size = 1.84

$$\left[\frac{3\,b^{\frac{3}{2}}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(4\,bx^2+a\right)\sqrt{bx^2+a}}{6\,x^3},-\frac{3\,\sqrt{-b}\,bx^3\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(4\,bx^2+a\right)\sqrt{bx^2+a}}{3\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(4*
b*x^2 + a)*sqrt(b*x^2 + a))/x^3, -1/3*(3*sqrt(-b)*b*x^3*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) + (4*b*x^2 + a)*sqrt(b*x^2 + a))/x^3]
```

giac [B] time = 0.71, size = 114, normalized size = 1.87

$$-\frac{1}{2}\,b^{\frac{3}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{4\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4ab^{\frac{3}{2}}-3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^2b^{\frac{3}{2}}+2\,a^3b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 4/3*(3*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*a*b^(3/2) - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(3/2) +
 2*a^3*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

maple [A] time = 0.00, size = 92, normalized size = 1.51

$$b^{\frac{3}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)+\frac{\sqrt{b\,x^2+a}\,b^2x}{a}+\frac{2\left(b\,x^2+a\right)^{\frac{3}{2}}b^2x}{3a^2}-\frac{2\left(b\,x^2+a\right)^{\frac{5}{2}}b}{3a^2x}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^4,x)
```

```
[Out] -1/3/a/x^3*(b*x^2+a)^(5/2)-2/3/a^2*b/x*(b*x^2+a)^(5/2)+2/3/a^2*b^2*x*(b*x^2
+a)^(3/2)+1/a*b^2*x*(b*x^2+a)^(1/2)+b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.32, size = 66, normalized size = 1.08

$$\frac{\sqrt{bx^2+a}\,b^2x}{a}+b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{2\left(bx^2+a\right)^{\frac{3}{2}}b}{3\,ax}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)*b^2*x/a + b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 2/3*(b*x^2 + a)^
(3/2)*b/(a*x) - 1/3*(b*x^2 + a)^(5/2)/(a*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^4,x)
```

```
[Out] int((a + b*x^2)^(3/2)/x^4, x)
```

sympy [A] time = 2.03, size = 78, normalized size = 1.28

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}-\frac{b^{\frac{3}{2}}\log\left(\frac{a}{bx^2}\right)}{2}+b^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**4,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - 4*b**(3/2)*sqrt(a/(b*x**2) + 1)/
3 - b**(3/2)*log(a/(b*x**2))/2 + b**(3/2)*log(sqrt(a/(b*x**2) + 1) + 1)
```

3.383 $\int \frac{(a+bx^2)^{3/2}}{x^6} dx$

Optimal. Leaf size=21

$$-\frac{(a+bx^2)^{5/2}}{5ax^5}$$

```
[Out] -1/5*(b*x^2+a)^(5/2)/a/x^5
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{(a+bx^2)^{5/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^6,x]
```

```
[Out] -(a + b*x^2)^(5/2)/(5*a*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{x^6} dx = -\frac{(a+bx^2)^{5/2}}{5ax^5}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{(a+bx^2)^{5/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^6,x]
```

```
[Out] -1/5*(a + b*x^2)^(5/2)/(a*x^5)
```

fricas [B] time = 0.86, size = 35, normalized size = 1.67

$$-\frac{(b^2x^4 + 2\,abx^2 + a^2)\sqrt{bx^2 + a}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/5*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/(a*x^5)
```

giac [B] time = 0.64, size = 86, normalized size = 4.10

$$\frac{2\left(5\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8b^{\frac{5}{2}}+10\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{5}{2}}+a^4b^{\frac{5}{2}}\right)}{5\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 2/5*(5*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2) + 10*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^2*b^(5/2) + a^4*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{5ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^6,x)
```

```
[Out] -1/5*(b*x^2+a)^(5/2)/a/x^5
```

maxima [A] time = 1.34, size = 17, normalized size = 0.81

$$-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*(b*x^2 + a)^(5/2)/(a*x^5)
```

mupad [B] time = 5.29, size = 17, normalized size = 0.81

$$-\frac{\left(b\,x^2+a\right)^{5/2}}{5\,a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^6,x)
```

```
[Out] -(a + b*x^2)^(5/2)/(5*a*x^5)
```

sympy [B] time = 0.91, size = 68, normalized size = 3.24

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2}-\frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**6,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(5*x**2) - b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a)
```

3.384 $\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=44

$$\frac{2b(a+bx^2)^{5/2}}{35a^2x^5}-\frac{(a+bx^2)^{5/2}}{7ax^7}$$

```
[Out] -1/7*(b*x^2+a)^(5/2)/a/x^7+2/35*b*(b*x^2+a)^(5/2)/a^2/x^5
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2b(a+bx^2)^{5/2}}{35a^2x^5}-\frac{(a+bx^2)^{5/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^8,x]
```

```
[Out] -(a + b*x^2)^(5/2)/(7*a*x^7) + (2*b*(a + b*x^2)^(5/2))/(35*a^2*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx &= -\frac{(a+bx^2)^{5/2}}{7ax^7}-\frac{(2b)\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx}{7a}\\ &= -\frac{(a+bx^2)^{5/2}}{7ax^7}+\frac{2b(a+bx^2)^{5/2}}{35a^2x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{(a+bx^2)^{5/2}(2bx^2-5a)}{35a^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^8,x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-5*a + 2*b*x^2))/(35*a^2*x^7)
```

fricas [A] time = 1.32, size = 49, normalized size = 1.11

$$\frac{(2\,b^3x^6-ab^2x^4-8\,a^2bx^2-5\,a^3)\sqrt{bx^2+a}}{35\,a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="fricas")

[Out] 1/35*(2*b^3*x^6 - a*b^2*x^4 - 8*a^2*b*x^2 - 5*a^3)*sqrt(b*x^2 + a)/(a^2*x^7
)

giac [B] time = 0.69, size = 166, normalized size = 3.77

$$\frac{4\left(35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{7}{2}}+35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}ab^{\frac{7}{2}}+70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^2b^{\frac{7}{2}}+14\left(\sqrt{b}x-\sqrt{bx^2\right.}{35\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="giac")

[Out] 4/35*(35*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2) + 35*(sqrt(b)*x - sqrt(b*
x^2 + a))^8*a*b^(7/2) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(7/2) + 14
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(7/2) + 7*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*a^4*b^(7/2) - a^5*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7

maple [A] time = 0.00, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\left(-2b\,x^2+5a\right)}{35a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(3/2)/x^8,x)

[Out] -1/35*(b*x^2+a)^(5/2)*(-2*b*x^2+5*a)/x^7/a^2

maxima [A] time = 1.39, size = 36, normalized size = 0.82

$$\frac{2\left(bx^2+a\right)^{\frac{5}{2}}b}{35\,a^2x^5}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="maxima")

[Out] 2/35*(b*x^2 + a)^(5/2)*b/(a^2*x^5) - 1/7*(b*x^2 + a)^(5/2)/(a*x^7)

mupad [B] time = 5.65, size = 71, normalized size = 1.61

$$\frac{2\,b^3\,\sqrt{b\,x^2+a}}{35\,a^2\,x}-\frac{8\,b\,\sqrt{b\,x^2+a}}{35\,x^5}-\frac{b^2\,\sqrt{b\,x^2+a}}{35\,a\,x^3}-\frac{a\,\sqrt{b\,x^2+a}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^(3/2)/x^8,x)

[Out] (2*b^3*(a + b*x^2)^(1/2))/(35*a^2*x) - (8*b*(a + b*x^2)^(1/2))/(35*x^5) - (
b^2*(a + b*x^2)^(1/2))/(35*a*x^3) - (a*(a + b*x^2)^(1/2))/(7*x^7)

sympy [B] time = 1.18, size = 94, normalized size = 2.14

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{7x^6}-\frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{35x^4}-\frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{35ax^2}+\frac{2b^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{35a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**8,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 8*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(35*x**4) - b**(5/2)*sqrt(a/(b*x**2) + 1)/(35*a*x**2) + 2*b**(7/2)*sqrt(a/(
b*x**2) + 1)/(35*a**2)
```

3.385 $\int \frac{(a+bx^2)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=68

$$-\frac{8b^2\left(a+bx^2\right)^{5/2}}{315a^3x^5}+\frac{4b\left(a+bx^2\right)^{5/2}}{63a^2x^7}-\frac{\left(a+bx^2\right)^{5/2}}{9ax^9}$$

```
[Out] -1/9*(b*x^2+a)^(5/2)/a/x^9+4/63*b*(b*x^2+a)^(5/2)/a^2/x^7-8/315*b^2*(b*x^2+
a)^(5/2)/a^3/x^5
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8b^2\left(a+bx^2\right)^{5/2}}{315a^3x^5}+\frac{4b\left(a+bx^2\right)^{5/2}}{63a^2x^7}-\frac{\left(a+bx^2\right)^{5/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^10,x]
```

```
[Out] -(a + b*x^2)^(5/2)/(9*a*x^9) + (4*b*(a + b*x^2)^(5/2))/(63*a^2*x^7) - (8*b^
2*(a + b*x^2)^(5/2))/(315*a^3*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^{10}}\,dx &= -\frac{\left(a+bx^2\right)^{5/2}}{9ax^9}-\frac{(4b)\int \frac{\left(a+bx^2\right)^{3/2}}{x^8}\,dx}{9a}\\
&= -\frac{\left(a+bx^2\right)^{5/2}}{9ax^9}+\frac{4b\left(a+bx^2\right)^{5/2}}{63a^2x^7}+\frac{\left(8b^2\right)\int \frac{\left(a+bx^2\right)^{3/2}}{x^6}\,dx}{63a^2}\\
&= -\frac{\left(a+bx^2\right)^{5/2}}{9ax^9}+\frac{4b\left(a+bx^2\right)^{5/2}}{63a^2x^7}-\frac{8b^2\left(a+bx^2\right)^{5/2}}{315a^3x^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^2\right)^{5/2}\left(35a^2-20abx^2+8b^2x^4\right)}{315a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^10,x]
```

```
[Out] -1/315*((a + b*x^2)^(5/2)*(35*a^2 - 20*a*b*x^2 + 8*b^2*x^4))/(a^3*x^9)
```

fricas [A] time = 0.89, size = 60, normalized size = 0.88

$$-\frac{\left(8\, b^{4} x^{8} - 4\, a b^{3} x^{6} + 3\, a^{2} b^{2} x^{4} + 50\, a^{3} b x^{2} + 35\, a^{4}\right) \sqrt{b x^{2} + a}}{315\, a^{3} x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(8*b^4*x^8 - 4*a*b^3*x^6 + 3*a^2*b^2*x^4 + 50*a^3*b*x^2 + 35*a^4)*sq
rt(b*x^2 + a)/(a^3*x^9)
```

giac [B] time = 0.64, size = 192, normalized size = 2.82

$$\frac{16\left(210\left(\sqrt{b}\,x - \sqrt{bx^{2} + a}\right)^{12} b^{\frac{9}{2}} + 315\left(\sqrt{b}\,x - \sqrt{bx^{2} + a}\right)^{10} a b^{\frac{9}{2}} + 441\left(\sqrt{b}\,x - \sqrt{bx^{2} + a}\right)^{8} a^{2} b^{\frac{9}{2}} + 126\left(\sqrt{b}\,x - \sqrt{b}\right.\right.}{315\left(\left(\sqrt{b}\,x - \sqrt{bx^{2} + a}\right)^{2} - a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(9/2) + 315*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*a*b^(9/2) + 441*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(9/2
) + 126*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(9/2) + 36*(sqrt(b)*x - sqrt(
b*x^2 + a))^4*a^4*b^(9/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(9/2) +
 a^6*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

maple [A] time = 0.00, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^{2} + a\right)^{\frac{5}{2}} \left(8b^{2} x^{4} - 20 a b\,x^{2} + 35 a^{2}\right)}{315 a^{3} x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^10,x)
```

```
[Out] -1/315*(b*x^2+a)^(5/2)*(8*b^2*x^4-20*a*b*x^2+35*a^2)/x^9/a^3
```

maxima [A] time = 1.37, size = 56, normalized size = 0.82

$$-\frac{8\left(bx^{2} + a\right)^{\frac{5}{2}} b^{2}}{315\, a^{3} x^{5}} + \frac{4\left(bx^{2} + a\right)^{\frac{5}{2}} b}{63\, a^{2} x^{7}} - \frac{\left(bx^{2} + a\right)^{\frac{5}{2}}}{9\, a x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] -8/315*(b*x^2 + a)^(5/2)*b^2/(a^3*x^5) + 4/63*(b*x^2 + a)^(5/2)*b/(a^2*x^7)
 - 1/9*(b*x^2 + a)^(5/2)/(a*x^9)
```

mupad [B] time = 5.71, size = 91, normalized size = 1.34

$$\frac{4\, b^{3}\, \sqrt{b\, x^{2} + a}}{315\, a^{2}\, x^{3}} - \frac{10\, b\, \sqrt{b\, x^{2} + a}}{63\, x^{7}} - \frac{b^{2}\, \sqrt{b\, x^{2} + a}}{105\, a\, x^{5}} - \frac{a\, \sqrt{b\, x^{2} + a}}{9\, x^{9}} - \frac{8\, b^{4}\, \sqrt{b\, x^{2} + a}}{315\, a^{3}\, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^10,x)
```

```
[Out] (4*b^3*(a + b*x^2)^(1/2))/(315*a^2*x^3) - (10*b*(a + b*x^2)^(1/2))/(63*x^7)
 - (b^2*(a + b*x^2)^(1/2))/(105*a*x^5) - (a*(a + b*x^2)^(1/2))/(9*x^9) - (8
*b^4*(a + b*x^2)^(1/2))/(315*a^3*x)
```

sympy [B] time = 1.61, size = 420, normalized size = 6.18

$$-\frac{35a^6b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{120a^5b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{138a^4b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+}}{315a^5b^4x^8+630a^4b^5x^{10}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**10,x)
```

```
[Out] -35*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*
x**10 + 315*a**3*b**6*x**12) - 120*a**5*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)
/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 138*a**
4*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x
**10 + 315*a**3*b**6*x**12) - 52*a**3*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(
315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 3*a**2*b*
*(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10
 + 315*a**3*b**6*x**12) - 12*a*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a*
*5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 8*b**(21/2)*x**
12*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**
3*b**6*x**12)
```

3.386 $\int \frac{(a+bx^2)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=92

$$\frac{16b^3\left(a+bx^2\right)^{5/2}}{1155a^4x^5}-\frac{8b^2\left(a+bx^2\right)^{5/2}}{231a^3x^7}+\frac{2b\left(a+bx^2\right)^{5/2}}{33a^2x^9}-\frac{\left(a+bx^2\right)^{5/2}}{11ax^{11}}$$

```
[Out] -1/11*(b*x^2+a)^(5/2)/a/x^11+2/33*b*(b*x^2+a)^(5/2)/a^2/x^9-8/231*b^2*(b*x^
2+a)^(5/2)/a^3/x^7+16/1155*b^3*(b*x^2+a)^(5/2)/a^4/x^5
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{16b^3\left(a+bx^2\right)^{5/2}}{1155a^4x^5}-\frac{8b^2\left(a+bx^2\right)^{5/2}}{231a^3x^7}+\frac{2b\left(a+bx^2\right)^{5/2}}{33a^2x^9}-\frac{\left(a+bx^2\right)^{5/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^12,x]
```

```
[Out] -(a + b*x^2)^(5/2)/(11*a*x^11) + (2*b*(a + b*x^2)^(5/2))/(33*a^2*x^9) - (8*
b^2*(a + b*x^2)^(5/2))/(231*a^3*x^7) + (16*b^3*(a + b*x^2)^(5/2))/(1155*a^4
*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^{12}}\,dx &= -\frac{\left(a+bx^2\right)^{5/2}}{11ax^{11}}-\frac{(6b)\int \frac{\left(a+bx^2\right)^{3/2}}{x^{10}}\,dx}{11a}\\
&= -\frac{\left(a+bx^2\right)^{5/2}}{11ax^{11}}+\frac{2b\left(a+bx^2\right)^{5/2}}{33a^2x^9}+\frac{\left(8b^2\right)\int \frac{\left(a+bx^2\right)^{3/2}}{x^8}\,dx}{33a^2}\\
&= -\frac{\left(a+bx^2\right)^{5/2}}{11ax^{11}}+\frac{2b\left(a+bx^2\right)^{5/2}}{33a^2x^9}-\frac{8b^2\left(a+bx^2\right)^{5/2}}{231a^3x^7}-\frac{\left(16b^3\right)\int \frac{\left(a+bx^2\right)^{3/2}}{x^6}\,dx}{231a^3}\\
&= -\frac{\left(a+bx^2\right)^{5/2}}{11ax^{11}}+\frac{2b\left(a+bx^2\right)^{5/2}}{33a^2x^9}-\frac{8b^2\left(a+bx^2\right)^{5/2}}{231a^3x^7}+\frac{16b^3\left(a+bx^2\right)^{5/2}}{1155a^4x^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.58

$$\frac{\left(a+bx^2\right)^{5/2}\left(-105a^3+70a^2bx^2-40ab^2x^4+16b^3x^6\right)}{1155a^4x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^12,x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-105*a^3 + 70*a^2*b*x^2 - 40*a*b^2*x^4 + 16*b^3*x^6))/(
1155*a^4*x^11)
```

fricas [A] time = 0.83, size = 71, normalized size = 0.77

$$\frac{\left(16\,b^5x^{10}-8\,ab^4x^8+6\,a^2b^3x^6-5\,a^3b^2x^4-140\,a^4bx^2-105\,a^5\right)\sqrt{bx^2+a}}{1155\,a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] 1/1155*(16*b^5*x^10 - 8*a*b^4*x^8 + 6*a^2*b^3*x^6 - 5*a^3*b^2*x^4 - 140*a^4
*b*x^2 - 105*a^5)*sqrt(b*x^2 + a)/(a^4*x^11)
```

giac [B] time = 0.69, size = 220, normalized size = 2.39

$$\frac{32\left(1155\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{14}b^{\frac{11}{2}}+2079\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}ab^{\frac{11}{2}}+2541\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{10}a^2b^{\frac{11}{2}}+825\left(\right.\right.}{1155\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] 32/1155*(1155*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(11/2) + 2079*(sqrt(b)*x -
 sqrt(b*x^2 + a))^12*a*b^(11/2) + 2541*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2
*b^(11/2) + 825*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(11/2) + 165*(sqrt(b)
*x - sqrt(b*x^2 + a))^6*a^4*b^(11/2) - 55*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a
^5*b^(11/2) + 11*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(11/2) - a^7*b^(11/2
))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^11
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\left(-16b^3x^6+40a\,b^2x^4-70a^2b\,x^2+105a^3\right)}{1155a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^12,x)
```

```
[Out] -1/1155*(b*x^2+a)^(5/2)*(-16*b^3*x^6+40*a*b^2*x^4-70*a^2*b*x^2+105*a^3)/x^1
1/a^4
```

maxima [A] time = 1.42, size = 76, normalized size = 0.83

$$\frac{16\left(bx^2+a\right)^{\frac{5}{2}}b^3}{1155\,a^4x^5}-\frac{8\left(bx^2+a\right)^{\frac{5}{2}}b^2}{231\,a^3x^7}+\frac{2\left(bx^2+a\right)^{\frac{5}{2}}b}{33\,a^2x^9}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{11\,ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] 16/1155*(b*x^2 + a)^(5/2)*b^3/(a^4*x^5) - 8/231*(b*x^2 + a)^(5/2)*b^2/(a^3*
x^7) + 2/33*(b*x^2 + a)^(5/2)*b/(a^2*x^9) - 1/11*(b*x^2 + a)^(5/2)/(a*x^11)
```

mupad [B] time = 5.78, size = 111, normalized size = 1.21

$$\frac{2\,b^3\,\sqrt{b\,x^2+a}}{385\,a^2\,x^5}-\frac{4\,b\,\sqrt{b\,x^2+a}}{33\,x^9}-\frac{b^2\,\sqrt{b\,x^2+a}}{231\,a\,x^7}-\frac{a\,\sqrt{b\,x^2+a}}{11\,x^{11}}-\frac{8\,b^4\,\sqrt{b\,x^2+a}}{1155\,a^3\,x^3}+\frac{16\,b^5\,\sqrt{b\,x^2+a}}{1155\,a^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^12,x)
```

```
[Out] (2*b^3*(a + b*x^2)^(1/2))/(385*a^2*x^5) - (4*b*(a + b*x^2)^(1/2))/(33*x^9)
- (b^2*(a + b*x^2)^(1/2))/(231*a*x^7) - (a*(a + b*x^2)^(1/2))/(11*x^11) - (
8*b^4*(a + b*x^2)^(1/2))/(1155*a^3*x^3) + (16*b^5*(a + b*x^2)^(1/2))/(1155*
a^4*x)
```

sympy [B] time = 2.12, size = 648, normalized size = 7.04

$$-\frac{105a^8b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}-\frac{455a^7b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**12,x)
```

```
[Out] -105*a**8*b**(19/2)*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*
b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 455*a**7*b**
(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x*
*12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 740*a**6*b**(23/2)*x
**4*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 34
65*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 534*a**5*b**(25/2)*x**6*sqrt
(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*
b**11*x**14 + 1155*a**4*b**12*x**16) - 145*a**4*b**(27/2)*x**8*sqrt(a/(b*x*
*2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x*
*14 + 1155*a**4*b**12*x**16) + 5*a**3*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/
(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 115
5*a**4*b**12*x**16) + 30*a**2*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(1155*a*
*7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b
**12*x**16) + 40*a*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**
10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16)
 + 16*b**(35/2)*x**16*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**
6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16)
```

3.387 $\int x^7 \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=80

$$-\frac{a^3\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{a^2\left(a+bx^2\right)^{9/2}}{3b^4}+\frac{\left(a+bx^2\right)^{13/2}}{13b^4}-\frac{3a\left(a+bx^2\right)^{11/2}}{11b^4}$$

```
[Out] -1/7*a^3*(b*x^2+a)^(7/2)/b^4+1/3*a^2*(b*x^2+a)^(9/2)/b^4-3/11*a*(b*x^2+a)^(
11/2)/b^4+1/13*(b*x^2+a)^(13/2)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^{9/2}}{3b^4}-\frac{a^3\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{\left(a+bx^2\right)^{13/2}}{13b^4}-\frac{3a\left(a+bx^2\right)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(5/2),x]
```

```
[Out] -(a^3*(a + b*x^2)^(7/2))/(7*b^4) + (a^2*(a + b*x^2)^(9/2))/(3*b^4) - (3*a*(
a + b*x^2)^(11/2))/(11*b^4) + (a + b*x^2)^(13/2)/(13*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a+bx^2\right)^{5/2} dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{5/2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3(a+bx)^{5/2}}{b^3}+\frac{3a^2(a+bx)^{7/2}}{b^3}-\frac{3a(a+bx)^{9/2}}{b^3}+\frac{(a+bx)^{11/2}}{b^3}\right)dx,x,x^2\right)\\
&= -\frac{a^3\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{a^2\left(a+bx^2\right)^{9/2}}{3b^4}-\frac{3a\left(a+bx^2\right)^{11/2}}{11b^4}+\frac{\left(a+bx^2\right)^{13/2}}{13b^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^{7/2}\left(-16a^3+56a^2bx^2-126ab^2x^4+231b^3x^6\right)}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(5/2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-16*a^3 + 56*a^2*b*x^2 - 126*a*b^2*x^4 + 231*b^3*x^6))/
(3003*b^4)
```

fricas [A] time = 0.90, size = 79, normalized size = 0.99

$$\frac{\left(231\,b^6x^{12}+567\,ab^5x^{10}+371\,a^2b^4x^8+5\,a^3b^3x^6-6\,a^4b^2x^4+8\,a^5bx^2-16\,a^6\right)\sqrt{bx^2+a}}{3003\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3003*(231*b^6*x^12 + 567*a*b^5*x^10 + 371*a^2*b^4*x^8 + 5*a^3*b^3*x^6 - 6
*a^4*b^2*x^4 + 8*a^5*b*x^2 - 16*a^6)*sqrt(b*x^2 + a)/b^4
```

giac [A] time = 0.66, size = 57, normalized size = 0.71

$$\frac{231\left(bx^2+a\right)^{\frac{13}{2}}-819\left(bx^2+a\right)^{\frac{11}{2}}a+1001\left(bx^2+a\right)^{\frac{9}{2}}a^2-429\left(bx^2+a\right)^{\frac{7}{2}}a^3}{3003\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3003*(231*(b*x^2 + a)^(13/2) - 819*(b*x^2 + a)^(11/2)*a + 1001*(b*x^2 + a
)^(9/2)*a^2 - 429*(b*x^2 + a)^(7/2)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}\left(-231b^3x^6+126a\,b^2x^4-56a^2b\,x^2+16a^3\right)}{3003b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3003*(b*x^2+a)^(7/2)*(-231*b^3*x^6+126*a*b^2*x^4-56*a^2*b*x^2+16*a^3)/b^
4
```

maxima [A] time = 1.33, size = 73, normalized size = 0.91

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}x^6}{13\,b}-\frac{6\left(bx^2+a\right)^{\frac{7}{2}}ax^4}{143\,b^2}+\frac{8\left(bx^2+a\right)^{\frac{7}{2}}a^2x^2}{429\,b^3}-\frac{16\left(bx^2+a\right)^{\frac{7}{2}}a^3}{3003\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/13*(b*x^2 + a)^(7/2)*x^6/b - 6/143*(b*x^2 + a)^(7/2)*a*x^4/b^2 + 8/429*(b
*x^2 + a)^(7/2)*a^2*x^2/b^3 - 16/3003*(b*x^2 + a)^(7/2)*a^3/b^4
```

mupad [B] time = 4.86, size = 75, normalized size = 0.94

$$\sqrt{b\,x^2+a}\,\left(\frac{53\,a^2\,x^8}{429}-\frac{16\,a^6}{3003\,b^4}+\frac{b^2\,x^{12}}{13}+\frac{5\,a^3\,x^6}{3003\,b}-\frac{2\,a^4\,x^4}{1001\,b^2}+\frac{8\,a^5\,x^2}{3003\,b^3}+\frac{27\,a\,b\,x^{10}}{143}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((53*a^2*x^8)/429 - (16*a^6)/(3003*b^4) + (b^2*x^12)/13 +
 (5*a^3*x^6)/(3003*b) - (2*a^4*x^4)/(1001*b^2) + (8*a^5*x^2)/(3003*b^3) + (
27*a*b*x^10)/143)
```

sympy [A] time = 9.13, size = 158, normalized size = 1.98

$$\begin{cases} -\frac{16a^6\sqrt{a+bx^2}}{3003b^4} + \frac{8a^5x^2\sqrt{a+bx^2}}{3003b^3} - \frac{2a^4x^4\sqrt{a+bx^2}}{1001b^2} + \frac{5a^3x^6\sqrt{a+bx^2}}{3003b} + \frac{53a^2x^8\sqrt{a+bx^2}}{429} + \frac{27abx^{10}\sqrt{a+bx^2}}{143} + \frac{b^2x^{12}\sqrt{a+bx^2}}{13} & \text{for } b \neq \\ \frac{a^{\frac{5}{2}}x^8}{8} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-16*a**6*sqrt(a + b*x**2)/(3003*b**4) + 8*a**5*x**2*sqrt(a + b*x
**2)/(3003*b**3) - 2*a**4*x**4*sqrt(a + b*x**2)/(1001*b**2) + 5*a**3*x**6*s
qrt(a + b*x**2)/(3003*b) + 53*a**2*x**8*sqrt(a + b*x**2)/429 + 27*a*b*x**10
*sqrt(a + b*x**2)/143 + b**2*x**12*sqrt(a + b*x**2)/13, Ne(b, 0)), (a**(5/2
)*x**8/8, True))
```

3.388 $\int x^5 \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{\left(a+bx^2\right)^{11/2}}{11b^3}-\frac{2a\left(a+bx^2\right)^{9/2}}{9b^3}$$

```
[Out] 1/7*a^2*(b*x^2+a)^(7/2)/b^3-2/9*a*(b*x^2+a)^(9/2)/b^3+1/11*(b*x^2+a)^(11/2)
/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{\left(a+bx^2\right)^{11/2}}{11b^3}-\frac{2a\left(a+bx^2\right)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(5/2),x]
```

```
[Out] (a^2*(a + b*x^2)^(7/2))/(7*b^3) - (2*a*(a + b*x^2)^(9/2))/(9*b^3) + (a + b*
x^2)^(11/2)/(11*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^{5/2} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^{5/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{a^2(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{7/2}}{b^2}+\frac{(a+bx)^{9/2}}{b^2}\right)dx,x,x^2\right)\\ &= \frac{a^2\left(a+bx^2\right)^{7/2}}{7b^3}-\frac{2a\left(a+bx^2\right)^{9/2}}{9b^3}+\frac{\left(a+bx^2\right)^{11/2}}{11b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^2\right)^{7/2}\left(8a^2-28abx^2+63b^2x^4\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(5/2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(8*a^2 - 28*a*b*x^2 + 63*b^2*x^4))/(693*b^3)
```

fricas [A] time = 0.74, size = 68, normalized size = 1.15

$$\frac{\left(63\,b^{5}x^{10}+161\,ab^{4}x^{8}+113\,a^{2}b^{3}x^{6}+3\,a^{3}b^{2}x^{4}-4\,a^{4}bx^{2}+8\,a^{5}\right)\sqrt{bx^{2}+a}}{693\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/693*(63*b^5*x^10 + 161*a*b^4*x^8 + 113*a^2*b^3*x^6 + 3*a^3*b^2*x^4 - 4*a^
4*b*x^2 + 8*a^5)*sqrt(b*x^2 + a)/b^3
```

giac [A] time = 0.92, size = 43, normalized size = 0.73

$$\frac{63\left(bx^{2}+a\right)^{\frac{11}{2}}-154\left(bx^{2}+a\right)^{\frac{9}{2}}a+99\left(bx^{2}+a\right)^{\frac{7}{2}}a^{2}}{693\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/693*(63*(b*x^2 + a)^(11/2) - 154*(b*x^2 + a)^(9/2)*a + 99*(b*x^2 + a)^(7/
2)*a^2)/b^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(63b^{2}x^{4}-28ab\,x^{2}+8a^{2}\right)}{693b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/693*(b*x^2+a)^(7/2)*(63*b^2*x^4-28*a*b*x^2+8*a^2)/b^3
```

maxima [A] time = 1.36, size = 53, normalized size = 0.90

$$\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}x^{4}}{11\,b}-\frac{4\left(bx^{2}+a\right)^{\frac{7}{2}}ax^{2}}{99\,b^{2}}+\frac{8\left(bx^{2}+a\right)^{\frac{7}{2}}a^{2}}{693\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^2 + a)^(7/2)*x^4/b - 4/99*(b*x^2 + a)^(7/2)*a*x^2/b^2 + 8/693*(b*
x^2 + a)^(7/2)*a^2/b^3
```

mupad [B] time = 4.75, size = 64, normalized size = 1.08

$$\sqrt{b\,x^{2}+a}\,\left(\frac{8\,a^{5}}{693\,b^{3}}+\frac{113\,a^{2}\,x^{6}}{693}+\frac{b^{2}\,x^{10}}{11}+\frac{a^{3}\,x^{4}}{231\,b}-\frac{4\,a^{4}\,x^{2}}{693\,b^{2}}+\frac{23\,a\,b\,x^{8}}{99}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a^5)/(693*b^3) + (113*a^2*x^6)/693 + (b^2*x^10)/11 +
(a^3*x^4)/(231*b) - (4*a^4*x^2)/(693*b^2) + (23*a*b*x^8)/99)
```

sympy [A] time = 6.28, size = 133, normalized size = 2.25

$$\begin{cases} \frac{8a^5\sqrt{a+bx^2}}{693b^3} - \frac{4a^4x^2\sqrt{a+bx^2}}{693b^2} + \frac{a^3x^4\sqrt{a+bx^2}}{231b} + \frac{113a^2x^6\sqrt{a+bx^2}}{693} + \frac{23abx^8\sqrt{a+bx^2}}{99} + \frac{b^2x^{10}\sqrt{a+bx^2}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((8*a**5*sqrt(a + b*x**2)/(693*b**3) - 4*a**4*x**2*sqrt(a + b*x**2
)/(693*b**2) + a**3*x**4*sqrt(a + b*x**2)/(231*b) + 113*a**2*x**6*sqrt(a +
b*x**2)/693 + 23*a*b*x**8*sqrt(a + b*x**2)/99 + b**2*x**10*sqrt(a + b*x**2)
/11, Ne(b, 0)), (a**(5/2)*x**6/6, True))
```

3.389 $\int x^3 \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^2\right)^{9/2}}{9b^2}-\frac{a\left(a+bx^2\right)^{7/2}}{7b^2}$$

```
[Out] -1/7*a*(b*x^2+a)^(7/2)/b^2+1/9*(b*x^2+a)^(9/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{9/2}}{9b^2}-\frac{a\left(a+bx^2\right)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(5/2),x]
```

```
[Out] -(a*(a + b*x^2)^(7/2))/(7*b^2) + (a + b*x^2)^(9/2)/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^2\right)^{5/2} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^{5/2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a(a+bx)^{5/2}}{b}+\frac{(a+bx)^{7/2}}{b}\right)dx, x, x^2\right)\\
&= -\frac{a\left(a+bx^2\right)^{7/2}}{7b^2}+\frac{\left(a+bx^2\right)^{9/2}}{9b^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{7/2}\left(7bx^2-2a\right)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(5/2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-2*a + 7*b*x^2))/(63*b^2)
```

fricas [A] time = 0.61, size = 56, normalized size = 1.47

$$\frac{\left(7\,b^4x^8+19\,ab^3x^6+15\,a^2b^2x^4+a^3bx^2-2\,a^4\right)\sqrt{bx^2+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^4*x^8 + 19*a*b^3*x^6 + 15*a^2*b^2*x^4 + a^3*b*x^2 - 2*a^4)*sqrt(b
*x^2 + a)/b^2
```

giac [A] time = 0.99, size = 29, normalized size = 0.76

$$\frac{7\left(bx^2+a\right)^{\frac{9}{2}}-9\left(bx^2+a\right)^{\frac{7}{2}}a}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/63*(7*(b*x^2 + a)^(9/2) - 9*(b*x^2 + a)^(7/2)*a)/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}\left(-7b\,x^2+2a\right)}{63b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(5/2),x)
```

```
[Out] -1/63*(b*x^2+a)^(7/2)*(-7*b*x^2+2*a)/b^2
```

maxima [A] time = 1.37, size = 33, normalized size = 0.87

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}x^2}{9\,b}-\frac{2\left(bx^2+a\right)^{\frac{7}{2}}a}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(7/2)*x^2/b - 2/63*(b*x^2 + a)^(7/2)*a/b^2
```

mupad [B] time = 4.74, size = 53, normalized size = 1.39

$$\sqrt{b\,x^2+a}\,\left(\frac{5\,a^2\,x^4}{21}-\frac{2\,a^4}{63\,b^2}+\frac{b^2\,x^8}{9}+\frac{a^3\,x^2}{63\,b}+\frac{19\,a\,b\,x^6}{63}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((5*a^2*x^4)/21 - (2*a^4)/(63*b^2) + (b^2*x^8)/9 + (a^3*x
^2)/(63*b) + (19*a*b*x^6)/63)
```

sympy [A] time = 3.83, size = 109, normalized size = 2.87

$$\begin{cases} -\frac{2a^4\sqrt{a+bx^2}}{63b^2}+\frac{a^3x^2\sqrt{a+bx^2}}{63b}+\frac{5a^2x^4\sqrt{a+bx^2}}{21}+\frac{19abx^6\sqrt{a+bx^2}}{63}+\frac{b^2x^8\sqrt{a+bx^2}}{9} & \text{for } b\neq 0 \\ \frac{a^{\frac{5}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-2*a**4*sqrt(a + b*x**2)/(63*b**2) + a**3*x**2*sqrt(a + b*x**2)/
(63*b) + 5*a**2*x**4*sqrt(a + b*x**2)/21 + 19*a*b*x**6*sqrt(a + b*x**2)/63
+ b**2*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(5/2)*x**4/4, True))
```

3.390 $\int x\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

[Out] 1/7*(b*x^2+a)^(7/2)/b

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x^2)^(5/2),x]

[Out] (a + b*x^2)^(7/2)/(7*b)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{5/2}\,dx=\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x^2)^(5/2),x]

[Out] (a + b*x^2)^(7/2)/(7*b)

fricas [B] time = 0.46, size = 43, normalized size = 2.39

$$\frac{\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{bx^2+a}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^2+a)^(5/2),x, algorithm="fricas")

[Out] 1/7*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 + a)/b

giac [A] time = 1.20, size = 14, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/7*(b*x^2 + a)^(7/2)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/7*(b*x^2+a)^(7/2)/b
```

maxima [A] time = 1.34, size = 14, normalized size = 0.78

$$\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(7/2)/b
```

mupad [B] time = 4.60, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^{2}+a\right)^{7/2}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(7/2)/(7*b)
```

sympy [A] time = 2.10, size = 85, normalized size = 4.72

$$\begin{cases} \frac{a^{3}\sqrt{a+bx^{2}}}{7b}+\frac{3a^{2}x^{2}\sqrt{a+bx^{2}}}{7}+\frac{3abx^{4}\sqrt{a+bx^{2}}}{7}+\frac{b^{2}x^{6}\sqrt{a+bx^{2}}}{7} & \text{for } b\neq 0\\ \frac{a^{\frac{5}{2}}x^{2}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((a**3*sqrt(a + b*x**2)/(7*b) + 3*a**2*x**2*sqrt(a + b*x**2)/7 + 3
*a*b*x**4*sqrt(a + b*x**2)/7 + b**2*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (a*
*(5/2)*x**2/2, True))
```

3.391 $\int \frac{(a+bx^2)^{5/2}}{x} dx$

Optimal. Leaf size=72

$$a^{5/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+a^2\sqrt{a+bx^2}+\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}$$

```
[Out] 1/3*a*(b*x^2+a)^(3/2)+1/5*(b*x^2+a)^(5/2)-a^(5/2)*arctanh((b*x^2+a)^(1/2)/a
^(1/2))+a^2*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 208}

$$a^2\sqrt{a+bx^2}+a^{5/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x,x]
```

```
[Out] a^2*Sqrt[a + b*x^2] + (a*(a + b*x^2)^(3/2))/3 + (a + b*x^2)^(5/2)/5 - a^(5/
2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{5}\left(a+bx^2\right)^{5/2}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^2\right)\\
&= a^2\sqrt{a+bx^2}+\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}+\frac{1}{2}a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= a^2\sqrt{a+bx^2}+\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}+\frac{a^3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\
&= a^2\sqrt{a+bx^2}+\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}-a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.86

$$\frac{1}{15}\sqrt{a+bx^2}\left(23a^2+11abx^2+3b^2x^4\right)-a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(23*a^2 + 11*a*b*x^2 + 3*b^2*x^4))/15 - a^(5/2)*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]]
```

fricas [A] time = 0.96, size = 126, normalized size = 1.75

$$\left[\frac{1}{2}a^{\frac{5}{2}}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+\frac{1}{15}\left(3b^2x^4+11abx^2+23a^2\right)\sqrt{bx^2+a},\sqrt{-a}\,a^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*a^(5/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/15*(3*
b^2*x^4 + 11*a*b*x^2 + 23*a^2)*sqrt(b*x^2 + a), sqrt(-a)*a^2*arctan(sqrt(-a
)/sqrt(b*x^2 + a)) + 1/15*(3*b^2*x^4 + 11*a*b*x^2 + 23*a^2)*sqrt(b*x^2 + a)
]
```

giac [A] time = 1.19, size = 62, normalized size = 0.86

$$\frac{a^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}a+\sqrt{bx^2+a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] a^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/5*(b*x^2 + a)^(5/2) + 1/3
*(b*x^2 + a)^(3/2)*a + sqrt(b*x^2 + a)*a^2
```

maple [A] time = 0.00, size = 66, normalized size = 0.92

$$-a^{\frac{5}{2}} \ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)+\sqrt{b\,x^{2}+a}\;a^{2}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} a}{3}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x,x)
```

```
[Out] 1/5*(b*x^2+a)^(5/2)+1/3*a*(b*x^2+a)^(3/2)-a^(5/2)*ln((2*a+2*(b*x^2+a)^(1/2)
*a^(1/2))/x)+a^2*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.33, size = 54, normalized size = 0.75

$$-a^{\frac{5}{2}} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{5}\left(bx^{2}+a\right)^{\frac{5}{2}}+\frac{1}{3}\left(bx^{2}+a\right)^{\frac{3}{2}}a+\sqrt{bx^{2}+a}\,a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] -a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2) + 1/3*(b*x^2
 + a)^(3/2)*a + sqrt(b*x^2 + a)*a^2
```

mupad [B] time = 4.68, size = 59, normalized size = 0.82

$$\frac{a\left(b\,x^{2}+a\right)^{3/2}}{3}+\frac{\left(b\,x^{2}+a\right)^{5/2}}{5}+a^{2}\sqrt{b\,x^{2}+a}+a^{5/2}\operatorname{atan}\left(\frac{\sqrt{b\,x^{2}+a}\;1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x,x)
```

```
[Out] a^(5/2)*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i + (a*(a + b*x^2)^(3/2))/3 +
 (a + b*x^2)^(5/2)/5 + a^2*(a + b*x^2)^(1/2)
```

sympy [A] time = 3.56, size = 105, normalized size = 1.46

$$\frac{23a^{\frac{5}{2}}\sqrt{1+\frac{bx^{2}}{a}}}{15}+\frac{a^{\frac{5}{2}}\log\left(\frac{bx^{2}}{a}\right)}{2}-a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)+\frac{11a^{\frac{3}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}{15}+\frac{\sqrt{a}\,b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x,x)
```

```
[Out] 23*a**(5/2)*sqrt(1 + b*x**2/a)/15 + a**(5/2)*log(b*x**2/a)/2 - a**(5/2)*log
(sqrt(1 + b*x**2/a) + 1) + 11*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a)/15 + sqrt(
a)*b**2*x**4*sqrt(1 + b*x**2/a)/5
```

3.392 $\int \frac{\left(a+bx^2\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=80

$$-\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{5/2}}{2x^2}+\frac{5}{6}b\left(a+bx^2\right)^{3/2}+\frac{5}{2}ab\sqrt{a+bx^2}$$

```
[Out] 5/6*b*(b*x^2+a)^(3/2)-1/2*(b*x^2+a)^(5/2)/x^2-5/2*a^(3/2)*b*arctanh((b*x^2+
a)^(1/2)/a^(1/2))+5/2*a*b*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$-\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{5/2}}{2x^2}+\frac{5}{6}b\left(a+bx^2\right)^{3/2}+\frac{5}{2}ab\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^3,x]
```

```
[Out] (5*a*b*Sqrt[a + b*x^2])/2 + (5*b*(a + b*x^2)^(3/2))/6 - (a + b*x^2)^(5/2)/(
2*x^2) - (5*a^(3/2)*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{5/2}}{2x^2}+\frac{1}{4}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right)\\
&= \frac{5}{6}b\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{5/2}}{2x^2}+\frac{1}{4}(5ab)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\
&= \frac{5}{2}ab\sqrt{a+bx^2}+\frac{5}{6}b\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{5/2}}{2x^2}+\frac{1}{4}\left(5a^2b\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \frac{5}{2}ab\sqrt{a+bx^2}+\frac{5}{6}b\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{5/2}}{2x^2}+\frac{1}{2}\left(5a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)\\
&= \frac{5}{2}ab\sqrt{a+bx^2}+\frac{5}{6}b\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{5/2}}{2x^2}-\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.46

$$\frac{b\left(a+bx^2\right)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^3,x]
[Out] (b*(a + b*x^2)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, 1 + (b*x^2)/a])/(7*a^2)
```

fricas [A] time = 0.97, size = 142, normalized size = 1.78

$$\left[\frac{15\,a^{\frac{3}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(2\,b^2x^4+14\,abx^2-3\,a^2\right)\sqrt{bx^2+a}}{12\,x^2},\ \frac{15\,\sqrt{-a}\,abx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(2\,b^2\right.}{6\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="fricas")
[Out] [1/12*(15*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 2*(2*b^2*x^4 + 14*a*b*x^2 - 3*a^2)*sqrt(b*x^2 + a))/x^2, 1/6*(15*sqrt(-a
)*a*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*b^2*x^4 + 14*a*b*x^2 - 3*a^
2)*sqrt(b*x^2 + a))/x^2]
```

giac [A] time = 1.13, size = 82, normalized size = 1.02

$$\frac{\frac{15\,a^2b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\left(bx^2+a\right)^{\frac{3}{2}}b^2+12\,\sqrt{bx^2+a}\,ab^2-\frac{3\,\sqrt{bx^2+a}\,a^2b}{x^2}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*a^2*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 2*(b*x^2 + a)^(
3/2)*b^2 + 12*sqrt(b*x^2 + a)*a*b^2 - 3*sqrt(b*x^2 + a)*a^2*b/x^2)/b
```

maple [A] time = 0.01, size = 88, normalized size = 1.10

$$-\frac{5a^{\frac{3}{2}}b\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2}+\frac{5\sqrt{b\,x^{2}+a}\,ab}{2}+\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}b}{6}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}b}{2a}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(7/2)+1/2/a*b*(b*x^2+a)^(5/2)+5/6*b*(b*x^2+a)^(3/2)-5/
2*a^(3/2)*b*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+5/2*a*b*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.38, size = 76, normalized size = 0.95

$$-\frac{5}{2}\,a^{\frac{3}{2}}b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{5}{6}\left(bx^{2}+a\right)^{\frac{3}{2}}b+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}b}{2\,a}+\frac{5}{2}\,\sqrt{bx^{2}+a}\,ab-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -5/2*a^(3/2)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 5/6*(b*x^2 + a)^(3/2)*b + 1/
2*(b*x^2 + a)^(5/2)*b/a + 5/2*sqrt(b*x^2 + a)*a*b - 1/2*(b*x^2 + a)^(7/2)/(
a*x^2)
```

mupad [B] time = 4.95, size = 66, normalized size = 0.82

$$\frac{b\left(b\,x^{2}+a\right)^{3/2}}{3}-\frac{a^{2}\,\sqrt{b\,x^{2}+a}}{2\,x^{2}}+2\,a\,b\,\sqrt{b\,x^{2}+a}\,+\frac{a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^{2}+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^3,x)
```

```
[Out] (b*(a + b*x^2)^(3/2))/3 - (a^2*(a + b*x^2)^(1/2))/(2*x^2) + (a^(3/2)*b*atan
(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/2 + 2*a*b*(a + b*x^2)^(1/2)
```

sympy [A] time = 3.22, size = 112, normalized size = 1.40

$$-\frac{a^{\frac{5}{2}}\sqrt{1+\frac{bx^{2}}{a}}}{2x^{2}}+\frac{7a^{\frac{3}{2}}b\sqrt{1+\frac{bx^{2}}{a}}}{3}+\frac{5a^{\frac{3}{2}}b\log\left(\frac{bx^{2}}{a}\right)}{4}-\frac{5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{2}+\frac{\sqrt{a}\,b^{2}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**3,x)
```

```
[Out] -a**(5/2)*sqrt(1 + b*x**2/a)/(2*x**2) + 7*a**(3/2)*b*sqrt(1 + b*x**2/a)/3 +
 5*a**(3/2)*b*log(b*x**2/a)/4 - 5*a**(3/2)*b*log(sqrt(1 + b*x**2/a) + 1)/2
+ sqrt(a)*b**2*x**2*sqrt(1 + b*x**2/a)/3
```

3.393 $\int \frac{(a+bx^2)^{5/2}}{x^5} dx$

Optimal. Leaf size=86

$$\frac{15}{8}b^2\sqrt{a+bx^2} - \frac{15}{8}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{5b\left(a+bx^2\right)^{3/2}}{8x^2} - \frac{\left(a+bx^2\right)^{5/2}}{4x^4}$$

```
[Out] -5/8*b*(b*x^2+a)^(3/2)/x^2-1/4*(b*x^2+a)^(5/2)/x^4-15/8*b^2*arctanh((b*x^2+
a)^(1/2)/a^(1/2))*a^(1/2)+15/8*b^2*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$\frac{15}{8}b^2\sqrt{a+bx^2} - \frac{15}{8}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{\left(a+bx^2\right)^{5/2}}{4x^4} - \frac{5b\left(a+bx^2\right)^{3/2}}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^5,x]
```

```
[Out] (15*b^2*Sqrt[a + b*x^2])/8 - (5*b*(a + b*x^2)^(3/2))/(8*x^2) - (a + b*x^2)^
(5/2)/(4*x^4) - (15*Sqrt[a]*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^{5/2}}{4x^4} + \frac{1}{8}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{5b\left(a+bx^2\right)^{3/2}}{8x^2} - \frac{\left(a+bx^2\right)^{5/2}}{4x^4} + \frac{1}{16}\left(15b^2\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)$$
$$= \frac{15}{8}b^2\sqrt{a+bx^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{8x^2} - \frac{\left(a+bx^2\right)^{5/2}}{4x^4} + \frac{1}{16}\left(15ab^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x,\right.$$
$$= \frac{15}{8}b^2\sqrt{a+bx^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{8x^2} - \frac{\left(a+bx^2\right)^{5/2}}{4x^4} + \frac{1}{8}(15ab)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+}\right.$$
$$= \frac{15}{8}b^2\sqrt{a+bx^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{8x^2} - \frac{\left(a+bx^2\right)^{5/2}}{4x^4} - \frac{15}{8}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.45

$$-\frac{b^2\left(a+bx^2\right)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^5,x]
```

```
[Out] -1/7*(b^2*(a + b*x^2)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, 1 + (b*x^2)/a])/
a^3
```

fricas [A] time = 0.90, size = 145, normalized size = 1.69

$$\left[\frac{15\sqrt{a}\,b^2x^4\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(8b^2x^4-9abx^2-2a^2\right)\sqrt{bx^2+a}}{16x^4}, \frac{15\sqrt{-a}\,b^2x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(\cdots}{8x\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(15*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*(8*b^2*x^4 - 9*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/x^4, 1/8*(15*sqrt(-
a)*b^2*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*b^2*x^4 - 9*a*b*x^2 - 2*a^
2)*sqrt(b*x^2 + a))/x^4]
```

giac [A] time = 1.16, size = 88, normalized size = 1.02

$$\frac{\frac{15\,ab^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 8\sqrt{bx^2+a}\,b^3 - \frac{9\left(bx^2+a\right)^{\frac{3}{2}}ab^3 - 7\sqrt{bx^2+a}\,a^2b^3}{b^2x^4}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(15*a*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 8*sqrt(b*x^2 + a)
*b^3 - (9*(b*x^2 + a)^(3/2)*a*b^3 - 7*sqrt(b*x^2 + a)*a^2*b^3)/(b^2*x^4))/b
```

maple [A] time = 0.01, size = 116, normalized size = 1.35

$$-\frac{15\sqrt{a}\, b^2 \ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{8}+\frac{15\sqrt{b\,x^2+a}\, b^2}{8}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}} b^2}{8a}+\frac{3\left(b\,x^2+a\right)^{\frac{5}{2}} b^2}{8a^2}-\frac{3\left(b\,x^2+a\right)^{\frac{7}{2}} b}{8a^2x^2}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(7/2)-3/8/a^2*b/x^2*(b*x^2+a)^(7/2)+3/8/a^2*b^2*(b*x^2
+a)^(5/2)+5/8/a*b^2*(b*x^2+a)^(3/2)-15/8*a^(1/2)*b^2*ln((2*a+2*(b*x^2+a)^(1
/2)*a^(1/2))/x)+15/8*b^2*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.35, size = 104, normalized size = 1.21

$$-\frac{15}{8}\sqrt{a}\, b^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{15}{8}\sqrt{bx^2+a}\, b^2+\frac{3\left(bx^2+a\right)^{\frac{5}{2}} b^2}{8\,a^2}+\frac{5\left(bx^2+a\right)^{\frac{3}{2}} b^2}{8\,a}-\frac{3\left(bx^2+a\right)^{\frac{7}{2}} b}{8\,a^2x^2}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] -15/8*sqrt(a)*b^2*arcsinh(a/(sqrt(a*b)*abs(x))) + 15/8*sqrt(b*x^2 + a)*b^2
+ 3/8*(b*x^2 + a)^(5/2)*b^2/a^2 + 5/8*(b*x^2 + a)^(3/2)*b^2/a - 3/8*(b*x^2
+ a)^(7/2)*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(7/2)/(a*x^4)
```

mupad [B] time = 5.01, size = 71, normalized size = 0.83

$$b^2\sqrt{b\,x^2+a}-\frac{9\,a\left(b\,x^2+a\right)^{3/2}}{8\,x^4}+\frac{7\,a^2\sqrt{b\,x^2+a}}{8\,x^4}+\frac{\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)15\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^5,x)
```

```
[Out] b^2*(a + b*x^2)^(1/2) + (a^(1/2)*b^2*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1
5i)/8 - (9*a*(a + b*x^2)^(3/2))/(8*x^4) + (7*a^2*(a + b*x^2)^(1/2))/(8*x^4)
```

sympy [A] time = 3.73, size = 117, normalized size = 1.36

$$-\frac{15\sqrt{a}\,b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8}-\frac{a^3}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{11a^2\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{ab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{5}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**5,x)
```

```
[Out] -15*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - a**3/(4*sqrt(b)*x**5*sqrt(a
/(b*x**2) + 1)) - 11*a**2*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - a*b**(3/2
)/(8*x*sqrt(a/(b*x**2) + 1)) + b**(5/2)*x/sqrt(a/(b*x**2) + 1)
```

3.394 $\int \frac{(a+bx^2)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=89

$$-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{(a+bx^2)^{5/2}}{6x^6}-\frac{5b(a+bx^2)^{3/2}}{24x^4}$$

```
[Out] -5/24*b*(b*x^2+a)^(3/2)/x^4-1/6*(b*x^2+a)^(5/2)/x^6-5/16*b^3*arctanh((b*x^2
+a)^(1/2)/a^(1/2))/a^(1/2)-5/16*b^2*(b*x^2+a)^(1/2)/x^2
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 208}

$$-\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{5b(a+bx^2)^{3/2}}{24x^4}-\frac{(a+bx^2)^{5/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^7,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x^2])/(16*x^2) - (5*b*(a + b*x^2)^(3/2))/(24*x^4) - (a +
 b*x^2)^(5/2)/(6*x^6) - (5*b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*Sqrt[a
])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{(a+bx^2)^{5/2}}{6x^6}+\frac{1}{12}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{5b(a+bx^2)^{3/2}}{24x^4}-\frac{(a+bx^2)^{5/2}}{6x^6}+\frac{1}{16}\left(5b^2\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{5b(a+bx^2)^{3/2}}{24x^4}-\frac{(a+bx^2)^{5/2}}{6x^6}+\frac{1}{32}\left(5b^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{5b(a+bx^2)^{3/2}}{24x^4}-\frac{(a+bx^2)^{5/2}}{6x^6}+\frac{1}{16}\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b
\end{aligned}
$$

$$
= -\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{5b(a+bx^2)^{3/2}}{24x^4}-\frac{(a+bx^2)^{5/2}}{6x^6}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}
$$

Mathematica [A] time = 0.04, size = 87, normalized size = 0.98

$$
-\frac{8a^3+34a^2bx^2+15b^3x^6\sqrt{\frac{bx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)+59ab^2x^4+33b^3x^6}{48x^6\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^7,x]
```

```
[Out] -1/48*(8*a^3 + 34*a^2*b*x^2 + 59*a*b^2*x^4 + 33*b^3*x^6 + 15*b^3*x^6*Sqrt[1
 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a]])/(x^6*Sqrt[a + b*x^2])
```

fricas [A] time = 0.97, size = 158, normalized size = 1.78

$$
\left[\frac{15\sqrt{a}\,b^3x^6\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\,\sqrt{a}+2a}{x^2}\right)-2\left(33ab^2x^4+26a^2bx^2+8a^3\right)\sqrt{bx^2+a}}{96ax^6},\ \frac{15\sqrt{-a}\,b^3x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-
\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(15*sqrt(a)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) - 2*(33*a*b^2*x^4 + 26*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a*x^6), 1/48
*(15*sqrt(-a)*b^3*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (33*a*b^2*x^4 + 26
*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a*x^6)]
```

giac [A] time = 1.22, size = 87, normalized size = 0.98

$$
\frac{\frac{15\,b^4\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{33\left(bx^2+a\right)^{\frac{5}{2}}b^4-40\left(bx^2+a\right)^{\frac{3}{2}}ab^4+15\sqrt{bx^2+a}\,a^2b^4}{b^3x^6}}{48\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(15*b^4*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (33*(b*x^2 + a)^(5
/2)*b^4 - 40*(b*x^2 + a)^(3/2)*a*b^4 + 15*sqrt(b*x^2 + a)*a^2*b^4)/(b^3*x^6
))/b
```

maple [A] time = 0.01, size = 139, normalized size = 1.56

$$-\frac{5b^3\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{16\sqrt{a}}+\frac{5\sqrt{b\,x^2+a}\,b^3}{16a}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}b^3}{48a^2}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}b^3}{16a^3}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}b^2}{16a^3x^2}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}b}{24a^2x^4}-\frac{(b\,x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^7,x)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(7/2)-1/24/a^2*b/x^4*(b*x^2+a)^(7/2)-1/16/a^3*b^2/x^2*
(b*x^2+a)^(7/2)+1/16/a^3*b^3*(b*x^2+a)^(5/2)+5/48/a^2*b^3*(b*x^2+a)^(3/2)-5
/16/a^(1/2)*b^3*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+5/16/a*b^3*(b*x^2+a)^
(1/2)
```

maxima [A] time = 1.47, size = 127, normalized size = 1.43

$$-\frac{5\,b^3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,\sqrt{a}}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}b^3}{16\,a^3}+\frac{5\left(bx^2+a\right)^{\frac{3}{2}}b^3}{48\,a^2}+\frac{5\,\sqrt{bx^2+a}\,b^3}{16\,a}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}b^2}{16\,a^3x^2}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}b}{24\,a^2x^4}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] -5/16*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/16*(b*x^2 + a)^(5/2)*b^
3/a^3 + 5/48*(b*x^2 + a)^(3/2)*b^3/a^2 + 5/16*sqrt(b*x^2 + a)*b^3/a - 1/16*
(b*x^2 + a)^(7/2)*b^2/(a^3*x^2) - 1/24*(b*x^2 + a)^(7/2)*b/(a^2*x^4) - 1/6*
(b*x^2 + a)^(7/2)/(a*x^6)
```

mupad [B] time = 5.20, size = 72, normalized size = 0.81

$$\frac{5\,a\,{\left(b\,x^2+a\right)}^{3/2}}{6\,x^6}-\frac{11\,{\left(b\,x^2+a\right)}^{5/2}}{16\,x^6}-\frac{5\,a^2\,\sqrt{b\,x^2+a}}{16\,x^6}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^7,x)
```

```
[Out] (b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(1/2)) - (11*(a + b*x^2
)^(5/2))/(16*x^6) + (5*a*(a + b*x^2)^(3/2))/(6*x^6) - (5*a^2*(a + b*x^2)^(1
/2))/(16*x^6)
```

sympy [A] time = 4.53, size = 99, normalized size = 1.11

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{6x^5}-\frac{13ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{24x^3}-\frac{11b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{16x}-\frac{5b^3\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**7,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(6*x**5) - 13*a*b**(3/2)*sqrt(a/(b*x**2)
 + 1)/(24*x**3) - 11*b**(5/2)*sqrt(a/(b*x**2) + 1)/(16*x) - 5*b**3*asinh(sq
rt(a)/(sqrt(b)*x))/(16*sqrt(a))
```

3.395 $\int \frac{(a+bx^2)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=113

$$\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}-\frac{5b^3\sqrt{a+bx^2}}{128ax^2}-\frac{5b^2\sqrt{a+bx^2}}{64x^4}-\frac{(a+bx^2)^{5/2}}{8x^8}-\frac{5b(a+bx^2)^{3/2}}{48x^6}$$

```
[Out] -5/48*b*(b*x^2+a)^(3/2)/x^6-1/8*(b*x^2+a)^(5/2)/x^8+5/128*b^4*arctanh((b*x^
2+a)^(1/2)/a^(1/2))/a^(3/2)-5/64*b^2*(b*x^2+a)^(1/2)/x^4-5/128*b^3*(b*x^2+a
)^(1/2)/a/x^2
```

Rubi [A] time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}-\frac{5b^3\sqrt{a+bx^2}}{128ax^2}-\frac{5b^2\sqrt{a+bx^2}}{64x^4}-\frac{5b(a+bx^2)^{3/2}}{48x^6}-\frac{(a+bx^2)^{5/2}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^9,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x^2])/(64*x^4) - (5*b^3*Sqrt[a + b*x^2])/(128*a*x^2) - (
5*b*(a + b*x^2)^(3/2))/(48*x^6) - (a + b*x^2)^(5/2)/(8*x^8) + (5*b^4*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^9}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^5}\,dx, x, x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^{5/2}}{8x^8} + \frac{1}{16}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{5b\left(a+bx^2\right)^{3/2}}{48x^6} - \frac{\left(a+bx^2\right)^{5/2}}{8x^8} + \frac{1}{32}\left(5b^2\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b\left(a+bx^2\right)^{3/2}}{48x^6} - \frac{\left(a+bx^2\right)^{5/2}}{8x^8} + \frac{1}{128}\left(5b^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x,\right.$$
$$= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{48x^6} - \frac{\left(a+bx^2\right)^{5/2}}{8x^8} - \frac{\left(5b^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.}{256a}$$
$$= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{48x^6} - \frac{\left(a+bx^2\right)^{5/2}}{8x^8} - \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{128a}$$
$$= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{48x^6} - \frac{\left(a+bx^2\right)^{5/2}}{8x^8} + \frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.35

$$-\frac{b^4\left(a+bx^2\right)^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^9,x]
```

```
[Out] -1/7*(b^4*(a + b*x^2)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, 1 + (b*x^2)/a])/
a^5
```

fricas [A] time = 0.86, size = 179, normalized size = 1.58

$$\left[\frac{15\sqrt{a}\,b^4x^8\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\,\sqrt{a}+2a}{x^2}\right) - 2\left(15\,ab^3x^6+118\,a^2b^2x^4+136\,a^3bx^2+48\,a^4\right)\sqrt{bx^2+a}}{768\,a^2x^8}, -\frac{15\sqrt{-a}\,b^4x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/768*(15*sqrt(a)*b^4*x^8*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) - 2*(15*a*b^3*x^6 + 118*a^2*b^2*x^4 + 136*a^3*b*x^2 + 48*a^4)*sqrt(b*x^
2 + a))/(a^2*x^8), -1/384*(15*sqrt(-a)*b^4*x^8*arctan(sqrt(-a)/sqrt(b*x^2 +
```

```
a)) + (15*a*b^3*x^6 + 118*a^2*b^2*x^4 + 136*a^3*b*x^2 + 48*a^4)*sqrt(b*x^2
+ a))/(a^2*x^8)]
```

giac [A] time = 1.11, size = 109, normalized size = 0.96

$$-\frac{\frac{15\, b^5 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\, a} + \frac{15\left(bx^2+a\right)^{\frac{7}{2}} b^5 + 73\left(bx^2+a\right)^{\frac{5}{2}} ab^5 - 55\left(bx^2+a\right)^{\frac{3}{2}} a^2 b^5 + 15\sqrt{bx^2+a}\, a^3 b^5}{ab^4x^8}}{384\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/384*(15*b^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (15*(b*x^2 +
a)^(7/2)*b^5 + 73*(b*x^2 + a)^(5/2)*a*b^5 - 55*(b*x^2 + a)^(3/2)*a^2*b^5 +
15*sqrt(b*x^2 + a)*a^3*b^5)/(a*b^4*x^8))/b
```

maple [A] time = 0.02, size = 159, normalized size = 1.41

$$\frac{5b^4 \ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{128a^{\frac{3}{2}}} - \frac{5\sqrt{b\,x^2+a}\; b^4}{128a^2} - \frac{5\left(b\,x^2+a\right)^{\frac{3}{2}} b^4}{384a^3} - \frac{\left(b\,x^2+a\right)^{\frac{5}{2}} b^4}{128a^4} + \frac{\left(b\,x^2+a\right)^{\frac{7}{2}} b^3}{128a^4x^2} + \frac{\left(b\,x^2+a\right)^{\frac{7}{2}} b^2}{192a^3x^4} + \frac{\left(b\,x^2+\right.}{48a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^9,x)
```

```
[Out] -1/8/a/x^8*(b*x^2+a)^(7/2)+1/48/a^2*b/x^6*(b*x^2+a)^(7/2)+1/192/a^3*b^2/x^4
*(b*x^2+a)^(7/2)+1/128/a^4*b^3/x^2*(b*x^2+a)^(7/2)-1/128/a^4*b^4*(b*x^2+a)^
(5/2)-5/384/a^3*b^4*(b*x^2+a)^(3/2)+5/128/a^(3/2)*b^4*ln((2*a+2*(b*x^2+a)^(
1/2)*a^(1/2))/x)-5/128/a^2*b^4*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.38, size = 147, normalized size = 1.30

$$\frac{5\, b^4 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\, a^{\frac{3}{2}}} - \frac{\left(bx^2+a\right)^{\frac{5}{2}} b^4}{128\, a^4} - \frac{5\left(bx^2+a\right)^{\frac{3}{2}} b^4}{384\, a^3} - \frac{5\sqrt{bx^2+a}\, b^4}{128\, a^2} + \frac{\left(bx^2+a\right)^{\frac{7}{2}} b^3}{128\, a^4x^2} + \frac{\left(bx^2+a\right)^{\frac{7}{2}} b^2}{192\, a^3x^4} + \frac{\left(bx^2+a\right)^{\frac{7}{2}} b}{48\, a^2x^6} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] 5/128*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/128*(b*x^2 + a)^(5/2)*b
^4/a^4 - 5/384*(b*x^2 + a)^(3/2)*b^4/a^3 - 5/128*sqrt(b*x^2 + a)*b^4/a^2 +
1/128*(b*x^2 + a)^(7/2)*b^3/(a^4*x^2) + 1/192*(b*x^2 + a)^(7/2)*b^2/(a^3*x^
4) + 1/48*(b*x^2 + a)^(7/2)*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(7/2)/(a*x^8)
```

mupad [B] time = 5.43, size = 89, normalized size = 0.79

$$\frac{55\, a\left(b\,x^2+a\right)^{3/2}}{384\, x^8} - \frac{73\left(b\,x^2+a\right)^{5/2}}{384\, x^8} - \frac{5\, a^2\sqrt{b\,x^2+a}}{128\, x^8} - \frac{5\left(b\,x^2+a\right)^{7/2}}{128\, a\, x^8} - \frac{b^4 \operatorname{atan}\left(\frac{\sqrt{b\,x^2+a}\; 1\mathrm{i}}{\sqrt{a}}\right) 5\mathrm{i}}{128\, a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^9,x)
```

```
[Out] (55*a*(a + b*x^2)^(3/2))/(384*x^8) - (b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/
2))*5i)/(128*a^(3/2)) - (73*(a + b*x^2)^(5/2))/(384*x^8) - (5*a^2*(a + b*x^
2)^(1/2))/(128*x^8) - (5*(a + b*x^2)^(7/2))/(128*a*x^8)
```

sympy [A] time = 7.58, size = 150, normalized size = 1.33

$$-\frac{a^3}{8\sqrt{b}\,x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{23a^2\sqrt{b}}{48x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{127ab^{\frac{3}{2}}}{192x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{133b^{\frac{5}{2}}}{384x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{5b^{\frac{7}{2}}}{128ax\sqrt{\frac{a}{bx^2}+1}}+\frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{128a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**9,x)
```

```
[Out] -a**3/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 23*a**2*sqrt(b)/(48*x**7*sqrt
(a/(b*x**2) + 1)) - 127*a*b**(3/2)/(192*x**5*sqrt(a/(b*x**2) + 1)) - 133*b*
*(5/2)/(384*x**3*sqrt(a/(b*x**2) + 1)) - 5*b**(7/2)/(128*a*x*sqrt(a/(b*x**2
) + 1)) + 5*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(3/2))
```

3.396 $\int \frac{(a+bx^2)^{5/2}}{x^{11}} \, dx$

Optimal. Leaf size=137

$$-\frac{3b^5 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}+\frac{3b^4\sqrt{a+bx^2}}{256a^2x^2}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}-\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{(a+bx^2)^{5/2}}{10x^{10}}-\frac{b(a+bx^2)^{3/2}}{16x^8}$$

```
[Out] -1/16*b*(b*x^2+a)^(3/2)/x^8-1/10*(b*x^2+a)^(5/2)/x^10-3/256*b^5*arctanh((b*
x^2+a)^(1/2)/a^(1/2))/a^(5/2)-1/32*b^2*(b*x^2+a)^(1/2)/x^6-1/128*b^3*(b*x^2
+a)^(1/2)/a/x^4+3/256*b^4*(b*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{3b^4\sqrt{a+bx^2}}{256a^2x^2}-\frac{3b^5 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}-\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b(a+bx^2)^{3/2}}{16x^8}-\frac{(a+bx^2)^{5/2}}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^11,x]
```

```
[Out] -(b^2*Sqrt[a + b*x^2])/(32*x^6) - (b^3*Sqrt[a + b*x^2])/(128*a*x^4) + (3*b^
4*Sqrt[a + b*x^2])/(256*a^2*x^2) - (b*(a + b*x^2)^(3/2))/(16*x^8) - (a + b*
x^2)^(5/2)/(10*x^10) - (3*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(5/2
))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{5/2}}{x^{11}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^6}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{5/2}}{10x^{10}}+\frac{1}{4}b\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx,x,x^2\right)\\
&= -\frac{b\left(a+bx^2\right)^{3/2}}{16x^8}-\frac{\left(a+bx^2\right)^{5/2}}{10x^{10}}+\frac{1}{32}\left(3b^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b\left(a+bx^2\right)^{3/2}}{16x^8}-\frac{\left(a+bx^2\right)^{5/2}}{10x^{10}}+\frac{1}{64}b^3\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}-\frac{b\left(a+bx^2\right)^{3/2}}{16x^8}-\frac{\left(a+bx^2\right)^{5/2}}{10x^{10}}-\frac{\left(3b^4\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,d\right.}{256a}\\
&= -\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}+\frac{3b^4\sqrt{a+bx^2}}{256a^2x^2}-\frac{b\left(a+bx^2\right)^{3/2}}{16x^8}-\frac{\left(a+bx^2\right)^{5/2}}{10x^{10}}+\frac{\left(3b^5\right)\operatorname{Sul}}{}\\
&= -\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}+\frac{3b^4\sqrt{a+bx^2}}{256a^2x^2}-\frac{b\left(a+bx^2\right)^{3/2}}{16x^8}-\frac{\left(a+bx^2\right)^{5/2}}{10x^{10}}+\frac{\left(3b^4\right)\operatorname{Sul}}{}\\
&= -\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}+\frac{3b^4\sqrt{a+bx^2}}{256a^2x^2}-\frac{b\left(a+bx^2\right)^{3/2}}{16x^8}-\frac{\left(a+bx^2\right)^{5/2}}{10x^{10}}-\frac{3b^5\tanh}{25}\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.28

$$\frac{b^5\left(a+bx^2\right)^{7/2}{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^11,x]
```

```
[Out] (b^5*(a + b*x^2)^(7/2)*Hypergeometric2F1[7/2, 6, 9/2, 1 + (b*x^2)/a])/(7*a^
6)
```

fricas [A] time = 0.70, size = 201, normalized size = 1.47

$$\left[\frac{15\sqrt{a}\,b^5x^{10}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(15\,ab^4x^8-10\,a^2b^3x^6-248\,a^3b^2x^4-336\,a^4bx^2-128\,a^5\right)\sqrt{bx^2+a}}{2560\,a^3x^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*sqrt(a)*b^5*x^10*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)
/x^2) + 2*(15*a*b^4*x^8 - 10*a^2*b^3*x^6 - 248*a^3*b^2*x^4 - 336*a^4*b*x^2
```

```
- 128*a^5)*sqrt(b*x^2 + a))/(a^3*x^10), 1/1280*(15*sqrt(-a)*b^5*x^10*arctan
(sqrt(-a)/sqrt(b*x^2 + a)) + (15*a*b^4*x^8 - 10*a^2*b^3*x^6 - 248*a^3*b^2*x
^4 - 336*a^4*b*x^2 - 128*a^5)*sqrt(b*x^2 + a))/(a^3*x^10)]
```

giac [A] time = 1.16, size = 126, normalized size = 0.92

$$\frac{\frac{15\, b^6 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\, a^2} + \frac{15\left(bx^2+a\right)^{\frac{9}{2}} b^6 - 70\left(bx^2+a\right)^{\frac{7}{2}} ab^6 - 128\left(bx^2+a\right)^{\frac{5}{2}} a^2 b^6 + 70\left(bx^2+a\right)^{\frac{3}{2}} a^3 b^6 - 15\sqrt{bx^2+a}\, a^4 b^6}{a^2 b^5 x^{10}}}{1280\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/1280*(15*b^6*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (15*(b*x^2
 + a)^(9/2)*b^6 - 70*(b*x^2 + a)^(7/2)*a*b^6 - 128*(b*x^2 + a)^(5/2)*a^2*b^
6 + 70*(b*x^2 + a)^(3/2)*a^3*b^6 - 15*sqrt(b*x^2 + a)*a^4*b^6)/(a^2*b^5*x^1
0))/b
```

maple [A] time = 0.04, size = 179, normalized size = 1.31

$$-\frac{3b^5 \ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{256a^{\frac{5}{2}}} + \frac{3\sqrt{b\,x^2+a}\, b^5}{256a^3} + \frac{\left(b\,x^2+a\right)^{\frac{3}{2}} b^5}{256a^4} + \frac{3\left(b\,x^2+a\right)^{\frac{5}{2}} b^5}{1280a^5} - \frac{3\left(b\,x^2+a\right)^{\frac{7}{2}} b^4}{1280a^5x^2} - \frac{\left(b\,x^2+a\right)^{\frac{7}{2}} b^3}{640a^4x^4} - \frac{\left(b\,x\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^11,x)
```

```
[Out] -1/10/a/x^10*(b*x^2+a)^(7/2)+3/80/a^2*b/x^8*(b*x^2+a)^(7/2)-1/160/a^3*b^2/x
^6*(b*x^2+a)^(7/2)-1/640/a^4*b^3/x^4*(b*x^2+a)^(7/2)-3/1280/a^5*b^4/x^2*(b*
x^2+a)^(7/2)+3/1280/a^5*b^5*(b*x^2+a)^(5/2)+1/256/a^4*b^5*(b*x^2+a)^(3/2)-3
/256/a^(5/2)*b^5*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/256/a^3*b^5*(b*x^2
+a)^(1/2)
```

maxima [A] time = 1.44, size = 167, normalized size = 1.22

$$-\frac{3\, b^5 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{256\, a^{\frac{5}{2}}} + \frac{3\left(bx^2+a\right)^{\frac{5}{2}} b^5}{1280\, a^5} + \frac{\left(bx^2+a\right)^{\frac{3}{2}} b^5}{256\, a^4} + \frac{3\sqrt{bx^2+a}\, b^5}{256\, a^3} - \frac{3\left(bx^2+a\right)^{\frac{7}{2}} b^4}{1280\, a^5 x^2} - \frac{\left(bx^2+a\right)^{\frac{7}{2}} b^3}{640\, a^4 x^4} - \frac{\left(bx^2+a\right)^{\frac{7}{2}}}{160\, a^3 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] -3/256*b^5*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 3/1280*(b*x^2 + a)^(5/2)
*b^5/a^5 + 1/256*(b*x^2 + a)^(3/2)*b^5/a^4 + 3/256*sqrt(b*x^2 + a)*b^5/a^3
- 3/1280*(b*x^2 + a)^(7/2)*b^4/(a^5*x^2) - 1/640*(b*x^2 + a)^(7/2)*b^3/(a^4
*x^4) - 1/160*(b*x^2 + a)^(7/2)*b^2/(a^3*x^6) + 3/80*(b*x^2 + a)^(7/2)*b/(a
^2*x^8) - 1/10*(b*x^2 + a)^(7/2)/(a*x^10)
```

mupad [B] time = 5.68, size = 106, normalized size = 0.77

$$\frac{7\,a\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^{10}} - \frac{{\left(b\,x^2+a\right)}^{5/2}}{10\,x^{10}} - \frac{3\,a^2\,\sqrt{b\,x^2+a}}{256\,x^{10}} - \frac{7\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a\,x^{10}} + \frac{3\,{\left(b\,x^2+a\right)}^{9/2}}{256\,a^2\,x^{10}} + \frac{b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,3\mathrm{i}}{256\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^11,x)
```

```
[Out] (b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(256*a^(5/2)) - (a + b*x^2)^(
5/2)/(10*x^10) + (7*a*(a + b*x^2)^(3/2))/(128*x^10) - (3*a^2*(a + b*x^2)^(1
/2))/(256*x^10) - (7*(a + b*x^2)^(7/2))/(128*a*x^10) + (3*(a + b*x^2)^(9/2)
)/(256*a^2*x^10)
```

sympy [**A**] time = 11.54, size = 175, normalized size = 1.28

$$-\frac{a^3}{10\sqrt{b}\,x^{11}\sqrt{\frac{a}{bx^2}+1}}-\frac{29a^2\sqrt{b}}{80x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{73ab^{\frac{3}{2}}}{160x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{129b^{\frac{5}{2}}}{640x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{7}{2}}}{256ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3b^{\frac{9}{2}}}{256a^2x\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**11,x)
```

```
[Out] -a**3/(10*sqrt(b)*x**11*sqrt(a/(b*x**2) + 1)) - 29*a**2*sqrt(b)/(80*x**9*sq
rt(a/(b*x**2) + 1)) - 73*a*b**(3/2)/(160*x**7*sqrt(a/(b*x**2) + 1)) - 129*b
**(5/2)/(640*x**5*sqrt(a/(b*x**2) + 1)) + b**(7/2)/(256*a*x**3*sqrt(a/(b*x*
*2) + 1)) + 3*b**(9/2)/(256*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*b**5*asinh(sqr
t(a)/(sqrt(b)*x))/(256*a**(5/2))
```

3.397 $\int x^4 \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=136

$$\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}$$

```
[Out] 1/16*a*x^5*(b*x^2+a)^(3/2)+1/10*x^5*(b*x^2+a)^(5/2)+3/256*a^5*arctanh(x*b^(
1/2)/(b*x^2+a)^(1/2))/b^(5/2)-3/256*a^4*x*(b*x^2+a)^(1/2)/b^2+1/128*a^3*x^3
*(b*x^2+a)^(1/2)/b+1/32*a^2*x^5*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(5/2),x]
```

```
[Out] (-3*a^4*x*Sqrt[a + b*x^2])/(256*b^2) + (a^3*x^3*Sqrt[a + b*x^2])/(128*b) +
(a^2*x^5*Sqrt[a + b*x^2])/32 + (a*x^5*(a + b*x^2)^(3/2))/16 + (x^5*(a + b*x
^2)^(5/2))/10 + (3*a^5*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^{5/2}dx=\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\frac{1}{2}a\int x^4\left(a+bx^2\right)^{3/2}dx$$
$$=\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(3a^2\right)\int x^4\sqrt{a+bx^2}\,dx$$
$$=\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\frac{1}{32}a^3\int\frac{x^4}{\sqrt{a+bx^2}}dx$$
$$=\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}-\frac{\left(3a^4\right)\int}{1}$$
$$=-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a\right.$$
$$=-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a\right.$$
$$=-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a\right.$$

Mathematica [A] time = 0.15, size = 105, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\left(\frac{15a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\sqrt{b}x\left(-15a^4+10a^3bx^2+248a^2b^2x^4+336ab^3x^6+128b^4x^8\right)\right)}{1280b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-15*a^4 + 10*a^3*b*x^2 + 248*a^2*b^2*x^4 + 336
*a*b^3*x^6 + 128*b^4*x^8) + (15*a^(9/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[
1 + (b*x^2)/a]))/(1280*b^(5/2))
```

fricas [A] time = 0.82, size = 190, normalized size = 1.40

$$\left[\frac{15\,a^5\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(128\,b^5x^9+336\,ab^4x^7+248\,a^2b^3x^5+10\,a^3b^2x^3-15\,a^4bx\right)\sqrt{}}{2560\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(15*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(128*b^5*x^9 + 336*a*b^4*x^7 + 248*a^2*b^3*x^5 + 10*a^3*b^2*x^3 - 15*a^4*b
*x)*sqrt(b*x^2 + a))/b^3, -1/1280*(15*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (128*b^5*x^9 + 336*a*b^4*x^7 + 248*a^2*b^3*x^5 + 10*a^3*b^2*x^
3 - 15*a^4*b*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 1.09, size = 91, normalized size = 0.67

$$-\frac{3\,a^5\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}+\frac{1}{1280}\left(2\left(4\left(2\left(8\,b^2x^2+21\,ab\right)x^2+31\,a^2\right)x^2+\frac{5\,a^3}{b}\right)x^2-\frac{15\,a^4}{b^2}\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -3/256*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) + 1/1280*(2*(4*(2
*(8*b^2*x^2 + 21*a*b)*x^2 + 31*a^2)*x^2 + 5*a^3/b)*x^2 - 15*a^4/b^2)*sqrt(b
*x^2 + a)*x
```

maple [A] time = 0.01, size = 113, normalized size = 0.83

$$\frac{3a^5\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{256b^{\frac{5}{2}}}+\frac{3\sqrt{b\,x^2+a}\,a^4x}{256b^2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}a^3x}{128b^2}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}x^3}{10b}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}a^2x}{160b^2}-\frac{3\left(b\,x^2+a\right)^{\frac{7}{2}}ax}{80b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/10*x^3*(b*x^2+a)^(7/2)/b-3/80*a/b^2*x*(b*x^2+a)^(7/2)+1/160*a^2/b^2*x*(b*
x^2+a)^(5/2)+1/128*a^3/b^2*x*(b*x^2+a)^(3/2)+3/256*a^4*x*(b*x^2+a)^(1/2)/b^
2+3/256*a^5/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.26, size = 105, normalized size = 0.77

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}x^3}{10\,b}-\frac{3\left(bx^2+a\right)^{\frac{7}{2}}ax}{80\,b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}a^2x}{160\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}a^3x}{128\,b^2}+\frac{3\sqrt{bx^2+a}\,a^4x}{256\,b^2}+\frac{3\,a^5\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(7/2)*x^3/b - 3/80*(b*x^2 + a)^(7/2)*a*x/b^2 + 1/160*(b*x^
2 + a)^(5/2)*a^2*x/b^2 + 1/128*(b*x^2 + a)^(3/2)*a^3*x/b^2 + 3/256*sqrt(b*x
^2 + a)*a^4*x/b^2 + 3/256*a^5*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(b\,x^2+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^4*(a + b*x^2)^(5/2), x)
```

sympy [A] time = 11.11, size = 175, normalized size = 1.29

$$-\frac{3a^{\frac{9}{2}}x}{256b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{7}{2}}x^3}{256b\sqrt{1+\frac{bx^2}{a}}}+\frac{129a^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{bx^2}{a}}}+\frac{73a^{\frac{3}{2}}bx^7}{160\sqrt{1+\frac{bx^2}{a}}}+\frac{29\sqrt{a}\,b^2x^9}{80\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^5\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}}+\frac{b^3x^{11}}{10\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(5/2),x)
```

```
[Out] -3*a**(9/2)*x/(256*b**2*sqrt(1 + b*x**2/a)) - a**(7/2)*x**3/(256*b*sqrt(1 +
 b*x**2/a)) + 129*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*a**(3/2)*b*x*
*7/(160*sqrt(1 + b*x**2/a)) + 29*sqrt(a)*b**2*x**9/(80*sqrt(1 + b*x**2/a))
+ 3*a**5*asinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + b**3*x**11/(10*sqrt(a)*s
qrt(1 + b*x**2/a))
```

3.398 $\int x^2 \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=112

$$-\frac{5a^4 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}$$

```
[Out] 5/48*a*x^3*(b*x^2+a)^(3/2)+1/8*x^3*(b*x^2+a)^(5/2)-5/128*a^4*arctanh(x*b^(1
/2)/(b*x^2+a)^(1/2))/b^(3/2)+5/128*a^3*x*(b*x^2+a)^(1/2)/b+5/64*a^2*x^3*(b*
x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{5a^4 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^3*x*Sqrt[a + b*x^2])/(128*b) + (5*a^2*x^3*Sqrt[a + b*x^2])/64 + (5*a*x
^3*(a + b*x^2)^(3/2))/48 + (x^3*(a + b*x^2)^(5/2))/8 - (5*a^4*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+bx^2\right)^{5/2}dx &= \frac{1}{8}x^3\left(a+bx^2\right)^{5/2}+\frac{1}{8}(5a)\int x^2\left(a+bx^2\right)^{3/2}dx \\
&= \frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(5a^2\right)\int x^2\sqrt{a+bx^2}\,dx \\
&= \frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}+\frac{1}{64}\left(5a^3\right)\int\frac{x^2}{\sqrt{a+bx^2}}dx \\
&= \frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}-\frac{\left(5a^4\right)\int\frac{}{\sqrt{a}}}{128b} \\
&= \frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}-\frac{\left(5a^4\right)\text{Subs}}{} \\
&= \frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}-\frac{5a^4\tanh^{-1}}{128b}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 94, normalized size = 0.84

$$
\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(15a^3+118a^2bx^2+136ab^2x^4+48b^3x^6\right)-\frac{15a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{384b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(15*a^3 + 118*a^2*b*x^2 + 136*a*b^2*x^4 + 48*b^
3*x^6) - (15*a^(7/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(3
84*b^(3/2))
```

fricas [A] time = 0.85, size = 167, normalized size = 1.49

$$
\left[\frac{15\,a^4\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(48\,b^4x^7+136\,ab^3x^5+118\,a^2b^2x^3+15\,a^3bx\right)\sqrt{bx^2+a}}{768\,b^2},\ \frac{15\,a^4\sqrt{}}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(48*b^4*x^7 + 136*a*b^3*x^5 + 118*a^2*b^2*x^3 + 15*a^3*b*x)*sqrt(b*x^2 + a)
)/b^2, 1/384*(15*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (48*b^4*
x^7 + 136*a*b^3*x^5 + 118*a^2*b^2*x^3 + 15*a^3*b*x)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 1.08, size = 77, normalized size = 0.69

$$
\frac{5\,a^4\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}+\frac{1}{384}\left(2\left(4\left(6\,b^2x^2+17\,ab\right)x^2+59\,a^2\right)x^2+\frac{15\,a^3}{b}\right)\sqrt{bx^2+a}\,x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/128*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/384*(2*(4*(6*b
^2*x^2 + 17*a*b)*x^2 + 59*a^2)*x^2 + 15*a^3/b)*sqrt(b*x^2 + a)*x
```

maple [A] time = 0.01, size = 93, normalized size = 0.83

$$-\frac{5a^4 \ln\left(\sqrt{b}\, x+\sqrt{b\,x^2+a}\right)}{128b^{\frac{3}{2}}}-\frac{5\sqrt{b\,x^2+a}\, a^3 x}{128b}-\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}} a^2 x}{192b}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}} a x}{48b}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}} x}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/8*x*(b*x^2+a)^(7/2)/b-1/48*a/b*x*(b*x^2+a)^(5/2)-5/192*a^2/b*x*(b*x^2+a)^
(3/2)-5/128*a^3*x*(b*x^2+a)^(1/2)/b-5/128*a^4/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2))
```

maxima [A] time = 1.34, size = 85, normalized size = 0.76

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}} x}{8\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}} ax}{48\,b}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}} a^2 x}{192\,b}-\frac{5\sqrt{bx^2+a}\,a^3 x}{128\,b}-\frac{5\,a^4 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(7/2)*x/b - 1/48*(b*x^2 + a)^(5/2)*a*x/b - 5/192*(b*x^2 + a
)^(3/2)*a^2*x/b - 5/128*sqrt(b*x^2 + a)*a^3*x/b - 5/128*a^4*arcsinh(b*x/sqr
t(a*b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(b\,x^2+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^(5/2), x)
```

sympy [A] time = 7.22, size = 150, normalized size = 1.34

$$\frac{5a^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133a^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127a^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23\sqrt{a}\,b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^4 \operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{b^3x^9}{8\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(5/2),x)
```

```
[Out] 5*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*a**(5/2)*x**3/(384*sqrt(1 + b
*x**2/a)) + 127*a**(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*sqrt(a)*b**2*
x**7/(48*sqrt(1 + b*x**2/a)) - 5*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(3/2
)) + b**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.399 $\int \left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=84

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

```
[Out] 5/24*a*x*(b*x^2+a)^(3/2)+1/6*x*(b*x^2+a)^(5/2)+5/16*a^3*arctanh(x*b^(1/2)/(
b*x^2+a)^(1/2))/b^(1/2)+5/16*a^2*x*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*x*Sqrt[a + b*x^2])/16 + (5*a*x*(a + b*x^2)^(3/2))/24 + (x*(a + b*x^2
)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^2\right)^{5/2}\,dx &= \frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{6}(5a)\int \left(a+bx^2\right)^{3/2}\,dx\\
&= \frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{8}\left(5a^2\right)\int \sqrt{a+bx^2}\,dx\\
&= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,-\right.\\
&= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 76, normalized size = 0.90

$$\frac{1}{48}\sqrt{a+bx^2}\left(\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}+33a^2x+26abx^3+8b^2x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(33*a^2*x + 26*a*b*x^3 + 8*b^2*x^5 + (15*a^(5/2)*ArcSinh[(
Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/48
```

fricas [A] time = 0.90, size = 146, normalized size = 1.74

$$\left[\frac{15\,a^3\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx\right)\sqrt{bx^2+a}}{96\,b},-\frac{15\,a^3\sqrt{-b}\,\arctan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
8*b^3*x^5 + 26*a*b^2*x^3 + 33*a^2*b*x)*sqrt(b*x^2 + a))/b, -1/48*(15*a^3*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 26*a*b^2*x^3 + 33*
a^2*b*x)*sqrt(b*x^2 + a))/b]
```

giac [A] time = 1.23, size = 63, normalized size = 0.75

$$-\frac{5\,a^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{48}\left(2\left(4\,b^2x^2+13\,ab\right)x^2+33\,a^2\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/48*(2*(4*b^2*x
^2 + 13*a*b)*x^2 + 33*a^2)*sqrt(b*x^2 + a)*x
```

maple [A] time = 0.00, size = 66, normalized size = 0.79

$$\frac{5a^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16\sqrt{b}}+\frac{5\sqrt{b\,x^2+a}\,a^2x}{16}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}ax}{24}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2),x)
```

```
[Out] 1/6*x*(b*x^2+a)^(5/2)+5/24*a*x*(b*x^2+a)^(3/2)+5/16*a^2*x*(b*x^2+a)^(1/2)+5
/16*a^3/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.32, size = 58, normalized size = 0.69

$$\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}x+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}ax+\frac{5}{16}\sqrt{bx^2+a}\,a^2x+\frac{5\,a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*x + 5/24*(b*x^2 + a)^(3/2)*a*x + 5/16*sqrt(b*x^2 + a)
*a^2*x + 5/16*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 4.46, size = 37, normalized size = 0.44

$$\frac{x\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2),x)
```

```
[Out] (x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(5/2)
```

sympy [A] time = 4.21, size = 97, normalized size = 1.15

$$\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{24}+\frac{\sqrt{a}\,b^2x^5\sqrt{1+\frac{bx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2),x)
```

```
[Out] 11*a**(5/2)*x*sqrt(1 + b*x**2/a)/16 + 13*a**(3/2)*b*x**3*sqrt(1 + b*x**2/a)
/24 + sqrt(a)*b**2*x**5*sqrt(1 + b*x**2/a)/6 + 5*a**3*asinh(sqrt(b)*x/sqrt(
a))/(16*sqrt(b))
```

3.400 $\int \frac{(a+bx^2)^{5/2}}{x^2} dx$

Optimal. Leaf size=83

$$\frac{15}{8}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{(a+bx^2)^{5/2}}{x}+\frac{5}{4}bx\left(a+bx^2\right)^{3/2}+\frac{15}{8}abx\sqrt{a+bx^2}$$

```
[Out] 5/4*b*x*(b*x^2+a)^(3/2)-(b*x^2+a)^(5/2)/x+15/8*a^2*arctanh(x*b^(1/2)/(b*x^2
+a)^(1/2))*b^(1/2)+15/8*a*b*x*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$\frac{15}{8}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{(a+bx^2)^{5/2}}{x}+\frac{5}{4}bx\left(a+bx^2\right)^{3/2}+\frac{15}{8}abx\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^2,x]
```

```
[Out] (15*a*b*x*Sqrt[a + b*x^2])/8 + (5*b*x*(a + b*x^2)^(3/2))/4 - (a + b*x^2)^(5
/2)/x + (15*a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx &= -\frac{(a+bx^2)^{5/2}}{x} + (5b)\int (a+bx^2)^{3/2}\,dx \\
&= \frac{5}{4}bx(a+bx^2)^{3/2} - \frac{(a+bx^2)^{5/2}}{x} + \frac{1}{4}(15ab)\int \sqrt{a+bx^2}\,dx \\
&= \frac{15}{8}abx\sqrt{a+bx^2} + \frac{5}{4}bx(a+bx^2)^{3/2} - \frac{(a+bx^2)^{5/2}}{x} + \frac{1}{8}\left(15a^2b\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= \frac{15}{8}abx\sqrt{a+bx^2} + \frac{5}{4}bx(a+bx^2)^{3/2} - \frac{(a+bx^2)^{5/2}}{x} + \frac{1}{8}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{}{\sqrt{a}\cdots}\right. \\
&= \frac{15}{8}abx\sqrt{a+bx^2} + \frac{5}{4}bx(a+bx^2)^{3/2} - \frac{(a+bx^2)^{5/2}}{x} + \frac{15}{8}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.63

$$-\frac{a^2\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^2,x]
```

```
[Out] -((a^2*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/2, -1/2, 1/2, -((b*x^2)/a)])/(x
*Sqrt[1 + (b*x^2)/a]))
```

fricas [A] time = 0.62, size = 140, normalized size = 1.69

$$\left[\frac{15\,a^2\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(2\,b^2x^4+9\,abx^2-8\,a^2\right)\sqrt{bx^2+a}}{16\,x}, -\frac{15\,a^2\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+\cdots}}\right.\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/16*(15*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(2*b^2*x^4 + 9*a*b*x^2 - 8*a^2)*sqrt(b*x^2 + a))/x, -1/8*(15*a^2*sqrt(-b)*
x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^4 + 9*a*b*x^2 - 8*a^2)*sqrt
(b*x^2 + a))/x]
```

giac [A] time = 1.11, size = 87, normalized size = 1.05

$$-\frac{15}{16}\,a^2\sqrt{b}\,\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,a^3\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{8}\left(2\,b^2x^2+9\,ab\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] -15/16*a^2*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a^3*sqrt(b)/((s
qrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/8*(2*b^2*x^2 + 9*a*b)*sqrt(b*x^2 + a
)*x
```

maple [A] time = 0.00, size = 85, normalized size = 1.02

$$\frac{15a^2\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8}+\frac{15\sqrt{b\,x^2+a}\,abx}{8}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}bx}{4}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}bx}{a}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^2,x)
```

```
[Out] -1/a/x*(b*x^2+a)^(7/2)+1/a*b*x*(b*x^2+a)^(5/2)+5/4*b*x*(b*x^2+a)^(3/2)+15/8
*a*b*x*(b*x^2+a)^(1/2)+15/8*a^2*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.32, size = 59, normalized size = 0.71

$$\frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}bx+\frac{15}{8}\sqrt{bx^2+a}\,abx+\frac{15}{8}\,a^2\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5/4*(b*x^2 + a)^(3/2)*b*x + 15/8*sqrt(b*x^2 + a)*a*b*x + 15/8*a^2*sqrt(b)*a
rcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(5/2)/x
```

mupad [B] time = 5.05, size = 40, normalized size = 0.48

$$-\frac{\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^2,x)
```

```
[Out] -((a + b*x^2)^(5/2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a
 + 1)^(5/2))
```

sympy [A] time = 3.63, size = 117, normalized size = 1.41

$$-\frac{a^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{11\sqrt{a}\,b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8}+\frac{b^3x^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**2,x)
```

```
[Out] -a**(5/2)/(x*sqrt(1 + b*x**2/a)) + a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 11
*sqrt(a)*b**2*x**3/(8*sqrt(1 + b*x**2/a)) + 15*a**2*sqrt(b)*asinh(sqrt(b)*x
/sqrt(a))/8 + b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.401 $\int \frac{(a+bx^2)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=86

$$\frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{5}{2}b^2x\sqrt{a+bx^2}-\frac{5b\left(a+bx^2\right)^{3/2}}{3x}-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}$$

```
[Out] -5/3*b*(b*x^2+a)^(3/2)/x-1/3*(b*x^2+a)^(5/2)/x^3+5/2*a*b^(3/2)*arctanh(x*b^
(1/2)/(b*x^2+a)^(1/2))+5/2*b^2*x*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = $\{277, 195, 217, 206\}$

$$\frac{5}{2}b^2x\sqrt{a+bx^2}+\frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}-\frac{5b\left(a+bx^2\right)^{3/2}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^4,x]
```

```
[Out] (5*b^2*x*Sqrt[a + b*x^2])/2 - (5*b*(a + b*x^2)^(3/2))/(3*x) - (a + b*x^2)^(
5/2)/(3*x^3) + (5*a*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{x^4}\,dx &= -\frac{\left(a+bx^2\right)^{5/2}}{3x^3}+\frac{1}{3}(5b)\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx \\
&= -\frac{5b\left(a+bx^2\right)^{3/2}}{3x}-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}+\left(5b^2\right)\int \sqrt{a+bx^2}\,dx \\
&= \frac{5}{2}b^2x\sqrt{a+bx^2}-\frac{5b\left(a+bx^2\right)^{3/2}}{3x}-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}+\frac{1}{2}\left(5ab^2\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= \frac{5}{2}b^2x\sqrt{a+bx^2}-\frac{5b\left(a+bx^2\right)^{3/2}}{3x}-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}+\frac{1}{2}\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{}{\sqrt{a+}}\right. \\
&= \frac{5}{2}b^2x\sqrt{a+bx^2}-\frac{5b\left(a+bx^2\right)^{3/2}}{3x}-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}+\frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.63

$$
-\frac{a^2\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^4,x]
```

```
[Out] -1/3*(a^2*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/2, -3/2, -1/2, -((b*x^2)/a)]
)/(x^3*Sqrt[1 + (b*x^2)/a])
```

fricas [A] time = 0.93, size = 141, normalized size = 1.64

$$
\left[\frac{15\,ab^{\frac{3}{2}}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(3\,b^2x^4-14\,abx^2-2\,a^2\right)\sqrt{bx^2+a}}{12\,x^3},-\frac{15\,a\sqrt{-b}\,bx^3\arctan\left(\right.}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(15*a*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(3*b^2*x^4 - 14*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/x^3, -1/6*(15*a*sqrt(-b)
*b*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*b^2*x^4 - 14*a*b*x^2 - 2*a^2
)*sqrt(b*x^2 + a))/x^3]
```

giac [A] time = 1.12, size = 132, normalized size = 1.53

$$
\frac{1}{2}\,\sqrt{bx^2+a}\,b^2x-\frac{5}{4}\,ab^{\frac{3}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(9\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{3}{2}}-12\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^3b\right.}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b^2*x - 5/4*a*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))
^2) + 2/3*(9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2) - 12*(sqrt(b)*x -
```

```
sqrt(b*x^2 + a))^2*a^3*b^(3/2) + 7*a^4*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 +
a))^2 - a)^3
```

maple [A] time = 0.01, size = 110, normalized size = 1.28

$$\frac{5a\,b^{\frac{3}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2}+\frac{5\sqrt{b\,x^2+a}\,b^2x}{2}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}b^2x}{3a}+\frac{4\left(b\,x^2+a\right)^{\frac{5}{2}}b^2x}{3a^2}-\frac{4\left(b\,x^2+a\right)^{\frac{7}{2}}b}{3a^2x}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^4,x)
```

```
[Out] -1/3/a/x^3*(b*x^2+a)^(7/2)-4/3/a^2*b/x*(b*x^2+a)^(7/2)+4/3/a^2*b^2*x*(b*x^2
+a)^(5/2)+5/3/a*b^2*x*(b*x^2+a)^(3/2)+5/2*b^2*x*(b*x^2+a)^(1/2)+5/2*a*b^(3/
2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.32, size = 84, normalized size = 0.98

$$\frac{5}{2}\sqrt{bx^2+a}\,b^2x+\frac{5\left(bx^2+a\right)^{\frac{3}{2}}b^2x}{3\,a}+\frac{5}{2}\,ab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{4\left(bx^2+a\right)^{\frac{5}{2}}b}{3\,ax}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 5/2*sqrt(b*x^2 + a)*b^2*x + 5/3*(b*x^2 + a)^(3/2)*b^2*x/a + 5/2*a*b^(3/2)*a
rcsinh(b*x/sqrt(a*b)) - 4/3*(b*x^2 + a)^(5/2)*b/(a*x) - 1/3*(b*x^2 + a)^(7/
2)/(a*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^4,x)
```

```
[Out] int((a + b*x^2)^(5/2)/x^4, x)
```

sympy [A] time = 3.16, size = 112, normalized size = 1.30

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{7ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}-\frac{5ab^{\frac{3}{2}}\log\left(\frac{a}{bx^2}\right)}{4}+\frac{5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)}{2}+\frac{b^{\frac{5}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**4,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - 7*a*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/3 - 5*a*b**(3/2)*log(a/(b*x**2))/4 + 5*a*b**(3/2)*log(sqrt(a/(b*x**2)
+ 1) + 1)/2 + b**(5/2)*x**2*sqrt(a/(b*x**2) + 1)/2
```

3.402 $\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=82

$$b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b^2\sqrt{a+bx^2}}{x}-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}-\frac{b\left(a+bx^2\right)^{3/2}}{3x^3}$$

```
[Out] -1/3*b*(b*x^2+a)^(3/2)/x^3-1/5*(b*x^2+a)^(5/2)/x^5+b^(5/2)*arctanh(x*b^(1/2
)/(b*x^2+a)^(1/2))-b^2*(b*x^2+a)^(1/2)/x
```

Rubi [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 217, 206}

$$-\frac{b^2\sqrt{a+bx^2}}{x}+b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^6,x]
```

```
[Out] -((b^2*Sqrt[a + b*x^2])/x) - (b*(a + b*x^2)^(3/2))/(3*x^3) - (a + b*x^2)^(5
/2)/(5*x^5) + b^(5/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{x^6}\,dx &= -\frac{\left(a+bx^2\right)^{5/2}}{5x^5}+b\int \frac{\left(a+bx^2\right)^{3/2}}{x^4}\,dx\\
&= -\frac{b\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}+b^2\int \frac{\sqrt{a+bx^2}}{x^2}\,dx\\
&= -\frac{b^2\sqrt{a+bx^2}}{x}-\frac{b\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}+b^3\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= -\frac{b^2\sqrt{a+bx^2}}{x}-\frac{b\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}+b^3\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= -\frac{b^2\sqrt{a+bx^2}}{x}-\frac{b\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}+b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.66

$$
-\frac{a^2\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^6,x]
```

```
[Out] -1/5*(a^2*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/2, -5/2, -3/2, -((b*x^2)/a)]
)/(x^5*Sqrt[1 + (b*x^2)/a])
```

fricas [A] time = 1.04, size = 140, normalized size = 1.71

$$
\left[\frac{15\,b^{\frac{5}{2}}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(23\,b^2x^4+11\,abx^2+3\,a^2\right)\sqrt{bx^2+a}}{30\,x^5},-\frac{15\,\sqrt{-b}\,b^2x^5\arctan\left(\frac{\sqrt{-b}}{\sqrt{bx^2}}\right.}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(15*b^(5/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(
23*b^2*x^4 + 11*a*b*x^2 + 3*a^2)*sqrt(b*x^2 + a))/x^5, -1/15*(15*sqrt(-b)*b
^2*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (23*b^2*x^4 + 11*a*b*x^2 + 3*a^
2)*sqrt(b*x^2 + a))/x^5]
```

giac [B] time = 1.14, size = 168, normalized size = 2.05

$$
-\frac{1}{2}\,b^{\frac{5}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(45\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8ab^{\frac{5}{2}}-90\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6a^2b^{\frac{5}{2}}+140\left(\sqrt{b}\,x-\sqrt{b}\right.\right.}{15\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/15*(45*(sqrt(b)*x - s
qrt(b*x^2 + a))^8*a*b^(5/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(5/2
) + 140*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(5/2) - 70*(sqrt(b)*x - sqrt(
```

```
b*x^2 + a))^2*a^4*b^(5/2) + 23*a^5*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^
2 - a)^5
```

maple [A] time = 0.01, size = 130, normalized size = 1.59

$$b^{\frac{5}{2}} \ln\left(\sqrt{b}\, x + \sqrt{b\,x^2 + a}\right) + \frac{\sqrt{b\,x^2 + a}\, b^3 x}{a} + \frac{2\left(b\,x^2 + a\right)^{\frac{3}{2}} b^3 x}{3a^2} + \frac{8\left(b\,x^2 + a\right)^{\frac{5}{2}} b^3 x}{15a^3} - \frac{8\left(b\,x^2 + a\right)^{\frac{7}{2}} b^2}{15a^3 x} - \frac{2\left(b\,x^2 + a\right)}{15a^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^6,x)
```

```
[Out] -1/5/a/x^5*(b*x^2+a)^(7/2)-2/15/a^2*b/x^3*(b*x^2+a)^(7/2)-8/15/a^3*b^2/x*(b
*x^2+a)^(7/2)+8/15/a^3*b^3*x*(b*x^2+a)^(5/2)+2/3/a^2*b^3*x*(b*x^2+a)^(3/2)+
1/a*b^3*x*(b*x^2+a)^(1/2)+b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.40, size = 104, normalized size = 1.27

$$\frac{2\left(bx^2 + a\right)^{\frac{3}{2}} b^3 x}{3\,a^2} + \frac{\sqrt{bx^2 + a}\, b^3 x}{a} + b^{\frac{5}{2}} \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{8\left(bx^2 + a\right)^{\frac{5}{2}} b^2}{15\,a^2 x} - \frac{2\left(bx^2 + a\right)^{\frac{7}{2}} b}{15\,a^2 x^3} - \frac{\left(bx^2 + a\right)^{\frac{7}{2}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 2/3*(b*x^2 + a)^(3/2)*b^3*x/a^2 + sqrt(b*x^2 + a)*b^3*x/a + b^(5/2)*arcsinh
(b*x/sqrt(a*b)) - 8/15*(b*x^2 + a)^(5/2)*b^2/(a^2*x) - 2/15*(b*x^2 + a)^(7/
2)*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(7/2)/(a*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2 + a\right)^{5/2}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^6,x)
```

```
[Out] int((a + b*x^2)^(5/2)/x^6, x)
```

sympy [A] time = 3.66, size = 105, normalized size = 1.28

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{5x^4} - \frac{11ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2} + 1}}{15x^2} - \frac{23b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2} + 1}}{15} - \frac{b^{\frac{5}{2}}\log\left(\frac{a}{bx^2}\right)}{2} + b^{\frac{5}{2}}\log\left(\sqrt{\frac{a}{bx^2} + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**6,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 11*a*b**(3/2)*sqrt(a/(b*x**2)
 + 1)/(15*x**2) - 23*b**(5/2)*sqrt(a/(b*x**2) + 1)/15 - b**(5/2)*log(a/(b*x
**2))/2 + b**(5/2)*log(sqrt(a/(b*x**2) + 1) + 1)
```

3.403 $\int \frac{\left(a+bx^2\right)^{5/2}}{x^8} \, dx$

Optimal. Leaf size=21

$$-\frac{\left(a+bx^2\right)^{7/2}}{7ax^7}$$

```
[Out] -1/7*(b*x^2+a)^(7/2)/a/x^7
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\left(a+bx^2\right)^{7/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^8,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(7*a*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^8} \, dx = -\frac{\left(a+bx^2\right)^{7/2}}{7ax^7}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{\left(a+bx^2\right)^{7/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^8,x]
```

```
[Out] -1/7*(a + b*x^2)^(7/2)/(a*x^7)
```

fricas [B] time = 0.99, size = 46, normalized size = 2.19

$$-\frac{\left(b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3\right)\sqrt{bx^2 + a}}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/7*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 + a)/(a*x^7)
```

giac [B] time = 0.99, size = 113, normalized size = 5.38

$$\frac{2\left(7\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}b^{\frac{7}{2}}+35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^2b^{\frac{7}{2}}+21\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^4b^{\frac{7}{2}}+a^6b^{\frac{7}{2}}\right)}{7\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 2/7*(7*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(7/2) + 35*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*a^2*b^(7/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(7/2) + a^
6*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{7a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^8,x)
```

```
[Out] -1/7*(b*x^2+a)^(7/2)/a/x^7
```

maxima [A] time = 1.41, size = 17, normalized size = 0.81

$$-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/7*(b*x^2 + a)^(7/2)/(a*x^7)
```

mupad [B] time = 5.07, size = 71, normalized size = 3.38

$$-\frac{a^2\,\sqrt{b\,x^2+a}}{7\,x^7}-\frac{3\,b^2\,\sqrt{b\,x^2+a}}{7\,x^3}-\frac{b^3\,\sqrt{b\,x^2+a}}{7\,a\,x}-\frac{3\,a\,b\,\sqrt{b\,x^2+a}}{7\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^8,x)
```

```
[Out] - (a^2*(a + b*x^2)^(1/2))/(7*x^7) - (3*b^2*(a + b*x^2)^(1/2))/(7*x^3) - (b^
3*(a + b*x^2)^(1/2))/(7*a*x) - (3*a*b*(a + b*x^2)^(1/2))/(7*x^5)
```

sympy [B] time = 1.26, size = 95, normalized size = 4.52

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{7x^6}-\frac{3ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{7x^4}-\frac{3b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{7x^2}-\frac{b^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{7a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**8,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 3*a*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/(7*x**4) - 3*b**(5/2)*sqrt(a/(b*x**2) + 1)/(7*x**2) - b**(7/2)*sqrt(a/
(b*x**2) + 1)/(7*a)
```

3.404 $\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx$

Optimal. Leaf size=44

$$\frac{2b(a+bx^2)^{7/2}}{63a^2x^7}-\frac{(a+bx^2)^{7/2}}{9ax^9}$$

[Out] -1/9*(b*x^2+a)^(7/2)/a/x^9+2/63*b*(b*x^2+a)^(7/2)/a^2/x^7

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2b(a+bx^2)^{7/2}}{63a^2x^7}-\frac{(a+bx^2)^{7/2}}{9ax^9}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(5/2)/x^10,x]

[Out] -(a + b*x^2)^(7/2)/(9*a*x^9) + (2*b*(a + b*x^2)^(7/2))/(63*a^2*x^7)

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx&=-\frac{(a+bx^2)^{7/2}}{9ax^9}-\frac{(2b)\int \frac{(a+bx^2)^{5/2}}{x^8}\,dx}{9a}\\&=-\frac{(a+bx^2)^{7/2}}{9ax^9}+\frac{2b(a+bx^2)^{7/2}}{63a^2x^7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{(a+bx^2)^{7/2}(2bx^2-7a)}{63a^2x^9}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(5/2)/x^10,x]

[Out] ((a + b*x^2)^(7/2)*(-7*a + 2*b*x^2))/(63*a^2*x^9)

fricas [A] time = 0.68, size = 60, normalized size = 1.36

$$\frac{(2\,b^4x^8-ab^3x^6-15\,a^2b^2x^4-19\,a^3bx^2-7\,a^4)\sqrt{bx^2+a}}{63\,a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(2*b^4*x^8 - a*b^3*x^6 - 15*a^2*b^2*x^4 - 19*a^3*b*x^2 - 7*a^4)*sqrt(b
*x^2 + a)/(a^2*x^9)
```

giac [B] time = 1.14, size = 220, normalized size = 5.00

$$\frac{4\left(63\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}b^{\frac{9}{2}}+105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}ab^{\frac{9}{2}}+315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^2b^{\frac{9}{2}}+189\left(\sqrt{b}x-\right.\right.}{63\left(\left(\sqrt{b}x-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 4/63*(63*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(9/2) + 105*(sqrt(b)*x - sqrt(b
*x^2 + a))^12*a*b^(9/2) + 315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(9/2)
+ 189*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(9/2) + 189*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*a^4*b^(9/2) + 27*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(9/2) +
 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(9/2) - a^7*b^(9/2))/((sqrt(b)*x -
 sqrt(b*x^2 + a))^2 - a)^9
```

maple [A] time = 0.01, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}\left(-2b\,x^2+7a\right)}{63a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^10,x)
```

```
[Out] -1/63*(b*x^2+a)^(7/2)*(-2*b*x^2+7*a)/x^9/a^2
```

maxima [A] time = 1.53, size = 36, normalized size = 0.82

$$\frac{2\left(bx^2+a\right)^{\frac{7}{2}}b}{63\,a^2x^7}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] 2/63*(b*x^2 + a)^(7/2)*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(7/2)/(a*x^9)
```

mupad [B] time = 5.37, size = 91, normalized size = 2.07

$$\frac{2\,b^4\,\sqrt{b\,x^2+a}}{63\,a^2\,x}-\frac{5\,b^2\,\sqrt{b\,x^2+a}}{21\,x^5}-\frac{b^3\,\sqrt{b\,x^2+a}}{63\,a\,x^3}-\frac{a^2\,\sqrt{b\,x^2+a}}{9\,x^9}-\frac{19\,a\,b\,\sqrt{b\,x^2+a}}{63\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^10,x)
```

```
[Out] (2*b^4*(a + b*x^2)^(1/2))/(63*a^2*x) - (5*b^2*(a + b*x^2)^(1/2))/(21*x^5) -
 (b^3*(a + b*x^2)^(1/2))/(63*a*x^3) - (a^2*(a + b*x^2)^(1/2))/(9*x^9) - (19
*a*b*(a + b*x^2)^(1/2))/(63*x^7)
```

sympy [B] time = 1.59, size = 121, normalized size = 2.75

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{9x^8}-\frac{19ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{63x^6}-\frac{5b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{21x^4}-\frac{b^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{63ax^2}+\frac{2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{63a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**10,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(9*x**8) - 19*a*b**(3/2)*sqrt(a/(b*x**2)
 + 1)/(63*x**6) - 5*b**(5/2)*sqrt(a/(b*x**2) + 1)/(21*x**4) - b**(7/2)*sqrt
(a/(b*x**2) + 1)/(63*a*x**2) + 2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(63*a**2)
```

3.405 $\int \frac{\left(a+bx^2\right)^{5/2}}{x^{12}}\,dx$

Optimal. Leaf size=68

$$-\frac{8b^2\left(a+bx^2\right)^{7/2}}{693a^3x^7}+\frac{4b\left(a+bx^2\right)^{7/2}}{99a^2x^9}-\frac{\left(a+bx^2\right)^{7/2}}{11ax^{11}}$$

```
[Out] -1/11*(b*x^2+a)^(7/2)/a/x^11+4/99*b*(b*x^2+a)^(7/2)/a^2/x^9-8/693*b^2*(b*x^
2+a)^(7/2)/a^3/x^7
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8b^2\left(a+bx^2\right)^{7/2}}{693a^3x^7}+\frac{4b\left(a+bx^2\right)^{7/2}}{99a^2x^9}-\frac{\left(a+bx^2\right)^{7/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^12,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(11*a*x^11) + (4*b*(a + b*x^2)^(7/2))/(99*a^2*x^9) - (8*
b^2*(a + b*x^2)^(7/2))/(693*a^3*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{x^{12}}\,dx &= -\frac{\left(a+bx^2\right)^{7/2}}{11ax^{11}}-\frac{(4b)\int \frac{\left(a+bx^2\right)^{5/2}}{x^{10}}\,dx}{11a}\\
&= -\frac{\left(a+bx^2\right)^{7/2}}{11ax^{11}}+\frac{4b\left(a+bx^2\right)^{7/2}}{99a^2x^9}+\frac{\left(8b^2\right)\int \frac{\left(a+bx^2\right)^{5/2}}{x^8}\,dx}{99a^2}\\
&= -\frac{\left(a+bx^2\right)^{7/2}}{11ax^{11}}+\frac{4b\left(a+bx^2\right)^{7/2}}{99a^2x^9}-\frac{8b^2\left(a+bx^2\right)^{7/2}}{693a^3x^7}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^2\right)^{7/2}\left(63a^2-28abx^2+8b^2x^4\right)}{693a^3x^{11}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(5/2)/x^12,x]

[Out] -1/693*((a + b*x^2)^(7/2)*(63*a^2 - 28*a*b*x^2 + 8*b^2*x^4))/(a^3*x^11)

fricas [A] time = 0.94, size = 71, normalized size = 1.04

$$-\frac{\left(8\,b^{5}x^{10}-4\,ab^{4}x^{8}+3\,a^{2}b^{3}x^{6}+113\,a^{3}b^{2}x^{4}+161\,a^{4}bx^{2}+63\,a^{5}\right)\sqrt{bx^{2}+a}}{693\,a^{3}x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="fricas")

[Out] -1/693*(8*b^5*x^10 - 4*a*b^4*x^8 + 3*a^2*b^3*x^6 + 113*a^3*b^2*x^4 + 161*a^4*b*x^2 + 63*a^5)*sqrt(b*x^2 + a)/(a^3*x^11)

giac [B] time = 1.16, size = 246, normalized size = 3.62

$$\frac{16\left(462\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{16}b^{\frac{11}{2}}+1155\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{14}ab^{\frac{11}{2}}+2541\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{12}a^{2}b^{\frac{11}{2}}+2079\left(\sqrt{b}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="giac")

[Out] 16/693*(462*(sqrt(b)*x - sqrt(b*x^2 + a))^16*b^(11/2) + 1155*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a*b^(11/2) + 2541*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b^(11/2) + 2079*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*b^(11/2) + 1485*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^4*b^(11/2) + 297*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*b^(11/2) + 55*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*b^(11/2) - 11*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^7*b^(11/2) + a^8*b^(11/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^11

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(8b^{2}x^{4}-28ab\,x^{2}+63a^{2}\right)}{693a^{3}x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(5/2)/x^12,x)

[Out] -1/693*(b*x^2+a)^(7/2)*(8*b^2*x^4-28*a*b*x^2+63*a^2)/x^11/a^3

maxima [A] time = 1.41, size = 56, normalized size = 0.82

$$-\frac{8\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}{693\,a^{3}x^{7}}+\frac{4\left(bx^{2}+a\right)^{\frac{7}{2}}b}{99\,a^{2}x^{9}}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}}{11\,ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="maxima")

[Out] -8/693*(b*x^2 + a)^(7/2)*b^2/(a^3*x^7) + 4/99*(b*x^2 + a)^(7/2)*b/(a^2*x^9) - 1/11*(b*x^2 + a)^(7/2)/(a*x^11)

mupad [B] time = 5.62, size = 111, normalized size = 1.63

$$\frac{4\,b^{4}\,\sqrt{b\,x^{2}+a}}{693\,a^{2}\,x^{3}}-\frac{113\,b^{2}\,\sqrt{b\,x^{2}+a}}{693\,x^{7}}-\frac{b^{3}\,\sqrt{b\,x^{2}+a}}{231\,a\,x^{5}}-\frac{a^{2}\,\sqrt{b\,x^{2}+a}}{11\,x^{11}}-\frac{8\,b^{5}\,\sqrt{b\,x^{2}+a}}{693\,a^{3}\,x}-\frac{23\,a\,b\,\sqrt{b\,x^{2}+a}}{99\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^12,x)
```

```
[Out] (4*b^4*(a + b*x^2)^(1/2))/(693*a^2*x^3) - (113*b^2*(a + b*x^2)^(1/2))/(693*
x^7) - (b^3*(a + b*x^2)^(1/2))/(231*a*x^5) - (a^2*(a + b*x^2)^(1/2))/(11*x^
11) - (8*b^5*(a + b*x^2)^(1/2))/(693*a^3*x) - (23*a*b*(a + b*x^2)^(1/2))/(9
9*x^9)
```

sympy [B] time = 2.16, size = 481, normalized size = 7.07

$$-\frac{63a^7b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^2\left(693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}\right)}-\frac{287a^6b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}-\frac{498a^5b^{\frac{13}{2}}x^2}{693a^5b^4x^8+1386a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**12,x)
```

```
[Out] -63*a**7*b**(9/2)*sqrt(a/(b*x**2) + 1)/(x**2*(693*a**5*b**4*x**8 + 1386*a**
4*b**5*x**10 + 693*a**3*b**6*x**12)) - 287*a**6*b**(11/2)*sqrt(a/(b*x**2) +
 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12) - 498
*a**5*b**(13/2)*x**2*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*b
**5*x**10 + 693*a**3*b**6*x**12) - 390*a**4*b**(15/2)*x**4*sqrt(a/(b*x**2)
+ 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12) - 11
5*a**3*b**(17/2)*x**6*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*
b**5*x**10 + 693*a**3*b**6*x**12) - 3*a**2*b**(19/2)*x**8*sqrt(a/(b*x**2) +
 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12) - 12*
a*b**(21/2)*x**10*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5
*x**10 + 693*a**3*b**6*x**12) - 8*b**(23/2)*x**12*sqrt(a/(b*x**2) + 1)/(693
*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12)
```

3.406 $\int \frac{(a+bx^2)^{5/2}}{x^{14}} dx$

Optimal. Leaf size=92

$$\frac{16b^3(a+bx^2)^{7/2}}{3003a^4x^7} - \frac{8b^2(a+bx^2)^{7/2}}{429a^3x^9} + \frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}} - \frac{(a+bx^2)^{7/2}}{13ax^{13}}$$

```
[Out] -1/13*(b*x^2+a)^(7/2)/a/x^13+6/143*b*(b*x^2+a)^(7/2)/a^2/x^11-8/429*b^2*(b*
x^2+a)^(7/2)/a^3/x^9+16/3003*b^3*(b*x^2+a)^(7/2)/a^4/x^7
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{16b^3(a+bx^2)^{7/2}}{3003a^4x^7} - \frac{8b^2(a+bx^2)^{7/2}}{429a^3x^9} + \frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}} - \frac{(a+bx^2)^{7/2}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^14,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(13*a*x^13) + (6*b*(a + b*x^2)^(7/2))/(143*a^2*x^11) - (
8*b^2*(a + b*x^2)^(7/2))/(429*a^3*x^9) + (16*b^3*(a + b*x^2)^(7/2))/(3003*a
^4*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{x^{14}} dx &= -\frac{(a+bx^2)^{7/2}}{13ax^{13}} - \frac{(6b)\int \frac{(a+bx^2)^{5/2}}{x^{12}} dx}{13a} \\
&= -\frac{(a+bx^2)^{7/2}}{13ax^{13}} + \frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}} + \frac{(24b^2)\int \frac{(a+bx^2)^{5/2}}{x^{10}} dx}{143a^2} \\
&= -\frac{(a+bx^2)^{7/2}}{13ax^{13}} + \frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}} - \frac{8b^2(a+bx^2)^{7/2}}{429a^3x^9} - \frac{(16b^3)\int \frac{(a+bx^2)^{5/2}}{x^8} dx}{429a^3} \\
&= -\frac{(a+bx^2)^{7/2}}{13ax^{13}} + \frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}} - \frac{8b^2(a+bx^2)^{7/2}}{429a^3x^9} + \frac{16b^3(a+bx^2)^{7/2}}{3003a^4x^7}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.58

$$\frac{(a+bx^2)^{7/2}(-231a^3+126a^2bx^2-56ab^2x^4+16b^3x^6)}{3003a^4x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^14,x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-231*a^3 + 126*a^2*b*x^2 - 56*a*b^2*x^4 + 16*b^3*x^6))/
(3003*a^4*x^13)
```

fricas [A] time = 1.09, size = 82, normalized size = 0.89

$$\frac{\left(16\,b^{6}x^{12}-8\,ab^{5}x^{10}+6\,a^{2}b^{4}x^{8}-5\,a^{3}b^{3}x^{6}-371\,a^{4}b^{2}x^{4}-567\,a^{5}bx^{2}-231\,a^{6}\right)\sqrt{bx^{2}+a}}{3003\,a^{4}x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="fricas")
```

```
[Out] 1/3003*(16*b^6*x^12 - 8*a*b^5*x^10 + 6*a^2*b^4*x^8 - 5*a^3*b^3*x^6 - 371*a^
4*b^2*x^4 - 567*a^5*b*x^2 - 231*a^6)*sqrt(b*x^2 + a)/(a^4*x^13)
```

giac [B] time = 1.18, size = 274, normalized size = 2.98

$$32\left(3003\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{18}b^{\frac{13}{2}}+9009\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{16}ab^{\frac{13}{2}}+18018\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{14}a^{2}b^{\frac{13}{2}}+163\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="giac")
```

```
[Out] 32/3003*(3003*(sqrt(b)*x - sqrt(b*x^2 + a))^18*b^(13/2) + 9009*(sqrt(b)*x -
 sqrt(b*x^2 + a))^16*a*b^(13/2) + 18018*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^
2*b^(13/2) + 16302*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(13/2) + 10296*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(13/2) + 2288*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a^5*b^(13/2) + 286*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(13/2) - 7
8*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^(13/2) + 13*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^8*b^(13/2) - a^9*b^(13/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
^13
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(-16b^{3}x^{6}+56a\,b^{2}x^{4}-126a^{2}b\,x^{2}+231a^{3}\right)}{3003a^{4}x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^14,x)
```

```
[Out] -1/3003*(b*x^2+a)^(7/2)*(-16*b^3*x^6+56*a*b^2*x^4-126*a^2*b*x^2+231*a^3)/x^
13/a^4
```

maxima [A] time = 1.39, size = 76, normalized size = 0.83

$$\frac{16\left(bx^{2}+a\right)^{\frac{7}{2}}b^{3}}{3003\,a^{4}x^{7}}-\frac{8\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}{429\,a^{3}x^{9}}+\frac{6\left(bx^{2}+a\right)^{\frac{7}{2}}b}{143\,a^{2}x^{11}}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}}{13\,ax^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="maxima")
```

```
[Out] 16/3003*(b*x^2 + a)^(7/2)*b^3/(a^4*x^7) - 8/429*(b*x^2 + a)^(7/2)*b^2/(a^3*
x^9) + 6/143*(b*x^2 + a)^(7/2)*b/(a^2*x^11) - 1/13*(b*x^2 + a)^(7/2)/(a*x^1
3)
```

mupad [B] time = 5.98, size = 131, normalized size = 1.42

$$\frac{2\,b^4\,\sqrt{b\,x^2+a}}{1001\,a^2\,x^5}-\frac{53\,b^2\,\sqrt{b\,x^2+a}}{429\,x^9}-\frac{5\,b^3\,\sqrt{b\,x^2+a}}{3003\,a\,x^7}-\frac{a^2\,\sqrt{b\,x^2+a}}{13\,x^{13}}-\frac{8\,b^5\,\sqrt{b\,x^2+a}}{3003\,a^3\,x^3}+\frac{16\,b^6\,\sqrt{b\,x^2+a}}{3003\,a^4\,x}-\frac{27\,a\,b\,\sqrt{b\,x^2+a}}{143\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^14,x)
```

```
[Out] (2*b^4*(a + b*x^2)^(1/2))/(1001*a^2*x^5) - (53*b^2*(a + b*x^2)^(1/2))/(429*
x^9) - (5*b^3*(a + b*x^2)^(1/2))/(3003*a*x^7) - (a^2*(a + b*x^2)^(1/2))/(13
*x^13) - (8*b^5*(a + b*x^2)^(1/2))/(3003*a^3*x^3) + (16*b^6*(a + b*x^2)^(1/
2))/(3003*a^4*x) - (27*a*b*(a + b*x^2)^(1/2))/(143*x^11)
```

sympy [B] time = 2.79, size = 721, normalized size = 7.84

$$-\frac{231a^9b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}-\frac{1260a^8b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**14,x)
```

```
[Out] -231*a**9*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*
b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 1260*a**8*b*
*(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x
**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 2765*a**7*b**(23/2)
*x**4*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 +
9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 3050*a**6*b**(25/2)*x**6*s
qrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a*
*5*b**11*x**16 + 3003*a**4*b**12*x**18) - 1689*a**5*b**(27/2)*x**8*sqrt(a/(
b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**1
1*x**16 + 3003*a**4*b**12*x**18) - 376*a**4*b**(29/2)*x**10*sqrt(a/(b*x**2)
 + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16
 + 3003*a**4*b**12*x**18) + 5*a**3*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(30
03*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a
**4*b**12*x**18) + 30*a**2*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(3003*a**7*
b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**1
2*x**18) + 40*a*b**(35/2)*x**16*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12
+ 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) +
16*b**(37/2)*x**18*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b
**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18)
```

3.407 $\int \frac{\left(a+bx^2\right)^{5/2}}{x^{16}}\,dx$

Optimal. Leaf size=116

$$-\frac{128b^4\left(a+bx^2\right)^{7/2}}{45045a^5x^7}+\frac{64b^3\left(a+bx^2\right)^{7/2}}{6435a^4x^9}-\frac{16b^2\left(a+bx^2\right)^{7/2}}{715a^3x^{11}}+\frac{8b\left(a+bx^2\right)^{7/2}}{195a^2x^{13}}-\frac{\left(a+bx^2\right)^{7/2}}{15ax^{15}}$$

```
[Out] -1/15*(b*x^2+a)^(7/2)/a/x^15+8/195*b*(b*x^2+a)^(7/2)/a^2/x^13-16/715*b^2*(b
*x^2+a)^(7/2)/a^3/x^11+64/6435*b^3*(b*x^2+a)^(7/2)/a^4/x^9-128/45045*b^4*(b
*x^2+a)^(7/2)/a^5/x^7
```

Rubi [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{271, 264\}$

$$-\frac{128b^4\left(a+bx^2\right)^{7/2}}{45045a^5x^7}+\frac{64b^3\left(a+bx^2\right)^{7/2}}{6435a^4x^9}-\frac{16b^2\left(a+bx^2\right)^{7/2}}{715a^3x^{11}}+\frac{8b\left(a+bx^2\right)^{7/2}}{195a^2x^{13}}-\frac{\left(a+bx^2\right)^{7/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^16,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(15*a*x^15) + (8*b*(a + b*x^2)^(7/2))/(195*a^2*x^13) - (
16*b^2*(a + b*x^2)^(7/2))/(715*a^3*x^11) + (64*b^3*(a + b*x^2)^(7/2))/(6435
*a^4*x^9) - (128*b^4*(a + b*x^2)^(7/2))/(45045*a^5*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{x^{16}}\,dx &= -\frac{\left(a+bx^2\right)^{7/2}}{15ax^{15}}-\frac{(8b)\int \frac{\left(a+bx^2\right)^{5/2}}{x^{14}}\,dx}{15a}\\
&= -\frac{\left(a+bx^2\right)^{7/2}}{15ax^{15}}+\frac{8b\left(a+bx^2\right)^{7/2}}{195a^2x^{13}}+\frac{\left(16b^2\right)\int \frac{\left(a+bx^2\right)^{5/2}}{x^{12}}\,dx}{65a^2}\\
&= -\frac{\left(a+bx^2\right)^{7/2}}{15ax^{15}}+\frac{8b\left(a+bx^2\right)^{7/2}}{195a^2x^{13}}-\frac{16b^2\left(a+bx^2\right)^{7/2}}{715a^3x^{11}}-\frac{\left(64b^3\right)\int \frac{\left(a+bx^2\right)^{5/2}}{x^{10}}\,dx}{715a^3}\\
&= -\frac{\left(a+bx^2\right)^{7/2}}{15ax^{15}}+\frac{8b\left(a+bx^2\right)^{7/2}}{195a^2x^{13}}-\frac{16b^2\left(a+bx^2\right)^{7/2}}{715a^3x^{11}}+\frac{64b^3\left(a+bx^2\right)^{7/2}}{6435a^4x^9}+\frac{\left(128b^4\right)\int \cdots}{6435a^4}\\
&= -\frac{\left(a+bx^2\right)^{7/2}}{15ax^{15}}+\frac{8b\left(a+bx^2\right)^{7/2}}{195a^2x^{13}}-\frac{16b^2\left(a+bx^2\right)^{7/2}}{715a^3x^{11}}+\frac{64b^3\left(a+bx^2\right)^{7/2}}{6435a^4x^9}-\frac{128b^4\left(a+bx^2\right)^{7/2}}{45045a^5x^7}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.55

$$-\frac{\left(a+bx^2\right)^{7/2}\left(3003a^4-1848a^3bx^2+1008a^2b^2x^4-448ab^3x^6+128b^4x^8\right)}{45045a^5x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^16,x]
```

```
[Out] -1/45045*((a + b*x^2)^(7/2)*(3003*a^4 - 1848*a^3*b*x^2 + 1008*a^2*b^2*x^4 -
 448*a*b^3*x^6 + 128*b^4*x^8))/(a^5*x^15)
```

fricas [A] time = 1.11, size = 93, normalized size = 0.80

$$-\frac{\left(128\,b^7x^{14}-64\,ab^6x^{12}+48\,a^2b^5x^{10}-40\,a^3b^4x^8+35\,a^4b^3x^6+4473\,a^5b^2x^4+7161\,a^6bx^2+3003\,a^7\right)\sqrt{bx^2+a}}{45045\,a^5x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(128*b^7*x^14 - 64*a*b^6*x^12 + 48*a^2*b^5*x^10 - 40*a^3*b^4*x^8 +
 35*a^4*b^3*x^6 + 4473*a^5*b^2*x^4 + 7161*a^6*b*x^2 + 3003*a^7)*sqrt(b*x^2
+ a)/(a^5*x^15)
```

giac [B] time = 1.23, size = 300, normalized size = 2.59

$$256\left(18018\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{20}b^{\frac{15}{2}}+60060\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{18}ab^{\frac{15}{2}}+115830\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{16}a^2b^{\frac{15}{2}}+10\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] 256/45045*(18018*(sqrt(b)*x - sqrt(b*x^2 + a))^20*b^(15/2) + 60060*(sqrt(b)
*x - sqrt(b*x^2 + a))^18*a*b^(15/2) + 115830*(sqrt(b)*x - sqrt(b*x^2 + a))^
16*a^2*b^(15/2) + 109395*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^3*b^(15/2) + 65
065*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(15/2) + 15015*(sqrt(b)*x - sqrt
(b*x^2 + a))^10*a^5*b^(15/2) + 1365*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(
15/2) - 455*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^7*b^(15/2) + 105*(sqrt(b)*x -
 sqrt(b*x^2 + a))^4*a^8*b^(15/2) - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^9*b
^(15/2) + a^10*b^(15/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^15
```

maple [A] time = 0.01, size = 61, normalized size = 0.53

$$-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}\left(128b^4x^8-448a\,b^3x^6+1008a^2b^2x^4-1848a^3b\,x^2+3003a^4\right)}{45045a^5x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^16,x)
```

```
[Out] -1/45045*(b*x^2+a)^(7/2)*(128*b^4*x^8-448*a*b^3*x^6+1008*a^2*b^2*x^4-1848*a
^3*b*x^2+3003*a^4)/x^15/a^5
```

maxima [A] time = 1.45, size = 96, normalized size = 0.83

$$-\frac{128\left(bx^2+a\right)^{\frac{7}{2}}b^4}{45045\,a^5x^7}+\frac{64\left(bx^2+a\right)^{\frac{7}{2}}b^3}{6435\,a^4x^9}-\frac{16\left(bx^2+a\right)^{\frac{7}{2}}b^2}{715\,a^3x^{11}}+\frac{8\left(bx^2+a\right)^{\frac{7}{2}}b}{195\,a^2x^{13}}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{15\,ax^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] -128/45045*(b*x^2 + a)^(7/2)*b^4/(a^5*x^7) + 64/6435*(b*x^2 + a)^(7/2)*b^3/
(a^4*x^9) - 16/715*(b*x^2 + a)^(7/2)*b^2/(a^3*x^11) + 8/195*(b*x^2 + a)^(7/
2)*b/(a^2*x^13) - 1/15*(b*x^2 + a)^(7/2)/(a*x^15)
```

mupad [B] time = 6.37, size = 151, normalized size = 1.30

$$\frac{8\,b^4\,\sqrt{b\,x^2+a}}{9009\,a^2\,x^7}-\frac{71\,b^2\,\sqrt{b\,x^2+a}}{715\,x^{11}}-\frac{b^3\,\sqrt{b\,x^2+a}}{1287\,a\,x^9}-\frac{a^2\,\sqrt{b\,x^2+a}}{15\,x^{15}}-\frac{16\,b^5\,\sqrt{b\,x^2+a}}{15015\,a^3\,x^5}+\frac{64\,b^6\,\sqrt{b\,x^2+a}}{45045\,a^4\,x^3}-\frac{128\,b^7\,\sqrt{b\,x}}{45045\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^16,x)
```

```
[Out] (8*b^4*(a + b*x^2)^(1/2))/(9009*a^2*x^7) - (71*b^2*(a + b*x^2)^(1/2))/(715*
x^11) - (b^3*(a + b*x^2)^(1/2))/(1287*a*x^9) - (a^2*(a + b*x^2)^(1/2))/(15*
x^15) - (16*b^5*(a + b*x^2)^(1/2))/(15015*a^3*x^5) + (64*b^6*(a + b*x^2)^(1
/2))/(45045*a^4*x^3) - (128*b^7*(a + b*x^2)^(1/2))/(45045*a^5*x) - (31*a*b*
(a + b*x^2)^(1/2))/(195*x^13)
```

sympy [B] time = 3.49, size = 1012, normalized size = 8.72

$$-\frac{3003a^{11}b^{\frac{33}{2}}\sqrt{\frac{a}{bx^2}+1}}{45045a^9b^{16}x^{14}+180180a^8b^{17}x^{16}+270270a^7b^{18}x^{18}+180180a^6b^{19}x^{20}+45045a^5b^{20}x^{22}}-\frac{}{45045a^9b^{16}x^{14}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**16,x)
```

```
[Out] -3003*a**11*b**(33/2)*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180
*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 450
45*a**5*b**20*x**22) - 19173*a**10*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(450
45*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 1
80180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 51135*a**9*b**(37/2)*x**
4*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 +
270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22)
 - 72905*a**8*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 +
 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**2
0 + 45045*a**5*b**20*x**22) - 58585*a**7*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1
)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**
18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 25151*a**6*b**(43/
2)*x**10*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x
**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20
*x**22) - 4501*a**5*b**(45/2)*x**12*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*
x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**
19*x**20 + 45045*a**5*b**20*x**22) - 35*a**4*b**(47/2)*x**14*sqrt(a/(b*x**2
) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**1
8*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 280*a**3*b**(
49/2)*x**16*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**1
7*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b*
*20*x**22) - 560*a**2*b**(51/2)*x**18*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**1
6*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b
**19*x**20 + 45045*a**5*b**20*x**22) - 448*a*b**(53/2)*x**20*sqrt(a/(b*x**2
) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**1
8*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 128*b**(55/2)
*x**22*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**
16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x
**22)
```

3.408 $\int \frac{\left(a+bx^2\right)^{5/2}}{x^{18}}\,dx$

Optimal. Leaf size=140

$$\frac{256b^5\left(a+bx^2\right)^{7/2}}{153153a^6x^7}-\frac{128b^4\left(a+bx^2\right)^{7/2}}{21879a^5x^9}+\frac{32b^3\left(a+bx^2\right)^{7/2}}{2431a^4x^{11}}-\frac{16b^2\left(a+bx^2\right)^{7/2}}{663a^3x^{13}}+\frac{2b\left(a+bx^2\right)^{7/2}}{51a^2x^{15}}-\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}}$$

```
[Out] -1/17*(b*x^2+a)^(7/2)/a/x^17+2/51*b*(b*x^2+a)^(7/2)/a^2/x^15-16/663*b^2*(b*
x^2+a)^(7/2)/a^3/x^13+32/2431*b^3*(b*x^2+a)^(7/2)/a^4/x^11-128/21879*b^4*(b
*x^2+a)^(7/2)/a^5/x^9+256/153153*b^5*(b*x^2+a)^(7/2)/a^6/x^7
```

Rubi [A] time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = $\{271, 264\}$

$$\frac{256b^5\left(a+bx^2\right)^{7/2}}{153153a^6x^7}-\frac{128b^4\left(a+bx^2\right)^{7/2}}{21879a^5x^9}+\frac{32b^3\left(a+bx^2\right)^{7/2}}{2431a^4x^{11}}-\frac{16b^2\left(a+bx^2\right)^{7/2}}{663a^3x^{13}}+\frac{2b\left(a+bx^2\right)^{7/2}}{51a^2x^{15}}-\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^18,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(17*a*x^17) + (2*b*(a + b*x^2)^(7/2))/(51*a^2*x^15) - (1
6*b^2*(a + b*x^2)^(7/2))/(663*a^3*x^13) + (32*b^3*(a + b*x^2)^(7/2))/(2431*
a^4*x^11) - (128*b^4*(a + b*x^2)^(7/2))/(21879*a^5*x^9) + (256*b^5*(a + b*x
^2)^(7/2))/(153153*a^6*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^{18}}\,dx = -\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}} - \frac{(10b)\int \frac{\left(a+bx^2\right)^{5/2}}{x^{16}}\,dx}{17a}$$
$$= -\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}} + \frac{2b\left(a+bx^2\right)^{7/2}}{51a^2x^{15}} + \frac{\left(16b^2\right)\int \frac{\left(a+bx^2\right)^{5/2}}{x^{14}}\,dx}{51a^2}$$
$$= -\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}} + \frac{2b\left(a+bx^2\right)^{7/2}}{51a^2x^{15}} - \frac{16b^2\left(a+bx^2\right)^{7/2}}{663a^3x^{13}} - \frac{\left(32b^3\right)\int \frac{\left(a+bx^2\right)^{5/2}}{x^{12}}\,dx}{221a^3}$$
$$= -\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}} + \frac{2b\left(a+bx^2\right)^{7/2}}{51a^2x^{15}} - \frac{16b^2\left(a+bx^2\right)^{7/2}}{663a^3x^{13}} + \frac{32b^3\left(a+bx^2\right)^{7/2}}{2431a^4x^{11}} + \frac{\left(128b^4\right)\int \frac{(a+b}{x}}{2431a^4}$$
$$= -\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}} + \frac{2b\left(a+bx^2\right)^{7/2}}{51a^2x^{15}} - \frac{16b^2\left(a+bx^2\right)^{7/2}}{663a^3x^{13}} + \frac{32b^3\left(a+bx^2\right)^{7/2}}{2431a^4x^{11}} - \frac{128b^4\left(a+bx^2\right.}{21879a^5x^9}$$
$$= -\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}} + \frac{2b\left(a+bx^2\right)^{7/2}}{51a^2x^{15}} - \frac{16b^2\left(a+bx^2\right)^{7/2}}{663a^3x^{13}} + \frac{32b^3\left(a+bx^2\right)^{7/2}}{2431a^4x^{11}} - \frac{128b^4\left(a+bx^2\right.}{21879a^5x^9}$$

Mathematica [**A**] time = 0.02, size = 75, normalized size = 0.54

$$\frac{\left(a+bx^2\right)^{7/2}\left(-9009a^5+6006a^4bx^2-3696a^3b^2x^4+2016a^2b^3x^6-896ab^4x^8+256b^5x^{10}\right)}{153153a^6x^{17}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^18,x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-9009*a^5 + 6006*a^4*b*x^2 - 3696*a^3*b^2*x^4 + 2016*a^
2*b^3*x^6 - 896*a*b^4*x^8 + 256*b^5*x^10))/(153153*a^6*x^17)
```

fricas [**A**] time = 1.41, size = 104, normalized size = 0.74

$$\frac{\left(256\,b^8x^{16} - 128\,ab^7x^{14} + 96\,a^2b^6x^{12} - 80\,a^3b^5x^{10} + 70\,a^4b^4x^8 - 63\,a^5b^3x^6 - 12705\,a^6b^2x^4 - 21021\,a^7bx^2 - 90\right.}{153153\,a^6x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="fricas")
```

```
[Out] 1/153153*(256*b^8*x^16 - 128*a*b^7*x^14 + 96*a^2*b^6*x^12 - 80*a^3*b^5*x^10
 + 70*a^4*b^4*x^8 - 63*a^5*b^3*x^6 - 12705*a^6*b^2*x^4 - 21021*a^7*b*x^2 -
9009*a^8)*sqrt(b*x^2 + a)/(a^6*x^17)
```

giac [**B**] time = 1.10, size = 328, normalized size = 2.34

$$\frac{512\left(102102\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{22}b^{\frac{17}{2}}+364650\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{20}ab^{\frac{17}{2}}+692835\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{18}a^2b^{\frac{1}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="giac")
```

```
[Out] 512/153153*(102102*(sqrt(b)*x - sqrt(b*x^2 + a))^22*b^(17/2) + 364650*(sqrt
(b)*x - sqrt(b*x^2 + a))^20*a*b^(17/2) + 692835*(sqrt(b)*x - sqrt(b*x^2 + a
```

```
))^18*a^2*b^(17/2) + 668525*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^3*b^(17/2) +
 384098*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^4*b^(17/2) + 89726*(sqrt(b)*x -
sqrt(b*x^2 + a))^12*a^5*b^(17/2) + 6188*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^
6*b^(17/2) - 2380*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*b^(17/2) + 680*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*a^8*b^(17/2) - 136*(sqrt(b)*x - sqrt(b*x^2 + a))^
4*a^9*b^(17/2) + 17*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*b^(17/2) - a^11*b^
(17/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^17
```

maple [A] time = 0.01, size = 72, normalized size = 0.51

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(-256b^{5}x^{10}+896a\,b^{4}x^{8}-2016a^{2}b^{3}x^{6}+3696a^{3}b^{2}x^{4}-6006a^{4}b\,x^{2}+9009a^{5}\right)}{153153a^{6}x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^18,x)
```

```
[Out] -1/153153*(b*x^2+a)^(7/2)*(-256*b^5*x^10+896*a*b^4*x^8-2016*a^2*b^3*x^6+369
6*a^3*b^2*x^4-6006*a^4*b*x^2+9009*a^5)/x^17/a^6
```

maxima [A] time = 1.54, size = 116, normalized size = 0.83

$$\frac{256\left(bx^{2}+a\right)^{\frac{7}{2}}b^{5}}{153153\,a^{6}x^{7}}-\frac{128\left(bx^{2}+a\right)^{\frac{7}{2}}b^{4}}{21879\,a^{5}x^{9}}+\frac{32\left(bx^{2}+a\right)^{\frac{7}{2}}b^{3}}{2431\,a^{4}x^{11}}-\frac{16\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}{663\,a^{3}x^{13}}+\frac{2\left(bx^{2}+a\right)^{\frac{7}{2}}b}{51\,a^{2}x^{15}}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}}{17\,ax^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="maxima")
```

```
[Out] 256/153153*(b*x^2 + a)^(7/2)*b^5/(a^6*x^7) - 128/21879*(b*x^2 + a)^(7/2)*b^
4/(a^5*x^9) + 32/2431*(b*x^2 + a)^(7/2)*b^3/(a^4*x^11) - 16/663*(b*x^2 + a)
^(7/2)*b^2/(a^3*x^13) + 2/51*(b*x^2 + a)^(7/2)*b/(a^2*x^15) - 1/17*(b*x^2 +
 a)^(7/2)/(a*x^17)
```

mupad [B] time = 6.87, size = 171, normalized size = 1.22

$$\frac{10\,b^{4}\,\sqrt{b\,x^{2}+a}}{21879\,a^{2}\,x^{9}}-\frac{55\,b^{2}\,\sqrt{b\,x^{2}+a}}{663\,x^{13}}-\frac{b^{3}\,\sqrt{b\,x^{2}+a}}{2431\,a\,x^{11}}-\frac{a^{2}\,\sqrt{b\,x^{2}+a}}{17\,x^{17}}-\frac{80\,b^{5}\,\sqrt{b\,x^{2}+a}}{153153\,a^{3}\,x^{7}}+\frac{32\,b^{6}\,\sqrt{b\,x^{2}+a}}{51051\,a^{4}\,x^{5}}-\frac{128\,b^{7}\,\sqrt{b\,x^{2}}}{153153\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^18,x)
```

```
[Out] (10*b^4*(a + b*x^2)^(1/2))/(21879*a^2*x^9) - (55*b^2*(a + b*x^2)^(1/2))/(66
3*x^13) - (b^3*(a + b*x^2)^(1/2))/(2431*a*x^11) - (a^2*(a + b*x^2)^(1/2))/(
17*x^17) - (80*b^5*(a + b*x^2)^(1/2))/(153153*a^3*x^7) + (32*b^6*(a + b*x^2
)^(1/2))/(51051*a^4*x^5) - (128*b^7*(a + b*x^2)^(1/2))/(153153*a^5*x^3) + (
256*b^8*(a + b*x^2)^(1/2))/(153153*a^6*x) - (7*a*b*(a + b*x^2)^(1/2))/(51*x
^15)
```

sympy [B] time = 4.42, size = 1346, normalized size = 9.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**18,x)
```

```
[Out] -9009*a**13*b**(51/2)*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 7657
65*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22
+ 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 66066*a**12*b**(53/2
```

```
)*x**2*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*
x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b
**29*x**24 + 153153*a**6*b**30*x**26) - 207900*a**11*b**(55/2)*x**4*sqrt(a/
(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 153153
0*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 1
53153*a**6*b**30*x**26) - 363888*a**10*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/
(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x
**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**
30*x**26) - 382550*a**9*b**(59/2)*x**8*sqrt(a/(b*x**2) + 1)/(153153*a**11*b
**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*
a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 241
524*a**8*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 7
65765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**
22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 84780*a**7*b**(63
/2)*x**12*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**
26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**
7*b**29*x**24 + 153153*a**6*b**30*x**26) - 12768*a**6*b**(65/2)*x**14*sqrt(
a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531
530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 +
 153153*a**6*b**30*x**26) + 63*a**5*b**(67/2)*x**16*sqrt(a/(b*x**2) + 1)/(1
53153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**
20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30
*x**26) + 630*a**4*b**(69/2)*x**18*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25
*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8
*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) + 1680*a*
*3*b**(71/2)*x**20*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*
a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 7
65765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) + 2016*a**2*b**(73/2)*x**
22*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**1
8 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29
*x**24 + 153153*a**6*b**30*x**26) + 1152*a*b**(75/2)*x**24*sqrt(a/(b*x**2)
+ 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b*
*27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**
6*b**30*x**26) + 256*b**(77/2)*x**26*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**
25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a*
*8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26)
```

3.409 $\int x^{15}\left(a+bx^2\right)^{9/2}\,dx$

Optimal. Leaf size=161

$$-\frac{a^7\left(a+bx^2\right)^{11/2}}{11b^8}+\frac{7a^6\left(a+bx^2\right)^{13/2}}{13b^8}-\frac{7a^5\left(a+bx^2\right)^{15/2}}{5b^8}+\frac{35a^4\left(a+bx^2\right)^{17/2}}{17b^8}-\frac{35a^3\left(a+bx^2\right)^{19/2}}{19b^8}+\frac{a^2\left(a+bx^2\right)^{21/2}}{b^8}-$$

```
[Out] -1/11*a^7*(b*x^2+a)^(11/2)/b^8+7/13*a^6*(b*x^2+a)^(13/2)/b^8-7/5*a^5*(b*x^2
+a)^(15/2)/b^8+35/17*a^4*(b*x^2+a)^(17/2)/b^8-35/19*a^3*(b*x^2+a)^(19/2)/b^
8+a^2*(b*x^2+a)^(21/2)/b^8-7/23*a*(b*x^2+a)^(23/2)/b^8+1/25*(b*x^2+a)^(25/2
)/b^8
```

Rubi [A] time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^{21/2}}{b^8}-\frac{35a^3\left(a+bx^2\right)^{19/2}}{19b^8}+\frac{35a^4\left(a+bx^2\right)^{17/2}}{17b^8}-\frac{7a^5\left(a+bx^2\right)^{15/2}}{5b^8}+\frac{7a^6\left(a+bx^2\right)^{13/2}}{13b^8}-\frac{a^7\left(a+bx^2\right)^{11/2}}{11b^8}+$$

Antiderivative was successfully verified.

```
[In] Int[x^15*(a + b*x^2)^(9/2),x]
```

```
[Out] -(a^7*(a + b*x^2)^(11/2))/(11*b^8) + (7*a^6*(a + b*x^2)^(13/2))/(13*b^8) -
(7*a^5*(a + b*x^2)^(15/2))/(5*b^8) + (35*a^4*(a + b*x^2)^(17/2))/(17*b^8) -
 (35*a^3*(a + b*x^2)^(19/2))/(19*b^8) + (a^2*(a + b*x^2)^(21/2))/b^8 - (7*a
*(a + b*x^2)^(23/2))/(23*b^8) + (a + b*x^2)^(25/2)/(25*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{15}\left(a+bx^2\right)^{9/2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^7(a+bx)^{9/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a^7(a+bx)^{9/2}}{b^7}+\frac{7a^6(a+bx)^{11/2}}{b^7}-\frac{21a^5(a+bx)^{13/2}}{b^7}+\frac{35a^4(a+bx)^{15/2}}{b^7}-\right.\right.\\ &= -\frac{a^7\left(a+bx^2\right)^{11/2}}{11b^8}+\frac{7a^6\left(a+bx^2\right)^{13/2}}{13b^8}-\frac{7a^5\left(a+bx^2\right)^{15/2}}{5b^8}+\frac{17a^4\left(a+bx^2\right)^{17/2}}{17b^8}-\frac{35a^3\left(a\right.}{}\end{aligned}$$

Mathematica [A] time = 0.06, size = 94, normalized size = 0.58

$$\frac{\left(a+bx^2\right)^{11/2}\left(-2048a^7+11264a^6bx^2-36608a^5b^2x^4+91520a^4b^3x^6-194480a^3b^4x^8+369512a^2b^5x^{10}-646646\right.}{26558675b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-2048*a^7 + 11264*a^6*b*x^2 - 36608*a^5*b^2*x^4 + 9152
0*a^4*b^3*x^6 - 194480*a^3*b^4*x^8 + 369512*a^2*b^5*x^10 - 646646*a*b^6*x^1
2 + 1062347*b^7*x^14))/(26558675*b^8)
```

fricas [A] time = 1.03, size = 145, normalized size = 0.90

$$\frac{\left(1062347\,b^{12}x^{24} + 4665089\,ab^{11}x^{22} + 7759752\,a^2b^{10}x^{20} + 5810090\,a^3b^9x^{18} + 1659515\,a^4b^8x^{16} + 429\,a^5b^7x^{14}\right.}{2655867}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/26558675*(1062347*b^12*x^24 + 4665089*a*b^11*x^22 + 7759752*a^2*b^10*x^20
 + 5810090*a^3*b^9*x^18 + 1659515*a^4*b^8*x^16 + 429*a^5*b^7*x^14 - 462*a^6
*b^6*x^12 + 504*a^7*b^5*x^10 - 560*a^8*b^4*x^8 + 640*a^9*b^3*x^6 - 768*a^10
*b^2*x^4 + 1024*a^11*b*x^2 - 2048*a^12)*sqrt(b*x^2 + a)/b^8
```

giac [A] time = 1.07, size = 113, normalized size = 0.70

$$\frac{1062347\left(bx^2+a\right)^{\frac{25}{2}} - 8083075\left(bx^2+a\right)^{\frac{23}{2}}a + 26558675\left(bx^2+a\right)^{\frac{21}{2}}a^2 - 48923875\left(bx^2+a\right)^{\frac{19}{2}}a^3 + 5467962}{26558675\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/26558675*(1062347*(b*x^2 + a)^(25/2) - 8083075*(b*x^2 + a)^(23/2)*a + 265
58675*(b*x^2 + a)^(21/2)*a^2 - 48923875*(b*x^2 + a)^(19/2)*a^3 + 54679625*(
b*x^2 + a)^(17/2)*a^4 - 37182145*(b*x^2 + a)^(15/2)*a^5 + 14300825*(b*x^2 +
 a)^(13/2)*a^6 - 2414425*(b*x^2 + a)^(11/2)*a^7)/b^8
```

maple [A] time = 0.01, size = 91, normalized size = 0.57

$$-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}\left(-1062347x^{14}b^7 + 646646a\,x^{12}b^6 - 369512a^2x^{10}b^5 + 194480a^3x^8b^4 - 91520a^4x^6b^3 + 36608a^5x\right.}{26558675b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(b*x^2+a)^(9/2),x)
```

```
[Out] -1/26558675*(b*x^2+a)^(11/2)*(-1062347*b^7*x^14+646646*a*b^6*x^12-369512*a^
2*b^5*x^10+194480*a^3*b^4*x^8-91520*a^4*b^3*x^6+36608*a^5*b^2*x^4-11264*a^6
*b*x^2+2048*a^7)/b^8
```

maxima [A] time = 1.47, size = 153, normalized size = 0.95

$$\frac{\left(bx^2+a\right)^{\frac{11}{2}}x^{14}}{25\,b} - \frac{14\left(bx^2+a\right)^{\frac{11}{2}}ax^{12}}{575\,b^2} + \frac{8\left(bx^2+a\right)^{\frac{11}{2}}a^2x^{10}}{575\,b^3} - \frac{16\left(bx^2+a\right)^{\frac{11}{2}}a^3x^8}{2185\,b^4} + \frac{128\left(bx^2+a\right)^{\frac{11}{2}}a^4x^6}{37145\,b^5} - \frac{256\left(bx^2\right.}{185}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/25*(b*x^2 + a)^(11/2)*x^14/b - 14/575*(b*x^2 + a)^(11/2)*a*x^12/b^2 + 8/5
75*(b*x^2 + a)^(11/2)*a^2*x^10/b^3 - 16/2185*(b*x^2 + a)^(11/2)*a^3*x^8/b^4
 + 128/37145*(b*x^2 + a)^(11/2)*a^4*x^6/b^5 - 256/185725*(b*x^2 + a)^(11/2)
*a^5*x^4/b^6 + 1024/2414425*(b*x^2 + a)^(11/2)*a^6*x^2/b^7 - 2048/26558675*
(b*x^2 + a)^(11/2)*a^7/b^8
```

mupad [B] time = 4.84, size = 141, normalized size = 0.88

$$\sqrt{b\,x^2+a}\,\left(\frac{2321\,a^4\,x^{16}}{37145}-\frac{2048\,a^{12}}{26558675\,b^8}+\frac{b^4\,x^{24}}{25}+\frac{478\,a^3\,b\,x^{18}}{2185}+\frac{101\,a\,b^3\,x^{22}}{575}+\frac{3\,a^5\,x^{14}}{185725\,b}-\frac{42\,a^6\,x^{12}}{2414425\,b^2}+\frac{504\,}{2655}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((2321*a^4*x^16)/37145 - (2048*a^12)/(26558675*b^8) + (b^
4*x^24)/25 + (478*a^3*b*x^18)/2185 + (101*a*b^3*x^22)/575 + (3*a^5*x^14)/(1
85725*b) - (42*a^6*x^12)/(2414425*b^2) + (504*a^7*x^10)/(26558675*b^3) - (1
12*a^8*x^8)/(5311735*b^4) + (128*a^9*x^6)/(5311735*b^5) - (768*a^10*x^4)/(2
6558675*b^6) + (1024*a^11*x^2)/(26558675*b^7) + (168*a^2*b^2*x^20)/575)
```

sympy [A] time = 103.13, size = 301, normalized size = 1.87

$$\begin{cases} -\frac{2048a^{12}\sqrt{a+bx^2}}{26558675b^8}+\frac{1024a^{11}x^2\sqrt{a+bx^2}}{26558675b^7}-\frac{768a^{10}x^4\sqrt{a+bx^2}}{26558675b^6}+\frac{128a^9x^6\sqrt{a+bx^2}}{5311735b^5}-\frac{112a^8x^8\sqrt{a+bx^2}}{5311735b^4}+\frac{504a^7x^{10}\sqrt{a+bx^2}}{26558675b^3}-\frac{42a^6x^{12}\sqrt{a+bx^2}}{2414425b^2} \\ \frac{a^{\frac{9}{2}}x^{16}}{16} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-2048*a**12*sqrt(a + b*x**2)/(26558675*b**8) + 1024*a**11*x**2*s
qrt(a + b*x**2)/(26558675*b**7) - 768*a**10*x**4*sqrt(a + b*x**2)/(26558675
*b**6) + 128*a**9*x**6*sqrt(a + b*x**2)/(5311735*b**5) - 112*a**8*x**8*sqrt
(a + b*x**2)/(5311735*b**4) + 504*a**7*x**10*sqrt(a + b*x**2)/(26558675*b**
3) - 42*a**6*x**12*sqrt(a + b*x**2)/(2414425*b**2) + 3*a**5*x**14*sqrt(a +
b*x**2)/(185725*b) + 2321*a**4*x**16*sqrt(a + b*x**2)/37145 + 478*a**3*b*x*
*18*sqrt(a + b*x**2)/2185 + 168*a**2*b**2*x**20*sqrt(a + b*x**2)/575 + 101*
a*b**3*x**22*sqrt(a + b*x**2)/575 + b**4*x**24*sqrt(a + b*x**2)/25, Ne(b, 0
)), (a**(9/2)*x**16/16, True))
```

3.410 $\int x^{13}\left(a+bx^2\right)^{9/2}\,dx$

Optimal. Leaf size=140

$$\frac{a^6\left(a+bx^2\right)^{11/2}}{11b^7}-\frac{6a^5\left(a+bx^2\right)^{13/2}}{13b^7}+\frac{a^4\left(a+bx^2\right)^{15/2}}{b^7}-\frac{20a^3\left(a+bx^2\right)^{17/2}}{17b^7}+\frac{15a^2\left(a+bx^2\right)^{19/2}}{19b^7}+\frac{\left(a+bx^2\right)^{23/2}}{23b^7}-\frac{2\!\!}{}$$

```
[Out] 1/11*a^6*(b*x^2+a)^(11/2)/b^7-6/13*a^5*(b*x^2+a)^(13/2)/b^7+a^4*(b*x^2+a)^(
15/2)/b^7-20/17*a^3*(b*x^2+a)^(17/2)/b^7+15/19*a^2*(b*x^2+a)^(19/2)/b^7-2/7
*a*(b*x^2+a)^(21/2)/b^7+1/23*(b*x^2+a)^(23/2)/b^7
```

Rubi [A] time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{15a^2\left(a+bx^2\right)^{19/2}}{19b^7}-\frac{20a^3\left(a+bx^2\right)^{17/2}}{17b^7}+\frac{a^4\left(a+bx^2\right)^{15/2}}{b^7}-\frac{6a^5\left(a+bx^2\right)^{13/2}}{13b^7}+\frac{a^6\left(a+bx^2\right)^{11/2}}{11b^7}+\frac{\left(a+bx^2\right)^{23/2}}{23b^7}-\frac{2\!\!}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a + b*x^2)^(9/2),x]
```

```
[Out] (a^6*(a + b*x^2)^(11/2))/(11*b^7) - (6*a^5*(a + b*x^2)^(13/2))/(13*b^7) + (
a^4*(a + b*x^2)^(15/2))/b^7 - (20*a^3*(a + b*x^2)^(17/2))/(17*b^7) + (15*a^
2*(a + b*x^2)^(19/2))/(19*b^7) - (2*a*(a + b*x^2)^(21/2))/(7*b^7) + (a + b*
x^2)^(23/2)/(23*b^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{13}\left(a+bx^2\right)^{9/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^6(a+bx)^{9/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^6(a+bx)^{9/2}}{b^6}-\frac{6a^5(a+bx)^{11/2}}{b^6}+\frac{15a^4(a+bx)^{13/2}}{b^6}-\frac{20a^3(a+bx)^{15/2}}{b^6}+\right.\right.\\ &= \frac{a^6\left(a+bx^2\right)^{11/2}}{11b^7}-\frac{6a^5\left(a+bx^2\right)^{13/2}}{13b^7}+\frac{a^4\left(a+bx^2\right)^{15/2}}{b^7}-\frac{20a^3\left(a+bx^2\right)^{17/2}}{17b^7}+\frac{15a^2\left(a\right.}{}\end{aligned}$$

Mathematica [A] time = 0.04, size = 83, normalized size = 0.59

$$\frac{\left(a+bx^2\right)^{11/2}\left(1024a^6-5632a^5bx^2+18304a^4b^2x^4-45760a^3b^3x^6+97240a^2b^4x^8-184756ab^5x^{10}+323323b^6\right.}{7436429b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(1024*a^6 - 5632*a^5*b*x^2 + 18304*a^4*b^2*x^4 - 45760*
a^3*b^3*x^6 + 97240*a^2*b^4*x^8 - 184756*a*b^5*x^10 + 323323*b^6*x^12))/(74
36429*b^7)
```

fricas [A] time = 0.90, size = 134, normalized size = 0.96

$$\frac{(323323\,b^{11}x^{22} + 1431859\,ab^{10}x^{20} + 2406690\,a^2b^9x^{18} + 1826110\,a^3b^8x^{16} + 530959\,a^4b^7x^{14} + 231\,a^5b^6x^{12} - 252\,a}{7436429\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7436429*(323323*b^11*x^22 + 1431859*a*b^10*x^20 + 2406690*a^2*b^9*x^18 +
1826110*a^3*b^8*x^16 + 530959*a^4*b^7*x^14 + 231*a^5*b^6*x^12 - 252*a^6*b^5
*x^10 + 280*a^7*b^4*x^8 - 320*a^8*b^3*x^6 + 384*a^9*b^2*x^4 - 512*a^10*b*x^
2 + 1024*a^11)*sqrt(b*x^2 + a)/b^7
```

giac [A] time = 1.10, size = 99, normalized size = 0.71

$$\frac{323323\left(bx^2+a\right)^{\frac{23}{2}} - 2124694\left(bx^2+a\right)^{\frac{21}{2}}a + 5870865\left(bx^2+a\right)^{\frac{19}{2}}a^2 - 8748740\left(bx^2+a\right)^{\frac{17}{2}}a^3 + 7436429\left(bx^2+\right.}{7436429\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/7436429*(323323*(b*x^2 + a)^(23/2) - 2124694*(b*x^2 + a)^(21/2)*a + 58708
65*(b*x^2 + a)^(19/2)*a^2 - 8748740*(b*x^2 + a)^(17/2)*a^3 + 7436429*(b*x^2
 + a)^(15/2)*a^4 - 3432198*(b*x^2 + a)^(13/2)*a^5 + 676039*(b*x^2 + a)^(11/
2)*a^6)/b^7
```

maple [A] time = 0.01, size = 80, normalized size = 0.57

$$\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}\left(323323x^{12}b^6 - 184756a\,x^{10}b^5 + 97240a^2x^8b^4 - 45760a^3x^6b^3 + 18304a^4x^4b^2 - 5632a^5x^2b + 1024a^6\right.}{7436429b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7436429*(b*x^2+a)^(11/2)*(323323*b^6*x^12-184756*a*b^5*x^10+97240*a^2*b^4
*x^8-45760*a^3*b^3*x^6+18304*a^4*b^2*x^4-5632*a^5*b*x^2+1024*a^6)/b^7
```

maxima [A] time = 1.45, size = 133, normalized size = 0.95

$$\frac{\left(bx^2+a\right)^{\frac{11}{2}}x^{12}}{23\,b} - \frac{4\left(bx^2+a\right)^{\frac{11}{2}}ax^{10}}{161\,b^2} + \frac{40\left(bx^2+a\right)^{\frac{11}{2}}a^2x^8}{3059\,b^3} - \frac{320\left(bx^2+a\right)^{\frac{11}{2}}a^3x^6}{52003\,b^4} + \frac{128\left(bx^2+a\right)^{\frac{11}{2}}a^4x^4}{52003\,b^5} - \frac{512\left(bx^2+\right.}{67603}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/23*(b*x^2 + a)^(11/2)*x^12/b - 4/161*(b*x^2 + a)^(11/2)*a*x^10/b^2 + 40/3
059*(b*x^2 + a)^(11/2)*a^2*x^8/b^3 - 320/52003*(b*x^2 + a)^(11/2)*a^3*x^6/b
^4 + 128/52003*(b*x^2 + a)^(11/2)*a^4*x^4/b^5 - 512/676039*(b*x^2 + a)^(11/
2)*a^5*x^2/b^6 + 1024/7436429*(b*x^2 + a)^(11/2)*a^6/b^7
```

mupad [B] time = 4.84, size = 130, normalized size = 0.93

$$\sqrt{b\,x^2+a}\,\left(\frac{1024\,a^{11}}{7436429\,b^7}+\frac{3713\,a^4\,x^{14}}{52003}+\frac{b^4\,x^{22}}{23}+\frac{12770\,a^3\,b\,x^{16}}{52003}+\frac{31\,a\,b^3\,x^{20}}{161}+\frac{3\,a^5\,x^{12}}{96577\,b}-\frac{36\,a^6\,x^{10}}{1062347\,b^2}+\frac{40}{106}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((1024*a^11)/(7436429*b^7) + (3713*a^4*x^14)/52003 + (b^4
*x^22)/23 + (12770*a^3*b*x^16)/52003 + (31*a*b^3*x^20)/161 + (3*a^5*x^12)/(
96577*b) - (36*a^6*x^10)/(1062347*b^2) + (40*a^7*x^8)/(1062347*b^3) - (320*
a^8*x^6)/(7436429*b^4) + (384*a^9*x^4)/(7436429*b^5) - (512*a^10*x^2)/(7436
429*b^6) + (990*a^2*b^2*x^18)/3059)
```

sympy [A] time = 79.37, size = 277, normalized size = 1.98

$$\begin{cases}\frac{1024a^{11}\sqrt{a+bx^2}}{7436429b^7}-\frac{512a^{10}x^2\sqrt{a+bx^2}}{7436429b^6}+\frac{384a^9x^4\sqrt{a+bx^2}}{7436429b^5}-\frac{320a^8x^6\sqrt{a+bx^2}}{7436429b^4}+\frac{40a^7x^8\sqrt{a+bx^2}}{1062347b^3}-\frac{36a^6x^{10}\sqrt{a+bx^2}}{1062347b^2}+\frac{3a^5x^{12}\sqrt{a+bx^2}}{96577b}+ \\ \frac{a^{\frac{9}{2}}x^{14}}{14}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((1024*a**11*sqrt(a + b*x**2)/(7436429*b**7) - 512*a**10*x**2*sqrt
(a + b*x**2)/(7436429*b**6) + 384*a**9*x**4*sqrt(a + b*x**2)/(7436429*b**5)
 - 320*a**8*x**6*sqrt(a + b*x**2)/(7436429*b**4) + 40*a**7*x**8*sqrt(a + b*
x**2)/(1062347*b**3) - 36*a**6*x**10*sqrt(a + b*x**2)/(1062347*b**2) + 3*a*
*5*x**12*sqrt(a + b*x**2)/(96577*b) + 3713*a**4*x**14*sqrt(a + b*x**2)/5200
3 + 12770*a**3*b*x**16*sqrt(a + b*x**2)/52003 + 990*a**2*b**2*x**18*sqrt(a
+ b*x**2)/3059 + 31*a*b**3*x**20*sqrt(a + b*x**2)/161 + b**4*x**22*sqrt(a +
 b*x**2)/23, Ne(b, 0)), (a**(9/2)*x**14/14, True))
```

3.411 $\int x^{11} \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=122

$$-\frac{a^5 \left(a+bx^2\right)^{11/2}}{11b^6}+\frac{5a^4 \left(a+bx^2\right)^{13/2}}{13b^6}-\frac{2a^3 \left(a+bx^2\right)^{15/2}}{3b^6}+\frac{10a^2 \left(a+bx^2\right)^{17/2}}{17b^6}+\frac{\left(a+bx^2\right)^{21/2}}{21b^6}-\frac{5a \left(a+bx^2\right)^{19/2}}{19b^6}$$

```
[Out] -1/11*a^5*(b*x^2+a)^(11/2)/b^6+5/13*a^4*(b*x^2+a)^(13/2)/b^6-2/3*a^3*(b*x^2
+a)^(15/2)/b^6+10/17*a^2*(b*x^2+a)^(17/2)/b^6-5/19*a*(b*x^2+a)^(19/2)/b^6+1
/21*(b*x^2+a)^(21/2)/b^6
```

Rubi [A] time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{10a^2 \left(a+bx^2\right)^{17/2}}{17b^6}-\frac{2a^3 \left(a+bx^2\right)^{15/2}}{3b^6}+\frac{5a^4 \left(a+bx^2\right)^{13/2}}{13b^6}-\frac{a^5 \left(a+bx^2\right)^{11/2}}{11b^6}+\frac{\left(a+bx^2\right)^{21/2}}{21b^6}-\frac{5a \left(a+bx^2\right)^{19/2}}{19b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^2)^(9/2),x]
```

```
[Out] -(a^5*(a + b*x^2)^(11/2))/(11*b^6) + (5*a^4*(a + b*x^2)^(13/2))/(13*b^6) -
(2*a^3*(a + b*x^2)^(15/2))/(3*b^6) + (10*a^2*(a + b*x^2)^(17/2))/(17*b^6) -
 (5*a*(a + b*x^2)^(19/2))/(19*b^6) + (a + b*x^2)^(21/2)/(21*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11} \left(a+bx^2\right)^{9/2} dx &= \frac{1}{2}\text{Subst}\left(\int x^5(a+bx)^{9/2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^5(a+bx)^{9/2}}{b^5}+\frac{5a^4(a+bx)^{11/2}}{b^5}-\frac{10a^3(a+bx)^{13/2}}{b^5}+\frac{10a^2(a+bx)^{15/2}}{b^5}-\right.\right.\\ &= -\frac{a^5 \left(a+bx^2\right)^{11/2}}{11b^6}+\frac{5a^4 \left(a+bx^2\right)^{13/2}}{13b^6}-\frac{2a^3 \left(a+bx^2\right)^{15/2}}{3b^6}+\frac{10a^2 \left(a+bx^2\right)^{17/2}}{17b^6}-\frac{5a\left(a+\right.}{19}\end{aligned}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 0.59

$$\frac{\left(a+bx^2\right)^{11/2} \left(-256a^5+1408a^4bx^2-4576a^3b^2x^4+11440a^2b^3x^6-24310ab^4x^8+46189b^5x^{10}\right)}{969969b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-256*a^5 + 1408*a^4*b*x^2 - 4576*a^3*b^2*x^4 + 11440*a
^2*b^3*x^6 - 24310*a*b^4*x^8 + 46189*b^5*x^10))/(969969*b^6)
```

fricas [A] time = 0.78, size = 123, normalized size = 1.01

$$\frac{\left(46189\, b^{10} x^{20} + 206635\, a b^{9} x^{18} + 351780\, a^{2} b^{8} x^{16} + 271414\, a^{3} b^{7} x^{14} + 80773\, a^{4} b^{6} x^{12} + 63\, a^{5} b^{5} x^{10} - 70\, a^{6} b^{4} x^{8}}{969969\, b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/969969*(46189*b^10*x^20 + 206635*a*b^9*x^18 + 351780*a^2*b^8*x^16 + 27141
4*a^3*b^7*x^14 + 80773*a^4*b^6*x^12 + 63*a^5*b^5*x^10 - 70*a^6*b^4*x^8 + 80
*a^7*b^3*x^6 - 96*a^8*b^2*x^4 + 128*a^9*b*x^2 - 256*a^10)*sqrt(b*x^2 + a)/b
^6
```

giac [A] time = 1.01, size = 85, normalized size = 0.70

$$\frac{46189 \left(bx^{2} + a\right)^{\frac{21}{2}} - 255255 \left(bx^{2} + a\right)^{\frac{19}{2}} a + 570570 \left(bx^{2} + a\right)^{\frac{17}{2}} a^{2} - 646646 \left(bx^{2} + a\right)^{\frac{15}{2}} a^{3} + 373065 \left(bx^{2} + a\right)}{969969\, b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/969969*(46189*(b*x^2 + a)^(21/2) - 255255*(b*x^2 + a)^(19/2)*a + 570570*(
b*x^2 + a)^(17/2)*a^2 - 646646*(b*x^2 + a)^(15/2)*a^3 + 373065*(b*x^2 + a)^
(13/2)*a^4 - 88179*(b*x^2 + a)^(11/2)*a^5)/b^6
```

maple [A] time = 0.01, size = 69, normalized size = 0.57

$$-\frac{\left(b\, x^{2} + a\right)^{\frac{11}{2}} \left(-46189 b^{5} x^{10} + 24310 a\, b^{4} x^{8} - 11440 a^{2} b^{3} x^{6} + 4576 a^{3} b^{2} x^{4} - 1408 a^{4} b\, x^{2} + 256 a^{5}\right)}{969969 b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^2+a)^(9/2),x)
```

```
[Out] -1/969969*(b*x^2+a)^(11/2)*(-46189*b^5*x^10+24310*a*b^4*x^8-11440*a^2*b^3*x
^6+4576*a^3*b^2*x^4-1408*a^4*b*x^2+256*a^5)/b^6
```

maxima [A] time = 1.48, size = 113, normalized size = 0.93

$$\frac{\left(bx^{2} + a\right)^{\frac{11}{2}} x^{10}}{21\, b} - \frac{10 \left(bx^{2} + a\right)^{\frac{11}{2}} a x^{8}}{399\, b^{2}} + \frac{80 \left(bx^{2} + a\right)^{\frac{11}{2}} a^{2} x^{6}}{6783\, b^{3}} - \frac{32 \left(bx^{2} + a\right)^{\frac{11}{2}} a^{3} x^{4}}{6783\, b^{4}} + \frac{128 \left(bx^{2} + a\right)^{\frac{11}{2}} a^{4} x^{2}}{88179\, b^{5}} - \frac{256 \left(bx^{2}\right.}{9699}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/21*(b*x^2 + a)^(11/2)*x^10/b - 10/399*(b*x^2 + a)^(11/2)*a*x^8/b^2 + 80/6
783*(b*x^2 + a)^(11/2)*a^2*x^6/b^3 - 32/6783*(b*x^2 + a)^(11/2)*a^3*x^4/b^4
 + 128/88179*(b*x^2 + a)^(11/2)*a^4*x^2/b^5 - 256/969969*(b*x^2 + a)^(11/2)
*a^5/b^6
```

mupad [B] time = 4.81, size = 119, normalized size = 0.98

$$\sqrt{b\, x^{2} + a}\, \left(\frac{1049\, a^{4}\, x^{12}}{12597} - \frac{256\, a^{10}}{969969\, b^{6}} + \frac{b^{4}\, x^{20}}{21} + \frac{1898\, a^{3}\, b\, x^{14}}{6783} + \frac{85\, a\, b^{3}\, x^{18}}{399} + \frac{3\, a^{5}\, x^{10}}{46189\, b} - \frac{10\, a^{6}\, x^{8}}{138567\, b^{2}} + \frac{80\, a^{7}\, x}{969969}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((1049*a^4*x^12)/12597 - (256*a^10)/(969969*b^6) + (b^4*x
^20)/21 + (1898*a^3*b*x^14)/6783 + (85*a*b^3*x^18)/399 + (3*a^5*x^10)/(4618
9*b) - (10*a^6*x^8)/(138567*b^2) + (80*a^7*x^6)/(969969*b^3) - (32*a^8*x^4)
/(323323*b^4) + (128*a^9*x^2)/(969969*b^5) + (820*a^2*b^2*x^16)/2261)
```

sympy [A] time = 62.85, size = 253, normalized size = 2.07

$$\begin{cases} -\frac{256a^{10}\sqrt{a+bx^2}}{969969b^6} + \frac{128a^9x^2\sqrt{a+bx^2}}{969969b^5} - \frac{32a^8x^4\sqrt{a+bx^2}}{323323b^4} + \frac{80a^7x^6\sqrt{a+bx^2}}{969969b^3} - \frac{10a^6x^8\sqrt{a+bx^2}}{138567b^2} + \frac{3a^5x^{10}\sqrt{a+bx^2}}{46189b} + \frac{1049a^4x^{12}\sqrt{a+bx^2}}{12597} + 18 \\ \frac{a^{\frac{9}{2}}x^{12}}{12} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-256*a**10*sqrt(a + b*x**2)/(969969*b**6) + 128*a**9*x**2*sqrt(a
 + b*x**2)/(969969*b**5) - 32*a**8*x**4*sqrt(a + b*x**2)/(323323*b**4) + 80
*a**7*x**6*sqrt(a + b*x**2)/(969969*b**3) - 10*a**6*x**8*sqrt(a + b*x**2)/(
138567*b**2) + 3*a**5*x**10*sqrt(a + b*x**2)/(46189*b) + 1049*a**4*x**12*sq
rt(a + b*x**2)/12597 + 1898*a**3*b*x**14*sqrt(a + b*x**2)/6783 + 820*a**2*b
**2*x**16*sqrt(a + b*x**2)/2261 + 85*a*b**3*x**18*sqrt(a + b*x**2)/399 + b*
*4*x**20*sqrt(a + b*x**2)/21, Ne(b, 0)), (a**(9/2)*x**12/12, True))
```

3.412 $\int x^9 \left(a+bx^2\right)^{9/2}\,dx$

Optimal. Leaf size=101

$$\frac{a^4\left(a+bx^2\right)^{11/2}}{11b^5}-\frac{4a^3\left(a+bx^2\right)^{13/2}}{13b^5}+\frac{2a^2\left(a+bx^2\right)^{15/2}}{5b^5}+\frac{\left(a+bx^2\right)^{19/2}}{19b^5}-\frac{4a\left(a+bx^2\right)^{17/2}}{17b^5}$$

```
[Out] 1/11*a^4*(b*x^2+a)^(11/2)/b^5-4/13*a^3*(b*x^2+a)^(13/2)/b^5+2/5*a^2*(b*x^2+
a)^(15/2)/b^5-4/17*a*(b*x^2+a)^(17/2)/b^5+1/19*(b*x^2+a)^(19/2)/b^5
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{2a^2\left(a+bx^2\right)^{15/2}}{5b^5}-\frac{4a^3\left(a+bx^2\right)^{13/2}}{13b^5}+\frac{a^4\left(a+bx^2\right)^{11/2}}{11b^5}+\frac{\left(a+bx^2\right)^{19/2}}{19b^5}-\frac{4a\left(a+bx^2\right)^{17/2}}{17b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^(9/2),x]
```

```
[Out] (a^4*(a + b*x^2)^(11/2))/(11*b^5) - (4*a^3*(a + b*x^2)^(13/2))/(13*b^5) + (
2*a^2*(a + b*x^2)^(15/2))/(5*b^5) - (4*a*(a + b*x^2)^(17/2))/(17*b^5) + (a
+ b*x^2)^(19/2)/(19*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^2\right)^{9/2}\,dx&=\frac{1}{2}\text{Subst}\left(\int x^4(a+bx)^{9/2}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{a^4(a+bx)^{9/2}}{b^4}-\frac{4a^3(a+bx)^{11/2}}{b^4}+\frac{6a^2(a+bx)^{13/2}}{b^4}-\frac{4a(a+bx)^{15/2}}{b^4}+\frac{(a\right.\right.\\&=\frac{a^4\left(a+bx^2\right)^{11/2}}{11b^5}-\frac{4a^3\left(a+bx^2\right)^{13/2}}{13b^5}+\frac{2a^2\left(a+bx^2\right)^{15/2}}{5b^5}-\frac{4a\left(a+bx^2\right)^{17/2}}{17b^5}+\frac{\left(a+bx^2\right)}{19b^5}\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.60

$$\frac{\left(a+bx^2\right)^{11/2}\left(128a^4-704a^3bx^2+2288a^2b^2x^4-5720ab^3x^6+12155b^4x^8\right)}{230945b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(128*a^4 - 704*a^3*b*x^2 + 2288*a^2*b^2*x^4 - 5720*a*b^
3*x^6 + 12155*b^4*x^8))/(230945*b^5)
```

fricas [A] time = 0.89, size = 112, normalized size = 1.11

$$\frac{\left(12155\,b^{9}x^{18}+55055\,ab^{8}x^{16}+95238\,a^{2}b^{7}x^{14}+75086\,a^{3}b^{6}x^{12}+23063\,a^{4}b^{5}x^{10}+35\,a^{5}b^{4}x^{8}-40\,a^{6}b^{3}x^{6}+48\,a^{7}b\right.}{230945\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/230945*(12155*b^9*x^18 + 55055*a*b^8*x^16 + 95238*a^2*b^7*x^14 + 75086*a^
3*b^6*x^12 + 23063*a^4*b^5*x^10 + 35*a^5*b^4*x^8 - 40*a^6*b^3*x^6 + 48*a^7*
b^2*x^4 - 64*a^8*b*x^2 + 128*a^9)*sqrt(b*x^2 + a)/b^5
```

giac [A] time = 1.08, size = 71, normalized size = 0.70

$$\frac{12155\left(bx^{2}+a\right)^{\frac{19}{2}}-54340\left(bx^{2}+a\right)^{\frac{17}{2}}a+92378\left(bx^{2}+a\right)^{\frac{15}{2}}a^{2}-71060\left(bx^{2}+a\right)^{\frac{13}{2}}a^{3}+20995\left(bx^{2}+a\right)^{\frac{11}{2}}a^{4}}{230945\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/230945*(12155*(b*x^2 + a)^(19/2) - 54340*(b*x^2 + a)^(17/2)*a + 92378*(b*
x^2 + a)^(15/2)*a^2 - 71060*(b*x^2 + a)^(13/2)*a^3 + 20995*(b*x^2 + a)^(11/
2)*a^4)/b^5
```

maple [A] time = 0.01, size = 58, normalized size = 0.57

$$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(12155b^{4}x^{8}-5720a\,b^{3}x^{6}+2288a^{2}b^{2}x^{4}-704a^{3}b\,x^{2}+128a^{4}\right)}{230945b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/230945*(b*x^2+a)^(11/2)*(12155*b^4*x^8-5720*a*b^3*x^6+2288*a^2*b^2*x^4-70
4*a^3*b*x^2+128*a^4)/b^5
```

maxima [A] time = 1.43, size = 93, normalized size = 0.92

$$\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}x^{8}}{19\,b}-\frac{8\left(bx^{2}+a\right)^{\frac{11}{2}}ax^{6}}{323\,b^{2}}+\frac{16\left(bx^{2}+a\right)^{\frac{11}{2}}a^{2}x^{4}}{1615\,b^{3}}-\frac{64\left(bx^{2}+a\right)^{\frac{11}{2}}a^{3}x^{2}}{20995\,b^{4}}+\frac{128\left(bx^{2}+a\right)^{\frac{11}{2}}a^{4}}{230945\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/19*(b*x^2 + a)^(11/2)*x^8/b - 8/323*(b*x^2 + a)^(11/2)*a*x^6/b^2 + 16/161
5*(b*x^2 + a)^(11/2)*a^2*x^4/b^3 - 64/20995*(b*x^2 + a)^(11/2)*a^3*x^2/b^4
+ 128/230945*(b*x^2 + a)^(11/2)*a^4/b^5
```

mupad [B] time = 4.74, size = 108, normalized size = 1.07

$$\sqrt{b\,x^{2}+a}\,\left(\frac{128\,a^{9}}{230945\,b^{5}}+\frac{23063\,a^{4}\,x^{10}}{230945}+\frac{b^{4}\,x^{18}}{19}+\frac{6826\,a^{3}\,b\,x^{12}}{20995}+\frac{77\,a\,b^{3}\,x^{16}}{323}+\frac{7\,a^{5}\,x^{8}}{46189\,b}-\frac{8\,a^{6}\,x^{6}}{46189\,b^{2}}+\frac{48\,a^{7}\,x^{4}}{230945\,b^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((128*a^9)/(230945*b^5) + (23063*a^4*x^10)/230945 + (b^4*
x^18)/19 + (6826*a^3*b*x^12)/20995 + (77*a*b^3*x^16)/323 + (7*a^5*x^8)/(461
89*b) - (8*a^6*x^6)/(46189*b^2) + (48*a^7*x^4)/(230945*b^3) - (64*a^8*x^2)/
(230945*b^4) + (666*a^2*b^2*x^14)/1615)
```

sympy **[A]** time = 48.28, size = 230, normalized size = 2.28

$$\begin{cases} \frac{128a^9\sqrt{a+bx^2}}{230945b^5} - \frac{64a^8x^2\sqrt{a+bx^2}}{230945b^4} + \frac{48a^7x^4\sqrt{a+bx^2}}{230945b^3} - \frac{8a^6x^6\sqrt{a+bx^2}}{46189b^2} + \frac{7a^5x^8\sqrt{a+bx^2}}{46189b} + \frac{23063a^4x^{10}\sqrt{a+bx^2}}{230945} + \frac{6826a^3bx^{12}\sqrt{a+bx^2}}{20995} + \\ \frac{a^{\frac{9}{2}}x^{10}}{10} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((128*a**9*sqrt(a + b*x**2)/(230945*b**5) - 64*a**8*x**2*sqrt(a +
b*x**2)/(230945*b**4) + 48*a**7*x**4*sqrt(a + b*x**2)/(230945*b**3) - 8*a**
6*x**6*sqrt(a + b*x**2)/(46189*b**2) + 7*a**5*x**8*sqrt(a + b*x**2)/(46189*
b) + 23063*a**4*x**10*sqrt(a + b*x**2)/230945 + 6826*a**3*b*x**12*sqrt(a +
b*x**2)/20995 + 666*a**2*b**2*x**14*sqrt(a + b*x**2)/1615 + 77*a*b**3*x**16
*sqrt(a + b*x**2)/323 + b**4*x**18*sqrt(a + b*x**2)/19, Ne(b, 0)), (a**(9/2
)*x**10/10, True))
```

3.413 $\int x^7 \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=80

$$-\frac{a^3 \left(a+bx^2\right)^{11/2}}{11b^4}+\frac{3a^2 \left(a+bx^2\right)^{13/2}}{13b^4}+\frac{\left(a+bx^2\right)^{17/2}}{17b^4}-\frac{a \left(a+bx^2\right)^{15/2}}{5b^4}$$

```
[Out] -1/11*a^3*(b*x^2+a)^(11/2)/b^4+3/13*a^2*(b*x^2+a)^(13/2)/b^4-1/5*a*(b*x^2+a
)^(15/2)/b^4+1/17*(b*x^2+a)^(17/2)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2 \left(a+bx^2\right)^{13/2}}{13b^4}-\frac{a^3 \left(a+bx^2\right)^{11/2}}{11b^4}+\frac{\left(a+bx^2\right)^{17/2}}{17b^4}-\frac{a \left(a+bx^2\right)^{15/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(9/2),x]
```

```
[Out] -(a^3*(a + b*x^2)^(11/2))/(11*b^4) + (3*a^2*(a + b*x^2)^(13/2))/(13*b^4) -
(a*(a + b*x^2)^(15/2))/(5*b^4) + (a + b*x^2)^(17/2)/(17*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a+bx^2\right)^{9/2} dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{9/2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3(a+bx)^{9/2}}{b^3}+\frac{3a^2(a+bx)^{11/2}}{b^3}-\frac{3a(a+bx)^{13/2}}{b^3}+\frac{(a+bx)^{15/2}}{b^3}\right) dx, x, x^2\right)\\
&= -\frac{a^3 \left(a+bx^2\right)^{11/2}}{11b^4}+\frac{3a^2 \left(a+bx^2\right)^{13/2}}{13b^4}-\frac{a \left(a+bx^2\right)^{15/2}}{5b^4}+\frac{\left(a+bx^2\right)^{17/2}}{17b^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^{11/2} \left(-16a^3+88a^2bx^2-286ab^2x^4+715b^3x^6\right)}{12155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-16*a^3 + 88*a^2*b*x^2 - 286*a*b^2*x^4 + 715*b^3*x^6))
/(12155*b^4)
```

fricas [A] time = 1.22, size = 101, normalized size = 1.26

$$\frac{\left(715\,b^{8}x^{16} + 3289\,ab^{7}x^{14} + 5808\,a^{2}b^{6}x^{12} + 4714\,a^{3}b^{5}x^{10} + 1515\,a^{4}b^{4}x^{8} + 5\,a^{5}b^{3}x^{6} - 6\,a^{6}b^{2}x^{4} + 8\,a^{7}bx^{2} - 16\,a}{12155\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/12155*(715*b^8*x^16 + 3289*a*b^7*x^14 + 5808*a^2*b^6*x^12 + 4714*a^3*b^5*
x^10 + 1515*a^4*b^4*x^8 + 5*a^5*b^3*x^6 - 6*a^6*b^2*x^4 + 8*a^7*b*x^2 - 16*
a^8)*sqrt(b*x^2 + a)/b^4
```

giac [A] time = 1.15, size = 57, normalized size = 0.71

$$\frac{715\left(bx^{2}+a\right)^{\frac{17}{2}} - 2431\left(bx^{2}+a\right)^{\frac{15}{2}}a + 2805\left(bx^{2}+a\right)^{\frac{13}{2}}a^{2} - 1105\left(bx^{2}+a\right)^{\frac{11}{2}}a^{3}}{12155\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/12155*(715*(b*x^2 + a)^(17/2) - 2431*(b*x^2 + a)^(15/2)*a + 2805*(b*x^2 +
 a)^(13/2)*a^2 - 1105*(b*x^2 + a)^(11/2)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-715b^{3}x^{6}+286a\,b^{2}x^{4}-88a^{2}b\,x^{2}+16a^{3}\right)}{12155b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(9/2),x)
```

```
[Out] -1/12155*(b*x^2+a)^(11/2)*(-715*b^3*x^6+286*a*b^2*x^4-88*a^2*b*x^2+16*a^3)/
b^4
```

maxima [A] time = 1.42, size = 73, normalized size = 0.91

$$\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}x^{6}}{17\,b} - \frac{2\left(bx^{2}+a\right)^{\frac{11}{2}}ax^{4}}{85\,b^{2}} + \frac{8\left(bx^{2}+a\right)^{\frac{11}{2}}a^{2}x^{2}}{1105\,b^{3}} - \frac{16\left(bx^{2}+a\right)^{\frac{11}{2}}a^{3}}{12155\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/17*(b*x^2 + a)^(11/2)*x^6/b - 2/85*(b*x^2 + a)^(11/2)*a*x^4/b^2 + 8/1105*
(b*x^2 + a)^(11/2)*a^2*x^2/b^3 - 16/12155*(b*x^2 + a)^(11/2)*a^3/b^4
```

mupad [B] time = 4.73, size = 97, normalized size = 1.21

$$\sqrt{b\,x^{2}+a}\,\left(\frac{303\,a^{4}\,x^{8}}{2431} - \frac{16\,a^{8}}{12155\,b^{4}} + \frac{b^{4}\,x^{16}}{17} + \frac{4714\,a^{3}\,b\,x^{10}}{12155} + \frac{23\,a\,b^{3}\,x^{14}}{85} + \frac{a^{5}\,x^{6}}{2431\,b} - \frac{6\,a^{6}\,x^{4}}{12155\,b^{2}} + \frac{8\,a^{7}\,x^{2}}{12155\,b^{3}} + \frac{52}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((303*a^4*x^8)/2431 - (16*a^8)/(12155*b^4) + (b^4*x^16)/1
7 + (4714*a^3*b*x^10)/12155 + (23*a*b^3*x^14)/85 + (a^5*x^6)/(2431*b) - (6*
a^6*x^4)/(12155*b^2) + (8*a^7*x^2)/(12155*b^3) + (528*a^2*b^2*x^12)/1105)
```

sympy [A] time = 37.20, size = 204, normalized size = 2.55

$$\begin{cases} -\frac{16a^8\sqrt{a+bx^2}}{12155b^4} + \frac{8a^7x^2\sqrt{a+bx^2}}{12155b^3} - \frac{6a^6x^4\sqrt{a+bx^2}}{12155b^2} + \frac{a^5x^6\sqrt{a+bx^2}}{2431b} + \frac{303a^4x^8\sqrt{a+bx^2}}{2431} + \frac{4714a^3bx^{10}\sqrt{a+bx^2}}{12155} + \frac{528a^2b^2x^{12}\sqrt{a+bx^2}}{1105} + \frac{23a \\ \frac{a^{\frac{9}{2}}x^8}{8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-16*a**8*sqrt(a + b*x**2)/(12155*b**4) + 8*a**7*x**2*sqrt(a + b*
x**2)/(12155*b**3) - 6*a**6*x**4*sqrt(a + b*x**2)/(12155*b**2) + a**5*x**6*
sqrt(a + b*x**2)/(2431*b) + 303*a**4*x**8*sqrt(a + b*x**2)/2431 + 4714*a**3
*b*x**10*sqrt(a + b*x**2)/12155 + 528*a**2*b**2*x**12*sqrt(a + b*x**2)/1105
 + 23*a*b**3*x**14*sqrt(a + b*x**2)/85 + b**4*x**16*sqrt(a + b*x**2)/17, Ne
(b, 0)), (a**(9/2)*x**8/8, True))
```

3.414 $\int x^5 \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^2\right)^{11/2}}{11b^3}+\frac{\left(a+bx^2\right)^{15/2}}{15b^3}-\frac{2a\left(a+bx^2\right)^{13/2}}{13b^3}$$

```
[Out] 1/11*a^2*(b*x^2+a)^(11/2)/b^3-2/13*a*(b*x^2+a)^(13/2)/b^3+1/15*(b*x^2+a)^(1
5/2)/b^3
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^{11/2}}{11b^3}+\frac{\left(a+bx^2\right)^{15/2}}{15b^3}-\frac{2a\left(a+bx^2\right)^{13/2}}{13b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(9/2),x]
```

```
[Out] (a^2*(a + b*x^2)^(11/2))/(11*b^3) - (2*a*(a + b*x^2)^(13/2))/(13*b^3) + (a
+ b*x^2)^(15/2)/(15*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^2\right)^{9/2} dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^{9/2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2(a+bx)^{9/2}}{b^2}-\frac{2a(a+bx)^{11/2}}{b^2}+\frac{(a+bx)^{13/2}}{b^2}\right)dx,x,x^2\right)\\
&= \frac{a^2\left(a+bx^2\right)^{11/2}}{11b^3}-\frac{2a\left(a+bx^2\right)^{13/2}}{13b^3}+\frac{\left(a+bx^2\right)^{15/2}}{15b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^2\right)^{11/2}\left(8a^2-44abx^2+143b^2x^4\right)}{2145b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(8*a^2 - 44*a*b*x^2 + 143*b^2*x^4))/(2145*b^3)
```

fricas [A] time = 0.87, size = 90, normalized size = 1.53

$$\frac{\left(143\,b^{7}x^{14}+671\,ab^{6}x^{12}+1218\,a^{2}b^{5}x^{10}+1030\,a^{3}b^{4}x^{8}+355\,a^{4}b^{3}x^{6}+3\,a^{5}b^{2}x^{4}-4\,a^{6}bx^{2}+8\,a^{7}\right)\sqrt{bx^{2}+a}}{2145\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/2145*(143*b^7*x^14 + 671*a*b^6*x^12 + 1218*a^2*b^5*x^10 + 1030*a^3*b^4*x^
8 + 355*a^4*b^3*x^6 + 3*a^5*b^2*x^4 - 4*a^6*b*x^2 + 8*a^7)*sqrt(b*x^2 + a)/
b^3
```

giac [A] time = 0.96, size = 43, normalized size = 0.73

$$\frac{143\,\left(bx^{2}+a\right)^{\frac{15}{2}}-330\,\left(bx^{2}+a\right)^{\frac{13}{2}}a+195\,\left(bx^{2}+a\right)^{\frac{11}{2}}a^{2}}{2145\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/2145*(143*(b*x^2 + a)^(15/2) - 330*(b*x^2 + a)^(13/2)*a + 195*(b*x^2 + a)
^(11/2)*a^2)/b^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(143b^{2}x^{4}-44ab\,x^{2}+8a^{2}\right)}{2145b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/2145*(b*x^2+a)^(11/2)*(143*b^2*x^4-44*a*b*x^2+8*a^2)/b^3
```

maxima [A] time = 1.39, size = 53, normalized size = 0.90

$$\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}x^{4}}{15\,b}-\frac{4\left(bx^{2}+a\right)^{\frac{11}{2}}ax^{2}}{195\,b^{2}}+\frac{8\left(bx^{2}+a\right)^{\frac{11}{2}}a^{2}}{2145\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/15*(b*x^2 + a)^(11/2)*x^4/b - 4/195*(b*x^2 + a)^(11/2)*a*x^2/b^2 + 8/2145
*(b*x^2 + a)^(11/2)*a^2/b^3
```

mupad [B] time = 4.60, size = 86, normalized size = 1.46

$$\sqrt{b\,x^{2}+a}\,\left(\frac{8\,a^{7}}{2145\,b^{3}}+\frac{71\,a^{4}\,x^{6}}{429}+\frac{b^{4}\,x^{14}}{15}+\frac{206\,a^{3}\,b\,x^{8}}{429}+\frac{61\,a\,b^{3}\,x^{12}}{195}+\frac{a^{5}\,x^{4}}{715\,b}-\frac{4\,a^{6}\,x^{2}}{2145\,b^{2}}+\frac{406\,a^{2}\,b^{2}\,x^{10}}{715}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a^7)/(2145*b^3) + (71*a^4*x^6)/429 + (b^4*x^14)/15 +
(206*a^3*b*x^8)/429 + (61*a*b^3*x^12)/195 + (a^5*x^4)/(715*b) - (4*a^6*x^2)
/(2145*b^2) + (406*a^2*b^2*x^10)/715)
```

sympy [A] time = 27.02, size = 180, normalized size = 3.05

$$\begin{cases} \frac{8a^7\sqrt{a+bx^2}}{2145b^3} - \frac{4a^6x^2\sqrt{a+bx^2}}{2145b^2} + \frac{a^5x^4\sqrt{a+bx^2}}{715b} + \frac{71a^4x^6\sqrt{a+bx^2}}{429} + \frac{206a^3bx^8\sqrt{a+bx^2}}{429} + \frac{406a^2b^2x^{10}\sqrt{a+bx^2}}{715} + \frac{61ab^3x^{12}\sqrt{a+bx^2}}{195} + b^4 \\ \frac{a^{\frac{9}{2}}x^6}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((8*a**7*sqrt(a + b*x**2)/(2145*b**3) - 4*a**6*x**2*sqrt(a + b*x**
2)/(2145*b**2) + a**5*x**4*sqrt(a + b*x**2)/(715*b) + 71*a**4*x**6*sqrt(a +
 b*x**2)/429 + 206*a**3*b*x**8*sqrt(a + b*x**2)/429 + 406*a**2*b**2*x**10*s
qrt(a + b*x**2)/715 + 61*a*b**3*x**12*sqrt(a + b*x**2)/195 + b**4*x**14*sqr
t(a + b*x**2)/15, Ne(b, 0)), (a**(9/2)*x**6/6, True))
```

3.415 $\int x^3\left(a+bx^2\right)^{9/2}\,dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^2\right)^{13/2}}{13b^2}-\frac{a\left(a+bx^2\right)^{11/2}}{11b^2}$$

```
[Out] -1/11*a*(b*x^2+a)^(11/2)/b^2+1/13*(b*x^2+a)^(13/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{13/2}}{13b^2}-\frac{a\left(a+bx^2\right)^{11/2}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(9/2),x]
```

```
[Out] -(a*(a + b*x^2)^(11/2))/(11*b^2) + (a + b*x^2)^(13/2)/(13*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+bx^2\right)^{9/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^{9/2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^{9/2}}{b}+\frac{(a+bx)^{11/2}}{b}\right)dx,x,x^2\right)\\
&= -\frac{a\left(a+bx^2\right)^{11/2}}{11b^2}+\frac{\left(a+bx^2\right)^{13/2}}{13b^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{11/2}\left(11bx^2-2a\right)}{143b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-2*a + 11*b*x^2))/(143*b^2)
```

fricas [B] time = 0.82, size = 78, normalized size = 2.05

$$\frac{\left(11\,b^6x^{12}+53\,ab^5x^{10}+100\,a^2b^4x^8+90\,a^3b^3x^6+35\,a^4b^2x^4+a^5bx^2-2\,a^6\right)\sqrt{bx^2+a}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/143*(11*b^6*x^12 + 53*a*b^5*x^10 + 100*a^2*b^4*x^8 + 90*a^3*b^3*x^6 + 35*
a^4*b^2*x^4 + a^5*b*x^2 - 2*a^6)*sqrt(b*x^2 + a)/b^2
```

giac [A] time = 1.06, size = 29, normalized size = 0.76

$$\frac{11\left(bx^2+a\right)^{\frac{13}{2}}-13\left(bx^2+a\right)^{\frac{11}{2}}a}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/143*(11*(b*x^2 + a)^(13/2) - 13*(b*x^2 + a)^(11/2)*a)/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}\left(-11b\,x^2+2a\right)}{143b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(9/2),x)
```

```
[Out] -1/143*(b*x^2+a)^(11/2)*(-11*b*x^2+2*a)/b^2
```

maxima [A] time = 1.29, size = 33, normalized size = 0.87

$$\frac{\left(bx^2+a\right)^{\frac{11}{2}}x^2}{13\,b}-\frac{2\left(bx^2+a\right)^{\frac{11}{2}}a}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/13*(b*x^2 + a)^(11/2)*x^2/b - 2/143*(b*x^2 + a)^(11/2)*a/b^2
```

mupad [B] time = 4.74, size = 29, normalized size = 0.76

$$-\frac{13\,a\left(b\,x^2+a\right)^{11/2}-11\left(b\,x^2+a\right)^{13/2}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(9/2),x)
```

```
[Out] -(13*a*(a + b*x^2)^(11/2) - 11*(a + b*x^2)^(13/2))/(143*b^2)
```

sympy [A] time = 18.50, size = 156, normalized size = 4.11

$$\begin{cases} -\frac{2a^6\sqrt{a+bx^2}}{143b^2}+\frac{a^5x^2\sqrt{a+bx^2}}{143b}+\frac{35a^4x^4\sqrt{a+bx^2}}{143}+\frac{90a^3bx^6\sqrt{a+bx^2}}{143}+\frac{100a^2b^2x^8\sqrt{a+bx^2}}{143}+\frac{53ab^3x^{10}\sqrt{a+bx^2}}{143}+\frac{b^4x^{12}\sqrt{a+bx^2}}{13} & \text{for} \\ \frac{a^{\frac{9}{2}}x^4}{4} & \text{otl} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-2*a**6*sqrt(a + b*x**2)/(143*b**2) + a**5*x**2*sqrt(a + b*x**2)
/(143*b) + 35*a**4*x**4*sqrt(a + b*x**2)/143 + 90*a**3*b*x**6*sqrt(a + b*x*
*2)/143 + 100*a**2*b**2*x**8*sqrt(a + b*x**2)/143 + 53*a*b**3*x**10*sqrt(a
+ b*x**2)/143 + b**4*x**12*sqrt(a + b*x**2)/13, Ne(b, 0)), (a**(9/2)*x**4/4
, True))
```

3.416 $\int x\left(a+bx^2\right)^{9/2}\,dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

```
[Out] 1/11*(b*x^2+a)^(11/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(9/2),x]
```

```
[Out] (a + b*x^2)^(11/2)/(11*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{9/2}\,dx=\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(9/2),x]
```

```
[Out] (a + b*x^2)^(11/2)/(11*b)
```

fricas [B] time = 0.71, size = 65, normalized size = 3.61

$$\frac{\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{bx^2+a}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/11*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^
2 + a^5)*sqrt(b*x^2 + a)/b
```

giac [A] time = 1.19, size = 14, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/11*(b*x^2 + a)^(11/2)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/11*(b*x^2+a)^(11/2)/b
```

maxima [A] time = 1.32, size = 14, normalized size = 0.78

$$\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^2 + a)^(11/2)/b
```

mupad [B] time = 4.81, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^{2}+a\right)^{11/2}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(11/2)/(11*b)
```

sympy [A] time = 12.96, size = 133, normalized size = 7.39

$$\begin{cases} \frac{a^{5}\sqrt{a+bx^{2}}}{11b} + \frac{5a^{4}x^{2}\sqrt{a+bx^{2}}}{11} + \frac{10a^{3}bx^{4}\sqrt{a+bx^{2}}}{11} + \frac{10a^{2}b^{2}x^{6}\sqrt{a+bx^{2}}}{11} + \frac{5ab^{3}x^{8}\sqrt{a+bx^{2}}}{11} + \frac{b^{4}x^{10}\sqrt{a+bx^{2}}}{11} & \text{for}\ b\neq 0 \\ \frac{a^{\frac{9}{2}}x^{2}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((a**5*sqrt(a + b*x**2)/(11*b) + 5*a**4*x**2*sqrt(a + b*x**2)/11 +
 10*a**3*b*x**4*sqrt(a + b*x**2)/11 + 10*a**2*b**2*x**6*sqrt(a + b*x**2)/11
 + 5*a*b**3*x**8*sqrt(a + b*x**2)/11 + b**4*x**10*sqrt(a + b*x**2)/11, Ne(b
, 0)), (a**(9/2)*x**2/2, True))
```

3.417 $\int \frac{(a+bx^2)^{9/2}}{x} dx$

Optimal. Leaf size=108

$$a^{9/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+a^4\sqrt{a+bx^2}+\frac{1}{3}a^3\left(a+bx^2\right)^{3/2}+\frac{1}{5}a^2\left(a+bx^2\right)^{5/2}+\frac{1}{7}a\left(a+bx^2\right)^{7/2}+\frac{1}{9}\left(a+bx^2\right)^{9/2}$$

```
[Out] 1/3*a^3*(b*x^2+a)^(3/2)+1/5*a^2*(b*x^2+a)^(5/2)+1/7*a*(b*x^2+a)^(7/2)+1/9*(
b*x^2+a)^(9/2)-a^(9/2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+a^4*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 208}

$$a^4\sqrt{a+bx^2}+\frac{1}{3}a^3\left(a+bx^2\right)^{3/2}+\frac{1}{5}a^2\left(a+bx^2\right)^{5/2}+a^{9/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{1}{7}a\left(a+bx^2\right)^{7/2}+\frac{1}{9}\left(a+bx^2\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x,x]
```

```
[Out] a^4*Sqrt[a + b*x^2] + (a^3*(a + b*x^2)^(3/2))/3 + (a^2*(a + b*x^2)^(5/2))/5
 + (a*(a + b*x^2)^(7/2))/7 + (a + b*x^2)^(9/2)/9 - a^(9/2)*ArcTanh[Sqrt[a +
 b*x^2]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x}dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x}dx, x, x^2\right)$$
$$= \frac{1}{9}(a+bx^2)^{9/2} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}dx, x, x^2\right)$$
$$= \frac{1}{7}a(a+bx^2)^{7/2} + \frac{1}{9}(a+bx^2)^{9/2} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}dx, x, x^2\right)$$
$$= \frac{1}{5}a^2(a+bx^2)^{5/2} + \frac{1}{7}a(a+bx^2)^{7/2} + \frac{1}{9}(a+bx^2)^{9/2} + \frac{1}{2}a^3\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}dx, x, x^2\right)$$
$$= \frac{1}{3}a^3(a+bx^2)^{3/2} + \frac{1}{5}a^2(a+bx^2)^{5/2} + \frac{1}{7}a(a+bx^2)^{7/2} + \frac{1}{9}(a+bx^2)^{9/2} + \frac{1}{2}a^4\,\text{Subst}\left(\int \frac{\sqrt{a}}{}\right.$$
$$= a^4\sqrt{a+bx^2} + \frac{1}{3}a^3(a+bx^2)^{3/2} + \frac{1}{5}a^2(a+bx^2)^{5/2} + \frac{1}{7}a(a+bx^2)^{7/2} + \frac{1}{9}(a+bx^2)^{9/2} + \frac{1}{2}a^5$$
$$= a^4\sqrt{a+bx^2} + \frac{1}{3}a^3(a+bx^2)^{3/2} + \frac{1}{5}a^2(a+bx^2)^{5/2} + \frac{1}{7}a(a+bx^2)^{7/2} + \frac{1}{9}(a+bx^2)^{9/2} + \frac{a^5 \text{S}}{}$$
$$= a^4\sqrt{a+bx^2} + \frac{1}{3}a^3(a+bx^2)^{3/2} + \frac{1}{5}a^2(a+bx^2)^{5/2} + \frac{1}{7}a(a+bx^2)^{7/2} + \frac{1}{9}(a+bx^2)^{9/2} - a^{9/2}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 0.78

$$\frac{1}{315}\sqrt{a+bx^2}\left(563a^4+506a^3bx^2+408a^2b^2x^4+185ab^3x^6+35b^4x^8\right)-a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(563*a^4 + 506*a^3*b*x^2 + 408*a^2*b^2*x^4 + 185*a*b^3*x^6
 + 35*b^4*x^8))/315 - a^(9/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

fricas [A] time = 0.97, size = 170, normalized size = 1.57

$$\left[\frac{1}{2}a^{\frac{9}{2}}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+\frac{1}{315}\left(35b^4x^8+185ab^3x^6+408a^2b^2x^4+506a^3bx^2+563a^4\right)\sqrt{bx^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*a^(9/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/315*(3
5*b^4*x^8 + 185*a*b^3*x^6 + 408*a^2*b^2*x^4 + 506*a^3*b*x^2 + 563*a^4)*sqrt
(b*x^2 + a), sqrt(-a)*a^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 1/315*(35*b^4*
x^8 + 185*a*b^3*x^6 + 408*a^2*b^2*x^4 + 506*a^3*b*x^2 + 563*a^4)*sqrt(b*x^2
 + a)]
```

giac [A] time = 1.17, size = 90, normalized size = 0.83

$$\frac{a^5\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{1}{9}\left(bx^2+a\right)^{\frac{9}{2}}+\frac{1}{7}\left(bx^2+a\right)^{\frac{7}{2}}a+\frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}a^2+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}a^3+\sqrt{bx^2+a}\,a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x,x, algorithm="giac")
```

```
[Out] a^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/9*(b*x^2 + a)^(9/2) + 1/7
*(b*x^2 + a)^(7/2)*a + 1/5*(b*x^2 + a)^(5/2)*a^2 + 1/3*(b*x^2 + a)^(3/2)*a^
3 + sqrt(b*x^2 + a)*a^4
```

maple [A] time = 0.00, size = 94, normalized size = 0.87

$$-a^{\frac{9}{2}} \ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)+\sqrt{b\,x^{2}+a}\,a^{4}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} a^{3}}{3}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} a^{2}}{5}+\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}} a}{7}+\frac{\left(b\,x^{2}+a\right)^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x,x)
```

```
[Out] 1/9*(b*x^2+a)^(9/2)+1/7*a*(b*x^2+a)^(7/2)+1/5*a^2*(b*x^2+a)^(5/2)+1/3*a^3*(
b*x^2+a)^(3/2)-a^(9/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+a^4*(b*x^2+a)^
(1/2)
```

maxima [A] time = 1.40, size = 82, normalized size = 0.76

$$-a^{\frac{9}{2}} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{9}\left(bx^{2}+a\right)^{\frac{9}{2}}+\frac{1}{7}\left(bx^{2}+a\right)^{\frac{7}{2}}a+\frac{1}{5}\left(bx^{2}+a\right)^{\frac{5}{2}}a^{2}+\frac{1}{3}\left(bx^{2}+a\right)^{\frac{3}{2}}a^{3}+\sqrt{bx^{2}+a}\,a^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x,x, algorithm="maxima")
```

```
[Out] -a^(9/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/9*(b*x^2 + a)^(9/2) + 1/7*(b*x^2
 + a)^(7/2)*a + 1/5*(b*x^2 + a)^(5/2)*a^2 + 1/3*(b*x^2 + a)^(3/2)*a^3 + sqr
t(b*x^2 + a)*a^4
```

mupad [B] time = 5.31, size = 87, normalized size = 0.81

$$\frac{a\left(b\,x^{2}+a\right)^{7/2}}{7}+\frac{\left(b\,x^{2}+a\right)^{9/2}}{9}+a^{4}\,\sqrt{b\,x^{2}+a}+\frac{a^{3}\left(b\,x^{2}+a\right)^{3/2}}{3}+\frac{a^{2}\left(b\,x^{2}+a\right)^{5/2}}{5}+a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^{2}+a}\,1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x,x)
```

```
[Out] a^(9/2)*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i + (a*(a + b*x^2)^(7/2))/7 +
 (a + b*x^2)^(9/2)/9 + a^4*(a + b*x^2)^(1/2) + (a^3*(a + b*x^2)^(3/2))/3 +
(a^2*(a + b*x^2)^(5/2))/5
```

sympy [A] time = 10.48, size = 160, normalized size = 1.48

$$\frac{563a^{\frac{9}{2}}\sqrt{1+\frac{bx^{2}}{a}}}{315}+\frac{a^{\frac{9}{2}}\log\left(\frac{bx^{2}}{a}\right)}{2}-a^{\frac{9}{2}}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)+\frac{506a^{\frac{7}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}{315}+\frac{136a^{\frac{5}{2}}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}}{105}+\frac{37a^{\frac{3}{2}}b^{3}x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x,x)
```

```
[Out] 563*a**(9/2)*sqrt(1 + b*x**2/a)/315 + a**(9/2)*log(b*x**2/a)/2 - a**(9/2)*l
og(sqrt(1 + b*x**2/a) + 1) + 506*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)/315 + 1
36*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a)/105 + 37*a**(3/2)*b**3*x**6*sqrt(1
 + b*x**2/a)/63 + sqrt(a)*b**4*x**8*sqrt(1 + b*x**2/a)/9
```

3.418 $\int \frac{\left(a+bx^2\right)^{9/2}}{x^3}\,dx$

Optimal. Leaf size=118

$$-\frac{9}{2}a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{9}{2}a^3b\sqrt{a+bx^2}+\frac{3}{2}a^2b\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{9/2}}{2x^2}+\frac{9}{14}b\left(a+bx^2\right)^{7/2}+\frac{9}{10}ab\left(a+bx^2\right)^{5/2}$$

```
[Out] 3/2*a^2*b*(b*x^2+a)^(3/2)+9/10*a*b*(b*x^2+a)^(5/2)+9/14*b*(b*x^2+a)^(7/2)-1
/2*(b*x^2+a)^(9/2)/x^2-9/2*a^(7/2)*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))+9/2*a
^3*b*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$\frac{3}{2}a^2b\left(a+bx^2\right)^{3/2}+\frac{9}{2}a^3b\sqrt{a+bx^2}-\frac{9}{2}a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{2x^2}+\frac{9}{14}b\left(a+bx^2\right)^{7/2}+\frac{9}{10}ab\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^3,x]
```

```
[Out] (9*a^3*b*Sqrt[a + b*x^2])/2 + (3*a^2*b*(a + b*x^2)^(3/2))/2 + (9*a*b*(a + b
*x^2)^(5/2))/10 + (9*b*(a + b*x^2)^(7/2))/14 - (a + b*x^2)^(9/2)/(2*x^2) -
(9*a^(7/2)*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{9/2}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{9/2}}{2x^2}+\frac{1}{4}(9b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}\,dx, x, x^2\right)\\
&= \frac{9}{14}b\left(a+bx^2\right)^{7/2}-\frac{\left(a+bx^2\right)^{9/2}}{2x^2}+\frac{1}{4}(9ab)\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, x^2\right)\\
&= \frac{9}{10}ab\left(a+bx^2\right)^{5/2}+\frac{9}{14}b\left(a+bx^2\right)^{7/2}-\frac{\left(a+bx^2\right)^{9/2}}{2x^2}+\frac{1}{4}\left(9a^2b\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,\right.\\
&= \frac{3}{2}a^2b\left(a+bx^2\right)^{3/2}+\frac{9}{10}ab\left(a+bx^2\right)^{5/2}+\frac{9}{14}b\left(a+bx^2\right)^{7/2}-\frac{\left(a+bx^2\right)^{9/2}}{2x^2}+\frac{1}{4}\left(9a^3b\right)\text{Subs}\\
&= \frac{9}{2}a^3b\sqrt{a+bx^2}+\frac{3}{2}a^2b\left(a+bx^2\right)^{3/2}+\frac{9}{10}ab\left(a+bx^2\right)^{5/2}+\frac{9}{14}b\left(a+bx^2\right)^{7/2}-\frac{\left(a+bx^2\right)^{9/2}}{2x^2}\\
&= \frac{9}{2}a^3b\sqrt{a+bx^2}+\frac{3}{2}a^2b\left(a+bx^2\right)^{3/2}+\frac{9}{10}ab\left(a+bx^2\right)^{5/2}+\frac{9}{14}b\left(a+bx^2\right)^{7/2}-\frac{\left(a+bx^2\right)^{9/2}}{2x^2}\\
&= \frac{9}{2}a^3b\sqrt{a+bx^2}+\frac{3}{2}a^2b\left(a+bx^2\right)^{3/2}+\frac{9}{10}ab\left(a+bx^2\right)^{5/2}+\frac{9}{14}b\left(a+bx^2\right)^{7/2}-\frac{\left(a+bx^2\right)^{9/2}}{2x^2}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.31

$$\frac{b\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(2,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^3,x]
```

```
[Out] (b*(a + b*x^2)^(11/2)*Hypergeometric2F1[2, 11/2, 13/2, 1 + (b*x^2)/a])/(11*
a^2)
```

fricas [A] time = 1.07, size = 188, normalized size = 1.59

$$\left[\frac{315\,a^{\frac{7}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(10\,b^4x^8+58\,ab^3x^6+156\,a^2b^2x^4+388\,a^3bx^2-35\,a^4\right)\sqrt{bx^2+a}}{140\,x^2},\ \frac{315}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/140*(315*a^(7/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*(10*b^4*x^8 + 58*a*b^3*x^6 + 156*a^2*b^2*x^4 + 388*a^3*b*x^2 - 35*a^
4)*sqrt(b*x^2 + a))/x^2, 1/70*(315*sqrt(-a)*a^3*b*x^2*arctan(sqrt(-a)/sqrt(
```

```
b*x^2 + a)) + (10*b^4*x^8 + 58*a*b^3*x^6 + 156*a^2*b^2*x^4 + 388*a^3*b*x^2
- 35*a^4)*sqrt(b*x^2 + a))/x^2]
```

giac [A] time = 1.14, size = 116, normalized size = 0.98

$$\frac{\frac{315\,a^4b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+10\left(bx^2+a\right)^{\frac{7}{2}}b^2+28\left(bx^2+a\right)^{\frac{5}{2}}ab^2+70\left(bx^2+a\right)^{\frac{3}{2}}a^2b^2+280\sqrt{bx^2+a}\,a^3b^2-\frac{35\sqrt{bx^2+a}\,a^4}{x^2}}{70\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/70*(315*a^4*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 10*(b*x^2 + a
)^(7/2)*b^2 + 28*(b*x^2 + a)^(5/2)*a*b^2 + 70*(b*x^2 + a)^(3/2)*a^2*b^2 + 2
80*sqrt(b*x^2 + a)*a^3*b^2 - 35*sqrt(b*x^2 + a)*a^4*b/x^2)/b
```

maple [A] time = 0.01, size = 118, normalized size = 1.00

$$-\frac{9a^{\frac{7}{2}}b\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2}+\frac{9\sqrt{b\,x^2+a}\,a^3b}{2}+\frac{3\left(b\,x^2+a\right)^{\frac{3}{2}}a^2b}{2}+\frac{9\left(b\,x^2+a\right)^{\frac{5}{2}}ab}{10}+\frac{9\left(b\,x^2+a\right)^{\frac{7}{2}}b}{14}+\frac{\left(b\,x^2+a\right)^{\frac{9}{2}}b}{2a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(11/2)+1/2/a*b*(b*x^2+a)^(9/2)+9/14*b*(b*x^2+a)^(7/2)+
9/10*a*b*(b*x^2+a)^(5/2)+3/2*a^2*b*(b*x^2+a)^(3/2)-9/2*a^(7/2)*b*ln((2*a+2*
(b*x^2+a)^(1/2)*a^(1/2))/x)+9/2*a^3*b*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.43, size = 106, normalized size = 0.90

$$-\frac{9}{2}\,a^{\frac{7}{2}}b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{9}{14}\left(bx^2+a\right)^{\frac{7}{2}}b+\frac{\left(bx^2+a\right)^{\frac{9}{2}}b}{2\,a}+\frac{9}{10}\left(bx^2+a\right)^{\frac{5}{2}}ab+\frac{3}{2}\left(bx^2+a\right)^{\frac{3}{2}}a^2b+\frac{9}{2}\sqrt{bx^2+a}\,a^3b-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="maxima")
```

```
[Out] -9/2*a^(7/2)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 9/14*(b*x^2 + a)^(7/2)*b + 1
/2*(b*x^2 + a)^(9/2)*b/a + 9/10*(b*x^2 + a)^(5/2)*a*b + 3/2*(b*x^2 + a)^(3/
2)*a^2*b + 9/2*sqrt(b*x^2 + a)*a^3*b - 1/2*(b*x^2 + a)^(11/2)/(a*x^2)
```

mupad [B] time = 5.45, size = 95, normalized size = 0.81

$$\frac{b\left(b\,x^2+a\right)^{7/2}}{7}+4\,a^3\,b\,\sqrt{b\,x^2+a}+a^2\,b\left(b\,x^2+a\right)^{3/2}-\frac{a^4\,\sqrt{b\,x^2+a}}{2\,x^2}+\frac{2\,a\,b\left(b\,x^2+a\right)^{5/2}}{5}+\frac{a^{7/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,9\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^3,x)
```

```
[Out] (b*(a + b*x^2)^(7/2))/7 + 4*a^3*b*(a + b*x^2)^(1/2) + a^2*b*(a + b*x^2)^(3/
2) - (a^4*(a + b*x^2)^(1/2))/(2*x^2) + (a^(7/2)*b*atan(((a + b*x^2)^(1/2)*1
i)/a^(1/2))*9i)/2 + (2*a*b*(a + b*x^2)^(5/2))/5
```

sympy [A] time = 9.27, size = 167, normalized size = 1.42

$$-\frac{a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}}{2x^2}+\frac{194a^{\frac{7}{2}}b\sqrt{1+\frac{bx^2}{a}}}{35}+\frac{9a^{\frac{7}{2}}b\log\left(\frac{bx^2}{a}\right)}{4}-\frac{9a^{\frac{7}{2}}b\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2}+\frac{78a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx^2}{a}}}{35}+\frac{29a^{\frac{3}{2}}b^3x^4\sqrt{1}}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**3,x)
```

```
[Out] -a**(9/2)*sqrt(1 + b*x**2/a)/(2*x**2) + 194*a**(7/2)*b*sqrt(1 + b*x**2/a)/3
5 + 9*a**(7/2)*b*log(b*x**2/a)/4 - 9*a**(7/2)*b*log(sqrt(1 + b*x**2/a) + 1)
/2 + 78*a**(5/2)*b**2*x**2*sqrt(1 + b*x**2/a)/35 + 29*a**(3/2)*b**3*x**4*sq
rt(1 + b*x**2/a)/35 + sqrt(a)*b**4*x**6*sqrt(1 + b*x**2/a)/7
```

3.419 $\int \frac{(a+bx^2)^{9/2}}{x^5} dx$

Optimal. Leaf size=126

$$-\frac{63}{8}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{63}{8}a^2b^2\sqrt{a+bx^2}+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}+\frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{(a+b}{4x}$$

```
[Out] 21/8*a*b^2*(b*x^2+a)^(3/2)+63/40*b^2*(b*x^2+a)^(5/2)-9/8*b*(b*x^2+a)^(7/2)/
x^2-1/4*(b*x^2+a)^(9/2)/x^4-63/8*a^(5/2)*b^2*arctanh((b*x^2+a)^(1/2)/a^(1/2
))+63/8*a^2*b^2*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$\frac{63}{8}a^2b^2\sqrt{a+bx^2}-\frac{63}{8}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}+\frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}-\frac{9b\left(a+bx^2\right.}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^5,x]
```

```
[Out] (63*a^2*b^2*Sqrt[a + b*x^2])/8 + (21*a*b^2*(a + b*x^2)^(3/2))/8 + (63*b^2*(
a + b*x^2)^(5/2))/40 - (9*b*(a + b*x^2)^(7/2))/(8*x^2) - (a + b*x^2)^(9/2)/
(4*x^4) - (63*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{9/2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{9/2}}{4x^4}+\frac{1}{8}(9b)\operatorname{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}+\frac{1}{16}\left(63b^2\right)\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx,x,x^2\right)\\
&= \frac{63}{40}b^2\left(a+bx^2\right)^{5/2}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}+\frac{1}{16}\left(63ab^2\right)\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx\right.\\
&= \frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}+\frac{1}{16}\left(63a^2b^2\right)\mathrm{Su}\\
&= \frac{63}{8}a^2b^2\sqrt{a+bx^2}+\frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+bx^2\right)}{4x^4}\\
&= \frac{63}{8}a^2b^2\sqrt{a+bx^2}+\frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+bx^2\right)}{4x^4}\\
&= \frac{63}{8}a^2b^2\sqrt{a+bx^2}+\frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+bx^2\right)}{4x^4}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.31

$$
-\frac{b^2\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(3,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^3}
$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(9/2)/x^5,x]

[Out] -1/11*(b^2*(a + b*x^2)^(11/2)*Hypergeometric2F1[3, 11/2, 13/2, 1 + (b*x^2)/a])/a^3

fricas [A] time = 0.89, size = 192, normalized size = 1.52

$$
\left[\frac{315\,a^{\frac{5}{2}}b^2x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(8\,b^4x^8+56\,ab^3x^6+288\,a^2b^2x^4-85\,a^3bx^2-10\,a^4\right)\sqrt{bx^2+a}}{80\,x^4},\ \frac{315}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="fricas")

[Out] [1/80*(315*a^(5/2)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(8*b^4*x^8 + 56*a*b^3*x^6 + 288*a^2*b^2*x^4 - 85*a^3*b*x^2 - 10*a^4)*sqrt(b*x^2 + a))/x^4, 1/40*(315*sqrt(-a)*a^2*b^2*x^4*arctan(sqrt(-a)/sqrt

```
(b*x^2 + a)) + (8*b^4*x^8 + 56*a*b^3*x^6 + 288*a^2*b^2*x^4 - 85*a^3*b*x^2 -
 10*a^4)*sqrt(b*x^2 + a))/x^4]
```

giac [A] time = 1.15, size = 124, normalized size = 0.98

$$\frac{\frac{315\,a^3b^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\left(bx^2+a\right)^{\frac{5}{2}}b^3+40\left(bx^2+a\right)^{\frac{3}{2}}ab^3+240\sqrt{bx^2+a}\,a^2b^3-\frac{5\left(17\left(bx^2+a\right)^{\frac{3}{2}}a^3b^3-15\sqrt{bx^2+a}\,a^4b^3\right)}{b^2x^4}}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/40*(315*a^3*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 8*(b*x^2 + a)
^(5/2)*b^3 + 40*(b*x^2 + a)^(3/2)*a*b^3 + 240*sqrt(b*x^2 + a)*a^2*b^3 - 5*(
17*(b*x^2 + a)^(3/2)*a^3*b^3 - 15*sqrt(b*x^2 + a)*a^4*b^3)/(b^2*x^4))/b
```

maple [A] time = 0.01, size = 148, normalized size = 1.17

$$-\frac{63a^{\frac{5}{2}}b^2\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{8}+\frac{63\sqrt{b\,x^2+a}\,a^2b^2}{8}+\frac{21\left(b\,x^2+a\right)^{\frac{3}{2}}a\,b^2}{8}+\frac{63\left(b\,x^2+a\right)^{\frac{5}{2}}b^2}{40}+\frac{9\left(b\,x^2+a\right)^{\frac{7}{2}}b^2}{8a}+\frac{7\left(b\,x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(11/2)-7/8/a^2*b/x^2*(b*x^2+a)^(11/2)+7/8/a^2*b^2*(b*x
^2+a)^(9/2)+9/8/a*b^2*(b*x^2+a)^(7/2)+63/40*b^2*(b*x^2+a)^(5/2)+21/8*a*b^2*
(b*x^2+a)^(3/2)-63/8*a^(5/2)*b^2*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+63/8
*a^2*b^2*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.46, size = 136, normalized size = 1.08

$$-\frac{63}{8}\,a^{\frac{5}{2}}b^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{63}{40}\left(bx^2+a\right)^{\frac{5}{2}}b^2+\frac{7\left(bx^2+a\right)^{\frac{9}{2}}b^2}{8\,a^2}+\frac{9\left(bx^2+a\right)^{\frac{7}{2}}b^2}{8\,a}+\frac{21}{8}\left(bx^2+a\right)^{\frac{3}{2}}ab^2+\frac{63}{8}\sqrt{bx^2+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="maxima")
```

```
[Out] -63/8*a^(5/2)*b^2*arcsinh(a/(sqrt(a*b)*abs(x))) + 63/40*(b*x^2 + a)^(5/2)*b
^2 + 7/8*(b*x^2 + a)^(9/2)*b^2/a^2 + 9/8*(b*x^2 + a)^(7/2)*b^2/a + 21/8*(b*
x^2 + a)^(3/2)*a*b^2 + 63/8*sqrt(b*x^2 + a)*a^2*b^2 - 7/8*(b*x^2 + a)^(11/2
)*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(11/2)/(a*x^4)
```

mupad [B] time = 5.65, size = 132, normalized size = 1.05

$$\frac{\frac{15\,a^4\,b^2\,\sqrt{b\,x^2+a}}{8}-\frac{17\,a^3\,b^2\,\left(b\,x^2+a\right)^{3/2}}{8}}{\left(b\,x^2+a\right)^2-2\,a\,\left(b\,x^2+a\right)+a^2}+\frac{b^2\,\left(b\,x^2+a\right)^{5/2}}{5}+a\,b^2\,\left(b\,x^2+a\right)^{3/2}+6\,a^2\,b^2\,\sqrt{b\,x^2+a}+\frac{a^{5/2}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,6}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^5,x)
```

```
[Out] ((15*a^4*b^2*(a + b*x^2)^(1/2))/8 - (17*a^3*b^2*(a + b*x^2)^(3/2))/8)/((a +
 b*x^2)^2 - 2*a*(a + b*x^2) + a^2) + (b^2*(a + b*x^2)^(5/2))/5 + (a^(5/2)*b
^2*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*63i)/8 + a*b^2*(a + b*x^2)^(3/2) +
6*a^2*b^2*(a + b*x^2)^(1/2)
```

sympy [A] time = 8.73, size = 175, normalized size = 1.39

$$-\frac{63a^{\frac{5}{2}}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8}-\frac{a^5}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{19a^4\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{203a^3b^{\frac{3}{2}}}{40x\sqrt{\frac{a}{bx^2}+1}}+\frac{43a^2b^{\frac{5}{2}}x}{5\sqrt{\frac{a}{bx^2}+1}}+\frac{8ab^{\frac{7}{2}}x^3}{5\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{9}{2}}x^5}{5\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**5,x)
```

```
[Out] -63*a**(5/2)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - a**5/(4*sqrt(b)*x**5*sqrt(
a/(b*x**2) + 1)) - 19*a**4*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) + 203*a**3
*b**(3/2)/(40*x*sqrt(a/(b*x**2) + 1)) + 43*a**2*b**(5/2)*x/(5*sqrt(a/(b*x**
2) + 1)) + 8*a*b**(7/2)*x**3/(5*sqrt(a/(b*x**2) + 1)) + b**(9/2)*x**5/(5*sq
rt(a/(b*x**2) + 1))
```

3.420 $\int \frac{(a+bx^2)^{9/2}}{x^7}\,dx$

Optimal. Leaf size=126

$$-\frac{105}{16}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{35}{16}b^3\left(a+bx^2\right)^{3/2}+\frac{105}{16}ab^3\sqrt{a+bx^2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}-\frac{3b\left(a+b\right.}{8x^4}$$

```
[Out] 35/16*b^3*(b*x^2+a)^(3/2)-21/16*b^2*(b*x^2+a)^(5/2)/x^2-3/8*b*(b*x^2+a)^(7/
2)/x^4-1/6*(b*x^2+a)^(9/2)/x^6-105/16*a^(3/2)*b^3*arctanh((b*x^2+a)^(1/2)/a
^(1/2))+105/16*a*b^3*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$-\frac{105}{16}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}+\frac{35}{16}b^3\left(a+bx^2\right)^{3/2}+\frac{105}{16}ab^3\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}-\frac{3b\left(a+b\right.}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^7,x]
```

```
[Out] (105*a*b^3*Sqrt[a + b*x^2])/16 + (35*b^3*(a + b*x^2)^(3/2))/16 - (21*b^2*(a
 + b*x^2)^(5/2))/(16*x^2) - (3*b*(a + b*x^2)^(7/2))/(8*x^4) - (a + b*x^2)^(
9/2)/(6*x^6) - (105*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/16
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{9/2}}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{9/2}}{6x^6}+\frac{1}{4}(3b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{3b\left(a+bx^2\right)^{7/2}}{8x^4}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}+\frac{1}{16}\left(21b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}-\frac{3b\left(a+bx^2\right)^{7/2}}{8x^4}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}+\frac{1}{32}\left(105b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,d\right.\\
&= \frac{35}{16}b^3\left(a+bx^2\right)^{3/2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}-\frac{3b\left(a+bx^2\right)^{7/2}}{8x^4}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}+\frac{1}{32}\left(105ab^3\right)\text{Sul}\\
&= \frac{105}{16}ab^3\sqrt{a+bx^2}+\frac{35}{16}b^3\left(a+bx^2\right)^{3/2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}-\frac{3b\left(a+bx^2\right)^{7/2}}{8x^4}-\frac{\left(a+bx^2\right)^{9/}}{6x^6}\\
&= \frac{105}{16}ab^3\sqrt{a+bx^2}+\frac{35}{16}b^3\left(a+bx^2\right)^{3/2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}-\frac{3b\left(a+bx^2\right)^{7/2}}{8x^4}-\frac{\left(a+bx^2\right)^{9/}}{6x^6}\\
&= \frac{105}{16}ab^3\sqrt{a+bx^2}+\frac{35}{16}b^3\left(a+bx^2\right)^{3/2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}-\frac{3b\left(a+bx^2\right)^{7/2}}{8x^4}-\frac{\left(a+bx^2\right)^{9/}}{6x^6}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.31

$$
\frac{b^3\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(4,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^7,x]
```

```
[Out] (b^3*(a + b*x^2)^(11/2)*Hypergeometric2F1[4, 11/2, 13/2, 1 + (b*x^2)/a])/(1
1*a^4)
```

fricas [A] time = 1.06, size = 190, normalized size = 1.51

$$
\left[\frac{315\,a^{\frac{3}{2}}b^3x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(16\,b^4x^8+208\,ab^3x^6-165\,a^2b^2x^4-50\,a^3bx^2-8\,a^4\right)\sqrt{bx^2+a}}{96\,x^6},\frac{315}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(315*a^(3/2)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) + 2*(16*b^4*x^8 + 208*a*b^3*x^6 - 165*a^2*b^2*x^4 - 50*a^3*b*x^2 - 8*a^
4)*sqrt(b*x^2 + a))/x^6, 1/48*(315*sqrt(-a)*a*b^3*x^6*arctan(sqrt(-a)/sqrt(
```

```
b*x^2 + a)) + (16*b^4*x^8 + 208*a*b^3*x^6 - 165*a^2*b^2*x^4 - 50*a^3*b*x^2
- 8*a^4)*sqrt(b*x^2 + a))/x^6]
```

giac [A] time = 1.10, size = 124, normalized size = 0.98

$$\frac{\frac{315\,a^2b^4\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+16\left(bx^2+a\right)^{\frac{3}{2}}b^4+192\,\sqrt{bx^2+a}\,ab^4-\frac{165\left(bx^2+a\right)^{\frac{5}{2}}a^2b^4-280\left(bx^2+a\right)^{\frac{3}{2}}a^3b^4+123\,\sqrt{bx^2+a}\,a^4b^4}{b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(315*a^2*b^4*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 16*(b*x^2 + a
)^(3/2)*b^4 + 192*sqrt(b*x^2 + a)*a*b^4 - (165*(b*x^2 + a)^(5/2)*a^2*b^4 -
280*(b*x^2 + a)^(3/2)*a^3*b^4 + 123*sqrt(b*x^2 + a)*a^4*b^4)/(b^3*x^6))/b
```

maple [A] time = 0.01, size = 168, normalized size = 1.33

$$-\frac{105a^{\frac{3}{2}}b^3\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{16}+\frac{105\sqrt{b\,x^2+a}\,\,a\,b^3}{16}+\frac{35\left(b\,x^2+a\right)^{\frac{3}{2}}b^3}{16}+\frac{21\left(b\,x^2+a\right)^{\frac{5}{2}}b^3}{16a}+\frac{15\left(b\,x^2+a\right)^{\frac{7}{2}}b^3}{16a^2}+\frac{35\,(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^7,x)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(11/2)-5/24/a^2*b/x^4*(b*x^2+a)^(11/2)-35/48/a^3*b^2/x
^2*(b*x^2+a)^(11/2)+35/48/a^3*b^3*(b*x^2+a)^(9/2)+15/16/a^2*b^3*(b*x^2+a)^(
7/2)+21/16/a*b^3*(b*x^2+a)^(5/2)+35/16*b^3*(b*x^2+a)^(3/2)-105/16*a^(3/2)*b
^3*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+105/16*a*b^3*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.47, size = 156, normalized size = 1.24

$$-\frac{105}{16}\,a^{\frac{3}{2}}b^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{35}{16}\left(bx^2+a\right)^{\frac{3}{2}}b^3+\frac{35\left(bx^2+a\right)^{\frac{9}{2}}b^3}{48\,a^3}+\frac{15\left(bx^2+a\right)^{\frac{7}{2}}b^3}{16\,a^2}+\frac{21\left(bx^2+a\right)^{\frac{5}{2}}b^3}{16\,a}+\frac{105}{16}\sqrt{bx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="maxima")
```

```
[Out] -105/16*a^(3/2)*b^3*arcsinh(a/(sqrt(a*b)*abs(x))) + 35/16*(b*x^2 + a)^(3/2)
*b^3 + 35/48*(b*x^2 + a)^(9/2)*b^3/a^3 + 15/16*(b*x^2 + a)^(7/2)*b^3/a^2 +
21/16*(b*x^2 + a)^(5/2)*b^3/a + 105/16*sqrt(b*x^2 + a)*a*b^3 - 35/48*(b*x^2
 + a)^(11/2)*b^2/(a^3*x^2) - 5/24*(b*x^2 + a)^(11/2)*b/(a^2*x^4) - 1/6*(b*x
^2 + a)^(11/2)/(a*x^6)
```

mupad [B] time = 5.99, size = 149, normalized size = 1.18

$$\frac{\frac{41\,a^4\,b^3\,\sqrt{b\,x^2+a}}{16}-\frac{35\,a^3\,b^3\,{\left(b\,x^2+a\right)}^{3/2}}{6}+\frac{55\,a^2\,b^3\,{\left(b\,x^2+a\right)}^{5/2}}{16}}{3\,a\,{\left(b\,x^2+a\right)}^2-3\,a^2\,\left(b\,x^2+a\right)-{\left(b\,x^2+a\right)}^3+a^3}+\frac{b^3\,{\left(b\,x^2+a\right)}^{3/2}}{3}+4\,a\,b^3\,\sqrt{b\,x^2+a}+\frac{a^{3/2}\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,10}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^7,x)
```

```
[Out] ((41*a^4*b^3*(a + b*x^2)^(1/2))/16 - (35*a^3*b^3*(a + b*x^2)^(3/2))/6 + (55
*a^2*b^3*(a + b*x^2)^(5/2))/16)/(3*a*(a + b*x^2)^2 - 3*a^2*(a + b*x^2) - (a
 + b*x^2)^3 + a^3) + (b^3*(a + b*x^2)^(3/2))/3 + (a^(3/2)*b^3*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*105i)/16 + 4*a*b^3*(a + b*x^2)^(1/2)
```

sympy [A] time = 7.12, size = 175, normalized size = 1.39

$$-\frac{105a^{\frac{3}{2}}b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16}-\frac{a^5}{6\sqrt{b}\,x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{29a^4\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{215a^3b^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{43a^2b^{\frac{5}{2}}}{48x\sqrt{\frac{a}{bx^2}+1}}+\frac{14ab^{\frac{7}{2}}x}{3\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{9}{2}}x^3}{3\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**7,x)
```

```
[Out] -105*a**(3/2)*b**3*asinh(sqrt(a)/(sqrt(b)*x))/16 - a**5/(6*sqrt(b)*x**7*sqr
t(a/(b*x**2) + 1)) - 29*a**4*sqrt(b)/(24*x**5*sqrt(a/(b*x**2) + 1)) - 215*a
**3*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) + 43*a**2*b**(5/2)/(48*x*sqrt(a
/(b*x**2) + 1)) + 14*a*b**(7/2)*x/(3*sqrt(a/(b*x**2) + 1)) + b**(9/2)*x**3/
(3*sqrt(a/(b*x**2) + 1))
```

3.421 $\int \frac{(a+bx^2)^{9/2}}{x^9} dx$

Optimal. Leaf size=128

$$\frac{315}{128}b^4\sqrt{a+bx^2}-\frac{315}{128}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{105b^3\left(a+bx^2\right)^{3/2}}{128x^2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4}-\frac{\left(a+bx^2\right)^{9/2}}{8x^8}-\frac{3b\left(a+bx^2\right.}{16x^6}$$

```
[Out] -105/128*b^3*(b*x^2+a)^(3/2)/x^2-21/64*b^2*(b*x^2+a)^(5/2)/x^4-3/16*b*(b*x^
2+a)^(7/2)/x^6-1/8*(b*x^2+a)^(9/2)/x^8-315/128*b^4*arctanh((b*x^2+a)^(1/2)/
a^(1/2))*a^(1/2)+315/128*b^4*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$-\frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4}-\frac{105b^3\left(a+bx^2\right)^{3/2}}{128x^2}+\frac{315}{128}b^4\sqrt{a+bx^2}-\frac{315}{128}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{8x^8}-\frac{3b\left(a+b\right.}{16x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^9,x]
```

```
[Out] (315*b^4*Sqrt[a + b*x^2])/128 - (105*b^3*(a + b*x^2)^(3/2))/(128*x^2) - (21
*b^2*(a + b*x^2)^(5/2))/(64*x^4) - (3*b*(a + b*x^2)^(7/2))/(16*x^6) - (a +
b*x^2)^(9/2)/(8*x^8) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/1
28
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{9/2}}{x^9}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^5}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{9/2}}{8x^8}+\frac{1}{16}(9b)\operatorname{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{3b\left(a+bx^2\right)^{7/2}}{16x^6}-\frac{\left(a+bx^2\right)^{9/2}}{8x^8}+\frac{1}{32}\left(21b^2\right)\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4}-\frac{3b\left(a+bx^2\right)^{7/2}}{16x^6}-\frac{\left(a+bx^2\right)^{9/2}}{8x^8}+\frac{1}{128}\left(105b^3\right)\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\right.\\
&= -\frac{105b^3\left(a+bx^2\right)^{3/2}}{128x^2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4}-\frac{3b\left(a+bx^2\right)^{7/2}}{16x^6}-\frac{\left(a+bx^2\right)^{9/2}}{8x^8}+\frac{1}{256}\left(315b^4\right)\\
&= \frac{315}{128}b^4\sqrt{a+bx^2}-\frac{105b^3\left(a+bx^2\right)^{3/2}}{128x^2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4}-\frac{3b\left(a+bx^2\right)^{7/2}}{16x^6}-\frac{\left(a+bx^2\right)^{9/2}}{8x^8}\\
&= \frac{315}{128}b^4\sqrt{a+bx^2}-\frac{105b^3\left(a+bx^2\right)^{3/2}}{128x^2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4}-\frac{3b\left(a+bx^2\right)^{7/2}}{16x^6}-\frac{\left(a+bx^2\right)^{9/2}}{8x^8}\\
&= \frac{315}{128}b^4\sqrt{a+bx^2}-\frac{105b^3\left(a+bx^2\right)^{3/2}}{128x^2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4}-\frac{3b\left(a+bx^2\right)^{7/2}}{16x^6}-\frac{\left(a+bx^2\right)^{9/2}}{8x^8}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.30

$$-\frac{b^4\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(5,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^9,x]
```

```
[Out] -1/11*(b^4*(a + b*x^2)^(11/2)*Hypergeometric2F1[5, 11/2, 13/2, 1 + (b*x^2)/
a])/a^5
```

fricas [A] time = 0.91, size = 189, normalized size = 1.48

$$\left[\frac{315\sqrt{a}\,b^4x^8\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(128\,b^4x^8-325\,ab^3x^6-210\,a^2b^2x^4-88\,a^3bx^2-16\,a^4\right)\sqrt{bx^2+a}}{256\,x^8},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/256*(315*sqrt(a)*b^4*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/
x^2) + 2*(128*b^4*x^8 - 325*a*b^3*x^6 - 210*a^2*b^2*x^4 - 88*a^3*b*x^2 - 16
```

```
*a^4)*sqrt(b*x^2 + a))/x^8, 1/128*(315*sqrt(-a)*b^4*x^8*arctan(sqrt(-a)/sqr
t(b*x^2 + a)) + (128*b^4*x^8 - 325*a*b^3*x^6 - 210*a^2*b^2*x^4 - 88*a^3*b*x
^2 - 16*a^4)*sqrt(b*x^2 + a))/x^8]
```

giac [A] time = 1.13, size = 122, normalized size = 0.95

$$\frac{\frac{315\,ab^5\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+128\,\sqrt{bx^2+a}\,b^5-\frac{325\,(bx^2+a)^{\frac{7}{2}}ab^5-765\,(bx^2+a)^{\frac{5}{2}}a^2b^5+643\,(bx^2+a)^{\frac{3}{2}}a^3b^5-187\,\sqrt{bx^2+a}\,a^4b^5}{b^4x^8}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/128*(315*a*b^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 128*sqrt(b*x^2
 + a)*b^5 - (325*(b*x^2 + a)^(7/2)*a*b^5 - 765*(b*x^2 + a)^(5/2)*a^2*b^5 +
643*(b*x^2 + a)^(3/2)*a^3*b^5 - 187*sqrt(b*x^2 + a)*a^4*b^5)/(b^4*x^8))/b
```

maple [A] time = 0.02, size = 190, normalized size = 1.48

$$-\frac{315\sqrt{a}\,b^4\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{128}+\frac{315\sqrt{b\,x^2+a}\,b^4}{128}+\frac{105\left(b\,x^2+a\right)^{\frac{3}{2}}b^4}{128a}+\frac{63\left(b\,x^2+a\right)^{\frac{5}{2}}b^4}{128a^2}+\frac{45\left(b\,x^2+a\right)^{\frac{7}{2}}b^4}{128a^3}+\frac{35(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^9,x)
```

```
[Out] -1/8/a/x^8*(b*x^2+a)^(11/2)-1/16/a^2*b/x^6*(b*x^2+a)^(11/2)-5/64/a^3*b^2/x^
4*(b*x^2+a)^(11/2)-35/128/a^4*b^3/x^2*(b*x^2+a)^(11/2)+35/128/a^4*b^4*(b*x^
2+a)^(9/2)+45/128/a^3*b^4*(b*x^2+a)^(7/2)+63/128/a^2*b^4*(b*x^2+a)^(5/2)+10
5/128/a*b^4*(b*x^2+a)^(3/2)-315/128*a^(1/2)*b^4*ln((2*a+2*(b*x^2+a)^(1/2)*a
^(1/2))/x)+315/128*b^4*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.45, size = 178, normalized size = 1.39

$$-\frac{315}{128}\sqrt{a}\,b^4\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{315}{128}\sqrt{bx^2+a}\,b^4+\frac{35\left(bx^2+a\right)^{\frac{9}{2}}b^4}{128\,a^4}+\frac{45\left(bx^2+a\right)^{\frac{7}{2}}b^4}{128\,a^3}+\frac{63\left(bx^2+a\right)^{\frac{5}{2}}b^4}{128\,a^2}+\frac{105\left(bx^2+\right.}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="maxima")
```

```
[Out] -315/128*sqrt(a)*b^4*arcsinh(a/(sqrt(a*b)*abs(x))) + 315/128*sqrt(b*x^2 + a
)*b^4 + 35/128*(b*x^2 + a)^(9/2)*b^4/a^4 + 45/128*(b*x^2 + a)^(7/2)*b^4/a^3
 + 63/128*(b*x^2 + a)^(5/2)*b^4/a^2 + 105/128*(b*x^2 + a)^(3/2)*b^4/a - 35/
128*(b*x^2 + a)^(11/2)*b^3/(a^4*x^2) - 5/64*(b*x^2 + a)^(11/2)*b^2/(a^3*x^4
) - 1/16*(b*x^2 + a)^(11/2)*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(11/2)/(a*x^8)
```

mupad [B] time = 6.20, size = 105, normalized size = 0.82

$$b^4\sqrt{b\,x^2+a}-\frac{325\,a\left(b\,x^2+a\right)^{7/2}}{128\,x^8}+\frac{187\,a^4\sqrt{b\,x^2+a}}{128\,x^8}-\frac{643\,a^3\left(b\,x^2+a\right)^{3/2}}{128\,x^8}+\frac{765\,a^2\left(b\,x^2+a\right)^{5/2}}{128\,x^8}+\frac{\sqrt{a}\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2}}{\sqrt{}}\right.}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^9,x)
```

```
[Out] b^4*(a + b*x^2)^(1/2) + (a^(1/2)*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3
15i)/128 - (325*a*(a + b*x^2)^(7/2))/(128*x^8) + (187*a^4*(a + b*x^2)^(1/2)
```

```
)/(128*x^8) - (643*a^3*(a + b*x^2)^(3/2))/(128*x^8) + (765*a^2*(a + b*x^2)^
(5/2))/(128*x^8)
```

sympy [A] time = 7.74, size = 173, normalized size = 1.35

$$-\frac{315\sqrt{a}\,b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{128}-\frac{a^5}{8\sqrt{b}\,x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{13a^4\sqrt{b}}{16x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{149a^3b^{\frac{3}{2}}}{64x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{535a^2b^{\frac{5}{2}}}{128x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{197ab^{\frac{7}{2}}}{128x\sqrt{\frac{a}{bx^2}+1}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**9,x)
```

```
[Out] -315*sqrt(a)*b**4*asinh(sqrt(a)/(sqrt(b)*x))/128 - a**5/(8*sqrt(b)*x**9*sqr
t(a/(b*x**2) + 1)) - 13*a**4*sqrt(b)/(16*x**7*sqrt(a/(b*x**2) + 1)) - 149*a
**3*b**(3/2)/(64*x**5*sqrt(a/(b*x**2) + 1)) - 535*a**2*b**(5/2)/(128*x**3*s
qrt(a/(b*x**2) + 1)) - 197*a*b**(7/2)/(128*x*sqrt(a/(b*x**2) + 1)) + b**(9/
2)*x/sqrt(a/(b*x**2) + 1)
```

3.422 $\int \frac{(a+bx^2)^{9/2}}{x^{11}}\,dx$

Optimal. Leaf size=131

$$-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}}-\frac{63b^4\sqrt{a+bx^2}}{256x^2}-\frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6}-\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}-\frac{9b\left(a+bx^2\right)^{7/2}}{80x^8}$$

```
[Out] -21/128*b^3*(b*x^2+a)^(3/2)/x^4-21/160*b^2*(b*x^2+a)^(5/2)/x^6-9/80*b*(b*x^
2+a)^(7/2)/x^8-1/10*(b*x^2+a)^(9/2)/x^10-63/256*b^5*arctanh((b*x^2+a)^(1/2)
/a^(1/2))/a^(1/2)-63/256*b^4*(b*x^2+a)^(1/2)/x^2
```

Rubi [A] time = 0.08, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = $\{266, 47, 63, 208\}$

$$-\frac{63b^4\sqrt{a+bx^2}}{256x^2}-\frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}}-\frac{9b\left(a+bx^2\right)^{7/2}}{80x^8}-\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^11,x]
```

```
[Out] (-63*b^4*Sqrt[a + b*x^2])/(256*x^2) - (21*b^3*(a + b*x^2)^(3/2))/(128*x^4)
- (21*b^2*(a + b*x^2)^(5/2))/(160*x^6) - (9*b*(a + b*x^2)^(7/2))/(80*x^8) -
 (a + b*x^2)^(9/2)/(10*x^10) - (63*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2
56*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{11}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^6}\,dx,x,x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}} + \frac{1}{20}(9b)\,\mathrm{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^5}\,dx,x,x^2\right)$$
$$= -\frac{9b\left(a+bx^2\right)^{7/2}}{80x^8} - \frac{\left(a+bx^2\right)^{9/2}}{10x^{10}} + \frac{1}{160}\left(63b^2\right)\mathrm{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^4}\,dx,x,x^2\right)$$
$$= -\frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6} - \frac{9b\left(a+bx^2\right)^{7/2}}{80x^8} - \frac{\left(a+bx^2\right)^{9/2}}{10x^{10}} + \frac{1}{64}\left(21b^3\right)\mathrm{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx$$
$$= -\frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6} - \frac{9b\left(a+bx^2\right)^{7/2}}{80x^8} - \frac{\left(a+bx^2\right)^{9/2}}{10x^{10}} + \frac{1}{256}\left(63b^4\right)\mathrm{Su}$$
$$= -\frac{63b^4\sqrt{a+bx^2}}{256x^2} - \frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6} - \frac{9b\left(a+bx^2\right)^{7/2}}{80x^8} - \frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}$$
$$= -\frac{63b^4\sqrt{a+bx^2}}{256x^2} - \frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6} - \frac{9b\left(a+bx^2\right)^{7/2}}{80x^8} - \frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}$$
$$= -\frac{63b^4\sqrt{a+bx^2}}{256x^2} - \frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6} - \frac{9b\left(a+bx^2\right)^{7/2}}{80x^8} - \frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}$$

Mathematica [A] time = 0.05, size = 109, normalized size = 0.83

$$-\frac{128a^5 + 784a^4bx^2 + 2024a^3b^2x^4 + 2858a^2b^3x^6 + 315b^5x^{10}\sqrt{\frac{bx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right) + 2455ab^4x^8 + 965b}{1280x^{10}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^11,x]
```

```
[Out] -1/1280*(128*a^5 + 784*a^4*b*x^2 + 2024*a^3*b^2*x^4 + 2858*a^2*b^3*x^6 + 24
55*a*b^4*x^8 + 965*b^5*x^10 + 315*b^5*x^10*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt
[1 + (b*x^2)/a]])/(x^10*Sqrt[a + b*x^2])
```

fricas [A] time = 1.05, size = 202, normalized size = 1.54

$$\left[\frac{315\sqrt{a}\,b^5x^{10}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\,\sqrt{a}+2a}{x^2}\right) - 2\left(965\,ab^4x^8 + 1490\,a^2b^3x^6 + 1368\,a^3b^2x^4 + 656\,a^4bx^2 + 128\,a^5\right)\sqrt{b}}{2560\,ax^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/2560*(315*sqrt(a)*b^5*x^10*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
)/x^2) - 2*(965*a*b^4*x^8 + 1490*a^2*b^3*x^6 + 1368*a^3*b^2*x^4 + 656*a^4*b
*x^2 + 128*a^5)*sqrt(b*x^2 + a))/(a*x^10), 1/1280*(315*sqrt(-a)*b^5*x^10*ar
ctan(sqrt(-a)/sqrt(b*x^2 + a)) - (965*a*b^4*x^8 + 1490*a^2*b^3*x^6 + 1368*a
^3*b^2*x^4 + 656*a^4*b*x^2 + 128*a^5)*sqrt(b*x^2 + a))/(a*x^10)]
```

giac [A] time = 1.09, size = 121, normalized size = 0.92

$$\frac{\frac{315\, b^6 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{965\, (bx^2+a)^{\frac{9}{2}} b^6 - 2370\, (bx^2+a)^{\frac{7}{2}} ab^6 + 2688\, (bx^2+a)^{\frac{5}{2}} a^2 b^6 - 1470\, (bx^2+a)^{\frac{3}{2}} a^3 b^6 + 315\, \sqrt{bx^2+a}\, a^4 b^6}{b^5 x^{10}}}{1280\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/1280*(315*b^6*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (965*(b*x^2 + a
)^(9/2)*b^6 - 2370*(b*x^2 + a)^(7/2)*a*b^6 + 2688*(b*x^2 + a)^(5/2)*a^2*b^6
 - 1470*(b*x^2 + a)^(3/2)*a^3*b^6 + 315*sqrt(b*x^2 + a)*a^4*b^6)/(b^5*x^10)
)/b
```

maple [B] time = 0.05, size = 213, normalized size = 1.63

$$-\frac{63b^5 \ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{256\sqrt{a}}+\frac{63\sqrt{b\,x^2+a}\;b^5}{256a}+\frac{21\left(b\,x^2+a\right)^{\frac{3}{2}} b^5}{256a^2}+\frac{63\left(b\,x^2+a\right)^{\frac{5}{2}} b^5}{1280a^3}+\frac{9\left(b\,x^2+a\right)^{\frac{7}{2}} b^5}{256a^4}+\frac{7\left(b\,x^2+a\right)^{\frac{9}{2}}}{256a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^11,x)
```

```
[Out] -1/10/a/x^10*(b*x^2+a)^(11/2)-1/80/a^2*b/x^8*(b*x^2+a)^(11/2)-1/160/a^3*b^2
/x^6*(b*x^2+a)^(11/2)-1/128/a^4*b^3/x^4*(b*x^2+a)^(11/2)-7/256/a^5*b^4/x^2*
(b*x^2+a)^(11/2)+7/256/a^5*b^5*(b*x^2+a)^(9/2)+9/256/a^4*b^5*(b*x^2+a)^(7/2
)+63/1280/a^3*b^5*(b*x^2+a)^(5/2)+21/256/a^2*b^5*(b*x^2+a)^(3/2)-63/256/a^(
1/2)*b^5*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+63/256/a*b^5*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.48, size = 201, normalized size = 1.53

$$-\frac{63\, b^5 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{256\,\sqrt{a}}+\frac{7\left(bx^2+a\right)^{\frac{9}{2}} b^5}{256\,a^5}+\frac{9\left(bx^2+a\right)^{\frac{7}{2}} b^5}{256\,a^4}+\frac{63\left(bx^2+a\right)^{\frac{5}{2}} b^5}{1280\,a^3}+\frac{21\left(bx^2+a\right)^{\frac{3}{2}} b^5}{256\,a^2}+\frac{63\,\sqrt{bx^2+a}\,b^5}{256\,a}-\frac{7\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="maxima")
```

```
[Out] -63/256*b^5*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 7/256*(b*x^2 + a)^(9/2)
*b^5/a^5 + 9/256*(b*x^2 + a)^(7/2)*b^5/a^4 + 63/1280*(b*x^2 + a)^(5/2)*b^5/
a^3 + 21/256*(b*x^2 + a)^(3/2)*b^5/a^2 + 63/256*sqrt(b*x^2 + a)*b^5/a - 7/2
56*(b*x^2 + a)^(11/2)*b^4/(a^5*x^2) - 1/128*(b*x^2 + a)^(11/2)*b^3/(a^4*x^4
) - 1/160*(b*x^2 + a)^(11/2)*b^2/(a^3*x^6) - 1/80*(b*x^2 + a)^(11/2)*b/(a^2
*x^8) - 1/10*(b*x^2 + a)^(11/2)/(a*x^10)
```

mupad [B] time = 6.62, size = 106, normalized size = 0.81

$$\frac{237\,a\left(b\,x^2+a\right)^{7/2}}{128\,x^{10}}-\frac{193\left(b\,x^2+a\right)^{9/2}}{256\,x^{10}}-\frac{63\,a^4\,\sqrt{b\,x^2+a}}{256\,x^{10}}+\frac{147\,a^3\left(b\,x^2+a\right)^{3/2}}{128\,x^{10}}-\frac{21\,a^2\left(b\,x^2+a\right)^{5/2}}{10\,x^{10}}+\frac{b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right.}{256\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^11,x)
```

```
[Out] (b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*63i)/(256*a^(1/2)) - (193*(a + b*
x^2)^(9/2))/(256*x^10) + (237*a*(a + b*x^2)^(7/2))/(128*x^10) - (63*a^4*(a
+ b*x^2)^(1/2))/(256*x^10) + (147*a^3*(a + b*x^2)^(3/2))/(128*x^10) - (21*a
^2*(a + b*x^2)^(5/2))/(10*x^10)
```

sympy [A] time = 9.31, size = 153, normalized size = 1.17

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{10x^9}-\frac{41a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{80x^7}-\frac{171a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{160x^5}-\frac{149ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{128x^3}-\frac{193b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{256x}-\frac{63b^5\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{256\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**11,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(10*x**9) - 41*a**3*b**(3/2)*sqrt(a/(b*x
**2) + 1)/(80*x**7) - 171*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(160*x**5) - 1
49*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(128*x**3) - 193*b**(9/2)*sqrt(a/(b*x**2
) + 1)/(256*x) - 63*b**5*asinh(sqrt(a)/(sqrt(b)*x))/(256*sqrt(a))
```

3.423 $\int \frac{(a+bx^2)^{9/2}}{x^{13}}\,dx$

Optimal. Leaf size=155

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{1024a^{3/2}}-\frac{21b^5\sqrt{a+bx^2}}{1024ax^2}-\frac{21b^4\sqrt{a+bx^2}}{512x^4}-\frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8}-\frac{\left(a+bx^2\right)^{9/2}}{12x^{12}}-\frac{3b\left(a\right.}{4}$$

```
[Out] -7/128*b^3*(b*x^2+a)^(3/2)/x^6-21/320*b^2*(b*x^2+a)^(5/2)/x^8-3/40*b*(b*x^2
+a)^(7/2)/x^10-1/12*(b*x^2+a)^(9/2)/x^12+21/1024*b^6*arctanh((b*x^2+a)^(1/2
)/a^(1/2))/a^(3/2)-21/512*b^4*(b*x^2+a)^(1/2)/x^4-21/1024*b^5*(b*x^2+a)^(1/
2)/a/x^2
```

Rubi [A] time = 0.10, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{1024a^{3/2}}-\frac{21b^5\sqrt{a+bx^2}}{1024ax^2}-\frac{21b^4\sqrt{a+bx^2}}{512x^4}-\frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8}-\frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}}-\frac{\left(a\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^13,x]
```

```
[Out] (-21*b^4*Sqrt[a + b*x^2])/(512*x^4) - (21*b^5*Sqrt[a + b*x^2])/(1024*a*x^2)
 - (7*b^3*(a + b*x^2)^(3/2))/(128*x^6) - (21*b^2*(a + b*x^2)^(5/2))/(320*x^
8) - (3*b*(a + b*x^2)^(7/2))/(40*x^10) - (a + b*x^2)^(9/2)/(12*x^12) + (21*
b^6*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(1024*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{13}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^7}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2)^{9/2}}{12x^{12}} + \frac{1}{8}(3b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{3b(a+bx^2)^{7/2}}{40x^{10}} - \frac{(a+bx^2)^{9/2}}{12x^{12}} + \frac{1}{80}(21b^2)\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{3b(a+bx^2)^{7/2}}{40x^{10}} - \frac{(a+bx^2)^{9/2}}{12x^{12}} + \frac{1}{128}(21b^3)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,d\right.$$

$$= -\frac{7b^3(a+bx^2)^{3/2}}{128x^6} - \frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{3b(a+bx^2)^{7/2}}{40x^{10}} - \frac{(a+bx^2)^{9/2}}{12x^{12}} + \frac{1}{256}(21b^4)\,\text{Sub}$$

$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{7b^3(a+bx^2)^{3/2}}{128x^6} - \frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{3b(a+bx^2)^{7/2}}{40x^{10}} - \frac{(a+bx^2)^{9/2}}{12x^{12}}$$

$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} - \frac{7b^3(a+bx^2)^{3/2}}{128x^6} - \frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{3b(a+bx^2)^{7/}}{40x^{10}}$$

$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} - \frac{7b^3(a+bx^2)^{3/2}}{128x^6} - \frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{3b(a+bx^2)^{7/}}{40x^{10}}$$

$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} - \frac{7b^3(a+bx^2)^{3/2}}{128x^6} - \frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{3b(a+bx^2)^{7/}}{40x^{10}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.25

$$-\frac{b^6(a+bx^2)^{11/2}\,{}_2F_1\left(\frac{11}{2},7;\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^13,x]
```

```
[Out] -1/11*(b^6*(a + b*x^2)^(11/2)*Hypergeometric2F1[11/2, 7, 13/2, 1 + (b*x^2)/
a])/a^7
```

fricas [A] time = 1.08, size = 223, normalized size = 1.44

$$\left[\frac{315\sqrt{a}\,b^6x^{12}\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right) - 2\left(315\,ab^5x^{10} + 4910\,a^2b^4x^8 + 11432\,a^3b^3x^6 + 12144\,a^4b^2x^4 + 6272\right.}{30720\,a^2x^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/30720*(315*sqrt(a)*b^6*x^12*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*
a)/x^2) - 2*(315*a*b^5*x^10 + 4910*a^2*b^4*x^8 + 11432*a^3*b^3*x^6 + 12144*
a^4*b^2*x^4 + 6272*a^5*b*x^2 + 1280*a^6)*sqrt(b*x^2 + a))/(a^2*x^12), -1/15
360*(315*sqrt(-a)*b^6*x^12*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (315*a*b^5*x^
10 + 4910*a^2*b^4*x^8 + 11432*a^3*b^3*x^6 + 12144*a^4*b^2*x^4 + 6272*a^5*b*
x^2 + 1280*a^6)*sqrt(b*x^2 + a))/(a^2*x^12)]
```

giac [A] time = 1.10, size = 143, normalized size = 0.92

$$-\frac{\frac{315\,b^7\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{315\left(bx^2+a\right)^{\frac{11}{2}}b^7+3335\left(bx^2+a\right)^{\frac{9}{2}}ab^7-5058\left(bx^2+a\right)^{\frac{7}{2}}a^2b^7+4158\left(bx^2+a\right)^{\frac{5}{2}}a^3b^7-1785\left(bx^2+a\right)^{\frac{3}{2}}a^4b^7+315\sqrt{bx^2+a}\,a^5b^7}{ab^6x^{12}}}{15360\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/15360*(315*b^7*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (315*(b*x
^2 + a)^(11/2)*b^7 + 3335*(b*x^2 + a)^(9/2)*a*b^7 - 5058*(b*x^2 + a)^(7/2)*
a^2*b^7 + 4158*(b*x^2 + a)^(5/2)*a^3*b^7 - 1785*(b*x^2 + a)^(3/2)*a^4*b^7 +
 315*sqrt(b*x^2 + a)*a^5*b^7)/(a*b^6*x^12))/b
```

maple [A] time = 0.13, size = 233, normalized size = 1.50

$$\frac{21b^6\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{1024a^{\frac{3}{2}}}-\frac{21\sqrt{b\,x^2+a}\;b^6}{1024a^2}-\frac{7\left(b\,x^2+a\right)^{\frac{3}{2}}b^6}{1024a^3}-\frac{21\left(b\,x^2+a\right)^{\frac{5}{2}}b^6}{5120a^4}-\frac{3\left(b\,x^2+a\right)^{\frac{7}{2}}b^6}{1024a^5}-\frac{7\left(b\,x^2+a\right)^{\frac{9}{2}}b^6}{3072a^6}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^13,x)
```

```
[Out] -1/12/a/x^12*(b*x^2+a)^(11/2)+1/120/a^2*b/x^10*(b*x^2+a)^(11/2)+1/960/a^3*b
^2/x^8*(b*x^2+a)^(11/2)+1/1920/a^4*b^3/x^6*(b*x^2+a)^(11/2)+1/1536/a^5*b^4/
x^4*(b*x^2+a)^(11/2)+7/3072/a^6*b^5/x^2*(b*x^2+a)^(11/2)-7/3072/a^6*b^6*(b*
x^2+a)^(9/2)-3/1024/a^5*b^6*(b*x^2+a)^(7/2)-21/5120/a^4*b^6*(b*x^2+a)^(5/2)
-7/1024/a^3*b^6*(b*x^2+a)^(3/2)+21/1024/a^(3/2)*b^6*ln((2*a+2*(b*x^2+a)^(1/
2)*a^(1/2))/x)-21/1024/a^2*b^6*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.51, size = 221, normalized size = 1.43

$$\frac{21\,b^6\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{1024\,a^{\frac{3}{2}}}-\frac{7\left(bx^2+a\right)^{\frac{9}{2}}b^6}{3072\,a^6}-\frac{3\left(bx^2+a\right)^{\frac{7}{2}}b^6}{1024\,a^5}-\frac{21\left(bx^2+a\right)^{\frac{5}{2}}b^6}{5120\,a^4}-\frac{7\left(bx^2+a\right)^{\frac{3}{2}}b^6}{1024\,a^3}-\frac{21\sqrt{bx^2+a}\,b^6}{1024\,a^2}+\frac{7\,(bx^2}{307}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="maxima")
```

```
[Out] 21/1024*b^6*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 7/3072*(b*x^2 + a)^(9/2
)*b^6/a^6 - 3/1024*(b*x^2 + a)^(7/2)*b^6/a^5 - 21/5120*(b*x^2 + a)^(5/2)*b^
6/a^4 - 7/1024*(b*x^2 + a)^(3/2)*b^6/a^3 - 21/1024*sqrt(b*x^2 + a)*b^6/a^2
+ 7/3072*(b*x^2 + a)^(11/2)*b^5/(a^6*x^2) + 1/1536*(b*x^2 + a)^(11/2)*b^4/(
a^5*x^4) + 1/1920*(b*x^2 + a)^(11/2)*b^3/(a^4*x^6) + 1/960*(b*x^2 + a)^(11/
2)*b^2/(a^3*x^8) + 1/120*(b*x^2 + a)^(11/2)*b/(a^2*x^10) - 1/12*(b*x^2 + a)
^(11/2)/(a*x^12)
```

mupad [B] time = 6.68, size = 123, normalized size = 0.79

$$\frac{843\,a\,\left(b\,x^2+a\right)^{7/2}}{2560\,x^{12}}-\frac{667\,\left(b\,x^2+a\right)^{9/2}}{3072\,x^{12}}-\frac{21\,a^4\,\sqrt{b\,x^2+a}}{1024\,x^{12}}+\frac{119\,a^3\,\left(b\,x^2+a\right)^{3/2}}{1024\,x^{12}}-\frac{693\,a^2\,\left(b\,x^2+a\right)^{5/2}}{2560\,x^{12}}-\frac{21\,\left(b\,x^2+a\right.}{1024\,a\,x^1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^13,x)
```

```
[Out] (843*a*(a + b*x^2)^(7/2))/(2560*x^12) - (b^6*atan(((a + b*x^2)^(1/2)*1i)/a^
(1/2))*21i)/(1024*a^(3/2)) - (667*(a + b*x^2)^(9/2))/(3072*x^12) - (21*a^4*
(a + b*x^2)^(1/2))/(1024*x^12) + (119*a^3*(a + b*x^2)^(3/2))/(1024*x^12) -
(693*a^2*(a + b*x^2)^(5/2))/(2560*x^12) - (21*(a + b*x^2)^(11/2))/(1024*a*x
^12)
```

sympy [A] time = 14.98, size = 204, normalized size = 1.32

$$-\frac{a^5}{12\sqrt{b}\,x^{13}\sqrt{\frac{a}{bx^2}+1}}-\frac{59a^4\sqrt{b}}{120x^{11}\sqrt{\frac{a}{bx^2}+1}}-\frac{1151a^3b^{\frac{3}{2}}}{960x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{2947a^2b^{\frac{5}{2}}}{1920x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{8171ab^{\frac{7}{2}}}{7680x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{1045b^{\frac{9}{2}}}{3072x^3\sqrt{\frac{a}{bx^2}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**13,x)
```

```
[Out] -a**5/(12*sqrt(b)*x***13*sqrt(a/(b*x**2) + 1)) - 59*a**4*sqrt(b)/(120*x**11*
sqrt(a/(b*x**2) + 1)) - 1151*a**3*b**(3/2)/(960*x***9*sqrt(a/(b*x**2) + 1))
- 2947*a**2*b**(5/2)/(1920*x**7*sqrt(a/(b*x**2) + 1)) - 8171*a*b**(7/2)/(76
80*x***5*sqrt(a/(b*x**2) + 1)) - 1045*b**(9/2)/(3072*x***3*sqrt(a/(b*x**2) +
1)) - 21*b**(11/2)/(1024*a*x*sqrt(a/(b*x**2) + 1)) + 21*b**6*asinh(sqrt(a)/
(sqrt(b)*x))/(1024*a**(3/2))
```

3.424 $\int \frac{\left(a+bx^2\right)^{9/2}}{x^{15}}\,dx$

Optimal. Leaf size=179

$$-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2048a^{5/2}}+\frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2}-\frac{3b^5\sqrt{a+bx^2}}{1024ax^4}-\frac{3b^4\sqrt{a+bx^2}}{256x^6}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8}-\frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}}-\frac{\left(a+bx\right.}{14x^1}$$

```
[Out] -3/128*b^3*(b*x^2+a)^(3/2)/x^8-3/80*b^2*(b*x^2+a)^(5/2)/x^10-3/56*b*(b*x^2+
a)^(7/2)/x^12-1/14*(b*x^2+a)^(9/2)/x^14-9/2048*b^7*arctanh((b*x^2+a)^(1/2)/
a^(1/2))/a^(5/2)-3/256*b^4*(b*x^2+a)^(1/2)/x^6-3/1024*b^5*(b*x^2+a)^(1/2)/a
/x^4+9/2048*b^6*(b*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.12, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2}-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2048a^{5/2}}-\frac{3b^5\sqrt{a+bx^2}}{1024ax^4}-\frac{3b^4\sqrt{a+bx^2}}{256x^6}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8}-\frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}}-\frac{3b\left(a+b\right.}{56x^1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^15,x]
```

```
[Out] (-3*b^4*Sqrt[a + b*x^2])/(256*x^6) - (3*b^5*Sqrt[a + b*x^2])/(1024*a*x^4) +
 (9*b^6*Sqrt[a + b*x^2])/(2048*a^2*x^2) - (3*b^3*(a + b*x^2)^(3/2))/(128*x^
8) - (3*b^2*(a + b*x^2)^(5/2))/(80*x^10) - (3*b*(a + b*x^2)^(7/2))/(56*x^12
) - (a + b*x^2)^(9/2)/(14*x^14) - (9*b^7*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(2048*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{15}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^8}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} + \frac{1}{28}(9b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^7}\,dx, x, x^2\right)$$

$$= -\frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} - \frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} + \frac{1}{16}\left(3b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} - \frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} + \frac{1}{32}\left(3b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx, x\right.$$

$$= -\frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} - \frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} + \frac{1}{256}\left(9b^4\right)\text{Subst}$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} - \frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} +$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} -$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} -$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} -$$

$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} -$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.22

$$\frac{b^7\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(\frac{11}{2},8;\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^15,x]
```

```
[Out] (b^7*(a + b*x^2)^(11/2)*Hypergeometric2F1[11/2, 8, 13/2, 1 + (b*x^2)/a])/(1
1*a^8)
```

fricas [A] time = 1.19, size = 245, normalized size = 1.37

$$\left[\frac{315\sqrt{a}\,b^7x^{14}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\,\sqrt{a}+2a}{x^2}\right)+2\left(315\,ab^6x^{12}-210\,a^2b^5x^{10}-14168\,a^3b^4x^8-39056\,a^4b^3x^6-44928\,a^5\right.}{143360\,a^3x^{14}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="fricas")
```

```
[Out] [1/143360*(315*sqrt(a)*b^7*x^14*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + 2*(315*a*b^6*x^12 - 210*a^2*b^5*x^10 - 14168*a^3*b^4*x^8 - 39056
*a^4*b^3*x^6 - 44928*a^5*b^2*x^4 - 24320*a^6*b*x^2 - 5120*a^7)*sqrt(b*x^2 +
 a))/(a^3*x^14), 1/71680*(315*sqrt(-a)*b^7*x^14*arctan(sqrt(-a)/sqrt(b*x^2
+ a)) + (315*a*b^6*x^12 - 210*a^2*b^5*x^10 - 14168*a^3*b^4*x^8 - 39056*a^4*
b^3*x^6 - 44928*a^5*b^2*x^4 - 24320*a^6*b*x^2 - 5120*a^7)*sqrt(b*x^2 + a))/
(a^3*x^14)]
```

giac [A] time = 1.09, size = 160, normalized size = 0.89

$$\frac{\frac{315\,b^8\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{315\left(bx^2+a\right)^{\frac{13}{2}}b^8-2100\left(bx^2+a\right)^{\frac{11}{2}}ab^8-8393\left(bx^2+a\right)^{\frac{9}{2}}a^2b^8+9216\left(bx^2+a\right)^{\frac{7}{2}}a^3b^8-5943\left(bx^2+a\right)^{\frac{5}{2}}a^4b^8+2100\left(bx^2+a\right)^{\frac{3}{2}}a^5b}{a^2b^7x^{14}}}{71680\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="giac")
```

```
[Out] 1/71680*(315*b^8*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (315*(b*
x^2 + a)^(13/2)*b^8 - 2100*(b*x^2 + a)^(11/2)*a*b^8 - 8393*(b*x^2 + a)^(9/2
)*a^2*b^8 + 9216*(b*x^2 + a)^(7/2)*a^3*b^8 - 5943*(b*x^2 + a)^(5/2)*a^4*b^8
 + 2100*(b*x^2 + a)^(3/2)*a^5*b^8 - 315*sqrt(b*x^2 + a)*a^6*b^8)/(a^2*b^7*x
^14))/b
```

maple [A] time = 0.34, size = 253, normalized size = 1.41

$$-\frac{9b^7\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2048a^{\frac{5}{2}}}+\frac{9\sqrt{b\,x^2+a}\,b^7}{2048a^3}+\frac{3\left(b\,x^2+a\right)^{\frac{3}{2}}b^7}{2048a^4}+\frac{9\left(b\,x^2+a\right)^{\frac{5}{2}}b^7}{10240a^5}+\frac{9\left(b\,x^2+a\right)^{\frac{7}{2}}b^7}{14336a^6}+\frac{\left(b\,x^2+a\right)^{\frac{9}{2}}b^7}{2048a^7}-\frac{(\ell}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^15,x)
```

```
[Out] -1/14/a/x^14*(b*x^2+a)^(11/2)+1/56/a^2*b/x^12*(b*x^2+a)^(11/2)-1/560/a^3*b^
2/x^10*(b*x^2+a)^(11/2)-1/4480/a^4*b^3/x^8*(b*x^2+a)^(11/2)-1/8960/a^5*b^4/
x^6*(b*x^2+a)^(11/2)-1/7168/a^6*b^5/x^4*(b*x^2+a)^(11/2)-1/2048/a^7*b^6/x^2
*(b*x^2+a)^(11/2)+1/2048/a^7*b^7*(b*x^2+a)^(9/2)+9/14336/a^6*b^7*(b*x^2+a)^
(7/2)+9/10240/a^5*b^7*(b*x^2+a)^(5/2)+3/2048/a^4*b^7*(b*x^2+a)^(3/2)-9/2048
/a^(5/2)*b^7*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+9/2048/a^3*b^7*(b*x^2+a)
^(1/2)
```

maxima [A] time = 1.51, size = 241, normalized size = 1.35

$$-\frac{9\,b^7\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2048\,a^{\frac{5}{2}}}+\frac{\left(bx^2+a\right)^{\frac{9}{2}}b^7}{2048\,a^7}+\frac{9\left(bx^2+a\right)^{\frac{7}{2}}b^7}{14336\,a^6}+\frac{9\left(bx^2+a\right)^{\frac{5}{2}}b^7}{10240\,a^5}+\frac{3\left(bx^2+a\right)^{\frac{3}{2}}b^7}{2048\,a^4}+\frac{9\sqrt{bx^2+a}\,b^7}{2048\,a^3}-\frac{\left(bx^2+a\right.}{2048\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="maxima")
```

```
[Out] -9/2048*b^7*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/2048*(b*x^2 + a)^(9/2
)*b^7/a^7 + 9/14336*(b*x^2 + a)^(7/2)*b^7/a^6 + 9/10240*(b*x^2 + a)^(5/2)*b
^7/a^5 + 3/2048*(b*x^2 + a)^(3/2)*b^7/a^4 + 9/2048*sqrt(b*x^2 + a)*b^7/a^3
- 1/2048*(b*x^2 + a)^(11/2)*b^6/(a^7*x^2) - 1/7168*(b*x^2 + a)^(11/2)*b^5/(
a^6*x^4) - 1/8960*(b*x^2 + a)^(11/2)*b^4/(a^5*x^6) - 1/4480*(b*x^2 + a)^(11
/2)*b^3/(a^4*x^8) - 1/560*(b*x^2 + a)^(11/2)*b^2/(a^3*x^10) + 1/56*(b*x^2 +
 a)^(11/2)*b/(a^2*x^12) - 1/14*(b*x^2 + a)^(11/2)/(a*x^14)
```

mupad [B] time = 7.23, size = 140, normalized size = 0.78

$$\frac{9\,a\,{\left(b\,x^2+a\right)}^{7/2}}{70\,x^{14}}-\frac{1199\,{\left(b\,x^2+a\right)}^{9/2}}{10240\,x^{14}}-\frac{9\,a^4\,\sqrt{b\,x^2+a}}{2048\,x^{14}}+\frac{15\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}{512\,x^{14}}-\frac{849\,a^2\,{\left(b\,x^2+a\right)}^{5/2}}{10240\,x^{14}}-\frac{15\,{\left(b\,x^2+a\right)}^{11/2}}{512\,a\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^15,x)
```

```
[Out] (b^7*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*9i)/(2048*a^(5/2)) - (1199*(a + b
*x^2)^(9/2))/(10240*x^14) + (9*a*(a + b*x^2)^(7/2))/(70*x^14) - (9*a^4*(a +
 b*x^2)^(1/2))/(2048*x^14) + (15*a^3*(a + b*x^2)^(3/2))/(512*x^14) - (849*a
^2*(a + b*x^2)^(5/2))/(10240*x^14) - (15*(a + b*x^2)^(11/2))/(512*a*x^14) +
 (9*(a + b*x^2)^(13/2))/(2048*a^2*x^14)
```

sympy [A] time = 21.82, size = 231, normalized size = 1.29

$$-\frac{a^5}{14\sqrt{b}\,x^{15}\sqrt{\frac{a}{bx^2}+1}}-\frac{23a^4\sqrt{b}}{56x^{13}\sqrt{\frac{a}{bx^2}+1}}-\frac{541a^3b^{\frac{3}{2}}}{560x^{11}\sqrt{\frac{a}{bx^2}+1}}-\frac{5249a^2b^{\frac{5}{2}}}{4480x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{6653ab^{\frac{7}{2}}}{8960x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{1027b^{\frac{9}{2}}}{5120x^5\sqrt{\frac{a}{bx^2}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**15,x)
```

```
[Out] -a**5/(14*sqrt(b)*x**15*sqrt(a/(b*x**2) + 1)) - 23*a**4*sqrt(b)/(56*x**13*s
qrt(a/(b*x**2) + 1)) - 541*a**3*b**(3/2)/(560*x**11*sqrt(a/(b*x**2) + 1)) -
 5249*a**2*b**(5/2)/(4480*x**9*sqrt(a/(b*x**2) + 1)) - 6653*a*b**(7/2)/(896
0*x**7*sqrt(a/(b*x**2) + 1)) - 1027*b**(9/2)/(5120*x**5*sqrt(a/(b*x**2) + 1
)) + 3*b**(11/2)/(2048*a*x**3*sqrt(a/(b*x**2) + 1)) + 9*b**(13/2)/(2048*a**
2*x*sqrt(a/(b*x**2) + 1)) - 9*b**7*asinh(sqrt(a)/(sqrt(b)*x))/(2048*a**(5/2
))
```

3.425 $\int x^6 \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=202

$$-\frac{45a^8\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32768b^{7/2}}+\frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+b\cdots\right.$$

```
[Out] 3/256*a^3*x^7*(b*x^2+a)^(3/2)+3/128*a^2*x^7*(b*x^2+a)^(5/2)+9/224*a*x^7*(b*
x^2+a)^(7/2)+1/16*x^7*(b*x^2+a)^(9/2)-45/32768*a^8*arctanh(x*b^(1/2)/(b*x^2
+a)^(1/2))/b^(7/2)+45/32768*a^7*x*(b*x^2+a)^(1/2)/b^3-15/16384*a^6*x^3*(b*x
^2+a)^(1/2)/b^2+3/4096*a^5*x^5*(b*x^2+a)^(1/2)/b+9/2048*a^4*x^7*(b*x^2+a)^(
1/2)
```

Rubi [A] time = 0.11, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = $\{279, 321, 217, 206\}$

$$\frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}-\frac{45a^8\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32768b^{7/2}}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^(9/2),x]
```

```
[Out] (45*a^7*x*Sqrt[a + b*x^2])/(32768*b^3) - (15*a^6*x^3*Sqrt[a + b*x^2])/(1638
4*b^2) + (3*a^5*x^5*Sqrt[a + b*x^2])/(4096*b) + (9*a^4*x^7*Sqrt[a + b*x^2])
/2048 + (3*a^3*x^7*(a + b*x^2)^(3/2))/256 + (3*a^2*x^7*(a + b*x^2)^(5/2))/1
28 + (9*a*x^7*(a + b*x^2)^(7/2))/224 + (x^7*(a + b*x^2)^(9/2))/16 - (45*a^8
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(32768*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^6 \left(a+bx^2\right)^{9/2} dx = \frac{1}{16}x^7\left(a+bx^2\right)^{9/2} + \frac{1}{16}(9a)\int x^6\left(a+bx^2\right)^{7/2} dx$$
$$= \frac{9}{224}ax^7\left(a+bx^2\right)^{7/2} + \frac{1}{16}x^7\left(a+bx^2\right)^{9/2} + \frac{1}{32}\left(9a^2\right)\int x^6\left(a+bx^2\right)^{5/2} dx$$
$$= \frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2} + \frac{9}{224}ax^7\left(a+bx^2\right)^{7/2} + \frac{1}{16}x^7\left(a+bx^2\right)^{9/2} + \frac{1}{128}\left(15a^3\right)\int x^6 (a$$
$$= \frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2} + \frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2} + \frac{9}{224}ax^7\left(a+bx^2\right)^{7/2} + \frac{1}{16}x^7\left(a+bx^2\right)^{9/}$$
$$= \frac{9a^4x^7\sqrt{a+bx^2}}{2048} + \frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2} + \frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2} + \frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}$$
$$= \frac{3a^5x^5\sqrt{a+bx^2}}{4096b} + \frac{9a^4x^7\sqrt{a+bx^2}}{2048} + \frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2} + \frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2} + \frac{}{2}$$
$$= -\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2} + \frac{3a^5x^5\sqrt{a+bx^2}}{4096b} + \frac{9a^4x^7\sqrt{a+bx^2}}{2048} + \frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2} + \frac{3}{128}$$
$$= \frac{45a^7x\sqrt{a+bx^2}}{32768b^3} - \frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2} + \frac{3a^5x^5\sqrt{a+bx^2}}{4096b} + \frac{9a^4x^7\sqrt{a+bx^2}}{2048} + \frac{3}{256}a^3x^7($$
$$= \frac{45a^7x\sqrt{a+bx^2}}{32768b^3} - \frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2} + \frac{3a^5x^5\sqrt{a+bx^2}}{4096b} + \frac{9a^4x^7\sqrt{a+bx^2}}{2048} + \frac{3}{256}a^3x^7($$
$$= \frac{45a^7x\sqrt{a+bx^2}}{32768b^3} - \frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2} + \frac{3a^5x^5\sqrt{a+bx^2}}{4096b} + \frac{9a^4x^7\sqrt{a+bx^2}}{2048} + \frac{3}{256}a^3x^7($$

Mathematica [A] time = 0.22, size = 138, normalized size = 0.68

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(315a^7-210a^6bx^2+168a^5b^2x^4+32624a^4b^3x^6+98432a^3b^4x^8+119040a^2b^5x^{10}+66560ab^6\right.\right.}{229376b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(315*a^7 - 210*a^6*b*x^2 + 168*a^5*b^2*x^4 + 32
624*a^4*b^3*x^6 + 98432*a^3*b^4*x^8 + 119040*a^2*b^5*x^10 + 66560*a*b^6*x^1
2 + 14336*b^7*x^14) - (315*a^(15/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 +
(b*x^2)/a]))/(229376*b^(7/2))
```

fricas [A] time = 1.24, size = 255, normalized size = 1.26

$$\left[\frac{315\,a^8\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(14336\,b^8x^{15}+66560\,ab^7x^{13}+119040\,a^2b^6x^{11}+98432\,a^3b\right.}{458752\,b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/458752*(315*a^8*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(14336*b^8*x^15 + 66560*a*b^7*x^13 + 119040*a^2*b^6*x^11 + 98432*a^3*b^
5*x^9 + 32624*a^4*b^4*x^7 + 168*a^5*b^3*x^5 - 210*a^6*b^2*x^3 + 315*a^7*b*x
)*sqrt(b*x^2 + a))/b^4, 1/229376*(315*a^8*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) + (14336*b^8*x^15 + 66560*a*b^7*x^13 + 119040*a^2*b^6*x^11 + 984
```

```
32*a^3*b^5*x^9 + 32624*a^4*b^4*x^7 + 168*a^5*b^3*x^5 - 210*a^6*b^2*x^3 + 31
5*a^7*b*x)*sqrt(b*x^2 + a))/b^4]
```

giac [A] time = 1.21, size = 133, normalized size = 0.66

$$\frac{45\,a^8\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{32768\,b^{\frac{7}{2}}}+\frac{1}{229376}\left(\frac{315\,a^7}{b^3}-2\left(\frac{105\,a^6}{b^2}-4\left(\frac{21\,a^5}{b}+2\left(2039\,a^4+8\left(769\,a^3b+2\left(465\,a^2b^2\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 45/32768*a^8*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2) + 1/229376*(315
*a^7/b^3 - 2*(105*a^6/b^2 - 4*(21*a^5/b + 2*(2039*a^4 + 8*(769*a^3*b + 2*(4
65*a^2*b^2 + 4*(14*b^4*x^2 + 65*a*b^3)*x^2)*x^2)*x^2)*x^2)*x^2)*x^2)*sqrt(b
*x^2 + a)*x
```

maple [A] time = 0.01, size = 169, normalized size = 0.84

$$-\frac{45a^8\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{32768b^{\frac{7}{2}}}-\frac{45\sqrt{b\,x^2+a}\,a^7x}{32768b^3}-\frac{15\left(b\,x^2+a\right)^{\frac{3}{2}}a^6x}{16384b^3}+\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}x^5}{16b}-\frac{3\left(b\,x^2+a\right)^{\frac{5}{2}}a^5x}{4096b^3}-\frac{9\left(b\,x^2\right.}{143}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/16*x^5*(b*x^2+a)^(11/2)/b-5/224*a/b^2*x^3*(b*x^2+a)^(11/2)+5/896*a^2/b^3*
x*(b*x^2+a)^(11/2)-1/1792*a^3/b^3*x*(b*x^2+a)^(9/2)-9/14336*a^4/b^3*x*(b*x^
2+a)^(7/2)-3/4096*a^5/b^3*x*(b*x^2+a)^(5/2)-15/16384*a^6/b^3*x*(b*x^2+a)^(3
/2)-45/32768*a^7*x*(b*x^2+a)^(1/2)/b^3-45/32768*a^8/b^(7/2)*ln(b^(1/2)*x+(b
*x^2+a)^(1/2))
```

maxima [A] time = 1.46, size = 161, normalized size = 0.80

$$\frac{\left(bx^2+a\right)^{\frac{11}{2}}x^5}{16\,b}-\frac{5\left(bx^2+a\right)^{\frac{11}{2}}ax^3}{224\,b^2}+\frac{5\left(bx^2+a\right)^{\frac{11}{2}}a^2x}{896\,b^3}-\frac{\left(bx^2+a\right)^{\frac{9}{2}}a^3x}{1792\,b^3}-\frac{9\left(bx^2+a\right)^{\frac{7}{2}}a^4x}{14336\,b^3}-\frac{3\left(bx^2+a\right)^{\frac{5}{2}}a^5x}{4096\,b^3}-\frac{15\left(bx^2\right.}{163}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/16*(b*x^2 + a)^(11/2)*x^5/b - 5/224*(b*x^2 + a)^(11/2)*a*x^3/b^2 + 5/896*
(b*x^2 + a)^(11/2)*a^2*x/b^3 - 1/1792*(b*x^2 + a)^(9/2)*a^3*x/b^3 - 9/14336
*(b*x^2 + a)^(7/2)*a^4*x/b^3 - 3/4096*(b*x^2 + a)^(5/2)*a^5*x/b^3 - 15/1638
4*(b*x^2 + a)^(3/2)*a^6*x/b^3 - 45/32768*sqrt(b*x^2 + a)*a^7*x/b^3 - 45/327
68*a^8*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\left(b\,x^2+a\right)^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^6*(a + b*x^2)^(9/2), x)
```

sympy [A] time = 30.60, size = 258, normalized size = 1.28

$$\frac{45a^{\frac{15}{2}}x}{32768b^3\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^{\frac{13}{2}}x^3}{32768b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{11}{2}}x^5}{16384b\sqrt{1+\frac{bx^2}{a}}}+\frac{4099a^{\frac{9}{2}}x^7}{28672\sqrt{1+\frac{bx^2}{a}}}+\frac{8191a^{\frac{7}{2}}bx^9}{14336\sqrt{1+\frac{bx^2}{a}}}+\frac{1699a^{\frac{5}{2}}b^2x^{11}}{1792\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**(9/2),x)
```

```
[Out] 45*a**(15/2)*x/(32768*b**3*sqrt(1 + b*x**2/a)) + 15*a**(13/2)*x**3/(32768*b
**2*sqrt(1 + b*x**2/a)) - 3*a**(11/2)*x**5/(16384*b*sqrt(1 + b*x**2/a)) + 4
099*a**(9/2)*x**7/(28672*sqrt(1 + b*x**2/a)) + 8191*a**(7/2)*b*x**9/(14336*
sqrt(1 + b*x**2/a)) + 1699*a**(5/2)*b**2*x**11/(1792*sqrt(1 + b*x**2/a)) +
725*a**(3/2)*b**3*x**13/(896*sqrt(1 + b*x**2/a)) + 79*sqrt(a)*b**4*x**15/(2
24*sqrt(1 + b*x**2/a)) - 45*a**8*asinh(sqrt(b)*x/sqrt(a))/(32768*b**(7/2))
+ b**5*x**17/(16*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.426 $\int x^4 \left(a+bx^2\right)^{9/2}\,dx$

Optimal. Leaf size=178

$$\frac{9a^7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2048b^{5/2}}-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+b\right.$$

```
[Out] 3/128*a^3*x^5*(b*x^2+a)^(3/2)+3/80*a^2*x^5*(b*x^2+a)^(5/2)+3/56*a*x^5*(b*x^
2+a)^(7/2)+1/14*x^5*(b*x^2+a)^(9/2)+9/2048*a^7*arctanh(x*b^(1/2)/(b*x^2+a)^
(1/2))/b^(5/2)-9/2048*a^6*x*(b*x^2+a)^(1/2)/b^2+3/1024*a^5*x^3*(b*x^2+a)^(1
/2)/b+3/256*a^4*x^5*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{9a^7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2048b^{5/2}}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(9/2),x]
```

```
[Out] (-9*a^6*x*Sqrt[a + b*x^2])/(2048*b^2) + (3*a^5*x^3*Sqrt[a + b*x^2])/(1024*b
) + (3*a^4*x^5*Sqrt[a + b*x^2])/256 + (3*a^3*x^5*(a + b*x^2)^(3/2))/128 + (
3*a^2*x^5*(a + b*x^2)^(5/2))/80 + (3*a*x^5*(a + b*x^2)^(7/2))/56 + (x^5*(a
+ b*x^2)^(9/2))/14 + (9*a^7*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2048*b^(
5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^{9/2}dx=\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}+\frac{1}{14}(9a)\int x^4\left(a+bx^2\right)^{7/2}dx$$
$$=\frac{3}{56}ax^5\left(a+bx^2\right)^{7/2}+\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}+\frac{1}{8}\left(3a^2\right)\int x^4\left(a+bx^2\right)^{5/2}dx$$
$$=\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+\frac{3}{56}ax^5\left(a+bx^2\right)^{7/2}+\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}+\frac{1}{16}\left(3a^3\right)\int x^4\left(a+bx\right.$$
$$=\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+\frac{3}{56}ax^5\left(a+bx^2\right)^{7/2}+\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}$$
$$=\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+\frac{3}{56}ax^5\left(a+bx^2\right)^{7/2}$$
$$=\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+$$
$$=-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}$$
$$=-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}$$
$$=-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}$$

Mathematica [A] time = 0.19, size = 127, normalized size = 0.71

$$\frac{\sqrt{a+bx^2}\left(\frac{315a^{13/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\sqrt{b}x\left(-315a^6+210a^5bx^2+14168a^4b^2x^4+39056a^3b^3x^6+44928a^2b^4x^8+243\right.\right.}{71680b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-315*a^6 + 210*a^5*b*x^2 + 14168*a^4*b^2*x^4 +
 39056*a^3*b^3*x^6 + 44928*a^2*b^4*x^8 + 24320*a*b^5*x^10 + 5120*b^6*x^12)
+ (315*a^(13/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(71680*
b^(5/2))
```

fricas [A] time = 1.06, size = 234, normalized size = 1.31

$$\left[\frac{315a^7\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(5120b^7x^{13}+24320ab^6x^{11}+44928a^2b^5x^9+39056a^3b^4x^7\right.}{143360b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/143360*(315*a^7*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(5120*b^7*x^13 + 24320*a*b^6*x^11 + 44928*a^2*b^5*x^9 + 39056*a^3*b^4*x
^7 + 14168*a^4*b^3*x^5 + 210*a^5*b^2*x^3 - 315*a^6*b*x)*sqrt(b*x^2 + a))/b^
3, -1/71680*(315*a^7*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (5120*b^
7*x^13 + 24320*a*b^6*x^11 + 44928*a^2*b^5*x^9 + 39056*a^3*b^4*x^7 + 14168*a
^4*b^3*x^5 + 210*a^5*b^2*x^3 - 315*a^6*b*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 1.17, size = 119, normalized size = 0.67

$$-\frac{9\,a^{7}\log\left(\left|-\sqrt{b}x+\sqrt{bx^{2}+a}\right|\right)}{2048\,b^{\frac{5}{2}}}-\frac{1}{71680}\left(\frac{315\,a^{6}}{b^{2}}-2\left(\frac{105\,a^{5}}{b}+4\left(1771\,a^{4}+2\left(2441\,a^{3}b+8\left(351\,a^{2}b^{2}+10\left(4\,b^{4}x\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -9/2048*a^7*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/71680*(315*a
^6/b^2 - 2*(105*a^5/b + 4*(1771*a^4 + 2*(2441*a^3*b + 8*(351*a^2*b^2 + 10*(
4*b^4*x^2 + 19*a*b^3)*x^2)*x^2)*x^2)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

maple [A] time = 0.01, size = 149, normalized size = 0.84

$$\frac{9a^{7}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2048b^{\frac{5}{2}}}+\frac{9\sqrt{b\,x^{2}+a}\;a^{6}x}{2048b^{2}}+\frac{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{5}x}{1024b^{2}}+\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{2}}a^{4}x}{1280b^{2}}+\frac{9\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{3}x}{4480b^{2}}+\frac{\left(b\,x^{2}+a\right)}{14b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/14*x^3*(b*x^2+a)^(11/2)/b-1/56*a/b^2*x*(b*x^2+a)^(11/2)+1/560*a^2/b^2*x*(
b*x^2+a)^(9/2)+9/4480*a^3/b^2*x*(b*x^2+a)^(7/2)+3/1280*a^4/b^2*x*(b*x^2+a)^
(5/2)+3/1024*a^5/b^2*x*(b*x^2+a)^(3/2)+9/2048*a^6*x*(b*x^2+a)^(1/2)/b^2+9/2
048*a^7/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.40, size = 141, normalized size = 0.79

$$\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}x^{3}}{14\,b}-\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}ax}{56\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{9}{2}}a^{2}x}{560\,b^{2}}+\frac{9\left(bx^{2}+a\right)^{\frac{7}{2}}a^{3}x}{4480\,b^{2}}+\frac{3\left(bx^{2}+a\right)^{\frac{5}{2}}a^{4}x}{1280\,b^{2}}+\frac{3\left(bx^{2}+a\right)^{\frac{3}{2}}a^{5}x}{1024\,b^{2}}+\frac{9\sqrt{bx^{2}+a}}{2048\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/14*(b*x^2 + a)^(11/2)*x^3/b - 1/56*(b*x^2 + a)^(11/2)*a*x/b^2 + 1/560*(b*
x^2 + a)^(9/2)*a^2*x/b^2 + 9/4480*(b*x^2 + a)^(7/2)*a^3*x/b^2 + 3/1280*(b*x
^2 + a)^(5/2)*a^4*x/b^2 + 3/1024*(b*x^2 + a)^(3/2)*a^5*x/b^2 + 9/2048*sqrt(
b*x^2 + a)*a^6*x/b^2 + 9/2048*a^7*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{4}\left(b\,x^{2}+a\right)^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^4*(a + b*x^2)^(9/2), x)
```

sympy [A] time = 20.00, size = 231, normalized size = 1.30

$$-\frac{9a^{\frac{13}{2}}x}{2048b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{3a^{\frac{11}{2}}x^{3}}{2048b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{1027a^{\frac{9}{2}}x^{5}}{5120\sqrt{1+\frac{bx^{2}}{a}}}+\frac{6653a^{\frac{7}{2}}bx^{7}}{8960\sqrt{1+\frac{bx^{2}}{a}}}+\frac{5249a^{\frac{5}{2}}b^{2}x^{9}}{4480\sqrt{1+\frac{bx^{2}}{a}}}+\frac{541a^{\frac{3}{2}}b^{3}x^{11}}{560\sqrt{1+\frac{bx^{2}}{a}}}+\frac{23\sqrt{a}\,b^{4}x}{56\sqrt{1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(9/2),x)
```

```
[Out] -9*a**(13/2)*x/(2048*b**2*sqrt(1 + b*x**2/a)) - 3*a**(11/2)*x**3/(2048*b*sq
rt(1 + b*x**2/a)) + 1027*a**(9/2)*x**5/(5120*sqrt(1 + b*x**2/a)) + 6653*a**
(7/2)*b*x**7/(8960*sqrt(1 + b*x**2/a)) + 5249*a**(5/2)*b**2*x**9/(4480*sqrt
(1 + b*x**2/a)) + 541*a**(3/2)*b**3*x**11/(560*sqrt(1 + b*x**2/a)) + 23*sqr
t(a)*b**4*x**13/(56*sqrt(1 + b*x**2/a)) + 9*a**7*asinh(sqrt(b)*x/sqrt(a))/(
2048*b**(5/2)) + b**5*x**15/(14*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.427 $\int x^2 \left(a+bx^2\right)^{9/2} \, dx$

Optimal. Leaf size=154

$$-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}}+\frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax$$

```
[Out] 7/128*a^3*x^3*(b*x^2+a)^(3/2)+21/320*a^2*x^3*(b*x^2+a)^(5/2)+3/40*a*x^3*(b*
x^2+a)^(7/2)+1/12*x^3*(b*x^2+a)^(9/2)-21/1024*a^6*arctanh(x*b^(1/2)/(b*x^2+
a)^(1/2))/b^(3/2)+21/1024*a^5*x*(b*x^2+a)^(1/2)/b+21/512*a^4*x^3*(b*x^2+a)^
(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}}+\frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(9/2),x]
```

```
[Out] (21*a^5*x*Sqrt[a + b*x^2])/(1024*b) + (21*a^4*x^3*Sqrt[a + b*x^2])/512 + (7
*a^3*x^3*(a + b*x^2)^(3/2))/128 + (21*a^2*x^3*(a + b*x^2)^(5/2))/320 + (3*a
*x^3*(a + b*x^2)^(7/2))/40 + (x^3*(a + b*x^2)^(9/2))/12 - (21*a^6*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^{9/2}dx = \frac{1}{12}x^3\left(a+bx^2\right)^{9/2}+\frac{1}{4}(3a)\int x^2\left(a+bx^2\right)^{7/2}dx$$
$$= \frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}+\frac{1}{40}\left(21a^2\right)\int x^2\left(a+bx^2\right)^{5/2}dx$$
$$= \frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}+\frac{1}{64}\left(21a^3\right)\int x^2\left(a+\right.$$
$$= \frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}$$
$$= \frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(a+bx^2\right)^{7/}$$
$$= \frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+$$
$$= \frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+$$
$$= \frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+$$

Mathematica [A] time = 0.18, size = 116, normalized size = 0.75

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(315a^5+4910a^4bx^2+11432a^3b^2x^4+12144a^2b^3x^6+6272ab^4x^8+1280b^5x^{10}\right)-\frac{315a^{11/2}\sinh^{-1}\left(\right.}{\sqrt{\frac{bx^2}{a}+1}}\right.}{15360b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(315*a^5 + 4910*a^4*b*x^2 + 11432*a^3*b^2*x^4 +
 12144*a^2*b^3*x^6 + 6272*a*b^4*x^8 + 1280*b^5*x^10) - (315*a^(11/2)*ArcSin
h[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(15360*b^(3/2))
```

fricas [A] time = 1.06, size = 211, normalized size = 1.37

$$\left[\frac{315\,a^6\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(1280\,b^6x^{11}+6272\,ab^5x^9+12144\,a^2b^4x^7+11432\,a^3b^3x^5+\right.}{30720\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(315*a^6*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(1280*b^6*x^11 + 6272*a*b^5*x^9 + 12144*a^2*b^4*x^7 + 11432*a^3*b^3*x^5
+ 4910*a^4*b^2*x^3 + 315*a^5*b*x)*sqrt(b*x^2 + a))/b^2, 1/15360*(315*a^6*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (1280*b^6*x^11 + 6272*a*b^5*x^9
 + 12144*a^2*b^4*x^7 + 11432*a^3*b^3*x^5 + 4910*a^4*b^2*x^3 + 315*a^5*b*x)*
sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 1.24, size = 105, normalized size = 0.68

$$\frac{21\,a^6\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{1024\,b^{\frac{3}{2}}}+\frac{1}{15360}\left(\frac{315\,a^5}{b}+2\left(2455\,a^4+4\left(1429\,a^3b+2\left(759\,a^2b^2+8\left(10\,b^4x^2+49\,ab\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 21/1024*a^6*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/15360*(315*a
^5/b + 2*(2455*a^4 + 4*(1429*a^3*b + 2*(759*a^2*b^2 + 8*(10*b^4*x^2 + 49*a*
b^3)*x^2)*x^2)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

maple [A] time = 0.01, size = 129, normalized size = 0.84

$$-\frac{21a^6\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{1024b^{\frac{3}{2}}}-\frac{21\sqrt{b\,x^2+a}\,a^5x}{1024b}-\frac{7\left(b\,x^2+a\right)^{\frac{3}{2}}a^4x}{512b}-\frac{7\left(b\,x^2+a\right)^{\frac{5}{2}}a^3x}{640b}-\frac{3\left(b\,x^2+a\right)^{\frac{7}{2}}a^2x}{320b}-\frac{\left(b\,x^2+\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/12*x*(b*x^2+a)^(11/2)/b-1/120*a/b*x*(b*x^2+a)^(9/2)-3/320*a^2/b*x*(b*x^2+
a)^(7/2)-7/640*a^3/b*x*(b*x^2+a)^(5/2)-7/512*a^4/b*x*(b*x^2+a)^(3/2)-21/102
4*a^5*x*(b*x^2+a)^(1/2)/b-21/1024*a^6/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.37, size = 121, normalized size = 0.79

$$\frac{\left(bx^2+a\right)^{\frac{11}{2}}x}{12\,b}-\frac{\left(bx^2+a\right)^{\frac{9}{2}}ax}{120\,b}-\frac{3\left(bx^2+a\right)^{\frac{7}{2}}a^2x}{320\,b}-\frac{7\left(bx^2+a\right)^{\frac{5}{2}}a^3x}{640\,b}-\frac{7\left(bx^2+a\right)^{\frac{3}{2}}a^4x}{512\,b}-\frac{21\sqrt{bx^2+a}\,a^5x}{1024\,b}-\frac{21\,a^6\,\mathrm{arsinh}}{1024\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b*x^2 + a)^(11/2)*x/b - 1/120*(b*x^2 + a)^(9/2)*a*x/b - 3/320*(b*x^2
+ a)^(7/2)*a^2*x/b - 7/640*(b*x^2 + a)^(5/2)*a^3*x/b - 7/512*(b*x^2 + a)^(3
/2)*a^4*x/b - 21/1024*sqrt(b*x^2 + a)*a^5*x/b - 21/1024*a^6*arcsinh(b*x/sqr
t(a*b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(b\,x^2+a\right)^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^(9/2), x)
```

sympy [A] time = 13.57, size = 204, normalized size = 1.32

$$\frac{21a^{\frac{11}{2}}x}{1024b\sqrt{1+\frac{bx^2}{a}}}+\frac{1045a^{\frac{9}{2}}x^3}{3072\sqrt{1+\frac{bx^2}{a}}}+\frac{8171a^{\frac{7}{2}}bx^5}{7680\sqrt{1+\frac{bx^2}{a}}}+\frac{2947a^{\frac{5}{2}}b^2x^7}{1920\sqrt{1+\frac{bx^2}{a}}}+\frac{1151a^{\frac{3}{2}}b^3x^9}{960\sqrt{1+\frac{bx^2}{a}}}+\frac{59\sqrt{a}\,b^4x^{11}}{120\sqrt{1+\frac{bx^2}{a}}}-\frac{21a^6\,\mathrm{asinh}\left(\frac{\sqrt{\ }}{\sqrt{\ }}\right.}{1024b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(9/2),x)
```

```
[Out] 21*a**(11/2)*x/(1024*b*sqrt(1 + b*x**2/a)) + 1045*a**(9/2)*x**3/(3072*sqrt(
1 + b*x**2/a)) + 8171*a**(7/2)*b*x**5/(7680*sqrt(1 + b*x**2/a)) + 2947*a**(
5/2)*b**2*x**7/(1920*sqrt(1 + b*x**2/a)) + 1151*a**(3/2)*b**3*x**9/(960*sqr
t(1 + b*x**2/a)) + 59*sqrt(a)*b**4*x**11/(120*sqrt(1 + b*x**2/a)) - 21*a**6
*asinh(sqrt(b)*x/sqrt(a))/(1024*b**(3/2)) + b**5*x**13/(12*sqrt(a)*sqrt(1 +
 b*x**2/a))
```

3.428 $\int \left(a+bx^2\right)^{9/2}\,dx$

Optimal. Leaf size=122

$$\frac{63a^5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}}+\frac{63}{256}a^4x\sqrt{a+bx^2}+\frac{21}{128}a^3x\left(a+bx^2\right)^{3/2}+\frac{21}{160}a^2x\left(a+bx^2\right)^{5/2}+\frac{9}{80}ax\left(a+bx^2\right)^{7/2}+\frac{1}{10}x$$

```
[Out] 21/128*a^3*x*(b*x^2+a)^(3/2)+21/160*a^2*x*(b*x^2+a)^(5/2)+9/80*a*x*(b*x^2+a
)^(7/2)+1/10*x*(b*x^2+a)^(9/2)+63/256*a^5*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2)
)/b^(1/2)+63/256*a^4*x*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{63}{256}a^4x\sqrt{a+bx^2}+\frac{21}{128}a^3x\left(a+bx^2\right)^{3/2}+\frac{21}{160}a^2x\left(a+bx^2\right)^{5/2}+\frac{63a^5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}}+\frac{9}{80}ax\left(a+bx^2\right)^{7/2}+\frac{1}{10}x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2),x]
```

```
[Out] (63*a^4*x*Sqrt[a + b*x^2])/256 + (21*a^3*x*(a + b*x^2)^(3/2))/128 + (21*a^2
*x*(a + b*x^2)^(5/2))/160 + (9*a*x*(a + b*x^2)^(7/2))/80 + (x*(a + b*x^2)^(
9/2))/10 + (63*a^5*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{9/2} dx = \frac{1}{10}x\left(a+bx^2\right)^{9/2}+\frac{1}{10}(9a)\int \left(a+bx^2\right)^{7/2} dx$$
$$= \frac{9}{80}ax\left(a+bx^2\right)^{7/2}+\frac{1}{10}x\left(a+bx^2\right)^{9/2}+\frac{1}{80}\left(63a^2\right)\int \left(a+bx^2\right)^{5/2} dx$$
$$= \frac{21}{160}a^2x\left(a+bx^2\right)^{5/2}+\frac{9}{80}ax\left(a+bx^2\right)^{7/2}+\frac{1}{10}x\left(a+bx^2\right)^{9/2}+\frac{1}{32}\left(21a^3\right)\int \left(a+bx^2\right)^{3/2} dx$$
$$= \frac{21}{128}a^3x\left(a+bx^2\right)^{3/2}+\frac{21}{160}a^2x\left(a+bx^2\right)^{5/2}+\frac{9}{80}ax\left(a+bx^2\right)^{7/2}+\frac{1}{10}x\left(a+bx^2\right)^{9/2}+\frac{1}{128}(63$$
$$= \frac{63}{256}a^4x\sqrt{a+bx^2}+\frac{21}{128}a^3x\left(a+bx^2\right)^{3/2}+\frac{21}{160}a^2x\left(a+bx^2\right)^{5/2}+\frac{9}{80}ax\left(a+bx^2\right)^{7/2}+\frac{1}{10}x($$
$$= \frac{63}{256}a^4x\sqrt{a+bx^2}+\frac{21}{128}a^3x\left(a+bx^2\right)^{3/2}+\frac{21}{160}a^2x\left(a+bx^2\right)^{5/2}+\frac{9}{80}ax\left(a+bx^2\right)^{7/2}+\frac{1}{10}x($$
$$= \frac{63}{256}a^4x\sqrt{a+bx^2}+\frac{21}{128}a^3x\left(a+bx^2\right)^{3/2}+\frac{21}{160}a^2x\left(a+bx^2\right)^{5/2}+\frac{9}{80}ax\left(a+bx^2\right)^{7/2}+\frac{1}{10}x($$

Mathematica [A] time = 0.14, size = 98, normalized size = 0.80

$$\frac{\sqrt{a+bx^2}\left(\frac{315a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}+965a^4x+1490a^3bx^3+1368a^2b^2x^5+656ab^3x^7+128b^4x^9\right)}{1280}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(965*a^4*x + 1490*a^3*b*x^3 + 1368*a^2*b^2*x^5 + 656*a*b^3
*x^7 + 128*b^4*x^9 + (315*a^(9/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*Sq
rt[1 + (b*x^2)/a])))/1280
```

fricas [A] time = 0.84, size = 190, normalized size = 1.56

$$\left[\frac{315\,a^5\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(128\,b^5x^9+656\,ab^4x^7+1368\,a^2b^3x^5+1490\,a^3b^2x^3+965\,a^4b\right.}{2560\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(315*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(128*b^5*x^9 + 656*a*b^4*x^7 + 1368*a^2*b^3*x^5 + 1490*a^3*b^2*x^3 + 965*
a^4*b*x)*sqrt(b*x^2 + a))/b, -1/1280*(315*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (128*b^5*x^9 + 656*a*b^4*x^7 + 1368*a^2*b^3*x^5 + 1490*a^3
*b^2*x^3 + 965*a^4*b*x)*sqrt(b*x^2 + a))/b]
```

giac [A] time = 1.12, size = 91, normalized size = 0.75

$$-\frac{63\,a^5\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{256\,\sqrt{b}}+\frac{1}{1280}\left(965\,a^4+2\left(745\,a^3b+4\left(171\,a^2b^2+2\left(8\,b^4x^2+41\,ab^3\right)x^2\right)x^2\right)x^2\right)\sqrt{bx^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -63/256*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/1280*(965*a^
4 + 2*(745*a^3*b + 4*(171*a^2*b^2 + 2*(8*b^4*x^2 + 41*a*b^3)*x^2)*x^2)*x^2)
*sqrt(b*x^2 + a)*x
```

maple [A] time = 0.00, size = 96, normalized size = 0.79

$$\frac{63a^5\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{256\sqrt{b}}+\frac{63\sqrt{b\,x^2+a}\;a^4x}{256}+\frac{21\left(b\,x^2+a\right)^{\frac{3}{2}}a^3x}{128}+\frac{21\left(b\,x^2+a\right)^{\frac{5}{2}}a^2x}{160}+\frac{9\left(b\,x^2+a\right)^{\frac{7}{2}}ax}{80}+\frac{(b\,x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2),x)
```

```
[Out] 1/10*x*(b*x^2+a)^(9/2)+9/80*a*x*(b*x^2+a)^(7/2)+21/160*a^2*x*(b*x^2+a)^(5/2
)+21/128*a^3*x*(b*x^2+a)^(3/2)+63/256*a^4*x*(b*x^2+a)^(1/2)+63/256*a^5/b^(1
/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.32, size = 88, normalized size = 0.72

$$\frac{1}{10}\left(bx^2+a\right)^{\frac{9}{2}}x+\frac{9}{80}\left(bx^2+a\right)^{\frac{7}{2}}ax+\frac{21}{160}\left(bx^2+a\right)^{\frac{5}{2}}a^2x+\frac{21}{128}\left(bx^2+a\right)^{\frac{3}{2}}a^3x+\frac{63}{256}\sqrt{bx^2+a}\,a^4x+\frac{63\,a^5\,\mathrm{arsinh}\left(\right.}{256\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(9/2)*x + 9/80*(b*x^2 + a)^(7/2)*a*x + 21/160*(b*x^2 + a)^
(5/2)*a^2*x + 21/128*(b*x^2 + a)^(3/2)*a^3*x + 63/256*sqrt(b*x^2 + a)*a^4*x
 + 63/256*a^5*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 4.62, size = 37, normalized size = 0.30

$$\frac{x\left(b\,x^2+a\right)^{9/2}{}_2\mathrm{F}_1\left(-\frac{9}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2),x)
```

```
[Out] (x*(a + b*x^2)^(9/2)*hypergeom([-9/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(9/2)
```

sympy [A] time = 8.35, size = 151, normalized size = 1.24

$$\frac{193a^{\frac{9}{2}}x\sqrt{1+\frac{bx^2}{a}}}{256}+\frac{149a^{\frac{7}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{128}+\frac{171a^{\frac{5}{2}}b^2x^5\sqrt{1+\frac{bx^2}{a}}}{160}+\frac{41a^{\frac{3}{2}}b^3x^7\sqrt{1+\frac{bx^2}{a}}}{80}+\frac{\sqrt{a}\,b^4x^9\sqrt{1+\frac{bx^2}{a}}}{10}+\frac{63a^5\,\mathrm{a}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2),x)
```

```
[Out] 193*a**(9/2)*x*sqrt(1 + b*x**2/a)/256 + 149*a**(7/2)*b*x**3*sqrt(1 + b*x**2
/a)/128 + 171*a**(5/2)*b**2*x**5*sqrt(1 + b*x**2/a)/160 + 41*a**(3/2)*b**3*
x**7*sqrt(1 + b*x**2/a)/80 + sqrt(a)*b**4*x**9*sqrt(1 + b*x**2/a)/10 + 63*a
**5*asinh(sqrt(b)*x/sqrt(a))/(256*sqrt(b))
```

3.429 $\int \frac{(a+bx^2)^{9/2}}{x^2}\,dx$

Optimal. Leaf size=123

$$\frac{315}{128}a^4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{315}{128}a^3bx\sqrt{a+bx^2}+\frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{9/2}}{x}+\frac{9}{8}bx\left(a+bx^2\right)^{7/2}+\frac{21}{16}abx$$

```
[Out] 105/64*a^2*b*x*(b*x^2+a)^(3/2)+21/16*a*b*x*(b*x^2+a)^(5/2)+9/8*b*x*(b*x^2+a
)^(7/2)-(b*x^2+a)^(9/2)/x+315/128*a^4*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^
(1/2)+315/128*a^3*b*x*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$\frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2}+\frac{315}{128}a^3bx\sqrt{a+bx^2}+\frac{315}{128}a^4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{x}+\frac{9}{8}bx\left(a+bx^2\right)^{7/2}+\frac{21}{16}abx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^2,x]
```

```
[Out] (315*a^3*b*x*Sqrt[a + b*x^2])/128 + (105*a^2*b*x*(a + b*x^2)^(3/2))/64 + (2
1*a*b*x*(a + b*x^2)^(5/2))/16 + (9*b*x*(a + b*x^2)^(7/2))/8 - (a + b*x^2)^(
9/2)/x + (315*a^4*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/128
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^2}\,dx = -\frac{\left(a+bx^2\right)^{9/2}}{x} + (9b)\int \left(a+bx^2\right)^{7/2}\,dx$$

$$= \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{x} + \frac{1}{8}(63ab)\int \left(a+bx^2\right)^{5/2}\,dx$$

$$= \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{x} + \frac{1}{16}\left(105a^2b\right)\int \left(a+bx^2\right)^{3/2}\,dx$$

$$= \frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2} + \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{x} + \frac{1}{64}\left(315a\right.$$

$$= \frac{315}{128}a^3bx\sqrt{a+bx^2} + \frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2} + \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{(a}{}$$

$$= \frac{315}{128}a^3bx\sqrt{a+bx^2} + \frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2} + \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{(a}{}$$

$$= \frac{315}{128}a^3bx\sqrt{a+bx^2} + \frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2} + \frac{21}{16}abx\left(a+bx^2\right)^{5/2} + \frac{9}{8}bx\left(a+bx^2\right)^{7/2} - \frac{(a}{}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.42

$$-\frac{a^4\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{9}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^2,x]
```

```
[Out] -((a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -1/2, 1/2, -((b*x^2)/a)])/(x
*Sqrt[1 + (b*x^2)/a]))
```

fricas [A] time = 0.92, size = 184, normalized size = 1.50

$$\left[\frac{315\,a^4\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(16\,b^4x^8+88\,ab^3x^6+210\,a^2b^2x^4+325\,a^3bx^2-128\,a^4\right)\sqrt{b}}{256\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/256*(315*a^4*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(16*b^4*x^8 + 88*a*b^3*x^6 + 210*a^2*b^2*x^4 + 325*a^3*b*x^2 - 128*a^4)*
sqrt(b*x^2 + a))/x, -1/128*(315*a^4*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (16*b^4*x^8 + 88*a*b^3*x^6 + 210*a^2*b^2*x^4 + 325*a^3*b*x^2 - 128
*a^4)*sqrt(b*x^2 + a))/x]
```

giac [A] time = 1.22, size = 115, normalized size = 0.93

$$-\frac{315}{256}\,a^4\sqrt{b}\,\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,a^5\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{128}\left(325\,a^3b+2\left(105\,a^2b^2+4\left(2\,b^4x^2+11\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="giac")
```

```
[Out] -315/256*a^4*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a^5*sqrt(b)/(
(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/128*(325*a^3*b + 2*(105*a^2*b^2 +
4*(2*b^4*x^2 + 11*a*b^3)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

maple [A] time = 0.00, size = 117, normalized size = 0.95

$$\frac{315a^4\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{128}+\frac{315\sqrt{b\,x^2+a}\,a^3bx}{128}+\frac{105\left(b\,x^2+a\right)^{\frac{3}{2}}a^2bx}{64}+\frac{21\left(b\,x^2+a\right)^{\frac{5}{2}}abx}{16}+\frac{9\left(b\,x^2+a\right)^{\frac{7}{2}}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^2,x)
```

```
[Out] -1/a/x*(b*x^2+a)^(11/2)+1/a*b*x*(b*x^2+a)^(9/2)+9/8*b*x*(b*x^2+a)^(7/2)+21/
16*a*b*x*(b*x^2+a)^(5/2)+105/64*a^2*b*x*(b*x^2+a)^(3/2)+315/128*a^3*b*x*(b*
x^2+a)^(1/2)+315/128*a^4*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.46, size = 91, normalized size = 0.74

$$\frac{9}{8}\left(bx^2+a\right)^{\frac{7}{2}}bx+\frac{21}{16}\left(bx^2+a\right)^{\frac{5}{2}}abx+\frac{105}{64}\left(bx^2+a\right)^{\frac{3}{2}}a^2bx+\frac{315}{128}\sqrt{bx^2+a}\,a^3bx+\frac{315}{128}a^4\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\left(bx^2+\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="maxima")
```

```
[Out] 9/8*(b*x^2 + a)^(7/2)*b*x + 21/16*(b*x^2 + a)^(5/2)*a*b*x + 105/64*(b*x^2 +
 a)^(3/2)*a^2*b*x + 315/128*sqrt(b*x^2 + a)*a^3*b*x + 315/128*a^4*sqrt(b)*a
rcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(9/2)/x
```

mupad [B] time = 6.03, size = 40, normalized size = 0.33

$$-\frac{\left(b\,x^2+a\right)^{9/2}\,{}_2\mathrm{F}_1\left(-\frac{9}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\left(\frac{b\,x^2}{a}+1\right)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^2,x)
```

```
[Out] -((a + b*x^2)^(9/2)*hypergeom([-9/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a
 + 1)^(9/2))
```

sympy [A] time = 7.47, size = 173, normalized size = 1.41

$$-\frac{a^{\frac{9}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{197a^{\frac{7}{2}}bx}{128\sqrt{1+\frac{bx^2}{a}}}+\frac{535a^{\frac{5}{2}}b^2x^3}{128\sqrt{1+\frac{bx^2}{a}}}+\frac{149a^{\frac{3}{2}}b^3x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{13\sqrt{a}\,b^4x^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{315a^4\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128}+\frac{b^5x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**2,x)
```

```
[Out] -a**(9/2)/(x*sqrt(1 + b*x**2/a)) + 197*a**(7/2)*b*x/(128*sqrt(1 + b*x**2/a)
) + 535*a**(5/2)*b**2*x**3/(128*sqrt(1 + b*x**2/a)) + 149*a**(3/2)*b**3*x**
5/(64*sqrt(1 + b*x**2/a)) + 13*sqrt(a)*b**4*x**7/(16*sqrt(1 + b*x**2/a)) +
315*a**4*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/128 + b**5*x**9/(8*sqrt(a)*sqrt(1
 + b*x**2/a))
```

3.430 $\int \frac{(a+bx^2)^{9/2}}{x^4}\,dx$

Optimal. Leaf size=128

$$\frac{105}{16}a^3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{105}{16}a^2b^2x\sqrt{a+bx^2}+\frac{7}{2}b^2x\left(a+bx^2\right)^{5/2}+\frac{35}{8}ab^2x\left(a+bx^2\right)^{3/2}-\frac{3b\left(a+bx^2\right)^{7/2}}{x}-$$

```
[Out] 35/8*a*b^2*x*(b*x^2+a)^(3/2)+7/2*b^2*x*(b*x^2+a)^(5/2)-3*b*(b*x^2+a)^(7/2)/
x-1/3*(b*x^2+a)^(9/2)/x^3+105/16*a^3*b^(3/2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1
/2))+105/16*a^2*b^2*x*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$\frac{105}{16}a^2b^2x\sqrt{a+bx^2}+\frac{105}{16}a^3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{7}{2}b^2x\left(a+bx^2\right)^{5/2}+\frac{35}{8}ab^2x\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{9/2}}{3x^3}-\frac{3b}{}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^4,x]
```

```
[Out] (105*a^2*b^2*x*Sqrt[a + b*x^2])/16 + (35*a*b^2*x*(a + b*x^2)^(3/2))/8 + (7*
b^2*x*(a + b*x^2)^(5/2))/2 - (3*b*(a + b*x^2)^(7/2))/x - (a + b*x^2)^(9/2)/
(3*x^3) + (105*a^3*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/16
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^4} dx = -\frac{(a+bx^2)^{9/2}}{3x^3} + (3b)\int \frac{(a+bx^2)^{7/2}}{x^2} dx$$
$$= -\frac{3b(a+bx^2)^{7/2}}{x} - \frac{(a+bx^2)^{9/2}}{3x^3} + (21b^2)\int (a+bx^2)^{5/2} dx$$
$$= \frac{7}{2}b^2x(a+bx^2)^{5/2} - \frac{3b(a+bx^2)^{7/2}}{x} - \frac{(a+bx^2)^{9/2}}{3x^3} + \frac{1}{2}(35ab^2)\int (a+bx^2)^{3/2} dx$$
$$= \frac{35}{8}ab^2x(a+bx^2)^{3/2} + \frac{7}{2}b^2x(a+bx^2)^{5/2} - \frac{3b(a+bx^2)^{7/2}}{x} - \frac{(a+bx^2)^{9/2}}{3x^3} + \frac{1}{8}(105a^2b^2)\int$$
$$= \frac{105}{16}a^2b^2x\sqrt{a+bx^2} + \frac{35}{8}ab^2x(a+bx^2)^{3/2} + \frac{7}{2}b^2x(a+bx^2)^{5/2} - \frac{3b(a+bx^2)^{7/2}}{x} - \frac{(a+bx^2}{3x^3}$$
$$= \frac{105}{16}a^2b^2x\sqrt{a+bx^2} + \frac{35}{8}ab^2x(a+bx^2)^{3/2} + \frac{7}{2}b^2x(a+bx^2)^{5/2} - \frac{3b(a+bx^2)^{7/2}}{x} - \frac{(a+bx^2}{3x^3}$$
$$= \frac{105}{16}a^2b^2x\sqrt{a+bx^2} + \frac{35}{8}ab^2x(a+bx^2)^{3/2} + \frac{7}{2}b^2x(a+bx^2)^{5/2} - \frac{3b(a+bx^2)^{7/2}}{x} - \frac{(a+bx^2}{3x^3}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.42

$$-\frac{a^4\sqrt{a+bx^2}\ {}_2F_1\left(-\frac{9}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^4,x]
```

```
[Out] -1/3*(a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -3/2, -1/2, -((b*x^2)/a)]
)/(x^3*Sqrt[1 + (b*x^2)/a])
```

fricas [A] time = 1.20, size = 189, normalized size = 1.48

$$\left[\frac{315\,a^3b^{\frac{3}{2}}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(8\,b^4x^8+50\,ab^3x^6+165\,a^2b^2x^4-208\,a^3bx^2-16\,a^4\right)\sqrt{bx^2+}}{96\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(315*a^3*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(8*b^4*x^8 + 50*a*b^3*x^6 + 165*a^2*b^2*x^4 - 208*a^3*b*x^2 - 16*a^4)*s
qrt(b*x^2 + a))/x^3, -1/48*(315*a^3*sqrt(-b)*b*x^3*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (8*b^4*x^8 + 50*a*b^3*x^6 + 165*a^2*b^2*x^4 - 208*a^3*b*x^2 -
16*a^4)*sqrt(b*x^2 + a))/x^3]
```

giac [A] time = 1.15, size = 160, normalized size = 1.25

$$-\frac{105}{32}a^3b^{\frac{3}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{1}{48}\left(165\,a^2b^2+2\left(4\,b^4x^2+25\,ab^3\right)x^2\right)\sqrt{bx^2+a}\,x+\frac{2\left(15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right.\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="giac")
```

```
[Out] -105/32*a^3*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 1/48*(165*a^2*b^
2 + 2*(4*b^4*x^2 + 25*a*b^3)*x^2)*sqrt(b*x^2 + a)*x + 2/3*(15*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*a^4*b^(3/2) - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(
3/2) + 13*a^6*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

maple [A] time = 0.01, size = 146, normalized size = 1.14

$$\frac{105a^3b^{\frac{3}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16}+\frac{105\sqrt{b\,x^2+a}\;a^2b^2x}{16}+\frac{35\left(b\,x^2+a\right)^{\frac{3}{2}}a\,b^2x}{8}+\frac{7\left(b\,x^2+a\right)^{\frac{5}{2}}b^2x}{2}+\frac{3\left(b\,x^2+a\right)^{\frac{7}{2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^4,x)
```

```
[Out] -1/3/a/x^3*(b*x^2+a)^(11/2)-8/3/a^2*b/x*(b*x^2+a)^(11/2)+8/3/a^2*b^2*x*(b*x
^2+a)^(9/2)+3/a*b^2*x*(b*x^2+a)^(7/2)+7/2*b^2*x*(b*x^2+a)^(5/2)+35/8*a*b^2*
x*(b*x^2+a)^(3/2)+105/16*a^2*b^2*x*(b*x^2+a)^(1/2)+105/16*a^3*b^(3/2)*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.42, size = 120, normalized size = 0.94

$$\frac{7}{2}\left(bx^2+a\right)^{\frac{5}{2}}b^2x+\frac{3\left(bx^2+a\right)^{\frac{7}{2}}b^2x}{a}+\frac{35}{8}\left(bx^2+a\right)^{\frac{3}{2}}ab^2x+\frac{105}{16}\sqrt{bx^2+a}\,a^2b^2x+\frac{105}{16}a^3b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{8\left(bx^2\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="maxima")
```

```
[Out] 7/2*(b*x^2 + a)^(5/2)*b^2*x + 3*(b*x^2 + a)^(7/2)*b^2*x/a + 35/8*(b*x^2 + a
)^(3/2)*a*b^2*x + 105/16*sqrt(b*x^2 + a)*a^2*b^2*x + 105/16*a^3*b^(3/2)*arc
sinh(b*x/sqrt(a*b)) - 8/3*(b*x^2 + a)^(9/2)*b/(a*x) - 1/3*(b*x^2 + a)^(11/2
)/(a*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{9/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^4,x)
```

```
[Out] int((a + b*x^2)^(9/2)/x^4, x)
```

sympy [A] time = 6.84, size = 175, normalized size = 1.37

$$-\frac{a^{\frac{9}{2}}}{3x^3\sqrt{1+\frac{bx^2}{a}}}-\frac{14a^{\frac{7}{2}}b}{3x\sqrt{1+\frac{bx^2}{a}}}-\frac{43a^{\frac{5}{2}}b^2x}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{215a^{\frac{3}{2}}b^3x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{29\sqrt{a}\,b^4x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{105a^3b^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16}+\frac{b^5x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**4,x)
```

```
[Out] -a**(9/2)/(3*x**3*sqrt(1 + b*x**2/a)) - 14*a**(7/2)*b/(3*x*sqrt(1 + b*x**2/
a)) - 43*a**(5/2)*b**2*x/(48*sqrt(1 + b*x**2/a)) + 215*a**(3/2)*b**3*x**3/(
48*sqrt(1 + b*x**2/a)) + 29*sqrt(a)*b**4*x**5/(24*sqrt(1 + b*x**2/a)) + 105
*a**3*b**(3/2)*asinh(sqrt(b)*x/sqrt(a))/16 + b**5*x**7/(6*sqrt(a)*sqrt(1 +
b*x**2/a))
```

3.431 $\int \frac{(a+bx^2)^{9/2}}{x^6} \, dx$

Optimal. Leaf size=129

$$\frac{63}{8}a^2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{21}{4}b^3x\left(a+bx^2\right)^{3/2}+\frac{63}{8}ab^3x\sqrt{a+bx^2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{5x}-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}-\frac{3b\left(a+bx^2\right)^{7/2}}{5x^3}$$

```
[Out] 21/4*b^3*x*(b*x^2+a)^(3/2)-21/5*b^2*(b*x^2+a)^(5/2)/x-3/5*b*(b*x^2+a)^(7/2)
/x^3-1/5*(b*x^2+a)^(9/2)/x^5+63/8*a^2*b^(5/2)*arctanh(x*b^(1/2)/(b*x^2+a)^(
1/2))+63/8*a*b^3*x*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$\frac{63}{8}a^2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{21b^2\left(a+bx^2\right)^{5/2}}{5x}+\frac{21}{4}b^3x\left(a+bx^2\right)^{3/2}+\frac{63}{8}ab^3x\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}-\frac{3b\left(a+bx^2\right)^{7/2}}{5x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^6,x]
```

```
[Out] (63*a*b^3*x*Sqrt[a + b*x^2])/8 + (21*b^3*x*(a + b*x^2)^(3/2))/4 - (21*b^2*(
a + b*x^2)^(5/2))/(5*x) - (3*b*(a + b*x^2)^(7/2))/(5*x^3) - (a + b*x^2)^(9/
2)/(5*x^5) + (63*a^2*b^(5/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^6}\,dx = -\frac{\left(a+bx^2\right)^{9/2}}{5x^5} + \frac{1}{5}(9b)\int \frac{\left(a+bx^2\right)^{7/2}}{x^4}\,dx$$
$$= -\frac{3b\left(a+bx^2\right)^{7/2}}{5x^3} - \frac{\left(a+bx^2\right)^{9/2}}{5x^5} + \frac{1}{5}\left(21b^2\right)\int \frac{\left(a+bx^2\right)^{5/2}}{x^2}\,dx$$
$$= -\frac{21b^2\left(a+bx^2\right)^{5/2}}{5x} - \frac{3b\left(a+bx^2\right)^{7/2}}{5x^3} - \frac{\left(a+bx^2\right)^{9/2}}{5x^5} + \left(21b^3\right)\int \left(a+bx^2\right)^{3/2}\,dx$$
$$= \frac{21}{4}b^3x\left(a+bx^2\right)^{3/2} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{5x} - \frac{3b\left(a+bx^2\right)^{7/2}}{5x^3} - \frac{\left(a+bx^2\right)^{9/2}}{5x^5} + \frac{1}{4}\left(63ab^3\right)\int \sqrt{}$$
$$= \frac{63}{8}ab^3x\sqrt{a+bx^2} + \frac{21}{4}b^3x\left(a+bx^2\right)^{3/2} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{5x} - \frac{3b\left(a+bx^2\right)^{7/2}}{5x^3} - \frac{\left(a+bx^2\right)}{5x^5}$$
$$= \frac{63}{8}ab^3x\sqrt{a+bx^2} + \frac{21}{4}b^3x\left(a+bx^2\right)^{3/2} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{5x} - \frac{3b\left(a+bx^2\right)^{7/2}}{5x^3} - \frac{\left(a+bx^2\right)}{5x^5}$$
$$= \frac{63}{8}ab^3x\sqrt{a+bx^2} + \frac{21}{4}b^3x\left(a+bx^2\right)^{3/2} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{5x} - \frac{3b\left(a+bx^2\right)^{7/2}}{5x^3} - \frac{\left(a+bx^2\right)}{5x^5}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.42

$$-\frac{a^4\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{9}{2},-\frac{5}{2};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^6,x]
```

```
[Out] -1/5*(a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -5/2, -3/2, -((b*x^2)/a)]
)/(x^5*Sqrt[1 + (b*x^2)/a])
```

fricas [A] time = 1.04, size = 191, normalized size = 1.48

$$\left[\frac{315\,a^2b^{\frac{5}{2}}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(10\,b^4x^8+85\,ab^3x^6-288\,a^2b^2x^4-56\,a^3bx^2-8\,a^4\right)\sqrt{bx^2}}{80\,x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/80*(315*a^2*b^(5/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(10*b^4*x^8 + 85*a*b^3*x^6 - 288*a^2*b^2*x^4 - 56*a^3*b*x^2 - 8*a^4)*sq
rt(b*x^2 + a))/x^5, -1/40*(315*a^2*sqrt(-b)*b^2*x^5*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) - (10*b^4*x^8 + 85*a*b^3*x^6 - 288*a^2*b^2*x^4 - 56*a^3*b*x^2 -
 8*a^4)*sqrt(b*x^2 + a))/x^5]
```

giac [A] time = 1.08, size = 200, normalized size = 1.55

$$-\frac{63}{16}\,a^2b^{\frac{5}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{1}{8}\left(2\,b^4x^2+17\,ab^3\right)\sqrt{bx^2+a}\,x+\frac{4\left(25\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8a^3b^{\frac{5}{2}}-75\left(\sqrt{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="giac")
```

```
[Out] -63/16*a^2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 1/8*(2*b^4*x^2 +
17*a*b^3)*sqrt(b*x^2 + a)*x + 4/5*(25*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b
^(5/2) - 75*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(5/2) + 105*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*a^5*b^(5/2) - 65*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(
5/2) + 18*a^7*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

maple [A] time = 0.01, size = 166, normalized size = 1.29

$$\frac{63a^2b^{\frac{5}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8}+\frac{63\sqrt{b\,x^2+a}\;a\,b^3x}{8}+\frac{21\left(b\,x^2+a\right)^{\frac{3}{2}}b^3x}{4}+\frac{21\left(b\,x^2+a\right)^{\frac{5}{2}}b^3x}{5a}+\frac{18\left(b\,x^2+a\right)^{\frac{7}{2}}b^3x}{5a^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^6,x)
```

```
[Out] -1/5/a/x^5*(b*x^2+a)^(11/2)-2/5/a^2*b/x^3*(b*x^2+a)^(11/2)-16/5/a^3*b^2/x*(
b*x^2+a)^(11/2)+16/5/a^3*b^3*x*(b*x^2+a)^(9/2)+18/5/a^2*b^3*x*(b*x^2+a)^(7/
2)+21/5/a*b^3*x*(b*x^2+a)^(5/2)+21/4*b^3*x*(b*x^2+a)^(3/2)+63/8*a*b^3*x*(b*
x^2+a)^(1/2)+63/8*a^2*b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.45, size = 140, normalized size = 1.09

$$\frac{21}{4}\left(bx^2+a\right)^{\frac{3}{2}}b^3x+\frac{18\left(bx^2+a\right)^{\frac{7}{2}}b^3x}{5\,a^2}+\frac{21\left(bx^2+a\right)^{\frac{5}{2}}b^3x}{5\,a}+\frac{63}{8}\sqrt{bx^2+a}\,ab^3x+\frac{63}{8}\,a^2b^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{16\left(bx^2+a\right.}{5\,a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="maxima")
```

```
[Out] 21/4*(b*x^2 + a)^(3/2)*b^3*x + 18/5*(b*x^2 + a)^(7/2)*b^3*x/a^2 + 21/5*(b*x
^2 + a)^(5/2)*b^3*x/a + 63/8*sqrt(b*x^2 + a)*a*b^3*x + 63/8*a^2*b^(5/2)*arc
sinh(b*x/sqrt(a*b)) - 16/5*(b*x^2 + a)^(9/2)*b^2/(a^2*x) - 2/5*(b*x^2 + a)^
(11/2)*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(11/2)/(a*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{9/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^6,x)
```

```
[Out] int((a + b*x^2)^(9/2)/x^6, x)
```

sympy [A] time = 7.82, size = 175, normalized size = 1.36

$$-\frac{a^{\frac{9}{2}}}{5x^5\sqrt{1+\frac{bx^2}{a}}}-\frac{8a^{\frac{7}{2}}b}{5x^3\sqrt{1+\frac{bx^2}{a}}}-\frac{43a^{\frac{5}{2}}b^2}{5x\sqrt{1+\frac{bx^2}{a}}}-\frac{203a^{\frac{3}{2}}b^3x}{40\sqrt{1+\frac{bx^2}{a}}}+\frac{19\sqrt{a}\,b^4x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{63a^2b^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8}+\frac{b^5x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**6,x)
```

```
[Out] -a**(9/2)/(5*x**5*sqrt(1 + b*x**2/a)) - 8*a**(7/2)*b/(5*x**3*sqrt(1 + b*x**
2/a)) - 43*a**(5/2)*b**2/(5*x*sqrt(1 + b*x**2/a)) - 203*a**(3/2)*b**3*x/(40
*sqrt(1 + b*x**2/a)) + 19*sqrt(a)*b**4*x**3/(8*sqrt(1 + b*x**2/a)) + 63*a**
2*b**(5/2)*asinh(sqrt(b)*x/sqrt(a))/8 + b**5*x**5/(4*sqrt(a)*sqrt(1 + b*x**
2/a))
```

3.432 $\int \frac{(a+bx^2)^{9/2}}{x^8} dx$

Optimal. Leaf size=126

$$\frac{9}{2}ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{9}{2}b^4x\sqrt{a+bx^2}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{x}-\frac{3b^2\left(a+bx^2\right)^{5/2}}{5x^3}-\frac{\left(a+bx^2\right)^{9/2}}{7x^7}-\frac{9b\left(a+bx^2\right)^{7/2}}{35x^5}$$

```
[Out] -3*b^3*(b*x^2+a)^(3/2)/x-3/5*b^2*(b*x^2+a)^(5/2)/x^3-9/35*b*(b*x^2+a)^(7/2)
/x^5-1/7*(b*x^2+a)^(9/2)/x^7+9/2*a*b^(7/2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2
))+9/2*b^4*x*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$-\frac{3b^2\left(a+bx^2\right)^{5/2}}{5x^3}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{x}+\frac{9}{2}b^4x\sqrt{a+bx^2}+\frac{9}{2}ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{7x^7}-\frac{9b\left(a+bx^2\right)^{7/2}}{35x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^8,x]
```

```
[Out] (9*b^4*x*Sqrt[a + b*x^2])/2 - (3*b^3*(a + b*x^2)^(3/2))/x - (3*b^2*(a + b*x
^2)^(5/2))/(5*x^3) - (9*b*(a + b*x^2)^(7/2))/(35*x^5) - (a + b*x^2)^(9/2)/(
7*x^7) + (9*a*b^(7/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^8}\,dx = -\frac{(a+bx^2)^{9/2}}{7x^7} + \frac{1}{7}(9b)\int \frac{(a+bx^2)^{7/2}}{x^6}\,dx$$
$$= -\frac{9b(a+bx^2)^{7/2}}{35x^5} - \frac{(a+bx^2)^{9/2}}{7x^7} + \frac{1}{5}(9b^2)\int \frac{(a+bx^2)^{5/2}}{x^4}\,dx$$
$$= -\frac{3b^2(a+bx^2)^{5/2}}{5x^3} - \frac{9b(a+bx^2)^{7/2}}{35x^5} - \frac{(a+bx^2)^{9/2}}{7x^7} + (3b^3)\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx$$
$$= -\frac{3b^3(a+bx^2)^{3/2}}{x} - \frac{3b^2(a+bx^2)^{5/2}}{5x^3} - \frac{9b(a+bx^2)^{7/2}}{35x^5} - \frac{(a+bx^2)^{9/2}}{7x^7} + (9b^4)\int \sqrt{a+bx^2}\,d$$
$$= \frac{9}{2}b^4x\sqrt{a+bx^2} - \frac{3b^3(a+bx^2)^{3/2}}{x} - \frac{3b^2(a+bx^2)^{5/2}}{5x^3} - \frac{9b(a+bx^2)^{7/2}}{35x^5} - \frac{(a+bx^2)^{9/2}}{7x^7} + \frac{1}{2}($$
$$= \frac{9}{2}b^4x\sqrt{a+bx^2} - \frac{3b^3(a+bx^2)^{3/2}}{x} - \frac{3b^2(a+bx^2)^{5/2}}{5x^3} - \frac{9b(a+bx^2)^{7/2}}{35x^5} - \frac{(a+bx^2)^{9/2}}{7x^7} + \frac{1}{2}($$
$$= \frac{9}{2}b^4x\sqrt{a+bx^2} - \frac{3b^3(a+bx^2)^{3/2}}{x} - \frac{3b^2(a+bx^2)^{5/2}}{5x^3} - \frac{9b(a+bx^2)^{7/2}}{35x^5} - \frac{(a+bx^2)^{9/2}}{7x^7} + \frac{9}{2}ab$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.43

$$-\frac{a^4\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{9}{2},-\frac{7}{2};-\frac{5}{2};-\frac{bx^2}{a}\right)}{7x^7\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^8,x]
```

```
[Out] -1/7*(a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -7/2, -5/2, -((b*x^2)/a)]
)/(x^7*Sqrt[1 + (b*x^2)/a])
```

fricas [A] time = 1.17, size = 187, normalized size = 1.48

$$\left[\frac{315\,ab^{\frac{7}{2}}x^7\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{b}\,x - a\right) + 2\left(35\,b^4x^8 - 388\,ab^3x^6 - 156\,a^2b^2x^4 - 58\,a^3bx^2 - 10\,a^4\right)\sqrt{bx^2+}}{140\,x^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/140*(315*a*b^(7/2)*x^7*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(35*b^4*x^8 - 388*a*b^3*x^6 - 156*a^2*b^2*x^4 - 58*a^3*b*x^2 - 10*a^4)*s
qrt(b*x^2 + a))/x^7, -1/70*(315*a*sqrt(-b)*b^3*x^7*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (35*b^4*x^8 - 388*a*b^3*x^6 - 156*a^2*b^2*x^4 - 58*a^3*b*x^2 -
 10*a^4)*sqrt(b*x^2 + a))/x^7]
```

giac [B] time = 1.20, size = 240, normalized size = 1.90

$$\frac{1}{2}\sqrt{bx^2+a}\,b^4x - \frac{9}{4}ab^{\frac{7}{2}}\log\left(\left(\sqrt{b}\,x - \sqrt{bx^2+a}\right)^2\right) + \frac{4\left(175\left(\sqrt{b}\,x - \sqrt{bx^2+a}\right)^{12}a^2b^{\frac{7}{2}} - 700\left(\sqrt{b}\,x - \sqrt{bx^2+a}\right)^{10}a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b^4*x - 9/4*a*b^(7/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))
^2) + 4/35*(175*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b^(7/2) - 700*(sqrt(b)
*x - sqrt(b*x^2 + a))^10*a^3*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^8
*a^4*b^(7/2) - 1820*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*b^(7/2) + 1337*(sqr
t(b)*x - sqrt(b*x^2 + a))^4*a^6*b^(7/2) - 504*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*a^7*b^(7/2) + 97*a^8*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

maple [A] time = 0.02, size = 186, normalized size = 1.48

$$\frac{9 a\, b^{\frac{7}{2}} \ln\left(\sqrt{b}\, x+\sqrt{b\, x^{2}+a}\right)}{2}+\frac{9 \sqrt{b\, x^{2}+a}\; b^{4} x}{2}+\frac{3 \left(b\, x^{2}+a\right)^{\frac{3}{2}} b^{4} x}{a}+\frac{12 \left(b\, x^{2}+a\right)^{\frac{5}{2}} b^{4} x}{5 a^{2}}+\frac{72 \left(b\, x^{2}+a\right)^{\frac{7}{2}} b^{4} x}{35 a^{3}}+64$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^8,x)
```

```
[Out] -1/7/a/x^7*(b*x^2+a)^(11/2)-4/35/a^2*b/x^5*(b*x^2+a)^(11/2)-8/35/a^3*b^2/x^
3*(b*x^2+a)^(11/2)-64/35/a^4*b^3/x*(b*x^2+a)^(11/2)+64/35/a^4*b^4*x*(b*x^2+
a)^(9/2)+72/35/a^3*b^4*x*(b*x^2+a)^(7/2)+12/5/a^2*b^4*x*(b*x^2+a)^(5/2)+3/a
*b^4*x*(b*x^2+a)^(3/2)+9/2*b^4*x*(b*x^2+a)^(1/2)+9/2*a*b^(7/2)*ln(b^(1/2)*x
+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.47, size = 160, normalized size = 1.27

$$\frac{9}{2} \sqrt{bx^{2}+a}\, b^{4} x+\frac{72 \left(bx^{2}+a\right)^{\frac{7}{2}} b^{4} x}{35\, a^{3}}+\frac{12 \left(bx^{2}+a\right)^{\frac{5}{2}} b^{4} x}{5\, a^{2}}+\frac{3 \left(bx^{2}+a\right)^{\frac{3}{2}} b^{4} x}{a}+\frac{9}{2}\, a b^{\frac{7}{2}} \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{64 \left(bx^{2}+a\right)^{\frac{9}{2}}}{35\, a^{3} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="maxima")
```

```
[Out] 9/2*sqrt(b*x^2 + a)*b^4*x + 72/35*(b*x^2 + a)^(7/2)*b^4*x/a^3 + 12/5*(b*x^2
 + a)^(5/2)*b^4*x/a^2 + 3*(b*x^2 + a)^(3/2)*b^4*x/a + 9/2*a*b^(7/2)*arcsinh
(b*x/sqrt(a*b)) - 64/35*(b*x^2 + a)^(9/2)*b^3/(a^3*x) - 8/35*(b*x^2 + a)^(1
1/2)*b^2/(a^3*x^3) - 4/35*(b*x^2 + a)^(11/2)*b/(a^2*x^5) - 1/7*(b*x^2 + a)^
(11/2)/(a*x^7)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{9/2}}{x^8} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^8,x)
```

```
[Out] int((a + b*x^2)^(9/2)/x^8, x)
```

sympy [A] time = 8.79, size = 167, normalized size = 1.33

$$-\frac{a^{4} \sqrt{b} \sqrt{\frac{a}{b x^{2}} + 1}}{7 x^{6}} - \frac{29 a^{3} b^{\frac{3}{2}} \sqrt{\frac{a}{b x^{2}} + 1}}{35 x^{4}} - \frac{78 a^{2} b^{\frac{5}{2}} \sqrt{\frac{a}{b x^{2}} + 1}}{35 x^{2}} - \frac{194 a b^{\frac{7}{2}} \sqrt{\frac{a}{b x^{2}} + 1}}{35} - \frac{9 a b^{\frac{7}{2}} \log{\left(\frac{a}{b x^{2}} \right)}}{4} + \frac{9 a b^{\frac{7}{2}} \log{\left(\sqrt{\frac{a}{b x^{2}} + 1} + \right.}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**8,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 29*a**3*b**(3/2)*sqrt(a/(b*x*
*2) + 1)/(35*x**4) - 78*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(35*x**2) - 194*
a*b**(7/2)*sqrt(a/(b*x**2) + 1)/35 - 9*a*b**(7/2)*log(a/(b*x**2))/4 + 9*a*b
**(7/2)*log(sqrt(a/(b*x**2) + 1) + 1)/2 + b**(9/2)*x**2*sqrt(a/(b*x**2) + 1
)/2
```

3.433 $\int \frac{(a+bx^2)^{9/2}}{x^{10}}\,dx$

Optimal. Leaf size=124

$$b^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b^4\sqrt{a+bx^2}}{x}-\frac{b^3\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{b^2\left(a+bx^2\right)^{5/2}}{5x^5}-\frac{\left(a+bx^2\right)^{9/2}}{9x^9}-\frac{b\left(a+bx^2\right)^{7/2}}{7x^7}$$

```
[Out] -1/3*b^3*(b*x^2+a)^(3/2)/x^3-1/5*b^2*(b*x^2+a)^(5/2)/x^5-1/7*b*(b*x^2+a)^(7
/2)/x^7-1/9*(b*x^2+a)^(9/2)/x^9+b^(9/2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))-
b^4*(b*x^2+a)^(1/2)/x
```

Rubi [A] time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 217, 206}

$$-\frac{b^4\sqrt{a+bx^2}}{x}-\frac{b^3\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{b^2\left(a+bx^2\right)^{5/2}}{5x^5}+b^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b\left(a+bx^2\right)^{7/2}}{7x^7}-\frac{\left(a+bx^2\right)^{9/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^10,x]
```

```
[Out] -((b^4*Sqrt[a + b*x^2])/x) - (b^3*(a + b*x^2)^(3/2))/(3*x^3) - (b^2*(a + b*
x^2)^(5/2))/(5*x^5) - (b*(a + b*x^2)^(7/2))/(7*x^7) - (a + b*x^2)^(9/2)/(9*
x^9) + b^(9/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{10}}\,dx = -\frac{(a+bx^2)^{9/2}}{9x^9} + b\int \frac{(a+bx^2)^{7/2}}{x^8}\,dx$$
$$= -\frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^2\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx$$
$$= -\frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^3\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx$$
$$= -\frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^4\int \frac{\sqrt{a+bx^2}}{x^2}\,dx$$
$$= -\frac{b^4\sqrt{a+bx^2}}{x} - \frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^5\int$$
$$= -\frac{b^4\sqrt{a+bx^2}}{x} - \frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^5\,\mathrm{Su}$$
$$= -\frac{b^4\sqrt{a+bx^2}}{x} - \frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^{9/2}\,\mathrm{ta}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.44

$$-\frac{a^4\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{9}{2},-\frac{9}{2};-\frac{7}{2};-\frac{bx^2}{a}\right)}{9x^9\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^10,x]
```

```
[Out] -1/9*(a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -9/2, -7/2, -((b*x^2)/a)]
)/(x^9*Sqrt[1 + (b*x^2)/a])
```

fricas [A] time = 1.04, size = 184, normalized size = 1.48

$$\left[\frac{315\,b^{\frac{9}{2}}x^9\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(563\,b^4x^8+506\,ab^3x^6+408\,a^2b^2x^4+185\,a^3bx^2+35\,a^4\right)\sqrt{b}}{630\,x^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/630*(315*b^(9/2)*x^9*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2
*(563*b^4*x^8 + 506*a*b^3*x^6 + 408*a^2*b^2*x^4 + 185*a^3*b*x^2 + 35*a^4)*s
qrt(b*x^2 + a))/x^9, -1/315*(315*sqrt(-b)*b^4*x^9*arctan(sqrt(-b)*x/sqrt(b*
x^2 + a)) + (563*b^4*x^8 + 506*a*b^3*x^6 + 408*a^2*b^2*x^4 + 185*a^3*b*x^2
+ 35*a^4)*sqrt(b*x^2 + a))/x^9]
```

giac [B] time = 1.12, size = 276, normalized size = 2.23

$$-\frac{1}{2}\,b^{\frac{9}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(1575\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{16}ab^{\frac{9}{2}}-6300\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{14}a^2b^{\frac{9}{2}}+21000\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/2*b^(9/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/315*(1575*(sqrt(b)*x
- sqrt(b*x^2 + a))^16*a*b^(9/2) - 6300*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^2
*b^(9/2) + 21000*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(9/2) - 31500*(sqrt
(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(9/2) + 39438*(sqrt(b)*x - sqrt(b*x^2 + a
))^8*a^5*b^(9/2) - 26292*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(9/2) + 1396
8*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^(9/2) - 3492*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a^8*b^(9/2) + 563*a^9*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
 a)^9
```

maple [B] time = 0.03, size = 206, normalized size = 1.66

$$b^{\frac{9}{2}} \ln\left(\sqrt{b}\, x + \sqrt{b\, x^2 + a}\right) + \frac{\sqrt{b\, x^2 + a}\; b^5 x}{a} + \frac{2\left(b\, x^2 + a\right)^{\frac{3}{2}} b^5 x}{3a^2} + \frac{8\left(b\, x^2 + a\right)^{\frac{5}{2}} b^5 x}{15a^3} + \frac{16\left(b\, x^2 + a\right)^{\frac{7}{2}} b^5 x}{35a^4} + \frac{128\left(b\, x^2 + \right.}{315a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^10,x)
```

```
[Out] -1/9/a/x^9*(b*x^2+a)^(11/2)-2/63/a^2*b/x^7*(b*x^2+a)^(11/2)-8/315/a^3*b^2/x
^5*(b*x^2+a)^(11/2)-16/315/a^4*b^3/x^3*(b*x^2+a)^(11/2)-128/315/a^5*b^4/x*(
b*x^2+a)^(11/2)+128/315/a^5*b^5*x*(b*x^2+a)^(9/2)+16/35/a^4*b^5*x*(b*x^2+a)
^(7/2)+8/15/a^3*b^5*x*(b*x^2+a)^(5/2)+2/3/a^2*b^5*x*(b*x^2+a)^(3/2)+1/a*b^5
*x*(b*x^2+a)^(1/2)+b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.51, size = 180, normalized size = 1.45

$$\frac{16\left(bx^2 + a\right)^{\frac{7}{2}} b^5 x}{35\, a^4} + \frac{8\left(bx^2 + a\right)^{\frac{5}{2}} b^5 x}{15\, a^3} + \frac{2\left(bx^2 + a\right)^{\frac{3}{2}} b^5 x}{3\, a^2} + \frac{\sqrt{bx^2 + a}\, b^5 x}{a} + b^{\frac{9}{2}} \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{128\left(bx^2 + a\right)^{\frac{9}{2}} b^4}{315\, a^4 x} - \frac{16\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="maxima")
```

```
[Out] 16/35*(b*x^2 + a)^(7/2)*b^5*x/a^4 + 8/15*(b*x^2 + a)^(5/2)*b^5*x/a^3 + 2/3*
(b*x^2 + a)^(3/2)*b^5*x/a^2 + sqrt(b*x^2 + a)*b^5*x/a + b^(9/2)*arcsinh(b*x
/sqrt(a*b)) - 128/315*(b*x^2 + a)^(9/2)*b^4/(a^4*x) - 16/315*(b*x^2 + a)^(1
1/2)*b^3/(a^4*x^3) - 8/315*(b*x^2 + a)^(11/2)*b^2/(a^3*x^5) - 2/63*(b*x^2 +
 a)^(11/2)*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(11/2)/(a*x^9)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\, x^2 + a\right)^{9/2}}{x^{10}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^10,x)
```

```
[Out] int((a + b*x^2)^(9/2)/x^10, x)
```

sympy [A] time = 9.86, size = 160, normalized size = 1.29

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{9x^8} - \frac{37a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{63x^6} - \frac{136a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{105x^4} - \frac{506ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{315x^2} - \frac{563b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{315} - \frac{b^{\frac{9}{2}}\log\left(\frac{a}{bx^2}\right)}{2} + b^{\frac{9}{2}}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**10,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(9*x**8) - 37*a**3*b**(3/2)*sqrt(a/(b*x*
*2) + 1)/(63*x**6) - 136*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(105*x**4) - 50
6*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(315*x**2) - 563*b**(9/2)*sqrt(a/(b*x**2)
 + 1)/315 - b**(9/2)*log(a/(b*x**2))/2 + b**(9/2)*log(sqrt(a/(b*x**2) + 1)
+ 1)
```

3.434 $\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx$

Optimal. Leaf size=21

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

```
[Out] -1/11*(b*x^2+a)^(11/2)/a/x^11
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^12,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(11*a*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx = -\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^12,x]
```

```
[Out] -1/11*(a + b*x^2)^(11/2)/(a*x^11)
```

fricas [B] time = 0.98, size = 68, normalized size = 3.24

$$-\frac{(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + 10\,a^3b^2x^4 + 5\,a^4bx^2 + a^5)\sqrt{bx^2+a}}{11\,ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/11*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x
^2 + a^5)*sqrt(b*x^2 + a)/(a*x^11)
```

giac [B] time = 1.06, size = 167, normalized size = 7.95

$$\frac{2\left(11\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}b^{\frac{11}{2}}+165\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}a^2b^{\frac{11}{2}}+462\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^4b^{\frac{11}{2}}+330\left(\sqrt{b}x\right.\right.}{11\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="giac")
```

```
[Out] 2/11*(11*(sqrt(b)*x - sqrt(b*x^2 + a))^20*b^(11/2) + 165*(sqrt(b)*x - sqrt(
b*x^2 + a))^16*a^2*b^(11/2) + 462*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(1
1/2) + 330*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(11/2) + 55*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*a^8*b^(11/2) + a^10*b^(11/2))/((sqrt(b)*x - sqrt(b*x^2 +
a))^2 - a)^11
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{11a\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^12,x)
```

```
[Out] -1/11*(b*x^2+a)^(11/2)/a/x^11
```

maxima [A] time = 1.50, size = 17, normalized size = 0.81

$$-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{11\,ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="maxima")
```

```
[Out] -1/11*(b*x^2 + a)^(11/2)/(a*x^11)
```

mupad [B] time = 6.28, size = 111, normalized size = 5.29

$$-\frac{a^4\,\sqrt{b\,x^2+a}}{11\,x^{11}}-\frac{5\,b^4\,\sqrt{b\,x^2+a}}{11\,x^3}-\frac{10\,a\,b^3\,\sqrt{b\,x^2+a}}{11\,x^5}-\frac{5\,a^3\,b\,\sqrt{b\,x^2+a}}{11\,x^9}-\frac{b^5\,\sqrt{b\,x^2+a}}{11\,a\,x}-\frac{10\,a^2\,b^2\,\sqrt{b\,x^2+a}}{11\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^12,x)
```

```
[Out] - (a^4*(a + b*x^2)^(1/2))/(11*x^11) - (5*b^4*(a + b*x^2)^(1/2))/(11*x^3) -
(10*a*b^3*(a + b*x^2)^(1/2))/(11*x^5) - (5*a^3*b*(a + b*x^2)^(1/2))/(11*x^9
) - (b^5*(a + b*x^2)^(1/2))/(11*a*x) - (10*a^2*b^2*(a + b*x^2)^(1/2))/(11*x
^7)
```

sympy [B] time = 2.45, size = 150, normalized size = 7.14

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{11x^{10}}-\frac{5a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^8}-\frac{10a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^6}-\frac{10ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^4}-\frac{5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^2}-\frac{b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{11a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**12,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(11*x**10) - 5*a**3*b**(3/2)*sqrt(a/(b*x
**2) + 1)/(11*x**8) - 10*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(11*x**6) - 10*
a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(11*x**4) - 5*b**(9/2)*sqrt(a/(b*x**2) + 1)
/(11*x**2) - b**(11/2)*sqrt(a/(b*x**2) + 1)/(11*a)
```

3.435 $\int \frac{(a+bx^2)^{9/2}}{x^{14}} \, dx$

Optimal. Leaf size=44

$$\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}} - \frac{(a+bx^2)^{11/2}}{13ax^{13}}$$

```
[Out] -1/13*(b*x^2+a)^(11/2)/a/x^13+2/143*b*(b*x^2+a)^(11/2)/a^2/x^11
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}} - \frac{(a+bx^2)^{11/2}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^14,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(13*a*x^13) + (2*b*(a + b*x^2)^(11/2))/(143*a^2*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{9/2}}{x^{14}} \, dx &= -\frac{(a+bx^2)^{11/2}}{13ax^{13}} - \frac{(2b)\int \frac{(a+bx^2)^{9/2}}{x^{12}} \, dx}{13a} \\
&= -\frac{(a+bx^2)^{11/2}}{13ax^{13}} + \frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{(a+bx^2)^{11/2}(2bx^2-11a)}{143a^2x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^14,x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-11*a + 2*b*x^2))/(143*a^2*x^13)
```

fricas [B] time = 0.94, size = 82, normalized size = 1.86

$$\frac{(2\,b^6x^{12} - ab^5x^{10} - 35\,a^2b^4x^8 - 90\,a^3b^3x^6 - 100\,a^4b^2x^4 - 53\,a^5bx^2 - 11\,a^6)\sqrt{bx^2+a}}{143\,a^2x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="fricas")
```

```
[Out] 1/143*(2*b^6*x^12 - a*b^5*x^10 - 35*a^2*b^4*x^8 - 90*a^3*b^3*x^6 - 100*a^4*
b^2*x^4 - 53*a^5*b*x^2 - 11*a^6)*sqrt(b*x^2 + a)/(a^2*x^13)
```

giac [B] time = 1.09, size = 328, normalized size = 7.45

$$\frac{4\left(143\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}b^{\frac{13}{2}}+429\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}ab^{\frac{13}{2}}+2145\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}a^2b^{\frac{13}{2}}+3003\left(\sqrt{b}x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="giac")
```

```
[Out] 4/143*(143*(sqrt(b)*x - sqrt(b*x^2 + a))^22*b^(13/2) + 429*(sqrt(b)*x - sqr
t(b*x^2 + a))^20*a*b^(13/2) + 2145*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^2*b^(
13/2) + 3003*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^3*b^(13/2) + 6006*(sqrt(b)*
x - sqrt(b*x^2 + a))^14*a^4*b^(13/2) + 4290*(sqrt(b)*x - sqrt(b*x^2 + a))^1
2*a^5*b^(13/2) + 4290*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^6*b^(13/2) + 1430*
(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*b^(13/2) + 715*(sqrt(b)*x - sqrt(b*x^2
+ a))^6*a^8*b^(13/2) + 65*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^9*b^(13/2) + 13
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*b^(13/2) - a^11*b^(13/2))/((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)^13
```

maple [A] time = 0.01, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}\left(-2b\,x^2+11a\right)}{143a^2x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^14,x)
```

```
[Out] -1/143*(b*x^2+a)^(11/2)*(-2*b*x^2+11*a)/x^13/a^2
```

maxima [A] time = 1.57, size = 36, normalized size = 0.82

$$\frac{2\left(bx^2+a\right)^{\frac{11}{2}}b}{143\,a^2x^{11}}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{13\,ax^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="maxima")
```

```
[Out] 2/143*(b*x^2 + a)^(11/2)*b/(a^2*x^11) - 1/13*(b*x^2 + a)^(11/2)/(a*x^13)
```

mupad [B] time = 6.85, size = 131, normalized size = 2.98

$$\frac{2\,b^6\,\sqrt{b\,x^2+a}}{143\,a^2\,x}-\frac{35\,b^4\,\sqrt{b\,x^2+a}}{143\,x^5}-\frac{90\,a\,b^3\,\sqrt{b\,x^2+a}}{143\,x^7}-\frac{53\,a^3\,b\,\sqrt{b\,x^2+a}}{143\,x^{11}}-\frac{b^5\,\sqrt{b\,x^2+a}}{143\,a\,x^3}-\frac{a^4\,\sqrt{b\,x^2+a}}{13\,x^{13}}-\frac{100\,a^2\,b^2\,\sqrt{}}{143}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^14,x)
```

```
[Out] (2*b^6*(a + b*x^2)^(1/2))/(143*a^2*x) - (35*b^4*(a + b*x^2)^(1/2))/(143*x^5
) - (90*a*b^3*(a + b*x^2)^(1/2))/(143*x^7) - (53*a^3*b*(a + b*x^2)^(1/2))/(
```

```
143*x^11) - (b^5*(a + b*x^2)^(1/2))/(143*a*x^3) - (a^4*(a + b*x^2)^(1/2))/(
13*x^13) - (100*a^2*b^2*(a + b*x^2)^(1/2))/(143*x^9)
```

sympy [B] time = 2.98, size = 175, normalized size = 3.98

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{13x^{12}}-\frac{53a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{143x^{10}}-\frac{100a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{143x^8}-\frac{90ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{143x^6}-\frac{35b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{143x^4}-\frac{b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{143ax^2}+\frac{2b^{\frac{13}{2}}\sqrt{\frac{a}{bx^2}+1}}{143a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**14,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(13*x**12) - 53*a**3*b**(3/2)*sqrt(a/(b*
x**2) + 1)/(143*x**10) - 100*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(143*x**8)
- 90*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(143*x**6) - 35*b**(9/2)*sqrt(a/(b*x**
2) + 1)/(143*x**4) - b**(11/2)*sqrt(a/(b*x**2) + 1)/(143*a*x**2) + 2*b**(13
/2)*sqrt(a/(b*x**2) + 1)/(143*a**2)
```

3.436 $\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx$

Optimal. Leaf size=68

$$-\frac{8b^2\left(a+bx^2\right)^{11/2}}{2145a^3x^{11}}+\frac{4b\left(a+bx^2\right)^{11/2}}{195a^2x^{13}}-\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}$$

```
[Out] -1/15*(b*x^2+a)^(11/2)/a/x^15+4/195*b*(b*x^2+a)^(11/2)/a^2/x^13-8/2145*b^2*
(b*x^2+a)^(11/2)/a^3/x^11
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8b^2\left(a+bx^2\right)^{11/2}}{2145a^3x^{11}}+\frac{4b\left(a+bx^2\right)^{11/2}}{195a^2x^{13}}-\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^16,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(15*a*x^15) + (4*b*(a + b*x^2)^(11/2))/(195*a^2*x^13) -
 (8*b^2*(a + b*x^2)^(11/2))/(2145*a^3*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{9/2}}{x^{16}}\,dx &= -\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}-\frac{(4b)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{14}}\,dx}{15a}\\
&= -\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}+\frac{4b\left(a+bx^2\right)^{11/2}}{195a^2x^{13}}+\frac{\left(8b^2\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{12}}\,dx}{195a^2}\\
&= -\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}+\frac{4b\left(a+bx^2\right)^{11/2}}{195a^2x^{13}}-\frac{8b^2\left(a+bx^2\right)^{11/2}}{2145a^3x^{11}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^2\right)^{11/2}\left(143a^2-44abx^2+8b^2x^4\right)}{2145a^3x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^16,x]
```

```
[Out] -1/2145*((a + b*x^2)^(11/2)*(143*a^2 - 44*a*b*x^2 + 8*b^2*x^4))/(a^3*x^15)
```

fricas [A] time = 1.15, size = 93, normalized size = 1.37

$$-\frac{\left(8\,b^{7}x^{14}-4\,ab^{6}x^{12}+3\,a^{2}b^{5}x^{10}+355\,a^{3}b^{4}x^{8}+1030\,a^{4}b^{3}x^{6}+1218\,a^{5}b^{2}x^{4}+671\,a^{6}bx^{2}+143\,a^{7}\right)\sqrt{bx^{2}+a}}{2145\,a^{3}x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/2145*(8*b^7*x^14 - 4*a*b^6*x^12 + 3*a^2*b^5*x^10 + 355*a^3*b^4*x^8 + 103
0*a^4*b^3*x^6 + 1218*a^5*b^2*x^4 + 671*a^6*b*x^2 + 143*a^7)*sqrt(b*x^2 + a)
/(a^3*x^15)
```

giac [B] time = 1.22, size = 354, normalized size = 5.21

$$\frac{16\left(1430\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{24}b^{\frac{15}{2}}+6435\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{22}ab^{\frac{15}{2}}+24453\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{20}a^{2}b^{\frac{15}{2}}+450\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="giac")
```

```
[Out] 16/2145*(1430*(sqrt(b)*x - sqrt(b*x^2 + a))^24*b^(15/2) + 6435*(sqrt(b)*x -
 sqrt(b*x^2 + a))^22*a*b^(15/2) + 24453*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^
2*b^(15/2) + 45045*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^3*b^(15/2) + 70785*(s
qrt(b)*x - sqrt(b*x^2 + a))^16*a^4*b^(15/2) + 64350*(sqrt(b)*x - sqrt(b*x^2
 + a))^14*a^5*b^(15/2) + 50050*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^6*b^(15/2
) + 21450*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^7*b^(15/2) + 7800*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*a^8*b^(15/2) + 975*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^9*
b^(15/2) + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^10*b^(15/2) - 15*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*a^11*b^(15/2) + a^12*b^(15/2))/((sqrt(b)*x - sqrt(b*
x^2 + a))^2 - a)^15
```

maple [A] time = 0.00, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(8b^{2}x^{4}-44ab\,x^{2}+143a^{2}\right)}{2145a^{3}x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^16,x)
```

```
[Out] -1/2145*(b*x^2+a)^(11/2)*(8*b^2*x^4-44*a*b*x^2+143*a^2)/x^15/a^3
```

maxima [A] time = 1.51, size = 56, normalized size = 0.82

$$-\frac{8\left(bx^{2}+a\right)^{\frac{11}{2}}b^{2}}{2145\,a^{3}x^{11}}+\frac{4\left(bx^{2}+a\right)^{\frac{11}{2}}b}{195\,a^{2}x^{13}}-\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}}{15\,ax^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="maxima")
```

```
[Out] -8/2145*(b*x^2 + a)^(11/2)*b^2/(a^3*x^11) + 4/195*(b*x^2 + a)^(11/2)*b/(a^2
*x^13) - 1/15*(b*x^2 + a)^(11/2)/(a*x^15)
```

mupad [B] time = 7.42, size = 151, normalized size = 2.22

$$\frac{4\,b^6\,\sqrt{b\,x^2+a}}{2145\,a^2\,x^3}-\frac{71\,b^4\,\sqrt{b\,x^2+a}}{429\,x^7}-\frac{206\,a\,b^3\,\sqrt{b\,x^2+a}}{429\,x^9}-\frac{61\,a^3\,b\,\sqrt{b\,x^2+a}}{195\,x^{13}}-\frac{b^5\,\sqrt{b\,x^2+a}}{715\,a\,x^5}-\frac{a^4\,\sqrt{b\,x^2+a}}{15\,x^{15}}-\frac{8\,b^7\,\sqrt{b\,x^2}}{2145\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^16,x)
```

```
[Out] (4*b^6*(a + b*x^2)^(1/2))/(2145*a^2*x^3) - (71*b^4*(a + b*x^2)^(1/2))/(429*
x^7) - (206*a*b^3*(a + b*x^2)^(1/2))/(429*x^9) - (61*a^3*b*(a + b*x^2)^(1/2
))/(195*x^13) - (b^5*(a + b*x^2)^(1/2))/(715*a*x^5) - (a^4*(a + b*x^2)^(1/2
))/(15*x^15) - (8*b^7*(a + b*x^2)^(1/2))/(2145*a^3*x) - (406*a^2*b^2*(a + b
*x^2)^(1/2))/(715*x^11)
```

sympy [B] time = 3.93, size = 604, normalized size = 8.88

$$-\frac{143a^9b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^6\left(2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}\right)}-\frac{957a^8b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^4\left(2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}\right)}-\frac{2}{x^2\left(2145a^5b^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**16,x)
```

```
[Out] -143*a**9*b**(9/2)*sqrt(a/(b*x**2) + 1)/(x**6*(2145*a**5*b**4*x**8 + 4290*a
**4*b**5*x**10 + 2145*a**3*b**6*x**12)) - 957*a**8*b**(11/2)*sqrt(a/(b*x**2
) + 1)/(x**4*(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x
**12)) - 2703*a**7*b**(13/2)*sqrt(a/(b*x**2) + 1)/(x**2*(2145*a**5*b**4*x**
8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12)) - 4137*a**6*b**(15/2)*sqr
t(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b
**6*x**12) - 3633*a**5*b**(17/2)*x**2*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*
x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 1743*a**4*b**(19/2)*x
**4*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145
*a**3*b**6*x**12) - 357*a**3*b**(21/2)*x**6*sqrt(a/(b*x**2) + 1)/(2145*a**5
*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 3*a**2*b**(23/2
)*x**8*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2
145*a**3*b**6*x**12) - 12*a*b**(25/2)*x**10*sqrt(a/(b*x**2) + 1)/(2145*a**5
*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 8*b**(27/2)*x**
12*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*
a**3*b**6*x**12)
```

3.437 $\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx$

Optimal. Leaf size=92

$$\frac{16b^3(a+bx^2)^{11/2}}{12155a^4x^{11}} - \frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}} + \frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}} - \frac{(a+bx^2)^{11/2}}{17ax^{17}}$$

```
[Out] -1/17*(b*x^2+a)^(11/2)/a/x^17+2/85*b*(b*x^2+a)^(11/2)/a^2/x^15-8/1105*b^2*(
b*x^2+a)^(11/2)/a^3/x^13+16/12155*b^3*(b*x^2+a)^(11/2)/a^4/x^11
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{271, 264\}$

$$\frac{16b^3(a+bx^2)^{11/2}}{12155a^4x^{11}} - \frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}} + \frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}} - \frac{(a+bx^2)^{11/2}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^18,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(17*a*x^17) + (2*b*(a + b*x^2)^(11/2))/(85*a^2*x^15) -
(8*b^2*(a + b*x^2)^(11/2))/(1105*a^3*x^13) + (16*b^3*(a + b*x^2)^(11/2))/(1
2155*a^4*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx &= -\frac{(a+bx^2)^{11/2}}{17ax^{17}} - \frac{(6b)\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx}{17a}\\
&= -\frac{(a+bx^2)^{11/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}} + \frac{(8b^2)\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx}{85a^2}\\
&= -\frac{(a+bx^2)^{11/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}} - \frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}} - \frac{(16b^3)\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx}{1105a^3}\\
&= -\frac{(a+bx^2)^{11/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}} - \frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}} + \frac{16b^3(a+bx^2)^{11/2}}{12155a^4x^{11}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.58

$$\frac{(a+bx^2)^{11/2}(-715a^3+286a^2bx^2-88ab^2x^4+16b^3x^6)}{12155a^4x^{17}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^18,x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-715*a^3 + 286*a^2*b*x^2 - 88*a*b^2*x^4 + 16*b^3*x^6))
/(12155*a^4*x^17)
```

fricas [A] time = 1.30, size = 104, normalized size = 1.13

$$\frac{\left(16\,b^8x^{16} - 8\,ab^7x^{14} + 6\,a^2b^6x^{12} - 5\,a^3b^5x^{10} - 1515\,a^4b^4x^8 - 4714\,a^5b^3x^6 - 5808\,a^6b^2x^4 - 3289\,a^7bx^2 - 715\,a^8\right)\sqrt{}}{12155\,a^4x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="fricas")
```

```
[Out] 1/12155*(16*b^8*x^16 - 8*a*b^7*x^14 + 6*a^2*b^6*x^12 - 5*a^3*b^5*x^10 - 151
5*a^4*b^4*x^8 - 4714*a^5*b^3*x^6 - 5808*a^6*b^2*x^4 - 3289*a^7*b*x^2 - 715*
a^8)*sqrt(b*x^2 + a)/(a^4*x^17)
```

giac [B] time = 1.30, size = 382, normalized size = 4.15

$$\frac{32\left(12155\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^{26} b^{\frac{17}{2}} + 65637\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^{24} ab^{\frac{17}{2}} + 233376\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^{22} a^2b^{\frac{17}{2}} + 46\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="giac")
```

```
[Out] 32/12155*(12155*(sqrt(b)*x - sqrt(b*x^2 + a))^26*b^(17/2) + 65637*(sqrt(b)*
x - sqrt(b*x^2 + a))^24*a*b^(17/2) + 233376*(sqrt(b)*x - sqrt(b*x^2 + a))^2
2*a^2*b^(17/2) + 466752*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^3*b^(17/2) + 692
835*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^4*b^(17/2) + 668525*(sqrt(b)*x - sqr
t(b*x^2 + a))^16*a^5*b^(17/2) + 486200*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^6
*b^(17/2) + 221000*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^7*b^(17/2) + 71825*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a^8*b^(17/2) + 9775*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a^9*b^(17/2) + 680*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^10*b^(17/2) -
136*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^11*b^(17/2) + 17*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a^12*b^(17/2) - a^13*b^(17/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2
 - a)^17
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^2 + a\right)^{\frac{11}{2}}\left(-16b^3x^6 + 88a\,b^2x^4 - 286a^2b\,x^2 + 715a^3\right)}{12155a^4x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^18,x)
```

```
[Out] -1/12155*(b*x^2+a)^(11/2)*(-16*b^3*x^6+88*a*b^2*x^4-286*a^2*b*x^2+715*a^3)/
x^17/a^4
```

maxima [A] time = 1.55, size = 76, normalized size = 0.83

$$\frac{16\left(bx^2 + a\right)^{\frac{11}{2}} b^3}{12155\,a^4x^{11}} - \frac{8\left(bx^2 + a\right)^{\frac{11}{2}} b^2}{1105\,a^3x^{13}} + \frac{2\left(bx^2 + a\right)^{\frac{11}{2}} b}{85\,a^2x^{15}} - \frac{\left(bx^2 + a\right)^{\frac{11}{2}}}{17\,ax^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="maxima")
```

```
[Out] 16/12155*(b*x^2 + a)^(11/2)*b^3/(a^4*x^11) - 8/1105*(b*x^2 + a)^(11/2)*b^2/
(a^3*x^13) + 2/85*(b*x^2 + a)^(11/2)*b/(a^2*x^15) - 1/17*(b*x^2 + a)^(11/2)
/(a*x^17)
```

mupad [B] time = 8.17, size = 171, normalized size = 1.86

$$\frac{6\,b^6\,\sqrt{b\,x^2+a}}{12155\,a^2\,x^5}-\frac{303\,b^4\,\sqrt{b\,x^2+a}}{2431\,x^9}-\frac{4714\,a\,b^3\,\sqrt{b\,x^2+a}}{12155\,x^{11}}-\frac{23\,a^3\,b\,\sqrt{b\,x^2+a}}{85\,x^{15}}-\frac{b^5\,\sqrt{b\,x^2+a}}{2431\,a\,x^7}-\frac{a^4\,\sqrt{b\,x^2+a}}{17\,x^{17}}-\frac{8\,b^7}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^18,x)
```

```
[Out] (6*b^6*(a + b*x^2)^(1/2))/(12155*a^2*x^5) - (303*b^4*(a + b*x^2)^(1/2))/(24
31*x^9) - (4714*a*b^3*(a + b*x^2)^(1/2))/(12155*x^11) - (23*a^3*b*(a + b*x^
2)^(1/2))/(85*x^15) - (b^5*(a + b*x^2)^(1/2))/(2431*a*x^7) - (a^4*(a + b*x^
2)^(1/2))/(17*x^17) - (8*b^7*(a + b*x^2)^(1/2))/(12155*a^3*x^3) + (16*b^8*(
a + b*x^2)^(1/2))/(12155*a^4*x) - (528*a^2*b^2*(a + b*x^2)^(1/2))/(1105*x^1
3)
```

sympy [B] time = 4.97, size = 867, normalized size = 9.42

$$-\frac{715a^{11}b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}}-\frac{5434a^{10}b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**18,x)
```

```
[Out] -715*a**11*b**(19/2)*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a*
*6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 5434*a*
*10*b**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6
*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 17820*a**
9*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b
**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 32720*a**8*
b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**
10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 36370*a**7*b*
*(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10
*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 24500*a**6*b**(
29/2)*x**10*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*
x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 9268*a**5*b**(31
/2)*x**12*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x*
*18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 1520*a**4*b**(33/2
)*x**14*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**1
8 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 5*a**3*b**(35/2)*x**
16*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 3
6465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 30*a**2*b**(37/2)*x**18*s
qrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465
*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 40*a*b**(39/2)*x**20*sqrt(a/(
b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465*a**5*b
**11*x**20 + 12155*a**4*b**12*x**22) + 16*b**(41/2)*x**22*sqrt(a/(b*x**2) +
 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465*a**5*b**11*x**2
0 + 12155*a**4*b**12*x**22)
```

3.438 $\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx$

Optimal. Leaf size=116

$$-\frac{128b^4(a+bx^2)^{11/2}}{230945a^5x^{11}}+\frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}}-\frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}}+\frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}$$

```
[Out] -1/19*(b*x^2+a)^(11/2)/a/x^19+8/323*b*(b*x^2+a)^(11/2)/a^2/x^17-16/1615*b^2
*(b*x^2+a)^(11/2)/a^3/x^15+64/20995*b^3*(b*x^2+a)^(11/2)/a^4/x^13-128/23094
5*b^4*(b*x^2+a)^(11/2)/a^5/x^11
```

Rubi [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{271, 264\}$

$$-\frac{128b^4(a+bx^2)^{11/2}}{230945a^5x^{11}}+\frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}}-\frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}}+\frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^20,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(19*a*x^19) + (8*b*(a + b*x^2)^(11/2))/(323*a^2*x^17) -
 (16*b^2*(a + b*x^2)^(11/2))/(1615*a^3*x^15) + (64*b^3*(a + b*x^2)^(11/2))/
(20995*a^4*x^13) - (128*b^4*(a + b*x^2)^(11/2))/(230945*a^5*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx &= -\frac{(a+bx^2)^{11/2}}{19ax^{19}}-\frac{(8b)\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx}{19a}\\
&= -\frac{(a+bx^2)^{11/2}}{19ax^{19}}+\frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}}+\frac{(48b^2)\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx}{323a^2}\\
&= -\frac{(a+bx^2)^{11/2}}{19ax^{19}}+\frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}}-\frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}}-\frac{(64b^3)\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx}{1615a^3}\\
&= -\frac{(a+bx^2)^{11/2}}{19ax^{19}}+\frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}}-\frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}}+\frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}}+\frac{(128b^4)\int \frac{(a+
}{20995
\\
&= -\frac{(a+bx^2)^{11/2}}{19ax^{19}}+\frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}}-\frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}}+\frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}}-\frac{128b^4(a+bx
}{230945a^5
\end{aligned}$$

Mathematica [**A**] time = 0.02, size = 64, normalized size = 0.55

$$-\frac{\left(a+bx^2\right)^{11/2}\left(12155a^4-5720a^3bx^2+2288a^2b^2x^4-704ab^3x^6+128b^4x^8\right)}{230945a^5x^{19}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^20,x]
```

```
[Out] -1/230945*((a + b*x^2)^(11/2)*(12155*a^4 - 5720*a^3*b*x^2 + 2288*a^2*b^2*x^
4 - 704*a*b^3*x^6 + 128*b^4*x^8))/(a^5*x^19)
```

fricas [**A**] time = 1.35, size = 115, normalized size = 0.99

$$-\frac{\left(128\,b^9x^{18}-64\,ab^8x^{16}+48\,a^2b^7x^{14}-40\,a^3b^6x^{12}+35\,a^4b^5x^{10}+23063\,a^5b^4x^8+75086\,a^6b^3x^6+95238\,a^7b^2\right.}{230945\,a^5x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/230945*(128*b^9*x^18 - 64*a*b^8*x^16 + 48*a^2*b^7*x^14 - 40*a^3*b^6*x^12
 + 35*a^4*b^5*x^10 + 23063*a^5*b^4*x^8 + 75086*a^6*b^3*x^6 + 95238*a^7*b^2*
x^4 + 55055*a^8*b*x^2 + 12155*a^9)*sqrt(b*x^2 + a)/(a^5*x^19)
```

giac [**B**] time = 1.17, size = 408, normalized size = 3.52

$$\frac{256\left(92378\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{28}b^{\frac{19}{2}}+554268\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{26}ab^{\frac{19}{2}}+1939938\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{24}a^2b^{\frac{1}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="giac")
```

```
[Out] 256/230945*(92378*(sqrt(b)*x - sqrt(b*x^2 + a))^28*b^(19/2) + 554268*(sqrt(
b)*x - sqrt(b*x^2 + a))^26*a*b^(19/2) + 1939938*(sqrt(b)*x - sqrt(b*x^2 + a
))^24*a^2*b^(19/2) + 4018443*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^3*b^(19/2)
+ 5866003*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^4*b^(19/2) + 5773625*(sqrt(b)*
x - sqrt(b*x^2 + a))^18*a^5*b^(19/2) + 4094025*(sqrt(b)*x - sqrt(b*x^2 + a)
)^16*a^6*b^(19/2) + 1889550*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^7*b^(19/2) +
 581400*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^8*b^(19/2) + 80750*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*a^9*b^(19/2) + 3876*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^1
0*b^(19/2) - 969*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^11*b^(19/2) + 171*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*a^12*b^(19/2) - 19*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*a^13*b^(19/2) + a^14*b^(19/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^19
```

maple [**A**] time = 0.01, size = 61, normalized size = 0.53

$$-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}\left(128b^4x^8-704a\,b^3x^6+2288a^2b^2x^4-5720a^3b\,x^2+12155a^4\right)}{230945a^5x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^20,x)
```

```
[Out] -1/230945*(b*x^2+a)^(11/2)*(128*b^4*x^8-704*a*b^3*x^6+2288*a^2*b^2*x^4-5720
*a^3*b*x^2+12155*a^4)/x^19/a^5
```

maxima [A] time = 1.57, size = 96, normalized size = 0.83

$$-\frac{128\left(bx^2+a\right)^{\frac{11}{2}}b^4}{230945\,a^5x^{11}}+\frac{64\left(bx^2+a\right)^{\frac{11}{2}}b^3}{20995\,a^4x^{13}}-\frac{16\left(bx^2+a\right)^{\frac{11}{2}}b^2}{1615\,a^3x^{15}}+\frac{8\left(bx^2+a\right)^{\frac{11}{2}}b}{323\,a^2x^{17}}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{19\,ax^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="maxima")
```

```
[Out] -128/230945*(b*x^2 + a)^(11/2)*b^4/(a^5*x^11) + 64/20995*(b*x^2 + a)^(11/2)
*b^3/(a^4*x^13) - 16/1615*(b*x^2 + a)^(11/2)*b^2/(a^3*x^15) + 8/323*(b*x^2
+ a)^(11/2)*b/(a^2*x^17) - 1/19*(b*x^2 + a)^(11/2)/(a*x^19)
```

mupad [B] time = 8.84, size = 191, normalized size = 1.65

$$\frac{8\,b^6\,\sqrt{b\,x^2+a}}{46189\,a^2\,x^7}-\frac{23063\,b^4\,\sqrt{b\,x^2+a}}{230945\,x^{11}}-\frac{6826\,a\,b^3\,\sqrt{b\,x^2+a}}{20995\,x^{13}}-\frac{77\,a^3\,b\,\sqrt{b\,x^2+a}}{323\,x^{17}}-\frac{7\,b^5\,\sqrt{b\,x^2+a}}{46189\,a\,x^9}-\frac{a^4\,\sqrt{b\,x^2+a}}{19\,x^{19}}-\frac{48}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^20,x)
```

```
[Out] (8*b^6*(a + b*x^2)^(1/2))/(46189*a^2*x^7) - (23063*b^4*(a + b*x^2)^(1/2))/(
230945*x^11) - (6826*a*b^3*(a + b*x^2)^(1/2))/(20995*x^13) - (77*a^3*b*(a +
 b*x^2)^(1/2))/(323*x^17) - (7*b^5*(a + b*x^2)^(1/2))/(46189*a*x^9) - (a^4*
(a + b*x^2)^(1/2))/(19*x^19) - (48*b^7*(a + b*x^2)^(1/2))/(230945*a^3*x^5)
+ (64*b^8*(a + b*x^2)^(1/2))/(230945*a^4*x^3) - (128*b^9*(a + b*x^2)^(1/2))
/(230945*a^5*x) - (666*a^2*b^2*(a + b*x^2)^(1/2))/(1615*x^15)
```

sympy [B] time = 6.26, size = 1182, normalized size = 10.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**20,x)
```

```
[Out] -12155*a**13*b**(33/2)*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 9237
80*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 +
230945*a**5*b**20*x**26) - 103675*a**12*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)
/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x*
*22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 388388*a**11*b**
(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**
17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5
*b**20*x**26) - 834988*a**10*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(230945*a*
*9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 92378
0*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 1127210*a**9*b**(41/2)*x**8
*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 +
1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**2
6) - 978810*a**8*b**(43/2)*x**10*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x*
*18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**1
9*x**24 + 230945*a**5*b**20*x**26) - 534060*a**7*b**(45/2)*x**12*sqrt(a/(b*
x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**
7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 167436
*a**6*b**(47/2)*x**14*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 92378
0*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 2
30945*a**5*b**20*x**26) - 23091*a**5*b**(49/2)*x**16*sqrt(a/(b*x**2) + 1)/(
230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**2
2 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 35*a**4*b**(51/2)*
x**18*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**
20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20
```

```
*x**26) - 280*a**3*b**(53/2)*x**20*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*
x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b*
*19*x**24 + 230945*a**5*b**20*x**26) - 560*a**2*b**(55/2)*x**22*sqrt(a/(b*x
**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7
*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 448*a*b
**(57/2)*x**24*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*
b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a
**5*b**20*x**26) - 128*b**(59/2)*x**26*sqrt(a/(b*x**2) + 1)/(230945*a**9*b*
*16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**
6*b**19*x**24 + 230945*a**5*b**20*x**26)
```

3.439 $\int \frac{\left(a+bx^2\right)^{9/2}}{x^{22}}\,dx$

Optimal. Leaf size=140

$$\frac{256b^5\left(a+bx^2\right)^{11/2}}{969969a^6x^{11}}-\frac{128b^4\left(a+bx^2\right)^{11/2}}{88179a^5x^{13}}+\frac{32b^3\left(a+bx^2\right)^{11/2}}{6783a^4x^{15}}-\frac{80b^2\left(a+bx^2\right)^{11/2}}{6783a^3x^{17}}+\frac{10b\left(a+bx^2\right)^{11/2}}{399a^2x^{19}}-\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}}$$

```
[Out] -1/21*(b*x^2+a)^(11/2)/a/x^21+10/399*b*(b*x^2+a)^(11/2)/a^2/x^19-80/6783*b^
2*(b*x^2+a)^(11/2)/a^3/x^17+32/6783*b^3*(b*x^2+a)^(11/2)/a^4/x^15-128/88179
*b^4*(b*x^2+a)^(11/2)/a^5/x^13+256/969969*b^5*(b*x^2+a)^(11/2)/a^6/x^11
```

Rubi [A] time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{271, 264\}$

$$\frac{256b^5\left(a+bx^2\right)^{11/2}}{969969a^6x^{11}}-\frac{128b^4\left(a+bx^2\right)^{11/2}}{88179a^5x^{13}}+\frac{32b^3\left(a+bx^2\right)^{11/2}}{6783a^4x^{15}}-\frac{80b^2\left(a+bx^2\right)^{11/2}}{6783a^3x^{17}}+\frac{10b\left(a+bx^2\right)^{11/2}}{399a^2x^{19}}-\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^22,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(21*a*x^21) + (10*b*(a + b*x^2)^(11/2))/(399*a^2*x^19)
- (80*b^2*(a + b*x^2)^(11/2))/(6783*a^3*x^17) + (32*b^3*(a + b*x^2)^(11/2))
/(6783*a^4*x^15) - (128*b^4*(a + b*x^2)^(11/2))/(88179*a^5*x^13) + (256*b^5
*(a + b*x^2)^(11/2))/(969969*a^6*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{22}}dx = -\frac{(a+bx^2)^{11/2}}{21ax^{21}} - \frac{(10b)\int \frac{(a+bx^2)^{9/2}}{x^{20}}dx}{21a}$$
$$= -\frac{(a+bx^2)^{11/2}}{21ax^{21}} + \frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}} + \frac{(80b^2)\int \frac{(a+bx^2)^{9/2}}{x^{18}}dx}{399a^2}$$
$$= -\frac{(a+bx^2)^{11/2}}{21ax^{21}} + \frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}} - \frac{80b^2(a+bx^2)^{11/2}}{6783a^3x^{17}} - \frac{(160b^3)\int \frac{(a+bx^2)^{9/2}}{x^{16}}dx}{2261a^3}$$
$$= -\frac{(a+bx^2)^{11/2}}{21ax^{21}} + \frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}} - \frac{80b^2(a+bx^2)^{11/2}}{6783a^3x^{17}} + \frac{32b^3(a+bx^2)^{11/2}}{6783a^4x^{15}} + \frac{(128b^4)}{6}$$
$$= -\frac{(a+bx^2)^{11/2}}{21ax^{21}} + \frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}} - \frac{80b^2(a+bx^2)^{11/2}}{6783a^3x^{17}} + \frac{32b^3(a+bx^2)^{11/2}}{6783a^4x^{15}} - \frac{128b^4(a}{8817}$$
$$= -\frac{(a+bx^2)^{11/2}}{21ax^{21}} + \frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}} - \frac{80b^2(a+bx^2)^{11/2}}{6783a^3x^{17}} + \frac{32b^3(a+bx^2)^{11/2}}{6783a^4x^{15}} - \frac{128b^4(a}{8817}$$

Mathematica [A] time = 0.02, size = 75, normalized size = 0.54

$$\frac{(a+bx^2)^{11/2}(-46189a^5 + 24310a^4bx^2 - 11440a^3b^2x^4 + 4576a^2b^3x^6 - 1408ab^4x^8 + 256b^5x^{10})}{969969a^6x^{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^22,x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-46189*a^5 + 24310*a^4*b*x^2 - 11440*a^3*b^2*x^4 + 457
6*a^2*b^3*x^6 - 1408*a*b^4*x^8 + 256*b^5*x^10))/(969969*a^6*x^21)
```

fricas [A] time = 1.81, size = 126, normalized size = 0.90

$$\frac{(256\,b^{10}x^{20} - 128\,ab^9x^{18} + 96\,a^2b^8x^{16} - 80\,a^3b^7x^{14} + 70\,a^4b^6x^{12} - 63\,a^5b^5x^{10} - 80773\,a^6b^4x^8 - 271414\,a^7b^3x^6}{969969\,a^6x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="fricas")
```

```
[Out] 1/969969*(256*b^10*x^20 - 128*a*b^9*x^18 + 96*a^2*b^8*x^16 - 80*a^3*b^7*x^1
4 + 70*a^4*b^6*x^12 - 63*a^5*b^5*x^10 - 80773*a^6*b^4*x^8 - 271414*a^7*b^3*
x^6 - 351780*a^8*b^2*x^4 - 206635*a^9*b*x^2 - 46189*a^10)*sqrt(b*x^2 + a)/(
a^6*x^21)
```

giac [B] time = 1.25, size = 436, normalized size = 3.11

$$\frac{512\left(646646\left(\sqrt{b}\,x - \sqrt{bx^2+a}\right)^{30} b^{\frac{21}{2}} + 4157010\left(\sqrt{b}\,x - \sqrt{bx^2+a}\right)^{28} ab^{\frac{21}{2}} + 14549535\left(\sqrt{b}\,x - \sqrt{bx^2+a}\right)^{26} a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="giac")
```

```
[Out] 512/969969*(646646*(sqrt(b)*x - sqrt(b*x^2 + a))^30*b^(21/2) + 4157010*(sqr
t(b)*x - sqrt(b*x^2 + a))^28*a*b^(21/2) + 14549535*(sqrt(b)*x - sqrt(b*x^2
```

```
+ a))^26*a^2*b^(21/2) + 30715685*(sqrt(b)*x - sqrt(b*x^2 + a))^24*a^3*b^(21
/2) + 44618574*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^4*b^(21/2) + 44265858*(sq
rt(b)*x - sqrt(b*x^2 + a))^20*a^5*b^(21/2) + 31009615*(sqrt(b)*x - sqrt(b*x
^2 + a))^18*a^6*b^(21/2) + 14346045*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^7*b^
(21/2) + 4273290*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^8*b^(21/2) + 592382*(sq
rt(b)*x - sqrt(b*x^2 + a))^12*a^9*b^(21/2) + 20349*(sqrt(b)*x - sqrt(b*x^2
+ a))^10*a^10*b^(21/2) - 5985*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^11*b^(21/2)
 + 1330*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^12*b^(21/2) - 210*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*a^13*b^(21/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^14*b^
(21/2) - a^15*b^(21/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^21
```

maple [A] time = 0.01, size = 72, normalized size = 0.51

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}} \left(-256 b^{5} x^{10}+1408 a\,b^{4} x^{8}-4576 a^{2} b^{3} x^{6}+11440 a^{3} b^{2} x^{4}-24310 a^{4} b\,x^{2}+46189 a^{5}\right)}{969969 a^{6} x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^22,x)
```

```
[Out] -1/969969*(b*x^2+a)^(11/2)*(-256*b^5*x^10+1408*a*b^4*x^8-4576*a^2*b^3*x^6+1
1440*a^3*b^2*x^4-24310*a^4*b*x^2+46189*a^5)/x^21/a^6
```

maxima [A] time = 1.53, size = 116, normalized size = 0.83

$$\frac{256 \left(bx^{2}+a\right)^{\frac{11}{2}} b^{5}}{969969\, a^{6} x^{11}}-\frac{128 \left(bx^{2}+a\right)^{\frac{11}{2}} b^{4}}{88179\, a^{5} x^{13}}+\frac{32 \left(bx^{2}+a\right)^{\frac{11}{2}} b^{3}}{6783\, a^{4} x^{15}}-\frac{80 \left(bx^{2}+a\right)^{\frac{11}{2}} b^{2}}{6783\, a^{3} x^{17}}+\frac{10 \left(bx^{2}+a\right)^{\frac{11}{2}} b}{399\, a^{2} x^{19}}-\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}}{21\, a x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="maxima")
```

```
[Out] 256/969969*(b*x^2 + a)^(11/2)*b^5/(a^6*x^11) - 128/88179*(b*x^2 + a)^(11/2)
*b^4/(a^5*x^13) + 32/6783*(b*x^2 + a)^(11/2)*b^3/(a^4*x^15) - 80/6783*(b*x^
2 + a)^(11/2)*b^2/(a^3*x^17) + 10/399*(b*x^2 + a)^(11/2)*b/(a^2*x^19) - 1/2
1*(b*x^2 + a)^(11/2)/(a*x^21)
```

mupad [B] time = 9.63, size = 211, normalized size = 1.51

$$\frac{10\, b^{6} \sqrt{b\,x^{2}+a}}{138567\, a^{2}\, x^{9}}-\frac{1049\, b^{4} \sqrt{b\,x^{2}+a}}{12597\, x^{13}}-\frac{1898\, a\, b^{3} \sqrt{b\,x^{2}+a}}{6783\, x^{15}}-\frac{85\, a^{3}\, b \sqrt{b\,x^{2}+a}}{399\, x^{19}}-\frac{3\, b^{5} \sqrt{b\,x^{2}+a}}{46189\, a\, x^{11}}-\frac{a^{4} \sqrt{b\,x^{2}+a}}{21\, x^{21}}-\frac{80\, b}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^22,x)
```

```
[Out] (10*b^6*(a + b*x^2)^(1/2))/(138567*a^2*x^9) - (1049*b^4*(a + b*x^2)^(1/2))/
(12597*x^13) - (1898*a*b^3*(a + b*x^2)^(1/2))/(6783*x^15) - (85*a^3*b*(a +
b*x^2)^(1/2))/(399*x^19) - (3*b^5*(a + b*x^2)^(1/2))/(46189*a*x^11) - (a^4*
(a + b*x^2)^(1/2))/(21*x^21) - (80*b^7*(a + b*x^2)^(1/2))/(969969*a^3*x^7)
+ (32*b^8*(a + b*x^2)^(1/2))/(323323*a^4*x^5) - (128*b^9*(a + b*x^2)^(1/2))
/(969969*a^5*x^3) + (256*b^10*(a + b*x^2)^(1/2))/(969969*a^6*x) - (820*a^2*
b^2*(a + b*x^2)^(1/2))/(2261*x^17)
```

sympy [B] time = 7.80, size = 1540, normalized size = 11.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**22,x)
```

```
[Out] -46189*a**15*b**(51/2)*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 484
9845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**2
6 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 437580*a**14*b**(
53/2)*x**2*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b
**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*
a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 1846845*a**13*b**(55/2)*x**4*
sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22
+ 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*
x**28 + 969969*a**6*b**30*x**30) - 4558554*a**12*b**(57/2)*x**6*sqrt(a/(b*x
**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a
**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969
969*a**6*b**30*x**30) - 7252938*a**11*b**(59/2)*x**8*sqrt(a/(b*x**2) + 1)/(
969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x
**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b*
*30*x**30) - 7715232*a**10*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(969969*a**
11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 969
9690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30)
 - 5487650*a**9*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x*
*20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b
**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 2516940*
a**8*b**(65/2)*x**14*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 48498
45*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26
+ 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 675513*a**7*b**(67/
2)*x**16*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**
26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a*
*7*b**29*x**28 + 969969*a**6*b**30*x**30) - 80836*a**6*b**(69/2)*x**18*sqrt
(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 96
99690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**2
8 + 969969*a**6*b**30*x**30) + 63*a**5*b**(71/2)*x**20*sqrt(a/(b*x**2) + 1)
/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27
*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*
b**30*x**30) + 630*a**4*b**(73/2)*x**22*sqrt(a/(b*x**2) + 1)/(969969*a**11*
b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 969969
0*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) +
1680*a**3*b**(75/2)*x**24*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 +
4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x
**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) + 2016*a**2*b**(
77/2)*x**26*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*
b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845
*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) + 1152*a*b**(79/2)*x**28*sqrt(
a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 969
9690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28
 + 969969*a**6*b**30*x**30) + 256*b**(81/2)*x**30*sqrt(a/(b*x**2) + 1)/(969
969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**2
4 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30
*x**30)
```

3.440 $\int \frac{(a+bx^2)^{9/2}}{x^{24}}\,dx$

Optimal. Leaf size=164

$$-\frac{1024b^6\left(a+bx^2\right)^{11/2}}{7436429a^7x^{11}}+\frac{512b^5\left(a+bx^2\right)^{11/2}}{676039a^6x^{13}}-\frac{128b^4\left(a+bx^2\right)^{11/2}}{52003a^5x^{15}}+\frac{320b^3\left(a+bx^2\right)^{11/2}}{52003a^4x^{17}}-\frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}}+\frac{4b\left(a\right.}{16}$$

```
[Out] -1/23*(b*x^2+a)^(11/2)/a/x^23+4/161*b*(b*x^2+a)^(11/2)/a^2/x^21-40/3059*b^2
*(b*x^2+a)^(11/2)/a^3/x^19+320/52003*b^3*(b*x^2+a)^(11/2)/a^4/x^17-128/5200
3*b^4*(b*x^2+a)^(11/2)/a^5/x^15+512/676039*b^5*(b*x^2+a)^(11/2)/a^6/x^13-10
24/7436429*b^6*(b*x^2+a)^(11/2)/a^7/x^11
```

Rubi [A] time = 0.07, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{1024b^6\left(a+bx^2\right)^{11/2}}{7436429a^7x^{11}}+\frac{512b^5\left(a+bx^2\right)^{11/2}}{676039a^6x^{13}}-\frac{128b^4\left(a+bx^2\right)^{11/2}}{52003a^5x^{15}}+\frac{320b^3\left(a+bx^2\right)^{11/2}}{52003a^4x^{17}}-\frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}}+\frac{4b\left(a\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^24,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(23*a*x^23) + (4*b*(a + b*x^2)^(11/2))/(161*a^2*x^21) -
 (40*b^2*(a + b*x^2)^(11/2))/(3059*a^3*x^19) + (320*b^3*(a + b*x^2)^(11/2))
/(52003*a^4*x^17) - (128*b^4*(a + b*x^2)^(11/2))/(52003*a^5*x^15) + (512*b^
5*(a + b*x^2)^(11/2))/(676039*a^6*x^13) - (1024*b^6*(a + b*x^2)^(11/2))/(74
36429*a^7*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{24}}\,dx = -\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}} - \frac{(12b)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{22}}dx}{23a}$$

$$= -\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}} + \frac{4b\left(a+bx^2\right)^{11/2}}{161a^2x^{21}} + \frac{\left(40b^2\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{20}}dx}{161a^2}$$

$$= -\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}} + \frac{4b\left(a+bx^2\right)^{11/2}}{161a^2x^{21}} - \frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}} - \frac{\left(320b^3\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{18}}dx}{3059a^3}$$

$$= -\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}} + \frac{4b\left(a+bx^2\right)^{11/2}}{161a^2x^{21}} - \frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}} + \frac{320b^3\left(a+bx^2\right)^{11/2}}{52003a^4x^{17}} + \frac{\left(1920b^4\right)\cdots}{5\cdots}$$

$$= -\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}} + \frac{4b\left(a+bx^2\right)^{11/2}}{161a^2x^{21}} - \frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}} + \frac{320b^3\left(a+bx^2\right)^{11/2}}{52003a^4x^{17}} - \frac{128b^4\left(a\cdots\right.}{5200\cdots}$$

$$= -\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}} + \frac{4b\left(a+bx^2\right)^{11/2}}{161a^2x^{21}} - \frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}} + \frac{320b^3\left(a+bx^2\right)^{11/2}}{52003a^4x^{17}} - \frac{128b^4\left(a\cdots\right.}{5200\cdots}$$

$$= -\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}} + \frac{4b\left(a+bx^2\right)^{11/2}}{161a^2x^{21}} - \frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}} + \frac{320b^3\left(a+bx^2\right)^{11/2}}{52003a^4x^{17}} - \frac{128b^4\left(a\cdots\right.}{5200\cdots}$$

Mathematica [A] time = 0.02, size = 86, normalized size = 0.52

$$-\frac{\left(a+bx^2\right)^{11/2}\left(323323a^6-184756a^5bx^2+97240a^4b^2x^4-45760a^3b^3x^6+18304a^2b^4x^8-5632ab^5x^{10}+1024\cdots\right.}{7436429a^7x^{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^24,x]
```

```
[Out] -1/7436429*((a + b*x^2)^(11/2)*(323323*a^6 - 184756*a^5*b*x^2 + 97240*a^4*b
^2*x^4 - 45760*a^3*b^3*x^6 + 18304*a^2*b^4*x^8 - 5632*a*b^5*x^10 + 1024*b^6
*x^12))/(a^7*x^23)
```

fricas [A] time = 1.74, size = 137, normalized size = 0.84

$$-\frac{\left(1024\,b^{11}x^{22}-512\,ab^{10}x^{20}+384\,a^2b^9x^{18}-320\,a^3b^8x^{16}+280\,a^4b^7x^{14}-252\,a^5b^6x^{12}+231\,a^6b^5x^{10}+53095\cdots\right.}{7436429\,a^7x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="fricas")
```

```
[Out] -1/7436429*(1024*b^11*x^22 - 512*a*b^10*x^20 + 384*a^2*b^9*x^18 - 320*a^3*b
^8*x^16 + 280*a^4*b^7*x^14 - 252*a^5*b^6*x^12 + 231*a^6*b^5*x^10 + 530959*a
^7*b^4*x^8 + 1826110*a^8*b^3*x^6 + 2406690*a^9*b^2*x^4 + 1431859*a^10*b*x^2
 + 323323*a^11)*sqrt(b*x^2 + a)/(a^7*x^23)
```

giac [B] time = 1.23, size = 462, normalized size = 2.82

$$\frac{2048\left(4249388\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{32}b^{\frac{23}{2}}+28683369\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{30}ab^{\frac{23}{2}}+100922965\left(\sqrt{b}\,x-\sqrt{bx^2+\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="giac")
```

```
[Out] 2048/7436429*(4249388*(sqrt(b)*x - sqrt(b*x^2 + a))^32*b^(23/2) + 28683369*
(sqrt(b)*x - sqrt(b*x^2 + a))^30*a*b^(23/2) + 100922965*(sqrt(b)*x - sqrt(b
*x^2 + a))^28*a^2*b^(23/2) + 215656441*(sqrt(b)*x - sqrt(b*x^2 + a))^26*a^3
*b^(23/2) + 313006057*(sqrt(b)*x - sqrt(b*x^2 + a))^24*a^4*b^(23/2) + 31165
3979*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^5*b^(23/2) + 216800507*(sqrt(b)*x -
 sqrt(b*x^2 + a))^20*a^6*b^(23/2) + 100105775*(sqrt(b)*x - sqrt(b*x^2 + a))
^18*a^7*b^(23/2) + 29173683*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^8*b^(23/2) +
 4004231*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^9*b^(23/2) + 100947*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*a^10*b^(23/2) - 33649*(sqrt(b)*x - sqrt(b*x^2 + a))^1
0*a^11*b^(23/2) + 8855*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^12*b^(23/2) - 1771
*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^13*b^(23/2) + 253*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*a^14*b^(23/2) - 23*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^15*b^(23/2)
+ a^16*b^(23/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^23
```

maple [A] time = 0.01, size = 83, normalized size = 0.51

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(1024x^{12}b^{6}-5632a\,x^{10}b^{5}+18304a^{2}x^{8}b^{4}-45760a^{3}x^{6}b^{3}+97240a^{4}x^{4}b^{2}-184756a^{5}x^{2}b+323323a^{6}\right)}{7436429a^{7}x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^24,x)
```

```
[Out] -1/7436429*(b*x^2+a)^(11/2)*(1024*b^6*x^12-5632*a*b^5*x^10+18304*a^2*b^4*x^
8-45760*a^3*b^3*x^6+97240*a^4*b^2*x^4-184756*a^5*b*x^2+323323*a^6)/x^23/a^7
```

maxima [A] time = 1.59, size = 136, normalized size = 0.83

$$-\frac{1024\left(bx^{2}+a\right)^{\frac{11}{2}}b^{6}}{7436429\,a^{7}x^{11}}+\frac{512\left(bx^{2}+a\right)^{\frac{11}{2}}b^{5}}{676039\,a^{6}x^{13}}-\frac{128\left(bx^{2}+a\right)^{\frac{11}{2}}b^{4}}{52003\,a^{5}x^{15}}+\frac{320\left(bx^{2}+a\right)^{\frac{11}{2}}b^{3}}{52003\,a^{4}x^{17}}-\frac{40\left(bx^{2}+a\right)^{\frac{11}{2}}b^{2}}{3059\,a^{3}x^{19}}+\frac{4\left(bx^{2}+a\right)^{\frac{11}{2}}}{161\,a^{2}x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="maxima")
```

```
[Out] -1024/7436429*(b*x^2 + a)^(11/2)*b^6/(a^7*x^11) + 512/676039*(b*x^2 + a)^(1
1/2)*b^5/(a^6*x^13) - 128/52003*(b*x^2 + a)^(11/2)*b^4/(a^5*x^15) + 320/520
03*(b*x^2 + a)^(11/2)*b^3/(a^4*x^17) - 40/3059*(b*x^2 + a)^(11/2)*b^2/(a^3*
x^19) + 4/161*(b*x^2 + a)^(11/2)*b/(a^2*x^21) - 1/23*(b*x^2 + a)^(11/2)/(a*
x^23)
```

mupad [B] time = 10.47, size = 231, normalized size = 1.41

$$\frac{36\,b^{6}\,\sqrt{b\,x^{2}+a}}{1062347\,a^{2}\,x^{11}}-\frac{3713\,b^{4}\,\sqrt{b\,x^{2}+a}}{52003\,x^{15}}-\frac{12770\,a\,b^{3}\,\sqrt{b\,x^{2}+a}}{52003\,x^{17}}-\frac{31\,a^{3}\,b\,\sqrt{b\,x^{2}+a}}{161\,x^{21}}-\frac{3\,b^{5}\,\sqrt{b\,x^{2}+a}}{96577\,a\,x^{13}}-\frac{a^{4}\,\sqrt{b\,x^{2}+a}}{23\,x^{23}}-\frac{40}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^24,x)
```

```
[Out] (36*b^6*(a + b*x^2)^(1/2))/(1062347*a^2*x^11) - (3713*b^4*(a + b*x^2)^(1/2)
)/(52003*x^15) - (12770*a*b^3*(a + b*x^2)^(1/2))/(52003*x^17) - (31*a^3*b*(
a + b*x^2)^(1/2))/(161*x^21) - (3*b^5*(a + b*x^2)^(1/2))/(96577*a*x^13) - (
a^4*(a + b*x^2)^(1/2))/(23*x^23) - (40*b^7*(a + b*x^2)^(1/2))/(1062347*a^3*
x^9) + (320*b^8*(a + b*x^2)^(1/2))/(7436429*a^4*x^7) - (384*b^9*(a + b*x^2)
^(1/2))/(7436429*a^5*x^5) + (512*b^10*(a + b*x^2)^(1/2))/(7436429*a^6*x^3)
- (1024*b^11*(a + b*x^2)^(1/2))/(7436429*a^7*x) - (990*a^2*b^2*(a + b*x^2)^
(1/2))/(3059*x^19)
```

<u>**sympy**</u> **[B]** time = 9.07, size = 1950, normalized size = 11.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**24,x)
```

```
[Out] -323323*a**17*b**(73/2)*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 4
4618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b
**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 74364
29*a**7*b**42*x**34) - 3371797*a**16*b**(75/2)*x**2*sqrt(a/(b*x**2) + 1)/(7
436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**
38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618
574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 15847689*a**15*b**(77/2)
*x**4*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**3
7*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 11154
6435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**3
4) - 44210595*a**14*b**(79/2)*x**6*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**3
6*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728
580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x*
*32 + 7436429*a**7*b**42*x**34) - 81074994*a**13*b**(81/2)*x**8*sqrt(a/(b*x
**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 1115464
35*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x
**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 102129258*a*
*12*b**(83/2)*x**10*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618
574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39
*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a
**7*b**42*x**34) - 89502546*a**11*b**(85/2)*x**12*sqrt(a/(b*x**2) + 1)/(743
6429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38
*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 4461857
4*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 53885062*a**10*b**(87/2)*x
**14*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37
*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546
435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34
) - 21329935*a**9*b**(89/2)*x**16*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36
*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 1487285
80*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**
32 + 7436429*a**7*b**42*x**34) - 5012953*a**8*b**(91/2)*x**18*sqrt(a/(b*x**
2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435
*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**
30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 531157*a**7*b*
*(93/2)*x**20*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a*
*12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28
 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b*
*42*x**34) - 231*a**6*b**(95/2)*x**22*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b
**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148
728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41
*x**32 + 7436429*a**7*b**42*x**34) - 2772*a**5*b**(97/2)*x**24*sqrt(a/(b*x*
*2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 11154643
5*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x*
*30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 9240*a**4*b**
(99/2)*x**26*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**
12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28
+ 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**
42*x**34) - 14784*a**3*b**(101/2)*x**28*sqrt(a/(b*x**2) + 1)/(7436429*a**13
*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 1
48728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**
41*x**32 + 7436429*a**7*b**42*x**34) - 12672*a**2*b**(103/2)*x**30*sqrt(a/(
b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 1115
```

```
46435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**4
0*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 5632*a*b*
*(105/2)*x**32*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a
**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**2
8 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b
**42*x**34) - 1024*b**(107/2)*x**34*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**
36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 14872
8580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x
**32 + 7436429*a**7*b**42*x**34)
```

3.441 $\int x^5\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{448}\left(4x^2+9\right)^{7/2}-\frac{9}{160}\left(4x^2+9\right)^{5/2}+\frac{27}{64}\left(4x^2+9\right)^{3/2}$$

```
[Out] 27/64*(4*x^2+9)^(3/2)-9/160*(4*x^2+9)^(5/2)+1/448*(4*x^2+9)^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{448}\left(4x^2+9\right)^{7/2}-\frac{9}{160}\left(4x^2+9\right)^{5/2}+\frac{27}{64}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[9 + 4*x^2],x]
```

```
[Out] (27*(9 + 4*x^2)^(3/2))/64 - (9*(9 + 4*x^2)^(5/2))/160 + (9 + 4*x^2)^(7/2)/4
48
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{9+4x^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int x^2\sqrt{9+4x}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{81}{16}\sqrt{9+4x}-\frac{9}{8}(9+4x)^{3/2}+\frac{1}{16}(9+4x)^{5/2}\right)dx,x,x^2\right)\\&=\frac{27}{64}\left(9+4x^2\right)^{3/2}-\frac{9}{160}\left(9+4x^2\right)^{5/2}+\frac{1}{448}\left(9+4x^2\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$\frac{1}{280}\left(4x^2+9\right)^{3/2}\left(10x^4-18x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[9 + 4*x^2],x]
```

```
[Out] ((9 + 4*x^2)^(3/2)*(27 - 18*x^2 + 10*x^4))/280
```

fricas [A] time = 0.87, size = 28, normalized size = 0.61

$$\frac{1}{280}\left(40\,x^6+18\,x^4-54\,x^2+243\right)\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/280*(40*x^6 + 18*x^4 - 54*x^2 + 243)*sqrt(4*x^2 + 9)
```

giac [A] time = 1.03, size = 34, normalized size = 0.74

$$\frac{1}{448}\left(4x^2+9\right)^{\frac{7}{2}}-\frac{9}{160}\left(4x^2+9\right)^{\frac{5}{2}}+\frac{27}{64}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/448*(4*x^2 + 9)^(7/2) - 9/160*(4*x^2 + 9)^(5/2) + 27/64*(4*x^2 + 9)^(3/2)
```

maple [A] time = 0.00, size = 24, normalized size = 0.52

$$\frac{\left(4x^2+9\right)^{\frac{3}{2}}\left(10x^4-18x^2+27\right)}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(4*x^2+9)^(1/2),x)
```

```
[Out] 1/280*(4*x^2+9)^(3/2)*(10*x^4-18*x^2+27)
```

maxima [A] time = 2.97, size = 40, normalized size = 0.87

$$\frac{1}{28}\left(4x^2+9\right)^{\frac{3}{2}}x^4-\frac{9}{140}\left(4x^2+9\right)^{\frac{3}{2}}x^2+\frac{27}{280}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/28*(4*x^2 + 9)^(3/2)*x^4 - 9/140*(4*x^2 + 9)^(3/2)*x^2 + 27/280*(4*x^2 +
9)^(3/2)
```

mupad [B] time = 4.55, size = 25, normalized size = 0.54

$$\sqrt{x^2+\frac{9}{4}}\left(\frac{2x^6}{7}+\frac{9x^4}{70}-\frac{27x^2}{70}+\frac{243}{140}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(4*x^2 + 9)^(1/2),x)
```

```
[Out] (x^2 + 9/4)^(1/2)*((9*x^4)/70 - (27*x^2)/70 + (2*x^6)/7 + 243/140)
```

sympy [A] time = 1.97, size = 61, normalized size = 1.33

$$\frac{x^6\sqrt{4x^2+9}}{7}+\frac{9x^4\sqrt{4x^2+9}}{140}-\frac{27x^2\sqrt{4x^2+9}}{140}+\frac{243\sqrt{4x^2+9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(4*x**2+9)**(1/2),x)
```

```
[Out] x**6*sqrt(4*x**2 + 9)/7 + 9*x**4*sqrt(4*x**2 + 9)/140 - 27*x**2*sqrt(4*x**2
 + 9)/140 + 243*sqrt(4*x**2 + 9)/280
```

3.442 $\int x^4\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=63

$$-\frac{81}{256}\sqrt{4x^2+9}\,x+\frac{1}{6}\sqrt{4x^2+9}\,x^5+\frac{3}{32}\sqrt{4x^2+9}\,x^3+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 729/512*arcsinh(2/3*x)-81/256*x*(4*x^2+9)^(1/2)+3/32*x^3*(4*x^2+9)^(1/2)+1/
6*x^5*(4*x^2+9)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {279, 321, 215}

$$\frac{1}{6}\sqrt{4x^2+9}\,x^5+\frac{3}{32}\sqrt{4x^2+9}\,x^3-\frac{81}{256}\sqrt{4x^2+9}\,x+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[9 + 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[9 + 4*x^2])/256 + (3*x^3*Sqrt[9 + 4*x^2])/32 + (x^5*Sqrt[9 + 4*
x^2])/6 + (729*ArcSinh[(2*x)/3])/512
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^4\sqrt{9+4x^2}\,dx &= \frac{1}{6}x^5\sqrt{9+4x^2}+\frac{3}{2}\int\frac{x^4}{\sqrt{9+4x^2}}\,dx\\
&= \frac{3}{32}x^3\sqrt{9+4x^2}+\frac{1}{6}x^5\sqrt{9+4x^2}-\frac{81}{32}\int\frac{x^2}{\sqrt{9+4x^2}}\,dx\\
&= -\frac{81}{256}x\sqrt{9+4x^2}+\frac{3}{32}x^3\sqrt{9+4x^2}+\frac{1}{6}x^5\sqrt{9+4x^2}+\frac{729}{256}\int\frac{1}{\sqrt{9+4x^2}}\,dx\\
&= -\frac{81}{256}x\sqrt{9+4x^2}+\frac{3}{32}x^3\sqrt{9+4x^2}+\frac{1}{6}x^5\sqrt{9+4x^2}+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.62

$$\frac{1}{768}x\sqrt{4x^2+9}\left(128x^4+72x^2-243\right)+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2]*(-243 + 72*x^2 + 128*x^4))/768 + (729*ArcSinh[(2*x)/3])/
512
```

fricas [A] time = 0.97, size = 42, normalized size = 0.67

$$\frac{1}{768}\left(128\,x^5+72\,x^3-243\,x\right)\sqrt{4\,x^2+9}-\frac{729}{512}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 + 72*x^3 - 243*x)*sqrt(4*x^2 + 9) - 729/512*log(-2*x + sqrt(
4*x^2 + 9))
```

giac [A] time = 1.17, size = 43, normalized size = 0.68

$$\frac{1}{768}\left(8\left(16\,x^2+9\right)x^2-243\right)\sqrt{4\,x^2+9}\,x-\frac{729}{512}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 + 9)*x^2 - 243)*sqrt(4*x^2 + 9)*x - 729/512*log(-2*x + sqr
t(4*x^2 + 9))
```

maple [A] time = 0.01, size = 46, normalized size = 0.73

$$\frac{\left(4x^2+9\right)^{\frac{3}{2}}x^3}{24}-\frac{9\left(4x^2+9\right)^{\frac{3}{2}}x}{128}+\frac{81\sqrt{4x^2+9}\,x}{256}+\frac{729\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^2+9)^(1/2),x)
```

```
[Out] 1/24*x^3*(4*x^2+9)^(3/2)-9/128*x*(4*x^2+9)^(3/2)+81/256*x*(4*x^2+9)^(1/2)+7
29/512*arcsinh(2/3*x)
```

maxima [A] time = 2.94, size = 45, normalized size = 0.71

$$\frac{1}{24}\left(4\,x^2+9\right)^{\frac{3}{2}}x^3-\frac{9}{128}\left(4\,x^2+9\right)^{\frac{3}{2}}x+\frac{81}{256}\,\sqrt{4\,x^2+9}\,x+\frac{729}{512}\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^2 + 9)^(3/2)*x^3 - 9/128*(4*x^2 + 9)^(3/2)*x + 81/256*sqrt(4*x^2
+ 9)*x + 729/512*arcsinh(2/3*x)
```

mupad [B] time = 0.03, size = 30, normalized size = 0.48

$$\frac{729\,\mathrm{asinh}\left(\frac{2\,x}{3}\right)}{512}+\frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{2\,x^5}{3}+\frac{3\,x^3}{8}-\frac{81\,x}{64}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^2 + 9)^(1/2),x)
```

```
[Out] (729*asinh((2*x)/3))/512 + ((x^2 + 9/4)^(1/2)*((3*x^3)/8 - (81*x)/64 + (2*x
^5)/3))/2
```

sympy [A] time = 4.56, size = 75, normalized size = 1.19

$$\frac{2x^7}{3\sqrt{4x^2+9}}+\frac{15x^5}{8\sqrt{4x^2+9}}-\frac{27x^3}{64\sqrt{4x^2+9}}-\frac{729x}{256\sqrt{4x^2+9}}+\frac{729\operatorname{asinh}\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(4*x**2+9)**(1/2),x)
```

```
[Out] 2*x**7/(3*sqrt(4*x**2 + 9)) + 15*x**5/(8*sqrt(4*x**2 + 9)) - 27*x**3/(64*sq
rt(4*x**2 + 9)) - 729*x/(256*sqrt(4*x**2 + 9)) + 729*asinh(2*x/3)/512
```

3.443 $\int x^3\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{80}\left(4x^2+9\right)^{5/2}-\frac{3}{16}\left(4x^2+9\right)^{3/2}$$

```
[Out] -3/16*(4*x^2+9)^(3/2)+1/80*(4*x^2+9)^(5/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{80}\left(4x^2+9\right)^{5/2}-\frac{3}{16}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[9 + 4*x^2],x]
```

```
[Out] (-3*(9 + 4*x^2)^(3/2))/16 + (9 + 4*x^2)^(5/2)/80
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{9+4x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x\sqrt{9+4x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{9}{4}\sqrt{9+4x}+\frac{1}{4}(9+4x)^{3/2}\right)dx,x,x^2\right)\\
&= -\frac{3}{16}\left(9+4x^2\right)^{3/2}+\frac{1}{80}\left(9+4x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(2x^2-3\right)\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[9 + 4*x^2],x]
```

```
[Out] ((-3 + 2*x^2)*(9 + 4*x^2)^(3/2))/40
```

fricas [A] time = 0.83, size = 23, normalized size = 0.74

$$\frac{1}{40}\left(8x^4+6x^2-27\right)\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/40*(8*x^4 + 6*x^2 - 27)*sqrt(4*x^2 + 9)
```

giac [A] time = 1.01, size = 23, normalized size = 0.74

$$\frac{1}{80}\left(4x^2+9\right)^{\frac{5}{2}}-\frac{3}{16}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/80*(4*x^2 + 9)^(5/2) - 3/16*(4*x^2 + 9)^(3/2)
```

maple [A] time = 0.00, size = 19, normalized size = 0.61

$$\frac{\left(4x^2+9\right)^{\frac{3}{2}}\left(2x^2-3\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*x^2+9)^(1/2),x)
[Out] 1/40*(4*x^2+9)^(3/2)*(2*x^2-3)
```

maxima [A] time = 2.96, size = 26, normalized size = 0.84

$$\frac{1}{20}\left(4x^2+9\right)^{\frac{3}{2}}x^2-\frac{3}{40}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/20*(4*x^2 + 9)^(3/2)*x^2 - 3/40*(4*x^2 + 9)^(3/2)
```

mupad [B] time = 0.02, size = 20, normalized size = 0.65

$$\sqrt{x^2+\frac{9}{4}}\left(\frac{2x^4}{5}+\frac{3x^2}{10}-\frac{27}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*x^2 + 9)^(1/2),x)
[Out] (x^2 + 9/4)^(1/2)*((3*x^2)/10 + (2*x^4)/5 - 27/20)
```

sympy [A] time = 0.63, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{4x^2+9}}{5}+\frac{3x^2\sqrt{4x^2+9}}{20}-\frac{27\sqrt{4x^2+9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(4*x**2+9)**(1/2),x)
[Out] x**4*sqrt(4*x**2 + 9)/5 + 3*x**2*sqrt(4*x**2 + 9)/20 - 27*sqrt(4*x**2 + 9)/
40
```

3.444 $\int x^2\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=45

$$\frac{9}{32}\sqrt{4x^2+9}\,x+\frac{1}{4}\sqrt{4x^2+9}\,x^3-\frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] -81/64*arcsinh(2/3*x)+9/32*x*(4*x^2+9)^(1/2)+1/4*x^3*(4*x^2+9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {279, 321, 215}

$$\frac{1}{4}\sqrt{4x^2+9}\,x^3+\frac{9}{32}\sqrt{4x^2+9}\,x-\frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[9 + 4*x^2],x]
```

```
[Out] (9*x*Sqrt[9 + 4*x^2])/32 + (x^3*Sqrt[9 + 4*x^2])/4 - (81*ArcSinh[(2*x)/3])/
64
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{9+4x^2}\,dx &= \frac{1}{4}x^3\sqrt{9+4x^2}+\frac{9}{4}\int\frac{x^2}{\sqrt{9+4x^2}}\,dx\\ &= \frac{9}{32}x\sqrt{9+4x^2}+\frac{1}{4}x^3\sqrt{9+4x^2}-\frac{81}{32}\int\frac{1}{\sqrt{9+4x^2}}\,dx\\ &= \frac{9}{32}x\sqrt{9+4x^2}+\frac{1}{4}x^3\sqrt{9+4x^2}-\frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.80

$$\sqrt{4x^2+9}\left(\frac{x^3}{4}+\frac{9x}{32}\right)-\frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[9 + 4*x^2],x]
[Out] Sqrt[9 + 4*x^2]*((9*x)/32 + x^3/4) - (81*ArcSinh[(2*x)/3])/64
```

fricas [A] time = 0.95, size = 37, normalized size = 0.82

$$\frac{1}{32}\left(8\,x^3+9\,x\right)\sqrt{4\,x^2+9}+\frac{81}{64}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/32*(8*x^3 + 9*x)*sqrt(4*x^2 + 9) + 81/64*log(-2*x + sqrt(4*x^2 + 9))
```

giac [A] time = 1.05, size = 36, normalized size = 0.80

$$\frac{1}{32}\left(8\,x^2+9\right)\sqrt{4\,x^2+9}\,x+\frac{81}{64}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/32*(8*x^2 + 9)*sqrt(4*x^2 + 9)*x + 81/64*log(-2*x + sqrt(4*x^2 + 9))
```

maple [A] time = 0.00, size = 32, normalized size = 0.71

$$\frac{\left(4x^2+9\right)^{\frac{3}{2}}x}{16}-\frac{9\sqrt{4x^2+9}\,x}{32}-\frac{81\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^2+9)^(1/2),x)
[Out] 1/16*(4*x^2+9)^(3/2)*x-9/32*(4*x^2+9)^(1/2)*x-81/64*arcsinh(2/3*x)
```

maxima [A] time = 2.97, size = 31, normalized size = 0.69

$$\frac{1}{16}\left(4\,x^2+9\right)^{\frac{3}{2}}x-\frac{9}{32}\,\sqrt{4\,x^2+9}\,x-\frac{81}{64}\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/16*(4*x^2 + 9)^(3/2)*x - 9/32*sqrt(4*x^2 + 9)*x - 81/64*arcsinh(2/3*x)
```

mupad [B] time = 0.03, size = 23, normalized size = 0.51

$$\frac{\left(x^3+\frac{9\,x}{8}\right)\,\sqrt{x^2+\frac{9}{4}}}{2}-\frac{81\,\mathrm{asinh}\left(\frac{2\,x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^2 + 9)^(1/2),x)
[Out] (((9*x)/8 + x^3)*(x^2 + 9/4)^(1/2))/2 - (81*asinh((2*x)/3))/64
```

sympy [A] time = 2.71, size = 54, normalized size = 1.20

$$\frac{x^5}{\sqrt{4x^2+9}}+\frac{27x^3}{8\sqrt{4x^2+9}}+\frac{81x}{32\sqrt{4x^2+9}}-\frac{81\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(4*x**2+9)**(1/2),x)
```

```
[Out] x**5/sqrt(4*x**2 + 9) + 27*x**3/(8*sqrt(4*x**2 + 9)) + 81*x/(32*sqrt(4*x**2
 + 9)) - 81*asinh(2*x/3)/64
```

3.445 $\int x\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=15

$$\frac{1}{12}\left(4x^2+9\right)^{3/2}$$

```
[Out] 1/12*(4*x^2+9)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{12}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[9 + 4*x^2],x]
```

```
[Out] (9 + 4*x^2)^(3/2)/12
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{9+4x^2}\,dx=\frac{1}{12}\left(9+4x^2\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{12}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[9 + 4*x^2],x]
```

```
[Out] (9 + 4*x^2)^(3/2)/12
```

fricas [A] time = 0.94, size = 11, normalized size = 0.73

$$\frac{1}{12}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^2 + 9)^(3/2)
```

giac [A] time = 1.04, size = 11, normalized size = 0.73

$$\frac{1}{12}\left(4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/12*(4*x^2 + 9)^(3/2)
```

maple [**A**] time = 0.00, size = 12, normalized size = 0.80

$$\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)*x,x)
[Out] 1/12*(4*x^2+9)^(3/2)
```

maxima [**A**] time = 1.31, size = 11, normalized size = 0.73

$$\frac{1}{12}\left(4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/12*(4*x^2 + 9)^(3/2)
```

mupad [**B**] time = 0.02, size = 16, normalized size = 1.07

$$\frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{4\,x^2}{3}+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2 + 9)^(1/2),x)
[Out] ((x^2 + 9/4)^(1/2)*((4*x^2)/3 + 3))/2
```

sympy [**B**] time = 0.20, size = 27, normalized size = 1.80

$$\frac{x^2\sqrt{4x^2+9}}{3}+\frac{3\sqrt{4x^2+9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2+9)**(1/2),x)
[Out] x**2*sqrt(4*x**2 + 9)/3 + 3*sqrt(4*x**2 + 9)/4
```

3.446 $\int \sqrt{9+4x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\sqrt{4x^2+9}\,x+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 9/4*arcsinh(2/3*x)+1/2*x*(4*x^2+9)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 215}

$$\frac{1}{2}\sqrt{4x^2+9}\,x+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2],x]
[Out] (x*Sqrt[9 + 4*x^2])/2 + (9*ArcSinh[(2*x)/3])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{9+4x^2}\,dx &= \frac{1}{2}x\sqrt{9+4x^2}+\frac{9}{2}\int \frac{1}{\sqrt{9+4x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{9+4x^2}+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{1}{2}\sqrt{4x^2+9}\,x+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2],x]
[Out] (x*Sqrt[9 + 4*x^2])/2 + (9*ArcSinh[(2*x)/3])/4
```

fricas [A] time = 0.95, size = 29, normalized size = 1.07

$$\frac{1}{2}\sqrt{4\,x^2+9}\,x-\frac{9}{4}\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

giac [A] time = 0.99, size = 29, normalized size = 1.07

$$\frac{1}{2}\sqrt{4x^2+9}\,x - \frac{9}{4}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

maple [A] time = 0.00, size = 20, normalized size = 0.74

$$\frac{\sqrt{4x^2+9}\,x}{2}+\frac{9\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2),x)
[Out] 9/4*arcsinh(2/3*x)+1/2*(4*x^2+9)^(1/2)*x
```

maxima [A] time = 2.98, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{4x^2+9}\,x + \frac{9}{4}\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(4*x^2 + 9)*x + 9/4*arcsinh(2/3*x)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.59

$$\frac{9\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{4}+x\sqrt{x^2+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2),x)
[Out] (9*asinh((2*x)/3))/4 + x*(x^2 + 9/4)^(1/2)
```

sympy [A] time = 0.21, size = 22, normalized size = 0.81

$$\frac{x\sqrt{4x^2+9}}{2}+\frac{9\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2),x)
[Out] x*sqrt(4*x**2 + 9)/2 + 9*asinh(2*x/3)/4
```

3.447 $\int \frac{\sqrt{9+4x^2}}{x} \, dx$

Optimal. Leaf size=30

$$\sqrt{4x^2+9}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

```
[Out] -3*arctanh(1/3*(4*x^2+9)^(1/2))+(4*x^2+9)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 207}

$$\sqrt{4x^2+9}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x,x]
```

```
[Out] Sqrt[9 + 4*x^2] - 3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{9+4x^2}}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{9+4x}}{x}\,dx, x, x^2\right)$$
$$= \sqrt{9+4x^2} + \frac{9}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)$$
$$= \sqrt{9+4x^2} + \frac{9}{4}\,\text{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)$$
$$= \sqrt{9+4x^2} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\sqrt{4x^2+9} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x,x]
```

```
[Out] Sqrt[9 + 4*x^2] - 3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

fricas [A] time = 1.00, size = 44, normalized size = 1.47

$$\sqrt{4x^2+9} - 3\log\left(-2x+\sqrt{4x^2+9}+3\right) + 3\log\left(-2x+\sqrt{4x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(4*x^2 + 9) - 3*log(-2*x + sqrt(4*x^2 + 9) + 3) + 3*log(-2*x + sqrt(4*x
^2 + 9) - 3)
```

giac [A] time = 1.03, size = 38, normalized size = 1.27

$$\sqrt{4x^2+9} - \frac{3}{2}\log\left(\sqrt{4x^2+9}+3\right) + \frac{3}{2}\log\left(\sqrt{4x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(4*x^2 + 9) - 3/2*log(sqrt(4*x^2 + 9) + 3) + 3/2*log(sqrt(4*x^2 + 9) -
3)
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-3\,\text{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right) + \sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x,x)
```

```
[Out] (4*x^2+9)^(1/2)-3*arctanh(3/(4*x^2+9)^(1/2))
```

maxima [A] time = 2.94, size = 19, normalized size = 0.63

$$\sqrt{4x^2+9} - 3\,\text{arsinh}\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(4*x^2 + 9) - 3*arcsinh(3/2/abs(x))
```

mupad [B] time = 0.03, size = 22, normalized size = 0.73

$$2\sqrt{x^2+\frac{9}{4}}-3\,\mathrm{atanh}\left(\frac{2\sqrt{x^2+\frac{9}{4}}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x,x)
```

```
[Out] 2*(x^2 + 9/4)^(1/2) - 3*atanh((2*(x^2 + 9/4)^(1/2))/3)
```

sympy [A] time = 1.25, size = 39, normalized size = 1.30

$$\frac{2x}{\sqrt{1+\frac{9}{4x^2}}}-3\,\mathrm{asinh}\left(\frac{3}{2x}\right)+\frac{9}{2x\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x,x)
```

```
[Out] 2*x/sqrt(1 + 9/(4*x**2)) - 3*asinh(3/(2*x)) + 9/(2*x*sqrt(1 + 9/(4*x**2)))
```

3.448 $\int \frac{\sqrt{9+4x^2}}{x^2} dx$

Optimal. Leaf size=25

$$2\sinh^{-1}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2+9}}{x}$$

```
[Out] 2*arcsinh(2/3*x)-(4*x^2+9)^(1/2)/x
```

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = $\{277, 215\}$

$$2\sinh^{-1}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2+9}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 + 4*x^2]/x) + 2*ArcSinh[(2*x)/3]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9+4x^2}}{x^2} dx &= -\frac{\sqrt{9+4x^2}}{x}+4\int \frac{1}{\sqrt{9+4x^2}} dx\\ &= -\frac{\sqrt{9+4x^2}}{x}+2\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$2\sinh^{-1}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2+9}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 + 4*x^2]/x) + 2*ArcSinh[(2*x)/3]
```

fricas [A] time = 0.73, size = 35, normalized size = 1.40

$$-\frac{2\,x\log\left(-2\,x+\sqrt{4\,x^2+9}\right)+2\,x+\sqrt{4\,x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*x*log(-2*x + sqrt(4*x^2 + 9)) + 2*x + sqrt(4*x^2 + 9))/x
```

giac [A] time = 1.15, size = 40, normalized size = 1.60

$$\frac{36}{\left(2\,x-\sqrt{4\,x^2+9}\right)^2-9}-2\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 36/((2*x - sqrt(4*x^2 + 9))^2 - 9) - 2*log(-2*x + sqrt(4*x^2 + 9))
```

maple [A] time = 0.00, size = 34, normalized size = 1.36

$$\frac{4\sqrt{4x^2+9}\,x}{9}+2\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)-\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^2,x)
```

```
[Out] -1/9/x*(4*x^2+9)^(3/2)+4/9*(4*x^2+9)^(1/2)*x+2*arcsinh(2/3*x)
```

maxima [A] time = 2.94, size = 21, normalized size = 0.84

$$-\frac{\sqrt{4\,x^2+9}}{x}+2\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(4*x^2 + 9)/x + 2*arcsinh(2/3*x)
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$2\,\mathrm{asinh}\left(\frac{2\,x}{3}\right)-\frac{2\,\sqrt{x^2+\frac{9}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x^2,x)
```

```
[Out] 2*asinh((2*x)/3) - (2*(x^2 + 9/4)^(1/2))/x
```

sympy [A] time = 0.25, size = 19, normalized size = 0.76

$$2\,\mathrm{asinh}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**2,x)
```

```
[Out] 2*asinh(2*x/3) - sqrt(4*x**2 + 9)/x
```

3.449 $\int \frac{\sqrt{9+4x^2}}{x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{4x^2+9}}{2x^2}-\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

```
[Out] -2/3*arctanh(1/3*(4*x^2+9)^(1/2))-1/2*(4*x^2+9)^(1/2)/x^2
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 207}

$$-\frac{\sqrt{4x^2+9}}{2x^2}-\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^3,x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(2*x^2) - (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{9+4x^2}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{9+4x}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{2x^2} + \text{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{2x^2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)\\
&= -\frac{\sqrt{9+4x^2}}{2x^2} - \frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.95

$$
-\frac{\sqrt{4x^2+9}}{2x^2} - \frac{2}{3}\tanh^{-1}\left(\sqrt{\frac{4x^2}{9}+1}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^3,x]
```

```
[Out] -1/2*Sqrt[9 + 4*x^2]/x^2 - (2*ArcTanh[Sqrt[1 + (4*x^2)/9]])/3
```

fricas [A] time = 1.00, size = 57, normalized size = 1.46

$$
-\frac{4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right) - 4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right) + 3\,\sqrt{4\,x^2+9}}{6\,x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*x^2*log(-2*x + sqrt(4*x^2 + 9) + 3) - 4*x^2*log(-2*x + sqrt(4*x^2 +
 9) - 3) + 3*sqrt(4*x^2 + 9))/x^2
```

giac [A] time = 1.21, size = 43, normalized size = 1.10

$$
-\frac{\sqrt{4\,x^2+9}}{2\,x^2} - \frac{1}{3}\log\left(\sqrt{4\,x^2+9}+3\right) + \frac{1}{3}\log\left(\sqrt{4\,x^2+9}-3\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(4*x^2 + 9)/x^2 - 1/3*log(sqrt(4*x^2 + 9) + 3) + 1/3*log(sqrt(4*x^
2 + 9) - 3)
```

maple [A] time = 0.01, size = 41, normalized size = 1.05

$$
-\frac{2\,\text{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{3} - \frac{\left(4x^2+9\right)^{\frac{3}{2}}}{18x^2} + \frac{2\sqrt{4x^2+9}}{9}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^3,x)
```

```
[Out] -1/18/x^2*(4*x^2+9)^(3/2)+2/9*(4*x^2+9)^(1/2)-2/3*arctanh(3/(4*x^2+9)^(1/2)
)
```

maxima [A] time = 2.81, size = 35, normalized size = 0.90

$$\frac{2}{9}\sqrt{4x^2+9} - \frac{\left(4x^2+9\right)^{\frac{3}{2}}}{18x^2} - \frac{2}{3}\,\mathrm{arsinh}\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(4*x^2 + 9) - 1/18*(4*x^2 + 9)^(3/2)/x^2 - 2/3*arcsinh(3/2/abs(x))
```

mupad [B] time = 0.03, size = 25, normalized size = 0.64

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\sqrt{x^2+\frac{9}{4}}}{3}\right)}{3} - \frac{\sqrt{x^2+\frac{9}{4}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x^3,x)
```

```
[Out] - (2*atanh((2*(x^2 + 9/4)^(1/2))/3))/3 - (x^2 + 9/4)^(1/2)/x^2
```

sympy [A] time = 1.67, size = 24, normalized size = 0.62

$$-\frac{2\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{3} - \frac{\sqrt{1+\frac{9}{4x^2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**3,x)
```

```
[Out] -2*asinh(3/(2*x))/3 - sqrt(1 + 9/(4*x**2))/x
```

3.450 $\int \frac{\sqrt{9+4x^2}}{x^4}\, dx$

Optimal. Leaf size=18

$$-\frac{\left(4x^2+9\right)^{3/2}}{27x^3}$$

```
[Out] -1/27*(4*x^2+9)^(3/2)/x^3
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\left(4x^2+9\right)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^4,x]
[Out] -(9 + 4*x^2)^(3/2)/(27*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{9+4x^2}}{x^4}\, dx = -\frac{\left(9+4x^2\right)^{3/2}}{27x^3}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{\left(4x^2+9\right)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^4,x]
[Out] -1/27*(9 + 4*x^2)^(3/2)/x^3
```

fricas [A] time = 0.76, size = 20, normalized size = 1.11

$$-\frac{8\,x^3+\left(4\,x^2+9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="fricas")
[Out] -1/27*(8*x^3 + (4*x^2 + 9)^(3/2))/x^3
```

giac [B] time = 1.01, size = 42, normalized size = 2.33

$$\frac{16\left(\left(2x-\sqrt{4x^2+9}\right)^4+27\right)}{\left(\left(2x-\sqrt{4x^2+9}\right)^2-9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="giac")
[Out] 16*((2*x - sqrt(4*x^2 + 9))^4 + 27)/((2*x - sqrt(4*x^2 + 9))^2 - 9)^3
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{(4x^2+9)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^4,x)
[Out] -1/27*(4*x^2+9)^(3/2)/x^3
```

maxima [A] time = 2.94, size = 14, normalized size = 0.78

$$-\frac{(4x^2+9)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="maxima")
[Out] -1/27*(4*x^2 + 9)^(3/2)/x^3
```

mupad [B] time = 0.03, size = 27, normalized size = 1.50

$$-\frac{18\sqrt{x^2+\frac{9}{4}}+8x^2\sqrt{x^2+\frac{9}{4}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x^4,x)
[Out] -(18*(x^2 + 9/4)^(1/2) + 8*x^2*(x^2 + 9/4)^(1/2))/(27*x^3)
```

sympy [B] time = 0.93, size = 34, normalized size = 1.89

$$-\frac{8\sqrt{1+\frac{9}{4x^2}}}{27}-\frac{2\sqrt{1+\frac{9}{4x^2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**4,x)
[Out] -8*sqrt(1 + 9/(4*x**2))/27 - 2*sqrt(1 + 9/(4*x**2))/(3*x**2)
```

3.451 $\int \frac{\sqrt{9+4x^2}}{x^5} dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{4x^2+9}}{18x^2}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{4x^4}$$

```
[Out] 2/27*arctanh(1/3*(4*x^2+9)^(1/2))-1/4*(4*x^2+9)^(1/2)/x^4-1/18*(4*x^2+9)^(1
/2)/x^2
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 207}

$$-\frac{\sqrt{4x^2+9}}{18x^2}-\frac{\sqrt{4x^2+9}}{4x^4}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^5,x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(4*x^4) - Sqrt[9 + 4*x^2]/(18*x^2) + (2*ArcTanh[Sqrt[9 + 4
*x^2]/3])/27
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{9+4x^2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{9+4x}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{4x^4} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{4x^4} - \frac{\sqrt{9+4x^2}}{18x^2} - \frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{4x^4} - \frac{\sqrt{9+4x^2}}{18x^2} - \frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)\\
&= -\frac{\sqrt{9+4x^2}}{4x^4} - \frac{\sqrt{9+4x^2}}{18x^2} + \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$-\frac{16\left(4x^2+9\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{4x^2}{9}+1\right)}{2187}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^5,x]
```

```
[Out] (-16*(9 + 4*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (4*x^2)/9])/2187
```

fricas [A] time = 1.10, size = 64, normalized size = 1.12

$$\frac{8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)-3\,\sqrt{4\,x^2+9}\left(2\,x^2+9\right)}{108\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/108*(8*x^4*log(-2*x + sqrt(4*x^2 + 9) + 3) - 8*x^4*log(-2*x + sqrt(4*x^2
+ 9) - 3) - 3*sqrt(4*x^2 + 9)*(2*x^2 + 9))/x^4
```

giac [A] time = 1.05, size = 55, normalized size = 0.96

$$-\frac{\left(4\,x^2+9\right)^{\frac{3}{2}}+9\,\sqrt{4\,x^2+9}}{72\,x^4}+\frac{1}{27}\,\log\left(\sqrt{4\,x^2+9}+3\right)-\frac{1}{27}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/72*((4*x^2 + 9)^(3/2) + 9*sqrt(4*x^2 + 9))/x^4 + 1/27*log(sqrt(4*x^2 + 9
) + 3) - 1/27*log(sqrt(4*x^2 + 9) - 3)
```

maple [A] time = 0.01, size = 55, normalized size = 0.96

$$\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{27}+\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{162x^2}-\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{36x^4}-\frac{2\sqrt{4x^2+9}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^5,x)
```

```
[Out] -1/36/x^4*(4*x^2+9)^(3/2)+1/162*(4*x^2+9)^(3/2)/x^2-2/81*(4*x^2+9)^(1/2)+2/
27*arctanh(3/(4*x^2+9)^(1/2))
```

maxima [A] time = 3.01, size = 49, normalized size = 0.86

$$-\frac{2}{81}\sqrt{4x^2+9}+\frac{(4x^2+9)^{\frac{3}{2}}}{162x^2}-\frac{(4x^2+9)^{\frac{3}{2}}}{36x^4}+\frac{2}{27}\operatorname{arsinh}\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(4*x^2 + 9) + 1/162*(4*x^2 + 9)^(3/2)/x^2 - 1/36*(4*x^2 + 9)^(3/2
)/x^4 + 2/27*arcsinh(3/2/abs(x))
```

mupad [B] time = 0.03, size = 45, normalized size = 0.79

$$\frac{2\operatorname{atanh}\left(\frac{2\sqrt{x^2+\frac{9}{4}}}{3}\right)}{27}+\frac{\sqrt{x^2+\frac{9}{4}}\left(\frac{2}{3x^2}-\frac{1}{x^4}\right)}{2}-\frac{4\sqrt{x^2+\frac{9}{4}}}{9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x^5,x)
```

```
[Out] (2*atanh((2*(x^2 + 9/4)^(1/2))/3))/27 + ((x^2 + 9/4)^(1/2)*(2/(3*x^2) - 1/x
^4))/2 - (4*(x^2 + 9/4)^(1/2))/(9*x^2)
```

sympy [A] time = 3.20, size = 63, normalized size = 1.11

$$\frac{2\operatorname{asinh}\left(\frac{3}{2x}\right)}{27}-\frac{1}{9x\sqrt{1+\frac{9}{4x^2}}}-\frac{3}{4x^3\sqrt{1+\frac{9}{4x^2}}}-\frac{9}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**5,x)
```

```
[Out] 2*asinh(3/(2*x))/27 - 1/(9*x*sqrt(1 + 9/(4*x**2))) - 3/(4*x**3*sqrt(1 + 9/(
4*x**2))) - 9/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

3.452 $\int x^5\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{448}\left(9-4x^2\right)^{7/2}+\frac{9}{160}\left(9-4x^2\right)^{5/2}-\frac{27}{64}\left(9-4x^2\right)^{3/2}$$

```
[Out] -27/64*(-4*x^2+9)^(3/2)+9/160*(-4*x^2+9)^(5/2)-1/448*(-4*x^2+9)^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{1}{448}\left(9-4x^2\right)^{7/2}+\frac{9}{160}\left(9-4x^2\right)^{5/2}-\frac{27}{64}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[9 - 4*x^2],x]
```

```
[Out] (-27*(9 - 4*x^2)^(3/2))/64 + (9*(9 - 4*x^2)^(5/2))/160 - (9 - 4*x^2)^(7/2)/
448
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5\sqrt{9-4x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \sqrt{9-4x}\,x^2\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16}\sqrt{9-4x}-\frac{9}{8}(9-4x)^{3/2}+\frac{1}{16}(9-4x)^{5/2}\right)dx, x, x^2\right)\\
&= -\frac{27}{64}\left(9-4x^2\right)^{3/2}+\frac{9}{160}\left(9-4x^2\right)^{5/2}-\frac{1}{448}\left(9-4x^2\right)^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$-\frac{1}{280}\left(9-4x^2\right)^{3/2}\left(10x^4+18x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[9 - 4*x^2],x]
```

```
[Out] -1/280*((9 - 4*x^2)^(3/2)*(27 + 18*x^2 + 10*x^4))
```

fricas [A] time = 0.95, size = 28, normalized size = 0.61

$$\frac{1}{280}\left(40\,x^6-18\,x^4-54\,x^2-243\right)\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/280*(40*x^6 - 18*x^4 - 54*x^2 - 243)*sqrt(-4*x^2 + 9)
```

giac [A] time = 1.02, size = 52, normalized size = 1.13

$$\frac{1}{448}\left(4x^2-9\right)^3\sqrt{-4x^2+9}+\frac{9}{160}\left(4x^2-9\right)^2\sqrt{-4x^2+9}-\frac{27}{64}\left(-4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/448*(4*x^2 - 9)^3*sqrt(-4*x^2 + 9) + 9/160*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9)
 - 27/64*(-4*x^2 + 9)^(3/2)
```

maple [A] time = 0.00, size = 34, normalized size = 0.74

$$\frac{(2x-3)(2x+3)\left(10x^4+18x^2+27\right)\sqrt{-4x^2+9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/280*(2*x-3)*(2*x+3)*(10*x^4+18*x^2+27)*(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.91, size = 40, normalized size = 0.87

$$-\frac{1}{28}\left(-4x^2+9\right)^{\frac{3}{2}}x^4-\frac{9}{140}\left(-4x^2+9\right)^{\frac{3}{2}}x^2-\frac{27}{280}\left(-4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/28*(-4*x^2 + 9)^(3/2)*x^4 - 9/140*(-4*x^2 + 9)^(3/2)*x^2 - 27/280*(-4*x^
2 + 9)^(3/2)
```

mupad [B] time = 4.53, size = 28, normalized size = 0.61

$$-\frac{\sqrt{\frac{9}{4}-x^2}\left(-\frac{4x^6}{7}+\frac{9x^4}{35}+\frac{27x^2}{35}+\frac{243}{70}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(9 - 4*x^2)^(1/2),x)
```

```
[Out] -((9/4 - x^2)^(1/2)*((27*x^2)/35 + (9*x^4)/35 - (4*x^6)/7 + 243/70))/2
```

sympy [A] time = 2.01, size = 61, normalized size = 1.33

$$\frac{x^6\sqrt{9-4x^2}}{7}-\frac{9x^4\sqrt{9-4x^2}}{140}-\frac{27x^2\sqrt{9-4x^2}}{140}-\frac{243\sqrt{9-4x^2}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-4*x**2+9)**(1/2),x)
```

```
[Out] x**6*sqrt(9 - 4*x**2)/7 - 9*x**4*sqrt(9 - 4*x**2)/140 - 27*x**2*sqrt(9 - 4*
x**2)/140 - 243*sqrt(9 - 4*x**2)/280
```

3.453 $\int x^4\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=63

$$-\frac{81}{256}\sqrt{9-4x^2}\,x+\frac{1}{6}\sqrt{9-4x^2}\,x^5-\frac{3}{32}\sqrt{9-4x^2}\,x^3+\frac{729}{512}\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 729/512*arcsin(2/3*x)-81/256*x*(-4*x^2+9)^(1/2)-3/32*x^3*(-4*x^2+9)^(1/2)+1
/6*x^5*(-4*x^2+9)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {279, 321, 216}

$$\frac{1}{6}\sqrt{9-4x^2}\,x^5-\frac{3}{32}\sqrt{9-4x^2}\,x^3-\frac{81}{256}\sqrt{9-4x^2}\,x+\frac{729}{512}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[9 - 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[9 - 4*x^2])/256 - (3*x^3*Sqrt[9 - 4*x^2])/32 + (x^5*Sqrt[9 - 4*
x^2])/6 + (729*ArcSin[(2*x)/3])/512
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^4\sqrt{9-4x^2}\,dx &= \frac{1}{6}x^5\sqrt{9-4x^2}+\frac{3}{2}\int\frac{x^4}{\sqrt{9-4x^2}}\,dx\\
&= -\frac{3}{32}x^3\sqrt{9-4x^2}+\frac{1}{6}x^5\sqrt{9-4x^2}+\frac{81}{32}\int\frac{x^2}{\sqrt{9-4x^2}}\,dx\\
&= -\frac{81}{256}x\sqrt{9-4x^2}-\frac{3}{32}x^3\sqrt{9-4x^2}+\frac{1}{6}x^5\sqrt{9-4x^2}+\frac{729}{256}\int\frac{1}{\sqrt{9-4x^2}}\,dx\\
&= -\frac{81}{256}x\sqrt{9-4x^2}-\frac{3}{32}x^3\sqrt{9-4x^2}+\frac{1}{6}x^5\sqrt{9-4x^2}+\frac{729}{512}\sin^{-1}\left(\frac{2x}{3}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.62

$$\frac{1}{768}x\sqrt{9-4x^2}\left(128x^4-72x^2-243\right)+\frac{729}{512}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[9 - 4*x^2]*(-243 - 72*x^2 + 128*x^4))/768 + (729*ArcSin[(2*x)/3])/5
12
```

fricas [A] time = 0.90, size = 45, normalized size = 0.71

$$\frac{1}{768}\left(128\,x^5-72\,x^3-243\,x\right)\sqrt{-4\,x^2+9}-\frac{729}{256}\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 - 72*x^3 - 243*x)*sqrt(-4*x^2 + 9) - 729/256*arctan(1/2*(sqr
t(-4*x^2 + 9) - 3)/x)
```

giac [A] time = 1.08, size = 33, normalized size = 0.52

$$\frac{1}{768}\left(8\left(16\,x^2-9\right)x^2-243\right)\sqrt{-4\,x^2+9}\,x+\frac{729}{512}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 - 9)*x^2 - 243)*sqrt(-4*x^2 + 9)*x + 729/512*arcsin(2/3*x)
```

maple [A] time = 0.01, size = 46, normalized size = 0.73

$$-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}x^3}{24}-\frac{9\left(-4x^2+9\right)^{\frac{3}{2}}x}{128}+\frac{81\sqrt{-4x^2+9}\;x}{256}+\frac{729\arcsin\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-4*x^2+9)^(1/2),x)
```

```
[Out] -1/24*x^3*(-4*x^2+9)^(3/2)-9/128*x*(-4*x^2+9)^(3/2)+81/256*x*(-4*x^2+9)^(1/
2)+729/512*arcsin(2/3*x)
```

maxima [A] time = 3.00, size = 45, normalized size = 0.71

$$-\frac{1}{24}\left(-4\,x^2+9\right)^{\frac{3}{2}}x^3-\frac{9}{128}\left(-4\,x^2+9\right)^{\frac{3}{2}}x+\frac{81}{256}\sqrt{-4\,x^2+9}\,x+\frac{729}{512}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(-4*x^2 + 9)^(3/2)*x^3 - 9/128*(-4*x^2 + 9)^(3/2)*x + 81/256*sqrt(-4*
x^2 + 9)*x + 729/512*arcsin(2/3*x)
```

mupad [B] time = 4.56, size = 32, normalized size = 0.51

$$\frac{729\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{512}-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(-\frac{2\,x^5}{3}+\frac{3\,x^3}{8}+\frac{81\,x}{64}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(9 - 4*x^2)^(1/2),x)
```

```
[Out] (729*asin((2*x)/3))/512 - ((9/4 - x^2)^(1/2)*((81*x)/64 + (3*x^3)/8 - (2*x^
5)/3))/2
```

sympy [A] time = 4.66, size = 167, normalized size = 2.65

$$\begin{cases} \frac{2ix^7}{3\sqrt{4x^2-9}} - \frac{15ix^5}{8\sqrt{4x^2-9}} - \frac{27ix^3}{64\sqrt{4x^2-9}} + \frac{729ix}{256\sqrt{4x^2-9}} - \frac{729i\operatorname{acosh}\left(\frac{2x}{3}\right)}{512} & \text{for } \frac{4|x^2|}{9} > 1 \\ -\frac{2x^7}{3\sqrt{9-4x^2}} + \frac{15x^5}{8\sqrt{9-4x^2}} + \frac{27x^3}{64\sqrt{9-4x^2}} - \frac{729x}{256\sqrt{9-4x^2}} + \frac{729\operatorname{asin}\left(\frac{2x}{3}\right)}{512} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((2*I*x**7/(3*sqrt(4*x**2 - 9)) - 15*I*x**5/(8*sqrt(4*x**2 - 9)) -
 27*I*x**3/(64*sqrt(4*x**2 - 9)) + 729*I*x/(256*sqrt(4*x**2 - 9)) - 729*I*a
cosh(2*x/3)/512, 4*Abs(x**2)/9 > 1), (-2*x**7/(3*sqrt(9 - 4*x**2)) + 15*x**
5/(8*sqrt(9 - 4*x**2)) + 27*x**3/(64*sqrt(9 - 4*x**2)) - 729*x/(256*sqrt(9
- 4*x**2)) + 729*asin(2*x/3)/512, True))
```

3.454 $\int x^3\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{80}\left(9-4x^2\right)^{5/2}-\frac{3}{16}\left(9-4x^2\right)^{3/2}$$

```
[Out] -3/16*(-4*x^2+9)^(3/2)+1/80*(-4*x^2+9)^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{80}\left(9-4x^2\right)^{5/2}-\frac{3}{16}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[9 - 4*x^2],x]
```

```
[Out] (-3*(9 - 4*x^2)^(3/2))/16 + (9 - 4*x^2)^(5/2)/80
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{9-4x^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int\sqrt{9-4x}\,x\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{9}{4}\sqrt{9-4x}-\frac{1}{4}(9-4x)^{3/2}\right)dx,x,x^2\right)\\&=-\frac{3}{16}\left(9-4x^2\right)^{3/2}+\frac{1}{80}\left(9-4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$-\frac{1}{40}\left(9-4x^2\right)^{3/2}\left(2x^2+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[9 - 4*x^2],x]
```

```
[Out] -1/40*((9 - 4*x^2)^(3/2)*(3 + 2*x^2))
```

fricas [A] time = 0.74, size = 23, normalized size = 0.74

$$\frac{1}{40}\left(8\,x^4-6\,x^2-27\right)\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 - 6*x^2 - 27)*sqrt(-4*x^2 + 9)
```

giac [A] time = 1.14, size = 32, normalized size = 1.03

$$\frac{1}{80} \left(4 x^2 - 9\right)^2 \sqrt{-4 x^2 + 9} - \frac{3}{16} \left(-4 x^2 + 9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/80*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9) - 3/16*(-4*x^2 + 9)^(3/2)
```

maple [A] time = 0.00, size = 29, normalized size = 0.94

$$\frac{(2x-3)(2x+3)\left(2x^2+3\right)\sqrt{-4x^2+9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/40*(2*x-3)*(2*x+3)*(2*x^2+3)*(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.97, size = 26, normalized size = 0.84

$$-\frac{1}{20} \left(-4 x^2 + 9\right)^{\frac{3}{2}} x^2 - \frac{3}{40} \left(-4 x^2 + 9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*(-4*x^2 + 9)^(3/2)*x^2 - 3/40*(-4*x^2 + 9)^(3/2)
```

mupad [B] time = 0.02, size = 23, normalized size = 0.74

$$-\frac{\sqrt{\frac{9}{4} - x^2}\,\left(-\frac{4\,x^4}{5} + \frac{3\,x^2}{5} + \frac{27}{10}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(9 - 4*x^2)^(1/2),x)
```

```
[Out] -((9/4 - x^2)^(1/2)*((3*x^2)/5 - (4*x^4)/5 + 27/10))/2
```

sympy [A] time = 0.67, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{9-4x^2}}{5} - \frac{3x^2\sqrt{9-4x^2}}{20} - \frac{27\sqrt{9-4x^2}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-4*x**2+9)**(1/2),x)
```

```
[Out] x**4*sqrt(9 - 4*x**2)/5 - 3*x**2*sqrt(9 - 4*x**2)/20 - 27*sqrt(9 - 4*x**2)/
40
```

3.455 $\int x^2\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=45

$$-\frac{9}{32}\sqrt{9-4x^2}\,x+\frac{1}{4}\sqrt{9-4x^2}\,x^3+\frac{81}{64}\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 81/64*arcsin(2/3*x)-9/32*x*(-4*x^2+9)^(1/2)+1/4*x^3*(-4*x^2+9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {279, 321, 216}

$$\frac{1}{4}\sqrt{9-4x^2}\,x^3-\frac{9}{32}\sqrt{9-4x^2}\,x+\frac{81}{64}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[9 - 4*x^2],x]
```

```
[Out] (-9*x*Sqrt[9 - 4*x^2])/32 + (x^3*Sqrt[9 - 4*x^2])/4 + (81*ArcSin[(2*x)/3])/
64
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{9-4x^2}\,dx&=\frac{1}{4}x^3\sqrt{9-4x^2}+\frac{9}{4}\int\frac{x^2}{\sqrt{9-4x^2}}\,dx\\&=-\frac{9}{32}x\sqrt{9-4x^2}+\frac{1}{4}x^3\sqrt{9-4x^2}+\frac{81}{32}\int\frac{1}{\sqrt{9-4x^2}}\,dx\\&=-\frac{9}{32}x\sqrt{9-4x^2}+\frac{1}{4}x^3\sqrt{9-4x^2}+\frac{81}{64}\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.80

$$\sqrt{9-4x^2}\left(\frac{x^3}{4}-\frac{9x}{32}\right)+\frac{81}{64}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[9 - 4*x^2],x]
[Out] Sqrt[9 - 4*x^2]*((-9*x)/32 + x^3/4) + (81*ArcSin[(2*x)/3])/64
```

fricas [A] time = 0.95, size = 40, normalized size = 0.89

$$\frac{1}{32}\left(8x^3 - 9x\right)\sqrt{-4x^2+9} - \frac{81}{32}\arctan\left(\frac{\sqrt{-4x^2+9}-3}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/32*(8*x^3 - 9*x)*sqrt(-4*x^2 + 9) - 81/32*arctan(1/2*(sqrt(-4*x^2 + 9) -
3)/x)
```

giac [A] time = 1.09, size = 26, normalized size = 0.58

$$\frac{1}{32}\left(8x^2 - 9\right)\sqrt{-4x^2+9}\,x + \frac{81}{64}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/32*(8*x^2 - 9)*sqrt(-4*x^2 + 9)*x + 81/64*arcsin(2/3*x)
```

maple [A] time = 0.01, size = 32, normalized size = 0.71

$$-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}x}{16} + \frac{9\sqrt{-4x^2+9}\,x}{32} + \frac{81\arcsin\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-4*x^2+9)^(1/2),x)
[Out] -1/16*(-4*x^2+9)^(3/2)*x+9/32*(-4*x^2+9)^(1/2)*x+81/64*arcsin(2/3*x)
```

maxima [A] time = 2.95, size = 31, normalized size = 0.69

$$-\frac{1}{16}\left(-4x^2+9\right)^{\frac{3}{2}}x + \frac{9}{32}\sqrt{-4x^2+9}\,x + \frac{81}{64}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/16*(-4*x^2 + 9)^(3/2)*x + 9/32*sqrt(-4*x^2 + 9)*x + 81/64*arcsin(2/3*x)
```

mupad [B] time = 0.03, size = 27, normalized size = 0.60

$$\frac{81\,\mathrm{asin}\left(\frac{2x}{3}\right)}{64} - \frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{9x}{8}-x^3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(9 - 4*x^2)^(1/2),x)
[Out] (81*asin((2*x)/3))/64 - ((9/4 - x^2)^(1/2)*((9*x)/8 - x^3))/2
```

sympy [A] time = 2.68, size = 124, normalized size = 2.76

$$\begin{cases} \frac{ix^5}{\sqrt{4x^2-9}} - \frac{27ix^3}{8\sqrt{4x^2-9}} + \frac{81ix}{32\sqrt{4x^2-9}} - \frac{81i\operatorname{acosh}\left(\frac{2x}{3}\right)}{64} & \text{for } \frac{4|x^2|}{9} > 1 \\ -\frac{x^5}{\sqrt{9-4x^2}} + \frac{27x^3}{8\sqrt{9-4x^2}} - \frac{81x}{32\sqrt{9-4x^2}} + \frac{81\operatorname{asin}\left(\frac{2x}{3}\right)}{64} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((I*x**5/sqrt(4*x**2 - 9) - 27*I*x**3/(8*sqrt(4*x**2 - 9)) + 81*I*
x/(32*sqrt(4*x**2 - 9)) - 81*I*acosh(2*x/3)/64, 4*Abs(x**2)/9 > 1), (-x**5/
sqrt(9 - 4*x**2) + 27*x**3/(8*sqrt(9 - 4*x**2)) - 81*x/(32*sqrt(9 - 4*x**2)
) + 81*asin(2*x/3)/64, True))
```

3.456 $\int x\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

```
[Out] -1/12*(-4*x^2+9)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[9 - 4*x^2],x]
[Out] -(9 - 4*x^2)^(3/2)/12
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{9-4x^2}\,dx=-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[9 - 4*x^2],x]
[Out] -1/12*(9 - 4*x^2)^(3/2)
```

fricas [A] time = 0.65, size = 18, normalized size = 1.20

$$\frac{1}{12}\left(4\,x^2-9\right)\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/12*(4*x^2 - 9)*sqrt(-4*x^2 + 9)
```

giac [A] time = 1.08, size = 11, normalized size = 0.73

$$-\frac{1}{12}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*(-4*x^2 + 9)^(3/2)
```

maple **[A]** time = 0.00, size = 22, normalized size = 1.47

$$\frac{(2x-3)(2x+3)\sqrt{-4x^2+9}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)*x,x)
```

```
[Out] 1/12*(2*x-3)*(2*x+3)*(-4*x^2+9)^(1/2)
```

maxima **[A]** time = 1.28, size = 11, normalized size = 0.73

$$-\frac{1}{12}\left(-4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-4*x^2 + 9)^(3/2)
```

mupad **[B]** time = 0.02, size = 18, normalized size = 1.20

$$\frac{\sqrt{\frac{9}{4}-x^2}\left(\frac{4x^2}{3}-3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(9 - 4*x^2)^(1/2),x)
```

```
[Out] ((9/4 - x^2)^(1/2)*((4*x^2)/3 - 3))/2
```

sympy **[B]** time = 0.20, size = 27, normalized size = 1.80

$$\frac{x^2\sqrt{9-4x^2}}{3}-\frac{3\sqrt{9-4x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x**2+9)**(1/2),x)
```

```
[Out] x**2*sqrt(9 - 4*x**2)/3 - 3*sqrt(9 - 4*x**2)/4
```

3.457 $\int \sqrt{9-4x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\sqrt{9-4x^2}\,x+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 9/4*arcsin(2/3*x)+1/2*x*(-4*x^2+9)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 216}

$$\frac{1}{2}\sqrt{9-4x^2}\,x+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[9 - 4*x^2])/2 + (9*ArcSin[(2*x)/3])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{9-4x^2}\,dx &= \frac{1}{2}x\sqrt{9-4x^2}+\frac{9}{2}\int \frac{1}{\sqrt{9-4x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{9-4x^2}+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{1}{2}\sqrt{9-4x^2}\,x+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[9 - 4*x^2])/2 + (9*ArcSin[(2*x)/3])/4
```

fricas [A] time = 1.47, size = 32, normalized size = 1.19

$$\frac{1}{2}\sqrt{-4\,x^2+9}\,x-\frac{9}{2}\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-4*x^2 + 9)*x - 9/2*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

giac [A] time = 1.00, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{-4\,x^2+9}\,x+\frac{9}{4}\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.74

$$\frac{\sqrt{-4x^2+9}\,x}{2}+\frac{9\arcsin\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2),x)
```

```
[Out] 9/4*arcsin(2/3*x)+1/2*(-4*x^2+9)^(1/2)*x
```

maxima [A] time = 2.94, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{-4\,x^2+9}\,x+\frac{9}{4}\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)
```

mupad [B] time = 0.02, size = 18, normalized size = 0.67

$$\frac{9\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{4}+x\,\sqrt{\frac{9}{4}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2),x)
```

```
[Out] (9*asin((2*x)/3))/4 + x*(9/4 - x^2)^(1/2)
```

sympy [A] time = 0.22, size = 22, normalized size = 0.81

$$\frac{x\sqrt{9-4x^2}}{2}+\frac{9\,\mathrm{asin}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2),x)
```

```
[Out] x*sqrt(9 - 4*x**2)/2 + 9*asin(2*x/3)/4
```

3.458 $\int \frac{\sqrt{9-4x^2}}{x} \, dx$

Optimal. Leaf size=30

$$\sqrt{9-4x^2} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

```
[Out] -3*arctanh(1/3*(-4*x^2+9)^(1/2))+(-4*x^2+9)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 206}

$$\sqrt{9-4x^2} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x,x]
```

```
[Out] Sqrt[9 - 4*x^2] - 3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{9-4x^2}}{x}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\sqrt{9-4x}}{x}\,dx, x, x^2\right)$$
$$= \sqrt{9-4x^2} + \frac{9}{2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx, x, x^2\right)$$
$$= \sqrt{9-4x^2} - \frac{9}{4}\,\mathrm{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)$$
$$= \sqrt{9-4x^2} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\sqrt{9-4x^2} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x,x]
```

```
[Out] Sqrt[9 - 4*x^2] - 3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

fricas [A] time = 0.85, size = 28, normalized size = 0.93

$$\sqrt{-4\,x^2+9} + 3\,\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(-4*x^2 + 9) + 3*log((sqrt(-4*x^2 + 9) - 3)/x)
```

giac [A] time = 1.18, size = 40, normalized size = 1.33

$$\sqrt{-4\,x^2+9} - \frac{3}{2}\,\log\left(\sqrt{-4\,x^2+9}+3\right) + \frac{3}{2}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(-4*x^2 + 9) - 3/2*log(sqrt(-4*x^2 + 9) + 3) + 3/2*log(-sqrt(-4*x^2 + 9
) + 3)
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$-3\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right) + \sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x,x)
```

```
[Out] (-4*x^2+9)^(1/2)-3*arctanh(3/(-4*x^2+9)^(1/2))
```

maxima [A] time = 2.86, size = 35, normalized size = 1.17

$$\sqrt{-4\,x^2+9} - 3\,\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(-4*x^2 + 9) - 3*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 4.55, size = 32, normalized size = 1.07

$$3\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1}-\frac{3\,\sqrt{\frac{1}{x^2}}}{2}\right)+2\,\sqrt{\frac{9}{4}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x,x)
```

```
[Out] 3*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2) + 2*(9/4 - x^2)^(1/2)
```

sympy [A] time = 1.32, size = 76, normalized size = 2.53

$$\begin{cases} i\sqrt{4x^2-9} - 3\log(x) + \frac{3\log(x^2)}{2} + 3i\operatorname{asin}\left(\frac{3}{2x}\right) & \text{for } \frac{4|x^2|}{9} > 1 \\ \sqrt{9-4x^2} + \frac{3\log(x^2)}{2} - 3\log\left(\sqrt{1-\frac{4x^2}{9}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x,x)
```

```
[Out] Piecewise((I*sqrt(4*x**2 - 9) - 3*log(x) + 3*log(x**2)/2 + 3*I*asin(3/(2*x)
), 4*Abs(x**2)/9 > 1), (sqrt(9 - 4*x**2) + 3*log(x**2)/2 - 3*log(sqrt(1 - 4
*x**2/9) + 1), True))
```

3.459 $\int \frac{\sqrt{9-4x^2}}{x^2}\,dx$

Optimal. Leaf size=25

$$-\frac{\sqrt{9-4x^2}}{x}-2\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] -2*arcsin(2/3*x)-(-4*x^2+9)^(1/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = $\{277, 216\}$

$$-\frac{\sqrt{9-4x^2}}{x}-2\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x^2,x]
[Out] -(Sqrt[9 - 4*x^2]/x) - 2*ArcSin[(2*x)/3]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9-4x^2}}{x^2}\,dx &= -\frac{\sqrt{9-4x^2}}{x}-4\int \frac{1}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{\sqrt{9-4x^2}}{x}-2\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{\sqrt{9-4x^2}}{x}-2\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^2,x]
[Out] -(Sqrt[9 - 4*x^2]/x) - 2*ArcSin[(2*x)/3]
```

fricas [A] time = 0.74, size = 36, normalized size = 1.44

$$\frac{4\,x\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)-\sqrt{-4\,x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (4*x*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x) - sqrt(-4*x^2 + 9))/x
```

giac [A] time = 1.14, size = 39, normalized size = 1.56

$$\frac{2\,x}{\sqrt{-4\,x^2+9}\,-3}-\frac{\sqrt{-4\,x^2+9}\,-3}{2\,x}-2\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*x/(sqrt(-4*x^2 + 9) - 3) - 1/2*(sqrt(-4*x^2 + 9) - 3)/x - 2*arcsin(2/3*x)
```

maple [A] time = 0.00, size = 34, normalized size = 1.36

$$-\frac{4\sqrt{-4x^2+9}\,x}{9}-2\arcsin\left(\frac{2x}{3}\right)-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^2,x)
```

```
[Out] -1/9/x*(-4*x^2+9)^(3/2)-4/9*(-4*x^2+9)^(1/2)*x-2*arcsin(2/3*x)
```

maxima [A] time = 2.92, size = 21, normalized size = 0.84

$$-\frac{\sqrt{-4\,x^2+9}}{x}-2\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(-4*x^2 + 9)/x - 2*arcsin(2/3*x)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$-2\,\mathrm{asin}\left(\frac{2\,x}{3}\right)-\frac{2\,\sqrt{\frac{9}{4}-x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x^2,x)
```

```
[Out] - 2*asin((2*x)/3) - (2*(9/4 - x^2)^(1/2))/x
```

sympy [A] time = 0.24, size = 20, normalized size = 0.80

$$-2\,\mathrm{asin}\left(\frac{2x}{3}\right)-\frac{\sqrt{9-4x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**2,x)
```

```
[Out] -2*asin(2*x/3) - sqrt(9 - 4*x**2)/x
```

3.460 $\int \frac{\sqrt{9-4x^2}}{x^3} dx$

Optimal. Leaf size=39

$$\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{2x^2}$$

[Out] 2/3*arctanh(1/3*(-4*x^2+9)^(1/2))-1/2*(-4*x^2+9)^(1/2)/x^2

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 206}

$$\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[9 - 4*x^2]/x^3,x]

[Out] -Sqrt[9 - 4*x^2]/(2*x^2) + (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/3

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{9-4x^2}}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{9-4x}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{9-4x^2}}{2x^2} - \text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{9-4x^2}}{2x^2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)$$
$$= -\frac{\sqrt{9-4x^2}}{2x^2} + \frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.95

$$\frac{2}{3}\tanh^{-1}\left(\sqrt{1-\frac{4x^2}{9}}\right) - \frac{\sqrt{9-4x^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^3,x]
[Out] -1/2*Sqrt[9 - 4*x^2]/x^2 + (2*ArcTanh[Sqrt[1 - (4*x^2)/9]])/3
```

fricas [A] time = 0.92, size = 38, normalized size = 0.97

$$-\frac{4\,x^2\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right) + 3\,\sqrt{-4\,x^2+9}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="fricas")
[Out] -1/6*(4*x^2*log((sqrt(-4*x^2 + 9) - 3)/x) + 3*sqrt(-4*x^2 + 9))/x^2
```

giac [A] time = 1.07, size = 45, normalized size = 1.15

$$-\frac{\sqrt{-4\,x^2+9}}{2\,x^2} + \frac{1}{3}\,\log\left(\sqrt{-4\,x^2+9}+3\right) - \frac{1}{3}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="giac")
[Out] -1/2*sqrt(-4*x^2 + 9)/x^2 + 1/3*log(sqrt(-4*x^2 + 9) + 3) - 1/3*log(-sqrt(-
4*x^2 + 9) + 3)
```

maple [A] time = 0.01, size = 41, normalized size = 1.05

$$\frac{2\,\text{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{3} - \frac{\left(-4x^2+9\right)^{\frac{3}{2}}}{18x^2} - \frac{2\sqrt{-4x^2+9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^3,x)
[Out] -1/18/x^2*(-4*x^2+9)^(3/2)-2/9*(-4*x^2+9)^(1/2)+2/3*arctanh(3/(-4*x^2+9)^(1
/2))
```

maxima [A] time = 2.99, size = 51, normalized size = 1.31

$$-\frac{2}{9}\sqrt{-4x^2+9}-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}}{18x^2}+\frac{2}{3}\log\left(\frac{6\sqrt{-4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(-4*x^2 + 9) - 1/18*(-4*x^2 + 9)^(3/2)/x^2 + 2/3*log(6*sqrt(-4*x^2
 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 4.66, size = 35, normalized size = 0.90

$$-\frac{2\ln\left(\sqrt{\frac{9}{4x^2}-1}-\frac{3\sqrt{\frac{1}{x^2}}}{2}\right)}{3}-\frac{\sqrt{\frac{9}{4}-x^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x^3,x)
```

```
[Out] - (2*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2))/3 - (9/4 - x^2)^(1/2
)/x^2
```

sympy [A] time = 1.73, size = 97, normalized size = 2.49

$$\begin{cases} \frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{3}+\frac{1}{x\sqrt{-1+\frac{9}{4x^2}}}-\frac{9}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{3}-\frac{i}{x\sqrt{1-\frac{9}{4x^2}}}+\frac{9i}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**3,x)
```

```
[Out] Piecewise((2*acosh(3/(2*x))/3 + 1/(x*sqrt(-1 + 9/(4*x**2))) - 9/(4*x**3*sqr
t(-1 + 9/(4*x**2))), 9/(4*Abs(x**2)) > 1), (-2*I*asin(3/(2*x))/3 - I/(x*sqr
t(1 - 9/(4*x**2))) + 9*I/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

3.461 $\int \frac{\sqrt{9-4x^2}}{x^4} \, dx$

Optimal. Leaf size=18

$$-\frac{(9-4x^2)^{3/2}}{27x^3}$$

```
[Out] -1/27*(-4*x^2+9)^(3/2)/x^3
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{(9-4x^2)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x^4,x]
```

```
[Out] -(9 - 4*x^2)^(3/2)/(27*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{9-4x^2}}{x^4} \, dx = -\frac{(9-4x^2)^{3/2}}{27x^3}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{(9-4x^2)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^4,x]
```

```
[Out] -1/27*(9 - 4*x^2)^(3/2)/x^3
```

fricas [A] time = 0.91, size = 21, normalized size = 1.17

$$\frac{(4x^2-9)\sqrt{-4x^2+9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(4*x^2 - 9)*sqrt(-4*x^2 + 9)/x^3
```

giac [B] time = 1.05, size = 73, normalized size = 4.06

$$-\frac{2\,x^3\left(\frac{3\left(\sqrt{-4\,x^2+9}-3\right)^2}{x^2}-4\right)}{27\left(\sqrt{-4\,x^2+9}-3\right)^3}+\frac{\sqrt{-4\,x^2+9}-3}{18\,x}-\frac{\left(\sqrt{-4\,x^2+9}-3\right)^3}{216\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -2/27*x^3*(3*(sqrt(-4*x^2 + 9) - 3)^2/x^2 - 4)/(sqrt(-4*x^2 + 9) - 3)^3 + 1
/18*(sqrt(-4*x^2 + 9) - 3)/x - 1/216*(sqrt(-4*x^2 + 9) - 3)^3/x^3
```

maple [A] time = 0.00, size = 25, normalized size = 1.39

$$\frac{(2x-3)(2x+3)\sqrt{-4x^2+9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^4,x)
```

```
[Out] 1/27/x^3*(2*x-3)*(2*x+3)*(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.90, size = 14, normalized size = 0.78

$$-\frac{\left(-4\,x^2+9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/27*(-4*x^2 + 9)^(3/2)/x^3
```

mupad [B] time = 0.03, size = 31, normalized size = 1.72

$$\frac{8\,x^2\,\sqrt{\frac{9}{4}-x^2}-18\,\sqrt{\frac{9}{4}-x^2}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x^4,x)
```

```
[Out] (8*x^2*(9/4 - x^2)^(1/2) - 18*(9/4 - x^2)^(1/2))/(27*x^3)
```

sympy [B] time = 0.97, size = 76, normalized size = 4.22

$$\begin{cases} \frac{8\sqrt{-1+\frac{9}{4x^2}}}{27}-\frac{2\sqrt{-1+\frac{9}{4x^2}}}{3x^2} & \text{for}\ \frac{9}{4|x^2|}>1 \\ \frac{8i\sqrt{1-\frac{9}{4x^2}}}{27}-\frac{2i\sqrt{1-\frac{9}{4x^2}}}{3x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**4,x)
```

```
[Out] Piecewise((8*sqrt(-1 + 9/(4*x**2))/27 - 2*sqrt(-1 + 9/(4*x**2))/(3*x**2), 9
/(4*Abs(x**2)) > 1), (8*I*sqrt(1 - 9/(4*x**2))/27 - 2*I*sqrt(1 - 9/(4*x**2)
)/(3*x**2), True))
```

3.462 $\int \frac{\sqrt{9-4x^2}}{x^5}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{9-4x^2}}{18x^2}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{4x^4}$$

```
[Out] 2/27*arctanh(1/3*(-4*x^2+9)^(1/2))-1/4*(-4*x^2+9)^(1/2)/x^4+1/18*(-4*x^2+9)
^(1/2)/x^2
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 206}

$$\frac{\sqrt{9-4x^2}}{18x^2}-\frac{\sqrt{9-4x^2}}{4x^4}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x^5,x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(4*x^4) + Sqrt[9 - 4*x^2]/(18*x^2) + (2*ArcTanh[Sqrt[9 - 4
*x^2]/3])/27
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{9-4x^2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{9-4x}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4} + \frac{\sqrt{9-4x^2}}{18x^2} - \frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4} + \frac{\sqrt{9-4x^2}}{18x^2} + \frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4} + \frac{\sqrt{9-4x^2}}{18x^2} + \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$
-\frac{16\left(9-4x^2\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-\frac{4x^2}{9}\right)}{2187}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^5,x]
[Out] (-16*(9 - 4*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 - (4*x^2)/9])/2187
```

fricas [A] time = 0.86, size = 45, normalized size = 0.79

$$
-\frac{8\,x^4\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right) - 3\left(2\,x^2-9\right)\sqrt{-4\,x^2+9}}{108\,x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="fricas")
[Out] -1/108*(8*x^4*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*(2*x^2 - 9)*sqrt(-4*x^2 + 9
))/x^4
```

giac [A] time = 1.06, size = 57, normalized size = 1.00

$$
-\frac{\left(-4\,x^2+9\right)^{\frac{3}{2}} + 9\sqrt{-4\,x^2+9}}{72\,x^4} + \frac{1}{27}\log\left(\sqrt{-4\,x^2+9}+3\right) - \frac{1}{27}\log\left(-\sqrt{-4\,x^2+9}+3\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="giac")
[Out] -1/72*((-4*x^2 + 9)^(3/2) + 9*sqrt(-4*x^2 + 9))/x^4 + 1/27*log(sqrt(-4*x^2
+ 9) + 3) - 1/27*log(-sqrt(-4*x^2 + 9) + 3)
```

maple [A] time = 0.00, size = 55, normalized size = 0.96

$$
\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{27} - \frac{\left(-4x^2+9\right)^{\frac{3}{2}}}{162x^2} - \frac{\left(-4x^2+9\right)^{\frac{3}{2}}}{36x^4} - \frac{2\sqrt{-4x^2+9}}{81}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^5,x)
```

```
[Out] -1/36/x^4*(-4*x^2+9)^(3/2)-1/162*(-4*x^2+9)^(3/2)/x^2-2/81*(-4*x^2+9)^(1/2)
+2/27*arctanh(3/(-4*x^2+9)^(1/2))
```

maxima [A] time = 3.02, size = 65, normalized size = 1.14

$$-\frac{2}{81}\sqrt{-4x^2+9}-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}}{162x^2}-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}}{36x^4}+\frac{2}{27}\log\left(\frac{6\sqrt{-4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(-4*x^2 + 9) - 1/162*(-4*x^2 + 9)^(3/2)/x^2 - 1/36*(-4*x^2 + 9)^(
3/2)/x^4 + 2/27*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 0.03, size = 49, normalized size = 0.86

$$\frac{\sqrt{\frac{9}{4}-x^2}}{9x^2}-\frac{2\ln\left(\sqrt{\frac{9}{4x^2}-1}-\frac{3\sqrt{\frac{1}{x^2}}}{2}\right)}{27}-\frac{\sqrt{\frac{9}{4}-x^2}}{2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x^5,x)
```

```
[Out] (9/4 - x^2)^(1/2)/(9*x^2) - (2*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2)
)/2))/27 - (9/4 - x^2)^(1/2)/(2*x^4)
```

sympy [A] time = 3.27, size = 139, normalized size = 2.44

$$\begin{cases} \frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{27}-\frac{1}{9x\sqrt{-1+\frac{9}{4x^2}}}+\frac{3}{4x^3\sqrt{-1+\frac{9}{4x^2}}}-\frac{9}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{27}+\frac{i}{9x\sqrt{1-\frac{9}{4x^2}}}-\frac{3i}{4x^3\sqrt{1-\frac{9}{4x^2}}}+\frac{9i}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**5,x)
```

```
[Out] Piecewise((2*acosh(3/(2*x))/27 - 1/(9*x*sqrt(-1 + 9/(4*x**2))) + 3/(4*x**3*
sqrt(-1 + 9/(4*x**2))) - 9/(8*x**5*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2))
> 1), (-2*I*asin(3/(2*x))/27 + I/(9*x*sqrt(1 - 9/(4*x**2))) - 3*I/(4*x**3*s
qrt(1 - 9/(4*x**2))) + 9*I/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

3.463 $\int x^5\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{448}\left(4x^2-9\right)^{7/2}+\frac{9}{160}\left(4x^2-9\right)^{5/2}+\frac{27}{64}\left(4x^2-9\right)^{3/2}$$

```
[Out] 27/64*(4*x^2-9)^(3/2)+9/160*(4*x^2-9)^(5/2)+1/448*(4*x^2-9)^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{448}\left(4x^2-9\right)^{7/2}+\frac{9}{160}\left(4x^2-9\right)^{5/2}+\frac{27}{64}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (27*(-9 + 4*x^2)^(3/2))/64 + (9*(-9 + 4*x^2)^(5/2))/160 + (-9 + 4*x^2)^(7/2
)/448
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{-9+4x^2}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int x^2\sqrt{-9+4x}\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{81}{16}\sqrt{-9+4x}+\frac{9}{8}(-9+4x)^{3/2}+\frac{1}{16}(-9+4x)^{5/2}\right)dx,x,x^2\right)\\&=\frac{27}{64}\left(-9+4x^2\right)^{3/2}+\frac{9}{160}\left(-9+4x^2\right)^{5/2}+\frac{1}{448}\left(-9+4x^2\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$\frac{1}{280}\left(4x^2-9\right)^{3/2}\left(10x^4+18x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[-9 + 4*x^2],x]
```

```
[Out] ((-9 + 4*x^2)^(3/2)*(27 + 18*x^2 + 10*x^4))/280
```

fricas [A] time = 0.92, size = 28, normalized size = 0.61

$$\frac{1}{280}\left(40\,x^6-18\,x^4-54\,x^2-243\right)\sqrt{4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/280*(40*x^6 - 18*x^4 - 54*x^2 - 243)*sqrt(4*x^2 - 9)
```

giac [A] time = 0.97, size = 34, normalized size = 0.74

$$\frac{1}{448}\left(4x^2-9\right)^{\frac{7}{2}}+\frac{9}{160}\left(4x^2-9\right)^{\frac{5}{2}}+\frac{27}{64}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/448*(4*x^2 - 9)^(7/2) + 9/160*(4*x^2 - 9)^(5/2) + 27/64*(4*x^2 - 9)^(3/2)
```

maple [A] time = 0.01, size = 34, normalized size = 0.74

$$\frac{(2x-3)(2x+3)\left(10x^4+18x^2+27\right)\sqrt{4x^2-9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(4*x^2-9)^(1/2),x)
[Out] 1/280*(2*x-3)*(2*x+3)*(10*x^4+18*x^2+27)*(4*x^2-9)^(1/2)
```

maxima [A] time = 2.94, size = 40, normalized size = 0.87

$$\frac{1}{28}\left(4x^2-9\right)^{\frac{3}{2}}x^4+\frac{9}{140}\left(4x^2-9\right)^{\frac{3}{2}}x^2+\frac{27}{280}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/28*(4*x^2 - 9)^(3/2)*x^4 + 9/140*(4*x^2 - 9)^(3/2)*x^2 + 27/280*(4*x^2 -
9)^(3/2)
```

mupad [B] time = 5.31, size = 28, normalized size = 0.61

$$-\sqrt{4x^2-9}\left(-\frac{x^6}{7}+\frac{9x^4}{140}+\frac{27x^2}{140}+\frac{243}{280}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(4*x^2 - 9)^(1/2),x)
[Out] -(4*x^2 - 9)^(1/2)*((27*x^2)/140 + (9*x^4)/140 - x^6/7 + 243/280)
```

sympy [A] time = 1.92, size = 61, normalized size = 1.33

$$\frac{x^6\sqrt{4x^2-9}}{7}-\frac{9x^4\sqrt{4x^2-9}}{140}-\frac{27x^2\sqrt{4x^2-9}}{140}-\frac{243\sqrt{4x^2-9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(4*x**2-9)**(1/2),x)
[Out] x**6*sqrt(4*x**2 - 9)/7 - 9*x**4*sqrt(4*x**2 - 9)/140 - 27*x**2*sqrt(4*x**2
 - 9)/140 - 243*sqrt(4*x**2 - 9)/280
```

3.464 $\int x^4\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{81}{256}\sqrt{4x^2-9}\,x-\frac{729}{512}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)+\frac{1}{6}\sqrt{4x^2-9}\,x^5-\frac{3}{32}\sqrt{4x^2-9}\,x^3$$

```
[Out] -729/512*arctanh(2*x/(4*x^2-9)^(1/2))-81/256*x*(4*x^2-9)^(1/2)-3/32*x^3*(4*
x^2-9)^(1/2)+1/6*x^5*(4*x^2-9)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$\frac{1}{6}\sqrt{4x^2-9}\,x^5-\frac{3}{32}\sqrt{4x^2-9}\,x^3-\frac{81}{256}\sqrt{4x^2-9}\,x-\frac{729}{512}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[-9 + 4*x^2])/256 - (3*x^3*Sqrt[-9 + 4*x^2])/32 + (x^5*Sqrt[-9 +
 4*x^2])/6 - (729*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/512
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^4\sqrt{-9+4x^2}\,dx &= \frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{3}{2}\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx\\ &= -\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{81}{32}\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx\\ &= -\frac{81}{256}x\sqrt{-9+4x^2}-\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{729}{256}\int \frac{1}{\sqrt{-9+4x^2}}\,dx\\ &= -\frac{81}{256}x\sqrt{-9+4x^2}-\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{729}{256}\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx,x,\frac{}{\sqrt{}}\right.\\ &= -\frac{81}{256}x\sqrt{-9+4x^2}-\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{729}{512}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 0.68

$$\frac{1}{768}x\sqrt{4x^2-9}\left(128x^4-72x^2-243\right)-\frac{729}{512}\log\left(\sqrt{4x^2-9}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 + 4*x^2]*(-243 - 72*x^2 + 128*x^4))/768 - (729*Log[2*x + Sqrt[-9
 + 4*x^2]])/512
```

fricas [A] time = 0.93, size = 42, normalized size = 0.58

$$\frac{1}{768}\left(128\,x^5-72\,x^3-243\,x\right)\sqrt{4\,x^2-9}+\frac{729}{512}\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 - 72*x^3 - 243*x)*sqrt(4*x^2 - 9) + 729/512*log(-2*x + sqrt(
4*x^2 - 9))
```

giac [A] time = 1.19, size = 44, normalized size = 0.61

$$\frac{1}{768}\left(8\left(16\,x^2-9\right)x^2-243\right)\sqrt{4\,x^2-9}\,x+\frac{729}{512}\log\left(\left|-2\,x+\sqrt{4\,x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 - 9)*x^2 - 243)*sqrt(4*x^2 - 9)*x + 729/512*log(abs(-2*x +
 sqrt(4*x^2 - 9)))
```

maple [A] time = 0.01, size = 61, normalized size = 0.85

$$\frac{\left(4x^2-9\right)^{\frac{3}{2}}x^3}{24}+\frac{9\left(4x^2-9\right)^{\frac{3}{2}}x}{128}+\frac{81\sqrt{4x^2-9}\,x}{256}-\frac{729\sqrt{4}\,\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^2-9)^(1/2),x)
```

```
[Out] 1/24*x^3*(4*x^2-9)^(3/2)+9/128*x*(4*x^2-9)^(3/2)+81/256*x*(4*x^2-9)^(1/2)-7
29/1024*ln(x*4^(1/2)+(4*x^2-9)^(1/2))*4^(1/2)
```

maxima [A] time = 2.92, size = 57, normalized size = 0.79

$$\frac{1}{24}\left(4x^2-9\right)^{\frac{3}{2}}x^3+\frac{9}{128}\left(4x^2-9\right)^{\frac{3}{2}}x+\frac{81}{256}\sqrt{4x^2-9}\,x-\frac{729}{512}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^2 - 9)^(3/2)*x^3 + 9/128*(4*x^2 - 9)^(3/2)*x + 81/256*sqrt(4*x^2
- 9)*x - 729/512*log(8*x + 4*sqrt(4*x^2 - 9))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\sqrt{4x^2-9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^2 - 9)^(1/2),x)
```

```
[Out] int(x^4*(4*x^2 - 9)^(1/2), x)
```

sympy [A] time = 4.56, size = 167, normalized size = 2.32

$$\begin{cases} \frac{2x^7}{3\sqrt{4x^2-9}}-\frac{15x^5}{8\sqrt{4x^2-9}}-\frac{27x^3}{64\sqrt{4x^2-9}}+\frac{729x}{256\sqrt{4x^2-9}}-\frac{729\operatorname{acosh}\left(\frac{2x}{3}\right)}{512} & \text{for}\ \frac{4|x^2|}{9}>1 \\ -\frac{2ix^7}{3\sqrt{9-4x^2}}+\frac{15ix^5}{8\sqrt{9-4x^2}}+\frac{27ix^3}{64\sqrt{9-4x^2}}-\frac{729ix}{256\sqrt{9-4x^2}}+\frac{729i\operatorname{asin}\left(\frac{2x}{3}\right)}{512} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*x**7/(3*sqrt(4*x**2 - 9)) - 15*x**5/(8*sqrt(4*x**2 - 9)) - 27*
x**3/(64*sqrt(4*x**2 - 9)) + 729*x/(256*sqrt(4*x**2 - 9)) - 729*acosh(2*x/3
)/512, 4*Abs(x**2)/9 > 1), (-2*I*x**7/(3*sqrt(9 - 4*x**2)) + 15*I*x**5/(8*s
qrt(9 - 4*x**2)) + 27*I*x**3/(64*sqrt(9 - 4*x**2)) - 729*I*x/(256*sqrt(9 -
4*x**2)) + 729*I*asin(2*x/3)/512, True))
```

3.465 $\int x^3\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{80}\left(4x^2-9\right)^{5/2}+\frac{3}{16}\left(4x^2-9\right)^{3/2}$$

```
[Out] 3/16*(4*x^2-9)^(3/2)+1/80*(4*x^2-9)^(5/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{80}\left(4x^2-9\right)^{5/2}+\frac{3}{16}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (3*(-9 + 4*x^2)^(3/2))/16 + (-9 + 4*x^2)^(5/2)/80
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{-9+4x^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int x\sqrt{-9+4x}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{9}{4}\sqrt{-9+4x}+\frac{1}{4}(-9+4x)^{3/2}\right)dx,x,x^2\right)\\&=\frac{3}{16}\left(-9+4x^2\right)^{3/2}+\frac{1}{80}\left(-9+4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(2x^2+3\right)\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[-9 + 4*x^2],x]
```

```
[Out] ((3 + 2*x^2)*(-9 + 4*x^2)^(3/2))/40
```

fricas [A] time = 0.88, size = 23, normalized size = 0.74

$$\frac{1}{40}\left(8\,x^4-6\,x^2-27\right)\sqrt{4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 - 6*x^2 - 27)*sqrt(4*x^2 - 9)
```

giac [A] time = 1.07, size = 23, normalized size = 0.74

$$\frac{1}{80}\left(4x^2-9\right)^{\frac{5}{2}}+\frac{3}{16}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/80*(4*x^2 - 9)^(5/2) + 3/16*(4*x^2 - 9)^(3/2)
```

maple [A] time = 0.00, size = 29, normalized size = 0.94

$$\frac{(2x-3)(2x+3)\left(2x^2+3\right)\sqrt{4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*x^2-9)^(1/2),x)
```

```
[Out] 1/40*(2*x-3)*(2*x+3)*(2*x^2+3)*(4*x^2-9)^(1/2)
```

maxima [A] time = 2.91, size = 26, normalized size = 0.84

$$\frac{1}{20}\left(4x^2-9\right)^{\frac{3}{2}}x^2+\frac{3}{40}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*(4*x^2 - 9)^(3/2)*x^2 + 3/40*(4*x^2 - 9)^(3/2)
```

mupad [B] time = 5.51, size = 23, normalized size = 0.74

$$-\sqrt{4x^2-9}\left(-\frac{x^4}{5}+\frac{3x^2}{20}+\frac{27}{40}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*x^2 - 9)^(1/2),x)
```

```
[Out] -(4*x^2 - 9)^(1/2)*((3*x^2)/20 - x^4/5 + 27/40)
```

sympy [A] time = 0.66, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{4x^2-9}}{5}-\frac{3x^2\sqrt{4x^2-9}}{20}-\frac{27\sqrt{4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(4*x**2-9)**(1/2),x)
```

```
[Out] x**4*sqrt(4*x**2 - 9)/5 - 3*x**2*sqrt(4*x**2 - 9)/20 - 27*sqrt(4*x**2 - 9)/
40
```

3.466 $\int x^2\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=54

$$-\frac{9}{32}\sqrt{4x^2-9}\,x-\frac{81}{64}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)+\frac{1}{4}\sqrt{4x^2-9}\,x^3$$

```
[Out] -81/64*arctanh(2*x/(4*x^2-9)^(1/2))-9/32*x*(4*x^2-9)^(1/2)+1/4*x^3*(4*x^2-9
)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$\frac{1}{4}\sqrt{4x^2-9}\,x^3-\frac{9}{32}\sqrt{4x^2-9}\,x-\frac{81}{64}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (-9*x*Sqrt[-9 + 4*x^2])/32 + (x^3*Sqrt[-9 + 4*x^2])/4 - (81*ArcTanh[(2*x)/S
qrt[-9 + 4*x^2]])/64
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{-9+4x^2}\,dx &= \frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{9}{4}\int\frac{x^2}{\sqrt{-9+4x^2}}\,dx\\ &= -\frac{9}{32}x\sqrt{-9+4x^2}+\frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{81}{32}\int\frac{1}{\sqrt{-9+4x^2}}\,dx\\ &= -\frac{9}{32}x\sqrt{-9+4x^2}+\frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{81}{32}\operatorname{Subst}\left(\int\frac{1}{1-4x^2}\,dx,x,\frac{x}{\sqrt{-9+4x^2}}\right)\\ &= -\frac{9}{32}x\sqrt{-9+4x^2}+\frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{81}{64}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.85

$$\sqrt{4x^2-9}\left(\frac{x^3}{4}-\frac{9x}{32}\right)-\frac{81}{64}\log\left(\sqrt{4x^2-9}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-9 + 4*x^2],x]
[Out] Sqrt[-9 + 4*x^2]*((-9*x)/32 + x^3/4) - (81*Log[2*x + Sqrt[-9 + 4*x^2]])/64
```

fricas [A] time = 0.85, size = 37, normalized size = 0.69

$$\frac{1}{32}\left(8\,x^3-9\,x\right)\sqrt{4\,x^2-9}+\frac{81}{64}\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/32*(8*x^3 - 9*x)*sqrt(4*x^2 - 9) + 81/64*log(-2*x + sqrt(4*x^2 - 9))
```

giac [A] time = 1.16, size = 37, normalized size = 0.69

$$\frac{1}{32}\left(8\,x^2-9\right)\sqrt{4\,x^2-9}\,x+\frac{81}{64}\log\left(\left|-2\,x+\sqrt{4\,x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/32*(8*x^2 - 9)*sqrt(4*x^2 - 9)*x + 81/64*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

maple [A] time = 0.01, size = 47, normalized size = 0.87

$$\frac{\left(4x^2-9\right)^{\frac{3}{2}}x}{16}+\frac{9\sqrt{4x^2-9}\,x}{32}-\frac{81\sqrt{4}\,\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^2-9)^(1/2),x)
[Out] 1/16*(4*x^2-9)^(3/2)*x+9/32*(4*x^2-9)^(1/2)*x-81/128*4^(1/2)*ln(4^(1/2)*x+(
4*x^2-9)^(1/2))
```

maxima [A] time = 2.89, size = 43, normalized size = 0.80

$$\frac{1}{16}\left(4\,x^2-9\right)^{\frac{3}{2}}x+\frac{9}{32}\sqrt{4\,x^2-9}\,x-\frac{81}{64}\log\left(8\,x+4\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*x^2 - 9)^(3/2)*x + 9/32*sqrt(4*x^2 - 9)*x - 81/64*log(8*x + 4*sqrt(
4*x^2 - 9))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \sqrt{4x^2-9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^2 - 9)^(1/2),x)
```

```
[Out] int(x^2*(4*x^2 - 9)^(1/2), x)
```

sympy [A] time = 2.80, size = 124, normalized size = 2.30

$$\begin{cases} \frac{x^5}{\sqrt{4x^2-9}} - \frac{27x^3}{8\sqrt{4x^2-9}} + \frac{81x}{32\sqrt{4x^2-9}} - \frac{81\operatorname{acosh}\left(\frac{2x}{3}\right)}{64} & \text{for } \frac{4|x^2|}{9} > 1 \\ -\frac{ix^5}{\sqrt{9-4x^2}} + \frac{27ix^3}{8\sqrt{9-4x^2}} - \frac{81ix}{32\sqrt{9-4x^2}} + \frac{81i\operatorname{asin}\left(\frac{2x}{3}\right)}{64} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((x**5/sqrt(4*x**2 - 9) - 27*x**3/(8*sqrt(4*x**2 - 9)) + 81*x/(32*
sqrt(4*x**2 - 9)) - 81*acosh(2*x/3)/64, 4*Abs(x**2)/9 > 1), (-I*x**5/sqrt(9
 - 4*x**2) + 27*I*x**3/(8*sqrt(9 - 4*x**2)) - 81*I*x/(32*sqrt(9 - 4*x**2))
+ 81*I*asin(2*x/3)/64, True))
```

3.467 $\int x\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=15

$$\frac{1}{12}\left(4x^2-9\right)^{3/2}$$

[Out] 1/12*(4*x^2-9)^(3/2)

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{12}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[-9 + 4*x^2],x]

[Out] (-9 + 4*x^2)^(3/2)/12

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{-9+4x^2}\,dx=\frac{1}{12}\left(-9+4x^2\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{12}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[-9 + 4*x^2],x]

[Out] (-9 + 4*x^2)^(3/2)/12

fricas [A] time = 0.84, size = 11, normalized size = 0.73

$$\frac{1}{12}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(4*x^2-9)^(1/2),x, algorithm="fricas")

[Out] 1/12*(4*x^2 - 9)^(3/2)

giac [A] time = 1.03, size = 11, normalized size = 0.73

$$\frac{1}{12}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(4*x^2 - 9)^(3/2)
```

maple [A] time = 0.00, size = 22, normalized size = 1.47

$$\frac{(2x-3)(2x+3)\sqrt{4x^2-9}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)*x,x)
```

```
[Out] 1/12*(2*x-3)*(2*x+3)*(4*x^2-9)^(1/2)
```

maxima [A] time = 1.30, size = 11, normalized size = 0.73

$$\frac{1}{12}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(4*x^2 - 9)^(3/2)
```

mupad [B] time = 0.06, size = 11, normalized size = 0.73

$$\frac{\left(4x^2-9\right)^{3/2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2 - 9)^(1/2),x)
```

```
[Out] (4*x^2 - 9)^(3/2)/12
```

sympy [B] time = 0.20, size = 27, normalized size = 1.80

$$\frac{x^2\sqrt{4x^2-9}}{3}-\frac{3\sqrt{4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2-9)**(1/2),x)
```

```
[Out] x**2*sqrt(4*x**2 - 9)/3 - 3*sqrt(4*x**2 - 9)/4
```

3.468 $\int \sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{4}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

```
[Out] -9/4*arctanh(2*x/(4*x^2-9)^(1/2))+1/2*x*(4*x^2-9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{4}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 + 4*x^2])/2 - (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-9+4x^2}\,dx &= \frac{1}{2}x\sqrt{-9+4x^2}-\frac{9}{2}\int \frac{1}{\sqrt{-9+4x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{-9+4x^2}-\frac{9}{2}\operatorname{Subst}\left(\int \frac{1}{1-4x^2}\,dx,x,\frac{x}{\sqrt{-9+4x^2}}\right)\\ &= \frac{1}{2}x\sqrt{-9+4x^2}-\frac{9}{4}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.03

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{4}\log\left(\sqrt{4x^2-9}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2],x]
[Out] (x*Sqrt[-9 + 4*x^2])/2 - (9*Log[2*x + Sqrt[-9 + 4*x^2]])/4
```

fricas [A] time = 0.74, size = 29, normalized size = 0.81

$$\frac{1}{2}\sqrt{4x^2-9}\,x+\frac{9}{4}\log\left(-2x+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(4*x^2 - 9)*x + 9/4*log(-2*x + sqrt(4*x^2 - 9))
```

giac [A] time = 1.11, size = 30, normalized size = 0.83

$$\frac{1}{2}\sqrt{4x^2-9}\,x+\frac{9}{4}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(4*x^2 - 9)*x + 9/4*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

maple [A] time = 0.00, size = 35, normalized size = 0.97

$$\frac{\sqrt{4x^2-9}\,x}{2}-\frac{9\sqrt{4}\,\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2),x)
[Out] 1/2*(4*x^2-9)^(1/2)*x-9/8*4^(1/2)*ln(4^(1/2)*x+(4*x^2-9)^(1/2))
```

maxima [A] time = 2.97, size = 31, normalized size = 0.86

$$\frac{1}{2}\sqrt{4x^2-9}\,x-\frac{9}{4}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(4*x^2 - 9)*x - 9/4*log(8*x + 4*sqrt(4*x^2 - 9))
```

mupad [B] time = 5.03, size = 29, normalized size = 0.81

$$\frac{x\sqrt{4x^2-9}}{2}-\frac{9\ln\left(2x+\sqrt{4x^2-9}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2),x)
[Out] (x*(4*x^2 - 9)^(1/2))/2 - (9*log(2*x + (4*x^2 - 9)^(1/2)))/4
```

sympy [A] time = 0.22, size = 22, normalized size = 0.61

$$\frac{x\sqrt{4x^2-9}}{2}-\frac{9\,\mathrm{acosh}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2),x)
[Out] x*sqrt(4*x**2 - 9)/2 - 9*acosh(2*x/3)/4
```

3.469 $\int \frac{\sqrt{-9+4x^2}}{x} \, dx$

Optimal. Leaf size=30

$$\sqrt{4x^2-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

```
[Out] -3*arctan(1/3*(4*x^2-9)^(1/2))+(4*x^2-9)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 203}

$$\sqrt{4x^2-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x,x]
```

```
[Out] Sqrt[-9 + 4*x^2] - 3*ArcTan[Sqrt[-9 + 4*x^2]/3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-9+4x^2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{-9+4x}}{x}\,dx,x,x^2\right)\\
&= \sqrt{-9+4x^2}-\frac{9}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx,x,x^2\right)\\
&= \sqrt{-9+4x^2}-\frac{9}{4}\operatorname{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx,x,\sqrt{-9+4x^2}\right)\\
&= \sqrt{-9+4x^2}-3\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\sqrt{4x^2-9}-3\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x,x]
[Out] Sqrt[-9 + 4*x^2] - 3*ArcTan[Sqrt[-9 + 4*x^2]/3]
```

fricas [A] time = 0.92, size = 28, normalized size = 0.93

$$\sqrt{4x^2-9}-6\arctan\left(-\frac{2}{3}x+\frac{1}{3}\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x,x, algorithm="fricas")
[Out] sqrt(4*x^2 - 9) - 6*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9))
```

giac [A] time = 1.01, size = 24, normalized size = 0.80

$$\sqrt{4x^2-9}-3\arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x,x, algorithm="giac")
[Out] sqrt(4*x^2 - 9) - 3*arctan(1/3*sqrt(4*x^2 - 9))
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$3\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)+\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x,x)
[Out] (4*x^2-9)^(1/2)+3*arctan(3/(4*x^2-9)^(1/2))
```

maxima [A] time = 3.01, size = 19, normalized size = 0.63

$$\sqrt{4x^2-9}+3\arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(4*x^2 - 9) + 3*arcsin(3/2/abs(x))
```

mupad [B] time = 5.34, size = 24, normalized size = 0.80

$$\sqrt{4x^2-9}-3\,\mathrm{atan}\left(\frac{\sqrt{4x^2-9}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x,x)
```

```
[Out] (4*x^2 - 9)^(1/2) - 3*atan((4*x^2 - 9)^(1/2)/3)
```

sympy [C] time = 1.37, size = 82, normalized size = 2.73

$$\begin{cases} \sqrt{4x^2-9}-3i\log(x)+\frac{3i\log(x^2)}{2}+3\,\mathrm{asin}\left(\frac{3}{2x}\right) & \text{for } \frac{4|x^2|}{9}>1 \\ i\sqrt{9-4x^2}+\frac{3i\log(x^2)}{2}-3i\log\left(\sqrt{1-\frac{4x^2}{9}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x,x)
```

```
[Out] Piecewise((sqrt(4*x**2 - 9) - 3*I*log(x) + 3*I*log(x**2)/2 + 3*asin(3/(2*x)
), 4*Abs(x**2)/9 > 1), (I*sqrt(9 - 4*x**2) + 3*I*log(x**2)/2 - 3*I*log(sqrt
(1 - 4*x**2/9) + 1), True))
```

3.470 $\int \frac{\sqrt{-9+4x^2}}{x^2}\,dx$

Optimal. Leaf size=34

$$2\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)-\frac{\sqrt{4x^2-9}}{x}$$

```
[Out] 2*arctanh(2*x/(4*x^2-9)^(1/2))-(4*x^2-9)^(1/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 217, 206}

$$2\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)-\frac{\sqrt{4x^2-9}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-9 + 4*x^2]/x) + 2*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-9+4x^2}}{x^2}\,dx &= -\frac{\sqrt{-9+4x^2}}{x}+4\int \frac{1}{\sqrt{-9+4x^2}}\,dx\\ &= -\frac{\sqrt{-9+4x^2}}{x}+4\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx,x,\frac{x}{\sqrt{-9+4x^2}}\right)\\ &= -\frac{\sqrt{-9+4x^2}}{x}+2\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.41

$$-\frac{\sqrt{4x^2-9}+\frac{2x\sqrt{4x^2-9}\sin^{-1}\left(\frac{2x}{3}\right)}{\sqrt{9-4x^2}}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^2,x]
```

```
[Out] -((Sqrt[-9 + 4*x^2] + (2*x*Sqrt[-9 + 4*x^2]*ArcSin[(2*x)/3])/Sqrt[9 - 4*x^2
])/x)
```

fricas [A] time = 0.63, size = 35, normalized size = 1.03

$$-\frac{2\,x\log\left(-2\,x+\sqrt{4\,x^2-9}\right)+2\,x+\sqrt{4\,x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*x*log(-2*x + sqrt(4*x^2 - 9)) + 2*x + sqrt(4*x^2 - 9))/x
```

giac [A] time = 1.23, size = 44, normalized size = 1.29

$$-\frac{36}{\left(2\,x-\sqrt{4\,x^2-9}\right)^2+9}-\log\left(\left(2\,x-\sqrt{4\,x^2-9}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -36/((2*x - sqrt(4*x^2 - 9))^2 + 9) - log((2*x - sqrt(4*x^2 - 9))^2)
```

maple [A] time = 0.00, size = 48, normalized size = 1.41

$$-\frac{4\sqrt{4x^2-9}\;x}{9}+\sqrt{4}\;\ln\left(\sqrt{4}\;x+\sqrt{4x^2-9}\right)+\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^2,x)
```

```
[Out] 1/9/x*(4*x^2-9)^(3/2)-4/9*(4*x^2-9)^(1/2)*x+4^(1/2)*ln(4^(1/2)*x+(4*x^2-9)^
(1/2))
```

maxima [A] time = 2.93, size = 33, normalized size = 0.97

$$-\frac{\sqrt{4\,x^2-9}}{x}+2\,\log\left(8\,x+4\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(4*x^2 - 9)/x + 2*log(8*x + 4*sqrt(4*x^2 - 9))
```

mupad [B] time = 5.55, size = 39, normalized size = 1.15

$$-\frac{\sqrt{4\,x^2-9}}{x}-\frac{2\,\mathrm{asin}\left(\frac{2\,x}{3}\right)\,\sqrt{4\,x^2-9}}{3\,\sqrt{1-\frac{4\,x^2}{9}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x^2,x)
```

```
[Out] - (4*x^2 - 9)^(1/2)/x - (2*asin((2*x)/3)*(4*x^2 - 9)^(1/2))/(3*(1 - (4*x^2)
/9)^(1/2))
```

sympy [A] time = 0.25, size = 19, normalized size = 0.56

$$2\,\text{acosh}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**2,x)
```

```
[Out] 2*acosh(2*x/3) - sqrt(4*x**2 - 9)/x
```

3.471 $\int \frac{\sqrt{-9+4x^2}}{x^3}\,dx$

Optimal. Leaf size=39

$$\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)-\frac{\sqrt{4x^2-9}}{2x^2}$$

```
[Out] 2/3*arctan(1/3*(4*x^2-9)^(1/2))-1/2*(4*x^2-9)^(1/2)/x^2
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 203}

$$\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)-\frac{\sqrt{4x^2-9}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^3,x]
```

```
[Out] -Sqrt[-9 + 4*x^2]/(2*x^2) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-9+4x^2}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{-9+4x}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{2x^2}+\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{2x^2}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx,x,\sqrt{-9+4x^2}\right)\\
&= -\frac{\sqrt{-9+4x^2}}{2x^2}+\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.41

$$-\frac{12x^2+4\sqrt{9-4x^2}\,x^2\tanh^{-1}\left(\sqrt{1-\frac{4x^2}{9}}\right)-27}{6x^2\sqrt{4x^2-9}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^3,x]
[Out] -1/6*(-27 + 12*x^2 + 4*x^2*Sqrt[9 - 4*x^2]*ArcTanh[Sqrt[1 - (4*x^2)/9]])/(x
^2*Sqrt[-9 + 4*x^2])
```

fricas [A] time = 1.18, size = 38, normalized size = 0.97

$$\frac{8\,x^2\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)-3\,\sqrt{4\,x^2-9}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="fricas")
[Out] 1/6*(8*x^2*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) - 3*sqrt(4*x^2 - 9))/x^2
```

giac [A] time = 1.07, size = 29, normalized size = 0.74

$$-\frac{\sqrt{4\,x^2-9}}{2\,x^2}+\frac{2}{3}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="giac")
[Out] -1/2*sqrt(4*x^2 - 9)/x^2 + 2/3*arctan(1/3*sqrt(4*x^2 - 9))
```

maple [A] time = 0.00, size = 41, normalized size = 1.05

$$-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{3}+\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{18x^2}-\frac{2\sqrt{4x^2-9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^3,x)
[Out] 1/18/x^2*(4*x^2-9)^(3/2)-2/9*(4*x^2-9)^(1/2)-2/3*arctan(3/(4*x^2-9)^(1/2))
```

maxima [A] time = 2.96, size = 35, normalized size = 0.90

$$-\frac{2}{9}\sqrt{4x^2-9}+\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{18x^2}-\frac{2}{3}\arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(4*x^2 - 9) + 1/18*(4*x^2 - 9)^(3/2)/x^2 - 2/3*arcsin(3/2/abs(x))
```

mupad [B] time = 5.33, size = 29, normalized size = 0.74

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{4x^2-9}}{3}\right)}{3}-\frac{\sqrt{4x^2-9}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x^3,x)
```

```
[Out] (2*atan((4*x^2 - 9)^(1/2)/3))/3 - (4*x^2 - 9)^(1/2)/(2*x^2)
```

sympy [A] time = 1.74, size = 97, normalized size = 2.49

$$\begin{cases} \frac{2i\,\mathrm{acosh}\left(\frac{3}{2x}\right)}{3}+\frac{i}{x\sqrt{-1+\frac{9}{4x^2}}}-\frac{9i}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2\,\mathrm{asin}\left(\frac{3}{2x}\right)}{3}-\frac{1}{x\sqrt{1-\frac{9}{4x^2}}}+\frac{9}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**3,x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/3 + I/(x*sqrt(-1 + 9/(4*x**2))) - 9*I/(4*x**3
*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2)) > 1), (-2*asin(3/(2*x))/3 - 1/(x*s
qrt(1 - 9/(4*x**2))) + 9/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

3.472 $\int \frac{\sqrt{-9+4x^2}}{x^4}\,dx$

Optimal. Leaf size=18

$$\frac{(4x^2-9)^{3/2}}{27x^3}$$

```
[Out] 1/27*(4*x^2-9)^(3/2)/x^3
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{(4x^2-9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^4,x]
```

```
[Out] (-9 + 4*x^2)^(3/2)/(27*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-9+4x^2}}{x^4}\,dx = \frac{(-9+4x^2)^{3/2}}{27x^3}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{(4x^2-9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^4,x]
```

```
[Out] (-9 + 4*x^2)^(3/2)/(27*x^3)
```

fricas [A] time = 0.90, size = 20, normalized size = 1.11

$$\frac{8\,x^3+(4\,x^2-9)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(8*x^3 + (4*x^2 - 9)^(3/2))/x^3
```

giac [B] time = 1.10, size = 42, normalized size = 2.33

$$\frac{16\left(\left(2x-\sqrt{4x^2-9}\right)^4+27\right)}{\left(\left(2x-\sqrt{4x^2-9}\right)^2+9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="giac")
[Out] 16*((2*x - sqrt(4*x^2 - 9))^4 + 27)/((2*x - sqrt(4*x^2 - 9))^2 + 9)^3
```

maple [A] time = 0.00, size = 25, normalized size = 1.39

$$\frac{(2x-3)(2x+3)\sqrt{4x^2-9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^4,x)
[Out] 1/27/x^3*(2*x-3)*(2*x+3)*(4*x^2-9)^(1/2)
```

maxima [A] time = 3.01, size = 14, normalized size = 0.78

$$\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="maxima")
[Out] 1/27*(4*x^2 - 9)^(3/2)/x^3
```

mupad [B] time = 5.10, size = 31, normalized size = 1.72

$$\frac{4x^2\sqrt{4x^2-9}-9\sqrt{4x^2-9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x^4,x)
[Out] (4*x^2*(4*x^2 - 9)^(1/2) - 9*(4*x^2 - 9)^(1/2))/(27*x^3)
```

sympy [B] time = 0.98, size = 76, normalized size = 4.22

$$\begin{cases} \frac{8i\sqrt{-1+\frac{9}{4x^2}}}{27}-\frac{2i\sqrt{-1+\frac{9}{4x^2}}}{3x^2} & \text{for}\ \frac{9}{4|x^2|}>1 \\ \frac{8\sqrt{1-\frac{9}{4x^2}}}{27}-\frac{2\sqrt{1-\frac{9}{4x^2}}}{3x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**4,x)
[Out] Piecewise((8*I*sqrt(-1 + 9/(4*x**2))/27 - 2*I*sqrt(-1 + 9/(4*x**2))/(3*x**2
), 9/(4*Abs(x**2)) > 1), (8*sqrt(1 - 9/(4*x**2))/27 - 2*sqrt(1 - 9/(4*x**2)
)/(3*x**2), True))
```

3.473 $\int \frac{\sqrt{-9+4x^2}}{x^5} dx$

Optimal. Leaf size=57

$$\frac{\sqrt{4x^2-9}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)-\frac{\sqrt{4x^2-9}}{4x^4}$$

```
[Out] 2/27*arctan(1/3*(4*x^2-9)^(1/2))-1/4*(4*x^2-9)^(1/2)/x^4+1/18*(4*x^2-9)^(1/
2)/x^2
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 203}

$$\frac{\sqrt{4x^2-9}}{18x^2}-\frac{\sqrt{4x^2-9}}{4x^4}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^5,x]
```

```
[Out] -Sqrt[-9 + 4*x^2]/(4*x^4) + Sqrt[-9 + 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 +
 4*x^2]/3])/27
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{-9+4x}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4}+\frac{\sqrt{-9+4x^2}}{18x^2}+\frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4}+\frac{\sqrt{-9+4x^2}}{18x^2}+\frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4}+\frac{\sqrt{-9+4x^2}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$\frac{16\left(4x^2-9\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-\frac{4x^2}{9}\right)}{2187}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^5,x]
[Out] (16*(-9 + 4*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 - (4*x^2)/9])/2187
```

fricas [A] time = 0.67, size = 45, normalized size = 0.79

$$\frac{16\,x^4\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)+3\,\sqrt{4\,x^2-9}\left(2\,x^2-9\right)}{108\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="fricas")
[Out] 1/108*(16*x^4*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9)*(2*x
^2 - 9))/x^4
```

giac [A] time = 0.98, size = 41, normalized size = 0.72

$$\frac{\left(4\,x^2-9\right)^{\frac{3}{2}}-9\,\sqrt{4\,x^2-9}}{72\,x^4}+\frac{2}{27}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="giac")
[Out] 1/72*((4*x^2 - 9)^(3/2) - 9*sqrt(4*x^2 - 9))/x^4 + 2/27*arctan(1/3*sqrt(4*x
^2 - 9))
```

maple [A] time = 0.00, size = 55, normalized size = 0.96

$$-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{27}+\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{162x^2}+\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{36x^4}-\frac{2\sqrt{4x^2-9}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^5,x)
```

```
[Out] 1/36/x^4*(4*x^2-9)^(3/2)+1/162*(4*x^2-9)^(3/2)/x^2-2/81*(4*x^2-9)^(1/2)-2/2
7*arctan(3/(4*x^2-9)^(1/2))
```

maxima [A] time = 2.96, size = 49, normalized size = 0.86

$$-\frac{2}{81}\sqrt{4x^2-9}+\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{162x^2}+\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{36x^4}-\frac{2}{27}\arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(4*x^2 - 9) + 1/162*(4*x^2 - 9)^(3/2)/x^2 + 1/36*(4*x^2 - 9)^(3/2
)/x^4 - 2/27*arcsin(3/2/abs(x))
```

mupad [B] time = 5.19, size = 43, normalized size = 0.75

$$\frac{2\operatorname{atan}\left(\frac{\sqrt{4x^2-9}}{3}\right)}{27}-\frac{\frac{\sqrt{4x^2-9}}{8}-\frac{\left(4x^2-9\right)^{3/2}}{72}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x^5,x)
```

```
[Out] (2*atan((4*x^2 - 9)^(1/2)/3))/27 - ((4*x^2 - 9)^(1/2)/8 - (4*x^2 - 9)^(3/2)
/72)/x^4
```

sympy [A] time = 3.36, size = 139, normalized size = 2.44

$$\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{3}{2x}\right)}{27}-\frac{i}{9x\sqrt{-1+\frac{9}{4x^2}}}+\frac{3i}{4x^3\sqrt{-1+\frac{9}{4x^2}}}-\frac{9i}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2\operatorname{asin}\left(\frac{3}{2x}\right)}{27}+\frac{1}{9x\sqrt{1-\frac{9}{4x^2}}}-\frac{3}{4x^3\sqrt{1-\frac{9}{4x^2}}}+\frac{9}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**5,x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/27 - I/(9*x*sqrt(-1 + 9/(4*x**2))) + 3*I/(4*x
**3*sqrt(-1 + 9/(4*x**2))) - 9*I/(8*x**5*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x
**2)) > 1), (-2*asin(3/(2*x))/27 + 1/(9*x*sqrt(1 - 9/(4*x**2))) - 3/(4*x**3
*sqrt(1 - 9/(4*x**2))) + 9/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

3.474 $\int x^5\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{448}\left(-4x^2-9\right)^{7/2}-\frac{9}{160}\left(-4x^2-9\right)^{5/2}-\frac{27}{64}\left(-4x^2-9\right)^{3/2}$$

```
[Out] -27/64*(-4*x^2-9)^(3/2)-9/160*(-4*x^2-9)^(5/2)-1/448*(-4*x^2-9)^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{1}{448}\left(-4x^2-9\right)^{7/2}-\frac{9}{160}\left(-4x^2-9\right)^{5/2}-\frac{27}{64}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (-27*(-9 - 4*x^2)^(3/2))/64 - (9*(-9 - 4*x^2)^(5/2))/160 - (-9 - 4*x^2)^(7/
2)/448
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5\sqrt{-9-4x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \sqrt{-9-4x}\,x^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16}\sqrt{-9-4x}+\frac{9}{8}(-9-4x)^{3/2}+\frac{1}{16}(-9-4x)^{5/2}\right)dx,x,x^2\right)\\
&= -\frac{27}{64}\left(-9-4x^2\right)^{3/2}-\frac{9}{160}\left(-9-4x^2\right)^{5/2}-\frac{1}{448}\left(-9-4x^2\right)^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$-\frac{1}{280}\left(-4x^2-9\right)^{3/2}\left(10x^4-18x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[-9 - 4*x^2],x]
```

```
[Out] -1/280*((-9 - 4*x^2)^(3/2)*(27 - 18*x^2 + 10*x^4))
```

fricas [A] time = 0.49, size = 28, normalized size = 0.61

$$\frac{1}{280}\left(40\,x^6+18\,x^4-54\,x^2+243\right)\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/280*(40*x^6 + 18*x^4 - 54*x^2 + 243)*sqrt(-4*x^2 - 9)
```

giac [A] time = 1.06, size = 37, normalized size = 0.80

$$\frac{1}{448}\left(4x^2+9\right)^{\frac{7}{2}}i-\frac{9}{160}\left(4x^2+9\right)^{\frac{5}{2}}i+\frac{27}{64}\left(4x^2+9\right)^{\frac{3}{2}}i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/448*(4*x^2 + 9)^(7/2)*i - 9/160*(4*x^2 + 9)^(5/2)*i + 27/64*(4*x^2 + 9)^(
3/2)*i
```

maple [A] time = 0.00, size = 24, normalized size = 0.52

$$-\frac{\left(10x^4-18x^2+27\right)\left(-4x^2-9\right)^{\frac{3}{2}}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/280*(10*x^4-18*x^2+27)*(-4*x^2-9)^(3/2)
```

maxima [A] time = 2.95, size = 40, normalized size = 0.87

$$-\frac{1}{28}\left(-4x^2-9\right)^{\frac{3}{2}}x^4+\frac{9}{140}\left(-4x^2-9\right)^{\frac{3}{2}}x^2-\frac{27}{280}\left(-4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/28*(-4*x^2 - 9)^(3/2)*x^4 + 9/140*(-4*x^2 - 9)^(3/2)*x^2 - 27/280*(-4*x^
2 - 9)^(3/2)
```

mupad [B] time = 5.13, size = 27, normalized size = 0.59

$$\sqrt{-4x^2-9}\left(\frac{x^6}{7}+\frac{9x^4}{140}-\frac{27x^2}{140}+\frac{243}{280}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] (- 4*x^2 - 9)^(1/2)*((9*x^4)/140 - (27*x^2)/140 + x^6/7 + 243/280)
```

sympy [A] time = 2.08, size = 68, normalized size = 1.48

$$\frac{x^6\sqrt{-4x^2-9}}{7}+\frac{9x^4\sqrt{-4x^2-9}}{140}-\frac{27x^2\sqrt{-4x^2-9}}{140}+\frac{243\sqrt{-4x^2-9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-4*x**2-9)**(1/2),x)
```

```
[Out] x**6*sqrt(-4*x**2 - 9)/7 + 9*x**4*sqrt(-4*x**2 - 9)/140 - 27*x**2*sqrt(-4*x
**2 - 9)/140 + 243*sqrt(-4*x**2 - 9)/280
```

3.475 $\int x^4\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{81}{256}\sqrt{-4x^2-9}\,x-\frac{729}{512}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)+\frac{1}{6}\sqrt{-4x^2-9}\,x^5+\frac{3}{32}\sqrt{-4x^2-9}\,x^3$$

```
[Out] -729/512*arctan(2*x/(-4*x^2-9)^(1/2))-81/256*x*(-4*x^2-9)^(1/2)+3/32*x^3*(-
4*x^2-9)^(1/2)+1/6*x^5*(-4*x^2-9)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 203}

$$\frac{1}{6}\sqrt{-4x^2-9}\,x^5+\frac{3}{32}\sqrt{-4x^2-9}\,x^3-\frac{81}{256}\sqrt{-4x^2-9}\,x-\frac{729}{512}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[-9 - 4*x^2])/256 + (3*x^3*Sqrt[-9 - 4*x^2])/32 + (x^5*Sqrt[-9 -
 4*x^2])/6 - (729*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/512
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int x^4\sqrt{-9-4x^2}\,dx &= \frac{1}{6}x^5\sqrt{-9-4x^2} - \frac{3}{2}\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx \\
&= \frac{3}{32}x^3\sqrt{-9-4x^2} + \frac{1}{6}x^5\sqrt{-9-4x^2} + \frac{81}{32}\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx \\
&= -\frac{81}{256}x\sqrt{-9-4x^2} + \frac{3}{32}x^3\sqrt{-9-4x^2} + \frac{1}{6}x^5\sqrt{-9-4x^2} - \frac{729}{256}\int \frac{1}{\sqrt{-9-4x^2}}\,dx \\
&= -\frac{81}{256}x\sqrt{-9-4x^2} + \frac{3}{32}x^3\sqrt{-9-4x^2} + \frac{1}{6}x^5\sqrt{-9-4x^2} - \frac{729}{256}\,\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{}{\sqrt{-}}\right. \\
&= -\frac{81}{256}x\sqrt{-9-4x^2} + \frac{3}{32}x^3\sqrt{-9-4x^2} + \frac{1}{6}x^5\sqrt{-9-4x^2} - \frac{729}{512}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.67

$$
\frac{1}{768}x\sqrt{-4x^2-9}\left(128x^4+72x^2-243\right) - \frac{729}{512}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 - 4*x^2]*(-243 + 72*x^2 + 128*x^4))/768 - (729*ArcTan[(2*x)/Sqrt
[-9 - 4*x^2]])/512
```

fricas [C] time = 0.63, size = 72, normalized size = 1.00

$$
\frac{1}{768}\left(128\,x^5+72\,x^3-243\,x\right)\sqrt{-4\,x^2-9} - \frac{729}{1024}i\,\log\left(-\frac{8\,x+4i\,\sqrt{-4\,x^2-9}}{x}\right) + \frac{729}{1024}i\,\log\left(-\frac{8\,x-4i\,\sqrt{-4\,x^2-9}}{x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 + 72*x^3 - 243*x)*sqrt(-4*x^2 - 9) - 729/1024*I*log(-(8*x +
4*I*sqrt(-4*x^2 - 9))/x) + 729/1024*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \sqrt{-4\,x^2-9}\,x^4\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 - 9)*x^4, x)
```

maple [A] time = 0.01, size = 55, normalized size = 0.76

$$
-\frac{\left(-4x^2-9\right)^{\frac{3}{2}}x^3}{24} + \frac{9\left(-4x^2-9\right)^{\frac{3}{2}}x}{128} + \frac{81\sqrt{-4x^2-9}\,x}{256} - \frac{729\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{512}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/24*x^3*(-4*x^2-9)^(3/2)+9/128*x*(-4*x^2-9)^(3/2)+81/256*x*(-4*x^2-9)^(1/
2)-729/512*arctan(2*x/(-4*x^2-9)^(1/2))
```

maxima [C] time = 2.94, size = 45, normalized size = 0.62

$$-\frac{1}{24}\left(-4x^{2}-9\right)^{\frac{3}{2}}x^{3}+\frac{9}{128}\left(-4x^{2}-9\right)^{\frac{3}{2}}x+\frac{81}{256}\sqrt{-4x^{2}-9}\,x+\frac{729}{512}i\,\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(-4*x^2 - 9)^(3/2)*x^3 + 9/128*(-4*x^2 - 9)^(3/2)*x + 81/256*sqrt(-4*
x^2 - 9)*x + 729/512*I*arcsinh(2/3*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{4}\sqrt{-4x^{2}-9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] int(x^4*(- 4*x^2 - 9)^(1/2), x)
```

sympy [C] time = 4.67, size = 83, normalized size = 1.15

$$\frac{2ix^{7}}{3\sqrt{4x^{2}+9}}+\frac{15ix^{5}}{8\sqrt{4x^{2}+9}}-\frac{27ix^{3}}{64\sqrt{4x^{2}+9}}-\frac{729ix}{256\sqrt{4x^{2}+9}}+\frac{729i\operatorname{asinh}\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-4*x**2-9)**(1/2),x)
```

```
[Out] 2*I*x**7/(3*sqrt(4*x**2 + 9)) + 15*I*x**5/(8*sqrt(4*x**2 + 9)) - 27*I*x**3/
(64*sqrt(4*x**2 + 9)) - 729*I*x/(256*sqrt(4*x**2 + 9)) + 729*I*asinh(2*x/3)
/512
```

3.476 $\int x^3\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{80}\left(-4x^2-9\right)^{5/2}+\frac{3}{16}\left(-4x^2-9\right)^{3/2}$$

```
[Out] 3/16*(-4*x^2-9)^(3/2)+1/80*(-4*x^2-9)^(5/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{80}\left(-4x^2-9\right)^{5/2}+\frac{3}{16}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (3*(-9 - 4*x^2)^(3/2))/16 + (-9 - 4*x^2)^(5/2)/80
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{-9-4x^2}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int\sqrt{-9-4x}\,x\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{9}{4}\sqrt{-9-4x}-\frac{1}{4}(-9-4x)^{3/2}\right)dx,x,x^2\right)\\&=\frac{3}{16}\left(-9-4x^2\right)^{3/2}+\frac{1}{80}\left(-9-4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(-4x^2-9\right)^{3/2}\left(3-2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[-9 - 4*x^2],x]
```

```
[Out] ((-9 - 4*x^2)^(3/2)*(3 - 2*x^2))/40
```

fricas [A] time = 0.91, size = 23, normalized size = 0.74

$$\frac{1}{40}\left(8\,x^4+6\,x^2-27\right)\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/40*(8*x^4 + 6*x^2 - 27)*sqrt(-4*x^2 - 9)
```

giac [A] time = 0.98, size = 25, normalized size = 0.81

$$\frac{1}{80}\left(4x^2+9\right)^{\frac{5}{2}}i-\frac{3}{16}\left(4x^2+9\right)^{\frac{3}{2}}i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/80*(4*x^2 + 9)^(5/2)*i - 3/16*(4*x^2 + 9)^(3/2)*i
```

maple [A] time = 0.00, size = 19, normalized size = 0.61

$$-\frac{\left(2x^2-3\right)\left(-4x^2-9\right)^{\frac{3}{2}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-4*x^2-9)^(1/2),x)
[Out] -1/40*(2*x^2-3)*(-4*x^2-9)^(3/2)
```

maxima [A] time = 3.03, size = 26, normalized size = 0.84

$$-\frac{1}{20}\left(-4x^2-9\right)^{\frac{3}{2}}x^2+\frac{3}{40}\left(-4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] -1/20*(-4*x^2 - 9)^(3/2)*x^2 + 3/40*(-4*x^2 - 9)^(3/2)
```

mupad [B] time = 5.15, size = 22, normalized size = 0.71

$$\sqrt{-4x^2-9}\left(\frac{x^4}{5}+\frac{3x^2}{20}-\frac{27}{40}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(- 4*x^2 - 9)^(1/2),x)
[Out] (- 4*x^2 - 9)^(1/2)*((3*x^2)/20 + x^4/5 - 27/40)
```

sympy [A] time = 0.69, size = 49, normalized size = 1.58

$$\frac{x^4\sqrt{-4x^2-9}}{5}+\frac{3x^2\sqrt{-4x^2-9}}{20}-\frac{27\sqrt{-4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-4*x**2-9)**(1/2),x)
[Out] x**4*sqrt(-4*x**2 - 9)/5 + 3*x**2*sqrt(-4*x**2 - 9)/20 - 27*sqrt(-4*x**2 -
9)/40
```

3.477 $\int x^2\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=54

$$\frac{9}{32}\sqrt{-4x^2-9}\,x+\frac{81}{64}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)+\frac{1}{4}\sqrt{-4x^2-9}\,x^3$$

```
[Out] 81/64*arctan(2*x/(-4*x^2-9)^(1/2))+9/32*x*(-4*x^2-9)^(1/2)+1/4*x^3*(-4*x^2-
9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 203}

$$\frac{1}{4}\sqrt{-4x^2-9}\,x^3+\frac{9}{32}\sqrt{-4x^2-9}\,x+\frac{81}{64}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (9*x*Sqrt[-9 - 4*x^2])/32 + (x^3*Sqrt[-9 - 4*x^2])/4 + (81*ArcTan[(2*x)/Sqr
t[-9 - 4*x^2]])/64
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{-9-4x^2}\,dx &= \frac{1}{4}x^3\sqrt{-9-4x^2} - \frac{9}{4}\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx\\ &= \frac{9}{32}x\sqrt{-9-4x^2} + \frac{1}{4}x^3\sqrt{-9-4x^2} + \frac{81}{32}\int \frac{1}{\sqrt{-9-4x^2}}\,dx\\ &= \frac{9}{32}x\sqrt{-9-4x^2} + \frac{1}{4}x^3\sqrt{-9-4x^2} + \frac{81}{32}\,\mathrm{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right)\\ &= \frac{9}{32}x\sqrt{-9-4x^2} + \frac{1}{4}x^3\sqrt{-9-4x^2} + \frac{81}{64}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.80

$$\frac{1}{64}\left(2x\sqrt{-4x^2-9}\left(8x^2+9\right)+81\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (2*x*Sqrt[-9 - 4*x^2]*(9 + 8*x^2) + 81*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/64
```

fricas [C] time = 0.79, size = 67, normalized size = 1.24

$$\frac{1}{32}\left(8\,x^3+9\,x\right)\sqrt{-4\,x^2-9}+\frac{81}{128}i\,\log\left(-\frac{8\,x+4i\,\sqrt{-4\,x^2-9}}{x}\right)-\frac{81}{128}i\,\log\left(-\frac{8\,x-4i\,\sqrt{-4\,x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 + 9*x)*sqrt(-4*x^2 - 9) + 81/128*I*log(-(8*x + 4*I*sqrt(-4*x^2
- 9))/x) - 81/128*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4\,x^2-9}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 - 9)*x^2, x)
```

maple [A] time = 0.00, size = 41, normalized size = 0.76

$$-\frac{\left(-4x^2-9\right)^{\frac{3}{2}}x}{16}-\frac{9\sqrt{-4x^2-9}\,x}{32}+\frac{81\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/16*(-4*x^2-9)^(3/2)*x-9/32*(-4*x^2-9)^(1/2)*x+81/64*arctan(2/(-4*x^2-9)^
(1/2)*x)
```

maxima [C] time = 2.98, size = 31, normalized size = 0.57

$$-\frac{1}{16}\left(-4\,x^2-9\right)^{\frac{3}{2}}x-\frac{9}{32}\,\sqrt{-4\,x^2-9}\,x-\frac{81}{64}i\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(-4*x^2 - 9)^(3/2)*x - 9/32*sqrt(-4*x^2 - 9)*x - 81/64*I*arcsinh(2/3*
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \sqrt{-4x^2-9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] int(x^2*(- 4*x^2 - 9)^(1/2), x)
```

sympy [C] time = 2.76, size = 61, normalized size = 1.13

$$\frac{ix^5}{\sqrt{4x^2+9}} + \frac{27ix^3}{8\sqrt{4x^2+9}} + \frac{81ix}{32\sqrt{4x^2+9}} - \frac{81i\operatorname{asinh}\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-4*x**2-9)**(1/2),x)
```

```
[Out] I*x**5/sqrt(4*x**2 + 9) + 27*I*x**3/(8*sqrt(4*x**2 + 9)) + 81*I*x/(32*sqrt(
4*x**2 + 9)) - 81*I*asinh(2*x/3)/64
```

3.478 $\int x\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{12}\left(-4x^2-9\right)^{3/2}$$

```
[Out] -1/12*(-4*x^2-9)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{12}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[-9 - 4*x^2],x]
```

```
[Out] -(-9 - 4*x^2)^(3/2)/12
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{-9-4x^2}\,dx=-\frac{1}{12}\left(-9-4x^2\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{12}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-9 - 4*x^2],x]
```

```
[Out] -1/12*(-9 - 4*x^2)^(3/2)
```

fricas [A] time = 0.99, size = 18, normalized size = 1.20

$$\frac{1}{12}\left(4\,x^2+9\right)\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^2 + 9)*sqrt(-4*x^2 - 9)
```

giac [A] time = 1.08, size = 12, normalized size = 0.80

$$\frac{1}{12}\left(4\,x^2+9\right)^{\frac{3}{2}}i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(4*x^2 + 9)^(3/2)*i
```

maple [A] time = 0.00, size = 12, normalized size = 0.80

$$-\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)*x,x)
```

```
[Out] -1/12*(-4*x^2-9)^(3/2)
```

maxima [A] time = 1.31, size = 11, normalized size = 0.73

$$-\frac{1}{12}\left(-4\,x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-4*x^2 - 9)^(3/2)
```

mupad [B] time = 0.07, size = 11, normalized size = 0.73

$$-\frac{\left(-4\,x^2-9\right)^{3/2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] -(- 4*x^2 - 9)^(3/2)/12
```

sympy [B] time = 0.22, size = 31, normalized size = 2.07

$$\frac{x^2\sqrt{-4x^2-9}}{3}+\frac{3\sqrt{-4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x**2-9)**(1/2),x)
```

```
[Out] x**2*sqrt(-4*x**2 - 9)/3 + 3*sqrt(-4*x**2 - 9)/4
```

3.479 $\int \sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}x\sqrt{-4x^2-9}-\frac{9}{4}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

```
[Out] -9/4*arctan(2*x/(-4*x^2-9)^(1/2))+1/2*x*(-4*x^2-9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 203}

$$\frac{1}{2}x\sqrt{-4x^2-9}-\frac{9}{4}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 - 4*x^2])/2 - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-9-4x^2}\,dx &= \frac{1}{2}x\sqrt{-9-4x^2}-\frac{9}{2}\int \frac{1}{\sqrt{-9-4x^2}}\,dx\\
&= \frac{1}{2}x\sqrt{-9-4x^2}-\frac{9}{2}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx,x,\frac{x}{\sqrt{-9-4x^2}}\right)\\
&= \frac{1}{2}x\sqrt{-9-4x^2}-\frac{9}{4}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{1}{4}\left(2x\sqrt{-4x^2-9}-9\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2],x]
```

```
[Out] (2*x*Sqrt[-9 - 4*x^2] - 9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/4
```

fricas [C] time = 0.88, size = 59, normalized size = 1.64

$$\frac{1}{2}\sqrt{-4x^2-9}\,x-\frac{9}{8}i\log\left(-\frac{8x+4i\sqrt{-4x^2-9}}{x}\right)+\frac{9}{8}i\log\left(-\frac{8x-4i\sqrt{-4x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-4*x^2 - 9)*x - 9/8*I*log(-(8*x + 4*I*sqrt(-4*x^2 - 9))/x) + 9/8*I
*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-4x^2-9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 - 9), x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.81

$$\frac{\sqrt{-4x^2-9}\,x}{2}-\frac{9\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2),x)
```

```
[Out] -9/4*arctan(2/(-4*x^2-9)^(1/2)*x)+1/2*(-4*x^2-9)^(1/2)*x
```

maxima [C] time = 2.99, size = 19, normalized size = 0.53

$$\frac{1}{2}\sqrt{-4x^2-9}\,x+\frac{9}{4}i\,\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-4*x^2 - 9)*x + 9/4*I*arcsinh(2/3*x)
```

mupad [B] time = 5.03, size = 28, normalized size = 0.78

$$\frac{x\sqrt{-4x^2-9}}{2}-\frac{9\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2),x)
```

```
[Out] (x*(- 4*x^2 - 9)^(1/2))/2 - (9*atan((2*x)/(- 4*x^2 - 9)^(1/2)))/4
```

sympy [A] time = 0.39, size = 34, normalized size = 0.94

$$\frac{x\sqrt{-4x^2-9}}{2}-\frac{9\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2),x)
```

```
[Out] x*sqrt(-4*x**2 - 9)/2 - 9*atan(2*x/sqrt(-4*x**2 - 9))/4
```

3.480 $\int \frac{\sqrt{-9-4x^2}}{x} \, dx$

Optimal. Leaf size=30

$$\sqrt{-4x^2-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

```
[Out] -3*arctan(1/3*(-4*x^2-9)^(1/2))+(-4*x^2-9)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 204}

$$\sqrt{-4x^2-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x,x]
```

```
[Out] Sqrt[-9 - 4*x^2] - 3*ArcTan[Sqrt[-9 - 4*x^2]/3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-9-4x^2}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-9-4x}}{x}\,dx, x, x^2\right)$$
$$= \sqrt{-9-4x^2} - \frac{9}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx, x, x^2\right)$$
$$= \sqrt{-9-4x^2} + \frac{9}{4}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)$$
$$= \sqrt{-9-4x^2} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\sqrt{-4x^2-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x,x]
[Out] Sqrt[-9 - 4*x^2] - 3*ArcTan[Sqrt[-9 - 4*x^2]/3]
```

fricas [C] time = 1.10, size = 52, normalized size = 1.73

$$\sqrt{-4\,x^2-9} - \frac{3}{2}i\,\log\left(-\frac{6\left(i\sqrt{-4\,x^2-9}-3\right)}{x}\right) + \frac{3}{2}i\,\log\left(-\frac{6\left(-i\sqrt{-4\,x^2-9}-3\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] sqrt(-4*x^2 - 9) - 3/2*I*log(-6*(I*sqrt(-4*x^2 - 9) - 3)/x) + 3/2*I*log(-6*
(-I*sqrt(-4*x^2 - 9) - 3)/x)
```

giac [A] time = 1.17, size = 24, normalized size = 0.80

$$\sqrt{-4\,x^2-9} - 3\,\arctan\left(\frac{1}{3}\,\sqrt{-4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] sqrt(-4*x^2 - 9) - 3*arctan(1/3*sqrt(-4*x^2 - 9))
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$3\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right) + \sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x*(-4*x^2-9)^(1/2),x)
[Out] (-4*x^2-9)^(1/2)+3*arctan(3/(-4*x^2-9)^(1/2))
```

maxima [C] time = 2.93, size = 35, normalized size = 1.17

$$\sqrt{-4\,x^2-9} + 3i\,\log\left(\frac{6\,\sqrt{4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-4*x^2 - 9) + 3*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 4.73, size = 24, normalized size = 0.80

$$\sqrt{-4\,x^2-9}-3\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2)/x,x)
```

```
[Out] (- 4*x^2 - 9)^(1/2) - 3*atan((- 4*x^2 - 9)^(1/2)/3)
```

sympy [C] time = 1.26, size = 44, normalized size = 1.47

$$\frac{2ix}{\sqrt{1+\frac{9}{4x^2}}}-3i\,\mathrm{asinh}\left(\frac{3}{2x}\right)+\frac{9i}{2x\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x**2-9)**(1/2),x)
```

```
[Out] 2*I*x/sqrt(1 + 9/(4*x**2)) - 3*I*asinh(3/(2*x)) + 9*I/(2*x*sqrt(1 + 9/(4*x*
*2)))
```

3.481 $\int \frac{\sqrt{-9-4x^2}}{x^2}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{-4x^2-9}}{x}-2\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

```
[Out] -2*arctan(2*x/(-4*x^2-9)^(1/2))-1/x*(-4*x^2-9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 217, 203}

$$-\frac{\sqrt{-4x^2-9}}{x}-2\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-9 - 4*x^2]/x) - 2*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-9-4x^2}}{x^2}\,dx &= -\frac{\sqrt{-9-4x^2}}{x}-4\int \frac{1}{\sqrt{-9-4x^2}}\,dx\\
&= -\frac{\sqrt{-9-4x^2}}{x}-4\,\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx,x,\frac{x}{\sqrt{-9-4x^2}}\right)\\
&= -\frac{\sqrt{-9-4x^2}}{x}-2\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.44

$$\frac{\sqrt{-4x^2-9}\left(2x\sinh^{-1}\left(\frac{2x}{3}\right)-\sqrt{4x^2+9}\right)}{x\sqrt{4x^2+9}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^2,x]
```

```
[Out] (Sqrt[-9 - 4*x^2]*(-Sqrt[9 + 4*x^2] + 2*x*ArcSinh[(2*x)/3]))/(x*Sqrt[9 + 4*
x^2])
```

fricas [C] time = 0.91, size = 64, normalized size = 1.88

$$\frac{-i\,x\log\left(-\frac{8\,x+4i\,\sqrt{-4\,x^2-9}}{x}\right)+i\,x\log\left(-\frac{8\,x-4i\,\sqrt{-4\,x^2-9}}{x}\right)-\sqrt{-4\,x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (-I*x*log(-(8*x + 4*I*sqrt(-4*x^2 - 9))/x) + I*x*log(-(8*x - 4*I*sqrt(-4*x^
2 - 9))/x) - sqrt(-4*x^2 - 9))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-4\,x^2-9}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 - 9)/x^2, x)
```

maple [A] time = 0.00, size = 43, normalized size = 1.26

$$\frac{4\sqrt{-4x^2-9}\,x}{9}-2\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)+\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^2,x)
```

```
[Out] 1/9/x*(-4*x^2-9)^(3/2)+4/9*(-4*x^2-9)^(1/2)*x-2*arctan(2/(-4*x^2-9)^(1/2)*x
)
```

maxima [C] time = 2.92, size = 21, normalized size = 0.62

$$-\frac{\sqrt{-4\,x^2-9}}{x}+2i\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(-4*x^2 - 9)/x + 2*I*arcsinh(2/3*x)
```

mupad [B] time = 4.77, size = 41, normalized size = 1.21

$$-\frac{\sqrt{-4\,x^2-9}}{x}-\frac{\operatorname{asin}\left(\frac{x\,2\mathrm{i}}{3}\right)\,\sqrt{-4\,x^2-9}\,2\mathrm{i}}{3\,\sqrt{\frac{4\,x^2}{9}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2)/x^2,x)
```

```
[Out] - (- 4*x^2 - 9)^(1/2)/x - (asin((x*2i)/3)*(- 4*x^2 - 9)^(1/2)*2i)/(3*((4*x^
2)/9 + 1)^(1/2))
```

sympy [**A**] time = 0.42, size = 32, normalized size = 0.94

$$-2\operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)-\frac{\sqrt{-4x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**2,x)
```

```
[Out] -2*atan(2*x/sqrt(-4*x**2 - 9)) - sqrt(-4*x**2 - 9)/x
```

3.482 $\int \frac{\sqrt{-9-4x^2}}{x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{-4x^2-9}}{2x^2}-\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

```
[Out] -2/3*arctan(1/3*(-4*x^2-9)^(1/2))-1/2*(-4*x^2-9)^(1/2)/x^2
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 204}

$$-\frac{\sqrt{-4x^2-9}}{2x^2}-\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^3,x]
```

```
[Out] -Sqrt[-9 - 4*x^2]/(2*x^2) - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-9-4x^2}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{-9-4x}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9-4x^2}}{2x^2} - \operatorname{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9-4x^2}}{2x^2} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\\
&= -\frac{\sqrt{-9-4x^2}}{2x^2} - \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.41

$$\frac{12x^2+4\sqrt{4x^2+9}\,x^2\tanh^{-1}\left(\sqrt{\frac{4x^2}{9}+1}\right)+27}{6x^2\sqrt{-4x^2-9}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^3,x]
[Out] (27 + 12*x^2 + 4*x^2*Sqrt[9 + 4*x^2]*ArcTanh[Sqrt[1 + (4*x^2)/9]])/(6*x^2*S
qrt[-9 - 4*x^2])
```

fricas [C] time = 0.89, size = 65, normalized size = 1.67

$$\frac{-2i\,x^2\log\left(-\frac{4\left(i\sqrt{-4x^2-9}-3\right)}{3x}\right)+2i\,x^2\log\left(-\frac{4\left(-i\sqrt{-4x^2-9}-3\right)}{3x}\right)-3\sqrt{-4x^2-9}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="fricas")
[Out] 1/6*(-2*I*x^2*log(-4/3*(I*sqrt(-4*x^2 - 9) - 3)/x) + 2*I*x^2*log(-4/3*(-I*s
qrt(-4*x^2 - 9) - 3)/x) - 3*sqrt(-4*x^2 - 9))/x^2
```

giac [A] time = 1.21, size = 29, normalized size = 0.74

$$-\frac{\sqrt{-4x^2-9}}{2x^2}-\frac{2}{3}\arctan\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="giac")
[Out] -1/2*sqrt(-4*x^2 - 9)/x^2 - 2/3*arctan(1/3*sqrt(-4*x^2 - 9))
```

maple [A] time = 0.00, size = 41, normalized size = 1.05

$$\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{3}+\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{18x^2}+\frac{2\sqrt{-4x^2-9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^3,x)
```

```
[Out] 1/18/x^2*(-4*x^2-9)^(3/2)+2/9*(-4*x^2-9)^(1/2)+2/3*arctan(3/(-4*x^2-9)^(1/2
))
```

maxima [C] time = 2.90, size = 51, normalized size = 1.31

$$\frac{2}{9}\sqrt{-4x^2-9}+\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{18x^2}+\frac{2}{3}i\log\left(\frac{6\sqrt{4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(-4*x^2 - 9) + 1/18*(-4*x^2 - 9)^(3/2)/x^2 + 2/3*I*log(6*sqrt(4*x^2
 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 4.79, size = 29, normalized size = 0.74

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-4x^2-9}}{3}\right)}{3}-\frac{\sqrt{-4x^2-9}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2)/x^3,x)
```

```
[Out] - (2*atan((- 4*x^2 - 9)^(1/2)/3))/3 - (- 4*x^2 - 9)^(1/2)/(2*x^2)
```

sympy [C] time = 1.70, size = 27, normalized size = 0.69

$$-\frac{2i\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{3}-\frac{i\sqrt{1+\frac{9}{4x^2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**3,x)
```

```
[Out] -2*I*asinh(3/(2*x))/3 - I*sqrt(1 + 9/(4*x**2))/x
```

3.483 $\int \frac{\sqrt{-9-4x^2}}{x^4} dx$

Optimal. Leaf size=18

$$\frac{(-4x^2-9)^{3/2}}{27x^3}$$

```
[Out] 1/27*(-4*x^2-9)^(3/2)/x^3
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{(-4x^2-9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^4,x]
```

```
[Out] (-9 - 4*x^2)^(3/2)/(27*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-9-4x^2}}{x^4} dx = \frac{(-9-4x^2)^{3/2}}{27x^3}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{(-4x^2-9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^4,x]
```

```
[Out] (-9 - 4*x^2)^(3/2)/(27*x^3)
```

fricas [A] time = 0.89, size = 14, normalized size = 0.78

$$\frac{(-4\,x^2-9)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(-4*x^2 - 9)^(3/2)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-4\,x^2-9}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 - 9)/x^4, x)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^4,x)
```

```
[Out] 1/27*(-4*x^2-9)^(3/2)/x^3
```

maxima [A] time = 2.89, size = 14, normalized size = 0.78

$$\frac{\left(-4\,x^2-9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/27*(-4*x^2 - 9)^(3/2)/x^3
```

mupad [B] time = 4.74, size = 31, normalized size = 1.72

$$-\frac{4\,x^2\,\sqrt{-4\,x^2-9}+9\,\sqrt{-4\,x^2-9}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2)/x^4,x)
```

```
[Out] -(4*x^2*(- 4*x^2 - 9)^(1/2) + 9*(- 4*x^2 - 9)^(1/2))/(27*x^3)
```

sympy [C] time = 1.00, size = 37, normalized size = 2.06

$$-\frac{8i\sqrt{1+\frac{9}{4x^2}}}{27}-\frac{2i\sqrt{1+\frac{9}{4x^2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**4,x)
```

```
[Out] -8*I*sqrt(1 + 9/(4*x**2))/27 - 2*I*sqrt(1 + 9/(4*x**2))/(3*x**2)
```

3.484 $\int \frac{\sqrt{-9-4x^2}}{x^5}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{-4x^2-9}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)-\frac{\sqrt{-4x^2-9}}{4x^4}$$

```
[Out] 2/27*arctan(1/3*(-4*x^2-9)^(1/2))-1/4*(-4*x^2-9)^(1/2)/x^4-1/18*(-4*x^2-9)^
(1/2)/x^2
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 204}

$$-\frac{\sqrt{-4x^2-9}}{18x^2}-\frac{\sqrt{-4x^2-9}}{4x^4}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^5,x]
```

```
[Out] -Sqrt[-9 - 4*x^2]/(4*x^4) - Sqrt[-9 - 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 -
 4*x^2]/3])/27
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-9-4x^2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{-9-4x}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{\sqrt{-9-4x^2}}{18x^2} + \frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{\sqrt{-9-4x^2}}{18x^2} - \frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{\sqrt{-9-4x^2}}{18x^2} + \frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$\frac{16\left(-4x^2-9\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{4x^2}{9}+1\right)}{2187}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^5,x]
[Out] (16*(-9 - 4*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (4*x^2)/9])/2187
```

fricas [C] time = 0.93, size = 72, normalized size = 1.26

$$\frac{-4i\,x^4\log\left(-\frac{4\left(i\sqrt{-4x^2-9}+3\right)}{27x}\right)+4i\,x^4\log\left(-\frac{4\left(-i\sqrt{-4x^2-9}+3\right)}{27x}\right)-3\left(2x^2+9\right)\sqrt{-4x^2-9}}{108x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="fricas")
[Out] 1/108*(-4*I*x^4*log(-4/27*(I*sqrt(-4*x^2 - 9) + 3)/x) + 4*I*x^4*log(-4/27*(
-I*sqrt(-4*x^2 - 9) + 3)/x) - 3*(2*x^2 + 9)*sqrt(-4*x^2 - 9))/x^4
```

giac [A] time = 1.11, size = 43, normalized size = 0.75

$$-\frac{\left(4x^2+9\right)^{\frac{3}{2}}i+9\sqrt{-4x^2-9}}{72x^4}+\frac{2}{27}\arctan\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="giac")
[Out] -1/72*((4*x^2 + 9)^(3/2)*i + 9*sqrt(-4*x^2 - 9))/x^4 + 2/27*arctan(1/3*sqrt
(-4*x^2 - 9))
```

maple [A] time = 0.01, size = 55, normalized size = 0.96

$$-\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{27}-\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{162x^2}+\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{36x^4}-\frac{2\sqrt{-4x^2-9}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^5,x)
```

```
[Out] 1/36/x^4*(-4*x^2-9)^(3/2)-1/162*(-4*x^2-9)^(3/2)/x^2-2/81*(-4*x^2-9)^(1/2)-
2/27*arctan(3/(-4*x^2-9)^(1/2))
```

maxima [C] time = 2.99, size = 65, normalized size = 1.14

$$-\frac{2}{81}\sqrt{-4x^2-9}-\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{162x^2}+\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{36x^4}-\frac{2}{27}i\log\left(\frac{6\sqrt{4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(-4*x^2 - 9) - 1/162*(-4*x^2 - 9)^(3/2)/x^2 + 1/36*(-4*x^2 - 9)^(
3/2)/x^4 - 2/27*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 4.69, size = 43, normalized size = 0.75

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-4x^2-9}}{3}\right)}{27}-\frac{\frac{\sqrt{-4x^2-9}}{8}-\frac{\left(-4x^2-9\right)^{3/2}}{72}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2)/x^5,x)
```

```
[Out] (2*atan((- 4*x^2 - 9)^(1/2)/3))/27 - ((- 4*x^2 - 9)^(1/2)/8 - (- 4*x^2 - 9)
^(3/2)/72)/x^4
```

sympy [C] time = 3.37, size = 68, normalized size = 1.19

$$\frac{2i\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{27}-\frac{i}{9x\sqrt{1+\frac{9}{4x^2}}}-\frac{3i}{4x^3\sqrt{1+\frac{9}{4x^2}}}-\frac{9i}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**5,x)
```

```
[Out] 2*I*asinh(3/(2*x))/27 - I/(9*x*sqrt(1 + 9/(4*x**2))) - 3*I/(4*x**3*sqrt(1 +
 9/(4*x**2))) - 9*I/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

3.485 $\int \frac{x^5}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=56

$$\frac{a^2\sqrt{a+bx^2}}{b^3}+\frac{\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{2a\left(a+bx^2\right)^{3/2}}{3b^3}$$

```
[Out] -2/3*a*(b*x^2+a)^(3/2)/b^3+1/5*(b*x^2+a)^(5/2)/b^3+a^2*(b*x^2+a)^(1/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2\sqrt{a+bx^2}}{b^3}+\frac{\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{2a\left(a+bx^2\right)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*Sqrt[a + b*x^2])/b^3 - (2*a*(a + b*x^2)^(3/2))/(3*b^3) + (a + b*x^2)^(
5/2)/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)dx,x,x^2\right)\\
&= \frac{a^2\sqrt{a+bx^2}}{b^3}-\frac{2a\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{\left(a+bx^2\right)^{5/2}}{5b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.70

$$\frac{\sqrt{a+bx^2}\left(8a^2-4abx^2+3b^2x^4\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2 - 4*a*b*x^2 + 3*b^2*x^4))/(15*b^3)
```

fricas [A] time = 0.87, size = 35, normalized size = 0.62

$$\frac{\left(3\,b^{2}x^{4}-4\,abx^{2}+8\,a^{2}\right)\sqrt{bx^{2}+a}}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*x^4 - 4*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/b^3
```

giac [A] time = 1.15, size = 46, normalized size = 0.82

$$\frac{\sqrt{bx^{2}+a}\,a^{2}}{b^{3}}+\frac{3\left(bx^{2}+a\right)^{\frac{5}{2}}-10\left(bx^{2}+a\right)^{\frac{3}{2}}a}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)*a^2/b^3 + 1/15*(3*(b*x^2 + a)^(5/2) - 10*(b*x^2 + a)^(3/2)*
a)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.64

$$\frac{\sqrt{b\,x^{2}+a}\,\left(3b^{2}x^{4}-4ab\,x^{2}+8a^{2}\right)}{15b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(3*b^2*x^4-4*a*b*x^2+8*a^2)/b^3
```

maxima [A] time = 1.32, size = 53, normalized size = 0.95

$$\frac{\sqrt{bx^{2}+a}\,x^{4}}{5\,b}-\frac{4\,\sqrt{bx^{2}+a}\,ax^{2}}{15\,b^{2}}+\frac{8\,\sqrt{bx^{2}+a}\,a^{2}}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(b*x^2 + a)*x^4/b - 4/15*sqrt(b*x^2 + a)*a*x^2/b^2 + 8/15*sqrt(b*x^
2 + a)*a^2/b^3
```

mupad [B] time = 4.67, size = 36, normalized size = 0.64

$$\sqrt{b\,x^{2}+a}\,\left(\frac{8\,a^{2}}{15\,b^{3}}+\frac{x^{4}}{5\,b}-\frac{4\,a\,x^{2}}{15\,b^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a^2)/(15*b^3) + x^4/(5*b) - (4*a*x^2)/(15*b^2))
```

sympy [A] time = 0.83, size = 68, normalized size = 1.21

$$\begin{cases}\frac{8a^{2}\sqrt{a+bx^{2}}}{15b^{3}}-\frac{4ax^{2}\sqrt{a+bx^{2}}}{15b^{2}}+\frac{x^{4}\sqrt{a+bx^{2}}}{5b} & \text{for}\ b\neq 0\\ \frac{x^{6}}{6\sqrt{a}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((8*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*a*x**2*sqrt(a + b*x**2)/(1
5*b**2) + x**4*sqrt(a + b*x**2)/(5*b), Ne(b, 0)), (x**6/(6*sqrt(a)), True))
```

3.486 $\int \frac{x^4}{\sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=73

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}} - \frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b}$$

```
[Out] 3/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-3/8*a*x*(b*x^2+a)^(1/2)/
b^2+1/4*x^3*(b*x^2+a)^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 217, 206}

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}} - \frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^2],x]
```

```
[Out] (-3*a*x*Sqrt[a + b*x^2])/(8*b^2) + (x^3*Sqrt[a + b*x^2])/(4*b) + (3*a^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{a+bx^2}} \, dx &= \frac{x^3\sqrt{a+bx^2}}{4b} - \frac{(3a)\int \frac{x^2}{\sqrt{a+bx^2}} \, dx}{4b} \\
&= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{\left(3a^2\right)\int \frac{1}{\sqrt{a+bx^2}} \, dx}{8b^2} \\
&= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8b^2} \\
&= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.85

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\sqrt{b}x\sqrt{a+bx^2}\,(2bx^2-3a)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-3*a + 2*b*x^2) + 3*a^2*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(8*b^(5/2))
```

fricas [A] time = 0.73, size = 124, normalized size = 1.70

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(2\,b^2x^3-3\,abx\right)\sqrt{bx^2+a}}{16\,b^3},\,-\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(2\,b^2x^3\right.}{8\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 1.04, size = 54, normalized size = 0.74

$$\frac{1}{8}\,\sqrt{bx^2+a}\,x\left(\frac{2\,x^2}{b}-\frac{3\,a}{b^2}\right)-\frac{3\,a^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*x*(2*x^2/b - 3*a/b^2) - 3/8*a^2*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 59, normalized size = 0.81

$$\frac{\sqrt{b\,x^2+a}\;x^3}{4b}+\frac{3a^2\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}-\frac{3\sqrt{b\,x^2+a}\;ax}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*x^3*(b*x^2+a)^(1/2)/b-3/8*a*x*(b*x^2+a)^(1/2)/b^2+3/8*a^2/b^(5/2)*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.35, size = 51, normalized size = 0.70

$$\frac{\sqrt{bx^2+a}\,x^3}{4\,b}-\frac{3\,\sqrt{bx^2+a}\,ax}{8\,b^2}+\frac{3\,a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*x^3/b - 3/8*sqrt(b*x^2 + a)*a*x/b^2 + 3/8*a^2*arcsinh(b
*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^4/(a + b*x^2)^(1/2), x)
```

sympy [A] time = 4.03, size = 95, normalized size = 1.30

$$-\frac{3a^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*x**3/(8*b*sqrt(1 + b*x*
*2/a)) + 3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + x**5/(4*sqrt(a)*sqr
t(1 + b*x**2/a))
```

3.487 $\int \frac{x^3}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

```
[Out] 1/3*(b*x^2+a)^(3/2)/b^2-a*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*Sqrt[a + b*x^2])/b^2) + (a + b*x^2)^(3/2)/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx,x,x^2\right)\\ &= -\frac{a\sqrt{a+bx^2}}{b^2}+\frac{\left(a+bx^2\right)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{\left(bx^2-2a\right)\sqrt{a+bx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2],x]
```

```
[Out] ((-2*a + b*x^2)*Sqrt[a + b*x^2])/(3*b^2)
```

fricas [A] time = 0.84, size = 23, normalized size = 0.64

$$\frac{\sqrt{bx^2+a}\left(bx^2-2\,a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(b*x^2 + a)*(b*x^2 - 2*a)/b^2
```

giac [A] time = 1.09, size = 30, normalized size = 0.83

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{3\,b^2}-\frac{\sqrt{bx^2+a}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/3*(b*x^2 + a)^(3/2)/b^2 - sqrt(b*x^2 + a)*a/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$-\frac{\sqrt{b\,x^2+a}\,\left(-b\,x^2+2a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/2),x)
[Out] -1/3*(b*x^2+a)^(1/2)*(-b*x^2+2*a)/b^2
```

maxima [A] time = 1.34, size = 33, normalized size = 0.92

$$\frac{\sqrt{bx^2+a}\,x^2}{3\,b}-\frac{2\,\sqrt{bx^2+a}\,a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(b*x^2 + a)*x^2/b - 2/3*sqrt(b*x^2 + a)*a/b^2
```

mupad [B] time = 4.73, size = 24, normalized size = 0.67

$$-\frac{\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(1/2),x)
[Out] -((a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

sympy [A] time = 0.49, size = 44, normalized size = 1.22

$$\begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for}\ b\neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne
(b, 0)), (x**4/(4*sqrt(a)), True))
```

3.488 $\int \frac{x^2}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/2*x*(b*x^2+a)^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 217, 206}

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a+bx^2}}\,dx &= \frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\
&= \frac{x\sqrt{a+bx^2}}{2b}-\frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b}\\
&= \frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

fricas [A] time = 0.94, size = 93, normalized size = 1.90

$$\left[\frac{2\sqrt{bx^2+a}\,bx + a\sqrt{b}\log\left(-2\,bx^2 + 2\sqrt{bx^2+a}\,\sqrt{b}\,x - a\right)}{4\,b^2}, \frac{\sqrt{bx^2+a}\,bx + a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*x + a*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)))/b^2]
```

giac [A] time = 1.18, size = 40, normalized size = 0.82

$$\frac{\sqrt{bx^2+a}\,x}{2\,b} + \frac{a\log\left(\left|-\sqrt{b}\,x + \sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b + 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3
/2)
```

maple [A] time = 0.00, size = 39, normalized size = 0.80

$$-\frac{a\ln\left(\sqrt{b}\,x + \sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}} + \frac{\sqrt{b\,x^2+a}\,x}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.26, size = 31, normalized size = 0.63

$$\frac{\sqrt{bx^2+a}\,x}{2\,b} - \frac{a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b - 1/2*a*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

mupad [B] time = 4.82, size = 56, normalized size = 1.14

$$\begin{cases} \frac{x^3}{3\sqrt{a}} & \text{if } b = 0 \\ \frac{x\sqrt{b\,x^2+a}}{2\,b} - \frac{a\ln\left(2\sqrt{b}\,x+2\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} & \text{if } b \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, x^3/(3*a^(1/2)), b ~= 0, (x*(a + b*x^2)^(1/2))/(2*b) - (a
*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)))
```

sympy [A] time = 2.25, size = 42, normalized size = 0.86

$$\frac{\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)
)
```

3.489 $\int \frac{x}{\sqrt{a+bx^2}} dx$

Optimal. Leaf size=15

$$\frac{\sqrt{a+bx^2}}{b}$$

```
[Out] (b*x^2+a)^(1/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^2],x]
[Out] Sqrt[a + b*x^2]/b
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^2}} dx = \frac{\sqrt{a+bx^2}}{b}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2],x]
[Out] Sqrt[a + b*x^2]/b
```

fricas [A] time = 0.96, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] sqrt(b*x^2 + a)/b
```

giac [A] time = 1.00, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] sqrt(b*x^2 + a)/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\sqrt{b\,x^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/2),x)
[Out] (b*x^2+a)^(1/2)/b
```

maxima [A] time = 1.40, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] sqrt(b*x^2 + a)/b
```

mupad [B] time = 4.58, size = 13, normalized size = 0.87

$$\frac{\sqrt{b\,x^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(1/2),x)
[Out] (a + b*x^2)^(1/2)/b
```

sympy [A] time = 0.38, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for}\ b \neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/2),x)
[Out] Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True))
```

3.490 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{217, 206\}$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*x^2],x]

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*x^2],x]

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]

fricas [A] time = 0.64, size = 59, normalized size = 2.36

$$\left[\frac{\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{b}\,x - a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

giac [A] time = 1.24, size = 23, normalized size = 0.92

$$-\frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

maple [A] time = 0.00, size = 21, normalized size = 0.84

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2),x)
```

```
[Out] ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

maxima [A] time = 1.36, size = 13, normalized size = 0.52

$$\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 0.12, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/2),x)
```

```
[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

sympy [A] time = 1.02, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

3.491 $\int \frac{1}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=25

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2]),x]
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

fricas [A] time = 1.04, size = 60, normalized size = 2.40

$$\left[\frac{\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)}{2\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)/sqrt(a), sqrt(-a)*
arctan(sqrt(-a)/sqrt(b*x^2 + a))/a]
```

giac [A] time = 0.99, size = 22, normalized size = 0.88

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$-\frac{\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/2),x)
[Out] -1/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.38, size = 17, normalized size = 0.68

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)
```

mupad [B] time = 4.87, size = 19, normalized size = 0.76

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(1/2)),x)
```

```
[Out] -atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(1/2)
```

sympy [A] time = 1.06, size = 19, normalized size = 0.76

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(1/2),x)
```

```
[Out] -asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

3.492 $\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sqrt{a+bx^2}}{ax}$$

```
[Out] -(b*x^2+a)^(1/2)/a/x
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^2]),x]
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}}{ax}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2]),x]
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

fricas [A] time = 1.25, size = 17, normalized size = 0.89

$$-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -sqrt(b*x^2 + a)/(a*x)
```

giac [A] time = 1.16, size = 30, normalized size = 1.58

$$\frac{2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$-\frac{\sqrt{b\,x^{2}+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] -(b*x^2+a)^(1/2)/a/x
```

maxima [A] time = 1.35, size = 17, normalized size = 0.89

$$-\frac{\sqrt{bx^{2}+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*x^2 + a)/(a*x)
```

mupad [B] time = 4.60, size = 17, normalized size = 0.89

$$-\frac{\sqrt{b\,x^{2}+a}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(1/2)),x)
```

```
[Out] -(a + b*x^2)^(1/2)/(a*x)
```

sympy [A] time = 0.69, size = 19, normalized size = 1.00

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

3.493 $\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=50

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

```
[Out] 1/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*(b*x^2+a)^(1/2)/a/x^2
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(2*a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2
))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{a+bx^2}}{2ax^2} - \frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a} \\
&= -\frac{\sqrt{a+bx^2}}{2ax^2} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a} \\
&= -\frac{\sqrt{a+bx^2}}{2ax^2} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.22

$$\frac{b\sqrt{a+bx^2}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2\sqrt{\frac{bx^2}{a}+1}} - \frac{a}{2bx^2}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] (b*Sqrt[a + b*x^2]*(-1/2*a/(b*x^2) + ArcTanh[Sqrt[1 + (b*x^2)/a]]/(2*Sqrt[1
 + (b*x^2)/a])))/a^2
```

fricas [A] time = 0.91, size = 105, normalized size = 2.10

$$\left[\frac{\sqrt{a}\,bx^2\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right) - 2\,\sqrt{bx^2+a}\,a}{4\,a^2x^2}, -\frac{\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) + \sqrt{bx^2+a}\,a}{2\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*sqrt(b*x^2 + a)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) + sqrt(b*x^2 + a)*a)/(a^2*x^2)]
```

giac [A] time = 1.09, size = 51, normalized size = 1.02

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} + \frac{\sqrt{bx^2+a}\,b}{ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^2 + a)*b
/(a*x^2))/b
```

maple [A] time = 0.00, size = 48, normalized size = 0.96

$$\frac{b\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}} - \frac{\sqrt{b\,x^2+a}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))
/x)
```

maxima [A] time = 1.35, size = 36, normalized size = 0.72

$$\frac{b \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2 a^{\frac{3}{2}}} - \frac{\sqrt{bx^2 + a}}{2 ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/2*sqrt(b*x^2 + a)/(a*x^2)
```

mupad [B] time = 4.75, size = 38, normalized size = 0.76

$$\frac{b \operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{\sqrt{b\,x^2 + a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (a + b*x^2)^(1/2)/(2*a*x
^2)
```

sympy [A] time = 2.25, size = 42, normalized size = 0.84

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{2ax} + \frac{b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**
(3/2))
```

3.494 $\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=44

$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

```
[Out] -1/3*(b*x^2+a)^(1/2)/a/x^3+2/3*b*(b*x^2+a)^(1/2)/a^2/x
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = $\{271, 264\}$

$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2]),x]
[Out] -Sqrt[a + b*x^2]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx&=-\frac{\sqrt{a+bx^2}}{3ax^3}-\frac{(2b)\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{3a}\\&=-\frac{\sqrt{a+bx^2}}{3ax^3}+\frac{2b\sqrt{a+bx^2}}{3a^2x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.66

$$-\frac{\left(a-2bx^2\right)\sqrt{a+bx^2}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2]),x]
[Out] -1/3*((a - 2*b*x^2)*Sqrt[a + b*x^2])/(a^2*x^3)
```

fricas [A] time = 0.86, size = 27, normalized size = 0.61

$$\frac{\left(2\,bx^2-a\right)\sqrt{bx^2+a}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*x^2 - a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

giac [A] time = 1.15, size = 55, normalized size = 1.25

$$\frac{4\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*b^(3/2)/((sqrt(b)*x - sqrt(b*x^
2 + a))^2 - a)^3
```

maple [A] time = 0.00, size = 26, normalized size = 0.59

$$-\frac{\sqrt{b\,x^2+a}\,\left(-2b\,x^2+a\right)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(-2*b*x^2+a)/a^2/x^3
```

maxima [A] time = 1.27, size = 36, normalized size = 0.82

$$\frac{2\sqrt{bx^2+a}\,b}{3\,a^2x}-\frac{\sqrt{bx^2+a}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(b*x^2 + a)*b/(a^2*x) - 1/3*sqrt(b*x^2 + a)/(a*x^3)
```

mupad [B] time = 4.62, size = 25, normalized size = 0.57

$$-\frac{\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(1/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3)
```

sympy [A] time = 0.90, size = 46, normalized size = 1.05

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(3*a**2)
```

3.495 $\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{3b\sqrt{a+bx^2}}{8a^2x^2}-\frac{\sqrt{a+bx^2}}{4ax^4}$$

```
[Out] -3/8*b^2*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)-1/4*(b*x^2+a)^(1/2)/a/x^4
+3/8*b*(b*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{3b\sqrt{a+bx^2}}{8a^2x^2}-\frac{\sqrt{a+bx^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(4*a*x^4) + (3*b*Sqrt[a + b*x^2])/(8*a^2*x^2) - (3*b^2*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)$$
$$= -\frac{\sqrt{a+bx^2}}{4ax^4} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{8a}$$
$$= -\frac{\sqrt{a+bx^2}}{4ax^4} + \frac{3b\sqrt{a+bx^2}}{8a^2x^2} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{16a^2}$$
$$= -\frac{\sqrt{a+bx^2}}{4ax^4} + \frac{3b\sqrt{a+bx^2}}{8a^2x^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{8a^2}$$
$$= -\frac{\sqrt{a+bx^2}}{4ax^4} + \frac{3b\sqrt{a+bx^2}}{8a^2x^2} - \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.50

$$-\frac{b^2\sqrt{a+bx^2}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx^2}{a}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] -((b^2*Sqrt[a + b*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x^2)/a])/a^3)
```

fricas [A] time = 1.00, size = 135, normalized size = 1.82

$$\left[\frac{3\sqrt{a}\,b^2x^4\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(3abx^2-2a^2\right)\sqrt{bx^2+a}}{16a^3x^4}, \frac{3\sqrt{-a}\,b^2x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(3abx^2-2a\right.}{8a^3x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + 2*(3*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/(a^3*x^4), 1/8*(3*sqrt(-a)*b^2*x
^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/
(a^3*x^4)]
```

giac [A] time = 1.11, size = 75, normalized size = 1.01

$$\frac{\frac{3b^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\left(bx^2+a\right)^{\frac{3}{2}}b^3-5\sqrt{bx^2+a}\,ab^3}{a^2b^2x^4}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a)
^(3/2)*b^3 - 5*sqrt(b*x^2 + a)*a*b^3)/(a^2*b^2*x^4))/b
```

maple [A] time = 0.00, size = 68, normalized size = 0.92

$$-\frac{3b^{2}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}+\frac{3\sqrt{b\,x^{2}+a}\;b}{8a^{2}x^{2}}-\frac{\sqrt{b\,x^{2}+a}}{4a\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)^(1/2)/a/x^4+3/8*b*(b*x^2+a)^(1/2)/a^2/x^2-3/8/a^(5/2)*b^2*ln
((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.29, size = 56, normalized size = 0.76

$$-\frac{3\,b^{2}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}}+\frac{3\,\sqrt{bx^{2}+a}\,b}{8\,a^{2}x^{2}}-\frac{\sqrt{bx^{2}+a}}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 3/8*sqrt(b*x^2 + a)*b/(a^2
*x^2) - 1/4*sqrt(b*x^2 + a)/(a*x^4)
```

mupad [B] time = 4.73, size = 57, normalized size = 0.77

$$\frac{3\left(b\,x^{2}+a\right)^{3/2}}{8\,a^{2}\,x^{4}}-\frac{5\,\sqrt{b\,x^{2}+a}}{8\,a\,x^{4}}-\frac{3\,b^{2}\operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^(1/2)),x)
```

```
[Out] (3*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (5*(a + b*x^2)^(1/2))/(8*a*x^4) - (3*b^
2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2))
```

sympy [A] time = 4.25, size = 97, normalized size = 1.31

$$-\frac{1}{4\sqrt{b}\,x^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{\sqrt{b}}{8ax^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3b^{\frac{3}{2}}}{8a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3b^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] -1/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)/(8*a*x**3*sqrt(a/(b*x**2
) + 1)) + 3*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*b**2*asinh(sqrt(a)
/(sqrt(b)*x))/(8*a**(5/2))
```

3.496 $\int \frac{x^5}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=55

$$-\frac{a^2}{b^3\sqrt{a+bx^2}}-\frac{2a\sqrt{a+bx^2}}{b^3}+\frac{\left(a+bx^2\right)^{3/2}}{3b^3}$$

```
[Out] 1/3*(b*x^2+a)^(3/2)/b^3-a^2/b^3/(b*x^2+a)^(1/2)-2*a*(b*x^2+a)^(1/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{a^2}{b^3\sqrt{a+bx^2}}-\frac{2a\sqrt{a+bx^2}}{b^3}+\frac{\left(a+bx^2\right)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(3/2),x]
```

```
[Out] -(a^2/(b^3*Sqrt[a + b*x^2])) - (2*a*Sqrt[a + b*x^2])/b^3 + (a + b*x^2)^(3/2
)/(3*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{3/2}}-\frac{2a}{b^2\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^2}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{b^3\sqrt{a+bx^2}}-\frac{2a\sqrt{a+bx^2}}{b^3}+\frac{\left(a+bx^2\right)^{3/2}}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.69

$$\frac{-8a^2-4abx^2+b^2x^4}{3b^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(3/2),x]
```

```
[Out] (-8*a^2 - 4*a*b*x^2 + b^2*x^4)/(3*b^3*Sqrt[a + b*x^2])
```

fricas [A] time = 0.93, size = 46, normalized size = 0.84

$$\frac{\left(b^{2} x^{4} - 4 \, a b x^{2} - 8 \, a^{2}\right) \sqrt{b x^{2} + a}}{3 \, \left(b^{4} x^{2} + a b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^4 - 4*a*b*x^2 - 8*a^2)*sqrt(b*x^2 + a)/(b^4*x^2 + a*b^3)
```

giac [A] time = 0.98, size = 52, normalized size = 0.95

$$-\frac{a^{2}}{\sqrt{b x^{2} + a} b^{3}} + \frac{{\left(b x^{2} + a\right)}^{\frac{3}{2}} b^{6} - 6 \, \sqrt{b x^{2} + a} a b^{6}}{3 \, b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -a^2/(sqrt(b*x^2 + a)*b^3) + 1/3*((b*x^2 + a)^(3/2)*b^6 - 6*sqrt(b*x^2 + a)
*a*b^6)/b^9
```

maple [A] time = 0.01, size = 36, normalized size = 0.65

$$-\frac{-b^{2} x^{4}+4 a b \,x^{2}+8 a^{2}}{3 \sqrt{b \,x^{2}+a}\, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/3*(-b^2*x^4+4*a*b*x^2+8*a^2)/(b*x^2+a)^(1/2)/b^3
```

maxima [A] time = 1.31, size = 53, normalized size = 0.96

$$\frac{x^{4}}{3 \, \sqrt{b x^{2} + a} b} - \frac{4 \, a x^{2}}{3 \, \sqrt{b x^{2} + a} b^{2}} - \frac{8 \, a^{2}}{3 \, \sqrt{b x^{2} + a} b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^4/(sqrt(b*x^2 + a)*b) - 4/3*a*x^2/(sqrt(b*x^2 + a)*b^2) - 8/3*a^2/(sq
rt(b*x^2 + a)*b^3)
```

mupad [B] time = 4.72, size = 41, normalized size = 0.75

$$-\frac{6\,a\,\left(b\,x^2+a\right)-{\left(b\,x^2+a\right)}^2+3\,a^2}{3\,b^3\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(3/2),x)
```

```
[Out] -(6*a*(a + b*x^2) - (a + b*x^2)^2 + 3*a^2)/(3*b^3*(a + b*x^2)^(1/2))
```

sympy [**A**] time = 0.94, size = 68, normalized size = 1.24

$$\begin{cases} -\frac{8a^2}{3b^3\sqrt{a+bx^2}} - \frac{4ax^2}{3b^2\sqrt{a+bx^2}} + \frac{x^4}{3b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-8*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*a*x**2/(3*b**2*sqrt(a + b*
x**2)) + x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(3/2)), True))
```

3.497 $\int \frac{x^4}{\left(a+bx^2\right)^{3/2}} dx$

Optimal. Leaf size=68

$$-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}+\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{x^3}{b\sqrt{a+bx^2}}$$

```
[Out] -3/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-x^3/b/(b*x^2+a)^(1/2)+3/2
*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {288, 321, 217, 206}

$$\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}-\frac{x^3}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(3/2),x]
```

```
[Out] -(x^3/(b*Sqrt[a + b*x^2])) + (3*x*Sqrt[a + b*x^2])/(2*b^2) - (3*a*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{x^3}{b\sqrt{a+bx^2}} + \frac{3\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{b}$$

$$= -\frac{x^3}{b\sqrt{a+bx^2}} + \frac{3x\sqrt{a+bx^2}}{2b^2} - \frac{(3a)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2}$$

$$= -\frac{x^3}{b\sqrt{a+bx^2}} + \frac{3x\sqrt{a+bx^2}}{2b^2} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b^2}$$

$$= -\frac{x^3}{b\sqrt{a+bx^2}} + \frac{3x\sqrt{a+bx^2}}{2b^2} - \frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 1.04

$$\frac{\sqrt{b}\,x\left(3a+bx^2\right) - 3a^{3/2}\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*x*(3*a + b*x^2) - 3*a^(3/2)*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x
)/Sqrt[a]])/(2*b^(5/2)*Sqrt[a + b*x^2])
```

fricas [A] time = 1.00, size = 159, normalized size = 2.34

$$\left[\frac{3\left(abx^2+a^2\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(b^2x^3+3\,abx\right)\sqrt{bx^2+a}}{4\left(b^4x^2+ab^3\right)}, \frac{3\left(abx^2+a^2\right)\sqrt{-b}\,\arctan}{2(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*b*x^2 + a^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
- a) + 2*(b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3), 1/2*(3*(a*
b*x^2 + a^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (b^2*x^3 + 3*a*b
*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

giac [A] time = 1.15, size = 51, normalized size = 0.75

$$\frac{x\left(\frac{x^2}{b}+\frac{3\,a}{b^2}\right)}{2\,\sqrt{bx^2+a}} + \frac{3\,a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x*(x^2/b + 3*a/b^2)/sqrt(b*x^2 + a) + 3/2*a*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 57, normalized size = 0.84

$$\frac{x^3}{2\sqrt{b\,x^2+a}\;b} + \frac{3ax}{2\sqrt{b\,x^2+a}\;b^2} - \frac{3a\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*x^3/b/(b*x^2+a)^(1/2)+3/2*a/b^2*x/(b*x^2+a)^(1/2)-3/2*a/b^(5/2)*ln(b^(1
/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.32, size = 49, normalized size = 0.72

$$\frac{x^3}{2\sqrt{bx^2+a}\,b}+\frac{3\,ax}{2\sqrt{bx^2+a}\,b^2}-\frac{3\,a\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/(sqrt(b*x^2 + a)*b) + 3/2*a*x/(sqrt(b*x^2 + a)*b^2) - 3/2*a*arcsinh
(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(3/2),x)
```

```
[Out] int(x^4/(a + b*x^2)^(3/2), x)
```

sympy [A] time = 3.26, size = 71, normalized size = 1.04

$$\frac{3\sqrt{a}\,x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(3/2),x)
```

```
[Out] 3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b
**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))
```

3.498 $\int \frac{x^3}{\left(a+bx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=32

$$\frac{a}{b^2\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^2}$$

```
[Out] a/b^2/(b*x^2+a)^(1/2)+(b*x^2+a)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a}{b^2\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(3/2),x]
```

```
[Out] a/(b^2*Sqrt[a + b*x^2]) + Sqrt[a + b*x^2]/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^{3/2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}} \, dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}\right) dx,x,x^2\right)\\
&= \frac{a}{b^2\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.75

$$\frac{2a+bx^2}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*a + b*x^2)/(b^2*Sqrt[a + b*x^2])
```

fricas [A] time = 0.92, size = 34, normalized size = 1.06

$$\frac{\left(bx^2 + 2a\right)\sqrt{bx^2 + a}}{b^3x^2 + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] (b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^3*x^2 + a*b^2)
```

giac [A] time = 1.06, size = 32, normalized size = 1.00

$$\frac{\frac{\sqrt{bx^2+a}}{b} + \frac{a}{\sqrt{bx^2+a}\,b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] (sqrt(b*x^2 + a)/b + a/(sqrt(b*x^2 + a)*b))/b
```

maple [A] time = 0.00, size = 23, normalized size = 0.72

$$\frac{b\,x^2 + 2a}{\sqrt{b\,x^2 + a}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(3/2),x)
[Out] (b*x^2+2*a)/(b*x^2+a)^(1/2)/b^2
```

maxima [A] time = 1.38, size = 32, normalized size = 1.00

$$\frac{x^2}{\sqrt{bx^2 + a}\,b} + \frac{2\,a}{\sqrt{bx^2 + a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] x^2/(sqrt(b*x^2 + a)*b) + 2*a/(sqrt(b*x^2 + a)*b^2)
```

mupad [B] time = 4.70, size = 22, normalized size = 0.69

$$\frac{b\,x^2 + 2\,a}{b^2\,\sqrt{b\,x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(3/2),x)
[Out] (2*a + b*x^2)/(b^2*(a + b*x^2)^(1/2))
```

sympy [A] time = 0.58, size = 41, normalized size = 1.28

$$\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b, 0
)), (x**4/(4*a**(3/2)), True))
```

3.499 $\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x}{b\sqrt{a+bx^2}}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-x/b/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {288, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(3/2),x]
```

```
[Out] -(x/(b*Sqrt[a + b*x^2])) + ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{x}{b\sqrt{a+bx^2}}+\frac{\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}\\
&= -\frac{x}{b\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b}\\
&= -\frac{x}{b\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 1.37

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\sqrt{b}\,x}{b^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(3/2),x]
[Out] (-(Sqrt[b]*x) + Sqrt[a]*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(
b^(3/2)*Sqrt[a + b*x^2])
```

fricas [A] time = 1.24, size = 130, normalized size = 3.02

$$\left[-\frac{2\sqrt{bx^2+a}\,bx-\left(bx^2+a\right)\sqrt{b}\,\log\left(-2\,bx^2-2\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\left(b^3x^2+ab^2\right)},-\frac{\sqrt{bx^2+a}\,bx+\left(bx^2+a\right)\sqrt{-b}\,\arctan\left(\right.}{b^3x^2+ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] [-1/2*(2*sqrt(b*x^2 + a)*b*x - (b*x^2 + a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a))/(b^3*x^2 + a*b^2), -(sqrt(b*x^2 + a)*b*x + (b*x^2
+ a)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(b^3*x^2 + a*b^2)]
```

giac [A] time = 1.23, size = 39, normalized size = 0.91

$$-\frac{x}{\sqrt{bx^2+a}\,b}-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] -x/(sqrt(b*x^2 + a)*b) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

maple [A] time = 0.01, size = 37, normalized size = 0.86

$$-\frac{x}{\sqrt{b\,x^2+a}\;b}+\frac{\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(3/2),x)
[Out] -x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.39, size = 29, normalized size = 0.67

$$-\frac{x}{\sqrt{bx^2+a}\,b}+\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] -x/(sqrt(b*x^2 + a)*b) + arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

mupad [B] time = 0.09, size = 36, normalized size = 0.84

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}-\frac{x}{b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(3/2),x)
[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(3/2) - x/(b*(a + b*x^2)^(1/2))
```

sympy [A] time = 1.71, size = 37, normalized size = 0.86

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(3/2),x)
[Out] asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))
```

3.500 $\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{b\sqrt{a+bx^2}}$$

```
[Out] -1/b/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(3/2),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2]))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{1}{b\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(3/2),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2]))
```

fricas [A] time = 0.70, size = 24, normalized size = 1.50

$$-\frac{\sqrt{bx^2+a}}{b^2x^2+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)/(b^2*x^2 + a*b)
```

giac [A] time = 1.05, size = 14, normalized size = 0.88

$$-\frac{1}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/(sqrt(b*x^2 + a)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{\sqrt{b\,x^2+a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/b/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.32, size = 14, normalized size = 0.88

$$-\frac{1}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/(sqrt(b*x^2 + a)*b)
```

mupad [B] time = 0.04, size = 14, normalized size = 0.88

$$-\frac{1}{b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(3/2),x)
```

```
[Out] -1/(b*(a + b*x^2)^(1/2))
```

sympy [A] time = 0.54, size = 24, normalized size = 1.50

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
```

3.501 $\int \frac{1}{\left(a+bx^2\right)^{3/2}} dx$

Optimal. Leaf size=16

$$\frac{x}{a\sqrt{a+bx^2}}$$

[Out] x/a/(b*x^2+a)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {191}

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(-3/2),x]

[Out] x/(a*Sqrt[a + b*x^2])

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}} dx = \frac{x}{a\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(-3/2),x]

[Out] x/(a*Sqrt[a + b*x^2])

fricas [A] time = 0.95, size = 23, normalized size = 1.44

$$\frac{\sqrt{bx^2+a}\,x}{abx^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="fricas")

[Out] sqrt(b*x^2 + a)*x/(a*b*x^2 + a^2)

giac [A] time = 1.12, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{bx^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(b*x^2 + a)*a)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$\frac{x}{\sqrt{b\,x^2+a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2),x)
```

```
[Out] x/a/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.33, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{bx^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(b*x^2 + a)*a)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.88

$$\frac{x}{a\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(3/2),x)
```

```
[Out] x/(a*(a + b*x^2)^(1/2))
```

sympy [A] time = 0.63, size = 17, normalized size = 1.06

$$\frac{x}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2),x)
```

```
[Out] x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

3.502 $\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{1}{a\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)+1/a/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$\frac{1}{a\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] 1/(a*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{a\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a}$$
$$= \frac{1}{a\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{ab}$$
$$= \frac{1}{a\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.80

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^2}{a}+1\right)}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^2)/a]/(a*Sqrt[a + b*x^2])
```

fricas [A] time = 0.92, size = 126, normalized size = 3.07

$$\left[\frac{\left(bx^2+a\right)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\sqrt{bx^2+a}\,a}{2\left(a^2bx^2+a^3\right)}, \frac{\left(bx^2+a\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}\,a}{a^2bx^2+a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*x^2 + a)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*sqrt(b*x^2 + a)*a)/(a^2*b*x^2 + a^3), ((b*x^2 + a)*sqrt(-a)*arctan(s
qrt(-a)/sqrt(b*x^2 + a)) + sqrt(b*x^2 + a)*a)/(a^2*b*x^2 + a^3)]
```

giac [A] time = 1.12, size = 39, normalized size = 0.95

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} + \frac{1}{\sqrt{bx^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 1/(sqrt(b*x^2 + a)*a)
```

maple [A] time = 0.00, size = 43, normalized size = 1.05

$$-\frac{\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{a^{\frac{3}{2}}} + \frac{1}{\sqrt{b\,x^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.36, size = 31, normalized size = 0.76

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}}+\frac{1}{\sqrt{bx^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 1/(sqrt(b*x^2 + a)*a)
```

mupad [B] time = 4.78, size = 33, normalized size = 0.80

$$\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(3/2)),x)
```

```
[Out] 1/(a*(a + b*x^2)^(1/2)) - atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(3/2)
```

sympy [B] time = 1.73, size = 184, normalized size = 4.49

$$\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(3/2),x)
```

```
[Out] 2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*x**
2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/
(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) +
2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2)
+ 2*a**(7/2)*b*x**2)
```

3.503 $\int \frac{1}{x^2(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{2bx}{a^2\sqrt{a+bx^2}}-\frac{1}{ax\sqrt{a+bx^2}}$$

```
[Out] -1/a/x/(b*x^2+a)^(1/2)-2*b*x/a^2/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 191}

$$-\frac{2bx}{a^2\sqrt{a+bx^2}}-\frac{1}{ax\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(1/(a*x*Sqrt[a + b*x^2])) - (2*b*x)/(a^2*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{1}{ax\sqrt{a+bx^2}}-\frac{(2b)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{a}\\ &= -\frac{1}{ax\sqrt{a+bx^2}}-\frac{2bx}{a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{a+2bx^2}{a^2x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] -((a + 2*b*x^2)/(a^2*x*Sqrt[a + b*x^2]))
```

fricas [A] time = 1.08, size = 35, normalized size = 0.92

$$-\frac{\left(2\,bx^2+a\right)\sqrt{bx^2+a}}{a^2bx^3+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*b*x^2 + a)*sqrt(b*x^2 + a)/(a^2*b*x^3 + a^3*x)
```

giac [A] time = 1.15, size = 50, normalized size = 1.32

$$-\frac{bx}{\sqrt{bx^2+a}\,a^2}+\frac{2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -b*x/(sqrt(b*x^2 + a)*a^2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
a)*a)
```

maple [A] time = 0.00, size = 26, normalized size = 0.68

$$-\frac{2b\,x^2+a}{\sqrt{b\,x^2+a}\;a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(3/2),x)
```

```
[Out] -(2*b*x^2+a)/x/(b*x^2+a)^(1/2)/a^2
```

maxima [A] time = 1.29, size = 34, normalized size = 0.89

$$-\frac{2\,bx}{\sqrt{bx^2+a}\,a^2}-\frac{1}{\sqrt{bx^2+a}\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b*x/(sqrt(b*x^2 + a)*a^2) - 1/(sqrt(b*x^2 + a)*a*x)
```

mupad [B] time = 4.64, size = 35, normalized size = 0.92

$$-\frac{\sqrt{b\,x^2+a}\,\left(\frac{1}{a}+\frac{2\,b\,x^2}{a^2}\right)}{b\,x^3+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(3/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(1/a + (2*b*x^2)/a^2))/(a*x + b*x^3)
```

sympy [A] time = 0.86, size = 46, normalized size = 1.21

$$-\frac{1}{a\sqrt{b}\,x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] -1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2)
+ 1))
```

3.504 $\int \frac{1}{x^3\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3b}{2a^2\sqrt{a+bx^2}}-\frac{1}{2ax^2\sqrt{a+bx^2}}$$

```
[Out] 3/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)-3/2*b/a^2/(b*x^2+a)^(1/2)-1/
2/a/x^2/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$-\frac{3\sqrt{a+bx^2}}{2a^2x^2}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{1}{ax^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] 1/(a*x^2*Sqrt[a + b*x^2]) - (3*Sqrt[a + b*x^2])/(2*a^2*x^2) + (3*b*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx,x,x^2\right)$$
$$= \frac{1}{ax^2\sqrt{a+bx^2}}+\frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}$$
$$= \frac{1}{ax^2\sqrt{a+bx^2}}-\frac{3\sqrt{a+bx^2}}{2a^2x^2}-\frac{(3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{4a^2}$$
$$= \frac{1}{ax^2\sqrt{a+bx^2}}-\frac{3\sqrt{a+bx^2}}{2a^2x^2}-\frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{2a^2}$$
$$= \frac{1}{ax^2\sqrt{a+bx^2}}-\frac{3\sqrt{a+bx^2}}{2a^2x^2}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [**C**] time = 0.01, size = 35, normalized size = 0.51

$$-\frac{b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx^2}{a}+1\right)}{a^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] -((b*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x^2)/a])/(a^2*Sqrt[a + b*x^2]))
```

fricas [**A**] time = 0.81, size = 171, normalized size = 2.48

$$\left[\frac{3\left(b^2x^4+abx^2\right)\sqrt{a}\,\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(3abx^2+a^2\right)\sqrt{bx^2+a}}{4\left(a^3bx^4+a^4x^2\right)},-\frac{3\left(b^2x^4+abx^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-}}{\sqrt{bx^2}}\right.}{2\left(a^3bx^4+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(b^2*x^4 + a*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a)
 + 2*a)/x^2) - 2*(3*a*b*x^2 + a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2),
-1/2*(3*(b^2*x^4 + a*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*
a*b*x^2 + a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)]
```

giac [**A**] time = 1.12, size = 72, normalized size = 1.04

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}\,a^2}-\frac{3\left(bx^2+a\right)b-2\,ab}{2\left(\left(bx^2+a\right)^{\frac{3}{2}}-\sqrt{bx^2+a}\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/2*(3*(b*x^2 + a)
*b - 2*a*b)/(((b*x^2 + a)^(3/2) - sqrt(b*x^2 + a)*a)*a^2)
```

maple [A] time = 0.01, size = 63, normalized size = 0.91

$$\frac{3b\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{5}{2}}}-\frac{3b}{2\sqrt{b\,x^{2}+a}\,a^{2}}-\frac{1}{2\sqrt{b\,x^{2}+a}\,a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/a/x^2/(b*x^2+a)^(1/2)-3/2*b/a^2/(b*x^2+a)^(1/2)+3/2/a^(5/2)*b*ln((2*a+
2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.31, size = 51, normalized size = 0.74

$$\frac{3\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{5}{2}}}-\frac{3\,b}{2\,\sqrt{bx^{2}+a}\,a^{2}}-\frac{1}{2\,\sqrt{bx^{2}+a}\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 3/2*b/(sqrt(b*x^2 + a)*a^2) -
 1/2/(sqrt(b*x^2 + a)*a*x^2)
```

mupad [B] time = 4.94, size = 53, normalized size = 0.77

$$\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}-\frac{1}{2\,a\,x^{2}\,\sqrt{b\,x^{2}+a}}-\frac{3\,b}{2\,a^{2}\,\sqrt{b\,x^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(3/2)),x)
```

```
[Out] (3*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2)) - 1/(2*a*x^2*(a + b*x^2)
^(1/2)) - (3*b)/(2*a^2*(a + b*x^2)^(1/2))
```

sympy [A] time = 3.39, size = 73, normalized size = 1.06

$$-\frac{1}{2a\sqrt{b}\,x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3\sqrt{b}}{2a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] -1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/(b*
x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))
```

3.505 $\int \frac{1}{x^4\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{8b^2x}{3a^3\sqrt{a+bx^2}}+\frac{4b}{3a^2x\sqrt{a+bx^2}}-\frac{1}{3ax^3\sqrt{a+bx^2}}$$

```
[Out] -1/3/a/x^3/(b*x^2+a)^(1/2)+4/3*b/a^2/x/(b*x^2+a)^(1/2)+8/3*b^2*x/a^3/(b*x^2
+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 191}

$$\frac{8b^2x}{3a^3\sqrt{a+bx^2}}+\frac{4b}{3a^2x\sqrt{a+bx^2}}-\frac{1}{3ax^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] -1/(3*a*x^3*Sqrt[a + b*x^2]) + (4*b)/(3*a^2*x*Sqrt[a + b*x^2]) + (8*b^2*x)/
(3*a^3*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{1}{3ax^3\sqrt{a+bx^2}}-\frac{(4b)\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{3a}\\
&= -\frac{1}{3ax^3\sqrt{a+bx^2}}+\frac{4b}{3a^2x\sqrt{a+bx^2}}+\frac{\left(8b^2\right)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^2}\\
&= -\frac{1}{3ax^3\sqrt{a+bx^2}}+\frac{4b}{3a^2x\sqrt{a+bx^2}}+\frac{8b^2x}{3a^3\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.61

$$-\frac{a^2-4abx^2-8b^2x^4}{3a^3x^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] -1/3*(a^2 - 4*a*b*x^2 - 8*b^2*x^4)/(a^3*x^3*Sqrt[a + b*x^2])
```

fricas [A] time = 0.93, size = 50, normalized size = 0.76

$$\frac{\left(8\,b^2x^4+4\,abx^2-a^2\right)\sqrt{bx^2+a}}{3\left(a^3bx^5+a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(8*b^2*x^4 + 4*a*b*x^2 - a^2)*sqrt(b*x^2 + a)/(a^3*b*x^5 + a^4*x^3)
```

giac [A] time = 1.16, size = 106, normalized size = 1.61

$$\frac{b^2x}{\sqrt{bx^2+a}\,a^3}-\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4b^{\frac{3}{2}}-12\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}+5\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] b^2*x/(sqrt(b*x^2 + a)*a^3) - 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2
) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 5*a^2*b^(3/2))/(((sqrt(b
)*x - sqrt(b*x^2 + a))^2 - a)^3*a^2)
```

maple [A] time = 0.01, size = 37, normalized size = 0.56

$$-\frac{-8b^2x^4-4ab\,x^2+a^2}{3\sqrt{b\,x^2+a}\;a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/3*(-8*b^2*x^4-4*a*b*x^2+a^2)/x^3/(b*x^2+a)^(1/2)/a^3
```

maxima [A] time = 1.31, size = 54, normalized size = 0.82

$$\frac{8\,b^2x}{3\,\sqrt{bx^2+a}\,a^3}+\frac{4\,b}{3\,\sqrt{bx^2+a}\,a^2x}-\frac{1}{3\,\sqrt{bx^2+a}\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*b^2*x/(sqrt(b*x^2 + a)*a^3) + 4/3*b/(sqrt(b*x^2 + a)*a^2*x) - 1/3/(sqrt
(b*x^2 + a)*a*x^3)
```

mupad [B] time = 5.12, size = 38, normalized size = 0.58

$$\frac{-a^2+4\,a\,b\,x^2+8\,b^2\,x^4}{3\,a^3\,x^3\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(3/2)),x)
```

```
[Out] (8*b^2*x^4 - a^2 + 4*a*b*x^2)/(3*a^3*x^3*(a + b*x^2)^(1/2))
```

sympy [B] time = 1.23, size = 233, normalized size = 3.53

$$-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{8b^{\frac{15}{2}}x^6\sqrt{}}{3a^5b^4x^2+6a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(3/2),x)
```

```
[Out] -a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 +
3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4
*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**4*sqrt(a/(
b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 8*b
**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3
*a**3*b**6*x**6)
```

3.506 $\int \frac{x^6}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}$$

```
[Out] -1/3*x^5/b/(b*x^2+a)^(3/2)-5/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)
-5/3*x^3/b^2/(b*x^2+a)^(1/2)+5/2*x*(b*x^2+a)^(1/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {288, 321, 217, 206}

$$-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(5/2),x]
```

```
[Out] -x^5/(3*b*(a + b*x^2)^(3/2)) - (5*x^3)/(3*b^2*Sqrt[a + b*x^2]) + (5*x*Sqrt[
a + b*x^2])/(2*b^3) - (5*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2)
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^{5/2}}\,dx = -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}+\frac{5\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx}{3b}$$
$$= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{b^2}$$
$$= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{(5a)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^3}$$
$$= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{(5a)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b^3}$$
$$= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}$$

Mathematica [A] time = 0.14, size = 90, normalized size = 0.99

$$\frac{\sqrt{b}\,x\left(15a^2+20abx^2+3b^2x^4\right)-15a^{3/2}\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6b^{7/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*x*(15*a^2 + 20*a*b*x^2 + 3*b^2*x^4) - 15*a^(3/2)*(a + b*x^2)*Sqrt[
1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(6*b^(7/2)*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.65, size = 227, normalized size = 2.49

$$\left[\frac{15\left(ab^2x^4+2\,a^2bx^2+a^3\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(3\,b^3x^5+20\,ab^2x^3+15\,a^2bx\right)\sqrt{bx^2+a}}{12\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) + 2*(3*b^3*x^5 + 20*a*b^2*x^3 + 15*a^2*b*x)*sqrt(b*x
^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/6*(15*(a*b^2*x^4 + 2*a^2*b*x^
2 + a^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*b^3*x^5 + 20*a*b^
2*x^3 + 15*a^2*b*x)*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

giac [A] time = 1.21, size = 65, normalized size = 0.71

$$\frac{\left(x^2\left(\frac{3\,x^2}{b}+\frac{20\,a}{b^2}\right)+\frac{15\,a^2}{b^3}\right)x}{6\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{5\,a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(x^2*(3*x^2/b + 20*a/b^2) + 15*a^2/b^3)*x/(b*x^2 + a)^(3/2) + 5/2*a*log
(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

maple [A] time = 0.01, size = 75, normalized size = 0.82

$$\frac{x^5}{2\left(b\,x^2+a\right)^{\frac{3}{2}} b}+\frac{5a\,x^3}{6\left(b\,x^2+a\right)^{\frac{3}{2}} b^2}+\frac{5ax}{2\sqrt{b\,x^2+a}\;b^3}-\frac{5a\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/2*x^5/b/(b*x^2+a)^(3/2)+5/6*a/b^2*x^3/(b*x^2+a)^(3/2)+5/2*a/b^3*x/(b*x^2+
a)^(1/2)-5/2*a/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.41, size = 89, normalized size = 0.98

$$\frac{x^5}{2\left(bx^2+a\right)^{\frac{3}{2}} b}+\frac{5\,ax\left(\frac{3\,x^2}{\left(bx^2+a\right)^{\frac{3}{2}} b}+\frac{2\,a}{\left(bx^2+a\right)^{\frac{3}{2}} b^2}\right)}{6\,b}+\frac{5\,ax}{6\,\sqrt{bx^2+a}\,b^3}-\frac{5\,a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^5/((b*x^2 + a)^(3/2)*b) + 5/6*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/
((b*x^2 + a)^(3/2)*b^2))/b + 5/6*a*x/(sqrt(b*x^2 + a)*b^3) - 5/2*a*arcsinh(
b*x/sqrt(a*b))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(b\,x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^6/(a + b*x^2)^(5/2), x)
```

sympy [B] time = 5.12, size = 367, normalized size = 4.03

$$-\frac{15a^{\frac{81}{2}}b^{22}\sqrt{1+\frac{bx^2}{a}}\ \mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{15a^{\frac{79}{2}}b^{23}x^2\sqrt{1+\frac{bx^2}{a}}\ \mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^{40}b^{\frac{45}{2}}x}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(5/2),x)
```

```
[Out] -15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2
)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2
/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/
(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt
(1 + b*x**2/a)) + 15*a**40*b**(45/2)*x/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x*
*2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 20*a**39*b**(47/2)
*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**
2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6*a**(79/2)*b**(51/2)*sqrt(
1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a))
```

3.507 $\int \frac{x^5}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=54

$$-\frac{a^2}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{2a}{b^3\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^3}$$

```
[Out] -1/3*a^2/b^3/(b*x^2+a)^(3/2)+2*a/b^3/(b*x^2+a)^(1/2)+(b*x^2+a)^(1/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{a^2}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{2a}{b^3\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(5/2),x]
```

```
[Out] -a^2/(3*b^3*(a + b*x^2)^(3/2)) + (2*a)/(b^3*Sqrt[a + b*x^2]) + Sqrt[a + b*x
^2]/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{5/2}}-\frac{2a}{b^2(a+bx)^{3/2}}+\frac{1}{b^2\sqrt{a+bx}}\right)dx,x,x^2\right)\\
&= -\frac{a^2}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{2a}{b^3\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.72

$$\frac{8a^2+12abx^2+3b^2x^4}{3b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(5/2),x]
```

```
[Out] (8*a^2 + 12*a*b*x^2 + 3*b^2*x^4)/(3*b^3*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.62, size = 58, normalized size = 1.07

$$\frac{\left(3\,b^{2}x^{4}+12\,abx^{2}+8\,a^{2}\right)\sqrt{bx^{2}+a}}{3\,\left(b^{5}x^{4}+2\,ab^{4}x^{2}+a^{2}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*x^4 + 12*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/(b^5*x^4 + 2*a*b^4*x^2
 + a^2*b^3)
```

giac [A] time = 1.02, size = 44, normalized size = 0.81

$$\frac{\sqrt{bx^{2}+a}}{b^{3}}+\frac{6\left(bx^{2}+a\right)a-a^{2}}{3\left(bx^{2}+a\right)^{\frac{3}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)/b^3 + 1/3*(6*(b*x^2 + a)*a - a^2)/((b*x^2 + a)^(3/2)*b^3)
```

maple [A] time = 0.00, size = 36, normalized size = 0.67

$$\frac{3b^{2}x^{4}+12ab\,x^{2}+8a^{2}}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(3*b^2*x^4+12*a*b*x^2+8*a^2)/(b*x^2+a)^(3/2)/b^3
```

maxima [A] time = 1.35, size = 52, normalized size = 0.96

$$\frac{x^{4}}{\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{4\,ax^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}+\frac{8\,a^{2}}{3\left(bx^{2}+a\right)^{\frac{3}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] x^4/((b*x^2 + a)^(3/2)*b) + 4*a*x^2/((b*x^2 + a)^(3/2)*b^2) + 8/3*a^2/((b*x
^2 + a)^(3/2)*b^3)
```

mupad [B] time = 5.20, size = 38, normalized size = 0.70

$$\frac{2\,a\,\left(b\,x^{2}+a\right)+{\left(b\,x^{2}+a\right)}^{2}-\frac{a^{2}}{3}}{b^{3}\,{\left(b\,x^{2}+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(5/2),x)
```

```
[Out] (2*a*(a + b*x^2) + (a + b*x^2)^2 - a^2/3)/(b^3*(a + b*x^2)^(3/2))
```

sympy [A] time = 1.09, size = 138, normalized size = 2.56

$$\begin{cases} \frac{8a^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{12abx^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{3b^2x^4}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((8*a**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)
) + 12*a*b*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2))
+ 3*b**2*x**4/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)), N
e(b, 0)), (x**6/(6*a**(5/2)), True))
```

3.508 $\int \frac{x^4}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=64

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}-\frac{x}{b^2\sqrt{a+bx^2}}-\frac{x^3}{3b\left(a+bx^2\right)^{3/2}}$$

```
[Out] -1/3*x^3/b/(b*x^2+a)^(3/2)+arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-x/b^2
/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {288, 217, 206}

$$-\frac{x}{b^2\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}-\frac{x^3}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(5/2),x]
```

```
[Out] -x^3/(3*b*(a + b*x^2)^(3/2)) - x/(b^2*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]]/b^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{5/2}}\,dx = -\frac{x^3}{3b\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx}{b}$$
$$= -\frac{x^3}{3b\left(a+bx^2\right)^{3/2}} - \frac{x}{b^2\sqrt{a+bx^2}} + \frac{\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}$$
$$= -\frac{x^3}{3b\left(a+bx^2\right)^{3/2}} - \frac{x}{b^2\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b^2}$$
$$= -\frac{x^3}{3b\left(a+bx^2\right)^{3/2}} - \frac{x}{b^2\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.12, size = 80, normalized size = 1.25

$$\frac{3\sqrt{a}\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\sqrt{b}\,x\left(3a+4bx^2\right)}{3b^{5/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(Sqrt[b]*x*(3*a + 4*b*x^2)) + 3*Sqrt[a]*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*A
rcSinh[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(5/2)*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.86, size = 199, normalized size = 3.11

$$\left[\frac{3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(4\,b^2x^3+3\,abx\right)\sqrt{bx^2+a}}{6\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}, -\frac{3\left(b^2x^4+2\,ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) - 2*(4*b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*
b^4*x^2 + a^2*b^3), -1/3*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) + (4*b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^5*x^4
+ 2*a*b^4*x^2 + a^2*b^3)]
```

giac [A] time = 1.05, size = 51, normalized size = 0.80

$$-\frac{x\left(\frac{4\,x^2}{b}+\frac{3\,a}{b^2}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(4*x^2/b + 3*a/b^2)/(b*x^2 + a)^(3/2) - log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 54, normalized size = 0.84

$$-\frac{x^{3}}{3 \left(b \,x^{2}+a \right)^{\frac{3}{2}} b}-\frac{x}{\sqrt{b \,x^{2}+a}\, b^{2}}+\frac{\ln \left(\sqrt{b}\, x +\sqrt{b \,x^{2}+a}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*x^3/b/(b*x^2+a)^(3/2)-x/b^2/(b*x^2+a)^(1/2)+1/b^(5/2)*ln(b^(1/2)*x+(b*
x^2+a)^(1/2))
```

maxima [A] time = 1.37, size = 65, normalized size = 1.02

$$-\frac{1}{3}\, x \left(\frac{3\, x^{2}}{{\left(b x^{2} + a\right)}^{\frac{3}{2}} b} + \frac{2\, a}{{\left(b x^{2} + a\right)}^{\frac{3}{2}} b^{2}}\right) - \frac{x}{3\, \sqrt{b x^{2} + a} b^{2}} + \frac{\operatorname{arsinh}\left(\frac{b x}{\sqrt{a b}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) - 1/3*x/
(sqrt(b*x^2 + a)*b^2) + arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{{\left(b\,x^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^4/(a + b*x^2)^(5/2), x)
```

sympy [B] time = 2.99, size = 303, normalized size = 4.73

$$\frac{3 a^{\frac{39}{2}} b^{11} \sqrt{1 + \frac{b x^{2}}{a}} \operatorname{asinh}{\left(\frac{\sqrt{b} x}{\sqrt{a}} \right)}}{3 a^{\frac{39}{2}} b^{\frac{27}{2}} \sqrt{1 + \frac{b x^{2}}{a}} + 3 a^{\frac{37}{2}} b^{\frac{29}{2}} x^{2} \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{3 a^{\frac{37}{2}} b^{12} x^{2} \sqrt{1 + \frac{b x^{2}}{a}} \operatorname{asinh}{\left(\frac{\sqrt{b} x}{\sqrt{a}} \right)}}{3 a^{\frac{39}{2}} b^{\frac{27}{2}} \sqrt{1 + \frac{b x^{2}}{a}} + 3 a^{\frac{37}{2}} b^{\frac{29}{2}} x^{2} \sqrt{1 + \frac{b x^{2}}{a}}} - \frac{3 a^{19} b^{\frac{23}{2}} x}{3 a^{\frac{39}{2}} b^{\frac{27}{2}} \sqrt{1 + \frac{b x^{2}}{a}} + 3 a^{\frac{37}{2}} b^{\frac{29}{2}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(5/2),x)
```

```
[Out] 3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*
b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a
)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*
a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1
+ b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a
) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3
/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqr
t(1 + b*x**2/a))
```

3.509 $\int \frac{x^3}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{a}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{1}{b^2\sqrt{a+bx^2}}$$

```
[Out] 1/3*a/b^2/(b*x^2+a)^(3/2)-1/b^2/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{1}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(5/2),x]
```

```
[Out] a/(3*b^2*(a + b*x^2)^(3/2)) - 1/(b^2*Sqrt[a + b*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^{5/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{5/2}}+\frac{1}{b(a+bx)^{3/2}}\right)dx,x,x^2\right)\\ &= \frac{a}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{1}{b^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{-2a-3bx^2}{3b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(5/2),x]
```

```
[Out] (-2*a - 3*b*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.88, size = 47, normalized size = 1.31

$$-\frac{\left(3\,bx^{2} + 2\,a\right)\sqrt{bx^{2} + a}}{3\,\left(b^{4}x^{4} + 2\,ab^{3}x^{2} + a^{2}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] -1/3*(3*b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 1.10, size = 24, normalized size = 0.67

$$-\frac{3\,bx^{2} + 2\,a}{3\,\left(bx^{2} + a\right)^{\frac{3}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] -1/3*(3*b*x^2 + 2*a)/((b*x^2 + a)^(3/2)*b^2)
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$-\frac{3b\,x^{2} + 2a}{3\left(b\,x^{2} + a\right)^{\frac{3}{2}}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(5/2),x)
[Out] -1/3*(3*b*x^2+2*a)/(b*x^2+a)^(3/2)/b^2
```

maxima [A] time = 1.27, size = 33, normalized size = 0.92

$$-\frac{x^{2}}{\left(bx^{2} + a\right)^{\frac{3}{2}}b} - \frac{2\,a}{3\,\left(bx^{2} + a\right)^{\frac{3}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] -x^2/((b*x^2 + a)^(3/2)*b) - 2/3*a/((b*x^2 + a)^(3/2)*b^2)
```

mupad [B] time = 5.17, size = 24, normalized size = 0.67

$$-\frac{3\,b\,x^{2} + 2\,a}{3\,b^{2}\,{\left(b\,x^{2} + a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(5/2),x)
[Out] -(2*a + 3*b*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

sympy [A] time = 1.07, size = 92, normalized size = 2.56

$$\begin{cases} -\frac{2a}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}} - \frac{3bx^{2}}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}} & \text{for}\ b \neq 0 \\ \frac{x^{4}}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2))
- 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne(b
, 0)), (x**4/(4*a**(5/2)), True))
```

3.510 $\int \frac{x^2}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=21

$$\frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] 1/3*x^3/a/(b*x^2+a)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$\frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(5/2),x]
[Out] x^3/(3*a*(a + b*x^2)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(5/2),x]
[Out] x^3/(3*a*(a + b*x^2)^(3/2))
```

fricas [B] time = 1.00, size = 37, normalized size = 1.76

$$\frac{\sqrt{bx^2+a}\,x^3}{3\left(ab^2x^4+2\,a^2bx^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] 1/3*sqrt(b*x^2 + a)*x^3/(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)
```

giac [A] time = 1.08, size = 17, normalized size = 0.81

$$\frac{x^3}{3\left(bx^2+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*x^3/((b*x^2 + a)^(3/2)*a)
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$\frac{x^3}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(5/2),x)
[Out] 1/3*x^3/a/(b*x^2+a)^(3/2)
```

maxima [A] time = 1.31, size = 34, normalized size = 1.62

$$-\frac{x}{3\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{x}{3\sqrt{bx^2+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] -1/3*x/((b*x^2 + a)^(3/2)*b) + 1/3*x/(sqrt(b*x^2 + a)*a*b)
```

mupad [B] time = 5.13, size = 17, normalized size = 0.81

$$\frac{x^3}{3\,a\left(b\,x^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(5/2),x)
[Out] x^3/(3*a*(a + b*x^2)^(3/2))
```

sympy [B] time = 0.76, size = 44, normalized size = 2.10

$$\frac{x^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(5/2),x)
[Out] x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

3.511 $\int \frac{x}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

```
[Out] -1/3/b/(b*x^2+a)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(5/2),x]
[Out] -1/(3*b*(a + b*x^2)^(3/2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{5/2}}\,dx=-\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(5/2),x]
[Out] -1/3*1/(b*(a + b*x^2)^(3/2))
```

fricas [B] time = 0.83, size = 35, normalized size = 1.94

$$-\frac{\sqrt{bx^2+a}}{3\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] -1/3*sqrt(b*x^2 + a)/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

giac [A] time = 1.02, size = 14, normalized size = 0.78

$$-\frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] -1/3/((b*x^2 + a)^(3/2)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{1}{3\left(b\,x^2+a\right)^{\frac{3}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(5/2),x)
[Out] -1/3/b/(b*x^2+a)^(3/2)
```

maxima [A] time = 1.33, size = 14, normalized size = 0.78

$$-\frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] -1/3/((b*x^2 + a)^(3/2)*b)
```

mupad [B] time = 4.94, size = 14, normalized size = 0.78

$$-\frac{1}{3\,b\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(5/2),x)
[Out] -1/(3*b*(a + b*x^2)^(3/2))
```

sympy [A] time = 1.00, size = 46, normalized size = 2.56

$$\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(5/2),x)
[Out] Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne(b
, 0)), (x**2/(2*a**(5/2)), True))
```

3.512 $\int \frac{1}{(a+bx^2)^{5/2}} dx$

Optimal. Leaf size=39

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] 1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-5/2),x]
```

```
[Out] x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^{5/2}} dx &= \frac{x}{3a\left(a+bx^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(a+bx^2\right)^{3/2}} dx}{3a}\\
&= \frac{x}{3a\left(a+bx^2\right)^{3/2}}+\frac{2x}{3a^2\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.74

$$\frac{x\left(3a+2bx^2\right)}{3a^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-5/2),x]
```

```
[Out] (x*(3*a + 2*b*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.92, size = 47, normalized size = 1.21

$$\frac{\left(2\,bx^3+3\,ax\right)\sqrt{bx^2+a}}{3\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] 1/3*(2*b*x^3 + 3*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

giac [A] time = 1.04, size = 27, normalized size = 0.69

$$\frac{x\left(\frac{2\,bx^2}{a^2}+\frac{3}{a}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*x*(2*b*x^2/a^2 + 3/a)/(b*x^2 + a)^(3/2)
```

maple [A] time = 0.00, size = 26, normalized size = 0.67

$$\frac{\left(2b\,x^2+3a\right)x}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2),x)
[Out] 1/3*x*(2*b*x^2+3*a)/(b*x^2+a)^(3/2)/a^2
```

maxima [A] time = 1.34, size = 31, normalized size = 0.79

$$\frac{2\,x}{3\,\sqrt{bx^2+a}\,a^2}+\frac{x}{3\left(bx^2+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] 2/3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*x/((b*x^2 + a)^(3/2)*a)
```

mupad [B] time = 4.89, size = 28, normalized size = 0.72

$$\frac{2\,x\,\left(b\,x^2+a\right)+a\,x}{3\,a^2\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(5/2),x)
[Out] (2*x*(a + b*x^2) + a*x)/(3*a^2*(a + b*x^2)^(3/2))
```

sympy [B] time = 0.83, size = 95, normalized size = 2.44

$$\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2),x)
```

```
[Out] 3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)
) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*
x**2/a))
```

3.513 $\int \frac{1}{x(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{1}{a^2\sqrt{a+bx^2}}+\frac{1}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] 1/3/a/(b*x^2+a)^(3/2)-arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+1/a^2/(b*x^2
+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$\frac{1}{a^2\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{1}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*(a + b*x^2)^(3/2)) + 1/(a^2*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]]/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}}+\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^2\right)}{2a}\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}}+\frac{1}{a^2\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}}+\frac{1}{a^2\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{a^2b}\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}}+\frac{1}{a^2\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.61

$$
\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{bx^2}{a}+1\right)}{3a\left(a+bx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(5/2)),x]
[Out] Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*x^2)/a]/(3*a*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.87, size = 197, normalized size = 3.34

$$
\left[\frac{3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(3\,abx^2+4\,a^2\right)\sqrt{bx^2+a}}{6\left(a^3b^2x^4+2\,a^4bx^2+a^5\right)},\ \frac{3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-a}\ \text{ar}}{3\left(a^3b^2\right.}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] [1/6*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(a) + 2*a)/x^2) + 2*(3*a*b*x^2 + 4*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4
+ 2*a^4*b*x^2 + a^5), 1/3*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a)*arctan(sq
rt(-a)/sqrt(b*x^2 + a)) + (3*a*b*x^2 + 4*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4
 + 2*a^4*b*x^2 + a^5)]
```

giac [A] time = 1.15, size = 50, normalized size = 0.85

$$
\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\,bx^2+4\,a}{3\left(bx^2+a\right)^{\frac{3}{2}}a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/3*(3*b*x^2 + 4*a)/((b*x
^2 + a)^(3/2)*a^2)
```

maple [A] time = 0.00, size = 57, normalized size = 0.97

$$\frac{1}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a}-\frac{\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{a^{\frac{5}{2}}}+\frac{1}{\sqrt{b\,x^{2}+a}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3/a/(b*x^2+a)^(3/2)+1/a^2/(b*x^2+a)^(1/2)-1/a^(5/2)*ln((2*a+2*(b*x^2+a)^(
1/2)*a^(1/2))/x)
```

maxima [A] time = 1.34, size = 45, normalized size = 0.76

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{5}{2}}}+\frac{1}{\sqrt{bx^{2}+a}\,a^{2}}+\frac{1}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/(sqrt(b*x^2 + a)*a^2) + 1/3/((b*
x^2 + a)^(3/2)*a)
```

mupad [B] time = 5.20, size = 47, normalized size = 0.80

$$\frac{\frac{b\,x^{2}+a}{a^{2}}+\frac{1}{3\,a}}{\left(b\,x^{2}+a\right)^{3/2}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(5/2)),x)
```

```
[Out] ((a + b*x^2)/a^2 + 1/(3*a))/(a + b*x^2)^(3/2) - atanh((a + b*x^2)^(1/2)/a^(
1/2))/a^(5/2)
```

sympy [B] time = 2.89, size = 740, normalized size = 12.54

$$\frac{8a^{7}\sqrt{1+\frac{bx^{2}}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{3a^{7}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{6a^{7}\log\left(\sqrt{1+\right.}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(5/2),x)
```

```
[Out] 8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)
*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2) + 1
8*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a*
*7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(
15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**2/a
)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)
```

```
*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**
2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log(sq
rt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**
2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a*
*(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x
**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(sqr
t(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2
*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(19/2
) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) -
 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b
*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6)
```

3.514 $\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{8bx}{3a^3\sqrt{a+bx^2}}-\frac{4bx}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{1}{ax\left(a+bx^2\right)^{3/2}}$$

```
[Out] -1/a/x/(b*x^2+a)^(3/2)-4/3*b*x/a^2/(b*x^2+a)^(3/2)-8/3*b*x/a^3/(b*x^2+a)^(1
/2)
```

Rubi [A] time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {271, 192, 191}

$$-\frac{8bx}{3a^3\sqrt{a+bx^2}}-\frac{4bx}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{1}{ax\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] -(1/(a*x*(a + b*x^2)^(3/2))) - (4*b*x)/(3*a^2*(a + b*x^2)^(3/2)) - (8*b*x)/
(3*a^3*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{1}{ax\left(a+bx^2\right)^{3/2}}-\frac{(4b)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{a}\\
&= -\frac{1}{ax\left(a+bx^2\right)^{3/2}}-\frac{4bx}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{(8b)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^2}\\
&= -\frac{1}{ax\left(a+bx^2\right)^{3/2}}-\frac{4bx}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{8bx}{3a^3\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.70

$$\frac{-3a^2 - 12abx^2 - 8b^2x^4}{3a^3x\left(a + bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(5/2)),x]
[Out] (-3*a^2 - 12*a*b*x^2 - 8*b^2*x^4)/(3*a^3*x*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.83, size = 59, normalized size = 0.98

$$-\frac{\left(8\,b^2x^4 + 12\,abx^2 + 3\,a^2\right)\sqrt{bx^2 + a}}{3\left(a^3b^2x^5 + 2\,a^4bx^3 + a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] -1/3*(8*b^2*x^4 + 12*a*b*x^2 + 3*a^2)*sqrt(b*x^2 + a)/(a^3*b^2*x^5 + 2*a^4*
b*x^3 + a^5*x)
```

giac [A] time = 1.20, size = 64, normalized size = 1.07

$$-\frac{x\left(\frac{5\,b^2x^2}{a^3} + \frac{6\,b}{a^2}\right)}{3\left(bx^2 + a\right)^{\frac{3}{2}}} + \frac{2\,\sqrt{b}}{\left(\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^2 - a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] -1/3*x*(5*b^2*x^2/a^3 + 6*b/a^2)/(b*x^2 + a)^(3/2) + 2*sqrt(b)/(((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)*a^2)
```

maple [A] time = 0.00, size = 39, normalized size = 0.65

$$-\frac{8b^2x^4 + 12ab\,x^2 + 3a^2}{3\left(b\,x^2 + a\right)^{\frac{3}{2}}\,a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(5/2),x)
[Out] -1/3*(8*b^2*x^4+12*a*b*x^2+3*a^2)/x/(b*x^2+a)^(3/2)/a^3
```

maxima [A] time = 1.34, size = 50, normalized size = 0.83

$$-\frac{8\,bx}{3\,\sqrt{bx^2 + a}\,a^3} - \frac{4\,bx}{3\left(bx^2 + a\right)^{\frac{3}{2}}a^2} - \frac{1}{\left(bx^2 + a\right)^{\frac{3}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] -8/3*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*b*x/((b*x^2 + a)^(3/2)*a^2) - 1/((b*x^
2 + a)^(3/2)*a*x)
```

mupad [B] time = 5.18, size = 42, normalized size = 0.70

$$\frac{4\,a\,\left(b\,x^2+a\right)-8\,{\left(b\,x^2+a\right)}^2+a^2}{3\,a^3\,x\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(5/2)),x)
```

```
[Out] (4*a*(a + b*x^2) - 8*(a + b*x^2)^2 + a^2)/(3*a^3*x*(a + b*x^2)^(3/2))
```

sympy [B] time = 1.29, size = 165, normalized size = 2.75

$$-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] -3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a
**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*
a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/
(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)
```

3.515 $\int \frac{1}{x^3\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{5b}{2a^3\sqrt{a+bx^2}}-\frac{5b}{6a^2\left(a+bx^2\right)^{3/2}}-\frac{1}{2ax^2\left(a+bx^2\right)^{3/2}}$$

```
[Out] -5/6*b/a^2/(b*x^2+a)^(3/2)-1/2/a/x^2/(b*x^2+a)^(3/2)+5/2*b*arctanh((b*x^2+a
)^(1/2)/a^(1/2))/a^(7/2)-5/2*b/a^3/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$-\frac{5\sqrt{a+bx^2}}{2a^3x^2}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*x^2*(a + b*x^2)^(3/2)) + 5/(3*a^2*x^2*Sqrt[a + b*x^2]) - (5*Sqrt[a +
 b*x^2])/(2*a^3*x^2) + (5*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx, x, x^2\right)$$

$$= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, x^2\right)}{6a}$$

$$= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5}{3a^2x^2\sqrt{a+bx^2}} + \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{2a^2}$$

$$= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5}{3a^2x^2\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{2a^3x^2} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a^3}$$

$$= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5}{3a^2x^2\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{2a^3x^2} - \frac{5\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a^3}$$

$$= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5}{3a^2x^2\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{2a^3x^2} + \frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.42

$$-\frac{b\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{bx^2}{a}+1\right)}{3a^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] -1/3*(b*Hypergeometric2F1[-3/2, 2, -1/2, 1 + (b*x^2)/a])/(a^2*(a + b*x^2)^(
3/2))
```

fricas [A] time = 1.01, size = 241, normalized size = 2.74

$$\left[\frac{15\left(b^3x^6+2\,ab^2x^4+a^2bx^2\right)\sqrt{a}\,\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\left(15\,ab^2x^4+20\,a^2bx^2+3\,a^3\right)\sqrt{bx^2+a}}{12\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}, -\frac{15\left(b^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(15*a*b^2*x^4 + 20*a^2*b*x^2 + 3*a^3)*sq
rt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), -1/6*(15*(b^3*x^6 + 2
*a*b^2*x^4 + a^2*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*a*b
^2*x^4 + 20*a^2*b*x^2 + 3*a^3)*sqrt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4
+ a^6*x^2)]
```

giac [A] time = 1.08, size = 73, normalized size = 0.83

$$-\frac{5\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^3} - \frac{6\left(bx^2+a\right)b+ab}{3\left(bx^2+a\right)^{\frac{3}{2}}a^3} - \frac{\sqrt{bx^2+a}}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) - 1/3*(6*(b*x^2 + a)
*b + a*b)/((b*x^2 + a)^(3/2)*a^3) - 1/2*sqrt(b*x^2 + a)/(a^3*x^2)
```

maple [A] time = 0.01, size = 78, normalized size = 0.89

$$-\frac{5b}{6\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}+\frac{5b\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{7}{2}}}-\frac{5b}{2\sqrt{b\,x^{2}+a}\,a^{3}}-\frac{1}{2\left(b\,x^{2}+a\right)^{\frac{3}{2}}a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/2/a/x^2/(b*x^2+a)^(3/2)-5/6*b/a^2/(b*x^2+a)^(3/2)-5/2*b/a^3/(b*x^2+a)^(1
/2)+5/2/a^(7/2)*b*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.35, size = 66, normalized size = 0.75

$$\frac{5\,b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{7}{2}}}-\frac{5\,b}{2\,\sqrt{bx^{2}+a}\,a^{3}}-\frac{5\,b}{6\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}}-\frac{1}{2\left(bx^{2}+a\right)^{\frac{3}{2}}ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 5/2*b/(sqrt(b*x^2 + a)*a^3) -
 5/6*b/((b*x^2 + a)^(3/2)*a^2) - 1/2/((b*x^2 + a)^(3/2)*a*x^2)
```

mupad [B] time = 5.25, size = 73, normalized size = 0.83

$$\frac{5\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{7/2}}-\frac{1}{2\,a\,x^2\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{10\,b}{3\,a^2\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{5\,b^2\,x^2}{2\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(5/2)),x)
```

```
[Out] (5*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(7/2)) - 1/(2*a*x^2*(a + b*x^2)
^(3/2)) - (10*b)/(3*a^2*(a + b*x^2)^(3/2)) - (5*b^2*x^2)/(2*a^3*(a + b*x^2)
^(3/2))
```

sympy [B] time = 5.17, size = 864, normalized size = 9.82

$$-\frac{6a^{17}\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{46a^{16}bx^2\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{15}{12a^{\frac{39}{2}}x^2+36a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] -6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a
**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 + b*x
**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 +
```

```
12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x**2
+ 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) +
30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/
2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*b**
2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**
(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x**2/
a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a
**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12*a*
*(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*
b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*
a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a*
*14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a
**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sqrt(
1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*
b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(12*
a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2
)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)
*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x*
*8)
```

3.516 $\int \frac{1}{x^4(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=86

$$\frac{16b^2x}{3a^4\sqrt{a+bx^2}}+\frac{8b^2x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{2b}{a^2x\left(a+bx^2\right)^{3/2}}-\frac{1}{3ax^3\left(a+bx^2\right)^{3/2}}$$

```
[Out] -1/3/a/x^3/(b*x^2+a)^(3/2)+2*b/a^2/x/(b*x^2+a)^(3/2)+8/3*b^2*x/a^3/(b*x^2+a
)^(3/2)+16/3*b^2*x/a^4/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {271, 192, 191}

$$\frac{16b^2x}{3a^4\sqrt{a+bx^2}}+\frac{8b^2x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{2b}{a^2x\left(a+bx^2\right)^{3/2}}-\frac{1}{3ax^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] -1/(3*a*x^3*(a + b*x^2)^(3/2)) + (2*b)/(a^2*x*(a + b*x^2)^(3/2)) + (8*b^2*x
)/(3*a^3*(a + b*x^2)^(3/2)) + (16*b^2*x)/(3*a^4*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a + bx^2\right)^{5/2}} \, dx = -\frac{1}{3ax^3 \left(a + bx^2\right)^{3/2}} - \frac{(2b) \int \frac{1}{x^2 \left(a+bx^2\right)^{5/2}} \, dx}{a}$$

$$= -\frac{1}{3ax^3 \left(a + bx^2\right)^{3/2}} + \frac{2b}{a^2 x \left(a + bx^2\right)^{3/2}} + \frac{\left(8b^2\right) \int \frac{1}{\left(a+bx^2\right)^{5/2}} \, dx}{a^2}$$

$$= -\frac{1}{3ax^3 \left(a + bx^2\right)^{3/2}} + \frac{2b}{a^2 x \left(a + bx^2\right)^{3/2}} + \frac{8b^2 x}{3a^3 \left(a + bx^2\right)^{3/2}} + \frac{\left(16b^2\right) \int \frac{1}{\left(a+bx^2\right)^{3/2}} \, dx}{3a^3}$$

$$= -\frac{1}{3ax^3 \left(a + bx^2\right)^{3/2}} + \frac{2b}{a^2 x \left(a + bx^2\right)^{3/2}} + \frac{8b^2 x}{3a^3 \left(a + bx^2\right)^{3/2}} + \frac{16b^2 x}{3a^4 \sqrt{a + bx^2}}$$

Mathematica [**A**] time = 0.01, size = 53, normalized size = 0.62

$$\frac{-a^3 + 6a^2 bx^2 + 24ab^2 x^4 + 16b^3 x^6}{3a^4 x^3 \left(a + bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-a^3 + 6*a^2*b*x^2 + 24*a*b^2*x^4 + 16*b^3*x^6)/(3*a^4*x^3*(a + b*x^2)^(3/
2))
```

fricas [**A**] time = 0.96, size = 72, normalized size = 0.84

$$\frac{\left(16 \, b^3 x^6 + 24 \, ab^2 x^4 + 6 \, a^2 bx^2 - a^3\right) \sqrt{bx^2 + a}}{3 \, \left(a^4 b^2 x^7 + 2 \, a^5 bx^5 + a^6 x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(16*b^3*x^6 + 24*a*b^2*x^4 + 6*a^2*b*x^2 - a^3)*sqrt(b*x^2 + a)/(a^4*b^
2*x^7 + 2*a^5*b*x^5 + a^6*x^3)
```

giac [**A**] time = 1.26, size = 121, normalized size = 1.41

$$\frac{x \left(\frac{8 \, b^3 x^2}{a^4} + \frac{9 \, b^2}{a^3}\right)}{3 \, \left(bx^2 + a\right)^{\frac{3}{2}}} - \frac{4 \, \left(3 \, \left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^4 b^{\frac{3}{2}} - 9 \, \left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^2 ab^{\frac{3}{2}} + 4 \, a^2 b^{\frac{3}{2}}\right)}{3 \, \left(\left(\sqrt{b} x - \sqrt{bx^2 + a}\right)^2 - a\right)^3 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(8*b^3*x^2/a^4 + 9*b^2/a^3)/(b*x^2 + a)^(3/2) - 4/3*(3*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*b^(3/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 4
*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a^3)
```

maple [**A**] time = 0.01, size = 48, normalized size = 0.56

$$-\frac{-16b^3 x^6 - 24a \, b^2 x^4 - 6a^2 b \, x^2 + a^3}{3 \left(b \, x^2 + a\right)^{\frac{3}{2}} a^4 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-16*b^3*x^6-24*a*b^2*x^4-6*a^2*b*x^2+a^3)/x^3/(b*x^2+a)^(3/2)/a^4
```

maxima [A] time = 1.40, size = 72, normalized size = 0.84

$$\frac{16\,b^2x}{3\,\sqrt{bx^2+a}\,a^4}+\frac{8\,b^2x}{3\,\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{2\,b}{\left(bx^2+a\right)^{\frac{3}{2}}a^2x}-\frac{1}{3\,\left(bx^2+a\right)^{\frac{3}{2}}ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 16/3*b^2*x/(sqrt(b*x^2 + a)*a^4) + 8/3*b^2*x/((b*x^2 + a)^(3/2)*a^3) + 2*b/
((b*x^2 + a)^(3/2)*a^2*x) - 1/3/((b*x^2 + a)^(3/2)*a*x^3)
```

mupad [B] time = 5.00, size = 78, normalized size = 0.91

$$-\frac{6\,a^2\,\left(b\,x^2+a\right)-24\,a\,\left(b\,x^2+a\right)^2+16\,\left(b\,x^2+a\right)^3+a^3}{\left(b\,x^2+a\right)^{3/2}\,\left(\frac{3\,a^5\,x}{b}-\frac{3\,a^4\,x\,\left(b\,x^2+a\right)}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(5/2)),x)
```

```
[Out] -(6*a^2*(a + b*x^2) - 24*a*(a + b*x^2)^2 + 16*(a + b*x^2)^3 + a^3)/((a + b*
x^2)^(3/2)*((3*a^5*x)/b - (3*a^4*x*(a + b*x^2))/b))
```

sympy [B] time = 1.73, size = 354, normalized size = 4.12

$$-\frac{a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{5a^3b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{30a^2b^{\frac{23}{2}}x^4}{3a^7b^9x^2+9a^6b^{10}x^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(5/2),x)
```

```
[Out] -a**4*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4
+ 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 5*a**3*b**(21/2)*x**2*sqrt(a/(b*
x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a*
*4*b**12*x**8) + 30*a**2*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x
**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 40*a*b**
(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*
a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 16*b**(27/2)*x**8*sqrt(a/(b*x**2) +
1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12
*x**8)
```

3.517 $\int \frac{x^{10}}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}+\frac{9x\sqrt{a+bx^2}}{2b^5}-\frac{3x^3}{b^4\sqrt{a+bx^2}}-\frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}}-\frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}$$

```
[Out] -1/7*x^9/b/(b*x^2+a)^(7/2)-9/35*x^7/b^2/(b*x^2+a)^(5/2)-3/5*x^5/b^3/(b*x^2+
a)^(3/2)-9/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(11/2)-3*x^3/b^4/(b*x^2
+a)^(1/2)+9/2*x*(b*x^2+a)^(1/2)/b^5
```

Rubi [A] time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {288, 321, 217, 206}

$$-\frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}}-\frac{3x^3}{b^4\sqrt{a+bx^2}}+\frac{9x\sqrt{a+bx^2}}{2b^5}-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}-\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^(9/2),x]
```

```
[Out] -x^9/(7*b*(a + b*x^2)^(7/2)) - (9*x^7)/(35*b^2*(a + b*x^2)^(5/2)) - (3*x^5)
/(5*b^3*(a + b*x^2)^(3/2)) - (3*x^3)/(b^4*Sqrt[a + b*x^2]) + (9*x*Sqrt[a +
b*x^2])/(2*b^5) - (9*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}} + \frac{9\int \frac{x^8}{\left(a+bx^2\right)^{7/2}}\,dx}{7b}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}} - \frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}} + \frac{9\int \frac{x^6}{\left(a+bx^2\right)^{5/2}}\,dx}{5b^2}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}} - \frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}} + \frac{3\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx}{b^3}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}} - \frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}} - \frac{3x^3}{b^4\sqrt{a+bx^2}} + \frac{9\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{b^4}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}} - \frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}} - \frac{3x^3}{b^4\sqrt{a+bx^2}} + \frac{9x\sqrt{a+bx^2}}{2b^5} - \frac{(9a}{}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}} - \frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}} - \frac{3x^3}{b^4\sqrt{a+bx^2}} + \frac{9x\sqrt{a+bx^2}}{2b^5} - \frac{(9a}{}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}} - \frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}} - \frac{3x^3}{b^4\sqrt{a+bx^2}} + \frac{9x\sqrt{a+bx^2}}{2b^5} - \frac{9a}{}$$

Mathematica [A] time = 0.20, size = 114, normalized size = 0.87

$$\frac{\sqrt{b}\,x\left(315a^4+1050a^3bx^2+1218a^2b^2x^4+528ab^3x^6+35b^4x^8\right)-315a^{3/2}\left(a+bx^2\right)^3\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{70b^{11/2}\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[b]*x*(315*a^4 + 1050*a^3*b*x^2 + 1218*a^2*b^2*x^4 + 528*a*b^3*x^6 + 3
5*b^4*x^8) - 315*a^(3/2)*(a + b*x^2)^3*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]
*x)/Sqrt[a]])/(70*b^(11/2)*(a + b*x^2)^(7/2))
```

fricas [A] time = 1.01, size = 359, normalized size = 2.74

$$\left[\frac{315\left(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(35\,b^5x^9+528\,ab^4x\right.}{140\left(b^{10}x^8+4\,ab^9x^6+6\,a^2b^8x^4+4\,a^3b^7x^2+a^4b^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)
*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(35*b^5*x^9 +
528*a*b^4*x^7 + 1218*a^2*b^3*x^5 + 1050*a^3*b^2*x^3 + 315*a^4*b*x)*sqrt(b*x
^2 + a))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)
, 1/70*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)
```

```
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (35*b^5*x^9 + 528*a*b^4*x^7
+ 1218*a^2*b^3*x^5 + 1050*a^3*b^2*x^3 + 315*a^4*b*x)*sqrt(b*x^2 + a))/(b^10
*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)]
```

giac [A] time = 1.19, size = 91, normalized size = 0.69

$$\frac{\left(\left(\left(x^2\left(\frac{35\,x^2}{b}+\frac{528\,a}{b^2}\right)+\frac{1218\,a^2}{b^3}\right)x^2+\frac{1050\,a^3}{b^4}\right)x^2+\frac{315\,a^4}{b^5}\right)x}{70\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{9\,a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/70*(((x^2*(35*x^2/b + 528*a/b^2) + 1218*a^2/b^3)*x^2 + 1050*a^3/b^4)*x^2
+ 315*a^4/b^5)*x/(b*x^2 + a)^(7/2) + 9/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2
+ a)))/b^(11/2)
```

maple [A] time = 0.03, size = 111, normalized size = 0.85

$$\frac{x^9}{2\left(b\,x^2+a\right)^{\frac{7}{2}}b}+\frac{9a\,x^7}{14\left(b\,x^2+a\right)^{\frac{7}{2}}b^2}+\frac{9a\,x^5}{10\left(b\,x^2+a\right)^{\frac{5}{2}}b^3}+\frac{3a\,x^3}{2\left(b\,x^2+a\right)^{\frac{3}{2}}b^4}+\frac{9ax}{2\sqrt{b\,x^2+a}\;b^5}-\frac{9a\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+}\right.}{2b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/2*x^9/b/(b*x^2+a)^(7/2)+9/14*a/b^2*x^7/(b*x^2+a)^(7/2)+9/10*a/b^3*x^5/(b*
x^2+a)^(5/2)+3/2*a/b^4*x^3/(b*x^2+a)^(3/2)+9/2*a/b^5*x/(b*x^2+a)^(1/2)-9/2*
a/b^(11/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [B] time = 1.69, size = 285, normalized size = 2.18

$$\frac{x^9}{2\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{9\left(\frac{35\,x^6}{\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{70\,ax^4}{\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{\left(bx^2+a\right)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{\left(bx^2+a\right)^{\frac{7}{2}}b^4}\right)ax}{70\,b}+\frac{3\,ax\left(\frac{15\,x^4}{\left(bx^2+a\right)^{\frac{5}{2}}b}+\frac{20\,ax^2}{\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{\left(bx^2+a\right)^{\frac{5}{2}}b^3}\right)}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^9/((b*x^2 + a)^(7/2)*b) + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x
^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((
b*x^2 + a)^(7/2)*b^4))*a*x/b + 3/10*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*
a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 + 3/2*a*
x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 + 9/2*a^2
*x^3/((b*x^2 + a)^(5/2)*b^4) - 417/70*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*a^2
*x/((b*x^2 + a)^(3/2)*b^5) + 261/70*a^3*x/((b*x^2 + a)^(5/2)*b^5) - 9/2*a*a
rcsinh(b*x/sqrt(a*b))/b^(11/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{10}}{\left(b\,x^2+a\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^10/(a + b*x^2)^(9/2), x)
```

sympy [B] time = 13.42, size = 3181, normalized size = 24.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**(9/2),x)
```

```
[Out] -315*a**(311/2)*b**66*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(3
09/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1
 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a*
*(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x*
*8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a)
+ 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 1890*a**(309/2)*b**6
7*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2
)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) +
1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(14
9/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x
**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)
*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 4725*a**(307/2)*b**68*x**4*sqrt(1 +
 b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x*
*2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)
*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(
1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a*
*(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**
12*sqrt(1 + b*x**2/a)) - 6300*a**(305/2)*b**69*x**6*sqrt(1 + b*x**2/a)*asin
h(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**
(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**
4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) +
 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(15
3/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x
**2/a)) - 4725*a**(303/2)*b**70*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqr
t(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145
/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x*
*2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2
)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt
(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 1890*
a**(301/2)*b**71*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(
309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(
1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a
**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x
**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a)
 + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 315*a**(299/2)*b**7
2*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/
2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) +
 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(1
49/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*
x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2
)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 315*a**155*b**(133/2)*x/(70*a**(30
9/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1
+ b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**
(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**
8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) +
 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 1995*a**154*b**(135/2
)*x**3/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(14
5/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x
**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/
2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqr
t(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 5313
```

```
*a**153*b**(137/2)*x**5/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*
a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*
x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a
) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**
(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 +
b*x**2/a)) + 7647*a**152*b**(139/2)*x**7/(70*a**(309/2)*b**(143/2)*sqrt(1 +
 b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(3
05/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*
sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 4
20*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2
)*x**12*sqrt(1 + b*x**2/a)) + 6323*a**151*b**(141/2)*x**9/(70*a**(309/2)*b*
*(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**
2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)
*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(
1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**
(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 2907*a**150*b**(143/2)*x**11
/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x
**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a)
 + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**
(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 +
b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 633*a**149
*b**(145/2)*x**13/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(30
7/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*s
qrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 10
50*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2
)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2
/a)) + 35*a**148*b**(147/2)*x**15/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2
/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b
**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1
+ b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(
299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12
*sqrt(1 + b*x**2/a))
```

3.518 $\int \frac{x^9}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{a^4}{7b^5\left(a+bx^2\right)^{7/2}}+\frac{4a^3}{5b^5\left(a+bx^2\right)^{5/2}}-\frac{2a^2}{b^5\left(a+bx^2\right)^{3/2}}+\frac{4a}{b^5\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^5}$$

```
[Out] -1/7*a^4/b^5/(b*x^2+a)^(7/2)+4/5*a^3/b^5/(b*x^2+a)^(5/2)-2*a^2/b^5/(b*x^2+a
)^(3/2)+4*a/b^5/(b*x^2+a)^(1/2)+(b*x^2+a)^(1/2)/b^5
```

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{a^4}{7b^5\left(a+bx^2\right)^{7/2}}+\frac{4a^3}{5b^5\left(a+bx^2\right)^{5/2}}-\frac{2a^2}{b^5\left(a+bx^2\right)^{3/2}}+\frac{4a}{b^5\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^(9/2),x]
```

```
[Out] -a^4/(7*b^5*(a + b*x^2)^(7/2)) + (4*a^3)/(5*b^5*(a + b*x^2)^(5/2)) - (2*a^2
)/(b^5*(a + b*x^2)^(3/2)) + (4*a)/(b^5*Sqrt[a + b*x^2]) + Sqrt[a + b*x^2]/b
^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{\left(a+bx^2\right)^{9/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx)^{9/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4}{b^4(a+bx)^{9/2}}-\frac{4a^3}{b^4(a+bx)^{7/2}}+\frac{6a^2}{b^4(a+bx)^{5/2}}-\frac{4a}{b^4(a+bx)^{3/2}}+\frac{1}{b^4\sqrt{a+bx}}\right)d\\
&= -\frac{a^4}{7b^5\left(a+bx^2\right)^{7/2}}+\frac{4a^3}{5b^5\left(a+bx^2\right)^{5/2}}-\frac{2a^2}{b^5\left(a+bx^2\right)^{3/2}}+\frac{4a}{b^5\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.65

$$\frac{128a^4+448a^3bx^2+560a^2b^2x^4+280ab^3x^6+35b^4x^8}{35b^5\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^(9/2),x]
```

```
[Out] (128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b^4*x^8)/(3
5*b^5*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.90, size = 102, normalized size = 1.09

$$\frac{\left(35\,b^4x^8 + 280\,ab^3x^6 + 560\,a^2b^2x^4 + 448\,a^3bx^2 + 128\,a^4\right)\sqrt{bx^2 + a}}{35\left(b^9x^8 + 4\,ab^8x^6 + 6\,a^2b^7x^4 + 4\,a^3b^6x^2 + a^4b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^4*x^8 + 280*a*b^3*x^6 + 560*a^2*b^2*x^4 + 448*a^3*b*x^2 + 128*a^
4)*sqrt(b*x^2 + a)/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 +
 a^4*b^5)
```

giac [A] time = 1.13, size = 72, normalized size = 0.77

$$\frac{\sqrt{bx^2 + a}}{b^5} + \frac{140\left(bx^2 + a\right)^3 a - 70\left(bx^2 + a\right)^2 a^2 + 28\left(bx^2 + a\right)a^3 - 5\,a^4}{35\left(bx^2 + a\right)^{\frac{7}{2}} b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)/b^5 + 1/35*(140*(b*x^2 + a)^3*a - 70*(b*x^2 + a)^2*a^2 + 28
*(b*x^2 + a)*a^3 - 5*a^4)/((b*x^2 + a)^(7/2)*b^5)
```

maple [A] time = 0.01, size = 58, normalized size = 0.62

$$\frac{35b^4x^8 + 280a\,b^3x^6 + 560a^2b^2x^4 + 448a^3b\,x^2 + 128a^4}{35\left(b\,x^2 + a\right)^{\frac{7}{2}}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/35*(35*b^4*x^8+280*a*b^3*x^6+560*a^2*b^2*x^4+448*a^3*b*x^2+128*a^4)/(b*x^
2+a)^(7/2)/b^5
```

maxima [A] time = 1.64, size = 92, normalized size = 0.98

$$\frac{x^8}{\left(bx^2 + a\right)^{\frac{7}{2}} b} + \frac{8\,ax^6}{\left(bx^2 + a\right)^{\frac{7}{2}} b^2} + \frac{16\,a^2x^4}{\left(bx^2 + a\right)^{\frac{7}{2}} b^3} + \frac{64\,a^3x^2}{5\left(bx^2 + a\right)^{\frac{7}{2}} b^4} + \frac{128\,a^4}{35\left(bx^2 + a\right)^{\frac{7}{2}} b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] x^8/((b*x^2 + a)^(7/2)*b) + 8*a*x^6/((b*x^2 + a)^(7/2)*b^2) + 16*a^2*x^4/((
b*x^2 + a)^(7/2)*b^3) + 64/5*a^3*x^2/((b*x^2 + a)^(7/2)*b^4) + 128/35*a^4/(
(b*x^2 + a)^(7/2)*b^5)
```

mupad [B] time = 4.94, size = 80, normalized size = 0.85

$$\frac{\sqrt{b\,x^2 + a}}{b^5} + \frac{4\,a}{b^5\,\sqrt{b\,x^2 + a}} - \frac{2\,a^2}{b^5\left(b\,x^2 + a\right)^{3/2}} + \frac{4\,a^3}{5\,b^5\left(b\,x^2 + a\right)^{5/2}} - \frac{a^4}{7\,b^5\left(b\,x^2 + a\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)/b^5 + (4*a)/(b^5*(a + b*x^2)^(1/2)) - (2*a^2)/(b^5*(a + b
*x^2)^(3/2)) + (4*a^3)/(5*b^5*(a + b*x^2)^(5/2)) - a^4/(7*b^5*(a + b*x^2)^(
7/2))
```

sympy [A] time = 6.39, size = 454, normalized size = 4.83

$$\begin{cases} \frac{128a^4}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}} + \frac{448a^3bx^2}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+b} \\ \frac{x^{10}}{10a^{\frac{9}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((128*a**4/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**2*sqr
t(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a + b*
x**2)) + 448*a**3*b*x**2/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**
2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a
 + b*x**2)) + 560*a**2*b**2*x**4/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*
b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**
6*sqrt(a + b*x**2)) + 280*a*b**3*x**6/(35*a**3*b**5*sqrt(a + b*x**2) + 105*
a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**
8*x**6*sqrt(a + b*x**2)) + 35*b**4*x**8/(35*a**3*b**5*sqrt(a + b*x**2) + 10
5*a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b
**8*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**10/(10*a**(9/2)), True))
```

3.519 $\int \frac{x^8}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=106

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}-\frac{x}{b^4\sqrt{a+bx^2}}-\frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}}-\frac{x^7}{7b\left(a+bx^2\right)^{7/2}}$$

```
[Out] -1/7*x^7/b/(b*x^2+a)^(7/2)-1/5*x^5/b^2/(b*x^2+a)^(5/2)-1/3*x^3/b^3/(b*x^2+a
)^(3/2)+arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)-x/b^4/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {288, 217, 206}

$$-\frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}}-\frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{x}{b^4\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}-\frac{x^7}{7b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^(9/2),x]
```

```
[Out] -x^7/(7*b*(a + b*x^2)^(7/2)) - x^5/(5*b^2*(a + b*x^2)^(5/2)) - x^3/(3*b^3*(
a + b*x^2)^(3/2)) - x/(b^4*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]]/b^(9/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{x^6}{\left(a+bx^2\right)^{7/2}}\,dx}{b}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^4}{\left(a+bx^2\right)^{5/2}}\,dx}{b^2}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} - \frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx}{b^3}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} - \frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}} - \frac{x}{b^4\sqrt{a+bx^2}} + \frac{\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^4}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} - \frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}} - \frac{x}{b^4\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x\right.}{b^4}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} - \frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}} - \frac{x}{b^4\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Mathematica [A] time = 0.13, size = 101, normalized size = 0.95

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}\sqrt{a+bx^2}} - \frac{x\left(105a^3+350a^2bx^2+406ab^2x^4+176b^3x^6\right)}{105b^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/105*(x*(105*a^3 + 350*a^2*b*x^2 + 406*a*b^2*x^4 + 176*b^3*x^6))/(b^4*(a
+ b*x^2)^(7/2)) + (Sqrt[a]*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]
)/(b^(9/2)*Sqrt[a + b*x^2])
```

fricas [A] time = 0.96, size = 331, normalized size = 3.12

$$\left[\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(176\,b^4x^7+406\,ab^3x^5\right.}{210\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqr
t(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(176*b^4*x^7 + 406
*a*b^3*x^5 + 350*a^2*b^2*x^3 + 105*a^3*b*x)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a
*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*(105*(b^4*x^8 +
 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-b)*arctan(sqrt(-b)*
x/sqrt(b*x^2 + a)) + (176*b^4*x^7 + 406*a*b^3*x^5 + 350*a^2*b^2*x^3 + 105*a
^3*b*x)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6
*x^2 + a^4*b^5)]
```

giac [A] time = 1.20, size = 78, normalized size = 0.74

$$-\frac{\left(2\left(x^2\left(\frac{88\,x^2}{b}+\frac{203\,a}{b^2}\right)+\frac{175\,a^2}{b^3}\right)x^2+\frac{105\,a^3}{b^4}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(2*(x^2*(88*x^2/b + 203*a/b^2) + 175*a^2/b^3)*x^2 + 105*a^3/b^4)*x/(
b*x^2 + a)^(7/2) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

maple [A] time = 0.02, size = 88, normalized size = 0.83

$$-\frac{x^7}{7\left(b\,x^2+a\right)^{\frac{7}{2}}b}-\frac{x^5}{5\left(b\,x^2+a\right)^{\frac{5}{2}}b^2}-\frac{x^3}{3\left(b\,x^2+a\right)^{\frac{3}{2}}b^3}-\frac{x}{\sqrt{b\,x^2+a}\,b^4}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*x^7/b/(b*x^2+a)^(7/2)-1/5*x^5/b^2/(b*x^2+a)^(5/2)-1/3*x^3/b^3/(b*x^2+a
)^(3/2)-x/b^4/(b*x^2+a)^(1/2)+1/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [B] time = 1.67, size = 255, normalized size = 2.41

$$-\frac{1}{35}\left(\frac{35\,x^6}{\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{70\,ax^4}{\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{\left(bx^2+a\right)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{\left(bx^2+a\right)^{\frac{7}{2}}b^4}\right)x-\frac{x\left(\frac{15\,x^4}{\left(bx^2+a\right)^{\frac{5}{2}}b}+\frac{20\,ax^2}{\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{\left(bx^2+a\right)^{\frac{5}{2}}b^3}\right)}{15\,b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*x - 1/15
*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2
/((b*x^2 + a)^(5/2)*b^3))/b - 1/3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*
x^2 + a)^(3/2)*b^2))/b^2 - a*x^3/((b*x^2 + a)^(5/2)*b^3) + 139/105*x/(sqrt(
b*x^2 + a)*b^4) + 17/105*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*a^2*x/((b*x^2
+ a)^(5/2)*b^4) + arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\left(b\,x^2+a\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^8/(a + b*x^2)^(9/2), x)
```

sympy [B] time = 9.04, size = 2980, normalized size = 28.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**(9/2),x)
```

```
[Out] 105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(2
05/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1
+ b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**
(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**
8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) +
 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(203/2)*b**46
*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)
*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1
575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105
/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x*
*2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)
*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b**47*x**4*sqrt(1 +
 b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x*
*2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)
*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(
1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a*
*(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x*
*12*sqrt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt(1 + b*x**2/a)*asi
nh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a*
*(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x*
*4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a)
+ 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(1
09/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b
*x**2/a)) + 1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/s
qrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(1
01/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*
x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197
/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sq
rt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 63
0*a**(195/2)*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a
**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqr
t(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100
*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)
*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/
a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 105*a**(193/2)*b
**51*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(
99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a
) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b*
*(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 +
 b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(1
93/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*b**(91/2)*x/(105*a*
*(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt
(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*
a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*
x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a
) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 665*a**101*b**(93
/2)*x**3/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(
101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b
*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(19
7/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*s
qrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 1
771*a**100*b**(95/2)*x**5/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 63
0*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2
)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2
/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b
**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1
```

```
 + b*x**2/a)) - 2549*a**99*b**(97/2)*x**7/(105*a**(205/2)*b**(99/2)*sqrt(1
+ b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(
201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6
*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) +
630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111
/2)*x**12*sqrt(1 + b*x**2/a)) - 2096*a**98*b**(99/2)*x**9/(105*a**(205/2)*b
**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**
2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)
*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(
1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a*
*(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 934*a**97*b**(101/2)*x**11/
(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x*
*2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a)
+ 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(
107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b
*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 176*a**96*
b**(103/2)*x**13/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203
/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sq
rt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 157
5*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)
*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2
/a))
```

3.520 $\int \frac{x^7}{\left(a+bx^2\right)^{9/2}} \, dx$

Optimal. Leaf size=75

$$\frac{a^3}{7b^4\left(a+bx^2\right)^{7/2}}-\frac{3a^2}{5b^4\left(a+bx^2\right)^{5/2}}+\frac{a}{b^4\left(a+bx^2\right)^{3/2}}-\frac{1}{b^4\sqrt{a+bx^2}}$$

```
[Out] 1/7*a^3/b^4/(b*x^2+a)^(7/2)-3/5*a^2/b^4/(b*x^2+a)^(5/2)+a/b^4/(b*x^2+a)^(3/
2)-1/b^4/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^3}{7b^4\left(a+bx^2\right)^{7/2}}-\frac{3a^2}{5b^4\left(a+bx^2\right)^{5/2}}+\frac{a}{b^4\left(a+bx^2\right)^{3/2}}-\frac{1}{b^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(9/2),x]
```

```
[Out] a^3/(7*b^4*(a + b*x^2)^(7/2)) - (3*a^2)/(5*b^4*(a + b*x^2)^(5/2)) + a/(b^4*
(a + b*x^2)^(3/2)) - 1/(b^4*Sqrt[a + b*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a+bx^2\right)^{9/2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{9/2}} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{9/2}}+\frac{3a^2}{b^3(a+bx)^{7/2}}-\frac{3a}{b^3(a+bx)^{5/2}}+\frac{1}{b^3(a+bx)^{3/2}}\right) dx, x, x^2\right)\\
&= \frac{a^3}{7b^4\left(a+bx^2\right)^{7/2}}-\frac{3a^2}{5b^4\left(a+bx^2\right)^{5/2}}+\frac{a}{b^4\left(a+bx^2\right)^{3/2}}-\frac{1}{b^4\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.67

$$\frac{-16a^3-56a^2bx^2-70ab^2x^4-35b^3x^6}{35b^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(9/2),x]
```

```
[Out] (-16*a^3 - 56*a^2*b*x^2 - 70*a*b^2*x^4 - 35*b^3*x^6)/(35*b^4*(a + b*x^2)^(7
/2))
```

fricas [A] time = 0.86, size = 91, normalized size = 1.21

$$-\frac{\left(35\,b^3x^6+70\,ab^2x^4+56\,a^2bx^2+16\,a^3\right)\sqrt{bx^2+a}}{35\left(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 70*a*b^2*x^4 + 56*a^2*b*x^2 + 16*a^3)*sqrt(b*x^2 + a)/(
b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

giac [A] time = 0.98, size = 55, normalized size = 0.73

$$-\frac{35\left(bx^2+a\right)^3-35\left(bx^2+a\right)^2a+21\left(bx^2+a\right)a^2-5\,a^3}{35\left(bx^2+a\right)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(35*(b*x^2 + a)^3 - 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 - 5*a^3)/
((b*x^2 + a)^(7/2)*b^4)
```

maple [A] time = 0.01, size = 47, normalized size = 0.63

$$-\frac{35b^3x^6+70a\,b^2x^4+56a^2b\,x^2+16a^3}{35\left(b\,x^2+a\right)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/35*(35*b^3*x^6+70*a*b^2*x^4+56*a^2*b*x^2+16*a^3)/(b*x^2+a)^(7/2)/b^4
```

maxima [A] time = 1.36, size = 73, normalized size = 0.97

$$-\frac{x^6}{\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{2\,ax^4}{\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{8\,a^2x^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b^3}-\frac{16\,a^3}{35\left(bx^2+a\right)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -x^6/((b*x^2 + a)^(7/2)*b) - 2*a*x^4/((b*x^2 + a)^(7/2)*b^2) - 8/5*a^2*x^2/
((b*x^2 + a)^(7/2)*b^3) - 16/35*a^3/((b*x^2 + a)^(7/2)*b^4)
```

mupad [B] time = 4.88, size = 63, normalized size = 0.84

$$\frac{a}{b^4\left(b\,x^2+a\right)^{3/2}}-\frac{1}{b^4\sqrt{b\,x^2+a}}-\frac{3\,a^2}{5\,b^4\left(b\,x^2+a\right)^{5/2}}+\frac{a^3}{7\,b^4\left(b\,x^2+a\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^(9/2),x)
```

```
[Out] a/(b^4*(a + b*x^2)^(3/2)) - 1/(b^4*(a + b*x^2)^(1/2)) - (3*a^2)/(5*b^4*(a +
 b*x^2)^(5/2)) + a^3/(7*b^4*(a + b*x^2)^(7/2))
```

sympy [A] time = 6.08, size = 364, normalized size = 4.85

$$\begin{cases} -\frac{16a^3}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+35b^7x^6\sqrt{a+bx^2}} - \frac{56a^2bx^2}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+35b^7x^6\sqrt{a+bx^2}} \\ \frac{x^8}{8a^{\frac{9}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*sqr
t(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a + b*
x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2
*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a
+ b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*
x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqr
t(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**
5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*s
qrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True))
```

3.521 $\int \frac{x^6}{(a+bx^2)^{9/2}}\, dx$

Optimal. Leaf size=21

$$\frac{x^7}{7a\left(a+bx^2\right)^{7/2}}$$

```
[Out] 1/7*x^7/a/(b*x^2+a)^(7/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{x^7}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(9/2),x]
[Out] x^7/(7*a*(a + b*x^2)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^{9/2}}\, dx = \frac{x^7}{7a\left(a+bx^2\right)^{7/2}}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{x^7}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(9/2),x]
[Out] x^7/(7*a*(a + b*x^2)^(7/2))
```

fricas [B] time = 0.62, size = 59, normalized size = 2.81

$$\frac{\sqrt{bx^2+a}\, x^7}{7\left(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="fricas")
[Out] 1/7*sqrt(b*x^2 + a)*x^7/(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*
b*x^2 + a^5)
```

giac [A] time = 1.22, size = 17, normalized size = 0.81

$$\frac{x^7}{7\left(bx^2+a\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/7*x^7/((b*x^2 + a)^(7/2)*a)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{x^7}{7\left(b\,x^2+a\right)^{\frac{7}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7*x^7/a/(b*x^2+a)^(7/2)
```

maxima [B] time = 1.33, size = 103, normalized size = 4.90

$$-\frac{x^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{5\,ax^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{x}{14\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{x}{7\sqrt{bx^2+a}\,ab^3}+\frac{3\,ax}{56\left(bx^2+a\right)^{\frac{5}{2}}b^3}-\frac{15\,a^2x}{56\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*a*x^3/((b*x^2 + a)^(7/2)*b^2) + 1/14*x
/((b*x^2 + a)^(3/2)*b^3) + 1/7*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*a*x/((b*x^2
 + a)^(5/2)*b^3) - 15/56*a^2*x/((b*x^2 + a)^(7/2)*b^3)
```

mupad [B] time = 4.76, size = 68, normalized size = 3.24

$$\frac{x}{7\,a\,b^3\,\sqrt{b\,x^2+a}}-\frac{3\,x}{7\,b^3\,\left(b\,x^2+a\right)^{3/2}}-\frac{a^2\,x}{7\,b^3\,\left(b\,x^2+a\right)^{7/2}}+\frac{3\,a\,x}{7\,b^3\,\left(b\,x^2+a\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(9/2),x)
```

```
[Out] x/(7*a*b^3*(a + b*x^2)^(1/2)) - (3*x)/(7*b^3*(a + b*x^2)^(3/2)) - (a^2*x)/(
7*b^3*(a + b*x^2)^(7/2)) + (3*a*x)/(7*b^3*(a + b*x^2)^(5/2))
```

sympy [B] time = 1.47, size = 95, normalized size = 4.52

$$\frac{x^7}{7a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+21a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+21a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+7a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(9/2),x)
```

```
[Out] x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 +
 b*x**2/a))
```

3.522 $\int \frac{x^5}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{a^2}{7b^3\left(a+bx^2\right)^{7/2}}+\frac{2a}{5b^3\left(a+bx^2\right)^{5/2}}-\frac{1}{3b^3\left(a+bx^2\right)^{3/2}}$$

```
[Out] -1/7*a^2/b^3/(b*x^2+a)^(7/2)+2/5*a/b^3/(b*x^2+a)^(5/2)-1/3/b^3/(b*x^2+a)^(3
/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{a^2}{7b^3\left(a+bx^2\right)^{7/2}}+\frac{2a}{5b^3\left(a+bx^2\right)^{5/2}}-\frac{1}{3b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(9/2),x]
```

```
[Out] -a^2/(7*b^3*(a + b*x^2)^(7/2)) + (2*a)/(5*b^3*(a + b*x^2)^(5/2)) - 1/(3*b^3
*(a + b*x^2)^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^{9/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{9/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{9/2}}-\frac{2a}{b^2(a+bx)^{7/2}}+\frac{1}{b^2(a+bx)^{5/2}}\right)dx,x,x^2\right)\\
&= -\frac{a^2}{7b^3\left(a+bx^2\right)^{7/2}}+\frac{2a}{5b^3\left(a+bx^2\right)^{5/2}}-\frac{1}{3b^3\left(a+bx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{-8a^2-28abx^2-35b^2x^4}{105b^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^2 - 28*a*b*x^2 - 35*b^2*x^4)/(105*b^3*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.88, size = 80, normalized size = 1.36

$$-\frac{\left(35\,b^2x^4+28\,abx^2+8\,a^2\right)\sqrt{bx^2+a}}{105\left(b^7x^8+4\,ab^6x^6+6\,a^2b^5x^4+4\,a^3b^4x^2+a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^2*x^4 + 28*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/(b^7*x^8 + 4*a*b^6
*x^6 + 6*a^2*b^5*x^4 + 4*a^3*b^4*x^2 + a^4*b^3)
```

giac [A] time = 0.95, size = 41, normalized size = 0.69

$$-\frac{35\left(bx^2+a\right)^2-42\left(bx^2+a\right)a+15\,a^2}{105\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(35*(b*x^2 + a)^2 - 42*(b*x^2 + a)*a + 15*a^2)/((b*x^2 + a)^(7/2)*b^
3)
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$-\frac{35b^2x^4+28ab\,x^2+8a^2}{105\left(b\,x^2+a\right)^{\frac{7}{2}}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(35*b^2*x^4+28*a*b*x^2+8*a^2)/(b*x^2+a)^(7/2)/b^3
```

maxima [A] time = 1.41, size = 53, normalized size = 0.90

$$-\frac{x^4}{3\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{4\,ax^2}{15\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{8\,a^2}{105\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/3*x^4/((b*x^2 + a)^(7/2)*b) - 4/15*a*x^2/((b*x^2 + a)^(7/2)*b^2) - 8/105
*a^2/((b*x^2 + a)^(7/2)*b^3)
```

mupad [B] time = 4.81, size = 41, normalized size = 0.69

$$-\frac{35\left(b\,x^2+a\right)^2-42\,a\,\left(b\,x^2+a\right)+15\,a^2}{105\,b^3\left(b\,x^2+a\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(9/2),x)
```

```
[Out] -(35*(a + b*x^2)^2 - 42*a*(a + b*x^2) + 15*a^2)/(105*b^3*(a + b*x^2)^(7/2))
```

sympy [A] time = 5.91, size = 272, normalized size = 4.61

$$\begin{cases} -\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{28abx^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105 \ldots} \\ \frac{x^6}{6a^{\frac{9}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqr
t(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b
*x**2)) - 28*a*b*x**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*
sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a
+ b*x**2)) - 35*b**2*x**4/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x
**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqr
t(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(9/2)), True))
```

3.523 $\int \frac{x^4}{(a+bx^2)^{9/2}} \, dx$

Optimal. Leaf size=44

$$\frac{2bx^7}{35a^2(a+bx^2)^{7/2}} + \frac{x^5}{5a(a+bx^2)^{7/2}}$$

```
[Out] 1/5*x^5/a/(b*x^2+a)^(7/2)+2/35*b*x^7/a^2/(b*x^2+a)^(7/2)
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{2bx^7}{35a^2(a+bx^2)^{7/2}} + \frac{x^5}{5a(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(9/2),x]
```

```
[Out] x^5/(5*a*(a + b*x^2)^(7/2)) + (2*b*x^7)/(35*a^2*(a + b*x^2)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(a+bx^2)^{9/2}} \, dx &= \frac{x^5}{5a(a+bx^2)^{7/2}} + \frac{(2b) \int \frac{x^6}{(a+bx^2)^{9/2}} \, dx}{5a} \\
&= \frac{x^5}{5a(a+bx^2)^{7/2}} + \frac{2bx^7}{35a^2(a+bx^2)^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{7ax^5 + 2bx^7}{35a^2(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(9/2),x]
```

```
[Out] (7*a*x^5 + 2*b*x^7)/(35*a^2*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.59, size = 71, normalized size = 1.61

$$\frac{\left(2\,bx^7+7\,ax^5\right)\sqrt{bx^2+a}}{35\,\left(a^2b^4x^8+4\,a^3b^3x^6+6\,a^4b^2x^4+4\,a^5bx^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(2*b*x^7 + 7*a*x^5)*sqrt(b*x^2 + a)/(a^2*b^4*x^8 + 4*a^3*b^3*x^6 + 6*a
^4*b^2*x^4 + 4*a^5*b*x^2 + a^6)
```

giac [A] time = 0.99, size = 29, normalized size = 0.66

$$\frac{x^5\left(\frac{2\,bx^2}{a^2}+\frac{7}{a}\right)}{35\,\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*x^5*(2*b*x^2/a^2 + 7/a)/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.00, size = 28, normalized size = 0.64

$$\frac{\left(2b\,x^2+7a\right)x^5}{35\left(b\,x^2+a\right)^{\frac{7}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/35*x^5*(2*b*x^2+7*a)/(b*x^2+a)^(7/2)/a^2
```

maxima [B] time = 1.38, size = 85, normalized size = 1.93

$$-\frac{x^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{3\,x}{140\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{2\,x}{35\,\sqrt{bx^2+a}\,a^2b^2}+\frac{x}{35\left(bx^2+a\right)^{\frac{3}{2}}ab^2}-\frac{3\,ax}{28\left(bx^2+a\right)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/((b*x^2 + a)^(7/2)*b) + 3/140*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*x/(
sqrt(b*x^2 + a)*a^2*b^2) + 1/35*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*a*x/((b*
x^2 + a)^(7/2)*b^2)
```

mupad [B] time = 4.88, size = 68, normalized size = 1.55

$$\frac{2\,x}{35\,a^2\,b^2\,\sqrt{b\,x^2+a}}-\frac{8\,x}{35\,b^2\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{x}{35\,a\,b^2\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{a\,x}{7\,b^2\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(9/2),x)
```

```
[Out] (2*x)/(35*a^2*b^2*(a + b*x^2)^(1/2)) - (8*x)/(35*b^2*(a + b*x^2)^(5/2)) + x
/(35*a*b^2*(a + b*x^2)^(3/2)) + (a*x)/(7*b^2*(a + b*x^2)^(7/2))
```

sympy [B] time = 1.60, size = 199, normalized size = 4.52

$$\frac{7ax^5}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}+\frac{}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(9/2),x)
```

```
[Out] 7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*
x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6
*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(
9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a)
+ 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))
```

3.524 $\int \frac{x^3}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=38

$$\frac{a}{7b^2\left(a+bx^2\right)^{7/2}}-\frac{1}{5b^2\left(a+bx^2\right)^{5/2}}$$

```
[Out] 1/7*a/b^2/(b*x^2+a)^(7/2)-1/5/b^2/(b*x^2+a)^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a}{7b^2\left(a+bx^2\right)^{7/2}}-\frac{1}{5b^2\left(a+bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(9/2),x]
```

```
[Out] a/(7*b^2*(a + b*x^2)^(7/2)) - 1/(5*b^2*(a + b*x^2)^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^{9/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{9/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{9/2}}+\frac{1}{b(a+bx)^{7/2}}\right)dx,x,x^2\right)\\
&= \frac{a}{7b^2\left(a+bx^2\right)^{7/2}}-\frac{1}{5b^2\left(a+bx^2\right)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{-2a-7bx^2}{35b^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(9/2),x]
```

```
[Out] (-2*a - 7*b*x^2)/(35*b^2*(a + b*x^2)^(7/2))
```

fricas [B] time = 0.85, size = 69, normalized size = 1.82

$$-\frac{\left(7\,bx^{2}+2\,a\right)\sqrt{bx^{2}+a}}{35\,\left(b^{6}x^{8}+4\,ab^{5}x^{6}+6\,a^{2}b^{4}x^{4}+4\,a^{3}b^{3}x^{2}+a^{4}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(7*b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x^
4 + 4*a^3*b^3*x^2 + a^4*b^2)
```

giac [A] time = 0.83, size = 24, normalized size = 0.63

$$-\frac{7\,bx^{2}+2\,a}{35\,\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(7*b*x^2 + 2*a)/((b*x^2 + a)^(7/2)*b^2)
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{7b\,x^{2}+2a}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/35*(7*b*x^2+2*a)/(b*x^2+a)^(7/2)/b^2
```

maxima [A] time = 1.30, size = 33, normalized size = 0.87

$$-\frac{x^{2}}{5\left(bx^{2}+a\right)^{\frac{7}{2}}b}-\frac{2\,a}{35\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/5*x^2/((b*x^2 + a)^(7/2)*b) - 2/35*a/((b*x^2 + a)^(7/2)*b^2)
```

mupad [B] time = 4.82, size = 24, normalized size = 0.63

$$-\frac{7\,b\,x^{2}+2\,a}{35\,b^{2}\,\left(b\,x^{2}+a\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(9/2),x)
```

```
[Out] -(2*a + 7*b*x^2)/(35*b^2*(a + b*x^2)^(7/2))
```

sympy [A] time = 5.86, size = 180, normalized size = 4.74

$$\begin{cases} -\frac{2a}{35a^{3}b^{2}\sqrt{a+bx^{2}}+105a^{2}b^{3}x^{2}\sqrt{a+bx^{2}}+105ab^{4}x^{4}\sqrt{a+bx^{2}}+35b^{5}x^{6}\sqrt{a+bx^{2}}}-\frac{7bx^{2}}{35a^{3}b^{2}\sqrt{a+bx^{2}}+105a^{2}b^{3}x^{2}\sqrt{a+bx^{2}}+105ab^{4}x^{4}\sqrt{a+bx^{2}}+35b^{5}x^{6}\sqrt{a+} \\ \frac{x^{4}}{4a^{\frac{9}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a
+ b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2
)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a +
b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2))
, Ne(b, 0)), (x**4/(4*a**(9/2)), True))
```

3.525 $\int \frac{x^2}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=68

$$\frac{8b^2x^7}{105a^3\left(a+bx^2\right)^{7/2}}+\frac{4bx^5}{15a^2\left(a+bx^2\right)^{7/2}}+\frac{x^3}{3a\left(a+bx^2\right)^{7/2}}$$

```
[Out] 1/3*x^3/a/(b*x^2+a)^(7/2)+4/15*b*x^5/a^2/(b*x^2+a)^(7/2)+8/105*b^2*x^7/a^3/
(b*x^2+a)^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{8b^2x^7}{105a^3\left(a+bx^2\right)^{7/2}}+\frac{4bx^5}{15a^2\left(a+bx^2\right)^{7/2}}+\frac{x^3}{3a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(9/2),x]
```

```
[Out] x^3/(3*a*(a + b*x^2)^(7/2)) + (4*b*x^5)/(15*a^2*(a + b*x^2)^(7/2)) + (8*b^2
*x^7)/(105*a^3*(a + b*x^2)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a+bx^2\right)^{9/2}}\,dx &= \frac{x^3}{3a\left(a+bx^2\right)^{7/2}}+\frac{(4b)\int \frac{x^4}{\left(a+bx^2\right)^{9/2}}\,dx}{3a}\\
&= \frac{x^3}{3a\left(a+bx^2\right)^{7/2}}+\frac{4bx^5}{15a^2\left(a+bx^2\right)^{7/2}}+\frac{\left(8b^2\right)\int \frac{x^6}{\left(a+bx^2\right)^{9/2}}\,dx}{15a^2}\\
&= \frac{x^3}{3a\left(a+bx^2\right)^{7/2}}+\frac{4bx^5}{15a^2\left(a+bx^2\right)^{7/2}}+\frac{8b^2x^7}{105a^3\left(a+bx^2\right)^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$\frac{x^3\left(35a^2+28abx^2+8b^2x^4\right)}{105a^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(9/2),x]
```

```
[Out] (x^3*(35*a^2 + 28*a*b*x^2 + 8*b^2*x^4))/(105*a^3*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.81, size = 82, normalized size = 1.21

$$\frac{\left(8\,b^2x^7 + 28\,abx^5 + 35\,a^2x^3\right)\sqrt{bx^2 + a}}{105\,\left(a^3b^4x^8 + 4\,a^4b^3x^6 + 6\,a^5b^2x^4 + 4\,a^6bx^2 + a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*b^2*x^7 + 28*a*b*x^5 + 35*a^2*x^3)*sqrt(b*x^2 + a)/(a^3*b^4*x^8 +
4*a^4*b^3*x^6 + 6*a^5*b^2*x^4 + 4*a^6*b*x^2 + a^7)
```

giac [A] time = 1.08, size = 43, normalized size = 0.63

$$\frac{\left(4\,x^2\left(\frac{2\,b^2x^2}{a^3} + \frac{7\,b}{a^2}\right) + \frac{35}{a}\right)x^3}{105\,\left(bx^2 + a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(4*x^2*(2*b^2*x^2/a^3 + 7*b/a^2) + 35/a)*x^3/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$\frac{\left(8b^2x^4 + 28ab\,x^2 + 35a^2\right)x^3}{105\left(b\,x^2 + a\right)^{\frac{7}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*x^3*(8*b^2*x^4+28*a*b*x^2+35*a^2)/(b*x^2+a)^(7/2)/a^3
```

maxima [A] time = 1.39, size = 70, normalized size = 1.03

$$-\frac{x}{7\left(bx^2 + a\right)^{\frac{7}{2}}b} + \frac{8\,x}{105\,\sqrt{bx^2 + a}\,a^3b} + \frac{4\,x}{105\left(bx^2 + a\right)^{\frac{3}{2}}a^2b} + \frac{x}{35\left(bx^2 + a\right)^{\frac{5}{2}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/7*x/((b*x^2 + a)^(7/2)*b) + 8/105*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*x/((
b*x^2 + a)^(3/2)*a^2*b) + 1/35*x/((b*x^2 + a)^(5/2)*a*b)
```

mupad [B] time = 4.76, size = 70, normalized size = 1.03

$$\frac{8\,x}{105\,a^3\,b\,\sqrt{b\,x^2 + a}} - \frac{x}{7\,b\,\left(b\,x^2 + a\right)^{7/2}} + \frac{4\,x}{105\,a^2\,b\,\left(b\,x^2 + a\right)^{3/2}} + \frac{x}{35\,a\,b\,\left(b\,x^2 + a\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(9/2),x)
```

```
[Out] (8*x)/(105*a^3*b*(a + b*x^2)^(1/2)) - x/(7*b*(a + b*x^2)^(7/2)) + (4*x)/(10
5*a^2*b*(a + b*x^2)^(3/2)) + x/(35*a*b*(a + b*x^2)^(5/2))
```

sympy [B] time = 1.87, size = 517, normalized size = 7.60

$$\frac{35a^5x^3}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] 35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(
1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a))
+ 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*s
qrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13
/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/
a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b
*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420
*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 +
b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(1
7/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a)
 + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqr
t(1 + b*x**2/a))
```

3.526 $\int \frac{x}{(a+bx^2)^{9/2}} dx$

Optimal. Leaf size=18

$$-\frac{1}{7b(a+bx^2)^{7/2}}$$

```
[Out] -1/7/b/(b*x^2+a)^(7/2)
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{7b(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/(7*b*(a + b*x^2)^(7/2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^{9/2}} dx = -\frac{1}{7b(a+bx^2)^{7/2}}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{7b(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*1/(b*(a + b*x^2)^(7/2))
```

fricas [B] time = 0.87, size = 57, normalized size = 3.17

$$-\frac{\sqrt{bx^2+a}}{7(b^5x^8+4ab^4x^6+6a^2b^3x^4+4a^3b^2x^2+a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/7*sqrt(b*x^2 + a)/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2
 + a^4*b)
```

giac [A] time = 0.97, size = 14, normalized size = 0.78

$$-\frac{1}{7\left(bx^2+a\right)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/7/((b*x^2 + a)^(7/2)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{1}{7\left(b\,x^2+a\right)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7/b/(b*x^2+a)^(7/2)
```

maxima [A] time = 1.33, size = 14, normalized size = 0.78

$$-\frac{1}{7\left(bx^2+a\right)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/7/((b*x^2 + a)^(7/2)*b)
```

mupad [B] time = 4.60, size = 14, normalized size = 0.78

$$-\frac{1}{7\,b\,\left(b\,x^2+a\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(9/2),x)
```

```
[Out] -1/(7*b*(a + b*x^2)^(7/2))
```

sympy [A] time = 5.73, size = 90, normalized size = 5.00

$$\begin{cases} -\frac{1}{7a^3b\sqrt{a+bx^2}+21a^2b^2x^2\sqrt{a+bx^2}+21ab^3x^4\sqrt{a+bx^2}+7b^4x^6\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*x**
2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), Ne(b,
 0)), (x**2/(2*a**(9/2)), True))
```

3.527 $\int \frac{1}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=77

$$\frac{16x}{35a^4\sqrt{a+bx^2}}+\frac{8x}{35a^3\left(a+bx^2\right)^{3/2}}+\frac{6x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}$$

```
[Out] 1/7*x/a/(b*x^2+a)^(7/2)+6/35*x/a^2/(b*x^2+a)^(5/2)+8/35*x/a^3/(b*x^2+a)^(3/
2)+16/35*x/a^4/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{16x}{35a^4\sqrt{a+bx^2}}+\frac{8x}{35a^3\left(a+bx^2\right)^{3/2}}+\frac{6x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-9/2),x]
```

```
[Out] x/(7*a*(a + b*x^2)^(7/2)) + (6*x)/(35*a^2*(a + b*x^2)^(5/2)) + (8*x)/(35*a^
3*(a + b*x^2)^(3/2)) + (16*x)/(35*a^4*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^{9/2}}\,dx &= \frac{x}{7a\left(a+bx^2\right)^{7/2}}+\frac{6\int \frac{1}{\left(a+bx^2\right)^{7/2}}\,dx}{7a}\\
&= \frac{x}{7a\left(a+bx^2\right)^{7/2}}+\frac{6x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{24\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2}\\
&= \frac{x}{7a\left(a+bx^2\right)^{7/2}}+\frac{6x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{8x}{35a^3\left(a+bx^2\right)^{3/2}}+\frac{16\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{35a^3}\\
&= \frac{x}{7a\left(a+bx^2\right)^{7/2}}+\frac{6x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{8x}{35a^3\left(a+bx^2\right)^{3/2}}+\frac{16x}{35a^4\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.66

$$\frac{x\left(35a^3+70a^2bx^2+56ab^2x^4+16b^3x^6\right)}{35a^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-9/2),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3*x^6))/(35*a^4*(a + b*x^2)
^(7/2))
```

fricas [A] time = 0.86, size = 91, normalized size = 1.18

$$\frac{\left(16\,b^3x^7+56\,ab^2x^5+70\,a^2bx^3+35\,a^3x\right)\sqrt{bx^2+a}}{35\,\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*b^3*x^7 + 56*a*b^2*x^5 + 70*a^2*b*x^3 + 35*a^3*x)*sqrt(b*x^2 + a)/
(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

giac [A] time = 1.12, size = 55, normalized size = 0.71

$$\frac{\left(2\left(4\,x^2\left(\frac{2\,b^3x^2}{a^4}+\frac{7\,b^2}{a^3}\right)+\frac{35\,b}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(2*(4*x^2*(2*b^3*x^2/a^4 + 7*b^2/a^3) + 35*b/a^2)*x^2 + 35/a)*x/(b*x^2
 + a)^(7/2)
```

maple [A] time = 0.00, size = 48, normalized size = 0.62

$$\frac{\left(16b^3x^6+56a\,b^2x^4+70a^2b\,x^2+35a^3\right)x}{35\left(b\,x^2+a\right)^{\frac{7}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/35*x*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/(b*x^2+a)^(7/2)/a^4
```

maxima [A] time = 1.37, size = 61, normalized size = 0.79

$$\frac{16\,x}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,x}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6\,x}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{x}{7\left(bx^2+a\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*x/(sqrt(b*x^2 + a)*a^4) + 8/35*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*x/((b
*x^2 + a)^(5/2)*a^2) + 1/7*x/((b*x^2 + a)^(7/2)*a)
```

mupad [B] time = 4.60, size = 61, normalized size = 0.79

$$\frac{16\,x}{35\,a^4\,\sqrt{b\,x^2+a}}+\frac{8\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{6\,x}{35\,a^2\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{x}{7\,a\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(9/2),x)
```

```
[Out] (16*x)/(35*a^4*(a + b*x^2)^(1/2)) + (8*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*x
)/(35*a^2*(a + b*x^2)^(5/2)) + x/(7*a*(a + b*x^2)^(7/2))
```

sympy [B] time = 2.19, size = 1265, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(9/2),x)
```

```
[Out] 35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 +
 b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**
3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 21
0*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1
+ b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(
35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a
) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sq
rt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/
2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*
x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**
2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6
*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(
27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x*
*2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/
2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(
1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*
b**6*x**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 +
 b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x
**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a
**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b
*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13
/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqr
t(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2
)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a
))
```

3.528 $\int \frac{1}{x\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{1}{a^4\sqrt{a+bx^2}}+\frac{1}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}+\frac{1}{7a\left(a+bx^2\right)^{7/2}}$$

```
[Out] 1/7/a/(b*x^2+a)^(7/2)+1/5/a^2/(b*x^2+a)^(5/2)+1/3/a^3/(b*x^2+a)^(3/2)-arcta
nh((b*x^2+a)^(1/2)/a^(1/2))/a^(9/2)+1/a^4/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$\frac{1}{a^4\sqrt{a+bx^2}}+\frac{1}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{1}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] 1/(7*a*(a + b*x^2)^(7/2)) + 1/(5*a^2*(a + b*x^2)^(5/2)) + 1/(3*a^3*(a + b*x
^2)^(3/2)) + 1/(a^4*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(
9/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{9/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{7a\left(a+bx^2\right)^{7/2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{7/2}}\,dx, x, x^2\right)}{2a}$$
$$= \frac{1}{7a\left(a+bx^2\right)^{7/2}} + \frac{1}{5a^2\left(a+bx^2\right)^{5/2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, x^2\right)}{2a^2}$$
$$= \frac{1}{7a\left(a+bx^2\right)^{7/2}} + \frac{1}{5a^2\left(a+bx^2\right)^{5/2}} + \frac{1}{3a^3\left(a+bx^2\right)^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right)}{2a^3}$$
$$= \frac{1}{7a\left(a+bx^2\right)^{7/2}} + \frac{1}{5a^2\left(a+bx^2\right)^{5/2}} + \frac{1}{3a^3\left(a+bx^2\right)^{3/2}} + \frac{1}{a^4\sqrt{a+bx^2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\ldots\right)}{2a^4}$$
$$= \frac{1}{7a\left(a+bx^2\right)^{7/2}} + \frac{1}{5a^2\left(a+bx^2\right)^{5/2}} + \frac{1}{3a^3\left(a+bx^2\right)^{3/2}} + \frac{1}{a^4\sqrt{a+bx^2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\ldots\right)}{a\ldots}$$
$$= \frac{1}{7a\left(a+bx^2\right)^{7/2}} + \frac{1}{5a^2\left(a+bx^2\right)^{5/2}} + \frac{1}{3a^3\left(a+bx^2\right)^{3/2}} + \frac{1}{a^4\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.38

$$\frac{{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(9/2)),x]
[Out] Hypergeometric2F1[-7/2, 1, -5/2, 1 + (b*x^2)/a]/(7*a*(a + b*x^2)^(7/2))
```

fricas [B] time = 0.82, size = 329, normalized size = 3.46

$$\left[\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(105\,ab^3x^6+350\,a^2b^2x^4+4\ldots\right.}{210\left(a^5b^4x^8+4\,a^6b^3x^6+6\,a^7b^2x^4+4\,a^8bx^2+a^9\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="fricas")
[Out] [1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqr
t(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(105*a*b^3*x^6
 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^2 + a))/(a^5*b^4*x^8
 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 + 4*a^8*b*x^2 + a^9), 1/105*(105*(b^4*x^8
+ 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a)*arctan(sqrt(-a)
/sqrt(b*x^2 + a)) + (105*a*b^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*
a^4)*sqrt(b*x^2 + a))/(a^5*b^4*x^8 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 + 4*a^8*
b*x^2 + a^9)]
```

giac [A] time = 1.08, size = 81, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4}+\frac{105\left(bx^2+a\right)^3+35\left(bx^2+a\right)^2a+21\left(bx^2+a\right)a^2+15\,a^3}{105\left(bx^2+a\right)^{\frac{7}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) + 1/105*(105*(b*x^2 + a)^3
+ 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 15*a^3)/((b*x^2 + a)^(7/2)*a^4)
```

maple [A] time = 0.01, size = 85, normalized size = 0.89

$$\frac{1}{7\left(b\,x^2+a\right)^{\frac{7}{2}}a}+\frac{1}{5\left(b\,x^2+a\right)^{\frac{5}{2}}a^2}+\frac{1}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a^3}-\frac{\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{a^{\frac{9}{2}}}+\frac{1}{\sqrt{b\,x^2+a}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7/a/(b*x^2+a)^(7/2)+1/5/a^2/(b*x^2+a)^(5/2)+1/3/a^3/(b*x^2+a)^(3/2)+1/a^4
/(b*x^2+a)^(1/2)-1/a^(9/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.36, size = 73, normalized size = 0.77

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{9}{2}}}+\frac{1}{\sqrt{bx^2+a}\,a^4}+\frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{1}{5\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{1}{7\left(bx^2+a\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 1/(sqrt(b*x^2 + a)*a^4) + 1/3/((b*
x^2 + a)^(3/2)*a^3) + 1/5/((b*x^2 + a)^(5/2)*a^2) + 1/7/((b*x^2 + a)^(7/2)*
a)
```

mupad [B] time = 4.85, size = 75, normalized size = 0.79

$$\frac{\frac{b\,x^2+a}{5\,a^2}+\frac{1}{7\,a}+\frac{\left(b\,x^2+a\right)^2}{3\,a^3}+\frac{\left(b\,x^2+a\right)^3}{a^4}}{\left(b\,x^2+a\right)^{7/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(9/2)),x)
```

```
[Out] ((a + b*x^2)/(5*a^2) + 1/(7*a) + (a + b*x^2)^2/(3*a^3) + (a + b*x^2)^3/a^4)
/(a + b*x^2)^(7/2) - atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(9/2)
```

sympy [B] time = 7.90, size = 5250, normalized size = 55.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(9/2),x)
```

```
[Out] 352*a**32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*
a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8
 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2
)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*
a**(53/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(
71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100
*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x*
*12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/
2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt(1 + b*x**2/
a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 +
 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b*
*5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a
**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20
) + 2924*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*
x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/
2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25
200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*
x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2*log(b*x**2/a)/(210*a
**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/
2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 441
00*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*
x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 2100*a**31
*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b*
*4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) + 10852*a**30*b**2*x**4*sqrt(1 + b*x**2/a)/(2
10*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**
(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 +
 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b
**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 4725*a
**30*b**2*x**4*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*
a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8
 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2
)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*
a**(53/2)*b**10*x**20) - 9450*a**30*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(
210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a*
*(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10
 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*
b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 23630
*a**29*b**3*x**6*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b*
*4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) + 12600*a**29*b**3*x**6*log(b*x**2/a)/(210*a*
*(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2
)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 4410
0*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x
**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 25200*a**29
*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x*
*2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)
*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2520
0*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x*
*18 + 210*a**(53/2)*b**10*x**20) + 33280*a**28*b**4*x**8*sqrt(1 + b*x**2/a)
/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*
a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**1
0 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2
)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 220
```

```
50*a**28*b**4*x**8*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9
450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*
x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(
59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 +
210*a**(53/2)*b**10*x**20) - 44100*a**28*b**4*x**8*log(sqrt(1 + b*x**2/a) +
 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 252
00*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x
**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(5
7/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) +
31442*a**27*b**5*x**10*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b
*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65
/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2
5200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9
*x**18 + 210*a**(53/2)*b**10*x**20) + 26460*a**27*b**5*x**10*log(b*x**2/a)/
(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a
**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10
 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)
*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 5292
0*a**27*b**5*x**10*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71
/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a
**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**1
2 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)
*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 19924*a**26*b**6*x**12*sqrt(1 +
b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4
 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*
b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450
*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**
20) + 22050*a**26*b**6*x**12*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*
b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(6
5/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 +
25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**
9*x**18 + 210*a**(53/2)*b**10*x**20) - 44100*a**26*b**6*x**12*log(sqrt(1 +
b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2
*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(6
3/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 +
 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**1
0*x**20) + 8162*a**25*b**7*x**14*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a
**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44
100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6
*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(
55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 12600*a**25*b**7*x**14*log(
b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4
 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*
b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450
*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**
20) - 25200*a**25*b**7*x**14*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2
100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6
 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)
*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100
*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1960*a**24*b**8*x**16*
sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*
b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a
**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**
14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*
b**10*x**20) + 4725*a**24*b**8*x**16*log(b*x**2/a)/(210*a**(73/2) + 2100*a*
*(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 441
00*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*
x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(5
5/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 9450*a**24*b**8*x**16*log(sq
```

```
rt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/
2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 5292
0*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*
x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/
2)*b**10*x**20) + 210*a**23*b**9*x**18*sqrt(1 + b*x**2/a)/(210*a**(73/2) +
2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**
6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2
)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 210
0*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**23*b**9*x**18
*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2
*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(6
3/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 +
 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**1
0*x**20) - 2100*a**23*b**9*x**18*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2)
 + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*
x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(6
1/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 105*a**22*b**10*x*
*20*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b
**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a*
*(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**1
4 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b
**10*x**20) - 210*a**22*b**10*x**20*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73
/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b*
*3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a*
*(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16
 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20)
```

3.529 $\int \frac{1}{x^2(a+bx^2)^{9/2}} dx$

Optimal. Leaf size=100

$$-\frac{128bx}{35a^5\sqrt{a+bx^2}}-\frac{64bx}{35a^4\left(a+bx^2\right)^{3/2}}-\frac{48bx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{8bx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}$$

```
[Out] -1/a/x/(b*x^2+a)^(7/2)-8/7*b*x/a^2/(b*x^2+a)^(7/2)-48/35*b*x/a^3/(b*x^2+a)^
(5/2)-64/35*b*x/a^4/(b*x^2+a)^(3/2)-128/35*b*x/a^5/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {271, 192, 191}

$$-\frac{128bx}{35a^5\sqrt{a+bx^2}}-\frac{64bx}{35a^4\left(a+bx^2\right)^{3/2}}-\frac{48bx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{8bx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(1/(a*x*(a + b*x^2)^(7/2))) - (8*b*x)/(7*a^2*(a + b*x^2)^(7/2)) - (48*b*x)
/(35*a^3*(a + b*x^2)^(5/2)) - (64*b*x)/(35*a^4*(a + b*x^2)^(3/2)) - (128*b*
x)/(35*a^5*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^2)^{9/2}}\,dx = -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{(8b)\int \frac{1}{(a+bx^2)^{9/2}}\,dx}{a}$$

$$= -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{(48b)\int \frac{1}{(a+bx^2)^{7/2}}\,dx}{7a^2}$$

$$= -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{48bx}{35a^3(a+bx^2)^{5/2}} - \frac{(192b)\int \frac{1}{(a+bx^2)^{5/2}}\,dx}{35a^3}$$

$$= -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{48bx}{35a^3(a+bx^2)^{5/2}} - \frac{64bx}{35a^4(a+bx^2)^{3/2}} - \frac{(128b)}{}$$

$$= -\frac{1}{ax(a+bx^2)^{7/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{48bx}{35a^3(a+bx^2)^{5/2}} - \frac{64bx}{35a^4(a+bx^2)^{3/2}} - \frac{128}{35a^5\sqrt{a}}$$

Mathematica [A] time = 0.01, size = 64, normalized size = 0.64

$$\frac{-35a^4-280a^3bx^2-560a^2b^2x^4-448ab^3x^6-128b^4x^8}{35a^5x(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-35*a^4 - 280*a^3*b*x^2 - 560*a^2*b^2*x^4 - 448*a*b^3*x^6 - 128*b^4*x^8)/(
35*a^5*x*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.83, size = 103, normalized size = 1.03

$$-\frac{\left(128\,b^4x^8+448\,ab^3x^6+560\,a^2b^2x^4+280\,a^3bx^2+35\,a^4\right)\sqrt{bx^2+a}}{35\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(128*b^4*x^8 + 448*a*b^3*x^6 + 560*a^2*b^2*x^4 + 280*a^3*b*x^2 + 35*a
^4)*sqrt(b*x^2 + a)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*
x^3 + a^9*x)
```

giac [A] time = 1.12, size = 90, normalized size = 0.90

$$-\frac{\left(\left(x^2\left(\frac{93\,b^4x^2}{a^5}+\frac{308\,b^3}{a^4}\right)+\frac{350\,b^2}{a^3}\right)x^2+\frac{140\,b}{a^2}\right)x}{35\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{2\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*((x^2*(93*b^4*x^2/a^5 + 308*b^3/a^4) + 350*b^2/a^3)*x^2 + 140*b/a^2)*
x/(b*x^2 + a)^(7/2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

maple [A] time = 0.01, size = 61, normalized size = 0.61

$$-\frac{128b^4x^8+448a\,b^3x^6+560a^2b^2x^4+280a^3b\,x^2+35a^4}{35\left(b\,x^2+a\right)^{\frac{7}{2}}a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/35*(128*b^4*x^8+448*a*b^3*x^6+560*a^2*b^2*x^4+280*a^3*b*x^2+35*a^4)/x/(b
*x^2+a)^(7/2)/a^5
```

maxima [A] time = 1.43, size = 82, normalized size = 0.82

$$-\frac{128\,bx}{35\,\sqrt{bx^2+a}\,a^5}-\frac{64\,bx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^4}-\frac{48\,bx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^3}-\frac{8\,bx}{7\left(bx^2+a\right)^{\frac{7}{2}}a^2}-\frac{1}{\left(bx^2+a\right)^{\frac{7}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -128/35*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/
35*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*b*x/((b*x^2 + a)^(7/2)*a^2) - 1/((b*x^
2 + a)^(7/2)*a*x)
```

mupad [B] time = 4.71, size = 76, normalized size = 0.76

$$-\frac{\frac{1}{a^4}+\frac{128\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}}-\frac{29\,b\,x}{35\,a^4\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{13\,b\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{b\,x}{7\,a^2\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(9/2)),x)
```

```
[Out] - (1/a^4 + (128*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - (29*b*x)/(35*a^4*(
a + b*x^2)^(3/2)) - (13*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (b*x)/(7*a^2*(a +
 b*x^2)^(7/2))
```

sympy [B] time = 2.93, size = 400, normalized size = 4.00

$$-\frac{35a^4b^{\frac{33}{2}}\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}+140a^8b^{17}x^2+210a^7b^{18}x^4+140a^6b^{19}x^6+35a^5b^{20}x^8}-\frac{280a^3b^{\frac{35}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}+140a^8b^{17}x^2+210a^7b^{18}x^4+140a^6b^{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] -35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 280*a
**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 560*a
**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 448*a
*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 +
 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b**(
41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*
a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
```

3.530 $\int \frac{1}{x^3(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=126

$$\frac{9b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}-\frac{9b}{2a^5\sqrt{a+bx^2}}-\frac{3b}{2a^4\left(a+bx^2\right)^{3/2}}-\frac{9b}{10a^3\left(a+bx^2\right)^{5/2}}-\frac{9b}{14a^2\left(a+bx^2\right)^{7/2}}-\frac{1}{2ax^2\left(a+bx^2\right)^{7/2}}$$

```
[Out] -9/14*b/a^2/(b*x^2+a)^(7/2)-1/2/a/x^2/(b*x^2+a)^(7/2)-9/10*b/a^3/(b*x^2+a)^
(5/2)-3/2*b/a^4/(b*x^2+a)^(3/2)+9/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1
1/2)-9/2*b/a^5/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$-\frac{9\sqrt{a+bx^2}}{2a^5x^2}+\frac{3}{a^4x^2\sqrt{a+bx^2}}+\frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}}+\frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}}+\frac{9b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}+\frac{1}{7ax^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] 1/(7*a*x^2*(a + b*x^2)^(7/2)) + 9/(35*a^2*x^2*(a + b*x^2)^(5/2)) + 3/(5*a^3
*x^2*(a + b*x^2)^(3/2)) + 3/(a^4*x^2*Sqrt[a + b*x^2]) - (9*Sqrt[a + b*x^2])
/(2*a^5*x^2) + (9*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(11/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^{9/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{9/2}}\,dx,x,x^2\right)$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{7/2}}\,dx,x,x^2\right)}{14a}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{9\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx,x,x^2\right)}{10a^2}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx,\right.}{2a^3}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} + \frac{9\,\text{Subst}}{}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} - \frac{9\sqrt{a+b}}{2a^5x}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} - \frac{9\sqrt{a+b}}{2a^5x}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} - \frac{9\sqrt{a+b}}{2a^5x}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.29

$$-\frac{b\,{}_2F_1\left(-\frac{7}{2},2;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/7*(b*Hypergeometric2F1[-7/2, 2, -5/2, 1 + (b*x^2)/a])/(a^2*(a + b*x^2)^(
7/2))
```

fricas [A] time = 1.11, size = 373, normalized size = 2.96

$$\left[\frac{315\left(b^5x^{10}+4\,ab^4x^8+6\,a^2b^3x^6+4\,a^3b^2x^4+a^4bx^2\right)\sqrt{a}\,\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\left(315\,ab^4x^8+1050\,a^2b^3x^6\right.}{140\left(a^6b^4x^{10}+4\,a^7b^3x^8+6\,a^8b^2x^6+4\,a^9bx^4+a^{10}x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^4 + a^4*b
*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(315*
a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2 + 35*a^5)*s
```

```
qrt(b*x^2 + a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4
 + a^10*x^2), -1/70*(315*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^
2*x^4 + a^4*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (315*a*b^4*x
^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2 + 35*a^5)*sqrt(b*x
^2 + a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10
*x^2)]
```

giac [A] time = 1.13, size = 104, normalized size = 0.83

$$-\frac{9\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^5}-\frac{\sqrt{bx^2+a}}{2\,a^5x^2}-\frac{140\,\left(bx^2+a\right)^3b+35\,\left(bx^2+a\right)^2ab+14\,\left(bx^2+a\right)a^2b+5\,a^3b}{35\,\left(bx^2+a\right)^{\frac{7}{2}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -9/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^5) - 1/2*sqrt(b*x^2 + a
)/(a^5*x^2) - 1/35*(140*(b*x^2 + a)^3*b + 35*(b*x^2 + a)^2*a*b + 14*(b*x^2
+ a)*a^2*b + 5*a^3*b)/((b*x^2 + a)^(7/2)*a^5)
```

maple [A] time = 0.01, size = 108, normalized size = 0.86

$$-\frac{9b}{14\left(b\,x^2+a\right)^{\frac{7}{2}}a^2}-\frac{9b}{10\left(b\,x^2+a\right)^{\frac{5}{2}}a^3}-\frac{1}{2\left(b\,x^2+a\right)^{\frac{7}{2}}a\,x^2}-\frac{3b}{2\left(b\,x^2+a\right)^{\frac{3}{2}}a^4}+\frac{9b\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{11}{2}}}-\frac{9b}{2\sqrt{b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/2/a/x^2/(b*x^2+a)^(7/2)-9/14*b/a^2/(b*x^2+a)^(7/2)-9/10*b/a^3/(b*x^2+a)^
(5/2)-3/2*b/a^4/(b*x^2+a)^(3/2)-9/2*b/a^5/(b*x^2+a)^(1/2)+9/2/a^(11/2)*b*ln
((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.39, size = 96, normalized size = 0.76

$$\frac{9\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{11}{2}}}-\frac{9\,b}{2\,\sqrt{bx^2+a}\,a^5}-\frac{3\,b}{2\,\left(bx^2+a\right)^{\frac{3}{2}}a^4}-\frac{9\,b}{10\,\left(bx^2+a\right)^{\frac{5}{2}}a^3}-\frac{9\,b}{14\,\left(bx^2+a\right)^{\frac{7}{2}}a^2}-\frac{1}{2\,\left(bx^2+a\right)^{\frac{7}{2}}ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 9/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(11/2) - 9/2*b/(sqrt(b*x^2 + a)*a^5)
- 3/2*b/((b*x^2 + a)^(3/2)*a^4) - 9/10*b/((b*x^2 + a)^(5/2)*a^3) - 9/14*b/(
(b*x^2 + a)^(7/2)*a^2) - 1/2/((b*x^2 + a)^(7/2)*a*x^2)
```

mupad [B] time = 4.96, size = 113, normalized size = 0.90

$$\frac{9\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{11/2}}-\frac{\frac{b}{7\,a}+\frac{3\,b\,{\left(b\,x^2+a\right)}^2}{5\,a^3}+\frac{3\,b\,{\left(b\,x^2+a\right)}^3}{a^4}-\frac{9\,b\,{\left(b\,x^2+a\right)}^4}{2\,a^5}+\frac{9\,b\,\left(b\,x^2+a\right)}{35\,a^2}}{a\,{\left(b\,x^2+a\right)}^{7/2}-{\left(b\,x^2+a\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(9/2)),x)
```

```
[Out] (9*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(11/2)) - (b/(7*a) + (3*b*(a +
b*x^2)^2)/(5*a^3) + (3*b*(a + b*x^2)^3)/a^4 - (9*b*(a + b*x^2)^4)/(2*a^5) +
 (9*b*(a + b*x^2))/(35*a^2))/(a*(a + b*x^2)^(7/2) - (a + b*x^2)^(9/2))
```

sympy [B] time = 12.43, size = 5540, normalized size = 43.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(9/2),x)
```

```
[Out] -70*a**49*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4
+ 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*
b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 1680
0*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x*
*20 + 140*a**(87/2)*b**10*x**22) - 1476*a**48*b*x**2*sqrt(1 + b*x**2/a)/(14
0*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16
800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**
5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a*
*(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22)
 - 315*a**48*b*x**2*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*
x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(9
9/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 +
 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b*
*9*x**20 + 140*a**(87/2)*b**10*x**22) + 630*a**48*b*x**2*log(sqrt(1 + b*x**
2/a) + 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b
**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*
a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x*
*16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)
*b**10*x**22) - 9822*a**47*b**2*x**4*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**
2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b
**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*
a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**
18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 3150*a**47*b*
*2*x**4*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*
a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x*
*10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(9
3/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 1
40*a**(87/2)*b**10*x**22) + 6300*a**47*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1
)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6
 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2
)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 63
00*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x
**22) - 33956*a**46*b**3*x**6*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 140
0*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**
8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/
2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 14
00*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 14175*a**46*b**3*x**
6*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(10
3/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 +
35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b
**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**
(87/2)*b**10*x**22) + 28350*a**46*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(14
0*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16
800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**
5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a*
*(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22)
 - 71940*a**45*b**4*x**8*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**
(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 2
9400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b*
```

```
*6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a*
*(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 37800*a**45*b**4*x**8*log
(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*
b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280
*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x
**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2
)*b**10*x**22) + 75600*a**45*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(140*a**
(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a
**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**
12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/
2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 10
0260*a**44*b**5*x**10*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(10
5/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 2940
0*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*
x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(8
9/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 66150*a**44*b**5*x**10*log(b
*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b*
*2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a
**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**
16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*
b**10*x**22) + 132300*a**44*b**5*x**10*log(sqrt(1 + b*x**2/a) + 1)/(140*a**
(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a
**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**
12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/
2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 94
396*a**43*b**6*x**12*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105
/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400
*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x
**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89
/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 79380*a**43*b**6*x**12*log(b*
x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**
2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a*
*(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**1
6 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b
**10*x**22) + 158760*a**43*b**6*x**12*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(
107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a*
*(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**1
2 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2
)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 597
72*a**42*b**7*x**14*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/
2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*
a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x*
*14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/
2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 66150*a**42*b**7*x**14*log(b*x
**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2
*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**
(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16
 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b*
*10*x**22) + 132300*a**42*b**7*x**14*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(1
07/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**
(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12
 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)
*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 2448
6*a**41*b**8*x**16*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2
)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a
**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**
14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2
)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 37800*a**41*b**8*x**16*log(b*x*
*2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*
```

```
x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(
97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16
+ 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**
10*x**22) + 75600*a**41*b**8*x**16*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107
/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(1
01/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 +
 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b
**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 5880*a
**40*b**9*x**18*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b
*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(
99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14
+ 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b
**9*x**20 + 140*a**(87/2)*b**10*x**22) - 14175*a**40*b**9*x**18*log(b*x**2/
a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**
6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/
2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6
300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*
x**22) + 28350*a**40*b**9*x**18*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)
*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/
2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29
400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8
*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 630*a**39
*b**10*x**20*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x*
*4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/
2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 1
6800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9
*x**20 + 140*a**(87/2)*b**10*x**22) - 3150*a**39*b**10*x**20*log(b*x**2/a)/
(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 +
 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*
b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300
*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**
22) + 6300*a**39*b**10*x**20*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)*x*
*2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*
b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400
*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x*
*18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 315*a**38*b*
*11*x**22*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 630
0*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*
x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**
(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 +
 140*a**(87/2)*b**10*x**22) + 630*a**38*b**11*x**22*log(sqrt(1 + b*x**2/a)
+ 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x
**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(9
7/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 +
 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**1
0*x**22)
```

3.531 $\int \frac{1}{x^4\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=132

$$\frac{256b^2x}{21a^6\sqrt{a+bx^2}}+\frac{128b^2x}{21a^5\left(a+bx^2\right)^{3/2}}+\frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}}+\frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}}+\frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}$$

```
[Out] -1/3/a/x^3/(b*x^2+a)^(7/2)+10/3*b/a^2/x/(b*x^2+a)^(7/2)+80/21*b^2*x/a^3/(b*
x^2+a)^(7/2)+32/7*b^2*x/a^4/(b*x^2+a)^(5/2)+128/21*b^2*x/a^5/(b*x^2+a)^(3/2
)+256/21*b^2*x/a^6/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {271, 192, 191}

$$\frac{256b^2x}{21a^6\sqrt{a+bx^2}}+\frac{128b^2x}{21a^5\left(a+bx^2\right)^{3/2}}+\frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}}+\frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}}+\frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/(3*a*x^3*(a + b*x^2)^(7/2)) + (10*b)/(3*a^2*x*(a + b*x^2)^(7/2)) + (80*b
^2*x)/(21*a^3*(a + b*x^2)^(7/2)) + (32*b^2*x)/(7*a^4*(a + b*x^2)^(5/2)) + (
128*b^2*x)/(21*a^5*(a + b*x^2)^(3/2)) + (256*b^2*x)/(21*a^6*Sqrt[a + b*x^2]
)
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{9/2}}\,dx = -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} - \frac{(10b)\int \frac{1}{x^2\left(a+bx^2\right)^{9/2}}\,dx}{3a}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80b^2\right)\int \frac{1}{\left(a+bx^2\right)^{9/2}}\,dx}{3a^2}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(160b^2\right)\int \frac{1}{\left(a+bx^2\right)^{7/2}}\,dx}{7a^3}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}} + \frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}} + \frac{\left(128b^2\right)}{}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}} + \frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}} + \frac{128}{21a^5\left(a\right.}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}} + \frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}} + \frac{128}{21a^5\left(a\right.}$$

Mathematica [A] time = 0.01, size = 75, normalized size = 0.57

$$\frac{-7a^5+70a^4bx^2+560a^3b^2x^4+1120a^2b^3x^6+896ab^4x^8+256b^5x^{10}}{21a^6x^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^3*x^6 + 896*a*b^4*x^8
 + 256*b^5*x^10)/(21*a^6*x^3*(a + b*x^2)^(7/2))
```

fricas [A] time = 1.06, size = 116, normalized size = 0.88

$$\frac{\left(256\,b^5x^{10}+896\,ab^4x^8+1120\,a^2b^3x^6+560\,a^3b^2x^4+70\,a^4bx^2-7\,a^5\right)\sqrt{bx^2+a}}{21\,\left(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(256*b^5*x^10 + 896*a*b^4*x^8 + 1120*a^2*b^3*x^6 + 560*a^3*b^2*x^4 + 7
0*a^4*b*x^2 - 7*a^5)*sqrt(b*x^2 + a)/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*
b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)
```

giac [A] time = 1.16, size = 147, normalized size = 1.11

$$\frac{\left(\left(x^2\left(\frac{158\,b^5x^2}{a^6}+\frac{511\,b^4}{a^5}\right)+\frac{560\,b^3}{a^4}\right)x^2+\frac{210\,b^2}{a^3}\right)x}{21\,\left(bx^2+a\right)^{\frac{7}{2}}} - \frac{4\left(6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4b^{\frac{3}{2}}-15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}+7\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/21*((x^2*(158*b^5*x^2/a^6 + 511*b^4/a^5) + 560*b^3/a^4)*x^2 + 210*b^2/a^3
)*x/(b*x^2 + a)^(7/2) - 4/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) - 15
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 7*a^2*b^(3/2))/(((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^3*a^5)
```

maple [A] time = 0.01, size = 72, normalized size = 0.55

$$-\frac{-256b^{5}x^{10}-896a\,b^{4}x^{8}-1120a^{2}b^{3}x^{6}-560a^{3}b^{2}x^{4}-70a^{4}b\,x^{2}+7a^{5}}{21\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{6}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/21*(-256*b^5*x^10-896*a*b^4*x^8-1120*a^2*b^3*x^6-560*a^3*b^2*x^4-70*a^4*
b*x^2+7*a^5)/x^3/(b*x^2+a)^(7/2)/a^6
```

maxima [A] time = 1.42, size = 108, normalized size = 0.82

$$\frac{256\,b^{2}x}{21\,\sqrt{bx^{2}+a}\,a^{6}}+\frac{128\,b^{2}x}{21\left(bx^{2}+a\right)^{\frac{3}{2}}a^{5}}+\frac{32\,b^{2}x}{7\left(bx^{2}+a\right)^{\frac{5}{2}}a^{4}}+\frac{80\,b^{2}x}{21\left(bx^{2}+a\right)^{\frac{7}{2}}a^{3}}+\frac{10\,b}{3\left(bx^{2}+a\right)^{\frac{7}{2}}a^{2}x}-\frac{1}{3\left(bx^{2}+a\right)^{\frac{7}{2}}ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 256/21*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*b^2*x/((b*x^2 + a)^(3/2)*a^5) +
 32/7*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*b^2*x/((b*x^2 + a)^(7/2)*a^3) +
 10/3*b/((b*x^2 + a)^(7/2)*a^2*x) - 1/3/((b*x^2 + a)^(7/2)*a*x^3)
```

mupad [B] time = 4.80, size = 97, normalized size = 0.73

$$\frac{\frac{128\,b}{21\,a^{5}}+\frac{256\,b^{2}\,x^{2}}{21\,a^{6}}}{x\,\sqrt{b\,x^{2}+a}}-\frac{\frac{1}{3\,a^{2}}+\frac{19\,b\,x^{2}}{21\,a^{3}}}{x^{3}\,{\left(b\,x^{2}+a\right)}^{5/2}}-\frac{32\,b}{21\,a^{4}\,x\,{\left(b\,x^{2}+a\right)}^{3/2}}+\frac{b^{2}\,x}{7\,a^{3}\,{\left(b\,x^{2}+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(9/2)),x)
```

```
[Out] ((128*b)/(21*a^5) + (256*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) - (1/(3*a
^2) + (19*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)^(5/2)) - (32*b)/(21*a^4*x*(a +
b*x^2)^(3/2)) + (b^2*x)/(7*a^3*(a + b*x^2)^(7/2))
```

sympy [B] time = 3.84, size = 668, normalized size = 5.06

$$-\frac{7a^{6}b^{\frac{51}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{21a^{11}b^{25}x^{2}+105a^{10}b^{26}x^{4}+210a^{9}b^{27}x^{6}+210a^{8}b^{28}x^{8}+105a^{7}b^{29}x^{10}+21a^{6}b^{30}x^{12}}+\frac{}{21a^{11}b^{25}x^{2}+105a^{10}b^{26}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(9/2),x)
```

```
[Out] -7*a**6*b**(51/2)*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**
26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10
+ 21*a**6*b**30*x**12) + 63*a**5*b**(53/2)*x**2*sqrt(a/(b*x**2) + 1)/(21*a*
*11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**2
8*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 630*a**4*b**(55/2)*x
**4*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*
```

```
a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**3
0*x**12) + 1680*a**3*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x*
*2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105
*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 2016*a**2*b**(59/2)*x**8*sqrt(a/
(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*
x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) +
1152*a*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**1
0*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x
**10 + 21*a**6*b**30*x**12) + 256*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/(21*
a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b*
*28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12)
```

3.532 $\int \frac{x^5}{\sqrt{9+4x^2}} \, dx$

Optimal. Leaf size=46

$$\frac{1}{320}\left(4x^2+9\right)^{5/2}-\frac{3}{32}\left(4x^2+9\right)^{3/2}+\frac{81}{64}\sqrt{4x^2+9}$$

```
[Out] -3/32*(4*x^2+9)^(3/2)+1/320*(4*x^2+9)^(5/2)+81/64*(4*x^2+9)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{320}\left(4x^2+9\right)^{5/2}-\frac{3}{32}\left(4x^2+9\right)^{3/2}+\frac{81}{64}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[9 + 4*x^2],x]
```

```
[Out] (81*Sqrt[9 + 4*x^2])/64 - (3*(9 + 4*x^2)^(3/2))/32 + (9 + 4*x^2)^(5/2)/320
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{9+4x^2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{9+4x}} \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16\sqrt{9+4x}}-\frac{9}{8}\sqrt{9+4x}+\frac{1}{16}(9+4x)^{3/2}\right) dx, x, x^2\right)\\ &= \frac{81}{64}\sqrt{9+4x^2}-\frac{3}{32}\left(9+4x^2\right)^{3/2}+\frac{1}{320}\left(9+4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$\frac{1}{40}\sqrt{4x^2+9}\left(2x^4-6x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[9 + 4*x^2],x]
```

```
[Out] (Sqrt[9 + 4*x^2]*(27 - 6*x^2 + 2*x^4))/40
```

fricas [A] time = 0.80, size = 23, normalized size = 0.50

$$\frac{1}{40}\left(2x^4-6x^2+27\right)\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(2*x^4 - 6*x^2 + 27)*sqrt(4*x^2 + 9)
```

giac [A] time = 1.13, size = 34, normalized size = 0.74

$$\frac{1}{320}\left(4x^2+9\right)^{\frac{5}{2}}-\frac{3}{32}\left(4x^2+9\right)^{\frac{3}{2}}+\frac{81}{64}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/320*(4*x^2 + 9)^(5/2) - 3/32*(4*x^2 + 9)^(3/2) + 81/64*sqrt(4*x^2 + 9)
```

maple [A] time = 0.00, size = 24, normalized size = 0.52

$$\frac{\sqrt{4x^2+9}\,\left(2x^4-6x^2+27\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/40*(4*x^2+9)^(1/2)*(2*x^4-6*x^2+27)
```

maxima [A] time = 2.89, size = 40, normalized size = 0.87

$$\frac{1}{20}\sqrt{4x^2+9}\,x^4-\frac{3}{20}\sqrt{4x^2+9}\,x^2+\frac{27}{40}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(4*x^2 + 9)*x^4 - 3/20*sqrt(4*x^2 + 9)*x^2 + 27/40*sqrt(4*x^2 + 9)
```

mupad [B] time = 0.02, size = 21, normalized size = 0.46

$$\frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{x^4}{5}-\frac{3x^2}{5}+\frac{27}{10}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^2 + 9)^(1/2),x)
```

```
[Out] ((x^2 + 9/4)^(1/2)*(x^4/5 - (3*x^2)/5 + 27/10))/2
```

sympy [A] time = 1.29, size = 44, normalized size = 0.96

$$\frac{x^4\sqrt{4x^2+9}}{20}-\frac{3x^2\sqrt{4x^2+9}}{20}+\frac{27\sqrt{4x^2+9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(4*x**2+9)**(1/2),x)
```

```
[Out] x**4*sqrt(4*x**2 + 9)/20 - 3*x**2*sqrt(4*x**2 + 9)/20 + 27*sqrt(4*x**2 + 9)
/40
```

3.533 $\int \frac{x^4}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=45

$$-\frac{27}{128}\sqrt{4x^2+9}\,x+\frac{1}{16}\sqrt{4x^2+9}\,x^3+\frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 243/256*arcsinh(2/3*x)-27/128*x*(4*x^2+9)^(1/2)+1/16*x^3*(4*x^2+9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 215}

$$\frac{1}{16}\sqrt{4x^2+9}\,x^3-\frac{27}{128}\sqrt{4x^2+9}\,x+\frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[9 + 4*x^2],x]
```

```
[Out] (-27*x*Sqrt[9 + 4*x^2])/128 + (x^3*Sqrt[9 + 4*x^2])/16 + (243*ArcSinh[(2*x)
/3])/256
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{9+4x^2}}\,dx &= \frac{1}{16}x^3\sqrt{9+4x^2}-\frac{27}{16}\int \frac{x^2}{\sqrt{9+4x^2}}\,dx\\ &= -\frac{27}{128}x\sqrt{9+4x^2}+\frac{1}{16}x^3\sqrt{9+4x^2}+\frac{243}{128}\int \frac{1}{\sqrt{9+4x^2}}\,dx\\ &= -\frac{27}{128}x\sqrt{9+4x^2}+\frac{1}{16}x^3\sqrt{9+4x^2}+\frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.76

$$\frac{1}{256}\left(2x\sqrt{4x^2+9}\left(8x^2-27\right)+243\sinh^{-1}\left(\frac{2x}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[9 + 4*x^2],x]
```

```
[Out] (2*x*Sqrt[9 + 4*x^2]*(-27 + 8*x^2) + 243*ArcSinh[(2*x)/3])/256
```

fricas [A] time = 0.75, size = 37, normalized size = 0.82

$$\frac{1}{128}\left(8x^3-27x\right)\sqrt{4x^2+9}-\frac{243}{256}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/128*(8*x^3 - 27*x)*sqrt(4*x^2 + 9) - 243/256*log(-2*x + sqrt(4*x^2 + 9))
```

giac [A] time = 1.09, size = 36, normalized size = 0.80

$$\frac{1}{128}\left(8x^2-27\right)\sqrt{4x^2+9}\,x-\frac{243}{256}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/128*(8*x^2 - 27)*sqrt(4*x^2 + 9)*x - 243/256*log(-2*x + sqrt(4*x^2 + 9))
```

maple [A] time = 0.01, size = 34, normalized size = 0.76

$$\frac{\sqrt{4x^2+9}\,x^3}{16}-\frac{27\sqrt{4x^2+9}\,x}{128}+\frac{243\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^2+9)^(1/2),x)
[Out] 243/256*arcsinh(2/3*x)-27/128*(4*x^2+9)^(1/2)*x+1/16*x^3*(4*x^2+9)^(1/2)
```

maxima [A] time = 2.91, size = 33, normalized size = 0.73

$$\frac{1}{16}\sqrt{4x^2+9}\,x^3-\frac{27}{128}\sqrt{4x^2+9}\,x+\frac{243}{256}\,\mathrm{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/16*sqrt(4*x^2 + 9)*x^3 - 27/128*sqrt(4*x^2 + 9)*x + 243/256*arcsinh(2/3*x
)
```

mupad [B] time = 0.03, size = 25, normalized size = 0.56

$$\frac{243\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{256}-\frac{\sqrt{x^2+\frac{9}{4}}\left(\frac{27x}{32}-\frac{x^3}{4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^2 + 9)^(1/2),x)
[Out] (243*asinh((2*x)/3))/256 - ((x^2 + 9/4)^(1/2)*((27*x)/32 - x^3/4))/2
```

sympy [A] time = 0.73, size = 39, normalized size = 0.87

$$\frac{x^3\sqrt{4x^2+9}}{16}-\frac{27x\sqrt{4x^2+9}}{128}+\frac{243\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(4*x**2+9)**(1/2),x)
[Out] x**3*sqrt(4*x**2 + 9)/16 - 27*x*sqrt(4*x**2 + 9)/128 + 243*asinh(2*x/3)/256
```

3.534 $\int \frac{x^3}{\sqrt{9+4x^2}}\, dx$

Optimal. Leaf size=31

$$\frac{1}{48}\left(4x^2+9\right)^{3/2}-\frac{9}{16}\sqrt{4x^2+9}$$

```
[Out] 1/48*(4*x^2+9)^(3/2)-9/16*(4*x^2+9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{48}\left(4x^2+9\right)^{3/2}-\frac{9}{16}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[9 + 4*x^2],x]
[Out] (-9*Sqrt[9 + 4*x^2])/16 + (9 + 4*x^2)^(3/2)/48
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{9+4x^2}}\, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{9+4x}}\, dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{9}{4\sqrt{9+4x}}+\frac{1}{4}\sqrt{9+4x}\right) dx, x, x^2\right)\\
&= -\frac{9}{16}\sqrt{9+4x^2}+\frac{1}{48}\left(9+4x^2\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{24}\left(2x^2-9\right)\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[9 + 4*x^2],x]
[Out] ((-9 + 2*x^2)*Sqrt[9 + 4*x^2])/24
```

fricas [A] time = 0.76, size = 18, normalized size = 0.58

$$\frac{1}{24}\sqrt{4x^2+9}\left(2x^2-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(4*x^2 + 9)*(2*x^2 - 9)
```

giac [A] time = 1.13, size = 23, normalized size = 0.74

$$\frac{1}{48}\left(4x^2+9\right)^{\frac{3}{2}}-\frac{9}{16}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(4*x^2 + 9)^(3/2) - 9/16*sqrt(4*x^2 + 9)
```

maple [A] time = 0.00, size = 19, normalized size = 0.61

$$\frac{\sqrt{4x^2+9}\,\left(2x^2-9\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/24*(4*x^2+9)^(1/2)*(2*x^2-9)
```

maxima [A] time = 2.91, size = 26, normalized size = 0.84

$$\frac{1}{12}\sqrt{4x^2+9}\,x^2-\frac{3}{8}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(4*x^2 + 9)*x^2 - 3/8*sqrt(4*x^2 + 9)
```

mupad [B] time = 0.02, size = 15, normalized size = 0.48

$$\sqrt{x^2+\frac{9}{4}}\,\left(\frac{x^2}{6}-\frac{3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2 + 9)^(1/2),x)
```

```
[Out] (x^2 + 9/4)^(1/2)*(x^2/6 - 3/4)
```

sympy [A] time = 0.38, size = 27, normalized size = 0.87

$$\frac{x^2\sqrt{4x^2+9}}{12}-\frac{3\sqrt{4x^2+9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**2+9)**(1/2),x)
```

```
[Out] x**2*sqrt(4*x**2 + 9)/12 - 3*sqrt(4*x**2 + 9)/8
```

3.535 $\int \frac{x^2}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=27

$$\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] -9/16*arcsinh(2/3*x)+1/8*x*(4*x^2+9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 215}

$$\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2])/8 - (9*ArcSinh[(2*x)/3])/16
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{9+4x^2}}\,dx &= \frac{1}{8}x\sqrt{9+4x^2}-\frac{9}{8}\int \frac{1}{\sqrt{9+4x^2}}\,dx\\ &= \frac{1}{8}x\sqrt{9+4x^2}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2])/8 - (9*ArcSinh[(2*x)/3])/16
```

fricas [A] time = 0.76, size = 29, normalized size = 1.07

$$\frac{1}{8}\sqrt{4x^2+9}\,x+\frac{9}{16}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(4*x^2 + 9)*x + 9/16*log(-2*x + sqrt(4*x^2 + 9))
```

giac [A] time = 1.07, size = 29, normalized size = 1.07

$$\frac{1}{8}\sqrt{4x^2+9}\,x+\frac{9}{16}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/8*sqrt(4*x^2 + 9)*x + 9/16*log(-2*x + sqrt(4*x^2 + 9))
```

maple [A] time = 0.01, size = 20, normalized size = 0.74

$$\frac{\sqrt{4x^2+9}\;x}{8}-\frac{9\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^2+9)^(1/2),x)
[Out] -9/16*arcsinh(2/3*x)+1/8*(4*x^2+9)^(1/2)*x
```

maxima [A] time = 2.92, size = 19, normalized size = 0.70

$$\frac{1}{8}\sqrt{4x^2+9}\,x-\frac{9}{16}\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/8*sqrt(4*x^2 + 9)*x - 9/16*arcsinh(2/3*x)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.63

$$\frac{x\,\sqrt{x^2+\frac{9}{4}}}{4}-\frac{9\,\mathrm{asinh}\left(\frac{2\,x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^2 + 9)^(1/2),x)
[Out] (x*(x^2 + 9/4)^(1/2))/4 - (9*asinh((2*x)/3))/16
```

sympy [A] time = 0.23, size = 22, normalized size = 0.81

$$\frac{x\sqrt{4x^2+9}}{8}-\frac{9\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(4*x**2+9)**(1/2),x)
[Out] x*sqrt(4*x**2 + 9)/8 - 9*asinh(2*x/3)/16
```

3.536 $\int \frac{x}{\sqrt{9+4x^2}} dx$

Optimal. Leaf size=15

$$\frac{1}{4}\sqrt{4x^2+9}$$

```
[Out] 1/4*(4*x^2+9)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{4}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[9 + 4*x^2],x]
```

```
[Out] Sqrt[9 + 4*x^2]/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{9+4x^2}} dx = \frac{1}{4}\sqrt{9+4x^2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{4}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[9 + 4*x^2],x]
```

```
[Out] Sqrt[9 + 4*x^2]/4
```

fricas [A] time = 0.61, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(4*x^2 + 9)
```

giac [A] time = 1.12, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(4*x^2 + 9)
```

maple [A] time = 0.00, size = 12, normalized size = 0.80

$$\frac{\sqrt{4x^2+9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/4*(4*x^2+9)^(1/2)
```

maxima [A] time = 1.32, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(4*x^2 + 9)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.60

$$\frac{\sqrt{x^2+\frac{9}{4}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2 + 9)^(1/2),x)
```

```
[Out] (x^2 + 9/4)^(1/2)/2
```

sympy [A] time = 0.15, size = 10, normalized size = 0.67

$$\frac{\sqrt{4x^2+9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x**2+9)**(1/2),x)
```

```
[Out] sqrt(4*x**2 + 9)/4
```

3.537 $\int \frac{1}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 1/2*arcsinh(2/3*x)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {215}

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 + 4*x^2],x]
```

```
[Out] ArcSinh[(2*x)/3]/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + 4*x^2],x]
```

```
[Out] ArcSinh[(2*x)/3]/2
```

fricas [B] time = 0.81, size = 16, normalized size = 1.60

$$-\frac{1}{2}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 9))
```

giac [B] time = 1.17, size = 16, normalized size = 1.60

$$-\frac{1}{2}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 9))
```

maple [A] time = 0.00, size = 7, normalized size = 0.70

$$\frac{\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/2*arcsinh(2/3*x)
```

maxima [A] time = 2.96, size = 6, normalized size = 0.60

$$\frac{1}{2} \operatorname{arsinh}\left(\frac{2}{3} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsinh(2/3*x)
```

mupad [B] time = 0.03, size = 6, normalized size = 0.60

$$\frac{\operatorname{asinh}\left(\frac{2\,x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 9)^(1/2),x)
```

```
[Out] asinh((2*x)/3)/2
```

sympy [A] time = 0.15, size = 7, normalized size = 0.70

$$\frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2+9)**(1/2),x)
```

```
[Out] asinh(2*x/3)/2
```

3.538 $\int \frac{1}{x\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

```
[Out] -1/3*arctanh(1/3*(4*x^2+9)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {266, 63, 207}

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -ArcTanh[Sqrt[9 + 4*x^2]/3]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{9+4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)\\ &= -\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[9 + 4*x^2]),x]
[Out] -1/3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

fricas [B] time = 0.93, size = 35, normalized size = 1.75

$$-\frac{1}{3}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\,+3\right)+\frac{1}{3}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\,-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(-2*x + sqrt(4*x^2 + 9) + 3) + 1/3*log(-2*x + sqrt(4*x^2 + 9) - 3)
```

giac [B] time = 1.06, size = 29, normalized size = 1.45

$$-\frac{1}{6}\,\log\left(\sqrt{4\,x^2+9}\,+3\right)+\frac{1}{6}\,\log\left(\sqrt{4\,x^2+9}\,-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] -1/6*log(sqrt(4*x^2 + 9) + 3) + 1/6*log(sqrt(4*x^2 + 9) - 3)
```

maple [A] time = 0.00, size = 15, normalized size = 0.75

$$-\frac{\operatorname{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4*x^2+9)^(1/2),x)
[Out] -1/3*arctanh(3/(4*x^2+9)^(1/2))
```

maxima [A] time = 2.96, size = 9, normalized size = 0.45

$$-\frac{1}{3}\,\operatorname{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/3*arcsinh(3/2/abs(x))
```

mupad [B] time = 0.04, size = 12, normalized size = 0.60

$$-\frac{\operatorname{atanh}\left(\frac{2\,\sqrt{x^2+\frac{9}{4}}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(4*x^2 + 9)^(1/2)),x)
[Out] -atanh((2*(x^2 + 9/4)^(1/2))/3)/3
```

sympy [A] time = 1.02, size = 8, normalized size = 0.40

$$-\frac{\operatorname{asinh}\left(\frac{3}{2x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x**2+9)**(1/2),x)
```

```
[Out] -asinh(3/(2*x))/3
```

3.539 $\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{4x^2+9}}{9x}$$

```
[Out] -1/9*(4*x^2+9)^(1/2)/x
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(9*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx = -\frac{\sqrt{9+4x^2}}{9x}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -1/9*Sqrt[9 + 4*x^2]/x
```

fricas [A] time = 0.78, size = 18, normalized size = 1.00

$$-\frac{2\,x+\sqrt{4\,x^2+9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(2*x + sqrt(4*x^2 + 9))/x
```

giac [A] time = 1.12, size = 23, normalized size = 1.28

$$\frac{4}{\left(2\,x-\sqrt{4\,x^2+9}\right)^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 4/((2*x - sqrt(4*x^2 + 9))^2 - 9)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4*x^2+9)^(1/2),x)
```

```
[Out] -1/9*(4*x^2+9)^(1/2)/x
```

maxima [A] time = 2.98, size = 14, normalized size = 0.78

$$-\frac{\sqrt{4\,x^2+9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(4*x^2 + 9)/x
```

mupad [B] time = 0.02, size = 12, normalized size = 0.67

$$-\frac{2\,\sqrt{x^2+\frac{9}{4}}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(4*x^2 + 9)^(1/2)),x)
```

```
[Out] -(2*(x^2 + 9/4)^(1/2))/(9*x)
```

sympy [A] time = 0.76, size = 15, normalized size = 0.83

$$-\frac{2\sqrt{1+\frac{9}{4x^2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4*x**2+9)**(1/2),x)
```

```
[Out] -2*sqrt(1 + 9/(4*x**2))/9
```

3.540 $\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{18x^2}$$

[Out] 2/27*arctanh(1/3*(4*x^2+9)^(1/2))-1/18*(4*x^2+9)^(1/2)/x^2

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 207}

$$\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{18x^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*Sqrt[9 + 4*x^2]),x]

[Out] -Sqrt[9 + 4*x^2]/(18*x^2) + (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/27

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{18x^2} - \frac{1}{9}\,\text{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{18x^2} - \frac{1}{18}\,\text{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)\\
&= -\frac{\sqrt{9+4x^2}}{18x^2} + \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)
\end{aligned}
$$

Mathematica [**A**] time = 0.01, size = 37, normalized size = 0.95

$$
\frac{1}{54}\left(4\tanh^{-1}\left(\sqrt{\frac{4x^2}{9}+1}\right)-\frac{3\sqrt{4x^2+9}}{x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[9 + 4*x^2]),x]
```

```
[Out] ((-3*Sqrt[9 + 4*x^2])/x^2 + 4*ArcTanh[Sqrt[1 + (4*x^2)/9]])/54
```

fricas [**A**] time = 0.81, size = 57, normalized size = 1.46

$$
\frac{4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)-3\,\sqrt{4\,x^2+9}}{54\,x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(4*x^2*log(-2*x + sqrt(4*x^2 + 9) + 3) - 4*x^2*log(-2*x + sqrt(4*x^2 +
 9) - 3) - 3*sqrt(4*x^2 + 9))/x^2
```

giac [**A**] time = 1.10, size = 43, normalized size = 1.10

$$
-\frac{\sqrt{4\,x^2+9}}{18\,x^2}+\frac{1}{27}\,\log\left(\sqrt{4\,x^2+9}+3\right)-\frac{1}{27}\,\log\left(\sqrt{4\,x^2+9}-3\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(4*x^2 + 9)/x^2 + 1/27*log(sqrt(4*x^2 + 9) + 3) - 1/27*log(sqrt(4
*x^2 + 9) - 3)
```

maple [**A**] time = 0.00, size = 30, normalized size = 0.77

$$
\frac{2\,\text{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{27}-\frac{\sqrt{4x^2+9}}{18x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4*x^2+9)^(1/2),x)
```

```
[Out] -1/18*(4*x^2+9)^(1/2)/x^2+2/27*arctanh(3/(4*x^2+9)^(1/2))
```

maxima [A] time = 2.94, size = 24, normalized size = 0.62

$$-\frac{\sqrt{4\,x^2+9}}{18\,x^2}+\frac{2}{27}\,\operatorname{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(4*x^2 + 9)/x^2 + 2/27*arcsinh(3/2/abs(x))
```

mupad [B] time = 0.03, size = 25, normalized size = 0.64

$$\frac{2\,\operatorname{atanh}\left(\frac{2\,\sqrt{x^2+\frac{9}{4}}}{3}\right)}{27}-\frac{\sqrt{x^2+\frac{9}{4}}}{9\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(4*x^2 + 9)^(1/2)),x)
```

```
[Out] (2*atanh((2*(x^2 + 9/4)^(1/2))/3))/27 - (x^2 + 9/4)^(1/2)/(9*x^2)
```

sympy [A] time = 2.10, size = 44, normalized size = 1.13

$$\frac{2\operatorname{asinh}\left(\frac{3}{2x}\right)}{27}-\frac{1}{9x\sqrt{1+\frac{9}{4x^2}}}-\frac{1}{4x^3\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4*x**2+9)**(1/2),x)
```

```
[Out] 2*asinh(3/(2*x))/27 - 1/(9*x*sqrt(1 + 9/(4*x**2))) - 1/(4*x**3*sqrt(1 + 9/(
4*x**2)))
```

3.541 $\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{8\sqrt{4x^2+9}}{243x}-\frac{\sqrt{4x^2+9}}{27x^3}$$

```
[Out] -1/27*(4*x^2+9)^(1/2)/x^3+8/243*(4*x^2+9)^(1/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{8\sqrt{4x^2+9}}{243x}-\frac{\sqrt{4x^2+9}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(27*x^3) + (8*Sqrt[9 + 4*x^2])/(243*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx &= -\frac{\sqrt{9+4x^2}}{27x^3}-\frac{8}{27}\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx\\ &= -\frac{\sqrt{9+4x^2}}{27x^3}+\frac{8\sqrt{9+4x^2}}{243x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 0.73

$$-\frac{\left(9-8x^2\right)\sqrt{\frac{4x^2}{9}+1}}{81x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -1/81*((9 - 8*x^2)*Sqrt[1 + (4*x^2)/9])/x^3
```

fricas [A] time = 0.76, size = 28, normalized size = 0.76

$$\frac{16\,x^3+\left(8\,x^2-9\right)\sqrt{4\,x^2+9}}{243\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/243*(16*x^3 + (8*x^2 - 9)*sqrt(4*x^2 + 9))/x^3
```

giac [A] time = 1.22, size = 42, normalized size = 1.14

$$\frac{32\left(\left(2\,x-\sqrt{4\,x^2+9}\right)^2-3\right)}{\left(\left(2\,x-\sqrt{4\,x^2+9}\right)^2-9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 32*((2*x - sqrt(4*x^2 + 9))^2 - 3)/((2*x - sqrt(4*x^2 + 9))^2 - 9)^3
```

maple [A] time = 0.00, size = 22, normalized size = 0.59

$$\frac{\sqrt{4x^2+9}\,\left(8x^2-9\right)}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/243*(4*x^2+9)^(1/2)*(8*x^2-9)/x^3
```

maxima [A] time = 2.86, size = 29, normalized size = 0.78

$$\frac{8\,\sqrt{4\,x^2+9}}{243\,x}-\frac{\sqrt{4\,x^2+9}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/243*sqrt(4*x^2 + 9)/x - 1/27*sqrt(4*x^2 + 9)/x^3
```

mupad [B] time = 0.02, size = 19, normalized size = 0.51

$$\sqrt{x^2+\frac{9}{4}}\,\left(\frac{16}{243\,x}-\frac{2}{27\,x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(4*x^2 + 9)^(1/2)),x)
```

```
[Out] (x^2 + 9/4)^(1/2)*(16/(243*x) - 2/(27*x^3))
```

sympy [A] time = 1.29, size = 32, normalized size = 0.86

$$\frac{16\sqrt{1+\frac{9}{4x^2}}}{243}-\frac{2\sqrt{1+\frac{9}{4x^2}}}{27x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4*x**2+9)**(1/2),x)
```

```
[Out] 16*sqrt(1 + 9/(4*x**2))/243 - 2*sqrt(1 + 9/(4*x**2))/(27*x**2)
```

3.542 $\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{4x^2+9}}{54x^2}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{36x^4}$$

```
[Out] -2/81*arctanh(1/3*(4*x^2+9)^(1/2))-1/36*(4*x^2+9)^(1/2)/x^4+1/54*(4*x^2+9)^
(1/2)/x^2
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 207}

$$\frac{\sqrt{4x^2+9}}{54x^2}-\frac{\sqrt{4x^2+9}}{36x^4}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(36*x^4) + Sqrt[9 + 4*x^2]/(54*x^2) - (2*ArcTanh[Sqrt[9 +
4*x^2]/3])/81
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{9+4x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{36x^4}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{x^2\sqrt{9+4x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{36x^4}+\frac{\sqrt{9+4x^2}}{54x^2}+\frac{1}{27}\text{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{36x^4}+\frac{\sqrt{9+4x^2}}{54x^2}+\frac{1}{54}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx,x,\sqrt{9+4x^2}\right)\\ &= -\frac{\sqrt{9+4x^2}}{36x^4}+\frac{\sqrt{9+4x^2}}{54x^2}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$-\frac{16}{729}\sqrt{4x^2+9}\ {}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{4x^2}{9}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[9 + 4*x^2]),x]
```

```
[Out] (-16*Sqrt[9 + 4*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (4*x^2)/9])/729
```

fricas [A] time = 0.83, size = 64, normalized size = 1.12

$$-\frac{8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)-3\,\sqrt{4\,x^2+9}\left(2\,x^2-3\right)}{324\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/324*(8*x^4*log(-2*x + sqrt(4*x^2 + 9) + 3) - 8*x^4*log(-2*x + sqrt(4*x^2
 + 9) - 3) - 3*sqrt(4*x^2 + 9)*(2*x^2 - 3))/x^4
```

giac [A] time = 1.18, size = 55, normalized size = 0.96

$$\frac{\left(4\,x^2+9\right)^{\frac{3}{2}}-15\,\sqrt{4\,x^2+9}}{216\,x^4}-\frac{1}{81}\,\log\left(\sqrt{4\,x^2+9}+3\right)+\frac{1}{81}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*((4*x^2 + 9)^(3/2) - 15*sqrt(4*x^2 + 9))/x^4 - 1/81*log(sqrt(4*x^2 +
9) + 3) + 1/81*log(sqrt(4*x^2 + 9) - 3)
```

maple [A] time = 0.00, size = 44, normalized size = 0.77

$$-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{81}+\frac{\sqrt{4x^2+9}}{54x^2}-\frac{\sqrt{4x^2+9}}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4*x^2+9)^(1/2),x)
```

```
[Out] -1/36*(4*x^2+9)^(1/2)/x^4+1/54*(4*x^2+9)^(1/2)/x^2-2/81*arctanh(3/(4*x^2+9)
^(1/2))
```

maxima [A] time = 2.95, size = 38, normalized size = 0.67

$$\frac{\sqrt{4\,x^2+9}}{54\,x^2}-\frac{\sqrt{4\,x^2+9}}{36\,x^4}-\frac{2}{81}\,\operatorname{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(4*x^2 + 9)/x^2 - 1/36*sqrt(4*x^2 + 9)/x^4 - 2/81*arcsinh(3/2/abs(
x))
```

mupad [B] time = 0.03, size = 33, normalized size = 0.58

$$\frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{2}{27\,x^2}-\frac{1}{9\,x^4}\right)}{2}-\frac{2\,\mathrm{atanh}\left(\frac{2\,\sqrt{x^2+\frac{9}{4}}}{3}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(4*x^2 + 9)^(1/2)),x)
```

```
[Out] ((x^2 + 9/4)^(1/2)*(2/(27*x^2) - 1/(9*x^4)))/2 - (2*atanh((2*(x^2 + 9/4)^(1
/2))/3))/81
```

sympy [A] time = 3.89, size = 63, normalized size = 1.11

$$-\frac{2\operatorname{asinh}\left(\frac{3}{2x}\right)}{81}+\frac{1}{27x\sqrt{1+\frac{9}{4x^2}}}+\frac{1}{36x^3\sqrt{1+\frac{9}{4x^2}}}-\frac{1}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4*x**2+9)**(1/2),x)
```

```
[Out] -2*asinh(3/(2*x))/81 + 1/(27*x*sqrt(1 + 9/(4*x**2))) + 1/(36*x**3*sqrt(1 +
9/(4*x**2))) - 1/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

3.543 $\int \frac{x^5}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{320}\left(9-4x^2\right)^{5/2}+\frac{3}{32}\left(9-4x^2\right)^{3/2}-\frac{81}{64}\sqrt{9-4x^2}$$

```
[Out] 3/32*(-4*x^2+9)^(3/2)-1/320*(-4*x^2+9)^(5/2)-81/64*(-4*x^2+9)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{1}{320}\left(9-4x^2\right)^{5/2}+\frac{3}{32}\left(9-4x^2\right)^{3/2}-\frac{81}{64}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[9 - 4*x^2],x]
```

```
[Out] (-81*Sqrt[9 - 4*x^2])/64 + (3*(9 - 4*x^2)^(3/2))/32 - (9 - 4*x^2)^(5/2)/320
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{9-4x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16\sqrt{9-4x}}-\frac{9}{8}\sqrt{9-4x}+\frac{1}{16}(9-4x)^{3/2}\right)dx,x,x^2\right)\\ &= -\frac{81}{64}\sqrt{9-4x^2}+\frac{3}{32}\left(9-4x^2\right)^{3/2}-\frac{1}{320}\left(9-4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$-\frac{1}{40}\sqrt{9-4x^2}\left(2x^4+6x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[9 - 4*x^2],x]
```

```
[Out] -1/40*(Sqrt[9 - 4*x^2]*(27 + 6*x^2 + 2*x^4))
```

fricas [A] time = 1.01, size = 23, normalized size = 0.50

$$-\frac{1}{40}\left(2\,x^4+6\,x^2+27\right)\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/40*(2*x^4 + 6*x^2 + 27)*sqrt(-4*x^2 + 9)
```

giac [A] time = 1.24, size = 43, normalized size = 0.93

$$-\frac{1}{320}\left(4x^2-9\right)^2\sqrt{-4x^2+9}+\frac{3}{32}\left(-4x^2+9\right)^{\frac{3}{2}}-\frac{81}{64}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/320*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9) + 3/32*(-4*x^2 + 9)^(3/2) - 81/64*sqr
t(-4*x^2 + 9)
```

maple [A] time = 0.00, size = 34, normalized size = 0.74

$$\frac{(2x-3)(2x+3)\left(2x^4+6x^2+27\right)}{40\sqrt{-4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/40*(2*x-3)*(2*x+3)*(2*x^4+6*x^2+27)/(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.95, size = 40, normalized size = 0.87

$$-\frac{1}{20}\sqrt{-4x^2+9}\,x^4-\frac{3}{20}\sqrt{-4x^2+9}\,x^2-\frac{27}{40}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(-4*x^2 + 9)*x^4 - 3/20*sqrt(-4*x^2 + 9)*x^2 - 27/40*sqrt(-4*x^2
+ 9)
```

mupad [B] time = 0.04, size = 23, normalized size = 0.50

$$-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{x^4}{5}+\frac{3x^2}{5}+\frac{27}{10}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(9 - 4*x^2)^(1/2),x)
```

```
[Out] -((9/4 - x^2)^(1/2)*((3*x^2)/5 + x^4/5 + 27/10))/2
```

sympy [A] time = 1.29, size = 46, normalized size = 1.00

$$-\frac{x^4\sqrt{9-4x^2}}{20}-\frac{3x^2\sqrt{9-4x^2}}{20}-\frac{27\sqrt{9-4x^2}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-4*x**2+9)**(1/2),x)
```

```
[Out] -x**4*sqrt(9 - 4*x**2)/20 - 3*x**2*sqrt(9 - 4*x**2)/20 - 27*sqrt(9 - 4*x**2
)/40
```

3.544 $\int \frac{x^4}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=45

$$-\frac{27}{128}\sqrt{9-4x^2}\,x-\frac{1}{16}\sqrt{9-4x^2}\,x^3+\frac{243}{256}\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 243/256*arcsin(2/3*x)-27/128*x*(-4*x^2+9)^(1/2)-1/16*x^3*(-4*x^2+9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 216}

$$-\frac{1}{16}\sqrt{9-4x^2}\,x^3-\frac{27}{128}\sqrt{9-4x^2}\,x+\frac{243}{256}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[9 - 4*x^2],x]
```

```
[Out] (-27*x*Sqrt[9 - 4*x^2])/128 - (x^3*Sqrt[9 - 4*x^2])/16 + (243*ArcSin[(2*x)/
3])/256
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{9-4x^2}}\,dx &= -\frac{1}{16}x^3\sqrt{9-4x^2}+\frac{27}{16}\int \frac{x^2}{\sqrt{9-4x^2}}\,dx\\
&= -\frac{27}{128}x\sqrt{9-4x^2}-\frac{1}{16}x^3\sqrt{9-4x^2}+\frac{243}{128}\int \frac{1}{\sqrt{9-4x^2}}\,dx\\
&= -\frac{27}{128}x\sqrt{9-4x^2}-\frac{1}{16}x^3\sqrt{9-4x^2}+\frac{243}{256}\sin^{-1}\left(\frac{2x}{3}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.76

$$\frac{1}{256}\left(243\sin^{-1}\left(\frac{2x}{3}\right)-2x\sqrt{9-4x^2}\,\left(8x^2+27\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[9 - 4*x^2],x]
```

```
[Out] (-2*x*Sqrt[9 - 4*x^2]*(27 + 8*x^2) + 243*ArcSin[(2*x)/3])/256
```

fricas [A] time = 0.94, size = 40, normalized size = 0.89

$$-\frac{1}{128}\left(8\,x^{3}+27\,x\right)\sqrt{-4\,x^{2}+9}-\frac{243}{128}\,\arctan\left(\frac{\sqrt{-4\,x^{2}+9}-3}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/128*(8*x^3 + 27*x)*sqrt(-4*x^2 + 9) - 243/128*arctan(1/2*(sqrt(-4*x^2 +
9) - 3)/x)
```

giac [A] time = 1.14, size = 26, normalized size = 0.58

$$-\frac{1}{128}\left(8\,x^{2}+27\right)\sqrt{-4\,x^{2}+9}\,x+\frac{243}{256}\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/128*(8*x^2 + 27)*sqrt(-4*x^2 + 9)*x + 243/256*arcsin(2/3*x)
```

maple [A] time = 0.01, size = 34, normalized size = 0.76

$$-\frac{\sqrt{-4x^{2}+9}\;x^{3}}{16}-\frac{27\sqrt{-4x^{2}+9}\;x}{128}+\frac{243\arcsin\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-4*x^2+9)^(1/2),x)
```

```
[Out] 243/256*arcsin(2/3*x)-27/128*(-4*x^2+9)^(1/2)*x-1/16*x^3*(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.93, size = 33, normalized size = 0.73

$$-\frac{1}{16}\,\sqrt{-4\,x^{2}+9}\,x^{3}-\frac{27}{128}\,\sqrt{-4\,x^{2}+9}\,x+\frac{243}{256}\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(-4*x^2 + 9)*x^3 - 27/128*sqrt(-4*x^2 + 9)*x + 243/256*arcsin(2/3
*x)
```

mupad [B] time = 0.03, size = 27, normalized size = 0.60

$$\frac{243\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{256}-\frac{\sqrt{\frac{9}{4}-x^{2}}\,\left(\frac{x^{3}}{4}+\frac{27\,x}{32}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(9 - 4*x^2)^(1/2),x)
```

```
[Out] (243*asin((2*x)/3))/256 - ((9/4 - x^2)^(1/2)*((27*x)/32 + x^3/4))/2
```

sympy [A] time = 0.73, size = 39, normalized size = 0.87

$$-\frac{x^{3}\sqrt{9-4x^{2}}}{16}-\frac{27x\sqrt{9-4x^{2}}}{128}+\frac{243\,\mathrm{asin}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-4*x**2+9)**(1/2),x)
```

```
[Out] -x**3*sqrt(9 - 4*x**2)/16 - 27*x*sqrt(9 - 4*x**2)/128 + 243*asin(2*x/3)/256
```

3.545 $\int \frac{x^3}{\sqrt{9-4x^2}} dx$

Optimal. Leaf size=31

$$\frac{1}{48}\left(9-4x^2\right)^{3/2}-\frac{9}{16}\sqrt{9-4x^2}$$

[Out] 1/48*(-4*x^2+9)^(3/2)-9/16*(-4*x^2+9)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{48}\left(9-4x^2\right)^{3/2}-\frac{9}{16}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[9 - 4*x^2],x]

[Out] (-9*Sqrt[9 - 4*x^2])/16 + (9 - 4*x^2)^(3/2)/48

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{9-4x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{9}{4\sqrt{9-4x}}-\frac{1}{4}\sqrt{9-4x}\right)dx,x,x^2\right)\\ &= -\frac{9}{16}\sqrt{9-4x^2}+\frac{1}{48}\left(9-4x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$-\frac{1}{24}\sqrt{9-4x^2}\left(2x^2+9\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[9 - 4*x^2],x]

[Out] -1/24*(Sqrt[9 - 4*x^2]*(9 + 2*x^2))

fricas [A] time = 0.94, size = 18, normalized size = 0.58

$$-\frac{1}{24}\left(2\,x^2+9\right)\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] -1/24*(2*x^2 + 9)*sqrt(-4*x^2 + 9)
```

giac [A] time = 0.98, size = 23, normalized size = 0.74

$$\frac{1}{48}\left(-4x^2+9\right)^{\frac{3}{2}}-\frac{9}{16}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/48*(-4*x^2 + 9)^(3/2) - 9/16*sqrt(-4*x^2 + 9)
```

maple [A] time = 0.01, size = 29, normalized size = 0.94

$$\frac{(2x-3)(2x+3)\left(2x^2+9\right)}{24\sqrt{-4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^2+9)^(1/2),x)
[Out] 1/24*(2*x-3)*(2*x+3)*(2*x^2+9)/(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.90, size = 26, normalized size = 0.84

$$-\frac{1}{12}\sqrt{-4x^2+9}\,x^2-\frac{3}{8}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/12*sqrt(-4*x^2 + 9)*x^2 - 3/8*sqrt(-4*x^2 + 9)
```

mupad [B] time = 0.02, size = 18, normalized size = 0.58

$$-\frac{\sqrt{\frac{9}{4}-x^2}\left(\frac{x^2}{3}+\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(9 - 4*x^2)^(1/2),x)
[Out] -((9/4 - x^2)^(1/2)*(x^2/3 + 3/2))/2
```

sympy [A] time = 0.38, size = 29, normalized size = 0.94

$$-\frac{x^2\sqrt{9-4x^2}}{12}-\frac{3\sqrt{9-4x^2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**2+9)**(1/2),x)
[Out] -x**2*sqrt(9 - 4*x**2)/12 - 3*sqrt(9 - 4*x**2)/8
```

3.546 $\int \frac{x^2}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=27

$$\frac{9}{16}\sin^{-1}\left(\frac{2x}{3}\right)-\frac{1}{8}x\sqrt{9-4x^2}$$

```
[Out] 9/16*arcsin(2/3*x)-1/8*x*(-4*x^2+9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 216}

$$\frac{9}{16}\sin^{-1}\left(\frac{2x}{3}\right)-\frac{1}{8}x\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[9 - 4*x^2],x]
[Out] -(x*Sqrt[9 - 4*x^2])/8 + (9*ArcSin[(2*x)/3])/16
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{9-4x^2}}\,dx &= -\frac{1}{8}x\sqrt{9-4x^2}+\frac{9}{8}\int \frac{1}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{1}{8}x\sqrt{9-4x^2}+\frac{9}{16}\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{9}{16}\sin^{-1}\left(\frac{2x}{3}\right)-\frac{1}{8}x\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[9 - 4*x^2],x]
[Out] -1/8*(x*Sqrt[9 - 4*x^2]) + (9*ArcSin[(2*x)/3])/16
```

fricas [A] time = 0.87, size = 32, normalized size = 1.19

$$-\frac{1}{8}\sqrt{-4x^2+9}\,x-\frac{9}{8}\arctan\left(\frac{\sqrt{-4x^2+9}-3}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^2 + 9)*x - 9/8*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

giac [A] time = 1.18, size = 19, normalized size = 0.70

$$-\frac{1}{8}\sqrt{-4\,x^2+9}\,x+\frac{9}{16}\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-4*x^2 + 9)*x + 9/16*arcsin(2/3*x)
```

maple [A] time = 0.01, size = 20, normalized size = 0.74

$$-\frac{\sqrt{-4x^2+9}\;x}{8}+\frac{9\arcsin\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*x^2+9)^(1/2),x)
```

```
[Out] 9/16*arcsin(2/3*x)-1/8*(-4*x^2+9)^(1/2)*x
```

maxima [A] time = 2.88, size = 19, normalized size = 0.70

$$-\frac{1}{8}\sqrt{-4\,x^2+9}\,x+\frac{9}{16}\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^2 + 9)*x + 9/16*arcsin(2/3*x)
```

mupad [B] time = 0.02, size = 19, normalized size = 0.70

$$\frac{9\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{16}-\frac{x\,\sqrt{\frac{9}{4}-x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(9 - 4*x^2)^(1/2),x)
```

```
[Out] (9*asin((2*x)/3))/16 - (x*(9/4 - x^2)^(1/2))/4
```

sympy [A] time = 0.23, size = 22, normalized size = 0.81

$$-\frac{x\sqrt{9-4x^2}}{8}+\frac{9\,\mathrm{asin}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*x**2+9)**(1/2),x)
```

```
[Out] -x*sqrt(9 - 4*x**2)/8 + 9*asin(2*x/3)/16
```

3.547 $\int \frac{x}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{4}\sqrt{9-4x^2}$$

```
[Out] -1/4*(-4*x^2+9)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{4}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[9 - 4*x^2],x]
```

```
[Out] -Sqrt[9 - 4*x^2]/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{9-4x^2}}\,dx = -\frac{1}{4}\sqrt{9-4x^2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{4}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[9 - 4*x^2],x]
```

```
[Out] -1/4*Sqrt[9 - 4*x^2]
```

fricas [A] time = 0.98, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-4*x^2 + 9)
```

giac [A] time = 1.13, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] -1/4*sqrt(-4*x^2 + 9)
```

maple [A] time = 0.00, size = 22, normalized size = 1.47

$$\frac{(2x-3)(2x+3)}{4\sqrt{-4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-4*x^2+9)^(1/2),x)
[Out] 1/4*(2*x-3)*(2*x+3)/(-4*x^2+9)^(1/2)
```

maxima [A] time = 1.33, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/4*sqrt(-4*x^2 + 9)
```

mupad [B] time = 4.56, size = 11, normalized size = 0.73

$$-\frac{\sqrt{\frac{9}{4}-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(9 - 4*x^2)^(1/2),x)
[Out] -(9/4 - x^2)^(1/2)/2
```

sympy [A] time = 0.15, size = 12, normalized size = 0.80

$$-\frac{\sqrt{9-4x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x**2+9)**(1/2),x)
[Out] -sqrt(9 - 4*x**2)/4
```

3.548 $\int \frac{1}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] 1/2*arcsin(2/3*x)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {216}

$$\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 - 4*x^2],x]
```

```
[Out] ArcSin[(2*x)/3]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 - 4*x^2],x]
```

```
[Out] ArcSin[(2*x)/3]/2
```

fricas [B] time = 0.94, size = 19, normalized size = 1.90

$$-\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

giac [A] time = 1.12, size = 6, normalized size = 0.60

$$\frac{1}{2}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(2/3*x)
```

maple [A] time = 0.00, size = 7, normalized size = 0.70

$$\frac{\arcsin\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/2*arcsin(2/3*x)
```

maxima [A] time = 2.98, size = 6, normalized size = 0.60

$$\frac{1}{2} \arcsin\left(\frac{2}{3} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsin(2/3*x)
```

mupad [B] time = 0.01, size = 6, normalized size = 0.60

$$\frac{\operatorname{asin}\left(\frac{2\,x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9 - 4*x^2)^(1/2),x)
```

```
[Out] asin((2*x)/3)/2
```

sympy [A] time = 0.15, size = 7, normalized size = 0.70

$$\frac{\operatorname{asin}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+9)**(1/2),x)
```

```
[Out] asin(2*x/3)/2
```

3.549 $\int \frac{1}{x\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

```
[Out] -1/3*arctanh(1/3*(-4*x^2+9)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {266, 63, 206}

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -ArcTanh[Sqrt[9 - 4*x^2]/3]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx, x, x^2\right)\\ &= -\left(\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)\right)\\ &= -\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[9 - 4*x^2]),x]
[Out] -1/3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

fricas [A] time = 0.57, size = 18, normalized size = 0.90

$$\frac{1}{3}\log\left(\frac{\sqrt{-4x^2+9}-3}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/3*log((sqrt(-4*x^2 + 9) - 3)/x)
```

giac [B] time = 0.94, size = 31, normalized size = 1.55

$$-\frac{1}{6}\log\left(\sqrt{-4x^2+9}+3\right)+\frac{1}{6}\log\left(-\sqrt{-4x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] -1/6*log(sqrt(-4*x^2 + 9) + 3) + 1/6*log(-sqrt(-4*x^2 + 9) + 3)
```

maple [A] time = 0.00, size = 15, normalized size = 0.75

$$-\frac{\operatorname{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-4*x^2+9)^(1/2),x)
[Out] -1/3*arctanh(3/(-4*x^2+9)^(1/2))
```

maxima [A] time = 2.99, size = 25, normalized size = 1.25

$$-\frac{1}{3}\log\left(\frac{6\sqrt{-4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/3*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 0.12, size = 20, normalized size = 1.00

$$\frac{\ln\left(\sqrt{\frac{9}{4x^2}-1}-\frac{3\sqrt{\frac{1}{x^2}}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(9 - 4*x^2)^(1/2)),x)
[Out] log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2)/3
```

sympy [A] time = 1.06, size = 26, normalized size = 1.30

$$\begin{cases} -\frac{\operatorname{acosh}\left(\frac{3}{2x}\right)}{3} & \text{for } \frac{9}{4|x^2|} > 1 \\ \frac{i \operatorname{asin}\left(\frac{3}{2x}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-acosh(3/(2*x))/3, 9/(4*Abs(x**2)) > 1), (I*asin(3/(2*x))/3, Tru
e))
```

3.550 $\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{9-4x^2}}{9x}$$

[Out] -1/9*(-4*x^2+9)^(1/2)/x

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\sqrt{9-4x^2}}{9x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[9 - 4*x^2]),x]

[Out] -Sqrt[9 - 4*x^2]/(9*x)

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx=-\frac{\sqrt{9-4x^2}}{9x}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{\sqrt{9-4x^2}}{9x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[9 - 4*x^2]),x]

[Out] -1/9*Sqrt[9 - 4*x^2]/x

fricas [A] time = 0.76, size = 14, normalized size = 0.78

$$-\frac{\sqrt{-4\,x^2+9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="fricas")

[Out] -1/9*sqrt(-4*x^2 + 9)/x

giac [B] time = 1.12, size = 33, normalized size = 1.83

$$\frac{2\,x}{9\left(\sqrt{-4\,x^2+9}-3\right)}-\frac{\sqrt{-4\,x^2+9}-3}{18\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*x/(sqrt(-4*x^2 + 9) - 3) - 1/18*(sqrt(-4*x^2 + 9) - 3)/x
```

maple [A] time = 0.00, size = 25, normalized size = 1.39

$$\frac{(2x-3)(2x+3)}{9\sqrt{-4x^2+9}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/9/x*(2*x-3)*(2*x+3)/(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.90, size = 14, normalized size = 0.78

$$-\frac{\sqrt{-4\,x^2+9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(-4*x^2 + 9)/x
```

mupad [B] time = 0.02, size = 14, normalized size = 0.78

$$-\frac{2\sqrt{\frac{9}{4}-x^2}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(9 - 4*x^2)^(1/2)),x)
```

```
[Out] -(2*(9/4 - x^2)^(1/2))/(9*x)
```

sympy [A] time = 0.80, size = 41, normalized size = 2.28

$$\begin{cases} -\frac{2\sqrt{-1+\frac{9}{4x^2}}}{9} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2i\sqrt{1-\frac{9}{4x^2}}}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 9/(4*x**2))/9, 9/(4*Abs(x**2)) > 1), (-2*I*sqrt(1 -
 9/(4*x**2))/9, True))
```

3.551 $\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{9-4x^2}}{18x^2}-\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

```
[Out] -2/27*arctanh(1/3*(-4*x^2+9)^(1/2))-1/18*(-4*x^2+9)^(1/2)/x^2
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 206}

$$-\frac{\sqrt{9-4x^2}}{18x^2}-\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(18*x^2) - (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/27
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x^2}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{18x^2}+\frac{1}{9}\,\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{18x^2}-\frac{1}{18}\,\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx,x,\sqrt{9-4x^2}\right)\\
&= -\frac{\sqrt{9-4x^2}}{18x^2}-\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.95

$$
\frac{1}{54}\left(-\frac{3\sqrt{9-4x^2}}{x^2}-4\tanh^{-1}\left(\sqrt{1-\frac{4x^2}{9}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[9 - 4*x^2]),x]
```

```
[Out] ((-3*Sqrt[9 - 4*x^2])/x^2 - 4*ArcTanh[Sqrt[1 - (4*x^2)/9]])/54
```

fricas [A] time = 0.91, size = 38, normalized size = 0.97

$$
\frac{4\,x^2\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)-3\,\sqrt{-4\,x^2+9}}{54\,x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(4*x^2*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*sqrt(-4*x^2 + 9))/x^2
```

giac [A] time = 1.21, size = 45, normalized size = 1.15

$$
-\frac{\sqrt{-4\,x^2+9}}{18\,x^2}-\frac{1}{27}\,\log\left(\sqrt{-4\,x^2+9}+3\right)+\frac{1}{27}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(-4*x^2 + 9)/x^2 - 1/27*log(sqrt(-4*x^2 + 9) + 3) + 1/27*log(-sqr
t(-4*x^2 + 9) + 3)
```

maple [A] time = 0.01, size = 30, normalized size = 0.77

$$
-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{27}-\frac{\sqrt{-4x^2+9}}{18x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-4*x^2+9)^(1/2),x)
```

```
[Out] -1/18*(-4*x^2+9)^(1/2)/x^2-2/27*arctanh(3/(-4*x^2+9)^(1/2))
```

maxima [A] time = 2.93, size = 40, normalized size = 1.03

$$-\frac{\sqrt{-4\,x^2+9}}{18\,x^2} - \frac{2}{27}\,\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(-4*x^2 + 9)/x^2 - 2/27*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x)
)
```

mupad [B] time = 0.03, size = 35, normalized size = 0.90

$$\frac{2\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1} - \frac{3\,\sqrt{\frac{1}{x^2}}}{2}\right)}{27} - \frac{\sqrt{\frac{9}{4}-x^2}}{9\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(9 - 4*x^2)^(1/2)),x)
```

```
[Out] (2*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2))/27 - (9/4 - x^2)^(1/2)
/(9*x^2)
```

sympy [A] time = 2.17, size = 99, normalized size = 2.54

$$\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{27} + \frac{1}{9x\sqrt{-1+\frac{9}{4x^2}}} - \frac{1}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\: \frac{9}{4|x^2|} > 1 \\ \frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{27} - \frac{i}{9x\sqrt{1-\frac{9}{4x^2}}} + \frac{i}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-2*acosh(3/(2*x))/27 + 1/(9*x*sqrt(-1 + 9/(4*x**2))) - 1/(4*x**3
*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2)) > 1), (2*I*asin(3/(2*x))/27 - I/(9
*x*sqrt(1 - 9/(4*x**2))) + I/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

3.552 $\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=37

$$-\frac{8\sqrt{9-4x^2}}{243x}-\frac{\sqrt{9-4x^2}}{27x^3}$$

```
[Out] -1/27*(-4*x^2+9)^(1/2)/x^3-8/243*(-4*x^2+9)^(1/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8\sqrt{9-4x^2}}{243x}-\frac{\sqrt{9-4x^2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[9 - 4*x^2]),x]
[Out] -Sqrt[9 - 4*x^2]/(27*x^3) - (8*Sqrt[9 - 4*x^2])/(243*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx &= -\frac{\sqrt{9-4x^2}}{27x^3}+\frac{8}{27}\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx\\ &= -\frac{\sqrt{9-4x^2}}{27x^3}-\frac{8\sqrt{9-4x^2}}{243x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 0.73

$$-\frac{\sqrt{1-\frac{4x^2}{9}}\left(8x^2+9\right)}{81x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[9 - 4*x^2]),x]
[Out] -1/81*(Sqrt[1 - (4*x^2)/9]*(9 + 8*x^2))/x^3
```

fricas [A] time = 0.92, size = 21, normalized size = 0.57

$$-\frac{\left(8\,x^2+9\right)\sqrt{-4\,x^2+9}}{243\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/243*(8*x^2 + 9)*sqrt(-4*x^2 + 9)/x^3
```

giac [B] time = 1.13, size = 73, normalized size = 1.97

$$\frac{2\,x^3\left(\frac{9\left(\sqrt{-4\,x^2+9}-3\right)^2}{x^2}+4\right)}{243\left(\sqrt{-4\,x^2+9}-3\right)^3}-\frac{\sqrt{-4\,x^2+9}-3}{54\,x}-\frac{\left(\sqrt{-4\,x^2+9}-3\right)^3}{1944\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 2/243*x^3*(9*(sqrt(-4*x^2 + 9) - 3)^2/x^2 + 4)/(sqrt(-4*x^2 + 9) - 3)^3 - 1
/54*(sqrt(-4*x^2 + 9) - 3)/x - 1/1944*(sqrt(-4*x^2 + 9) - 3)^3/x^3
```

maple [A] time = 0.00, size = 32, normalized size = 0.86

$$\frac{(2x-3)\,(2x+3)\left(8x^2+9\right)}{243\sqrt{-4x^2+9}\;x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/243*(2*x-3)*(2*x+3)*(8*x^2+9)/x^3/(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.97, size = 29, normalized size = 0.78

$$-\frac{8\sqrt{-4\,x^2+9}}{243\,x}-\frac{\sqrt{-4\,x^2+9}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -8/243*sqrt(-4*x^2 + 9)/x - 1/27*sqrt(-4*x^2 + 9)/x^3
```

mupad [B] time = 0.02, size = 22, normalized size = 0.59

$$-\sqrt{\frac{9}{4}-x^2}\,\left(\frac{16}{243\,x}+\frac{2}{27\,x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(9 - 4*x^2)^(1/2)),x)
```

```
[Out] -(9/4 - x^2)^(1/2)*(16/(243*x) + 2/(27*x^3))
```

sympy [A] time = 1.33, size = 80, normalized size = 2.16

$$\begin{cases} -\frac{16\sqrt{-1+\frac{9}{4x^2}}}{243}-\frac{2\sqrt{-1+\frac{9}{4x^2}}}{27x^2} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{16i\sqrt{1-\frac{9}{4x^2}}}{243}-\frac{2i\sqrt{1-\frac{9}{4x^2}}}{27x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-16*sqrt(-1 + 9/(4*x**2))/243 - 2*sqrt(-1 + 9/(4*x**2))/(27*x**2
), 9/(4*Abs(x**2)) > 1), (-16*I*sqrt(1 - 9/(4*x**2))/243 - 2*I*sqrt(1 - 9/(
4*x**2))/(27*x**2), True))
```

3.553 $\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{9-4x^2}}{54x^2}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{36x^4}$$

```
[Out] -2/81*arctanh(1/3*(-4*x^2+9)^(1/2))-1/36*(-4*x^2+9)^(1/2)/x^4-1/54*(-4*x^2+
9)^(1/2)/x^2
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 206}

$$-\frac{\sqrt{9-4x^2}}{54x^2}-\frac{\sqrt{9-4x^2}}{36x^4}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(36*x^4) - Sqrt[9 - 4*x^2]/(54*x^2) - (2*ArcTanh[Sqrt[9 -
4*x^2]/3])/81
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x^3}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{36x^4}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x^2}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{36x^4}-\frac{\sqrt{9-4x^2}}{54x^2}+\frac{1}{27}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{36x^4}-\frac{\sqrt{9-4x^2}}{54x^2}-\frac{1}{54}\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx,x,\sqrt{9-4x^2}\right)\\
&= -\frac{\sqrt{9-4x^2}}{36x^4}-\frac{\sqrt{9-4x^2}}{54x^2}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$
-\frac{16}{729}\sqrt{9-4x^2}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{4x^2}{9}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[9 - 4*x^2]),x]
```

```
[Out] (-16*Sqrt[9 - 4*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 - (4*x^2)/9])/729
```

fricas [A] time = 0.86, size = 45, normalized size = 0.79

$$
\frac{8\,x^4\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)-3\left(2\,x^2+3\right)\sqrt{-4\,x^2+9}}{324\,x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(8*x^4*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*(2*x^2 + 3)*sqrt(-4*x^2 + 9)
)/x^4
```

giac [A] time = 0.98, size = 57, normalized size = 1.00

$$
\frac{\left(-4\,x^2+9\right)^{\frac{3}{2}}-15\,\sqrt{-4\,x^2+9}}{216\,x^4}-\frac{1}{81}\,\log\left(\sqrt{-4\,x^2+9}+3\right)+\frac{1}{81}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*((-4*x^2 + 9)^(3/2) - 15*sqrt(-4*x^2 + 9))/x^4 - 1/81*log(sqrt(-4*x^2
 + 9) + 3) + 1/81*log(-sqrt(-4*x^2 + 9) + 3)
```

maple [A] time = 0.00, size = 44, normalized size = 0.77

$$
-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{81}-\frac{\sqrt{-4x^2+9}}{54x^2}-\frac{\sqrt{-4x^2+9}}{36x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-4*x^2+9)^(1/2),x)
```

```
[Out] -1/36*(-4*x^2+9)^(1/2)/x^4-1/54*(-4*x^2+9)^(1/2)/x^2-2/81*arctanh(3/(-4*x^2
+9)^(1/2))
```

maxima [A] time = 2.93, size = 54, normalized size = 0.95

$$-\frac{\sqrt{-4\,x^2+9}}{54\,x^2}-\frac{\sqrt{-4\,x^2+9}}{36\,x^4}-\frac{2}{81}\,\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/54*sqrt(-4*x^2 + 9)/x^2 - 1/36*sqrt(-4*x^2 + 9)/x^4 - 2/81*log(6*sqrt(-4
*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 4.51, size = 49, normalized size = 0.86

$$\frac{2\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1}-\sqrt{\frac{9}{4\,x^2}}\right)}{81}-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{2}{27\,x^2}+\frac{1}{9\,x^4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(9 - 4*x^2)^(1/2)),x)
```

```
[Out] (2*log((9/(4*x^2) - 1)^(1/2) - (9/(4*x^2))^(1/2)))/81 - ((9/4 - x^2)^(1/2)*
(2/(27*x^2) + 1/(9*x^4)))/2
```

sympy [A] time = 3.96, size = 136, normalized size = 2.39

$$\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{81}+\frac{1}{27x\sqrt{-1+\frac{9}{4x^2}}}-\frac{1}{36x^3\sqrt{-1+\frac{9}{4x^2}}}-\frac{1}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ \frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{81}-\frac{i}{27x\sqrt{1-\frac{9}{4x^2}}}+\frac{i}{36x^3\sqrt{1-\frac{9}{4x^2}}}+\frac{i}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-2*acosh(3/(2*x))/81 + 1/(27*x*sqrt(-1 + 9/(4*x**2))) - 1/(36*x*
*3*sqrt(-1 + 9/(4*x**2))) - 1/(8*x**5*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2
)) > 1), (2*I*asin(3/(2*x))/81 - I/(27*x*sqrt(1 - 9/(4*x**2))) + I/(36*x**3
*sqrt(1 - 9/(4*x**2))) + I/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

3.554 $\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=46

$$\frac{1}{320}\left(4x^2-9\right)^{5/2}+\frac{3}{32}\left(4x^2-9\right)^{3/2}+\frac{81}{64}\sqrt{4x^2-9}$$

```
[Out] 3/32*(4*x^2-9)^(3/2)+1/320*(4*x^2-9)^(5/2)+81/64*(4*x^2-9)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{320}\left(4x^2-9\right)^{5/2}+\frac{3}{32}\left(4x^2-9\right)^{3/2}+\frac{81}{64}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (81*Sqrt[-9 + 4*x^2])/64 + (3*(-9 + 4*x^2)^(3/2))/32 + (-9 + 4*x^2)^(5/2)/3
20
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{-9+4x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{81}{16\sqrt{-9+4x}}+\frac{9}{8}\sqrt{-9+4x}+\frac{1}{16}(-9+4x)^{3/2}\right)dx,x,x^2\right)\\ &= \frac{81}{64}\sqrt{-9+4x^2}+\frac{3}{32}\left(-9+4x^2\right)^{3/2}+\frac{1}{320}\left(-9+4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$\frac{1}{40}\sqrt{4x^2-9}\left(2x^4+6x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (Sqrt[-9 + 4*x^2]*(27 + 6*x^2 + 2*x^4))/40
```

fricas [A] time = 0.96, size = 23, normalized size = 0.50

$$\frac{1}{40}\left(2\,x^4+6\,x^2+27\right)\sqrt{4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(2*x^4 + 6*x^2 + 27)*sqrt(4*x^2 - 9)
```

giac [A] time = 1.12, size = 34, normalized size = 0.74

$$\frac{1}{320}\left(4x^2-9\right)^{\frac{5}{2}}+\frac{3}{32}\left(4x^2-9\right)^{\frac{3}{2}}+\frac{81}{64}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/320*(4*x^2 - 9)^(5/2) + 3/32*(4*x^2 - 9)^(3/2) + 81/64*sqrt(4*x^2 - 9)
```

maple [A] time = 0.00, size = 34, normalized size = 0.74

$$\frac{(2x-3)(2x+3)\left(2x^4+6x^2+27\right)}{40\sqrt{4x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/40*(2*x-3)*(2*x+3)*(2*x^4+6*x^2+27)/(4*x^2-9)^(1/2)
```

maxima [A] time = 2.93, size = 40, normalized size = 0.87

$$\frac{1}{20}\sqrt{4x^2-9}\,x^4+\frac{3}{20}\sqrt{4x^2-9}\,x^2+\frac{27}{40}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(4*x^2 - 9)*x^4 + 3/20*sqrt(4*x^2 - 9)*x^2 + 27/40*sqrt(4*x^2 - 9)
```

mupad [B] time = 4.74, size = 22, normalized size = 0.48

$$\sqrt{4x^2-9}\left(\frac{x^4}{20}+\frac{3x^2}{20}+\frac{27}{40}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^2 - 9)^(1/2),x)
```

```
[Out] (4*x^2 - 9)^(1/2)*((3*x^2)/20 + x^4/20 + 27/40)
```

sympy [A] time = 1.25, size = 44, normalized size = 0.96

$$\frac{x^4\sqrt{4x^2-9}}{20}+\frac{3x^2\sqrt{4x^2-9}}{20}+\frac{27\sqrt{4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(4*x**2-9)**(1/2),x)
```

```
[Out] x**4*sqrt(4*x**2 - 9)/20 + 3*x**2*sqrt(4*x**2 - 9)/20 + 27*sqrt(4*x**2 - 9)
/40
```

3.555 $\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{27}{128}\sqrt{4x^2-9}\,x+\frac{243}{256}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)+\frac{1}{16}\sqrt{4x^2-9}\,x^3$$

```
[Out] 243/256*arctanh(2*x/(4*x^2-9)^(1/2))+27/128*x*(4*x^2-9)^(1/2)+1/16*x^3*(4*x
^2-9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 217, 206}

$$\frac{1}{16}\sqrt{4x^2-9}\,x^3+\frac{27}{128}\sqrt{4x^2-9}\,x+\frac{243}{256}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (27*x*Sqrt[-9 + 4*x^2])/128 + (x^3*Sqrt[-9 + 4*x^2])/16 + (243*ArcTanh[(2*x
)/Sqrt[-9 + 4*x^2]])/256
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx &= \frac{1}{16}x^3\sqrt{-9+4x^2}+\frac{27}{16}\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx\\
&= \frac{27}{128}x\sqrt{-9+4x^2}+\frac{1}{16}x^3\sqrt{-9+4x^2}+\frac{243}{128}\int \frac{1}{\sqrt{-9+4x^2}}\,dx\\
&= \frac{27}{128}x\sqrt{-9+4x^2}+\frac{1}{16}x^3\sqrt{-9+4x^2}+\frac{243}{128}\operatorname{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x}{\sqrt{-9+4x^2}}\right)\\
&= \frac{27}{128}x\sqrt{-9+4x^2}+\frac{1}{16}x^3\sqrt{-9+4x^2}+\frac{243}{256}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.80

$$\frac{1}{256}\left(2x\sqrt{4x^2-9}\left(8x^2+27\right)+243\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (2*x*Sqrt[-9 + 4*x^2]*(27 + 8*x^2) + 243*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/2
56
```

fricas [A] time = 0.94, size = 37, normalized size = 0.69

$$\frac{1}{128}\left(8\,x^3+27\,x\right)\sqrt{4\,x^2-9}-\frac{243}{256}\,\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/128*(8*x^3 + 27*x)*sqrt(4*x^2 - 9) - 243/256*log(-2*x + sqrt(4*x^2 - 9))
```

giac [A] time = 1.24, size = 37, normalized size = 0.69

$$\frac{1}{128}\left(8\,x^2+27\right)\sqrt{4\,x^2-9}\,x-\frac{243}{256}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/128*(8*x^2 + 27)*sqrt(4*x^2 - 9)*x - 243/256*log(abs(-2*x + sqrt(4*x^2 -
9)))
```

maple [A] time = 0.01, size = 49, normalized size = 0.91

$$\frac{\sqrt{4x^2-9}\,x^3}{16}+\frac{27\sqrt{4x^2-9}\,x}{128}+\frac{243\sqrt{4}\,\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/16*x^3*(4*x^2-9)^(1/2)+27/128*(4*x^2-9)^(1/2)*x+243/512*4^(1/2)*ln(4^(1/2
)*x+(4*x^2-9)^(1/2))
```

maxima [A] time = 2.81, size = 45, normalized size = 0.83

$$\frac{1}{16}\,\sqrt{4\,x^2-9}\,x^3+\frac{27}{128}\,\sqrt{4\,x^2-9}\,x+\frac{243}{256}\,\log\left(8\,x+4\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(4*x^2 - 9)*x^3 + 27/128*sqrt(4*x^2 - 9)*x + 243/256*log(8*x + 4*s
qrt(4*x^2 - 9))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\sqrt{4\,x^2-9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^2 - 9)^(1/2),x)
```

```
[Out] int(x^4/(4*x^2 - 9)^(1/2), x)
```

sympy [A] time = 0.72, size = 39, normalized size = 0.72

$$\frac{x^{3}\sqrt{4x^{2}-9}}{16}+\frac{27x\sqrt{4x^{2}-9}}{128}+\frac{243\operatorname{acosh}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(4*x**2-9)**(1/2),x)
```

```
[Out] x**3*sqrt(4*x**2 - 9)/16 + 27*x*sqrt(4*x**2 - 9)/128 + 243*acosh(2*x/3)/256
```

3.556 $\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{48}\left(4x^2-9\right)^{3/2}+\frac{9}{16}\sqrt{4x^2-9}$$

[Out] 1/48*(4*x^2-9)^(3/2)+9/16*(4*x^2-9)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{48}\left(4x^2-9\right)^{3/2}+\frac{9}{16}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[-9 + 4*x^2],x]

[Out] (9*Sqrt[-9 + 4*x^2])/16 + (-9 + 4*x^2)^(3/2)/48

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{-9+4x}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{9}{4\sqrt{-9+4x}}+\frac{1}{4}\sqrt{-9+4x}\right)dx,x,x^2\right)\\
&= \frac{9}{16}\sqrt{-9+4x^2}+\frac{1}{48}\left(-9+4x^2\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{24}\left(2x^2+9\right)\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[-9 + 4*x^2],x]

[Out] ((9 + 2*x^2)*Sqrt[-9 + 4*x^2])/24

fricas [A] time = 0.91, size = 18, normalized size = 0.58

$$\frac{1}{24}\sqrt{4\,x^2-9}\left(2\,x^2+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/24*sqrt(4*x^2 - 9)*(2*x^2 + 9)
```

giac [A] time = 0.97, size = 23, normalized size = 0.74

$$\frac{1}{48}\left(4x^2-9\right)^{\frac{3}{2}}+\frac{9}{16}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/48*(4*x^2 - 9)^(3/2) + 9/16*sqrt(4*x^2 - 9)
```

maple [A] time = 0.00, size = 29, normalized size = 0.94

$$\frac{(2x-3)(2x+3)\left(2x^2+9\right)}{24\sqrt{4x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2-9)^(1/2),x)
[Out] 1/24*(2*x-3)*(2*x+3)*(2*x^2+9)/(4*x^2-9)^(1/2)
```

maxima [A] time = 2.92, size = 26, normalized size = 0.84

$$\frac{1}{12}\sqrt{4x^2-9}\,x^2+\frac{3}{8}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/12*sqrt(4*x^2 - 9)*x^2 + 3/8*sqrt(4*x^2 - 9)
```

mupad [B] time = 4.86, size = 18, normalized size = 0.58

$$\frac{\left(2x^2+9\right)\sqrt{4x^2-9}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2 - 9)^(1/2),x)
[Out] ((2*x^2 + 9)*(4*x^2 - 9)^(1/2))/24
```

sympy [A] time = 0.37, size = 27, normalized size = 0.87

$$\frac{x^2\sqrt{4x^2-9}}{12}+\frac{3\sqrt{4x^2-9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**2-9)**(1/2),x)
[Out] x**2*sqrt(4*x**2 - 9)/12 + 3*sqrt(4*x**2 - 9)/8
```

3.557 $\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=36

$$\frac{1}{8}\sqrt{4x^2-9}\,x+\frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

```
[Out] 9/16*arctanh(2*x/(4*x^2-9)^(1/2))+1/8*x*(4*x^2-9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = $\{321, 217, 206\}$

$$\frac{1}{8}\sqrt{4x^2-9}\,x+\frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 + 4*x^2])/8 + (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/16
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx &= \frac{1}{8}x\sqrt{-9+4x^2}+\frac{9}{8}\int \frac{1}{\sqrt{-9+4x^2}}\,dx\\ &= \frac{1}{8}x\sqrt{-9+4x^2}+\frac{9}{8}\operatorname{Subst}\left(\int \frac{1}{1-4x^2}\,dx,x,\frac{x}{\sqrt{-9+4x^2}}\right)\\ &= \frac{1}{8}x\sqrt{-9+4x^2}+\frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{1}{8}\sqrt{4x^2-9}\,x+\frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[-9 + 4*x^2],x]
[Out] (x*Sqrt[-9 + 4*x^2])/8 + (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/16
```

fricas [A] time = 0.90, size = 29, normalized size = 0.81

$$\frac{1}{8}\sqrt{4x^2-9}\,x-\frac{9}{16}\log\left(-2x+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(4*x^2 - 9)*x - 9/16*log(-2*x + sqrt(4*x^2 - 9))
```

giac [A] time = 1.18, size = 30, normalized size = 0.83

$$\frac{1}{8}\sqrt{4x^2-9}\,x-\frac{9}{16}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/8*sqrt(4*x^2 - 9)*x - 9/16*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

maple [A] time = 0.01, size = 35, normalized size = 0.97

$$\frac{\sqrt{4x^2-9}\,x}{8}+\frac{9\sqrt{4}\,\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^2-9)^(1/2),x)
[Out] 1/8*(4*x^2-9)^(1/2)*x+9/32*4^(1/2)*ln(4^(1/2)*x+(4*x^2-9)^(1/2))
```

maxima [A] time = 2.91, size = 31, normalized size = 0.86

$$\frac{1}{8}\sqrt{4x^2-9}\,x+\frac{9}{16}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/8*sqrt(4*x^2 - 9)*x + 9/16*log(8*x + 4*sqrt(4*x^2 - 9))
```

mupad [B] time = 0.10, size = 29, normalized size = 0.81

$$\frac{9\ln\left(x+\frac{\sqrt{4x^2-9}}{2}\right)}{16}+\frac{x\sqrt{4x^2-9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^2 - 9)^(1/2),x)
[Out] (9*log(x + (4*x^2 - 9)^(1/2)/2))/16 + (x*(4*x^2 - 9)^(1/2))/8
```

sympy [A] time = 0.22, size = 22, normalized size = 0.61

$$\frac{x\sqrt{4x^2-9}}{8}+\frac{9\operatorname{acosh}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(4*x**2-9)**(1/2),x)
[Out] x*sqrt(4*x**2 - 9)/8 + 9*acosh(2*x/3)/16
```

3.558 $\int \frac{x}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{1}{4}\sqrt{4x^2-9}$$

```
[Out] 1/4*(4*x^2-9)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{4}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-9 + 4*x^2],x]
```

```
[Out] Sqrt[-9 + 4*x^2]/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{-9+4x^2}}\,dx = \frac{1}{4}\sqrt{-9+4x^2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{4}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-9 + 4*x^2],x]
```

```
[Out] Sqrt[-9 + 4*x^2]/4
```

fricas [A] time = 0.76, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(4*x^2 - 9)
```

giac [A] time = 1.07, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(4*x^2 - 9)
```

maple [A] time = 0.00, size = 22, normalized size = 1.47

$$\frac{(2x-3)(2x+3)}{4\sqrt{4x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/4*(2*x-3)*(2*x+3)/(4*x^2-9)^(1/2)
```

maxima [A] time = 1.27, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(4*x^2 - 9)
```

mupad [B] time = 0.14, size = 11, normalized size = 0.73

$$\frac{\sqrt{4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2 - 9)^(1/2),x)
```

```
[Out] (4*x^2 - 9)^(1/2)/4
```

sympy [A] time = 0.15, size = 10, normalized size = 0.67

$$\frac{\sqrt{4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x**2-9)**(1/2),x)
```

```
[Out] sqrt(4*x**2 - 9)/4
```

3.559 $\int \frac{1}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

```
[Out] 1/2*arctanh(2*x/(4*x^2-9)^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{1}{2}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-9 + 4*x^2],x]
```

```
[Out] ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9+4x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x}{\sqrt{-9+4x^2}}\right)\\ &= \frac{1}{2}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.00, size = 43, normalized size = 2.26

$$\frac{1}{4}\log\left(\frac{2x}{\sqrt{4x^2-9}}+1\right)-\frac{1}{4}\log\left(1-\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-9 + 4*x^2],x]
```

```
[Out] -1/4*Log[1 - (2*x)/Sqrt[-9 + 4*x^2]] + Log[1 + (2*x)/Sqrt[-9 + 4*x^2]]/4
```

fricas [A] time = 1.03, size = 16, normalized size = 0.84

$$-\frac{1}{2}\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(-2*x + sqrt(4*x^2 - 9))
```

giac [A] time = 0.98, size = 17, normalized size = 0.89

$$-\frac{1}{2}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] -1/2*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

maple [A] time = 0.00, size = 22, normalized size = 1.16

$$\frac{\sqrt{4}\,\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2-9)^(1/2),x)
[Out] 1/4*4^(1/2)*ln(4^(1/2)*x+(4*x^2-9)^(1/2))
```

maxima [A] time = 2.96, size = 18, normalized size = 0.95

$$\frac{1}{2}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/2*log(8*x + 4*sqrt(4*x^2 - 9))
```

mupad [B] time = 4.75, size = 16, normalized size = 0.84

$$\frac{\ln\left(2x+\sqrt{4x^2-9}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 - 9)^(1/2),x)
[Out] log(2*x + (4*x^2 - 9)^(1/2))/2
```

sympy [A] time = 0.16, size = 7, normalized size = 0.37

$$\frac{\operatorname{acosh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2-9)**(1/2),x)
[Out] acosh(2*x/3)/2
```

3.560 $\int \frac{1}{x\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

```
[Out] 1/3*arctan(1/3*(4*x^2-9)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {266, 63, 203}

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] ArcTan[Sqrt[-9 + 4*x^2]/3]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\
&= \frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-9 + 4*x^2]),x]
[Out] ArcTan[Sqrt[-9 + 4*x^2]/3]/3
```

fricas [A] time = 0.55, size = 18, normalized size = 0.90

$$\frac{2}{3}\,\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 2/3*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9))
```

giac [A] time = 1.06, size = 14, normalized size = 0.70

$$\frac{1}{3}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/3*arctan(1/3*sqrt(4*x^2 - 9))
```

maple [A] time = 0.00, size = 15, normalized size = 0.75

$$-\frac{\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4*x^2-9)^(1/2),x)
[Out] -1/3*arctan(3/(4*x^2-9)^(1/2))
```

maxima [A] time = 2.91, size = 9, normalized size = 0.45

$$-\frac{1}{3}\,\arcsin\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] -1/3*arcsin(3/2/abs(x))
```

mupad [B] time = 0.12, size = 20, normalized size = 1.00

$$\frac{\ln\left(\frac{\sqrt{4\,x^2-9}+3{}\mathrm{i}}{x}\right)\,1{}\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(4*x^2 - 9)^(1/2)),x)
[Out] (log(((4*x^2 - 9)^(1/2) + 3i)/x)*1i)/3
```

sympy [A] time = 1.07, size = 26, normalized size = 1.30

$$\begin{cases} \frac{i \operatorname{acosh}\left(\frac{3}{2x}\right)}{3} & \text{for}\ \frac{9}{4|x^2|} > 1 \\ -\frac{\operatorname{asin}\left(\frac{3}{2x}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(3/(2*x))/3, 9/(4*Abs(x**2)) > 1), (-asin(3/(2*x))/3, Tru
e))
```

3.561 $\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{4x^2-9}}{9x}$$

```
[Out] 1/9*(4*x^2-9)^(1/2)/x
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{\sqrt{4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(9*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx=\frac{\sqrt{-9+4x^2}}{9x}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\sqrt{4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(9*x)
```

fricas [A] time = 0.85, size = 18, normalized size = 1.00

$$\frac{2\,x+\sqrt{4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*x + sqrt(4*x^2 - 9))/x
```

giac [A] time = 1.06, size = 23, normalized size = 1.28

$$\frac{4}{\left(2\,x-\sqrt{4\,x^2-9}\right)^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 4/((2*x - sqrt(4*x^2 - 9))^2 + 9)
```

maple [A] time = 0.00, size = 25, normalized size = 1.39

$$\frac{(2x-3)(2x+3)}{9\sqrt{4x^2-9}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/9/x*(2*x-3)*(2*x+3)/(4*x^2-9)^(1/2)
```

maxima [A] time = 2.84, size = 14, normalized size = 0.78

$$\frac{\sqrt{4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(4*x^2 - 9)/x
```

mupad [B] time = 4.75, size = 14, normalized size = 0.78

$$\frac{\sqrt{4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(4*x^2 - 9)^(1/2)),x)
```

```
[Out] (4*x^2 - 9)^(1/2)/(9*x)
```

sympy [A] time = 0.81, size = 37, normalized size = 2.06

$$\begin{cases} \frac{2i\sqrt{-1+\frac{9}{4x^2}}}{9} & \text{for } \frac{9}{4|x^2|} > 1 \\ \frac{2\sqrt{1-\frac{9}{4x^2}}}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*I*sqrt(-1 + 9/(4*x**2))/9, 9/(4*Abs(x**2)) > 1), (2*sqrt(1 - 9
/(4*x**2))/9, True))
```

3.562 $\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{4x^2-9}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

```
[Out] 2/27*arctan(1/3*(4*x^2-9)^(1/2))+1/18*(4*x^2-9)^(1/2)/x^2
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 203}

$$\frac{\sqrt{4x^2-9}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/27
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{-9+4x}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9+4x^2}}{18x^2}+\frac{1}{9}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9+4x^2}}{18x^2}+\frac{1}{18}\,\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx,x,\sqrt{-9+4x^2}\right)\\ &= \frac{\sqrt{-9+4x^2}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.38

$$\frac{4}{81}\sqrt{4x^2-9}\left(\frac{9}{8x^2}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{4x^2}{9}}\right)}{2\sqrt{1-\frac{4x^2}{9}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] (4*Sqrt[-9 + 4*x^2]*(9/(8*x^2) + ArcTanh[Sqrt[1 - (4*x^2)/9]]/(2*Sqrt[1 - (
4*x^2)/9])))/81
```

fricas [A] time = 0.92, size = 38, normalized size = 0.97

$$\frac{8\,x^2\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)+3\,\sqrt{4\,x^2-9}}{54\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(8*x^2*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9))/x^2
```

giac [A] time = 1.10, size = 29, normalized size = 0.74

$$\frac{\sqrt{4\,x^2-9}}{18\,x^2}+\frac{2}{27}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(4*x^2 - 9)/x^2 + 2/27*arctan(1/3*sqrt(4*x^2 - 9))
```

maple [A] time = 0.01, size = 30, normalized size = 0.77

$$-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{27}+\frac{\sqrt{4x^2-9}}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/18*(4*x^2-9)^(1/2)/x^2-2/27*arctan(3/(4*x^2-9)^(1/2))
```

maxima [A] time = 2.93, size = 24, normalized size = 0.62

$$\frac{\sqrt{4\,x^2-9}}{18\,x^2}-\frac{2}{27}\,\arcsin\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(4*x^2 - 9)/x^2 - 2/27*arcsin(3/2/abs(x))
```

mupad [B] time = 4.88, size = 29, normalized size = 0.74

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)}{27}+\frac{\sqrt{4\,x^2-9}}{18\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(4*x^2 - 9)^(1/2)),x)
```

```
[Out] (2*atan((4*x^2 - 9)^(1/2)/3))/27 + (4*x^2 - 9)^(1/2)/(18*x^2)
```

sympy [A] time = 2.13, size = 99, normalized size = 2.54

$$\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{3}{2x}\right)}{27}-\frac{i}{9x\sqrt{-1+\frac{9}{4x^2}}}+\frac{i}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2\operatorname{asin}\left(\frac{3}{2x}\right)}{27}+\frac{1}{9x\sqrt{1-\frac{9}{4x^2}}}-\frac{1}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/27 - I/(9*x*sqrt(-1 + 9/(4*x**2))) + I/(4*x**
3*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2)) > 1), (-2*asin(3/(2*x))/27 + 1/(9
*x*sqrt(1 - 9/(4*x**2))) - 1/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

3.563 $\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{8\sqrt{4x^2-9}}{243x}+\frac{\sqrt{4x^2-9}}{27x^3}$$

```
[Out] 1/27*(4*x^2-9)^(1/2)/x^3+8/243*(4*x^2-9)^(1/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = $\{271, 264\}$

$$\frac{8\sqrt{4x^2-9}}{243x}+\frac{\sqrt{4x^2-9}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(27*x^3) + (8*Sqrt[-9 + 4*x^2])/(243*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx &= \frac{\sqrt{-9+4x^2}}{27x^3}+\frac{8}{27}\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx\\ &= \frac{\sqrt{-9+4x^2}}{27x^3}+\frac{8\sqrt{-9+4x^2}}{243x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 0.68

$$\frac{\sqrt{4x^2-9}\left(8x^2+9\right)}{243x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] (Sqrt[-9 + 4*x^2]*(9 + 8*x^2))/(243*x^3)
```

fricas [A] time = 0.84, size = 28, normalized size = 0.76

$$\frac{16\,x^3+\left(8\,x^2+9\right)\sqrt{4\,x^2-9}}{243\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/243*(16*x^3 + (8*x^2 + 9)*sqrt(4*x^2 - 9))/x^3
```

giac [A] time = 1.13, size = 42, normalized size = 1.14

$$\frac{32\left(\left(2\,x-\sqrt{4\,x^2-9}\right)^2+3\right)}{\left(\left(2\,x-\sqrt{4\,x^2-9}\right)^2+9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 32*((2*x - sqrt(4*x^2 - 9))^2 + 3)/((2*x - sqrt(4*x^2 - 9))^2 + 9)^3
```

maple [A] time = 0.00, size = 32, normalized size = 0.86

$$\frac{(2x-3)\,(2x+3)\left(8x^2+9\right)}{243\sqrt{4x^2-9}\;x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4*x^2-9)^(1/2),x)
[Out] 1/243*(2*x-3)*(2*x+3)*(8*x^2+9)/x^3/(4*x^2-9)^(1/2)
```

maxima [A] time = 2.96, size = 29, normalized size = 0.78

$$\frac{8\,\sqrt{4\,x^2-9}}{243\,x}+\frac{\sqrt{4\,x^2-9}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 8/243*sqrt(4*x^2 - 9)/x + 1/27*sqrt(4*x^2 - 9)/x^3
```

mupad [B] time = 4.87, size = 31, normalized size = 0.84

$$\frac{8\,x^2\,\sqrt{4\,x^2-9}+9\,\sqrt{4\,x^2-9}}{243\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(4*x^2 - 9)^(1/2)),x)
[Out] (8*x^2*(4*x^2 - 9)^(1/2) + 9*(4*x^2 - 9)^(1/2))/(243*x^3)
```

sympy [A] time = 1.35, size = 76, normalized size = 2.05

$$\begin{cases} \frac{16i\sqrt{-1+\frac{9}{4x^2}}}{243}+\frac{2i\sqrt{-1+\frac{9}{4x^2}}}{27x^2} & \text{for}\ \frac{9}{4|x^2|}>1 \\ \frac{16\sqrt{1-\frac{9}{4x^2}}}{243}+\frac{2\sqrt{1-\frac{9}{4x^2}}}{27x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((16*I*sqrt(-1 + 9/(4*x**2))/243 + 2*I*sqrt(-1 + 9/(4*x**2))/(27*x
**2), 9/(4*Abs(x**2)) > 1), (16*sqrt(1 - 9/(4*x**2))/243 + 2*sqrt(1 - 9/(4*
x**2))/(27*x**2), True))
```

3.564 $\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{4x^2-9}}{54x^2}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)+\frac{\sqrt{4x^2-9}}{36x^4}$$

```
[Out] 2/81*arctan(1/3*(4*x^2-9)^(1/2))+1/36*(4*x^2-9)^(1/2)/x^4+1/54*(4*x^2-9)^(1
/2)/x^2
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 203}

$$\frac{\sqrt{4x^2-9}}{54x^2}+\frac{\sqrt{4x^2-9}}{36x^4}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(36*x^4) + Sqrt[-9 + 4*x^2]/(54*x^2) + (2*ArcTan[Sqrt[-9 +
 4*x^2]/3])/81
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-9+4x^2}}{36x^4} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-9+4x^2}}{36x^4} + \frac{\sqrt{-9+4x^2}}{54x^2} + \frac{1}{27}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-9+4x^2}}{36x^4} + \frac{\sqrt{-9+4x^2}}{54x^2} + \frac{1}{54}\operatorname{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\
&= \frac{\sqrt{-9+4x^2}}{36x^4} + \frac{\sqrt{-9+4x^2}}{54x^2} + \frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$\frac{16}{729}\sqrt{4x^2-9}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{4x^2}{9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] (16*Sqrt[-9 + 4*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 - (4*x^2)/9])/729
```

fricas [A] time = 0.76, size = 45, normalized size = 0.79

$$\frac{16\,x^4\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)+3\,\sqrt{4\,x^2-9}\left(2\,x^2+3\right)}{324\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(16*x^4*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9)*(2*x
^2 + 3))/x^4
```

giac [A] time = 1.04, size = 41, normalized size = 0.72

$$\frac{\left(4\,x^2-9\right)^{\frac{3}{2}}+15\,\sqrt{4\,x^2-9}}{216\,x^4}+\frac{2}{81}\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*((4*x^2 - 9)^(3/2) + 15*sqrt(4*x^2 - 9))/x^4 + 2/81*arctan(1/3*sqrt(4
*x^2 - 9))
```

maple [A] time = 0.01, size = 44, normalized size = 0.77

$$-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{81}+\frac{\sqrt{4x^2-9}}{54x^2}+\frac{\sqrt{4x^2-9}}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/36*(4*x^2-9)^(1/2)/x^4+1/54*(4*x^2-9)^(1/2)/x^2-2/81*arctan(3/(4*x^2-9)^(
1/2))
```

maxima [A] time = 2.92, size = 38, normalized size = 0.67

$$\frac{\sqrt{4\,x^2-9}}{54\,x^2}+\frac{\sqrt{4\,x^2-9}}{36\,x^4}-\frac{2}{81}\,\arcsin\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(4*x^2 - 9)/x^2 + 1/36*sqrt(4*x^2 - 9)/x^4 - 2/81*arcsin(3/2/abs(x
))
```

mupad [B] time = 5.32, size = 57, normalized size = 1.00

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)}{81}+\frac{\frac{10\,\sqrt{4\,x^2-9}}{9}+\frac{2\,\left(4\,x^2-9\right)^{3/2}}{27}}{72\,x^2+\left(4\,x^2-9\right)^2-81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(4*x^2 - 9)^(1/2)),x)
```

```
[Out] (2*atan((4*x^2 - 9)^(1/2)/3))/81 + ((10*(4*x^2 - 9)^(1/2))/9 + (2*(4*x^2 -
9)^(3/2))/27)/(72*x^2 + (4*x^2 - 9)^2 - 81)
```

sympy [A] time = 3.82, size = 136, normalized size = 2.39

$$\begin{cases} \frac{2i\,\mathrm{acosh}\left(\frac{3}{2x}\right)}{81}-\frac{i}{27x\sqrt{-1+\frac{9}{4x^2}}}+\frac{i}{36x^3\sqrt{-1+\frac{9}{4x^2}}}+\frac{i}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2\,\mathrm{asin}\left(\frac{3}{2x}\right)}{81}+\frac{1}{27x\sqrt{1-\frac{9}{4x^2}}}-\frac{1}{36x^3\sqrt{1-\frac{9}{4x^2}}}-\frac{1}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/81 - I/(27*x*sqrt(-1 + 9/(4*x**2))) + I/(36*x
**3*sqrt(-1 + 9/(4*x**2))) + I/(8*x**5*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**
2)) > 1), (-2*asin(3/(2*x))/81 + 1/(27*x*sqrt(1 - 9/(4*x**2))) - 1/(36*x**3
*sqrt(1 - 9/(4*x**2))) - 1/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

3.565 $\int \frac{x^5}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{320}\left(-4x^2-9\right)^{5/2}-\frac{3}{32}\left(-4x^2-9\right)^{3/2}-\frac{81}{64}\sqrt{-4x^2-9}$$

```
[Out] -3/32*(-4*x^2-9)^(3/2)-1/320*(-4*x^2-9)^(5/2)-81/64*(-4*x^2-9)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{1}{320}\left(-4x^2-9\right)^{5/2}-\frac{3}{32}\left(-4x^2-9\right)^{3/2}-\frac{81}{64}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (-81*Sqrt[-9 - 4*x^2])/64 - (3*(-9 - 4*x^2)^(3/2))/32 - (-9 - 4*x^2)^(5/2)/
320
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{-9-4x}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16\sqrt{-9-4x}}+\frac{9}{8}\sqrt{-9-4x}+\frac{1}{16}(-9-4x)^{3/2}\right)dx,x,x^2\right)\\
&= -\frac{81}{64}\sqrt{-9-4x^2}-\frac{3}{32}\left(-9-4x^2\right)^{3/2}-\frac{1}{320}\left(-9-4x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$-\frac{1}{40}\sqrt{-4x^2-9}\left(2x^4-6x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -1/40*(Sqrt[-9 - 4*x^2]*(27 - 6*x^2 + 2*x^4))
```

fricas [A] time = 0.69, size = 23, normalized size = 0.50

$$-\frac{1}{40}\left(2\,x^4-6\,x^2+27\right)\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/40*(2*x^4 - 6*x^2 + 27)*sqrt(-4*x^2 - 9)
```

giac [A] time = 1.00, size = 36, normalized size = 0.78

$$-\frac{1}{320}\left(4x^2+9\right)^{\frac{5}{2}}i+\frac{3}{32}\left(4x^2+9\right)^{\frac{3}{2}}i-\frac{81}{64}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/320*(4*x^2 + 9)^(5/2)*i + 3/32*(4*x^2 + 9)^(3/2)*i - 81/64*sqrt(-4*x^2 -
 9)
```

maple [A] time = 0.00, size = 24, normalized size = 0.52

$$-\frac{\left(2x^4-6x^2+27\right)\sqrt{-4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/40*(2*x^4-6*x^2+27)*(-4*x^2-9)^(1/2)
```

maxima [A] time = 2.91, size = 40, normalized size = 0.87

$$-\frac{1}{20}\sqrt{-4x^2-9}\,x^4+\frac{3}{20}\sqrt{-4x^2-9}\,x^2-\frac{27}{40}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(-4*x^2 - 9)*x^4 + 3/20*sqrt(-4*x^2 - 9)*x^2 - 27/40*sqrt(-4*x^2
- 9)
```

mupad [B] time = 5.29, size = 23, normalized size = 0.50

$$-\sqrt{-4x^2-9}\left(\frac{x^4}{20}-\frac{3x^2}{20}+\frac{27}{40}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] -(- 4*x^2 - 9)^(1/2)*(x^4/20 - (3*x^2)/20 + 27/40)
```

sympy [A] time = 1.21, size = 49, normalized size = 1.07

$$-\frac{x^4\sqrt{-4x^2-9}}{20}+\frac{3x^2\sqrt{-4x^2-9}}{20}-\frac{27\sqrt{-4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x**4*sqrt(-4*x**2 - 9)/20 + 3*x**2*sqrt(-4*x**2 - 9)/20 - 27*sqrt(-4*x**2
- 9)/40
```

3.566 $\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{27}{128}\sqrt{-4x^2-9}\,x+\frac{243}{256}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)-\frac{1}{16}\sqrt{-4x^2-9}\,x^3$$

```
[Out] 243/256*arctan(2*x/(-4*x^2-9)^(1/2))+27/128*x*(-4*x^2-9)^(1/2)-1/16*x^3*(-4
*x^2-9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 217, 203}

$$-\frac{1}{16}\sqrt{-4x^2-9}\,x^3+\frac{27}{128}\sqrt{-4x^2-9}\,x+\frac{243}{256}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (27*x*Sqrt[-9 - 4*x^2])/128 - (x^3*Sqrt[-9 - 4*x^2])/16 + (243*ArcTan[(2*x)
/Sqrt[-9 - 4*x^2]])/256
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx &= -\frac{1}{16}x^3\sqrt{-9-4x^2}-\frac{27}{16}\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx\\
&= \frac{27}{128}x\sqrt{-9-4x^2}-\frac{1}{16}x^3\sqrt{-9-4x^2}+\frac{243}{128}\int \frac{1}{\sqrt{-9-4x^2}}\,dx\\
&= \frac{27}{128}x\sqrt{-9-4x^2}-\frac{1}{16}x^3\sqrt{-9-4x^2}+\frac{243}{128}\operatorname{Subst}\left(\int \frac{1}{1+4x^2}\,dx,x,\frac{x}{\sqrt{-9-4x^2}}\right)\\
&= \frac{27}{128}x\sqrt{-9-4x^2}-\frac{1}{16}x^3\sqrt{-9-4x^2}+\frac{243}{256}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.80

$$\frac{1}{256}\left(2x\sqrt{-4x^2-9}\left(27-8x^2\right)+243\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (2*x*(27 - 8*x^2)*Sqrt[-9 - 4*x^2] + 243*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/25
6
```

fricas [C] time = 1.23, size = 67, normalized size = 1.24

$$-\frac{1}{128}\left(8x^3-27x\right)\sqrt{-4x^2-9}+\frac{243}{512}i\log\left(-\frac{8x+4i\sqrt{-4x^2-9}}{x}\right)-\frac{243}{512}i\log\left(-\frac{8x-4i\sqrt{-4x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/128*(8*x^3 - 27*x)*sqrt(-4*x^2 - 9) + 243/512*I*log(-(8*x + 4*I*sqrt(-4*
x^2 - 9))/x) - 243/512*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{-4x^2-9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(-4*x^2 - 9), x)
```

maple [A] time = 0.00, size = 43, normalized size = 0.80

$$-\frac{\sqrt{-4x^2-9}\,x^3}{16}+\frac{27\sqrt{-4x^2-9}\,x}{128}+\frac{243\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-4*x^2-9)^(1/2),x)
```

```
[Out] 243/256*arctan(2/(-4*x^2-9)^(1/2)*x)+27/128*(-4*x^2-9)^(1/2)*x-1/16*x^3*(-4
*x^2-9)^(1/2)
```

maxima [C] time = 2.95, size = 33, normalized size = 0.61

$$-\frac{1}{16}\sqrt{-4x^2-9}\,x^3+\frac{27}{128}\sqrt{-4x^2-9}\,x-\frac{243}{256}i\,\mathrm{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(-4*x^2 - 9)*x^3 + 27/128*sqrt(-4*x^2 - 9)*x - 243/256*I*arcsinh(
2/3*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^4}{\sqrt{-4x^2-9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] int(x^4/(- 4*x^2 - 9)^(1/2), x)
```

sympy [**A**] time = 0.89, size = 53, normalized size = 0.98

$$-\frac{x^3\sqrt{-4x^2-9}}{16}+\frac{27x\sqrt{-4x^2-9}}{128}+\frac{243\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x**3*sqrt(-4*x**2 - 9)/16 + 27*x*sqrt(-4*x**2 - 9)/128 + 243*atan(2*x/sqrt
(-4*x**2 - 9))/256
```

3.567 $\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{48}\left(-4x^2-9\right)^{3/2}+\frac{9}{16}\sqrt{-4x^2-9}$$

```
[Out] 1/48*(-4*x^2-9)^(3/2)+9/16*(-4*x^2-9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{48}\left(-4x^2-9\right)^{3/2}+\frac{9}{16}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-9 - 4*x^2],x]
[Out] (9*Sqrt[-9 - 4*x^2])/16 + (-9 - 4*x^2)^(3/2)/48
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{-9-4x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{9}{4\sqrt{-9-4x}}-\frac{1}{4}\sqrt{-9-4x}\right)dx,x,x^2\right)\\ &= \frac{9}{16}\sqrt{-9-4x^2}+\frac{1}{48}\left(-9-4x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{24}\sqrt{-4x^2-9}\left(9-2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-9 - 4*x^2],x]
[Out] (Sqrt[-9 - 4*x^2]*(9 - 2*x^2))/24
```

fricas [A] time = 0.90, size = 18, normalized size = 0.58

$$-\frac{1}{24}\left(2\,x^2-9\right)\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*x^2 - 9)*sqrt(-4*x^2 - 9)
```

giac [A] time = 1.08, size = 24, normalized size = 0.77

$$-\frac{1}{48}\left(4x^2+9\right)^{\frac{3}{2}}i+\frac{9}{16}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*(4*x^2 + 9)^(3/2)*i + 9/16*sqrt(-4*x^2 - 9)
```

maple [A] time = 0.00, size = 19, normalized size = 0.61

$$-\frac{\left(2x^2-9\right)\sqrt{-4x^2-9}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/24*(2*x^2-9)*(-4*x^2-9)^(1/2)
```

maxima [A] time = 2.96, size = 26, normalized size = 0.84

$$-\frac{1}{12}\sqrt{-4x^2-9}\,x^2+\frac{3}{8}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(-4*x^2 - 9)*x^2 + 3/8*sqrt(-4*x^2 - 9)
```

mupad [B] time = 5.07, size = 18, normalized size = 0.58

$$-\frac{\left(2x^2-9\right)\sqrt{-4x^2-9}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] -((2*x^2 - 9)*(- 4*x^2 - 9)^(1/2))/24
```

sympy [A] time = 0.37, size = 31, normalized size = 1.00

$$-\frac{x^2\sqrt{-4x^2-9}}{12}+\frac{3\sqrt{-4x^2-9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x**2*sqrt(-4*x**2 - 9)/12 + 3*sqrt(-4*x**2 - 9)/8
```

3.568 $\int \frac{x^2}{\sqrt{-9-4x^2}} dx$

Optimal. Leaf size=36

$$-\frac{1}{8}\sqrt{-4x^2-9}\,x-\frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

```
[Out] -9/16*arctan(2*x/(-4*x^2-9)^(1/2))-1/8*x*(-4*x^2-9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 217, 203}

$$-\frac{1}{8}\sqrt{-4x^2-9}\,x-\frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -(x*Sqrt[-9 - 4*x^2])/8 - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/16
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx &= -\frac{1}{8}x\sqrt{-9-4x^2}-\frac{9}{8}\int \frac{1}{\sqrt{-9-4x^2}}\,dx\\
&= -\frac{1}{8}x\sqrt{-9-4x^2}-\frac{9}{8}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right)\\
&= -\frac{1}{8}x\sqrt{-9-4x^2}-\frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$-\frac{1}{8}\sqrt{-4x^2-9}\,x-\frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[-9 - 4*x^2],x]
[Out] -1/8*(x*Sqrt[-9 - 4*x^2]) - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/16
```

fricas [C] time = 0.88, size = 59, normalized size = 1.64

$$-\frac{1}{8}\sqrt{-4x^2-9}\,x-\frac{9}{32}i\log\left(-\frac{8x+4i\sqrt{-4x^2-9}}{x}\right)+\frac{9}{32}i\log\left(-\frac{8x-4i\sqrt{-4x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] -1/8*sqrt(-4*x^2 - 9)*x - 9/32*I*log(-(8*x + 4*I*sqrt(-4*x^2 - 9))/x) + 9/3
2*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{-4x^2-9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(-4*x^2 - 9), x)
```

maple [A] time = 0.01, size = 29, normalized size = 0.81

$$-\frac{\sqrt{-4x^2-9}\,x}{8}-\frac{9\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*x^2-9)^(1/2),x)
[Out] -9/16*arctan(2/(-4*x^2-9)^(1/2)*x)-1/8*(-4*x^2-9)^(1/2)*x
```

maxima [C] time = 2.93, size = 19, normalized size = 0.53

$$-\frac{1}{8}\sqrt{-4x^2-9}\,x+\frac{9}{16}i\,\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] -1/8*sqrt(-4*x^2 - 9)*x + 9/16*I*arcsinh(2/3*x)
```

mupad [B] time = 0.12, size = 31, normalized size = 0.86

$$-\frac{x\sqrt{-4x^2-9}}{8}+\frac{\ln\left(x-\frac{\sqrt{-4x^2-9}\,1\mathrm{i}}{2}\right)9\mathrm{i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(- 4*x^2 - 9)^(1/2),x)
[Out] (log(x - ((- 4*x^2 - 9)^(1/2)*1i)/2)*9i)/16 - (x*(- 4*x^2 - 9)^(1/2))/8
```

sympy [A] time = 0.39, size = 36, normalized size = 1.00

$$-\frac{x\sqrt{-4x^2-9}}{8}-\frac{9\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x*sqrt(-4*x**2 - 9)/8 - 9*atan(2*x/sqrt(-4*x**2 - 9))/16
```

3.569 $\int \frac{x}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

```
[Out] -1/4*(-4*x^2-9)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-9 - 4*x^2],x]
[Out] -Sqrt[-9 - 4*x^2]/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{4}\sqrt{-9-4x^2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-9 - 4*x^2],x]
[Out] -1/4*Sqrt[-9 - 4*x^2]
```

fricas [A] time = 0.69, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(-4*x^2 - 9)
```

giac [A] time = 1.13, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-4*x^2 - 9)
```

maple [A] time = 0.00, size = 12, normalized size = 0.80

$$-\frac{\sqrt{-4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2-9)^(1/2)*x,x)
```

```
[Out] -1/4*(-4*x^2-9)^(1/2)
```

maxima [A] time = 1.32, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-4*x^2 - 9)
```

mupad [B] time = 4.99, size = 11, normalized size = 0.73

$$-\frac{\sqrt{-4\,x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] -(- 4*x^2 - 9)^(1/2)/4
```

sympy [A] time = 0.16, size = 14, normalized size = 0.93

$$-\frac{\sqrt{-4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x**2-9)**(1/2),x)
```

```
[Out] -sqrt(-4*x**2 - 9)/4
```

3.570 $\int \frac{1}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

[Out] 1/2*arctan(2*x/(-4*x^2-9)^(1/2))

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 203}

$$\frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-9 - 4*x^2],x]

[Out] ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]/2

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9-4x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right)\\ &= \frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-9 - 4*x^2],x]

[Out] ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]/2

fricas [C] time = 0.79, size = 47, normalized size = 2.47

$$\frac{1}{4}i\log\left(-\frac{8x+4i\sqrt{-4x^2-9}}{x}\right)-\frac{1}{4}i\log\left(-\frac{8x-4i\sqrt{-4x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*I*log(-(8*x + 4*I*sqrt(-4*x^2 - 9))/x) - 1/4*I*log(-(8*x - 4*I*sqrt(-4*
x^2 - 9))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4\,x^2-9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-4*x^2 - 9), x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2-9)^(1/2),x)
```

```
[Out] 1/2*arctan(2/(-4*x^2-9)^(1/2)*x)
```

maxima [C] time = 2.98, size = 6, normalized size = 0.32

$$-\frac{1}{2}i\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*arcsinh(2/3*x)
```

mupad [B] time = 0.11, size = 15, normalized size = 0.79

$$\frac{\operatorname{atan}\left(\frac{2\,x}{\sqrt{-4\,x^2-9}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] atan((2*x)/(- 4*x^2 - 9)^(1/2))/2
```

sympy [A] time = 0.32, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2-9)**(1/2),x)
```

```
[Out] atan(2*x/sqrt(-4*x**2 - 9))/2
```

3.571 $\int \frac{1}{x\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

```
[Out] 1/3*arctan(1/3*(-4*x^2-9)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {266, 63, 204}

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] ArcTan[Sqrt[-9 - 4*x^2]/3]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx, x, x^2\right)\\
&= -\left(\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\right)\\
&= \frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] ArcTan[Sqrt[-9 - 4*x^2]/3]/3
```

fricas [C] time = 0.76, size = 43, normalized size = 2.15

$$-\frac{1}{6} i \log\left(-\frac{2\left(i \sqrt{-4 x^2-9}+3\right)}{3 x}\right)+\frac{1}{6} i \log\left(-\frac{2\left(-i \sqrt{-4 x^2-9}+3\right)}{3 x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*I*log(-2/3*(I*sqrt(-4*x^2 - 9) + 3)/x) + 1/6*I*log(-2/3*(-I*sqrt(-4*x^
2 - 9) + 3)/x)
```

giac [A] time = 1.13, size = 14, normalized size = 0.70

$$\frac{1}{3} \arctan\left(\frac{1}{3} \sqrt{-4 x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*arctan(1/3*sqrt(-4*x^2 - 9))
```

maple [A] time = 0.00, size = 15, normalized size = 0.75

$$-\frac{\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/3*arctan(3/(-4*x^2-9)^(1/2))
```

maxima [C] time = 2.94, size = 25, normalized size = 1.25

$$-\frac{1}{3} i \log\left(\frac{6 \sqrt{4 x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 5.27, size = 14, normalized size = 0.70

$$\frac{\operatorname{atan}\left(\frac{\sqrt{-4 x^2-9}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(- 4*x^2 - 9)^(1/2)),x)
```

```
[Out] atan((- 4*x^2 - 9)^(1/2)/3)/3
```

sympy [C] time = 1.02, size = 8, normalized size = 0.40

$$\frac{i\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x**2-9)**(1/2),x)
```

```
[Out] I*asinh(3/(2*x))/3
```

3.572 $\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{-4x^2-9}}{9x}$$

```
[Out] 1/9/x*(-4*x^2-9)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{\sqrt{-4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] Sqrt[-9 - 4*x^2]/(9*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-9-4x^2}}{9x}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\sqrt{-4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] Sqrt[-9 - 4*x^2]/(9*x)
```

fricas [A] time = 0.87, size = 14, normalized size = 0.78

$$\frac{\sqrt{-4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(-4*x^2 - 9)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4\,x^2-9}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-4*x^2 - 9)*x^2), x)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\sqrt{-4x^2-9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-4*x^2-9)^(1/2),x)
[Out] 1/9/x*(-4*x^2-9)^(1/2)
```

maxima [A] time = 2.95, size = 14, normalized size = 0.78

$$\frac{\sqrt{-4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/9*sqrt(-4*x^2 - 9)/x
```

mupad [B] time = 5.04, size = 14, normalized size = 0.78

$$\frac{\sqrt{-4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(- 4*x^2 - 9)^(1/2)),x)
[Out] (- 4*x^2 - 9)^(1/2)/(9*x)
```

sympy [C] time = 0.77, size = 15, normalized size = 0.83

$$\frac{2i\sqrt{1+\frac{9}{4x^2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-4*x**2-9)**(1/2),x)
[Out] 2*I*sqrt(1 + 9/(4*x**2))/9
```

3.573 $\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{-4x^2-9}}{18x^2}-\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

```
[Out] -2/27*arctan(1/3*(-4*x^2-9)^(1/2))+1/18*(-4*x^2-9)^(1/2)/x^2
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 204}

$$\frac{\sqrt{-4x^2-9}}{18x^2}-\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] Sqrt[-9 - 4*x^2]/(18*x^2) - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/27
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x^2}\,dx,x,x^2\right)\\
&= \frac{\sqrt{-9-4x^2}}{18x^2}-\frac{1}{9}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx,x,x^2\right)\\
&= \frac{\sqrt{-9-4x^2}}{18x^2}+\frac{1}{18}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx,x,\sqrt{-9-4x^2}\right)\\
&= \frac{\sqrt{-9-4x^2}}{18x^2}-\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.38

$$
-\frac{4}{81}\sqrt{-4x^2-9}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{4x^2}{9}+1}\right)}{2\sqrt{\frac{4x^2}{9}+1}}-\frac{9}{8x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] (-4*Sqrt[-9 - 4*x^2]*(-9/(8*x^2) + ArcTanh[Sqrt[1 + (4*x^2)/9]]/(2*Sqrt[1 +
 (4*x^2)/9])))/81
```

fricas [C] time = 0.87, size = 65, normalized size = 1.67

$$
\frac{-2i\,x^2\log\left(-\frac{4\left(i\sqrt{-4\,x^2-9}-3\right)}{27\,x}\right)+2i\,x^2\log\left(-\frac{4\left(-i\sqrt{-4\,x^2-9}-3\right)}{27\,x}\right)+3\sqrt{-4\,x^2-9}}{54\,x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(-2*I*x^2*log(-4/27*(I*sqrt(-4*x^2 - 9) - 3)/x) + 2*I*x^2*log(-4/27*(-
I*sqrt(-4*x^2 - 9) - 3)/x) + 3*sqrt(-4*x^2 - 9))/x^2
```

giac [A] time = 1.07, size = 29, normalized size = 0.74

$$
\frac{\sqrt{-4\,x^2-9}}{18\,x^2}-\frac{2}{27}\arctan\left(\frac{1}{3}\sqrt{-4\,x^2-9}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(-4*x^2 - 9)/x^2 - 2/27*arctan(1/3*sqrt(-4*x^2 - 9))
```

maple [A] time = 0.00, size = 30, normalized size = 0.77

$$
\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{27}+\frac{\sqrt{-4x^2-9}}{18x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-4*x^2-9)^(1/2),x)
```

```
[Out] 1/18*(-4*x^2-9)^(1/2)/x^2+2/27*arctan(3/(-4*x^2-9)^(1/2))
```

maxima [C] time = 2.92, size = 40, normalized size = 1.03

$$\frac{\sqrt{-4\,x^2-9}}{18\,x^2}+\frac{2}{27}i\,\log\left(\frac{6\,\sqrt{4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(-4*x^2 - 9)/x^2 + 2/27*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x)
)
```

mupad [B] time = 5.18, size = 29, normalized size = 0.74

$$\frac{\sqrt{-4\,x^2-9}}{18\,x^2}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(- 4*x^2 - 9)^(1/2)),x)
```

```
[Out] (- 4*x^2 - 9)^(1/2)/(18*x^2) - (2*atan((- 4*x^2 - 9)^(1/2)/3))/27
```

sympy [C] time = 2.06, size = 46, normalized size = 1.18

$$-\frac{2i\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{27}+\frac{i}{9x\sqrt{1+\frac{9}{4x^2}}}+\frac{i}{4x^3\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-4*x**2-9)**(1/2),x)
```

```
[Out] -2*I*asinh(3/(2*x))/27 + I/(9*x*sqrt(1 + 9/(4*x**2))) + I/(4*x**3*sqrt(1 +
9/(4*x**2)))
```

3.574 $\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{-4x^2-9}}{27x^3}-\frac{8\sqrt{-4x^2-9}}{243x}$$

```
[Out] 1/27*(-4*x^2-9)^(1/2)/x^3-8/243/x*(-4*x^2-9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{\sqrt{-4x^2-9}}{27x^3}-\frac{8\sqrt{-4x^2-9}}{243x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] Sqrt[-9 - 4*x^2]/(27*x^3) - (8*Sqrt[-9 - 4*x^2])/(243*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx &= \frac{\sqrt{-9-4x^2}}{27x^3}-\frac{8}{27}\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx\\ &= \frac{\sqrt{-9-4x^2}}{27x^3}-\frac{8\sqrt{-9-4x^2}}{243x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.68

$$\frac{\left(9-8x^2\right)\sqrt{-4x^2-9}}{243x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] ((9 - 8*x^2)*Sqrt[-9 - 4*x^2])/(243*x^3)
```

fricas [A] time = 0.57, size = 21, normalized size = 0.57

$$-\frac{\left(8\,x^2-9\right)\sqrt{-4\,x^2-9}}{243\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/243*(8*x^2 - 9)*sqrt(-4*x^2 - 9)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4\,x^2-9}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-4*x^2 - 9)*x^4), x)
```

maple [A] time = 0.00, size = 22, normalized size = 0.59

$$-\frac{\left(8x^2-9\right)\sqrt{-4x^2-9}}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/243*(8*x^2-9)/x^3*(-4*x^2-9)^(1/2)
```

maxima [A] time = 2.96, size = 29, normalized size = 0.78

$$-\frac{8\sqrt{-4\,x^2-9}}{243\,x}+\frac{\sqrt{-4\,x^2-9}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -8/243*sqrt(-4*x^2 - 9)/x + 1/27*sqrt(-4*x^2 - 9)/x^3
```

mupad [B] time = 5.08, size = 31, normalized size = 0.84

$$-\frac{8\,x^2\,\sqrt{-4\,x^2-9}-9\,\sqrt{-4\,x^2-9}}{243\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(- 4*x^2 - 9)^(1/2)),x)
```

```
[Out] -(8*x^2*(- 4*x^2 - 9)^(1/2) - 9*(- 4*x^2 - 9)^(1/2))/(243*x^3)
```

sympy [C] time = 1.29, size = 36, normalized size = 0.97

$$-\frac{16i\sqrt{1+\frac{9}{4x^2}}}{243}+\frac{2i\sqrt{1+\frac{9}{4x^2}}}{27x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-4*x**2-9)**(1/2),x)
```

```
[Out] -16*I*sqrt(1 + 9/(4*x**2))/243 + 2*I*sqrt(1 + 9/(4*x**2))/(27*x**2)
```

3.575 $\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{-4x^2-9}}{54x^2}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)+\frac{\sqrt{-4x^2-9}}{36x^4}$$

```
[Out] 2/81*arctan(1/3*(-4*x^2-9)^(1/2))+1/36*(-4*x^2-9)^(1/2)/x^4-1/54*(-4*x^2-9)
^(1/2)/x^2
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 204}

$$-\frac{\sqrt{-4x^2-9}}{54x^2}+\frac{\sqrt{-4x^2-9}}{36x^4}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] Sqrt[-9 - 4*x^2]/(36*x^4) - Sqrt[-9 - 4*x^2]/(54*x^2) + (2*ArcTan[Sqrt[-9 -
 4*x^2]/3])/81
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x^3}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9-4x^2}}{36x^4}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x^2}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9-4x^2}}{36x^4}-\frac{\sqrt{-9-4x^2}}{54x^2}+\frac{1}{27}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-9-4x^2}}{36x^4}-\frac{\sqrt{-9-4x^2}}{54x^2}-\frac{1}{54}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx,x,\sqrt{-9-4x^2}\right)\\ &= \frac{\sqrt{-9-4x^2}}{36x^4}-\frac{\sqrt{-9-4x^2}}{54x^2}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$\frac{16}{729}\sqrt{-4x^2-9}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{4x^2}{9}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] (16*Sqrt[-9 - 4*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (4*x^2)/9])/729
```

fricas [C] time = 0.99, size = 72, normalized size = 1.26

$$\frac{-4i\,x^4\log\left(-\frac{4\left(i\sqrt{-4x^2-9}+3\right)}{81x}\right)+4i\,x^4\log\left(-\frac{4\left(-i\sqrt{-4x^2-9}+3\right)}{81x}\right)-3\left(2x^2-3\right)\sqrt{-4x^2-9}}{324x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(-4*I*x^4*log(-4/81*(I*sqrt(-4*x^2 - 9) + 3)/x) + 4*I*x^4*log(-4/81*(
-I*sqrt(-4*x^2 - 9) + 3)/x) - 3*(2*x^2 - 3)*sqrt(-4*x^2 - 9))/x^4
```

giac [A] time = 1.11, size = 43, normalized size = 0.75

$$-\frac{\left(4x^2+9\right)^{\frac{3}{2}}i-15\sqrt{-4x^2-9}}{216x^4}+\frac{2}{81}\arctan\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/216*((4*x^2 + 9)^(3/2)*i - 15*sqrt(-4*x^2 - 9))/x^4 + 2/81*arctan(1/3*sq
rt(-4*x^2 - 9))
```

maple [A] time = 0.01, size = 44, normalized size = 0.77

$$-\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{81}-\frac{\sqrt{-4x^2-9}}{54x^2}+\frac{\sqrt{-4x^2-9}}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-4*x^2-9)^(1/2),x)
```

```
[Out] 1/36*(-4*x^2-9)^(1/2)/x^4-1/54*(-4*x^2-9)^(1/2)/x^2-2/81*arctan(3/(-4*x^2-9
)^(1/2))
```

maxima [C] time = 2.89, size = 54, normalized size = 0.95

$$-\frac{\sqrt{-4\,x^2-9}}{54\,x^2}+\frac{\sqrt{-4\,x^2-9}}{36\,x^4}-\frac{2}{81}i\,\log\left(\frac{6\,\sqrt{4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/54*sqrt(-4*x^2 - 9)/x^2 + 1/36*sqrt(-4*x^2 - 9)/x^4 - 2/81*I*log(6*sqrt(
4*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 5.07, size = 60, normalized size = 1.05

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)}{81}-\frac{\frac{10\,\sqrt{-4\,x^2-9}}{9}+\frac{2\,{\left(-4\,x^2-9\right)}^{3/2}}{27}}{72\,x^2-{\left(4\,x^2+9\right)}^2+81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(- 4*x^2 - 9)^(1/2)),x)
```

```
[Out] (2*atan((- 4*x^2 - 9)^(1/2)/3))/81 - ((10*(- 4*x^2 - 9)^(1/2))/9 + (2*(- 4*
x^2 - 9)^(3/2))/27)/(72*x^2 - (4*x^2 + 9)^2 + 81)
```

sympy [C] time = 3.88, size = 65, normalized size = 1.14

$$\frac{2i\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{81}-\frac{i}{27x\sqrt{1+\frac{9}{4x^2}}}-\frac{i}{36x^3\sqrt{1+\frac{9}{4x^2}}}+\frac{i}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-4*x**2-9)**(1/2),x)
```

```
[Out] 2*I*asinh(3/(2*x))/81 - I/(27*x*sqrt(1 + 9/(4*x**2))) - I/(36*x**3*sqrt(1 +
 9/(4*x**2))) + I/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

3.576 $\int \frac{1}{\sqrt{9+bx^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

```
[Out] arcsinh(1/3*x*b^(1/2))/b^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {215}

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 + b*x^2],x]
```

```
[Out] ArcSinh[(Sqrt[b]*x)/3]/Sqrt[b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9+bx^2}}\,dx = \frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + b*x^2],x]
```

```
[Out] ArcSinh[(Sqrt[b]*x)/3]/Sqrt[b]
```

fricas [B] time = 0.98, size = 65, normalized size = 3.82

$$\left[\frac{\log\left(-\sqrt{b}\,x-\sqrt{bx^2+9}\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+9}\,\sqrt{-b}-3\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(-sqrt(b)*x - sqrt(b*x^2 + 9))/sqrt(b), -2*sqrt(-b)*arctan((sqrt(b*x^2
+ 9)*sqrt(-b) - 3*sqrt(-b))/(b*x))/b]
```

giac [**A**] time = 1.14, size = 22, normalized size = 1.29

$$-\frac{\log\left(-\sqrt{b}x+\sqrt{bx^2+9}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-sqrt(b)*x + sqrt(b*x^2 + 9))/sqrt(b)
```

maple [**A**] time = 0.00, size = 21, normalized size = 1.24

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+9}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+9)^(1/2),x)
```

```
[Out] ln(b^(1/2)*x+(b*x^2+9)^(1/2))/b^(1/2)
```

maxima [**A**] time = 1.30, size = 11, normalized size = 0.65

$$\frac{\operatorname{arsinh}\left(\frac{1}{3}\sqrt{b}x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/3*sqrt(b)*x)/sqrt(b)
```

mupad [**B**] time = 0.04, size = 11, normalized size = 0.65

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + 9)^(1/2),x)
```

```
[Out] asinh((b^(1/2)*x)/3)/b^(1/2)
```

sympy [**A**] time = 0.95, size = 14, normalized size = 0.82

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+9)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/3)/sqrt(b)
```

3.577 $\int \frac{1}{\sqrt{9-bx^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

```
[Out] arcsin(1/3*x*b^(1/2))/b^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {216}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 - b*x^2],x]
```

```
[Out] ArcSin[(Sqrt[b]*x)/3]/Sqrt[b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9-bx^2}}\,dx = \frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 - b*x^2],x]
```

```
[Out] ArcSin[(Sqrt[b]*x)/3]/Sqrt[b]
```

fricas [B] time = 0.96, size = 58, normalized size = 3.41

$$\left[-\frac{\sqrt{-b}\log\left(-\sqrt{-b}\,x-\sqrt{-bx^2+9}\right)}{b}, -\frac{2\arctan\left(\frac{\sqrt{-bx^2+9}-3}{\sqrt{b}\,x}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-sqrt(-b)*x - sqrt(-b*x^2 + 9))/b, -2*arctan((sqrt(-b*x^2 +
9) - 3)/(sqrt(b)*x))/sqrt(b)]
```

giac [B] time = 1.11, size = 27, normalized size = 1.59

$$-\frac{\log\left(-\sqrt{-b}\,x+\sqrt{-bx^2+9}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-sqrt(-b)*x + sqrt(-b*x^2 + 9))/sqrt(-b)
```

maple [A] time = 0.00, size = 21, normalized size = 1.24

$$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2+9}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+9)^(1/2),x)
```

```
[Out] 1/b^(1/2)*arctan(b^(1/2)*x/(-b*x^2+9)^(1/2))
```

maxima [A] time = 2.83, size = 11, normalized size = 0.65

$$\frac{\arcsin\left(\frac{1}{3}\sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(1/3*sqrt(b)*x)/sqrt(b)
```

mupad [B] time = 0.04, size = 15, normalized size = 0.88

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{-b}\,x}{3}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9 - b*x^2)^(1/2),x)
```

```
[Out] asinh(((-b)^(1/2)*x)/3)/(-b)^(1/2)
```

sympy [A] time = 1.01, size = 39, normalized size = 2.29

$$\begin{cases} -\frac{i\operatorname{acosh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx^2|}{9}>1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(b)*x/3)/sqrt(b), Abs(b*x**2)/9 > 1), (asin(sqrt(b)
*x/3)/sqrt(b), True))
```

3.578 $\int \frac{1}{\sqrt{-9+bx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

[Out] arctanh(x*b^(1/2)/(b*x^2-9)^(1/2))/b^(1/2)

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-9 + b*x^2],x]

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-9 + b*x^2]]/Sqrt[b]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{-9+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-9+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-9 + b*x^2],x]

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-9 + b*x^2]]/Sqrt[b]

fricas [A] time = 0.88, size = 57, normalized size = 2.28

$$\left[\frac{\log\left(2\,bx^2+2\,\sqrt{bx^2-9}\,\sqrt{b}\,x-9\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2-9}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*b*x^2 + 2*sqrt(b*x^2 - 9)*sqrt(b)*x - 9)/sqrt(b), -sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 - 9))/b]
```

giac [A] time = 1.14, size = 23, normalized size = 0.92

$$-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2-9}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 - 9)))/sqrt(b)
```

maple [A] time = 0.00, size = 21, normalized size = 0.84

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2-9}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-9)^(1/2),x)
```

```
[Out] ln(b^(1/2)*x+(b*x^2-9)^(1/2))/b^(1/2)
```

maxima [A] time = 1.33, size = 24, normalized size = 0.96

$$\frac{\log\left(2\,bx+2\,\sqrt{bx^2-9}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + 2*sqrt(b*x^2 - 9)*sqrt(b))/sqrt(b)
```

mupad [B] time = 0.08, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b\,x^2-9}+\sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 - 9)^(1/2),x)
```

```
[Out] log((b*x^2 - 9)^(1/2) + b^(1/2)*x)/b^(1/2)
```

sympy [A] time = 1.03, size = 39, normalized size = 1.56

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}} & \text{for}\ \frac{\left|bx^2\right|}{9}>1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(b)*x/3)/sqrt(b), Abs(b*x**2)/9 > 1), (-I*asin(sqrt(b)
*x/3)/sqrt(b), True))
```

3.579 $\int \frac{1}{\sqrt{-9-bx^2}}\,dx$

Optimal. Leaf size=26

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/(-b*x^2-9)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {217, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-9 - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-9 - b*x^2]]/Sqrt[b]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9-bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-9-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-9-bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-9 - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-9 - b*x^2]]/Sqrt[b]
```

fricas [A] time = 0.71, size = 68, normalized size = 2.62

$$\left[-\frac{\sqrt{-b}\,\log\left(-2\,bx^2+2\,\sqrt{-bx^2-9}\,\sqrt{-b}\,x-9\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{-bx^2-9}\,\sqrt{b}\,x}{bx^2+9}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-2*b*x^2 + 2*sqrt(-b*x^2 - 9)*sqrt(-b)*x - 9)/b, -arctan
(sqrt(-b*x^2 - 9)*sqrt(b)*x/(b*x^2 + 9))/sqrt(b)]
```

giac [A] time = 1.20, size = 28, normalized size = 1.08

$$-\frac{\log\left(\left|-\sqrt{-b}\,x+\sqrt{-bx^2-9}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(-b)*x + sqrt(-b*x^2 - 9)))/sqrt(-b)
```

maple [A] time = 0.00, size = 21, normalized size = 0.81

$$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2-9}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-9)^(1/2),x)
```

```
[Out] arctan(x*b^(1/2)/(-b*x^2-9)^(1/2))/b^(1/2)
```

maxima [C] time = 1.38, size = 12, normalized size = 0.46

$$-\frac{i\,\operatorname{arsinh}\left(\frac{1}{3}\,\sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*arcsinh(1/3*sqrt(b)*x)/sqrt(b)
```

mupad [B] time = 0.09, size = 25, normalized size = 0.96

$$\frac{\ln\left(\sqrt{-b\,x^2-9}\,+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- b*x^2 - 9)^(1/2),x)
```

```
[Out] log((- b*x^2 - 9)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

sympy [C] time = 0.97, size = 17, normalized size = 0.65

$$-\frac{i\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-9)**(1/2),x)
```

```
[Out] -I*asinh(sqrt(b)*x/3)/sqrt(b)
```

3.580 $\int \frac{1}{\sqrt{\pi+bx^2}}\,dx$

Optimal. Leaf size=19

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

```
[Out] arcsinh(x*b^(1/2)/Pi^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {215}

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Pi + b*x^2],x]
```

```
[Out] ArcSinh[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\pi+bx^2}}\,dx=\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Pi + b*x^2],x]
```

```
[Out] ArcSinh[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

fricas [B] time = 0.92, size = 59, normalized size = 3.11

$$\left[\frac{\log\left(-\pi-2\,bx^2-2\,\sqrt{\pi+bx^2}\,\sqrt{b}\,x\right)}{2\,\sqrt{b}},-\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{\pi+bx^2}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-pi - 2*b*x^2 - 2*sqrt(pi + b*x^2)*sqrt(b)*x)/sqrt(b), -sqrt(-b)*a
rctan(sqrt(-b)*x/sqrt(pi + b*x^2))/b]
```

giac [A] time = 1.12, size = 22, normalized size = 1.16

$$-\frac{\log\left(-\sqrt{b}x + \sqrt{\pi + bx^2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+pi)^(1/2),x, algorithm="giac")
[Out] -log(-sqrt(b)*x + sqrt(pi + b*x^2))/sqrt(b)
```

maple [A] time = 0.00, size = 21, normalized size = 1.11

$$\frac{\ln\left(\sqrt{b}\,x + \sqrt{b\,x^2 + \pi}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+Pi)^(1/2),x)
[Out] ln(b^(1/2)*x+(b*x^2+Pi)^(1/2))/b^(1/2)
```

maxima [A] time = 1.33, size = 13, normalized size = 0.68

$$\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{\pi b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+pi)^(1/2),x, algorithm="maxima")
[Out] arcsinh(b*x/sqrt(pi*b))/sqrt(b)
```

mupad [B] time = 5.12, size = 20, normalized size = 1.05

$$\frac{\ln\left(\sqrt{b}\,x + \sqrt{b\,x^2 + \Pi}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi + b*x^2)^(1/2),x)
[Out] log(b^(1/2)*x + (Pi + b*x^2)^(1/2))/b^(1/2)
```

sympy [A] time = 0.98, size = 17, normalized size = 0.89

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+pi)**(1/2),x)
[Out] asinh(sqrt(b)*x/sqrt(pi))/sqrt(b)
```

3.581 $\int \frac{1}{\sqrt{\pi - bx^2}} \, dx$

Optimal. Leaf size=19

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

[Out] arcsin(x*b^(1/2)/Pi^(1/2))/b^(1/2)

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {216}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Pi - b*x^2],x]

[Out] ArcSin[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\pi - bx^2}} \, dx = \frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[Pi - b*x^2],x]

[Out] ArcSin[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]

fricas [A] time = 0.96, size = 62, normalized size = 3.26

$$\left[-\frac{\sqrt{-b}\,\log\left(-\pi + 2\,bx^2 - 2\,\sqrt{\pi - bx^2}\,\sqrt{-b}\,x\right)}{2\,b}, -\frac{\arctan\left(-\frac{\sqrt{b}\,x}{\sqrt{\pi - bx^2}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-b)*log(-pi + 2*b*x^2 - 2*sqrt(pi - b*x^2)*sqrt(-b)*x)/b, -arcta
n(-sqrt(b)*x/sqrt(pi - b*x^2))/sqrt(b)]

giac [B] time = 1.05, size = 28, normalized size = 1.47

$$-\frac{\log\left(\left|-\sqrt{-b}\,x+\sqrt{\pi-bx^2}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(-b)*x + sqrt(pi - b*x^2)))/sqrt(-b)
```

maple [A] time = 0.01, size = 21, normalized size = 1.11

$$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2+\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+Pi)^(1/2),x)
```

```
[Out] 1/b^(1/2)*arctan(b^(1/2)*x/(-b*x^2+Pi)^(1/2))
```

maxima [A] time = 2.86, size = 13, normalized size = 0.68

$$\frac{\arcsin\left(\frac{bx}{\sqrt{\pi b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(b*x/sqrt(pi*b))/sqrt(b)
```

mupad [B] time = 0.10, size = 25, normalized size = 1.32

$$\frac{\ln\left(\sqrt{\Pi-b\,x^2}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi - b*x^2)^(1/2),x)
```

```
[Out] log((Pi - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

sympy [A] time = 1.04, size = 46, normalized size = 2.42

$$\begin{cases} -\frac{i\operatorname{acosh}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx^2|}{\pi}>1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(b)*x/sqrt(pi))/sqrt(b), Abs(b*x**2)/pi > 1), (asin
(sqrt(b)*x/sqrt(pi))/sqrt(b), True))
```

3.582 $\int \frac{1}{\sqrt{-\pi+bx^2}} \, dx$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-\pi}}\right)}{\sqrt{b}}$$

[Out] arctanh(x*b^(1/2)/(b*x^2-Pi)^(1/2))/b^(1/2)

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{217, 206\}$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-Pi + b*x^2],x]

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-Pi + b*x^2]]/Sqrt[b]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-\pi+bx^2}} \, dx &= \text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x}{\sqrt{-\pi+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-\pi+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-Pi + b*x^2],x]

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-Pi + b*x^2]]/Sqrt[b]

fricas [A] time = 0.64, size = 74, normalized size = 2.74

$$\left[\frac{\log\left(-\pi+2\,bx^2+2\,\sqrt{-\pi+bx^2}\,\sqrt{b}\,x\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(-\frac{\sqrt{-\pi+bx^2}\,\sqrt{-b}\,x}{\pi-bx^2}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-pi)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-pi + 2*b*x^2 + 2*sqrt(-pi + b*x^2)*sqrt(b)*x)/sqrt(b), -sqrt(-b)*
arctan(-sqrt(-pi + b*x^2)*sqrt(-b)*x/(pi - b*x^2))/b]
```

giac [A] time = 1.14, size = 25, normalized size = 0.93

$$-\frac{\log\left(\left|-\sqrt{b}x+\sqrt{-\pi+bx^2}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-pi)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(-pi + b*x^2)))/sqrt(b)
```

maple [A] time = 0.00, size = 23, normalized size = 0.85

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2-\pi}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-Pi)^(1/2),x)
```

```
[Out] ln(b^(1/2)*x+(b*x^2-Pi)^(1/2))/b^(1/2)
```

maxima [A] time = 1.39, size = 26, normalized size = 0.96

$$\frac{\log\left(2\,bx+2\,\sqrt{-\pi+bx^2}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-pi)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + 2*sqrt(-pi + b*x^2)*sqrt(b))/sqrt(b)
```

mupad [B] time = 0.13, size = 22, normalized size = 0.81

$$\frac{\ln\left(\sqrt{b\,x^2-\Pi}+\sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 - Pi)^(1/2),x)
```

```
[Out] log((b*x^2 - Pi)^(1/2) + b^(1/2)*x)/b^(1/2)
```

sympy [A] time = 1.05, size = 46, normalized size = 1.70

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{for}\ \frac{\left|bx^2\right|}{\pi}>1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-pi)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(b)*x/sqrt(pi))/sqrt(b), Abs(b*x**2)/pi > 1), (-I*asin
(sqrt(b)*x/sqrt(pi))/sqrt(b), True))
```

3.583 $\int \frac{1}{\sqrt{-\pi-bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/(-b*x^2-Pi)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {217, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Pi - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-Pi - b*x^2]]/Sqrt[b]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-\pi-bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-\pi-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-\pi-bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Pi - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-Pi - b*x^2]]/Sqrt[b]
```

fricas [A] time = 0.99, size = 74, normalized size = 2.64

$$\left[-\frac{\sqrt{-b}\,\log\left(-\pi-2\,bx^2+2\,\sqrt{-\pi-bx^2}\,\sqrt{-b}\,x\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{-\pi-bx^2}\,\sqrt{b}\,x}{\pi+bx^2}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-pi - 2*b*x^2 + 2*sqrt(-pi - b*x^2)*sqrt(-b)*x)/b, -arct
an(sqrt(-pi - b*x^2)*sqrt(b)*x/(pi + b*x^2))/sqrt(b)]
```

giac [A] time = 1.16, size = 30, normalized size = 1.07

$$-\frac{\log\left(\left|-\sqrt{-b}\,x+\sqrt{-\pi-bx^2}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(-b)*x + sqrt(-pi - b*x^2)))/sqrt(-b)
```

maple [A] time = 0.00, size = 23, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2-\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-Pi)^(1/2),x)
```

```
[Out] arctan(x*b^(1/2)/(-b*x^2-Pi)^(1/2))/b^(1/2)
```

maxima [C] time = 1.35, size = 14, normalized size = 0.50

$$-\frac{i\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{\pi b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*arcsinh(b*x/sqrt(pi*b))/sqrt(b)
```

mupad [B] time = 5.09, size = 27, normalized size = 0.96

$$\frac{\ln\left(\sqrt{-b\,x^2-\Pi}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- Pi - b*x^2)^(1/2),x)
```

```
[Out] log((- Pi - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

sympy [C] time = 1.00, size = 20, normalized size = 0.71

$$-\frac{i\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-pi)**(1/2),x)
```

```
[Out] -I*asinh(sqrt(b)*x/sqrt(pi))/sqrt(b)
```

3.584 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*x^2],x]

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*x^2],x]

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]

fricas [A] time = 0.96, size = 59, normalized size = 2.36

$$\left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

giac [A] time = 1.08, size = 23, normalized size = 0.92

$$-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

maple [A] time = 0.00, size = 21, normalized size = 0.84

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.35, size = 13, normalized size = 0.52

$$\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 0.00, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/2),x)
```

```
[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

sympy [A] time = 1.01, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

3.585 $\int \frac{1}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=26

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {217, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]]/Sqrt[b]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a-bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{a-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]]/Sqrt[b]
```

fricas [A] time = 0.77, size = 72, normalized size = 2.77

$$\left[-\frac{\sqrt{-b}\log\left(2\,bx^2 - 2\,\sqrt{-bx^2+a}\,\sqrt{-b}\,x - a\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{-bx^2+a}\,\sqrt{b}\,x}{bx^2-a}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(2*b*x^2 - 2*sqrt(-b*x^2 + a)*sqrt(-b)*x - a)/b, -arctan(
sqrt(-b*x^2 + a)*sqrt(b)*x/(b*x^2 - a))/sqrt(b)]
```

giac [A] time = 1.15, size = 28, normalized size = 1.08

$$-\frac{\log\left(\left|-\sqrt{-b}\,x+\sqrt{-bx^2+a}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(-b)*x + sqrt(-b*x^2 + a)))/sqrt(-b)
```

maple [A] time = 0.00, size = 21, normalized size = 0.81

$$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2+a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/2),x)
```

```
[Out] arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))/b^(1/2)
```

maxima [A] time = 2.96, size = 13, normalized size = 0.50

$$\frac{\arcsin\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 0.11, size = 25, normalized size = 0.96

$$\frac{\ln\left(\sqrt{a-b\,x^2}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(1/2),x)
```

```
[Out] log((a - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

sympy [A] time = 1.07, size = 46, normalized size = 1.77

$$\begin{cases} -\frac{i\operatorname{acosh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for}\ \left|\frac{bx^2}{a}\right|>1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(b)*x/sqrt(a))/sqrt(b), Abs(b*x**2/a) > 1), (asin(s
qrt(b)*x/sqrt(a))/sqrt(b), True))
```

3.586 $\int \frac{1}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-a}}\right)}{\sqrt{b}}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2-a)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-a}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-a + b*x^2]]/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{-a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-a}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-a + b*x^2]]/Sqrt[b]
```

fricas [A] time = 0.88, size = 63, normalized size = 2.33

$$\left[\frac{\log\left(2\,bx^2+2\,\sqrt{bx^2-a}\,\sqrt{b}\,x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2-a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*b*x^2 + 2*sqrt(b*x^2 - a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 - a))/b]
```

giac [A] time = 1.05, size = 25, normalized size = 0.93

$$-\frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2-a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 - a)))/sqrt(b)
```

maple [A] time = 0.00, size = 23, normalized size = 0.85

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2-a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-a)^(1/2),x)
```

```
[Out] ln(b^(1/2)*x+(b*x^2-a)^(1/2))/b^(1/2)
```

maxima [A] time = 1.32, size = 26, normalized size = 0.96

$$\frac{\log\left(2\,bx+2\,\sqrt{bx^2-a}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + 2*sqrt(b*x^2 - a)*sqrt(b))/sqrt(b)
```

mupad [B] time = 0.12, size = 22, normalized size = 0.81

$$\frac{\ln\left(\sqrt{b\,x^2-a}+\sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 - a)^(1/2),x)
```

```
[Out] log((b*x^2 - a)^(1/2) + b^(1/2)*x)/b^(1/2)
```

sympy [A] time = 1.09, size = 46, normalized size = 1.70

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for}\ \left|\frac{bx^2}{a}\right|>1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-a)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(b)*x/sqrt(a))/sqrt(b), Abs(b*x**2/a) > 1), (-I*asin(s
qrt(b)*x/sqrt(a))/sqrt(b), True))
```

3.587 $\int \frac{1}{\sqrt{-a-bx^2}} \, dx$

Optimal. Leaf size=28

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

[Out] arctan(x*b^(1/2)/(-b*x^2-a)^(1/2))/b^(1/2)

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{217, 203\}$

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-a - b*x^2],x]

[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-a - b*x^2]]/Sqrt[b]

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a-bx^2}} \, dx &= \text{Subst}\left(\int \frac{1}{1+bx^2} \, dx, x, \frac{x}{\sqrt{-a-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-a - b*x^2],x]

[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-a - b*x^2]]/Sqrt[b]

fricas [A] time = 0.90, size = 74, normalized size = 2.64

$$\left[-\frac{\sqrt{-b}\log\left(-2\,bx^2+2\,\sqrt{-bx^2-a}\,\sqrt{-b}\,x-a\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{-bx^2-a}\,\sqrt{b}\,x}{bx^2+a}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-2*b*x^2 + 2*sqrt(-b*x^2 - a)*sqrt(-b)*x - a)/b, -arctan
(sqrt(-b*x^2 - a)*sqrt(b)*x/(b*x^2 + a))/sqrt(b)]
```

giac [A] time = 1.27, size = 30, normalized size = 1.07

$$-\frac{\log\left(\left|-\sqrt{-b}\,x+\sqrt{-bx^2-a}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(-b)*x + sqrt(-b*x^2 - a)))/sqrt(-b)
```

maple [A] time = 0.00, size = 23, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2-a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-a)^(1/2),x)
```

```
[Out] arctan(x*b^(1/2)/(-b*x^2-a)^(1/2))/b^(1/2)
```

maxima [C] time = 1.32, size = 14, normalized size = 0.50

$$-\frac{i\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 5.13, size = 27, normalized size = 0.96

$$\frac{\ln\left(\sqrt{-b\,x^2-a}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- a - b*x^2)^(1/2),x)
```

```
[Out] log((- a - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

sympy [C] time = 1.02, size = 20, normalized size = 0.71

$$-\frac{i\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-a)**(1/2),x)
```

```
[Out] -I*asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

3.588 $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=16

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] arctan(x/(a^2-x^2)^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{217, 203\}$

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 - x^2],x]
```

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2-x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{a^2-x^2}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 - x^2],x]
```

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

fricas [A] time = 0.83, size = 23, normalized size = 1.44

$$-2\,\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="fricas")
[Out] -2*arctan(-(a - sqrt(a^2 - x^2))/x)
```

giac [A] time = 1.07, size = 9, normalized size = 0.56

$$\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="giac")
[Out] arcsin(x/a)*sgn(a)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2-x^2)^(1/2),x)
[Out] arctan(x/(a^2-x^2)^(1/2))
```

maxima [A] time = 2.89, size = 6, normalized size = 0.38

$$\arcsin\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="maxima")
[Out] arcsin(x/a)
```

mupad [B] time = 4.74, size = 14, normalized size = 0.88

$$\operatorname{atan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 - x^2)^(1/2),x)
[Out] atan(x/(a^2 - x^2)^(1/2))
```

sympy [A] time = 1.02, size = 19, normalized size = 1.19

$$\begin{cases} -i\operatorname{acosh}\left(\frac{x}{a}\right) & \text{for}\ \left|\frac{x^2}{a^2}\right| > 1 \\ \operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2-x**2)**(1/2),x)
[Out] Piecewise((-I*acosh(x/a), Abs(x**2/a**2) > 1), (asin(x/a), True))
```

3.589 $\int (cx)^{7/2}\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=184

$$\frac{10a^{11/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\ \text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}-\frac{20a^2c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}+$$

```
[Out] 4/77*a*c*(c*x)^(5/2)*(b*x^2+a)^(1/2)/b+2/11*(c*x)^(9/2)*(b*x^2+a)^(1/2)/c-2
0/231*a^2*c^3*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^2+10/231*a^(11/4)*c^(7/2)*(cos(
2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4
)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/
a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^
(1/2))^2)^(1/2)/b^(9/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {279, 321, 329, 220}

$$-\frac{20a^2c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{10a^{11/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}+\frac{4ac(cx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (-20*a^2*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/(231*b^2) + (4*a*c*(c*x)^(5/2)*Sqrt
[a + b*x^2])/(77*b) + (2*(c*x)^(9/2)*Sqrt[a + b*x^2])/(11*c) + (10*a^(11/4)
*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(231*b^(9/4)
*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (cx)^{7/2}\sqrt{a+bx^2}\,dx = \frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}+\frac{1}{11}(2a)\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b}+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}-\frac{\left(10a^2c^2\right)\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx}{77b}$$
$$= -\frac{20a^2c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b}+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}+\frac{\left(10a^3c^4\right)\int \frac{}{\sqrt{cx}\ }}{231b^2}$$
$$= -\frac{20a^2c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b}+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}+\frac{\left(20a^3c^3\right)\mathrm{Subst}}{}$$
$$= -\frac{20a^2c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b}+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}+\frac{10a^{11/4}c^{7/2}\left(\sqrt{a}\right.}{}$$

Mathematica [C] time = 0.06, size = 103, normalized size = 0.56

$$\frac{2c^3\sqrt{cx}\,\sqrt{a+bx^2}\left(\sqrt{\frac{bx^2}{a}+1}\left(-5a^2+2abx^2+7b^2x^4\right)+5a^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{77b^2\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)*Sqrt[a + b*x^2],x]
[Out] (2*c^3*Sqrt[c*x]*Sqrt[a + b*x^2]*(Sqrt[1 + (b*x^2)/a]*(-5*a^2 + 2*a*b*x^2 +
 7*b^2*x^4) + 5*a^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(77*b
^2*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{bx^2+a}\,\sqrt{cx}\,c^3x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c^3*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^2+a}\,(cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(7/2), x)
```

maple [A] time = 0.06, size = 152, normalized size = 0.83

$$\frac{2\sqrt{cx}\left(21b^4x^7+27a\,b^3x^5-4a^2b^2x^3-10a^3bx+5\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a^3\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.\right.}{231\sqrt{b\,x^2+a}\;b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(7/2)*(b*x^2+a)^(1/2),x)

[Out] 2/231*c^3/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(21*x^7*b^4+5*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+27*x^5*a*b^3-4*x^3*a^2*b^2-10*x*a^3*b)/b^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^2+a}\,(cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{7/2}\,\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(7/2)*(a + b*x^2)^(1/2),x)

[Out] int((c*x)^(7/2)*(a + b*x^2)^(1/2), x)

sympy [C] time = 27.19, size = 46, normalized size = 0.25

$$\frac{\sqrt{a}\,c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(7/2)*(b*x**2+a)**(1/2),x)

[Out] sqrt(a)*c**(7/2)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(13/4))

3.590 $\int (cx)^{5/2}\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=301

$$-\frac{2a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}}+\frac{4a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\right.}{15b^{7/4}\sqrt{a+bx^2}}$$

```
[Out] 4/45*a*c*(c*x)^(3/2)*(b*x^2+a)^(1/2)/b+2/9*(c*x)^(7/2)*(b*x^2+a)^(1/2)/c-4/
15*a^2*c^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(3/2)/(a^(1/2)+x*b^(1/2))+4/15*a^(
9/4)*c^(5/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^
(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^
2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2+a)^(1/2)-2/15*a^(9/4)*c^(5
/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arct
an(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*
x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(
1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {279, 321, 329, 305, 220, 1196}

$$-\frac{4a^2c^2\sqrt{cx}\,\sqrt{a+bx^2}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}}+\frac{4a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\cdots}}{15b^{7/4}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (4*a*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(45*b) + (2*(c*x)^(7/2)*Sqrt[a + b*x^2]
)/(9*c) - (4*a^2*c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(15*b^(3/2)*(Sqrt[a] + Sqrt
[b]*x)) + (4*a^(9/4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a
] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]
, 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2]) - (2*a^(9/4)*c^(5/2)*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*
Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int (cx)^{5/2}\sqrt{a+bx^2}\,dx &= \frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}+\frac{1}{9}(2a)\int\frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx\\
&= \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}-\frac{\left(2a^2c^2\right)\int\frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{15b}\\
&= \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}-\frac{\left(4a^2c\right)\operatorname{Subst}\left(\int\frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{15b}\\
&= \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}-\frac{\left(4a^{5/2}c^2\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{15b^{3/2}}+\frac{(4a\cdots}{\cdots}\\
&= \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}-\frac{4a^2c^2\sqrt{cx}\sqrt{a+bx^2}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{4a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\cdots}{1\cdots}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 85, normalized size = 0.28

$$\frac{2c(cx)^{3/2}\sqrt{a+bx^2}\left(\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{9b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*(c*x)^(3/2)*Sqrt[a + b*x^2]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hyper
geometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(9*b*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^2+a}\,\sqrt{cx}\,c^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^2+a}\,(cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(5/2), x)
```

maple [A] time = 0.03, size = 221, normalized size = 0.73

$$-\frac{2\sqrt{cx}\left(-5b^3x^6-7a\,b^2x^4-2a^2b\,x^2+6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a^3\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-\right.}{45\sqrt{b\,x^2+a}\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(b*x^2+a)^(1/2),x)
[Out] -2/45*c^2/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(-5*b^3*x^6+6*((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1
/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),
1/2*2^(1/2))*a^3-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3-7*a*b^2*x^4-2*a^2*b*x^
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^2+a}\,(cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{5/2}\,\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(a + b*x^2)^(1/2),x)
[Out] int((c*x)^(5/2)*(a + b*x^2)^(1/2), x)
```

sympy [C] time = 9.38, size = 46, normalized size = 0.15

$$\frac{\sqrt{a}\, c^{\frac{5}{2}} x^{\frac{7}{2}} \Gamma\left(\frac{7}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(11/4))
```

3.591 $\int (cx)^{3/2}\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=153

$$-\frac{2a^{7/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\ \text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{a+bx^2}}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}+\frac{4ac\sqrt{cx}\,\sqrt{a+bx^2}}{21b}$$

```
[Out] 2/7*(c*x)^(5/2)*(b*x^2+a)^(1/2)/c+4/21*a*c*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b-2/
21*a^(7/4)*c^(3/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(
1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arc
tan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*
((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = $\{279, 321, 329, 220\}$

$$-\frac{2a^{7/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{a+bx^2}}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}+\frac{4ac\sqrt{cx}\,\sqrt{a+bx^2}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*Sqrt[a + b*x^2],x]
```

```
[Out] (4*a*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(21*b) + (2*(c*x)^(5/2)*Sqrt[a + b*x^2])/
(7*c) - (2*a^(7/4)*c^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
+ Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])],
1/2])/(21*b^(5/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (cx)^{3/2}\sqrt{a+bx^2}\,dx = \frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}+\frac{1}{7}(2a)\int\frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{4ac\sqrt{cx}\sqrt{a+bx^2}}{21b}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}-\frac{\left(2a^2c^2\right)\int\frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{21b}$$
$$=\frac{4ac\sqrt{cx}\sqrt{a+bx^2}}{21b}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}-\frac{\left(4a^2c\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{21b}$$
$$=\frac{4ac\sqrt{cx}\sqrt{a+bx^2}}{21b}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}-\frac{2a^{7/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{21b^{5/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.04, size = 85, normalized size = 0.56

$$\frac{2c\sqrt{cx}\sqrt{a+bx^2}\left(\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{7b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*Sqrt[a + b*x^2],x]
[Out] (2*c*Sqrt[c*x]*Sqrt[a + b*x^2]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hyperge
ometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(7*b*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{bx^2+a}\,\sqrt{cx}\,cx,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{bx^2+a}\,(cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(3/2), x)
```

maple [A] time = 0.03, size = 138, normalized size = 0.90

$$-\frac{2\sqrt{cx}\left(-3b^3x^5-5a\,b^2x^3-2a^2bx+\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{-ab}\,a^2\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\right)}{21\sqrt{b\,x^2+a}\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(b*x^2+a)^(1/2),x)
```

```
[Out] -2/21*c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^
(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)
^(1/2)*a^2-3*b^3*x^5-5*a*b^2*x^3-2*a^2*b*x)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^2 + a}\,(cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(c*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{3/2}\,\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(a + b*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(3/2)*(a + b*x^2)^(1/2), x)
```

sympy [C] time = 2.76, size = 46, normalized size = 0.30

$$\frac{\sqrt{a}\,c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*exp_
polar(I*pi)/a)/(2*gamma(9/4))
```

3.592 $\int \sqrt{cx}\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=269

$$\frac{2a^{5/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}}-\frac{4a^{5/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\right.}{5b^{3/4}\sqrt{a+bx^2}}$$

```
[Out] 2/5*(c*x)^(3/2)*(b*x^2+a)^(1/2)/c+4/5*a*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)
/(a^(1/2)+x*b^(1/2))-4/5*a^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/
c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*Ellip
ticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/
2)+x*b^(1/2))*c^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^
2+a)^(1/2)+2/5*a^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2
*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/
2))*c^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {279, 329, 305, 220, 1196}

$$\frac{2a^{5/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}}-\frac{4a^{5/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right.\right.}{5b^{3/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*Sqrt[a + b*x^2],x]
```

```
[Out] (2*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*c) + (4*a*Sqrt[c*x]*Sqrt[a + b*x^2])/(5*
Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) - (4*a^(5/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^2]) + (2*a^(5/4)*Sqr
t[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(3/4)*Sqrt[
a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \sqrt{cx}\sqrt{a+bx^2}\,dx &= \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}+\frac{1}{5}(2a)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx\\ &= \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}+\frac{(4a)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{5c}\\ &= \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}+\frac{\left(4a^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{5\sqrt{b}}-\frac{\left(4a^{3/2}\right)\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,d\right.}{5\sqrt{b}}\\ &= \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}+\frac{4a\sqrt{cx}\sqrt{a+bx^2}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{4a^{5/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\tan^{-}\right.}{5b^{3/4}\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.21

$$\frac{2x\sqrt{cx}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*Sqrt[a + b*x^2],x]
[Out] (2*x*Sqrt[c*x]*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/
a)])/(3*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{bx^2+a}\sqrt{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^2 + a}\,\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(c*x), x)
```

maple [A] time = 0.03, size = 205, normalized size = 0.76

$$\frac{2\sqrt{cx}\left(b^2x^4 + ab\,x^2 + 2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a^2\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right) - \sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\cdots}\right.}{5\sqrt{b\,x^2 + a}\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^2+a)^(1/2),x)
```

```
[Out] 2/5*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1
/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-((b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2),1/2*2^(1/2))*a^2+b^2*x^4+a*b*x^2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^2 + a}\,\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{c\,x}\,\sqrt{b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a + b*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)*(a + b*x^2)^(1/2), x)
```

sympy [C] time = 1.07, size = 46, normalized size = 0.17

$$\frac{\sqrt{a}\,\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*exp_p
olar(I*pi)/a)/(2*gamma(7/4))
```

3.593 $\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}} \, dx$

Optimal. Leaf size=126

$$\frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\ \text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}+\frac{2\sqrt{cx}\,\sqrt{a+bx^2}}{3c}$$

```
[Out] 2/3*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c+2/3*a^(3/4)*(cos(2*arctan(b^(1/4)*(c*x)^(
1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^
(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^
(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(1/4)/
c^(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {279, 329, 220}

$$\frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}+\frac{2\sqrt{cx}\,\sqrt{a+bx^2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[c*x],x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*c) + (2*a^(3/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x]
)/(a^(1/4)*Sqrt[c])], 1/2])/(3*b^(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx = \frac{2\sqrt{cx}\,\sqrt{a+bx^2}}{3c} + \frac{1}{3}(2a)\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx$$
$$= \frac{2\sqrt{cx}\,\sqrt{a+bx^2}}{3c} + \frac{(4a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3c}$$
$$= \frac{2\sqrt{cx}\,\sqrt{a+bx^2}}{3c} + \frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.43

$$\frac{2x\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\,\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[c*x],x]
[Out] (2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)])/(Sqrt
[c*x]*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}}{cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(c*x), x)
```

maple [A] time = 0.03, size = 119, normalized size = 0.94

$$\frac{\frac{2b^2x^3}{3}+\frac{2abx}{3}+\frac{2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{-ab}\,a\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3}}{\sqrt{b\,x^2+a}\,\sqrt{cx}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] 2/3/(b*x^2+a)^(1/2)*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a+b^2*x^3+a
*b*x)/(c*x)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+a}}{\sqrt{c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c*x)^(1/2), x)
```

sympy [C] time = 0.99, size = 46, normalized size = 0.37

$$\frac{\sqrt{a}\,\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(c*x)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(I*pi
)/a)/(2*sqrt(c)*gamma(5/4))
```

3.594 $\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}} \, dx$

Optimal. Leaf size=263

$$\frac{2\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{c^{3/2}\sqrt{a+bx^2}}-\frac{4\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan\right.}{c^{3/2}\sqrt{a+bx^2}}$$

```
[Out] -2*(b*x^2+a)^(1/2)/c/(c*x)^(1/2)+4*b^(1/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c^2/
(a^(1/2)+x*b^(1/2))-4*a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(
1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*
EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(
a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/c^(3/2)/(b*x^2+a
)^(1/2)+2*a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(si
n(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^
(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/c^(3/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {277, 329, 305, 220, 1196}

$$\frac{4\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{c^2\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{c^{3/2}\sqrt{a+bx^2}}-\frac{4\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}}{c^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(c*Sqrt[c*x]) + (4*Sqrt[b]*Sqrt[c*x]*Sqrt[a + b*x^2])/
(c^2*(Sqrt[a] + Sqrt[b]*x)) - (4*a^(1/4)*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt
[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x]
)/(a^(1/4)*Sqrt[c])], 1/2])/(c^(3/2)*Sqrt[a + b*x^2]) + (2*a^(1/4)*b^(1/4)*
(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2
*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(c^(3/2)*Sqrt[a + b*x
^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx &= -\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}+\frac{(2b)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{c^2}\\
&= -\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}+\frac{(4b)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{c^3}\\
&= -\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}+\frac{\left(4\sqrt{a}\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{c^2}-\frac{\left(4\sqrt{a}\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,\ldots\right.}{c^2}\\
&= -\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}+\frac{4\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{c^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{4\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{c\ldots}}{\sqrt[4]{a}\sqrt{c\ldots}}\right.\right.}{c^{3/2}\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.21

$$-\frac{2x\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c*x)^(3/2),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^2)/a)])/((c
*x)^(3/2)*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}}{c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(3/2), x)
```

maple [A] time = 0.04, size = 194, normalized size = 0.74

$$\frac{-2b\,x^2+4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-}}{\sqrt{b\,x^2+a}\,\sqrt{cx}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(c*x)^(3/2),x)
```

```
[Out] 2*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1
/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-b*x^2-
a)/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^2+a}}{(c\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c*x)^(3/2),x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c*x)^(3/2), x)
```

sympy [C] time = 1.35, size = 49, normalized size = 0.19

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(c*x)**(3/2),x)
```

```
[Out] sqrt(a)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(
2*c**(3/2)*sqrt(x)*gamma(3/4))
```

3.595 $\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}} dx$

Optimal. Leaf size=126

$$\frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\ \text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\,c^{5/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}}$$

```
[Out] -2/3*(b*x^2+a)^(1/2)/c/(c*x)^(3/2)+2/3*b^(3/4)*(cos(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2
^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)
/c^(5/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {277, 329, 220}

$$\frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,c^{5/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(3*c*(c*x)^(3/2)) + (2*b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*a^(1/4)*c^(5/2)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx = -\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}} + \frac{(2b)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{3c^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}} + \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3c^3}$$

$$= -\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}} + \frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,c^{5/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.44

$$-\frac{2x\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c*x)^(5/2),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^2)/a)])/(3*
(c*x)^(5/2)*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{c^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(5/2), x)
```

maple [A] time = 0.03, size = 120, normalized size = 0.95

$$\frac{-\frac{2b\,x^2}{3}+\frac{2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{-ab}\,x\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3}-\frac{2a}{3}}{\sqrt{b\,x^2+a}\,\sqrt{cx}\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(c*x)^(5/2),x)
```

[Out] 2/3/(b*x^2+a)^(1/2)/x*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x-b*x^2-a)/c^2/(c*x)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+a}}{{\left(c\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^(1/2)/(c*x)^(5/2),x)

[Out] int((a + b*x^2)^(1/2)/(c*x)^(5/2), x)

sympy [C] time = 2.86, size = 49, normalized size = 0.39

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{3}{4}\right)\,{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(1/2)/(c*x)**(5/2),x)

[Out] sqrt(a)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*c**(5/2)*x**(3/2)*gamma(1/4))

3.596 $\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx$

Optimal. Leaf size=303

$$\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}}-\frac{4b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right.\right.}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}}$$

```
[Out] -2/5*(b*x^2+a)^(1/2)/c/(c*x)^(5/2)-4/5*b*(b*x^2+a)^(1/2)/a/c^3/(c*x)^(1/2)+
4/5*b^(3/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a/c^4/(a^(1/2)+x*b^(1/2))-4/5*b^(5/
4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x
)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1
/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/c^(7/2)/(b*x^2+a)^(1/2)+2/5*b^(5/4)*(cos(2*
arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^
(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1
/2))^2)^(1/2)/a^(3/4)/c^(7/2)/(b*x^2+a)^(1/2)
```

Rubi **[A]** time = 0.22, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {277, 325, 329, 305, 220, 1196}

$$\frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}}-\frac{4b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(c*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(5*c*(c*x)^(5/2)) - (4*b*Sqrt[a + b*x^2])/(5*a*c^3*Sqr
t[c*x]) + (4*b^(3/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(5*a*c^4*(Sqrt[a] + Sqrt[b]
*x)) - (4*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*
a^(3/4)*c^(7/2)*Sqrt[a + b*x^2]) + (2*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(
a^(1/4)*Sqrt[c])], 1/2])/(5*a^(3/4)*c^(7/2)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx &= -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}+\frac{(2b)\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{5c^2}\\
&= -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}-\frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}}+\frac{\left(2b^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5ac^4}\\
&= -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}-\frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}}+\frac{\left(4b^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{5ac^5}\\
&= -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}-\frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}}+\frac{\left(4b^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{5\sqrt{a}\,c^4}-\frac{\left(4b^{3/2}\right)\text{Subst}\left(\int \frac{1-\cdots}{\sqrt{a\cdots}}\right.}{5\sqrt{a}\,c\cdots}\\
&= -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}-\frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}}+\frac{4b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{5ac^4\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{4b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\,\mathrm{t}\cdots\right.}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.18

$$
-\frac{2x\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c*x)^(7/2),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^2)/a)])/(5
*(c*x)^(7/2)*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}}{c^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(c^4*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(7/2), x)
```

maple [A] time = 0.03, size = 219, normalized size = 0.72

$$\frac{-\frac{4b^2x^4}{5}+\frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,ab\,x^2\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{5}-\frac{2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,ab\,x^2\,\mathrm{EllipticF}\left(\cdots\right)}{5}}{\sqrt{b\,x^2+a}\,\sqrt{cx}\,a\,c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(c*x)^(7/2),x)
```

```
[Out] 2/5/x^2*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-((b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(
1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/
2))*x^2*a*b-2*b^2*x^4-3*a*b*x^2-a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(c*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^2+a}}{{\left(c\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(c*x)^(7/2),x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c*x)^(7/2), x)
```

sympy [C] time = 9.75, size = 53, normalized size = 0.17

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(c*x)**(7/2),x)
```

```
[Out] sqrt(a)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(7/2)*x**(5/2)*gamma(-1/4))
```

3.597 $\int (cx)^{7/2} \left(a+bx^2\right)^{3/2} \, dx$

Optimal. Leaf size=212

$$\frac{4a^{15/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\ \text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}-\frac{8a^3c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}$$

```
[Out] 2/15*(c*x)^(9/2)*(b*x^2+a)^(3/2)/c+8/385*a^2*c*(c*x)^(5/2)*(b*x^2+a)^(1/2)/
b+4/55*a*(c*x)^(9/2)*(b*x^2+a)^(1/2)/c-8/231*a^3*c^3*(c*x)^(1/2)*(b*x^2+a)^
(1/2)/b^2+4/231*a^(15/4)*c^(7/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/
c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*Ellip
ticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/
2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^2+a)^(1/
2)
```

Rubi [A] time = 0.13, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {279, 321, 329, 220}

$$-\frac{8a^3c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{4a^{15/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{2(cx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (-8*a^3*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/(231*b^2) + (8*a^2*c*(c*x)^(5/2)*Sqr
t[a + b*x^2])/(385*b) + (4*a*(c*x)^(9/2)*Sqrt[a + b*x^2])/(55*c) + (2*(c*x)
^(9/2)*(a + b*x^2)^(3/2))/(15*c) + (4*a^(15/4)*c^(7/2)*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqr
t[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(231*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (cx)^{7/2}\left(a+bx^2\right)^{3/2}\,dx = \frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}+\frac{1}{5}(2a)\int (cx)^{7/2}\sqrt{a+bx^2}\,dx$$

$$= \frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}+\frac{1}{55}\left(4a^2\right)\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}-\frac{\left(4a^3c^2\right)\int \frac{(cx}{\sqrt{a+}}}{77b}$$

$$= -\frac{8a^3c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)^{9/2}\left(a+\right.}{15c}$$

$$= -\frac{8a^3c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)^{9/2}\left(a+\right.}{15c}$$

$$= -\frac{8a^3c^3\sqrt{cx}\,\sqrt{a+bx^2}}{231b^2}+\frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b}+\frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}+\frac{2(cx)^{9/2}\left(a+\right.}{15c}$$

Mathematica [C] time = 0.08, size = 102, normalized size = 0.48

$$\frac{2c^3\sqrt{cx}\,\sqrt{a+bx^2}\left(5a^3\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-\left(5a-11bx^2\right)\left(a+bx^2\right)^2\sqrt{\frac{bx^2}{a}+1}\right)}{165b^2\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (2*c^3*Sqrt[c*x]*Sqrt[a + b*x^2]*(-((5*a - 11*b*x^2)*(a + b*x^2)^2*Sqrt[1 +
 (b*x^2)/a]) + 5*a^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(165
*b^2*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bc^3x^5+ac^3x^3\right)\sqrt{bx^2+a}\,\sqrt{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*c^3*x^5 + a*c^3*x^3)*sqrt(b*x^2 + a)*sqrt(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(7/2), x)
```

maple [A] time = 0.03, size = 163, normalized size = 0.77

$$\frac{2\sqrt{cx}\left(77b^5x^9+196a\,b^4x^7+131a^2b^3x^5-8a^3b^2x^3-20a^4bx+10\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{-ab}\,a^4\right.}{1155\sqrt{b\,x^2+a}\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(b*x^2+a)^(3/2),x)
```

```
[Out] 2/1155*c^3/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(77*b^5*x^9+196*a*b^4*x^7+10*((b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^4+131*a^2*b^3*x^5-8*a^3*b^2*x^3-20*
a^4*b*x)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{7/2}\left(b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(a + b*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)*(a + b*x^2)^(3/2), x)
```

sympy [C] time = 50.26, size = 46, normalized size = 0.22

$$\frac{a^{\frac{3}{2}}c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(3/2)*c**(7/2)*x**(9/2)*gamma(9/4)*hyper((-3/2, 9/4), (13/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(13/4))
```

3.598 $\int (cx)^{5/2} \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=329

$$-\frac{4a^{13/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^2}}+\frac{8a^{13/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E}{65b^{7/4}\sqrt{a+bx^2}}$$

```
[Out] 2/13*(c*x)^(7/2)*(b*x^2+a)^(3/2)/c+8/195*a^2*c*(c*x)^(3/2)*(b*x^2+a)^(1/2)/
b+4/39*a*(c*x)^(7/2)*(b*x^2+a)^(1/2)/c-8/65*a^3*c^2*(c*x)^(1/2)*(b*x^2+a)^(
1/2)/b^(3/2)/(a^(1/2)+x*b^(1/2))+8/65*a^(13/4)*c^(5/2)*(cos(2*arctan(b^(1/4
)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a
^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)
)),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)
/b^(7/4)/(b*x^2+a)^(1/2)-4/65*a^(13/4)*c^(5/2)*(cos(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2
^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)
/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {279, 321, 329, 305, 220, 1196}

$$-\frac{8a^3c^2\sqrt{cx}\sqrt{a+bx^2}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{4a^{13/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^2}}+\frac{8a^{13/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{}}{65b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (8*a^2*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(195*b) + (4*a*(c*x)^(7/2)*Sqrt[a + b
*x^2])/(39*c) - (8*a^3*c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(65*b^(3/2)*(Sqrt[a]
+ Sqrt[b]*x)) + (2*(c*x)^(7/2)*(a + b*x^2)^(3/2))/(13*c) + (8*a^(13/4)*c^(5
/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(65*b^(7/4)*Sqrt[
a + b*x^2]) - (4*a^(13/4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt
[c])], 1/2])/(65*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int (cx)^{5/2}\left(a+bx^2\right)^{3/2}\,dx &= \frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}+\frac{1}{13}(6a)\int (cx)^{5/2}\sqrt{a+bx^2}\,dx\\
&= \frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}+\frac{1}{39}\left(4a^2\right)\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx\\
&= \frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b}+\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}-\frac{\left(4a^3c^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}}{65b}\\
&= \frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b}+\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}-\frac{\left(8a^3c\right)\mathrm{Subst}\left(\int\right.}{}\\
&= \frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b}+\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}-\frac{\left(8a^{7/2}c^2\right)\mathrm{Subst}}{}\\
&= \frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b}+\frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}-\frac{8a^3c^2\sqrt{cx}\,\sqrt{a+bx^2}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2(cx)^{7/2}\left(a+bx^2\right.}{13c}
\end{aligned}
$$

Mathematica [C] time = 0.06, size = 89, normalized size = 0.27

$$
\frac{2c(cx)^{3/2}\sqrt{a+bx^2}\left(\left(a+bx^2\right)^2\sqrt{\frac{bx^2}{a}+1}-a^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{13b\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^(5/2)*(a + b*x^2)^(3/2),x]

[Out] (2*c*(c*x)^(3/2)*Sqrt[a + b*x^2]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]))/(13*b*Sqrt[1 + (b*x^2)/a])

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bc^2x^4 + ac^2x^2\right)\sqrt{bx^2 + a}\,\sqrt{cx}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(5/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")

[Out] integral((b*c^2*x^4 + a*c^2*x^2)*sqrt(b*x^2 + a)*sqrt(c*x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{3}{2}} (cx)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(5/2)*(b*x^2+a)^(3/2),x, algorithm="giac")

[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(5/2), x)

maple [A] time = 0.03, size = 232, normalized size = 0.71

$$-\frac{2\sqrt{cx}\,\left(-15b^4x^8 - 40a\,b^3x^6 - 29a^2b^2x^4 - 4a^3b\,x^2 + 12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a^4\,\text{EllipticE}\left(\sqrt{\right.\right.}{195\sqrt{b\,x^2 + a}\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(5/2)*(b*x^2+a)^(3/2),x)

[Out] -2/195*c^2/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(-15*b^4*x^8-40*a*b^3*x^6+12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^4-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^4-29*a^2*b^2*x^4-4*a^3*b*x^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{3}{2}} (cx)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(5/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{5/2}\,\left(b\,x^2 + a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(a + b*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(5/2)*(a + b*x^2)^(3/2), x)
```

sympy **[C]** time = 18.19, size = 46, normalized size = 0.14

$$\frac{a^{\frac{3}{2}}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(3/2)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-3/2, 7/4), (11/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(11/4))
```

3.599 $\int (cx)^{3/2} \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=181

$$-\frac{4a^{11/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\;\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{77b^{5/4}\sqrt{a+bx^2}}+\frac{8a^2c\sqrt{cx}\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+b\ldots}}{77c}$$

```
[Out] 2/11*(c*x)^(5/2)*(b*x^2+a)^(3/2)/c+12/77*a*(c*x)^(5/2)*(b*x^2+a)^(1/2)/c+8/
77*a^2*c*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b-4/77*a^(11/4)*c^(3/2)*(cos(2*arctan(
b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(
1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)
^(1/2)/b^(5/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {279, 321, 329, 220}

$$-\frac{4a^{11/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\;F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{a+bx^2}}+\frac{8a^2c\sqrt{cx}\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*(a + b*x^2)^(3/2),x]
```

```
[Out] (8*a^2*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(77*b) + (12*a*(c*x)^(5/2)*Sqrt[a + b*x
^2])/(77*c) + (2*(c*x)^(5/2)*(a + b*x^2)^(3/2))/(11*c) - (4*a^(11/4)*c^(3/2
)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF
[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(77*b^(5/4)*Sqrt[a
+ b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int (cx)^{3/2}\left(a+bx^2\right)^{3/2}\,dx &= \frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}+\frac{1}{11}(6a)\int (cx)^{3/2}\sqrt{a+bx^2}\,dx\\
&= \frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}+\frac{1}{77}\left(12a^2\right)\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx\\
&= \frac{8a^2c\sqrt{cx}\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}-\frac{\left(4a^3c^2\right)\int \frac{1}{\sqrt{cx}\sqrt{a+}}}{77b}\\
&= \frac{8a^2c\sqrt{cx}\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}-\frac{\left(8a^3c\right)\mathrm{Subst}\left(\int\right.}{7}\\
&= \frac{8a^2c\sqrt{cx}\sqrt{a+bx^2}}{77b}+\frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c}+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}-\frac{4a^{11/4}c^{3/2}\left(\sqrt{a}+\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 89, normalized size = 0.49

$$\frac{2c\sqrt{cx}\sqrt{a+bx^2}\left(\left(a+bx^2\right)^2\sqrt{\frac{bx^2}{a}+1}-a^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{11b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*(a + b*x^2)^(3/2),x]
[Out] (2*c*Sqrt[c*x]*Sqrt[a + b*x^2]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hyp
ergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(11*b*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bcx^3+acx\right)\sqrt{bx^2+a}\sqrt{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral((b*c*x^3 + a*c*x)*sqrt(b*x^2 + a)*sqrt(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(3/2), x)
```

maple [A] time = 0.01, size = 150, normalized size = 0.83

$$-\frac{2\sqrt{cx}\,\left(-7b^4x^7-20a\,b^3x^5-17a^2b^2x^3-4a^3bx+2\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a^3\,\mathrm{EllipticF}\left(\right.\right.}{77\sqrt{b\,x^2+a}\;b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(b*x^2+a)^(3/2),x)
```

```
[Out] -2/77*c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(-7*b^4*x^7+2*(-a*b)^(1/2)*((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2),1/2*2^(1/2))*a^3-20*a*b^3*x^5-17*a^2*b^2*x^3-4*a^3*b*x)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(c*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{3/2}\,\left(b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(a + b*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(3/2)*(a + b*x^2)^(3/2), x)
```

sympy [C] time = 6.12, size = 46, normalized size = 0.25

$$\frac{a^{\frac{3}{2}}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(3/2)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-3/2, 5/4), (9/4,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(9/4))
```

3.600 $\int \sqrt{cx}\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=297

$$\frac{4a^{9/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^2}}-\frac{8a^{9/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan\right.}{15b^{3/4}\sqrt{a+bx^2}}$$

```
[Out] 2/9*(c*x)^(3/2)*(b*x^2+a)^(3/2)/c+4/15*a*(c*x)^(3/2)*(b*x^2+a)^(1/2)/c+8/15
*a^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(a^(1/2)+x*b^(1/2))-8/15*a^(9/4)*(
cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1
/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1/2)*((b*x^2+a)/(
a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^2+a)^(1/2)+4/15*a^(9/4)*(cos(2*arc
tan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*
x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/
4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1/2)*((b*x^2+a)/(a^(1/2)+x
*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {279, 329, 305, 220, 1196}

$$\frac{4a^{9/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^2}}-\frac{8a^{9/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right.\right.}{15b^{3/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a + b*x^2)^(3/2),x]
```

```
[Out] (4*a*(c*x)^(3/2)*Sqrt[a + b*x^2])/(15*c) + (8*a^2*Sqrt[c*x]*Sqrt[a + b*x^2]
)/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) + (2*(c*x)^(3/2)*(a + b*x^2)^(3/2))/(9
*c) - (8*a^(9/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/
2])/(15*b^(3/4)*Sqrt[a + b*x^2]) + (4*a^(9/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)
*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{cx}\left(a+bx^2\right)^{3/2}dx &= \frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}+\frac{1}{3}(2a)\int \sqrt{cx}\sqrt{a+bx^2}\,dx\\
&= \frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}+\frac{1}{15}\left(4a^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx\\
&= \frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}+\frac{\left(8a^2\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{15c}\\
&= \frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}+\frac{\left(8a^{5/2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{15\sqrt{b}}-\\
&= \frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{8a^2\sqrt{cx}\sqrt{a+bx^2}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}-\frac{8a^{9/4}\sqrt{c}\left(\sqrt{a}+\right.}{}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.19

$$
\frac{2ax\sqrt{cx}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a + b*x^2)^(3/2),x]
```

```
[Out] (2*a*x*Sqrt[c*x]*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2
)/a)])/(3*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(c*x), x)
```

maple [A] time = 0.01, size = 218, normalized size = 0.73

$$\frac{2\sqrt{cx}\left(5b^3x^6+16a\,b^2x^4+11a^2b\,x^2+12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a^3\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-6\right.}{45\sqrt{b\,x^2+a}\;bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^2+a)^(3/2),x)
```

```
[Out] 2/45*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b*(5*b^3*x^6+12*((b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(
1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/
2))*a^3-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+16*a*b^2*x^4+11*a^2*b*x^2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{c\,x}\,\left(b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a + b*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(1/2)*(a + b*x^2)^(3/2), x)
```

sympy [C] time = 3.40, size = 46, normalized size = 0.15

$$\frac{a^{\frac{3}{2}}\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(3/2)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-3/2, 3/4), (7/4,), b*x**2*exp_
polar(I*pi)/a)/(2*gamma(7/4))
```

3.601 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=152

$$\frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{7\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}+\frac{4a\sqrt{cx}\,\sqrt{a+bx^2}}{7c}+\frac{2\sqrt{cx}\,\left(a+bx^2\right)^{3/2}}{7c}$$

```
[Out] 2/7*(b*x^2+a)^(3/2)*(c*x)^(1/2)/c+4/7*a*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c+4/7*a
^(7/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*
(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(
a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(1/4)/c^(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {279, 329, 220}

$$\frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}+\frac{4a\sqrt{cx}\,\sqrt{a+bx^2}}{7c}+\frac{2\sqrt{cx}\,\left(a+bx^2\right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/Sqrt[c*x],x]
```

```
[Out] (4*a*Sqrt[c*x]*Sqrt[a + b*x^2])/(7*c) + (2*Sqrt[c*x]*(a + b*x^2)^(3/2))/(7*
c) + (4*a^(7/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(7*b^
(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{\sqrt{cx}}\,dx &= \frac{2\sqrt{cx}\left(a+bx^2\right)^{3/2}}{7c}+\frac{1}{7}(6a)\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx \\
&= \frac{4a\sqrt{cx}\sqrt{a+bx^2}}{7c}+\frac{2\sqrt{cx}\left(a+bx^2\right)^{3/2}}{7c}+\frac{1}{7}\left(4a^2\right)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx \\
&= \frac{4a\sqrt{cx}\sqrt{a+bx^2}}{7c}+\frac{2\sqrt{cx}\left(a+bx^2\right)^{3/2}}{7c}+\frac{\left(8a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{7c} \\
&= \frac{4a\sqrt{cx}\sqrt{a+bx^2}}{7c}+\frac{2\sqrt{cx}\left(a+bx^2\right)^{3/2}}{7c}+\frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right.\right.}{7\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.36

$$
\frac{2ax\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/Sqrt[c*x],x]
[Out] (2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)])/(Sq
rt[c*x]*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}{cx},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/2)*sqrt(c*x)/(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{cx}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(c*x), x)
```

maple [A] time = 0.01, size = 134, normalized size = 0.88

$$
\frac{\frac{2b^3x^5}{7}+\frac{8a\,b^2x^3}{7}+\frac{6a^2bx}{7}+\frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{-ab}\,a^2\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7}}{\sqrt{b\,x^2+a}\,\sqrt{cx}\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(1/2),x)
```

```
[Out] 2/7/(b*x^2+a)^(1/2)*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^2+b^3*x
^5+4*a*b^2*x^3+3*a^2*b*x)/b/(c*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{\sqrt{c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c*x)^(1/2),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c*x)^(1/2), x)
```

sympy [C] time = 2.63, size = 46, normalized size = 0.30

$$\frac{a^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{3}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(1/2),x)
```

```
[Out] a**(3/2)*sqrt(x)*gamma(1/4)*hyper((-3/2, 1/4), (5/4,), b*x**2*exp_polar(I*p
i)/a)/(2*sqrt(c)*gamma(5/4))
```

3.602 $\int \frac{\left(a+bx^2\right)^{3/2}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=296

$$\frac{12a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\mathrm{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{5c^{3/2}\sqrt{a+bx^2}}-\frac{24a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(\right.}{5c^{3/2}\sqrt{a+bx^2}}$$

```
[Out] -2*(b*x^2+a)^(3/2)/c/(c*x)^(1/2)+12/5*b*(c*x)^(3/2)*(b*x^2+a)^(1/2)/c^3+24/
5*a*b^(1/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c^2/(a^(1/2)+x*b^(1/2))-24/5*a^(5/4
)*b^(1/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/
4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a
)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/c^(3/2)/(b*x^2+a)^(1/2)+12/5*a^(5/4)*b^(1/4)
*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(
b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2
)+x*b^(1/2))^2)^(1/2)/c^(3/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {277, 279, 329, 305, 220, 1196}

$$\frac{12a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{5c^{3/2}\sqrt{a+bx^2}}-\frac{24a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{5c^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c*x)^(3/2),x]
```

```
[Out] (12*b*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*c^3) + (24*a*Sqrt[b]*Sqrt[c*x]*Sqrt[a
 + b*x^2])/(5*c^2*(Sqrt[a] + Sqrt[b]*x)) - (2*(a + b*x^2)^(3/2))/(c*Sqrt[c*
x]) - (24*a^(5/4)*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] +
 Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1
/2])/(5*c^(3/2)*Sqrt[a + b*x^2]) + (12*a^(5/4)*b^(1/4)*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqr
t[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*c^(3/2)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{(cx)^{3/2}}\,dx &= -\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}+\frac{(6b)\int \sqrt{cx}\,\sqrt{a+bx^2}\,dx}{c^2}\\
&= \frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}+\frac{(12ab)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5c^2}\\
&= \frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}+\frac{(24ab)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{5c^3}\\
&= \frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}+\frac{\left(24a^{3/2}\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{5c^2}-\frac{\left(24a^{3/2}\cdots\right.}{\cdots}\\
&= \frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3}+\frac{24a\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{5c^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}-\frac{24a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\cdots}}{5c^{3/2}\cdots}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.19

$$-\frac{2ax\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(3/2),x]
```

```
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^2)/a)])/(
(c*x)^(3/2)*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}{c^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(c*x)/(c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(3/2), x)
```

maple [A] time = 0.02, size = 208, normalized size = 0.70

$$\frac{\frac{2b^2x^4}{5}-\frac{8abx^2}{5}+\frac{24\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a^2\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{5}-\frac{12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a^2\,\mathrm{E}}{5}}{\sqrt{b\,x^2+a}\,\sqrt{cx}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(3/2),x)
```

```
[Out] 2/5*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*
b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a
^2+b^2*x^4-4*a*b*x^2-5*a^2)/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{{\left(c\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c*x)^(3/2),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c*x)^(3/2), x)
```

sympy [C] time = 2.48, size = 49, normalized size = 0.17

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(3/2),x)
```

```
[Out] a**(3/2)*gamma(-1/4)*hyper((-3/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

3.603 $\int \frac{\left(a+bx^2\right)^{3/2}}{(cx)^{5/2}}\,dx$

Optimal. Leaf size=152

$$\frac{4a^{3/4}b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{3c^{5/2}\sqrt{a+bx^2}}+\frac{4b\sqrt{cx}\,\sqrt{a+bx^2}}{3c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{3c(cx)^{3/2}}$$

[Out] -2/3*(b*x^2+a)^(3/2)/c/(c*x)^(3/2)+4/3*b*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c^3+4/3*a^(3/4)*b^(3/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/c^(5/2)/(b*x^2+a)^(1/2)

Rubi [A] time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {277, 279, 329, 220}

$$\frac{4a^{3/4}b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3c^{5/2}\sqrt{a+bx^2}}+\frac{4b\sqrt{cx}\,\sqrt{a+bx^2}}{3c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(3/2)/(c*x)^(5/2),x]

[Out] (4*b*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*c^3) - (2*(a + b*x^2)^(3/2))/(3*c*(c*x)^(3/2)) + (4*a^(3/4)*b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*c^(5/2)*Sqrt[a + b*x^2])

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{(cx)^{5/2}}\,dx &= -\frac{2\left(a+bx^2\right)^{3/2}}{3c(cx)^{3/2}}+\frac{(2b)\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx}{c^2}\\
&= \frac{4b\sqrt{cx}\,\sqrt{a+bx^2}}{3c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{3c(cx)^{3/2}}+\frac{(4ab)\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx}{3c^2}\\
&= \frac{4b\sqrt{cx}\,\sqrt{a+bx^2}}{3c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{3c(cx)^{3/2}}+\frac{(8ab)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{3c^3}\\
&= \frac{4b\sqrt{cx}\,\sqrt{a+bx^2}}{3c^3}-\frac{2\left(a+bx^2\right)^{3/2}}{3c(cx)^{3/2}}+\frac{4a^{3/4}b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)\right.}{3c^{5/2}\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.38

$$-\frac{2ax\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^2)/a)])/(
3*(c*x)^(5/2)*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}{c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(c*x)/(c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(5/2), x)
```

maple [A] time = 0.01, size = 125, normalized size = 0.82

$$\frac{\frac{2b^2x^4}{3}+\frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{-ab}\,ax\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3}-\frac{2a^2}{3}}{\sqrt{b\,x^2+a}\,\sqrt{cx}\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(5/2),x)
```

```
[Out] 2/3/(b*x^2+a)^(1/2)/x*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((
-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*Elliptic
F(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x*a+b^2
*x^4-a^2)/c^2/(c*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\,x^2+a\right)^{3/2}}{(c\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c*x)^(5/2),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c*x)^(5/2), x)
```

sympy [C] time = 4.50, size = 49, normalized size = 0.32

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(5/2),x)
```

```
[Out] a**(3/2)*gamma(-3/4)*hyper((-3/2, -3/4), (1/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(5/2)*x**(3/2)*gamma(1/4))
```

3.604 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx$

Optimal. Leaf size=297

$$\frac{12\sqrt[4]{a}\,b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{5c^{7/2}\sqrt{a+bx^2}}-\frac{24\sqrt[4]{a}\,b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\,\text{ta}\right.}{5c^{7/2}\sqrt{a+bx^2}}$$

```
[Out] -2/5*(b*x^2+a)^(3/2)/c/(c*x)^(5/2)-12/5*b*(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)+2
4/5*b^(3/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c^4/(a^(1/2)+x*b^(1/2))-24/5*a^(1/4
)*b^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/
4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a
)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/c^(7/2)/(b*x^2+a)^(1/2)+12/5*a^(1/4)*b^(5/4)
*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(
b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2
)+x*b^(1/2))^2)^(1/2)/c^(7/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {277, 329, 305, 220, 1196}

$$\frac{24b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{5c^4\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{12\sqrt[4]{a}\,b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{5c^{7/2}\sqrt{a+bx^2}}-\frac{24\sqrt[4]{a}\,b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\left(\sqrt{a}\right.}}{5c^{7/2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c*x)^(7/2),x]
```

```
[Out] (-12*b*Sqrt[a + b*x^2])/(5*c^3*Sqrt[c*x]) + (24*b^(3/2)*Sqrt[c*x]*Sqrt[a +
b*x^2])/(5*c^4*(Sqrt[a] + Sqrt[b]*x)) - (2*(a + b*x^2)^(3/2))/(5*c*(c*x)^(5
/2)) - (24*a^(1/4)*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
+ Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])],
1/2])/(5*c^(7/2)*Sqrt[a + b*x^2]) + (12*a^(1/4)*b^(5/4)*(Sqrt[a] + Sqrt[b]*
x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sq
rt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*c^(7/2)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{(cx)^{7/2}}\,dx &= -\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}+\frac{(6b)\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx}{5c^2}\\
&= -\frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}}-\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}+\frac{\left(12b^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5c^4}\\
&= -\frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}}-\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}+\frac{\left(24b^2\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{5c^5}\\
&= -\frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}}-\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}+\frac{\left(24\sqrt{a}\,b^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{5c^4}-\frac{\left(24\sqrt{a}\,b^{3/\cdots}\right.}{\cdots}\\
&= -\frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}}+\frac{24b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{5c^4\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}-\frac{24\sqrt[4]{a}\,b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+\cdots}{\left(\sqrt{a}+\cdots\right.}}}{5c^{7/2}\sqrt{a\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.19

$$
-\frac{2ax\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(7/2),x]
```

```
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^2)/a)])/
(5*(c*x)^(7/2)*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}{c^4x^4},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(c*x)/(c^4*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(7/2), x)
```

maple [A] time = 0.02, size = 216, normalized size = 0.73

$$\frac{-\frac{14b^2x^4}{5}+\frac{24\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,ab\,x^2\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{5}-\frac{12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,ab\,x^2\,\mathrm{Elliptic}}{5}}{\sqrt{b\,x^2+a}\,\sqrt{cx}\,c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(7/2),x)
```

```
[Out] 2/5/x^2*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-6*((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b
)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))*x^2*a*b-7*b^2*x^4-8*a*b*x^2-a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{{\left(c\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c*x)^(7/2),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c*x)^(7/2), x)
```

sympy [C] time = 10.93, size = 53, normalized size = 0.18

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(7/2),x)
```

```
[Out] a**(3/2)*gamma(-5/4)*hyper((-3/2, -5/4), (-1/4,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(7/2)*x**(5/2)*gamma(-1/4))
```

3.605 $\int \frac{\left(a+bx^2\right)^{3/2}}{(cx)^{9/2}}\,dx$

Optimal. Leaf size=152

$$\frac{4b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\ \text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{7\sqrt[4]{a}\,c^{9/2}\sqrt{a+bx^2}}-\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}}-\frac{2\left(a+bx^2\right)^{3/2}}{7c(cx)^{7/2}}$$

```
[Out] -2/7*(b*x^2+a)^(3/2)/c/(c*x)^(7/2)-4/7*b*(b*x^2+a)^(1/2)/c^3/(c*x)^(3/2)+4/
7*b^(7/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/
4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a
)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/c^(9/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {277, 329, 220}

$$\frac{4b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}\,c^{9/2}\sqrt{a+bx^2}}-\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}}-\frac{2\left(a+bx^2\right)^{3/2}}{7c(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c*x)^(9/2),x]
```

```
[Out] (-4*b*Sqrt[a + b*x^2])/(7*c^3*(c*x)^(3/2)) - (2*(a + b*x^2)^(3/2))/(7*c*(c*
x)^(7/2)) + (4*b^(7/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sq
rt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2]
)/(7*a^(1/4)*c^(9/2)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx = -\frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}} + \frac{(6b)\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx}{7c^2}$$
$$= -\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}} - \frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}} + \frac{(4b^2)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{7c^4}$$
$$= -\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}} - \frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}} + \frac{(8b^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{7c^5}$$
$$= -\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}} - \frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}} + \frac{4b^{7/4}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}\,c^{9/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.38

$$-\frac{2ax\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{7}{4},-\frac{3}{2};-\frac{3}{4};-\frac{bx^2}{a}\right)}{7(cx)^{9/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(9/2),x]
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^2)/a)])/
(7*(c*x)^(9/2)*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(bx^2+a)^{\frac{3}{2}}\sqrt{cx}}{c^5x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(9/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/2)*sqrt(c*x)/(c^5*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(9/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(9/2), x)
```

maple [A] time = 0.04, size = 135, normalized size = 0.89

$$\frac{-\frac{6b^2x^4}{7} + \frac{4\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,b\,x^3\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)}{7} - \frac{8ab\,x^2}{7} - \frac{2a^2}{7}}{\sqrt{b\,x^2+a}\,\sqrt{cx}\,c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(9/2),x)
```

```
[Out] 2/7/(b*x^2+a)^(1/2)/x^3*(2*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)
^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^3*b
-3*b^2*x^4-4*a*b*x^2-a^2)/c^4/(c*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{(c\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c*x)^(9/2),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c*x)^(9/2), x)
```

sympy [C] time = 33.12, size = 53, normalized size = 0.35

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{3}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{9}{2}}x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(9/2),x)
```

```
[Out] a**(3/2)*gamma(-7/4)*hyper((-7/4, -3/2), (-3/4,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(9/2)*x**(7/2)*gamma(-3/4))
```

3.606 $\int \frac{\left(a+bx^2\right)^{3/2}}{(cx)^{11/2}}\,dx$

Optimal. Leaf size=331

$$\frac{4b^{9/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}}-\frac{8b^{9/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\cdot\right.\right.}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}}$$

```
[Out] -2/9*(b*x^2+a)^(3/2)/c/(c*x)^(9/2)-4/15*b*(b*x^2+a)^(1/2)/c^3/(c*x)^(5/2)-8
/15*b^2*(b*x^2+a)^(1/2)/a/c^5/(c*x)^(1/2)+8/15*b^(5/2)*(c*x)^(1/2)*(b*x^2+a
)^(1/2)/a/c^6/(a^(1/2)+x*b^(1/2))-8/15*b^(9/4)*(cos(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2
^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)
/c^(11/2)/(b*x^2+a)^(1/2)+4/15*b^(9/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^
(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))
*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*
(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/c^(11/2
)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {277, 325, 329, 305, 220, 1196}

$$\frac{4b^{9/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}}-\frac{8b^{9/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\cdot\right.}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (-4*b*Sqrt[a + b*x^2])/(15*c^3*(c*x)^(5/2)) - (8*b^2*Sqrt[a + b*x^2])/(15*a
*c^5*Sqrt[c*x]) + (8*b^(5/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(15*a*c^6*(Sqrt[a]
+ Sqrt[b]*x)) - (2*(a + b*x^2)^(3/2))/(9*c*(c*x)^(9/2)) - (8*b^(9/4)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(15*a^(3/4)*c^(11/2)*Sqrt[a
 + b*x^2]) + (4*b^(9/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2
])/(15*a^(3/4)*c^(11/2)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{(cx)^{11/2}}\,dx &= -\frac{2\left(a+bx^2\right)^{3/2}}{9c(cx)^{9/2}}+\frac{(2b)\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx}{3c^2}\\
&= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}}-\frac{2\left(a+bx^2\right)^{3/2}}{9c(cx)^{9/2}}+\frac{\left(4b^2\right)\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{15c^4}\\
&= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}}-\frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}}-\frac{2\left(a+bx^2\right)^{3/2}}{9c(cx)^{9/2}}+\frac{\left(4b^3\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{15ac^6}\\
&= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}}-\frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}}-\frac{2\left(a+bx^2\right)^{3/2}}{9c(cx)^{9/2}}+\frac{\left(8b^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{15ac^7}\\
&= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}}-\frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}}-\frac{2\left(a+bx^2\right)^{3/2}}{9c(cx)^{9/2}}+\frac{\left(8b^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{15\sqrt{a}\,c^6}-\frac{(8
\\
&= -\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}}-\frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}}+\frac{8b^{5/2}\sqrt{cx}\,\sqrt{a+bx^2}}{15ac^6\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{2\left(a+bx^2\right)^{3/2}}{9c(cx)^{9/2}}-\frac{8b^{9/4}\left(\sqrt{a}+\sqrt{b}\,x\right)}{15
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.17

$$-\frac{2ax\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{bx^2}{a}\right)}{9(cx)^{11/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^2)/a)])/
(9*(c*x)^(11/2)*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}{c^6x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(c*x)/(c^6*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(11/2), x)
```

maple [A] time = 0.04, size = 234, normalized size = 0.71

$$\frac{-\frac{8b^3x^6}{15}+\frac{8\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a\,b^2x^4\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{15}-\frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a\,b^2x^4\,\mathrm{Ellip}}{15}}{\sqrt{b\,x^2+a}\,\sqrt{cx}\,a\,c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(11/2),x)
```

```
[Out] 2/45/x^4*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a*b^2-6*((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-
a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*x^4*a*b^2-12*b^3*x^6-23*a*b^2*x^4-16*a^2*b*x^2-5*a^3)/(b*x^2+a)^(
1/2)/a/c^5/(c*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(c*x)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(c*x)^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{{\left(c\,x\right)}^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(c*x)^(11/2),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(c*x)^(11/2), x)
```

sympy [C] time = 83.79, size = 53, normalized size = 0.16

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{9}{4},-\frac{3}{2}\\ -\frac{5}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{11}{2}}x^{\frac{9}{2}}\Gamma\left(-\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(c*x)**(11/2),x)
```

```
[Out] a**(3/2)*gamma(-9/4)*hyper((-9/4, -3/2), (-5/4,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(11/2)*x**(9/2)*gamma(-5/4))
```

3.607 $\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx$

Optimal. Leaf size=128

$$-\frac{3\sqrt[4]{6}\,ac^2\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\,\sqrt{3a-2ax^2}}+\frac{2\sqrt{3a-2ax^2}\,(cx)^{7/2}}{9c}-\frac{2}{15}c\sqrt{3a-2ax^2}\,(cx)^{3/2}$$

```
[Out] -3/5*6^(1/4)*a*c^2*EllipticE(1/6*(3-x*6^(1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)
^(1/2)*(-2*x^2+3)^(1/2)/x^(1/2)/(-2*a*x^2+3*a)^(1/2)-2/15*c*(c*x)^(3/2)*(-2
*a*x^2+3*a)^(1/2)+2/9*(c*x)^(7/2)*(-2*a*x^2+3*a)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {279, 321, 320, 319, 318, 424}

$$-\frac{3\sqrt[4]{6}\,ac^2\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\,\sqrt{3a-2ax^2}}+\frac{2\sqrt{3a-2ax^2}\,(cx)^{7/2}}{9c}-\frac{2}{15}c\sqrt{3a-2ax^2}\,(cx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (-2*c*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2])/15 + (2*(c*x)^(7/2)*Sqrt[3*a - 2*a*x
^2])/(9*c) - (3*6^(1/4)*a*c^2*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSin[Sq
rt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*Sqrt[x]*Sqrt[3*a - 2*a*x^2])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx = \frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c}+\frac{1}{3}(2a)\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx$$

$$= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2}+\frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c}+\frac{1}{5}\left(3ac^2\right)\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx$$

$$= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2}+\frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c}+\frac{\left(3ac^2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{5\sqrt{x}}$$

$$= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2}+\frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c}+\frac{\left(3ac^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{5\sqrt{x}\sqrt{3a-2ax^2}}$$

$$= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2}+\frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c}-\frac{\left(3\sqrt[4]{2}\,3^{3/4}ac^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\Bigg(}{5\sqrt{x}\sqrt{3a-2a}$$

$$= -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2}+\frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c}-\frac{3\sqrt[4]{6}\,ac^2\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-}}{\sqrt{}}\right.\right.}{5\sqrt{x}\sqrt{3a-2ax^2}}$$

Mathematica [C] time = 0.03, size = 74, normalized size = 0.58

$$\frac{c\sqrt{a\left(3-2x^2\right)}(cx)^{3/2}\left(3\sqrt{3}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{2x^2}{3}\right)-\left(3-2x^2\right)^{3/2}\right)}{9\sqrt{3-2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (c*(c*x)^(3/2)*Sqrt[a*(3 - 2*x^2)]*(-(3 - 2*x^2)^(3/2) + 3*Sqrt[3]*Hypergeo
metric2F1[-1/2, 3/4, 7/4, (2*x^2)/3]))/(9*Sqrt[3 - 2*x^2])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,c^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2), x)
```

maple [B] time = 0.06, size = 237, normalized size = 1.85

$$\sqrt{cx}\,\sqrt{-\left(2x^2-3\right)a}\,\left(80x^6-168x^4+72x^2+18\sqrt{2}\,\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] 1/180*c^2/x*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(80*x^6+18*2^(1/2)*((2*x+2^(1/
2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))
^(1/2)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2
*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-9*2^(1/2)*((2*x+2^(
1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2
))^(1/2)*3^(1/2)*(-x*2^(1/2)*3^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(
(2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-168*x^4+72*x^2)/(
2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{5/2}\,\sqrt{3\,a-2\,a\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(3*a - 2*a*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(5/2)*(3*a - 2*a*x^2)^(1/2), x)
```

sympy [C] time = 10.49, size = 53, normalized size = 0.41

$$\frac{\sqrt{3}\,\sqrt{a}\,c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{2x^2e^{2i\pi}}{3}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(a)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), 2*
x**2*exp_polar(2*I*pi)/3)/(2*gamma(11/4))
```

3.608 $\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx$

Optimal. Leaf size=117

$$\frac{6^{3/4}ac^{3/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{7\sqrt{a\left(3-2x^2\right)}}+\frac{2\sqrt{3a-2ax^2}\,(cx)^{5/2}}{7c}-\frac{2}{7}c\sqrt{3a-2ax^2}\,\sqrt{cx}$$

```
[Out] 1/7*6^(3/4)*a*c^(3/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*
(-2*x^2+3)^(1/2)/(a*(-2*x^2+3))^(1/2)+2/7*(c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2)/
c-2/7*c*(c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {279, 321, 329, 224, 221}

$$\frac{6^{3/4}ac^{3/2}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{7\sqrt{a\left(3-2x^2\right)}}+\frac{2\sqrt{3a-2ax^2}\,(cx)^{5/2}}{7c}-\frac{2}{7}c\sqrt{3a-2ax^2}\,\sqrt{cx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (-2*c*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2])/7 + (2*(c*x)^(5/2)*Sqrt[3*a - 2*a*x^2]
)/(7*c) + (6^(3/4)*a*c^(3/2)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4)*
Sqrt[c*x])/Sqrt[c]], -1])/(7*Sqrt[a*(3 - 2*x^2)])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx &= \frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{1}{7}(6a)\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx\\
&= -\frac{2}{7}c\sqrt{cx}\,\sqrt{3a-2ax^2}+\frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{1}{7}\left(3ac^2\right)\int \frac{1}{\sqrt{cx}\,\sqrt{3a-2ax^2}}\,dx\\
&= -\frac{2}{7}c\sqrt{cx}\,\sqrt{3a-2ax^2}+\frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{1}{7}(6ac)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx,x,\right.\\
&= -\frac{2}{7}c\sqrt{cx}\,\sqrt{3a-2ax^2}+\frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{\left(2\sqrt{3}\,ac\sqrt{3-2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\right.}{7\sqrt{a\left(3-2x^2\right)}}\\
&= -\frac{2}{7}c\sqrt{cx}\,\sqrt{3a-2ax^2}+\frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{6^{3/4}ac^{3/2}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\right.}{7\sqrt{a\left(3-2x^2\right)}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 74, normalized size = 0.63

$$
\frac{c\sqrt{a\left(3-2x^2\right)}\sqrt{cx}\left(3\sqrt{3}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};\frac{2x^2}{3}\right)-\left(3-2x^2\right)^{3/2}\right)}{7\sqrt{3-2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (c*Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)]*(-(3 - 2*x^2)^(3/2) + 3*Sqrt[3]*Hypergeome
tric2F1[-1/2, 1/4, 5/4, (2*x^2)/3]))/(7*Sqrt[3 - 2*x^2])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,cx,x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2), x)
```

maple [A] time = 0.03, size = 133, normalized size = 1.14

$$-\frac{\sqrt{cx}\,\sqrt{-\left(2x^{2}-3\right)a}\,\left(-8x^{5}+20x^{3}-12x+\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-\sqrt{2}\,\sqrt{3}\,x}\right.}{14\left(2x^{2}-3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] -1/14*c*(c*x)^(1/2)*(-(2*x^2-3)*a)^(1/2)*(-8*x^5+((2*x+2^(1/2)*3^(1/2))*2^(
1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-2^(1/2
)*3^(1/2)*x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(
1/2)*3^(1/2))^(1/2),1/2*2^(1/2))+20*x^3-12*x)/x/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-2\,ax^{2}+3\,a}\,\left(cx\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,x\right)}^{3/2}\,\sqrt{3\,a-2\,a\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(3*a - 2*a*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(3/2)*(3*a - 2*a*x^2)^(1/2), x)
```

sympy [A] time = 2.73, size = 53, normalized size = 0.45

$$\frac{\sqrt{3}\,\sqrt{a}\,c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{2x^{2}e^{2i\pi}}{3}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(a)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), 2*x
**2*exp_polar(2*I*pi)/3)/(2*gamma(9/4))
```

3.609 $\int \sqrt{cx}\sqrt{3a-2ax^2}\,dx$

Optimal. Leaf size=99

$$\frac{2\sqrt{3a-2ax^2}\,(cx)^{3/2}}{5c}-\frac{6\sqrt[4]{6}\,a\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\,\sqrt{3a-2ax^2}}$$

```
[Out] -6/5*6^(1/4)*a*EllipticE(1/6*(3-x*6^(1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)^(1/
2)*(-2*x^2+3)^(1/2)/x^(1/2)/(-2*a*x^2+3*a)^(1/2)+2/5*(c*x)^(3/2)*(-2*a*x^2+
3*a)^(1/2)/c
```

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {279, 320, 319, 318, 424}

$$\frac{2\sqrt{3a-2ax^2}\,(cx)^{3/2}}{5c}-\frac{6\sqrt[4]{6}\,a\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\,\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (2*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2])/(5*c) - (6*6^(1/4)*a*Sqrt[c*x]*Sqrt[3 -
 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*Sqrt[x]*Sqrt[
3*a - 2*a*x^2])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{cx}\sqrt{3a-2ax^2}\,dx &= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}+\frac{1}{5}(6a)\int\frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx\\
&= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}+\frac{\left(6a\sqrt{cx}\right)\int\frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{5\sqrt{x}}\\
&= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}+\frac{\left(6a\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int\frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{5\sqrt{x}\sqrt{3a-2ax^2}}\\
&= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}-\frac{\left(6\sqrt[4]{2}\,3^{3/4}a\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\mathrm{Subst}\left(\int\frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{\frac{2}{3}}\,x}}{\sqrt{2}}\right)}{5\sqrt{x}\sqrt{3a-2ax^2}}\\
&= \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c}-\frac{6\sqrt[4]{6}\,a\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 51, normalized size = 0.52

$$\frac{2x\sqrt{a\left(3-2x^2\right)}\sqrt{cx}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{2x^2}{3}\right)}{\sqrt{9-6x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*Sqrt[3*a - 2*a*x^2],x]
[Out] (2*x*Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, (2*x^2
)/3])/Sqrt[9 - 6*x^2]
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x)
```

maple [B] time = 0.03, size = 229, normalized size = 2.31

$$\frac{\sqrt{cx}\,\sqrt{-\left(2x^2-3\right)a}\,\left(8x^4-12x^2+2\sqrt{2}\,\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-\sqrt{2}\,\sqrt{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] 1/10*(c*x)^(1/2)*(-(2*x^2-3)*a)^(1/2)*(2*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(
1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*
(-2^(1/2)*3^(1/2)*x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1
/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(
1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*
(-2^(1/2)*3^(1/2)*x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1
/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))+8*x^4-12*x^2)/x/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x}\,\sqrt{3\,a-2\,a\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(3*a - 2*a*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)*(3*a - 2*a*x^2)^(1/2), x)
```

sympy [C] time = 1.05, size = 53, normalized size = 0.54

$$\frac{\sqrt{3}\,\sqrt{a}\,\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{2x^2e^{2i\pi}}{3}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(a)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), 2*x*
*2*exp_polar(2*I*pi)/3)/(2*gamma(7/4))
```

3.610 $\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=94

$$\frac{2\,2^{3/4}a\sqrt{3-2x^2}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a\left(3-2x^2\right)}}+\frac{2\sqrt{3a-2ax^2}\sqrt{cx}}{3c}$$

```
[Out] 2/3*2^(3/4)*a*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2+
3)^(1/2)*3^(3/4)/c^(1/2)/(a*(-2*x^2+3))^(1/2)+2/3*(c*x)^(1/2)*(-2*a*x^2+3*a
)^(1/2)/c
```

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {279, 329, 224, 221}

$$\frac{2\sqrt{3a-2ax^2}\sqrt{cx}}{3c}+\frac{2\,2^{3/4}a\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*a - 2*a*x^2]/Sqrt[c*x],x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2])/(3*c) + (2*2^(3/4)*a*Sqrt[3 - 2*x^2]*Elli
pticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3^(1/4)*Sqrt[c]*Sqrt[a
*(3 - 2*x^2)])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx = \frac{2\sqrt{cx}\,\sqrt{3a-2ax^2}}{3c} + (2a)\int \frac{1}{\sqrt{cx}\,\sqrt{3a-2ax^2}}\,dx$$

$$= \frac{2\sqrt{cx}\,\sqrt{3a-2ax^2}}{3c} + \frac{(4a)\,\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{2\sqrt{cx}\,\sqrt{3a-2ax^2}}{3c} + \frac{\left(4a\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{3}\,c\sqrt{a\left(3-2x^2\right)}}$$

$$= \frac{2\sqrt{cx}\,\sqrt{3a-2ax^2}}{3c} + \frac{2\,2^{3/4}a\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\,\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{3}\,\sqrt{c}\,\sqrt{a\left(3-2x^2\right)}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.54

$$\frac{2x\sqrt{a\left(9-6x^2\right)}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};\frac{2x^2}{3}\right)}{\sqrt{3-2x^2}\,\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*a - 2*a*x^2]/Sqrt[c*x],x]
[Out] (2*x*Sqrt[a*(9 - 6*x^2)]*Hypergeometric2F1[-1/2, 1/4, 5/4, (2*x^2)/3])/(Sqr
t[c*x]*Sqrt[3 - 2*x^2])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}}{cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-2\,ax^2+3\,a}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/sqrt(c*x), x)
```

maple [A] time = 0.03, size = 124, normalized size = 1.32

$$-\frac{\sqrt{-\left(2x^2-3\right)a}\,\left(-4x^3+6x+\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-\sqrt{2}\,\sqrt{3}\,x}\,\text{EllipticF}\left(\right.\right.}{3\sqrt{cx}\,\left(2x^2-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] -1/3*(-(2*x^2-3)*a)^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((
-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-2^(1/2)*3^(1/2)*x)^(1/2)*Ell
ipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/
2*2^(1/2))-4*x^3+6*x)/(c*x)^(1/2)/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-2\,ax^2+3\,a}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,a-2\,a\,x^2}}{\sqrt{c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a - 2*a*x^2)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] int((3*a - 2*a*x^2)^(1/2)/(c*x)^(1/2), x)
```

sympy [A] time = 0.93, size = 53, normalized size = 0.56

$$\frac{\sqrt{3}\,\sqrt{a}\,\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{2x^2e^{2i\pi}}{3}\right)}{2\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x**2+3*a)**(1/2)/(c*x)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), 2*x**2*exp_po
lar(2*I*pi)/3)/(2*sqrt(c)*gamma(5/4))
```

3.611 $\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{4\sqrt[4]{6}\,a\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{c^2\sqrt{x}\,\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}$$

```
[Out] 4*6^(1/4)*a*EllipticE(1/6*(3-x*6^(1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)^(1/2)*
(-2*x^2+3)^(1/2)/c^2/x^(1/2)/(-2*a*x^2+3*a)^(1/2)-2*(-2*a*x^2+3*a)^(1/2)/c/
(c*x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {277, 320, 319, 318, 424}

$$\frac{4\sqrt[4]{6}\,a\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{c^2\sqrt{x}\,\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*a - 2*a*x^2]/(c*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(c*Sqrt[c*x]) + (4*6^(1/4)*a*Sqrt[c*x]*Sqrt[3 - 2*
x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(c^2*Sqrt[x]*Sqrt[3
*a - 2*a*x^2])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx &= -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}-\frac{(4a)\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{c^2}\\
&= -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}-\frac{\left(4a\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{c^2\sqrt{x}}\\
&= -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}-\frac{\left(4a\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{c^2\sqrt{x}\sqrt{3a-2ax^2}}\\
&= -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}+\frac{\left(4\sqrt[4]{2}\,3^{3/4}a\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{\frac{2}{3}}\,x}}{\sqrt{2}}\right)}{c^2\sqrt{x}\sqrt{3a-2ax^2}}\\
&= -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}+\frac{4\sqrt[4]{6}\,a\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{c^2\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.52

$$-\frac{2x\sqrt{a\left(9-6x^2\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\frac{2x^2}{3}\right)}{\sqrt{3-2x^2}\,(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*a - 2*a*x^2]/(c*x)^(3/2),x]
```

```
[Out] (-2*x*Sqrt[a*(9 - 6*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, (2*x^2)/3])/((
c*x)^(3/2)*Sqrt[3 - 2*x^2])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}}{c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(3/2), x)
```

maple [B] time = 0.04, size = 225, normalized size = 2.30

$$-\frac{\sqrt{-\left(2x^2-3\right)a}\,\left(12x^2+2\sqrt{2}\,\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-\sqrt{2}\,\sqrt{3}\,x}\,\mathrm{Ellipti}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x)
```

```
[Out] -1/3*(-(2*x^2-3)*a)^(1/2)*(2*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2)
)^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-2^(1/2)*3^
(1/2)*x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)
*3^(1/2))^(1/2),1/2*2^(1/2))-2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2)
)^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-2^(1/2)*3^
(1/2)*x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)
*3^(1/2))^(1/2),1/2*2^(1/2))+12*x^2-18)/c/(c*x)^(1/2)/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,a-2\,a\,x^2}}{{(c\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a - 2*a*x^2)^(1/2)/(c*x)^(3/2),x)
```

```
[Out] int((3*a - 2*a*x^2)^(1/2)/(c*x)^(3/2), x)
```

sympy [C] time = 1.29, size = 56, normalized size = 0.57

$$\frac{\sqrt{3}\,\sqrt{a}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{2x^2e^{2i\pi}}{3}\right)}{2c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x**2+3*a)**(1/2)/(c*x)**(3/2),x)
```

```
[Out] sqrt(3)*sqrt(a)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), 2*x**2*exp_polar(2*
I*pi)/3)/(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

3.612 $\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}} \, dx$

Optimal. Leaf size=96

$$-\frac{4\, 2^{3/4} a \sqrt{3-2x^2}\, \text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{3\sqrt[4]{3}\, c^{5/2}\sqrt{a\left(3-2x^2\right)}}-\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}$$

```
[Out] -4/9*2^(3/4)*a*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2
+3)^(1/2)*3^(3/4)/c^(5/2)/(a*(-2*x^2+3))^(1/2)-2/3*(-2*a*x^2+3*a)^(1/2)/c/(
c*x)^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {277, 329, 224, 221}

$$-\frac{4\, 2^{3/4} a \sqrt{3-2x^2}\, F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3\sqrt[4]{3}\, c^{5/2}\sqrt{a\left(3-2x^2\right)}}-\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*a - 2*a*x^2]/(c*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(3*c*(c*x)^(3/2)) - (4*2^(3/4)*a*Sqrt[3 - 2*x^2]*E
llipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3*3^(1/4)*c^(5/2)*S
qrt[a*(3 - 2*x^2)])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx = -\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}} - \frac{(4a)\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx}{3c^2}$$

$$= -\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}} - \frac{(8a)\,\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3c^3}$$

$$= -\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}} - \frac{\left(8a\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx, x, \sqrt{cx}\right)}{3\sqrt{3}\,c^3\sqrt{a\left(3-2x^2\right)}}$$

$$= -\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}} - \frac{4\,2^{3/4}a\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3\sqrt[4]{3}\,c^{5/2}\sqrt{a\left(3-2x^2\right)}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.53

$$-\frac{2x\sqrt{a\left(3-2x^2\right)}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};\frac{2x^2}{3}\right)}{\sqrt{9-6x^2}\,(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*a - 2*a*x^2]/(c*x)^(5/2),x]
```

```
[Out] (-2*x*Sqrt[a*(3 - 2*x^2)]*Hypergeometric2F1[-3/4, -1/2, 1/4, (2*x^2)/3])/((
c*x)^(5/2)*Sqrt[9 - 6*x^2])
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}}{c^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(5/2), x)
```

maple [A] time = 0.03, size = 129, normalized size = 1.34

$$\frac{2\sqrt{-\left(2x^2-3\right)a}\left(-6x^2+\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-\sqrt{2}\,\sqrt{3}\,x}\,x\,\text{EllipticF}\left(\frac{\sqrt{3}\,\cdots}{\cdots}\right.\right.}{9\sqrt{cx}\,\left(2x^2-3\right)c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x)
```

```
[Out] 2/9*(-(2*x^2-3)*a)^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-
2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-2^(1/2)*3^(1/2)*x)^(1/2)*Elli
pticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2
*2^(1/2))*x-6*x^2+9)/x/c^2/(c*x)^(1/2)/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,a-2\,a\,x^2}}{{(c\,x)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a - 2*a*x^2)^(1/2)/(c*x)^(5/2),x)
```

```
[Out] int((3*a - 2*a*x^2)^(1/2)/(c*x)^(5/2), x)
```

sympy [A] time = 2.86, size = 49, normalized size = 0.51

$$\frac{\sqrt{2}\,i\sqrt{a}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{3}{2x^2}\right)}{2c^{\frac{5}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*x**2+3*a)**(1/2)/(c*x)**(5/2),x)
```

```
[Out] sqrt(2)*I*sqrt(a)*gamma(-1/4)*hyper((-1/2, 1/4), (5/4,), 3/(2*x**2))/(2*c**
(5/2)*sqrt(x)*gamma(3/4))
```

3.613 $\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=156

$$\frac{5a^{7/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}}-\frac{10ac^3\sqrt{cx}\,\sqrt{a+bx^2}}{21b^2}+\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}$$

```
[Out] 2/7*c*(c*x)^(5/2)*(b*x^2+a)^(1/2)/b-10/21*a*c^3*(c*x)^(1/2)*(b*x^2+a)^(1/2)
/b^2+5/21*a^(7/4)*c^(7/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(si
n(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^
(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {321, 329, 220}

$$\frac{5a^{7/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}}-\frac{10ac^3\sqrt{cx}\,\sqrt{a+bx^2}}{21b^2}+\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (-10*a*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/(21*b^2) + (2*c*(c*x)^(5/2)*Sqrt[a +
b*x^2])/(7*b) + (5*a^(7/4)*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqr
t[c])], 1/2])/(21*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx = \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b} - \frac{\left(5ac^2\right)\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx}{7b}$$
$$= -\frac{10ac^3\sqrt{cx}\,\sqrt{a+bx^2}}{21b^2} + \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b} + \frac{\left(5a^2c^4\right)\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx}{21b^2}$$
$$= -\frac{10ac^3\sqrt{cx}\,\sqrt{a+bx^2}}{21b^2} + \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b} + \frac{\left(10a^2c^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{21b^2}$$
$$= -\frac{10ac^3\sqrt{cx}\,\sqrt{a+bx^2}}{21b^2} + \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b} + \frac{5a^{7/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{21b^{9/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.03, size = 87, normalized size = 0.56

$$\frac{2c^3\sqrt{cx}\left(5a^2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right) - 5a^2 - 2abx^2 + 3b^2x^4\right)}{21b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/Sqrt[a + b*x^2],x]
[Out] (2*c^3*Sqrt[c*x]*(-5*a^2 - 2*a*b*x^2 + 3*b^2*x^4 + 5*a^2*Sqrt[1 + (b*x^2)/a
]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(21*b^2*Sqrt[a + b*x^2])
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx}\,c^3x^3}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x)*c^3*x^3/sqrt(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((c*x)^(7/2)/sqrt(b*x^2 + a), x)
```

maple [A] time = 0.03, size = 141, normalized size = 0.90

$$\frac{\sqrt{cx}\left(6b^3x^5 - 4a\,b^2x^3 - 10a^2bx + 5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{-ab}\,a^2\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)\right)c^3}{21\sqrt{b\,x^2+a}\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/21*c^3/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*
x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a
*b)^(1/2)*a^2+6*b^3*x^5-4*a*b^2*x^3-10*a^2*b*x)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/sqrt(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{7/2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(7/2)/(a + b*x^2)^(1/2), x)
```

sympy [C] time = 19.57, size = 44, normalized size = 0.28

$$\frac{c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] c**(7/2)*x**(9/2)*gamma(9/4)*hyper((1/2, 9/4), (13/4,), b*x**2*exp_polar(I*
pi)/a)/(2*sqrt(a)*gamma(13/4))
```

3.614 $\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=273

$$-\frac{3a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}+\frac{6a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan\right.}{5b^{7/4}\sqrt{a+bx^2}}$$

```
[Out] 2/5*c*(c*x)^(3/2)*(b*x^2+a)^(1/2)/b-6/5*a*c^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b
^(3/2)/(a^(1/2)+x*b^(1/2))+6/5*a^(5/4)*c^(5/2)*(cos(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2
^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)
/(b*x^2+a)^(1/2)-3/5*a^(5/4)*c^(5/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1
/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*E
llipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a
^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2+a)
^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = $\{321, 329, 305, 220, 1196\}$

$$-\frac{3a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}+\frac{6a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right.\right.}{5b^{7/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*b) - (6*a*c^2*Sqrt[c*x]*Sqrt[a + b*x^2
])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) + (6*a^(5/4)*c^(5/2)*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*
Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^2]) - (3*a^(5/
4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*b^(7/4)
*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx = \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{\left(3ac^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5b}$$

$$= \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{(6ac)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5b}$$

$$= \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{\left(6a^{3/2}c^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5b^{3/2}} + \frac{\left(6a^{3/2}c^2\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,d\right.}{5b^{3/2}}$$

$$= \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{6ac^2\sqrt{cx}\sqrt{a+bx^2}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)} + \frac{6a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{5b^{7/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.03, size = 69, normalized size = 0.25

$$\frac{2c(cx)^{3/2}\left(-a\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+a+bx^2\right)}{5b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*(c*x)^(3/2)*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2,
3/4, 7/4, -((b*x^2)/a)]))/(5*b*Sqrt[a + b*x^2])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx}\,c^2x^2}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)*c^2*x^2/sqrt(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\sqrt{bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/sqrt(b*x^2 + a), x)
```

maple [A] time = 0.03, size = 210, normalized size = 0.77

$$-\frac{\sqrt{cx}\left(-2b^2x^4 - 2ab\,x^2 + 6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a^2\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right) - 3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{}\right.}{5\sqrt{b\,x^2 + a}\;b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/5*c^2/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2
)*b*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
*a^2-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-2*b^2*x^4-2*a*b*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\sqrt{bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/sqrt(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x)^{5/2}}{\sqrt{b\,x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(5/2)/(a + b*x^2)^(1/2), x)
```

sympy [C] time = 6.83, size = 44, normalized size = 0.16

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*sqrt(a)*gamma(11/4))
```

3.615 $\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=127

$$\frac{2c\sqrt{cx}\,\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}$$

```
[Out] 2/3*c*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b-1/3*a^(3/4)*c^(3/2)*(cos(2*arctan(b^(1/
4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/
a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2
))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2
)/b^(5/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {321, 329, 220}

$$\frac{2c\sqrt{cx}\,\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*b) - (a^(3/4)*c^(3/2)*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*S
qrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*b^(5/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx = \frac{2c\sqrt{cx}\,\sqrt{a+bx^2}}{3b} - \frac{(ac^2)\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx}{3b}$$

$$= \frac{2c\sqrt{cx}\,\sqrt{a+bx^2}}{3b} - \frac{(2ac)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3b}$$

$$= \frac{2c\sqrt{cx}\,\sqrt{a+bx^2}}{3b} - \frac{a^{3/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.02, size = 69, normalized size = 0.54

$$\frac{2c\sqrt{cx}\left(-a\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+a+bx^2\right)}{3b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*c*Sqrt[c*x]*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/
2, 5/4, -((b*x^2)/a)]))/(3*b*Sqrt[a + b*x^2])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx}\,cx}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)*c*x/sqrt(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/sqrt(b*x^2 + a), x)
```

maple [A] time = 0.01, size = 125, normalized size = 0.98

$$-\frac{\sqrt{cx}\left(-2b^2x^3-2abx+\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{-ab}\,a\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\right)c}{3\sqrt{b\,x^2+a}\;b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(
1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^
(1/2)*a-2*b^2*x^3-2*a*b*x)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/sqrt(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(3/2)/(a + b*x^2)^(1/2), x)
```

sympy [C] time = 1.93, size = 44, normalized size = 0.35

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*sqrt(a)*gamma(9/4))
```

3.616 $\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\ \text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}}-\frac{2\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan\cdots\right.}{b^{3/4}\sqrt{a+bx^2}}$$

```
[Out] 2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(a^(1/2)+x*b^(1/2))-2*a^(1/4)*(cos(2*
arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^
(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1/2)*((b*x^2+a)/(a^(1/2
)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^2+a)^(1/2)+a^(1/4)*(cos(2*arctan(b^(1/4)
*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^
(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))
),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2
)^(1/2)/b^(3/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {329, 305, 220, 1196}

$$\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}}-\frac{2\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right.\right.}{b^{3/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/Sqrt[a + b*x^2],x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) - (2*a^(1/4)*
Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(3/4)*Sqrt
[a + b*x^2]) + (a^(1/4)*Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c
])], 1/2])/(b^(3/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$
$$= \frac{\left(2\sqrt{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{b}} - \frac{\left(2\sqrt{a}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{b}}$$
$$= \frac{2\sqrt{cx}\sqrt{a+bx^2}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{2\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}\right.}{}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.24

$$\frac{2x\sqrt{cx}\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/Sqrt[a + b*x^2],x]
[Out] (2*x*Sqrt[c*x]*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^
2)/a)])/(3*Sqrt[a + b*x^2])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx}}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x)/sqrt(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x)/sqrt(b*x^2 + a), x)
```

maple [A] time = 0.01, size = 132, normalized size = 0.56

$$\frac{\sqrt{cx}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\right)a}{\sqrt{b\,x^{2}+a}\;bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (c*x)^(1/2)/(b*x^2+a)^(1/2)*a/b*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(
1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*(
2*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-EllipticF(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/sqrt(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)/(a + b*x^2)^(1/2), x)
```

sympy [C] time = 0.93, size = 44, normalized size = 0.19

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*sqrt(a)*gamma(7/4))
```

3.617 $\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=97

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\ \text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}$$

```
[Out] (cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b
^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(
1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)
+x*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c^(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {329, 220}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*Sqrt[a + b*x^2]),x]
```

```
[Out] ((Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[
2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(a^(1/4)*b^(1/4)*Sqr
t[c]*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{c}\\
&= \frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.56

$$\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\,\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*Sqrt[a + b*x^2]),x]
[Out] (2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/(S
qrt[c*x]*Sqrt[a + b*x^2])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}}{bcx^3+acx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b*c*x^3 + a*c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+a}\,\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(c*x)), x)
```

maple [A] time = 0.02, size = 104, normalized size = 1.07

$$\frac{\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{b\,x^2+a}\,\sqrt{cx}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x)
[Out] 1/(b*x^2+a)^(1/2)*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1
/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*El
lipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))/b/(c*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+a}\,\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(c*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x}\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a + b*x^2)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(a + b*x^2)^(1/2)), x)
```

sympy [C] time = 1.09, size = 44, normalized size = 0.45

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*s
qrt(a)*sqrt(c)*gamma(5/4))
```

3.618 $\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=268

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\mathrm{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{a^{3/4}c^{3/2}\sqrt{a+bx^2}}-\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)\right.}{a^{3/4}c^{3/2}\sqrt{a+bx^2}}$$

```
[Out] -2*(b*x^2+a)^(1/2)/a/c/(c*x)^(1/2)+2*b^(1/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a/
c^2/(a^(1/2)+x*b^(1/2))-2*b^(1/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)
/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*Elli
pticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1
/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/c^(3/2)/(b*x
^2+a)^(1/2)+b^(1/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*ar
ctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))
*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/c^(3/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {325, 329, 305, 220, 1196}

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}c^{3/2}\sqrt{a+bx^2}}-\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}c^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(a*c*Sqrt[c*x]) + (2*Sqrt[b]*Sqrt[c*x]*Sqrt[a + b*x^2]
)/(a*c^2*(Sqrt[a] + Sqrt[b]*x)) - (2*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a
^(1/4)*Sqrt[c])], 1/2])/(a^(3/4)*c^(3/2)*Sqrt[a + b*x^2]) + (b^(1/4)*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(a^(3/4)*c^(3/2)*Sqrt[a + b
*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}} + \frac{b\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{ac^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}} + \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{ac^3}$$

$$= -\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}} + \frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{a}\,c^2} - \frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{\right.}{\sqrt{a}\,c^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}} + \frac{2\sqrt{b}\,\sqrt{cx}\,\sqrt{a+bx^2}}{ac^2\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right.\right.}{a^{3/4}c^{3/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.20

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^2)/a)])/
((c*x)^(3/2)*Sqrt[a + b*x^2])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}}{bc^2x^4+ac^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b*c^2*x^4 + a*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(3/2)), x)
```

maple [A] time = 0.02, size = 196, normalized size = 0.73

$$\frac{-2b\,x^2+2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{}}{\sqrt{b\,x^2+a}\,\sqrt{cx}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2
)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-2*b*x^2-
2*a)/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{3/2}\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a + b*x^2)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(3/2)*(a + b*x^2)^(1/2)), x)
```

sympy [C] time = 1.64, size = 48, normalized size = 0.18

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)
*c**(3/2)*sqrt(x)*gamma(3/4))
```

3.619 $\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=129

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{3a^{5/4}c^{5/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}$$

```
[Out] -2/3*(b*x^2+a)^(1/2)/a/c/(c*x)^(3/2)-1/3*b^(3/4)*(cos(2*arctan(b^(1/4)*(c*x
)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)
/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2
*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(5/
4)/c^(5/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = $\{325, 329, 220\}$

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{3a^{5/4}c^{5/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(3*a*c*(c*x)^(3/2)) - (b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*a^(5/4)*c^(5/2)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}} - \frac{b\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{3ac^2}$$
$$= -\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}} - \frac{(2b)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3ac^3}$$
$$= -\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}} - \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{3a^{5/4}c^{5/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.43

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*Sqrt[a + b*x^2]),x]
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^2)/a)])/
(3*(c*x)^(5/2)*Sqrt[a + b*x^2])
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}}{bc^3x^5+ac^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b*c^3*x^5 + a*c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(5/2)), x)
```

maple [A] time = 0.02, size = 123, normalized size = 0.95

$$-\frac{2b\,x^2+\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{-ab}\,x\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)+2a}{3\sqrt{b\,x^2+a}\,\sqrt{cx}\,a\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3/(b*x^2+a)^(1/2)/x*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x+2*b*x^2+2*a)/a/c^2/(c*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{5/2}\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a + b*x^2)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(5/2)*(a + b*x^2)^(1/2)), x)
```

sympy [C] time = 3.89, size = 48, normalized size = 0.37

$$\frac{\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*c**(5/2)*x**(3/2)*gamma(1/4))
```

3.620 $\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=306

$$-\frac{3b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\ \text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}}+\frac{6b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\right.}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}}$$

```
[Out] -2/5*(b*x^2+a)^(1/2)/a/c/(c*x)^(5/2)+6/5*b*(b*x^2+a)^(1/2)/a^2/c^3/(c*x)^(1
/2)-6/5*b^(3/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a^2/c^4/(a^(1/2)+x*b^(1/2))+6/5
*b^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4
)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)
/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/c^(7/2)/(b*x^2+a)^(1/2)-3/5*b^(5/4)*(
cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1
/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+
x*b^(1/2))^2)^(1/2)/a^(7/4)/c^(7/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {325, 329, 305, 220, 1196}

$$-\frac{6b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^2c^4\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{3b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}}+\frac{6b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)}}}{5a^{7/4}c^{7/2}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*Sqrt[a + b*x^2])/(5*a*c*(c*x)^(5/2)) + (6*b*Sqrt[a + b*x^2])/(5*a^2*c^3
*Sqrt[c*x]) - (6*b^(3/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(5*a^2*c^4*(Sqrt[a] + S
qrt[b]*x)) + (6*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2
])/(5*a^(7/4)*c^(7/2)*Sqrt[a + b*x^2]) - (3*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*a^(7/4)*c^(7/2)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} - \frac{(3b)\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{5ac^2}$$

$$= -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} - \frac{\left(3b^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{5a^2c^4}$$

$$= -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} - \frac{\left(6b^2\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5a^2c^5}$$

$$= -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} - \frac{\left(6b^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{5a^{3/2}c^4} + \frac{\left(6b^{3/2}\right)\mathrm{Subst}\left(\int\right.}{5a}$$

$$= -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} - \frac{6b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^2c^4\left(\sqrt{a}+\sqrt{b}\,x\right)} + \frac{6b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E}{5a^{7/4}c^{7/2}\sqrt{a+b}}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.18

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*Sqrt[a + b*x^2]),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^2)/a)])
/(5*(c*x)^(7/2)*Sqrt[a + b*x^2])
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}}{bc^4x^6+ac^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b*c^4*x^6 + a*c^4*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(7/2)), x)
```

maple [A] time = 0.02, size = 219, normalized size = 0.72

$$-\frac{-6b^2x^4+6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,ab\,x^2\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\cdots}{5\sqrt{b\,x^2+a}\,\sqrt{cx}\,a^2c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/5/x^2*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-3*((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b
)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))*x^2*a*b-6*b^2*x^4-4*a*b*x^2+2*a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)/a
^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(c*x)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{7/2}\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a + b*x^2)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(7/2)*(a + b*x^2)^(1/2)), x)
```

sympy [C] time = 12.69, size = 51, normalized size = 0.17

$$\frac{\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a
)*c**(7/2)*x**(5/2)*gamma(-1/4))
```

3.621 $\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{5a^{3/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{6b^{9/4}\sqrt{a+bx^2}}+\frac{5c^3\sqrt{cx}\,\sqrt{a+bx^2}}{3b^2}-\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}$$

```
[Out] -c*(c*x)^(5/2)/b/(b*x^2+a)^(1/2)+5/3*c^3*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^2-5/
6*a^(3/4)*c^(7/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arct
an(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*(
(b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {288, 321, 329, 220}

$$-\frac{5a^{3/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{a+bx^2}}+\frac{5c^3\sqrt{cx}\,\sqrt{a+bx^2}}{3b^2}-\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((c*(c*x)^(5/2))/(b*Sqrt[a + b*x^2])) + (5*c^3*Sqrt[c*x]*Sqrt[a + b*x^2])/
(3*b^2) - (5*a^(3/4)*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a
] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]
, 1/2])/(6*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{7/2}}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}} + \frac{\left(5c^2\right)\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx}{2b}$$

$$= -\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}} + \frac{5c^3\sqrt{cx}\,\sqrt{a+bx^2}}{3b^2} - \frac{\left(5ac^4\right)\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx}{6b^2}$$

$$= -\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}} + \frac{5c^3\sqrt{cx}\,\sqrt{a+bx^2}}{3b^2} - \frac{\left(5ac^3\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3b^2}$$

$$= -\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}} + \frac{5c^3\sqrt{cx}\,\sqrt{a+bx^2}}{3b^2} - \frac{5a^{3/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)\right.}{6b^{9/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.03, size = 74, normalized size = 0.48

$$\frac{c^3\sqrt{cx}\left(-5a\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+5a+2bx^2\right)}{3b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] (c^3*Sqrt[c*x]*(5*a + 2*b*x^2 - 5*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1
/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*b^2*Sqrt[a + b*x^2])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}\,c^3x^3}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c^3*x^3/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(3/2), x)
```

maple [A] time = 0.04, size = 128, normalized size = 0.84

$$-\frac{\sqrt{cx}\,\left(-4b^{2}x^{3}-10abx+5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{-ab}\,a\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\right)c^{3}}{6\sqrt{b\,x^{2}+a}\;b^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/6*c^3/x*(c*x)^(1/2)*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*(
(-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*Ellipti
cF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a-4*b^
2*x^3-10*a*b*x)/(b*x^2+a)^(1/2)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(bx^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{7/2}}{\left(b\,x^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)/(a + b*x^2)^(3/2), x)
```

sympy [C] time = 22.73, size = 44, normalized size = 0.29

$$\frac{c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] c**(7/2)*x**(9/2)*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(3/2)*gamma(13/4))
```

3.622 $\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=266

$$\frac{3\sqrt[4]{a}\,c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\;\mathrm{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^2}}-\frac{3\sqrt[4]{a}\,c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\;E\left(2\tan\right.}{b^{7/4}\sqrt{a+bx^2}}$$

```
[Out] -c*(c*x)^(3/2)/b/(b*x^2+a)^(1/2)+3*c^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(3/2)/
(a^(1/2)+x*b^(1/2))-3*a^(1/4)*c^(5/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(
1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*
EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(
a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2+a
)^(1/2)+3/2*a^(1/4)*c^(5/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/
2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(
sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*
b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {288, 329, 305, 220, 1196}

$$\frac{3c^2\sqrt{cx}\,\sqrt{a+bx^2}}{b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{3\sqrt[4]{a}\,c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\;F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^2}}-\frac{3\sqrt[4]{a}\,c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right.}}}{b^{7/4}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((c*(c*x)^(3/2))/(b*Sqrt[a + b*x^2])) + (3*c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/
(b^(3/2)*(Sqrt[a] + Sqrt[b]*x)) - (3*a^(1/4)*c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*
Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[
c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(7/4)*Sqrt[a + b*x^2]) + (3*a^(1/4)*c^(5
/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(7/4)*Sqrt[a
 + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}+\frac{\left(3c^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{2b}\\
&= -\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}+\frac{(3c)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{b}\\
&= -\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}+\frac{\left(3\sqrt{a}\,c^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{b^{3/2}}-\frac{\left(3\sqrt{a}\,c^2\right)\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}\,c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x \ldots\right.}{b^{3/2}}\\
&= -\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}+\frac{3c^2\sqrt{cx}\,\sqrt{a+bx^2}}{b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{3\sqrt[4]{a}\,c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{c \ldots}}{\sqrt[4]{a}\,\sqrt{\ldots}}\right.\right.}{b^{7/4}\sqrt{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 60, normalized size = 0.23

$$
-\frac{2c(cx)^{3/2}\left(\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)-1\right)}{b\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] (-2*c*(c*x)^(3/2)*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4
, -((b*x^2)/a)]))/(b*Sqrt[a + b*x^2])
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}\,c^2x^2}{b^2x^4+2\,abx^2+a^2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c^2*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/2), x)
```

maple [A] time = 0.04, size = 197, normalized size = 0.74

$$\frac{\sqrt{cx}\left(-2b\,x^2+6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-a}}{\sqrt{-ab}}}\right.}{2\sqrt{b\,x^2+a}\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(3/2),x)
[Out] 1/2*c^2/x*(c*x)^(1/2)*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((
-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*Elliptic
E(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-3*((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-
1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))*a-2*b*x^2)/(b*x^2+a)^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x)^{5/2}}{\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a + b*x^2)^(3/2),x)
[Out] int((c*x)^(5/2)/(a + b*x^2)^(3/2), x)
```

sympy [C] time = 7.30, size = 44, normalized size = 0.17

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(3/2)*gamma(11/4))
```

3.623 $\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=125

$$\frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\,b^{5/4}\sqrt{a+bx^2}}-\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}$$

```
[Out] -c*(c*x)^(1/2)/b/(b*x^2+a)^(1/2)+1/2*c^(3/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1
/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(
1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(
1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/b
^(5/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {288, 329, 220}

$$\frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,b^{5/4}\sqrt{a+bx^2}}-\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((c*Sqrt[c*x])/(b*Sqrt[a + b*x^2])) + (c^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])
/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}} + \frac{c^2\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx}{2b}$$

$$= -\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{b}$$

$$= -\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}} + \frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,b^{5/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.03, size = 59, normalized size = 0.47

$$\frac{c\sqrt{cx}\left(\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-1\right)}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(3/2),x]
[Out] (c*Sqrt[c*x]*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((
b*x^2)/a)]))/(b*Sqrt[a + b*x^2])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}\,cx}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c*x/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(3/2), x)
```

maple [A] time = 0.01, size = 115, normalized size = 0.92

$$\frac{\sqrt{cx}\left(-2bx+\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{-ab}\;\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)\right)c}{2\sqrt{b\,x^2+a}\;b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*c/x*(c*x)^(1/2)*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)-2*b*x)/(b*x
^2+a)^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(3/2)/(a + b*x^2)^(3/2), x)
```

sympy [C] time = 2.05, size = 44, normalized size = 0.35

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(3/2)*gamma(9/4))
```

3.624 $\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=266

$$-\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^2}}+\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{a^{3/4}b^{3/4}\sqrt{a+bx^2}}$$

```
[Out] (c*x)^(3/2)/a/c/(b*x^2+a)^(1/2)-(c*x)^(1/2)*(b*x^2+a)^(1/2)/a/b^(1/2)/(a^(1
/2)+x*b^(1/2))+(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(
b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1
/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^2+a)^(1/2)
-1/2*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c
*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1/2)*((b*x^
2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {290, 329, 305, 220, 1196}

$$-\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^2}}+\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right.}{a^{3/4}b^{3/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(3/2),x]
```

```
[Out] (c*x)^(3/2)/(a*c*Sqrt[a + b*x^2]) - (Sqrt[c*x]*Sqrt[a + b*x^2])/(a*Sqrt[b]*
(Sqrt[a] + Sqrt[b]*x)) + (Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt
[c])], 1/2])/(a^(3/4)*b^(3/4)*Sqrt[a + b*x^2]) - (Sqrt[c]*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*
Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{2a}\\
&= \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{ac}\\
&= \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{\sqrt{a}\sqrt{b}}+\frac{\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{\sqrt{a}\sqrt{b}}\\
&= \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\sqrt{cx}\sqrt{a+bx^2}}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{a+bx^2}}-
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.22

$$
\frac{2x\sqrt{cx}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)}{3a\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*x*Sqrt[c*x]*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^
2)/a)])/(3*a*Sqrt[a + b*x^2])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^2x^4+2\,abx^2+a^2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(3/2), x)
```

maple [A] time = 0.01, size = 197, normalized size = 0.74

$$-\frac{\sqrt{cx}\left(-2b\,x^2+2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\cdots\right.}{2\sqrt{b\,x^2+a}\;abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*(c*x)^(1/2)*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b
)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))*a-2*b*x^2)/(b*x^2+a)^(1/2)/b/x/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x}}{\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(1/2)/(a + b*x^2)^(3/2), x)
```

sympy [C] time = 1.28, size = 44, normalized size = 0.17

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(3/2)*gamma(7/4))
```

3.625 $\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}+\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}$$

```
[Out] (c*x)^(1/2)/a/c/(b*x^2+a)^(1/2)+1/2*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/
4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*El
lipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^
(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(5/4)/b^(1/4)/c^
(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {290, 329, 220}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}+\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c*x]/(a*c*Sqrt[a + b*x^2]) + ((Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqr
t[c])], 1/2])/(2*a^(5/4)*b^(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/2}}\,dx = \frac{\sqrt{cx}}{ac\sqrt{a+bx^2}} + \frac{\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{2a}$$

$$= \frac{\sqrt{cx}}{ac\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{ac}$$

$$= \frac{\sqrt{cx}}{ac\sqrt{a+bx^2}} + \frac{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.02, size = 59, normalized size = 0.47

$$\frac{x\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+x}{a\sqrt{cx}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(3/2)),x]
```

```
[Out] (x + x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/
(a*Sqrt[c*x]*Sqrt[a + b*x^2])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^2cx^5+2\,abcx^3+a^2cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^2*c*x^5 + 2*a*b*c*x^3 + a^2*c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(c*x)), x)
```

maple [A] time = 0.02, size = 114, normalized size = 0.90

$$\frac{2bx+\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{-ab}\;\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)}{2\sqrt{b\,x^2+a}\,\sqrt{cx}\;ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)+2*b*x)/(b*x^2+a)^(1/2)/b/a/
(c*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(c*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x}\,\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a + b*x^2)^(3/2)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(a + b*x^2)^(3/2)), x)
```

sympy [C] time = 1.80, size = 44, normalized size = 0.35

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a
**(3/2)*sqrt(c)*gamma(5/4))
```

3.626 $\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=296

$$\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2a^{7/4}c^{3/2}\sqrt{a+bx^2}}-\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cdots}{\sqrt[4]{a}\,\cdots}\right.\right.}{a^{7/4}c^{3/2}\sqrt{a+bx^2}}$$

```
[Out] 1/a/c/(c*x)^(1/2)/(b*x^2+a)^(1/2)-3*(b*x^2+a)^(1/2)/a^2/c/(c*x)^(1/2)+3*b^(
1/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a^2/c^2/(a^(1/2)+x*b^(1/2))-3*b^(1/4)*(cos
(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/
4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)
/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b
^(1/2))^2)^(1/2)/a^(7/4)/c^(3/2)/(b*x^2+a)^(1/2)+3/2*b^(1/4)*(cos(2*arctan(
b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(
1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)
^(1/2)/a^(7/4)/c^(3/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {290, 325, 329, 305, 220, 1196}

$$\frac{3\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{a^2c^2\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{7/4}c^{3/2}\sqrt{a+bx^2}}-\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(\cdots\right.}{a^{7/4}c^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] 1/(a*c*Sqrt[c*x]*Sqrt[a + b*x^2]) - (3*Sqrt[a + b*x^2])/(a^2*c*Sqrt[c*x]) +
 (3*Sqrt[b]*Sqrt[c*x]*Sqrt[a + b*x^2])/(a^2*c^2*(Sqrt[a] + Sqrt[b]*x)) - (3
*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(a^(7/4)*c^(
3/2)*Sqrt[a + b*x^2]) + (3*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqr
t[c])], 1/2])/(2*a^(7/4)*c^(3/2)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx = \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}} + \frac{3\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{2a}$$

$$= \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}} - \frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}} + \frac{(3b)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{2a^2c^2}$$

$$= \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}} - \frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{a^2c^3}$$

$$= \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}} - \frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}} + \frac{\left(3\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{a^{3/2}c^2} - \frac{\left(3\sqrt{b}\right)\mathrm{S}\cdots}{\cdots}$$

$$= \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}} - \frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}} + \frac{3\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{a^2c^2\left(\sqrt{a}+\sqrt{b}\,x\right)} - \frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+\cdots}{\left(\sqrt{a}+\cdots\right.}}}{a^{7/4}c^{3/2}\cdots}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.19

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{a(cx)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^2)/a)])/
(a*(c*x)^(3/2)*Sqrt[a + b*x^2])
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^2c^2x^6+2\,abc^2x^4+a^2c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^2*c^2*x^6 + 2*a*b*c^2*x^4 + a^2*c^2*x
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/2)), x)
```

maple [A] time = 0.02, size = 197, normalized size = 0.67

$$\frac{-6b\,x^2+6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-}}{2\sqrt{b\,x^2+a}\,\sqrt{cx}\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x
)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-6*
b*x^2-4*a)/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{3/2}\,\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a + b*x^2)^(3/2)),x)
```

```
[Out] int(1/((c*x)^(3/2)*(a + b*x^2)^(3/2)), x)
```

sympy [C] time = 3.16, size = 48, normalized size = 0.16

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4}, \frac{3}{2}\\ \frac{3}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2
)*c**(3/2)*sqrt(x)*gamma(3/4))
```

3.627 $\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{6a^{9/4}c^{5/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}}+\frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}$$

```
[Out] 1/a/c/(c*x)^(3/2)/(b*x^2+a)^(1/2)-5/3*(b*x^2+a)^(1/2)/a^2/c/(c*x)^(3/2)-5/6
*b^(3/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4
)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)
/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(9/4)/c^(5/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {290, 325, 329, 220}

$$-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{6a^{9/4}c^{5/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}}+\frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] 1/(a*c*(c*x)^(3/2)*Sqrt[a + b*x^2]) - (5*Sqrt[a + b*x^2])/(3*a^2*c*(c*x)^(3
/2)) - (5*b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(6*
a^(9/4)*c^(5/2)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx = \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}} + \frac{5\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx}{2a}$$

$$= \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}} - \frac{(5b)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{6a^2c^2}$$

$$= \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}} - \frac{(5b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3a^2c^3}$$

$$= \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}} - \frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right.\right.}{6a^{9/4}c^{5/2}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.38

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3a(cx)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(3/2)),x]
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^2)/a)])/
(3*a*(c*x)^(5/2)*Sqrt[a + b*x^2])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}}{b^2c^3x^7+2\,abc^3x^5+a^2c^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^2*c^3*x^7 + 2*a*b*c^3*x^5 + a^2*c^3*x
^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(5/2)), x)
```

maple [A] time = 0.02, size = 124, normalized size = 0.81

$$-\frac{10b\,x^{2}+5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{-ab}\,x\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+4a}{6\sqrt{b\,x^{2}+a}\,\sqrt{cx}\,a^{2}c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/6/x*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x+10*b*x^2+4*a)/(b*x^2
+a)^(1/2)/a^2/c^2/(c*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{3}{2}}\left(cx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c\,x\right)^{5/2}\,\left(b\,x^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a + b*x^2)^(3/2)),x)
```

```
[Out] int(1/((c*x)^(5/2)*(a + b*x^2)^(3/2)), x)
```

sympy [C] time = 7.96, size = 48, normalized size = 0.31

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2
)*c**(5/2)*x**(3/2)*gamma(1/4))
```

3.628 $\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\ \text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{10a^{11/4}c^{7/2}\sqrt{a+bx^2}}+\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan\right.}{5a^{11/4}c^{7/2}\sqrt{a+bx^2}}$$

```
[Out] 1/a/c/(c*x)^(5/2)/(b*x^2+a)^(1/2)-7/5*(b*x^2+a)^(1/2)/a^2/c/(c*x)^(5/2)+21/
5*b*(b*x^2+a)^(1/2)/a^3/c^3/(c*x)^(1/2)-21/5*b^(3/2)*(c*x)^(1/2)*(b*x^2+a)^
(1/2)/a^3/c^4/(a^(1/2)+x*b^(1/2))+21/5*b^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2
^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4
)/c^(7/2)/(b*x^2+a)^(1/2)-21/10*b^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a
^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))
)*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))
*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/c^(7/
2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {290, 325, 329, 305, 220, 1196}

$$-\frac{21b^{3/2}\sqrt{cx}\,\sqrt{a+bx^2}}{5a^3c^4\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{10a^{11/4}c^{7/2}\sqrt{a+bx^2}}+\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+b}{\left(\sqrt{a}+\right.}}}{5a^{11/4}c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] 1/(a*c*(c*x)^(5/2)*Sqrt[a + b*x^2]) - (7*Sqrt[a + b*x^2])/(5*a^2*c*(c*x)^(5
/2)) + (21*b*Sqrt[a + b*x^2])/(5*a^3*c^3*Sqrt[c*x]) - (21*b^(3/2)*Sqrt[c*x]
*Sqrt[a + b*x^2])/(5*a^3*c^4*(Sqrt[a] + Sqrt[b]*x)) + (21*b^(5/4)*(Sqrt[a]
+ Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(
b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(5*a^(11/4)*c^(7/2)*Sqrt[a + b
*x^2]) - (21*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt
[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/
(10*a^(11/4)*c^(7/2)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx = \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}} + \frac{7\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx}{2a}$$

$$= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}} - \frac{(21b)\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{10a^2c^2}$$

$$= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}} + \frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}} - \frac{\left(21b^2\right)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{10a^3c^4}$$

$$= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}} + \frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}} - \frac{\left(21b^2\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{c}\right.}{5a^3c^5}$$

$$= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}} + \frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}} - \frac{\left(21b^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \right.}{5a^{5/2}c^4}$$

$$= \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}} + \frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}} - \frac{21b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^3c^4\left(\sqrt{a}+\sqrt{b}\,x\right)} + \frac{21b^{5/4}\left(\sqrt{\right.}{}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.18

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a(cx)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((b*x^2)/a)])
/(5*a*(c*x)^(7/2)*Sqrt[a + b*x^2])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}}{b^2c^4x^8+2\,abc^4x^6+a^2c^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^2*c^4*x^8 + 2*a*b*c^4*x^6 + a^2*c^4*x
^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(7/2)), x)
```

maple [A] time = 0.02, size = 219, normalized size = 0.66

$$-\frac{-42b^2x^4+42\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,ab\,x^2\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-21\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{-}}{10\sqrt{b\,x^2+a}\,\sqrt{cx}\,a^3c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/10/x^2*(42*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-21*((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-
a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*x^2*a*b-42*b^2*x^4-28*a*b*x^2+4*a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1
/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{7/2}\,\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a + b*x^2)^(3/2)),x)
```

```
[Out] int(1/((c*x)^(7/2)*(a + b*x^2)^(3/2)), x)
```

sympy [C] time = 24.98, size = 51, normalized size = 0.15

$$\frac{\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{5}{4}, \frac{3}{2} \\ -\frac{1}{4} \end{matrix}\right| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] gamma(-5/4)*hyper((-5/4, 3/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/
2)*c**(7/2)*x**(5/2)*gamma(-1/4))
```

3.629 $\int \frac{(cx)^{7/2}}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=155

$$\frac{5c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\mathrm{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{12\sqrt[4]{a}\,b^{9/4}\sqrt{a+bx^2}}-\frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}}-\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}}$$

```
[Out] -1/3*c*(c*x)^(5/2)/b/(b*x^2+a)^(3/2)-5/6*c^3*(c*x)^(1/2)/b^2/(b*x^2+a)^(1/2
)+5/12*c^(7/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(
b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*
x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/b^(9/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {288, 329, 220}

$$-\frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}}+\frac{5c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{12\sqrt[4]{a}\,b^{9/4}\sqrt{a+bx^2}}-\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] -(c*(c*x)^(5/2))/(3*b*(a + b*x^2)^(3/2)) - (5*c^3*Sqrt[c*x])/(6*b^2*Sqrt[a
+ b*x^2]) + (5*c^(7/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sq
rt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2]
)/(12*a^(1/4)*b^(9/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{7/2}}{\left(a+bx^2\right)^{5/2}}\,dx = -\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}} + \frac{\left(5c^2\right)\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx}{6b}$$

$$= -\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}} - \frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}} + \frac{\left(5c^4\right)\int \frac{1}{\sqrt{cx}\,\sqrt{a+bx^2}}\,dx}{12b^2}$$

$$= -\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}} - \frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}} + \frac{\left(5c^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{6b^2}$$

$$= -\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}} - \frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}} + \frac{5c^{7/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{12\sqrt[4]{a}\,b^{9/4}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.06, size = 80, normalized size = 0.52

$$\frac{c^3\sqrt{cx}\left(5\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-5a-7bx^2\right)}{6b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (c^3*Sqrt[c*x]*(-5*a - 7*b*x^2 + 5*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeo
metric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(6*b^2*(a + b*x^2)^(3/2))
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}\,c^3x^3}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c^3*x^3/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b
*x^2 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

maple [A] time = 0.04, size = 219, normalized size = 1.41

$$\frac{\left(-14b^2x^3+5\sqrt{-ab}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,b\,x^2\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-10abx+5\sqrt{\frac{bx+\sqrt{-}}{\sqrt{-a}}}\right.}{12\left(b\,x^2+a\right)^{\frac{3}{2}}b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/12*(5*(-a*b)^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(
-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*x^2*b+5*((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-
1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))*(-a*b)^(1/2)*a-14*b^2*x^3-10*a*b*x)*c^3/x*(c*x)^(1/2)/b^3/(b*
x^2+a)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{7/2}}{\left(b\,x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c*x)^(7/2)/(a + b*x^2)^(5/2), x)
```

sympy [C] time = 22.87, size = 44, normalized size = 0.28

$$\frac{c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{9}{4},\frac{5}{2}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] c**(7/2)*x**(9/2)*gamma(9/4)*hyper((9/4, 5/2), (13/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/2)*gamma(13/4))
```

3.630 $\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=304

$$-\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{7/4}\sqrt{a+bx^2}}+\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\ldots\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}}$$

```
[Out] -1/3*c*(c*x)^(3/2)/b/(b*x^2+a)^(3/2)+1/2*c*(c*x)^(3/2)/a/b/(b*x^2+a)^(1/2)-
1/2*c^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a/b^(3/2)/(a^(1/2)+x*b^(1/2))+1/2*c^(5/
2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x
)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1
/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/b^(7/4)/(b*x^2+a)^(1/2)-1/4*c^(5/2)*(cos(2*
arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^
(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1
/2))^2)^(1/2)/a^(3/4)/b^(7/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {288, 290, 329, 305, 220, 1196}

$$-\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{7/4}\sqrt{a+bx^2}}+\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] -(c*(c*x)^(3/2))/(3*b*(a + b*x^2)^(3/2)) + (c*(c*x)^(3/2))/(2*a*b*Sqrt[a +
b*x^2]) - (c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(2*a*b^(3/2)*(Sqrt[a] + Sqrt[b]*x
)) + (c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(3
/4)*b^(7/4)*Sqrt[a + b*x^2]) - (c^(5/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x
^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4
)*Sqrt[c])], 1/2])/(4*a^(3/4)*b^(7/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}+\frac{c^2\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/2}}\,dx}{2b}\\
&= -\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}+\frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}}-\frac{c^2\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{4ab}\\
&= -\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}+\frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}}-\frac{c\,\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{2ab}\\
&= -\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}+\frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}}-\frac{c^2\,\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{2\sqrt{a}\,b^{3/2}}+\frac{c^2\,\operatorname{Subst}\left(\int \frac{1-\frac{\sqrt{b}\,x}{\sqrt{a}}}{\sqrt{a+\frac{bx^4}{c^2}}}\cdots\right.}{2\sqrt{a}\,b\cdots}\\
&= -\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}+\frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}}-\frac{c^2\sqrt{cx}\,\sqrt{a+bx^2}}{2ab^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\cdots}{2a^{3/4}b^{7/4}\sqrt{a}\cdots}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 74, normalized size = 0.24

$$\frac{2c(cx)^{3/2}\left(\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};-\frac{bx^2}{a}\right)-a\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*c*(c*x)^(3/2)*(-a + (a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/
4, 5/2, 7/4, -((b*x^2)/a)]))/(3*a*b*(a + b*x^2)^(3/2))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}\,c^2x^2}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c^2*x^2/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b
*x^2 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(5/2), x)
```

maple [A] time = 0.04, size = 385, normalized size = 1.27

$$-\frac{\left(-6b^2x^4+6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,ab\,x^2\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-}}{\sqrt{-ab}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/12*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-3*((b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(
1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/
2))*x^2*a*b+6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-3*((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^
(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*a^2-6*b^2*x^4-2*a*b*x^2)*c^2/x*(c*x)^(1/2)/b^2/a/(b*x^2+a)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x)^{5/2}}{\left(b\,x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c*x)^(5/2)/(a + b*x^2)^(5/2), x)
```

sympy [C] time = 7.33, size = 44, normalized size = 0.14

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{5}{2}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((7/4, 5/2), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/2)*gamma(11/4))
```

3.631 $\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=156

$$\frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{12a^{5/4}b^{5/4}\sqrt{a+bx^2}}+\frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}}-\frac{c\sqrt{cx}}{3b\left(a+bx^2\right)^{3/2}}$$

```
[Out] -1/3*c*(c*x)^(1/2)/b/(b*x^2+a)^(3/2)+1/6*c*(c*x)^(1/2)/a/b/(b*x^2+a)^(1/2)+
1/12*c^(3/2)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^
(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^
2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(5/4)/b^(5/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {288, 290, 329, 220}

$$\frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{12a^{5/4}b^{5/4}\sqrt{a+bx^2}}+\frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}}-\frac{c\sqrt{cx}}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] -(c*Sqrt[c*x])/(3*b*(a + b*x^2)^(3/2)) + (c*Sqrt[c*x])/(6*a*b*Sqrt[a + b*x^
2]) + (c^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(12*a^
(5/4)*b^(5/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{c\sqrt{cx}}{3b\left(a+bx^2\right)^{3/2}}+\frac{c^2\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/2}}\,dx}{6b}\\
&= -\frac{c\sqrt{cx}}{3b\left(a+bx^2\right)^{3/2}}+\frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}}+\frac{c^2\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{12ab}\\
&= -\frac{c\sqrt{cx}}{3b\left(a+bx^2\right)^{3/2}}+\frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}}+\frac{c\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{6ab}\\
&= -\frac{c\sqrt{cx}}{3b\left(a+bx^2\right)^{3/2}}+\frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}}+\frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{12a^{5/4}b^{5/4}\sqrt{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.51

$$
\frac{c\sqrt{cx}\left(\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-a+bx^2\right)}{6ab\left(a+bx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (c*Sqrt[c*x]*(-a + b*x^2 + (a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F
1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(6*a*b*(a + b*x^2)^(3/2))
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}\,cx}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)*c*x/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2
 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(5/2), x)

maple [A] time = 0.02, size = 218, normalized size = 1.40

$$\frac{\left(2b^2x^3 + \sqrt{-ab}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,b\,x^2\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right) - 2abx + \sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{}\right.}{12\left(b\,x^2+a\right)^{\frac{3}{2}} a\,b^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(3/2)/(b*x^2+a)^(5/2),x)

[Out] 1/12*((-a*b)^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*x^2*b+((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a+2*b^2*x^3-2*a*b*x)*c/x*(c*x)^(1/2)/a/b^2/(b*x^2+a)^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")

[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{\left(b\,x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(3/2)/(a + b*x^2)^(5/2),x)

[Out] int((c*x)^(3/2)/(a + b*x^2)^(5/2), x)

sympy [C] time = 3.83, size = 44, normalized size = 0.28

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{5}{2}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(3/2)/(b*x**2+a)**(5/2),x)

[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/2), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*gamma(9/4))

3.632 $\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=302

$$-\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{4a^{7/4}b^{3/4}\sqrt{a+bx^2}}+\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\right.}{2a^{7/4}b^{3/4}\sqrt{a+bx^2}}$$

```
[Out] 1/3*(c*x)^(3/2)/a/c/(b*x^2+a)^(3/2)+1/2*(c*x)^(3/2)/a^2/c/(b*x^2+a)^(1/2)-1
/2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a^2/b^(1/2)/(a^(1/2)+x*b^(1/2))+1/2*(cos(2*a
rctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(
c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(
1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1/2)*((b*x^2+a)/(a^(1/2)
+x*b^(1/2))^2)^(1/2)/a^(7/4)/b^(3/4)/(b*x^2+a)^(1/2)-1/4*(cos(2*arctan(b^(1
/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)
/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/
2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*c^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2)
)^2)^(1/2)/a^(7/4)/b^(3/4)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {290, 329, 305, 220, 1196}

$$-\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{4a^{7/4}b^{3/4}\sqrt{a+bx^2}}+\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{3/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(5/2),x]
```

```
[Out] (c*x)^(3/2)/(3*a*c*(a + b*x^2)^(3/2)) + (c*x)^(3/2)/(2*a^2*c*Sqrt[a + b*x^2
]) - (Sqrt[c*x]*Sqrt[a + b*x^2])/(2*a^2*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)) + (S
qrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(7/4)*b^(
3/4)*Sqrt[a + b*x^2]) - (Sqrt[c]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sq
rt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[
c])], 1/2])/(4*a^(7/4)*b^(3/4)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}}+\frac{\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/2}}\,dx}{2a} \\
&= \frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}}+\frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{4a^2} \\
&= \frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}}+\frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}}-\frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{2a^2c} \\
&= \frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}}+\frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{2a^{3/2}\sqrt{b}}+\frac{\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}c}}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx,x,\right.}{2a^{3/2}\sqrt{b}} \\
&= \frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}}+\frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}}-\frac{\sqrt{cx}\sqrt{a+bx^2}}{2a^2\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E}{2a^{7/4}b^{3/4}\sqrt{a+}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.20

$$\frac{2x\sqrt{cx}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*x*Sqrt[c*x]*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 5/2, 7/4, -((b*x^
2)/a)])/(3*a^2*Sqrt[a + b*x^2])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a
^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(5/2), x)
```

maple [A] time = 0.02, size = 382, normalized size = 1.26

$$-\frac{\left(-6b^2x^4+6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,ab\,x^2\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+}{\sqrt{}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/12*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-3*((b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(
1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/
2))*x^2*a*b+6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-3*((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^
(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*a^2-6*b^2*x^4-10*a*b*x^2)*(c*x)^(1/2)/b/a^2/x/(b*x^2+a)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x}}{{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c*x)^(1/2)/(a + b*x^2)^(5/2), x)
```

sympy [C] time = 2.82, size = 44, normalized size = 0.15

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(5/2)*gamma(7/4))
```

3.633 $\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{12a^{9/4}\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}+\frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}}+\frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}}$$

```
[Out] 1/3*(c*x)^(1/2)/a/c/(b*x^2+a)^(3/2)+5/6*(c*x)^(1/2)/a^2/c/(b*x^2+a)^(1/2)+5
/12*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arct
an(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*
x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(
1/2)+x*b^(1/2))^2)^(1/2)/a^(9/4)/b^(1/4)/c^(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {290, 329, 220}

$$\frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}}+\frac{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt[4]{b}\,\sqrt{c}\,\sqrt{a+bx^2}}+\frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(5/2)),x]
```

```
[Out] Sqrt[c*x]/(3*a*c*(a + b*x^2)^(3/2)) + (5*Sqrt[c*x])/(6*a^2*c*Sqrt[a + b*x^2
]) + (5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(12*a^(9/4)*b
^(1/4)*Sqrt[c]*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{5/2}}\,dx = \frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}} + \frac{5\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/2}}\,dx}{6a}$$

$$= \frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}} + \frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}} + \frac{5\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx}{12a^2}$$

$$= \frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}} + \frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}} + \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{6a^2c}$$

$$= \frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}} + \frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}} + \frac{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.50

$$\frac{5x\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+7ax+5bx^3}{6a^2\sqrt{cx}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(5/2)),x]
```

```
[Out] (7*a*x + 5*b*x^3 + 5*x*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/
4, 1/2, 5/4, -((b*x^2)/a)])/(6*a^2*Sqrt[c*x]*(a + b*x^2)^(3/2))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{cx}}{b^3cx^7+3\,ab^2cx^5+3\,a^2bcx^3+a^3cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^3*c*x^7 + 3*a*b^2*c*x^5 + 3*a^2*b*c*x
^3 + a^3*c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*sqrt(c*x)), x)
```

maple [A] time = 0.02, size = 216, normalized size = 1.38

$$\frac{10b^2x^3+5\sqrt{-ab}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,b\,x^2\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+14abx+5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{}}{12\sqrt{cx}\,\left(b\,x^2+a\right)^{\frac{3}{2}}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/12*(5*(-a*b)^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(
-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*x^2*b+5*((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-
1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))*(-a*b)^(1/2)*a+10*b^2*x^3+14*a*b*x)/(c*x)^(1/2)/a^2/b/(b*x^2+
a)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*sqrt(c*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x}\,{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a + b*x^2)^(5/2)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(a + b*x^2)^(5/2)), x)
```

sympy [C] time = 4.28, size = 44, normalized size = 0.28

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a
**(5/2)*sqrt(c)*gamma(5/4))
```

3.634 $\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=333

$$\frac{7\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{4a^{11/4}c^{3/2}\sqrt{a+bx^2}}-\frac{7\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right.\right.}{2a^{11/4}c^{3/2}\sqrt{a+bx^2}}$$

```
[Out] 1/3/a/c/(b*x^2+a)^(3/2)/(c*x)^(1/2)+7/6/a^2/c/(c*x)^(1/2)/(b*x^2+a)^(1/2)-7
/2*(b*x^2+a)^(1/2)/a^3/c/(c*x)^(1/2)+7/2*b^(1/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2
)/a^3/c^2/(a^(1/2)+x*b^(1/2))-7/2*b^(1/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)
/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2
)))*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2
))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/c^(
3/2)/(b*x^2+a)^(1/2)+7/4*b^(1/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/
c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*Ellip
ticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/
2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/c^(3/2)/(b*x
^2+a)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {290, 325, 329, 305, 220, 1196}

$$\frac{7\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{2a^3c^2\left(\sqrt{a}+\sqrt{b}\,x\right)}+\frac{7\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{4a^{11/4}c^{3/2}\sqrt{a+bx^2}}-\frac{7\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(\right.}{2a^{11/4}c^{3/2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*c*Sqrt[c*x]*(a + b*x^2)^(3/2)) + 7/(6*a^2*c*Sqrt[c*x]*Sqrt[a + b*x^2
]) - (7*Sqrt[a + b*x^2])/(2*a^3*c*Sqrt[c*x]) + (7*Sqrt[b]*Sqrt[c*x]*Sqrt[a
+ b*x^2])/(2*a^3*c^2*(Sqrt[a] + Sqrt[b]*x)) - (7*b^(1/4)*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*S
qrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(11/4)*c^(3/2)*Sqrt[a + b*x^2]) +
(7*b^(1/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*
EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(4*a^(11/4
)*c^(3/2)*Sqrt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{3ac\sqrt{cx}\left(a+bx^2\right)^{3/2}}+\frac{7\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx}{6a}\\
&= \frac{1}{3ac\sqrt{cx}\left(a+bx^2\right)^{3/2}}+\frac{7}{6a^2c\sqrt{cx}\sqrt{a+bx^2}}+\frac{7\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx}{4a^2}\\
&= \frac{1}{3ac\sqrt{cx}\left(a+bx^2\right)^{3/2}}+\frac{7}{6a^2c\sqrt{cx}\sqrt{a+bx^2}}-\frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}}+\frac{(7b)\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx}{4a^3c^2}\\
&= \frac{1}{3ac\sqrt{cx}\left(a+bx^2\right)^{3/2}}+\frac{7}{6a^2c\sqrt{cx}\sqrt{a+bx^2}}-\frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}}+\frac{(7b)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^4}{c^2}}}\right.}{2a^3c^3}\\
&= \frac{1}{3ac\sqrt{cx}\left(a+bx^2\right)^{3/2}}+\frac{7}{6a^2c\sqrt{cx}\sqrt{a+bx^2}}-\frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}}+\frac{\left(7\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b}{c}}}\right.}{2a^{5/2}c^2}\\
&= \frac{1}{3ac\sqrt{cx}\left(a+bx^2\right)^{3/2}}+\frac{7}{6a^2c\sqrt{cx}\sqrt{a+bx^2}}-\frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}}+\frac{7\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{2a^3c^2\left(\sqrt{a}+\sqrt{b}\,x\right)}-
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.17

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{a^2(cx)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 5/2, 3/4, -((b*x^2)/a)])/
(a^2*(c*x)^(3/2)*Sqrt[a + b*x^2])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}}{b^3c^2x^8+3\,ab^2c^2x^6+3\,a^2bc^2x^4+a^3c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^3*c^2*x^8 + 3*a*b^2*c^2*x^6 + 3*a^2*b
*c^2*x^4 + a^3*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(3/2)), x)
```

maple [A] time = 0.02, size = 384, normalized size = 1.15

$$\frac{-42b^2x^4+42\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,ab\,x^2\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-21\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{}}{\sqrt{-a}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/12*(42*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-21*((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^
(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*x^2*a*b+42*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-21*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*
b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*a^2-42*b^2*x^4-70*a*b*x^2-24*a^2)/a^3/c/(c*x)^(1/2)/(b*x^2+a)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{3/2}\,\left(b\,x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a + b*x^2)^(5/2)),x)
```

```
[Out] int(1/((c*x)^(3/2)*(a + b*x^2)^(5/2)), x)
```

sympy [C] time = 7.66, size = 48, normalized size = 0.14

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 5/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2
)*c**(3/2)*sqrt(x)*gamma(3/4))
```

3.635 $\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right),\frac{1}{2}\right)}{4a^{13/4}c^{5/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}}+\frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}}+\frac{}{3ac(cx)^{3/2}}$$

```
[Out] 1/3/a/c/(c*x)^(3/2)/(b*x^2+a)^(3/2)+3/2/a^2/c/(c*x)^(3/2)/(b*x^2+a)^(1/2)-5
/2*(b*x^2+a)^(1/2)/a^3/c/(c*x)^(3/2)-5/4*b^(3/4)*(cos(2*arctan(b^(1/4)*(c*x
)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)
/c^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2
*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(13
/4)/c^(5/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {290, 325, 329, 220}

$$-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a}\,\sqrt{c}}\right)|\frac{1}{2}\right)}{4a^{13/4}c^{5/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}}+\frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}}+\frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*c*(c*x)^(3/2)*(a + b*x^2)^(3/2)) + 3/(2*a^2*c*(c*x)^(3/2)*Sqrt[a + b
*x^2]) - (5*Sqrt[a + b*x^2])/(2*a^3*c*(c*x)^(3/2)) - (5*b^(3/4)*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^
(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(4*a^(13/4)*c^(5/2)*Sqrt[a + b*x
^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}}+\frac{3\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx}{2a}$$

$$= \frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}}+\frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}}+\frac{15\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx}{4a^2}$$

$$= \frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}}+\frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}}-\frac{(5b)\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,d}{4a^3c^2}$$

$$= \frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}}+\frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}}-\frac{(5b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+}}\right.}{2a^3c}$$

$$= \frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}}+\frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}}-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x\right)}{4}$$

Mathematica [C] time = 0.02, size = 59, normalized size = 0.32

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{3}{4},\frac{5}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^2(cx)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 5/2, 1/4, -((b*x^2)/a)])/
(3*a^2*(c*x)^(5/2)*Sqrt[a + b*x^2])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}}{b^3c^3x^9+3\,ab^2c^3x^7+3\,a^2bc^3x^5+a^3c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^3*c^3*x^9 + 3*a*b^2*c^3*x^7 + 3*a^2*b
*c^3*x^5 + a^3*c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(5/2)), x)
```

maple [A] time = 0.02, size = 227, normalized size = 1.23

$$-\frac{30b^2x^4+15\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,b\,x^3\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+42ab\,x^2+15\sqrt{\frac{bx+\sqrt{-}}{\sqrt{-a}}}}{12\sqrt{cx}\,\left(b\,x^2+a\right)^{\frac{3}{2}}a^3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/12*(15*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^3*b+15*((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2),1/2*2^(1/2))*(-a*b)^(1/2)*x*a+30*b^2*x^4+42*a*b*x^2+8*a^2)/x/c^2/(c*x)^
(1/2)/a^3/(b*x^2+a)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{5/2}\,\left(b\,x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a + b*x^2)^(5/2)),x)
```

```
[Out] int(1/((c*x)^(5/2)*(a + b*x^2)^(5/2)), x)
```

sympy [C] time = 13.97, size = 48, normalized size = 0.26

$$\frac{\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{5}{2}\\ \frac{1}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 5/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2
)*c**(5/2)*x**(3/2)*gamma(1/4))
```

3.636 $\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=362

$$-\frac{77b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\ \text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{20a^{15/4}c^{7/2}\sqrt{a+bx^2}}+\frac{77b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan\right.}{10a^{15/4}c^{7/2}\sqrt{a+bx^2}}$$

```
[Out] 1/3/a/c/(c*x)^(5/2)/(b*x^2+a)^(3/2)+11/6/a^2/c/(c*x)^(5/2)/(b*x^2+a)^(1/2)-
77/30*(b*x^2+a)^(1/2)/a^3/c/(c*x)^(5/2)+77/10*b*(b*x^2+a)^(1/2)/a^4/c^3/(c*
x)^(1/2)-77/10*b^(3/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a^4/c^4/(a^(1/2)+x*b^(1/
2))+77/10*b^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticE(sin(2*arct
an(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*(
(b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(15/4)/c^(7/2)/(b*x^2+a)^(1/2)-77/
20*b^(5/4)*(cos(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))^2)^(1/2)/cos
(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)))*EllipticF(sin(2*arctan(b^(1
/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+
a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(15/4)/c^(7/2)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {290, 325, 329, 305, 220, 1196}

$$-\frac{77b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{10a^4c^4\left(\sqrt{a}+\sqrt{b}\,x\right)}-\frac{77b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{20a^{15/4}c^{7/2}\sqrt{a+bx^2}}+\frac{77b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+b}{\left(\sqrt{a}+\right.}}}{10a^{15/4}c^{7/}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*c*(c*x)^(5/2)*(a + b*x^2)^(3/2)) + 11/(6*a^2*c*(c*x)^(5/2)*Sqrt[a +
b*x^2]) - (77*Sqrt[a + b*x^2])/(30*a^3*c*(c*x)^(5/2)) + (77*b*Sqrt[a + b*x^
2])/(10*a^4*c^3*Sqrt[c*x]) - (77*b^(3/2)*Sqrt[c*x]*Sqrt[a + b*x^2])/(10*a^4
*c^4*(Sqrt[a] + Sqrt[b]*x)) + (77*b^(5/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b
*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1
/4)*Sqrt[c])], 1/2])/(10*a^(15/4)*c^(7/2)*Sqrt[a + b*x^2]) - (77*b^(5/4)*(S
qrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*A
rcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(20*a^(15/4)*c^(7/2)*Sq
rt[a + b*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}} + \frac{11\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx}{6a}$$

$$= \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} + \frac{77\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx}{12a^2}$$

$$= \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} - \frac{(77b)\int \frac{1}{(cx)^{3/2}\sqrt{a+b}}}{20a^3c^2}$$

$$= \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} + \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}} - $$

$$= \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} + \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}} - $$

$$= \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} + \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}} - $$

$$= \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} + \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}} - $$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.16

$$-\frac{2x\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{5}{4},\frac{5}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^2(cx)^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 5/2, -1/4, -((b*x^2)/a)])
/(5*a^2*(c*x)^(7/2)*Sqrt[a + b*x^2])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\,\sqrt{cx}}{b^3c^4x^{10}+3\,ab^2c^4x^8+3\,a^2bc^4x^6+a^3c^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(c*x)/(b^3*c^4*x^10 + 3*a*b^2*c^4*x^8 + 3*a^2*
b*c^4*x^6 + a^3*c^4*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(7/2)), x)
```

maple [A] time = 0.02, size = 410, normalized size = 1.13

$$-\frac{-462b^3x^6+462\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a\,b^2x^4\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-231\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/60*(462*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE(((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^4*a*b^2-231*((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-
a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*x^4*a*b^2+462*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticE
(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a^2*b-231*((b*x+(
-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2),1/2*2^(1/2))*x^2*a^2*b-462*b^3*x^6-770*a*b^2*x^4-264*a^2*b*x^2+24
*a^3)/x^2/a^4/c^3/(c*x)^(1/2)/(b*x^2+a)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{7/2}\,\left(b\,x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a + b*x^2)^(5/2)),x)
```

```
[Out] int(1/((c*x)^(7/2)*(a + b*x^2)^(5/2)), x)
```

sympy [C] time = 41.15, size = 51, normalized size = 0.14

$$\frac{\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},\frac{5}{2}\\-\frac{1}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] gamma(-5/4)*hyper((-5/4, 5/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/
2)*c**(7/2)*x**(5/2)*gamma(-1/4))
```

3.637 $\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{9\sqrt[4]{3}\,c^2\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{5\,2^{3/4}\sqrt{x}\,\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}\,(cx)^{3/2}}{5a}$$

```
[Out] -9/10*3^(1/4)*c^2*EllipticE(1/6*(3-x*6^(1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)^
(1/2)*(-2*x^2+3)^(1/2)*2^(1/4)/x^(1/2)/(-2*a*x^2+3*a)^(1/2)-1/5*c*(c*x)^(3/
2)*(-2*a*x^2+3*a)^(1/2)/a
```

Rubi [A] time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {321, 320, 319, 318, 424}

$$-\frac{9\sqrt[4]{3}\,c^2\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{5\,2^{3/4}\sqrt{x}\,\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}\,(cx)^{3/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] -(c*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2])/(5*a) - (9*3^(1/4)*c^2*Sqrt[c*x]*Sqrt[
3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*2^(3/4)*Sq
rt[x]*Sqrt[3*a - 2*a*x^2])
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx &= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}+\frac{1}{10}\left(9c^2\right)\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx\\
&= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}+\frac{\left(9c^2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{10\sqrt{x}}\\
&= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}+\frac{\left(9c^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{10\sqrt{x}\sqrt{3a-2ax^2}}\\
&= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}-\frac{\left(9\left(\frac{3}{2}\right)^{3/4}c^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{\frac{2}{3}}x}}{\sqrt{2}}\right)}{5\sqrt{x}\sqrt{3a-2ax^2}}\\
&= -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a}-\frac{9\sqrt[4]{3}\,c^2\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\,2^{3/4}\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 61, normalized size = 0.57

$$\frac{c(cx)^{3/2}\left(\sqrt{9-6x^2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{2x^2}{3}\right)+2x^2-3\right)}{5\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/Sqrt[3*a - 2*a*x^2],x]
[Out] (c*(c*x)^(3/2)*(-3 + 2*x^2 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/2, 3/4, 7/
4, (2*x^2)/3]))/(5*Sqrt[a*(3 - 2*x^2)])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,c^2x^2}{2\,ax^2-3\,a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c^2*x^2/(2*a*x^2 - 3*a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/sqrt(-2*a*x^2 + 3*a), x)
```

maple [B] time = 0.04, size = 235, normalized size = 2.20

$$\frac{\sqrt{cx}\,\sqrt{-\left(2x^{2}-3\right)a}\,\left(-16x^{4}+24x^{2}+6\sqrt{2}\,\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-\sqrt{2}\,\sqrt{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] 1/40*c^2/x*(c*x)^(1/2)*(-(2*x^2-3)*a)^(1/2)/a*(6*2^(1/2)*((2*x+2^(1/2)*3^(1
/2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*
3^(1/2)*(-2^(1/2)*3^(1/2)*x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1
/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-3*2^(1/2)*((2*x+2^(1/2)*3^
(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2
)*3^(1/2)*(-2^(1/2)*3^(1/2)*x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^
(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-16*x^4+24*x^2)/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\sqrt{-2\,ax^{2}+3\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/sqrt(-2*a*x^2 + 3*a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{5/2}}{\sqrt{3\,a-2\,a\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(3*a - 2*a*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(5/2)/(3*a - 2*a*x^2)^(1/2), x)
```

sympy [C] time = 6.44, size = 51, normalized size = 0.48

$$\frac{\sqrt{3}\,c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{2x^{2}e^{2i\pi}}{3}\right)}{6\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), 2*x**2*exp_
polar(2*I*pi)/3)/(6*sqrt(a)*gamma(11/4))
```

3.638 $\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=88

$$\frac{c^{3/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt[4]{6}\sqrt{a\left(3-2x^2\right)}}-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}$$

```
[Out] 1/6*c^(3/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2+3)
^(1/2)*6^(3/4)/(a*(-2*x^2+3))^(1/2)-1/3*c*(c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2)/
a
```

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {321, 329, 224, 221}

$$\frac{c^{3/2}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{6}\sqrt{a\left(3-2x^2\right)}}-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] -(c*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2])/(3*a) + (c^(3/2)*Sqrt[3 - 2*x^2]*Ellipti
cF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(6^(1/4)*Sqrt[a*(3 - 2*x^2
)])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx = -\frac{c\sqrt{cx}\,\sqrt{3a-2ax^2}}{3a}+\frac{1}{2}c^2\int \frac{1}{\sqrt{cx}\,\sqrt{3a-2ax^2}}\,dx$$
$$= -\frac{c\sqrt{cx}\,\sqrt{3a-2ax^2}}{3a}+c\,\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx,x,\sqrt{cx}\right)$$
$$= -\frac{c\sqrt{cx}\,\sqrt{3a-2ax^2}}{3a}+\frac{\left(c\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx,x,\sqrt{cx}\right)}{\sqrt{3}\,\sqrt{a\left(3-2x^2\right)}}$$
$$= -\frac{c\sqrt{cx}\,\sqrt{3a-2ax^2}}{3a}+\frac{c^{3/2}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\,\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{6}\,\sqrt{a\left(3-2x^2\right)}}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.69

$$\frac{c\sqrt{cx}\left(\sqrt{9-6x^2}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{2x^2}{3}\right)+2x^2-3\right)}{3\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] (c*Sqrt[c*x]*(-3 + 2*x^2 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4,
 (2*x^2)/3]))/(3*Sqrt[a*(3 - 2*x^2)])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,cx}{2\,ax^2-3\,a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c*x/(2*a*x^2 - 3*a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/sqrt(-2*a*x^2 + 3*a), x)
```

maple [A] time = 0.03, size = 131, normalized size = 1.49

$$-\frac{\sqrt{cx}\,\sqrt{-\left(2x^2-3\right)a}\,\left(8x^3-12x+\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-\sqrt{2}\,\sqrt{3}\,x}\,\text{EllipticF}\right.}{12\left(2x^2-3\right)ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] -1/12*c*(c*x)^(1/2)*(-(2*x^2-3)*a)^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^
(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-2^(1/2)*3^(1/
2)*x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^
(1/2))^(1/2),1/2*2^(1/2))+8*x^3-12*x)/x/a/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/sqrt(-2*a*x^2 + 3*a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{\sqrt{3\,a-2\,a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(3*a - 2*a*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(3/2)/(3*a - 2*a*x^2)^(1/2), x)
```

sympy [A] time = 1.87, size = 51, normalized size = 0.58

$$\frac{\sqrt{3}\,c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), 2*x**2*exp_p
olar(2*I*pi)/3)/(6*sqrt(a)*gamma(9/4))
```

3.639 $\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt[4]{6}\sqrt{3-2x^2}\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{\sqrt{x}\sqrt{3a-2ax^2}}$$

```
[Out] -6^(1/4)*EllipticE(1/6*(3-x*6^(1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)^(1/2)*(-2
*x^2+3)^(1/2)/x^(1/2)/(-2*a*x^2+3*a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {320, 319, 318, 424}

$$-\frac{\sqrt[4]{6}\sqrt{3-2x^2}\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{\sqrt{x}\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/Sqrt[3*a - 2*a*x^2],x]
```

```
[Out] -((6^(1/4)*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/S
qrt[6]], 2])/(Sqrt[x]*Sqrt[3*a - 2*a*x^2]))
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx = \frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{\sqrt{x}}$$
$$= \frac{\left(\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{\sqrt{x}\sqrt{3a-2ax^2}}$$
$$= -\frac{\left(\sqrt[4]{2}\,3^{3/4}\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{\frac{2}{3}}\,x}}{\sqrt{2}}\right)}{\sqrt{x}\sqrt{3a-2ax^2}}$$
$$= -\frac{\sqrt[4]{6}\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|\,2\right)}{\sqrt{x}\sqrt{3a-2ax^2}}$$

Mathematica [C] time = 0.02, size = 53, normalized size = 0.79

$$\frac{2x\sqrt{3-2x^2}\sqrt{cx}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{2x^2}{3}\right)}{3\sqrt{a\left(9-6x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/Sqrt[3*a - 2*a*x^2],x]
[Out] (2*x*Sqrt[c*x]*Sqrt[3 - 2*x^2]*Hypergeometric2F1[1/2, 3/4, 7/4, (2*x^2)/3])
/(3*Sqrt[a*(9 - 6*x^2)])
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}{2\,ax^2-3\,a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(2*a*x^2 - 3*a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x)/sqrt(-2*a*x^2 + 3*a), x)
```

maple [B] time = 0.02, size = 165, normalized size = 2.46

$$\frac{\sqrt{cx}\sqrt{-\left(2x^2-3\right)a}\sqrt{2}\sqrt{\left(2x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{\left(-2x+\sqrt{2}\sqrt{3}\right)\sqrt{2}\sqrt{3}}\sqrt{3}\sqrt{-\sqrt{2}\sqrt{3}\,x}\left(2\,\text{EllipticE}\left(\right.\right.}{12\left(2x^2-3\right)ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] 1/12*(c*x)^(1/2)*(-(2*x^2-3)*a)^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2
)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-2
^(1/2)*3^(1/2)*x)^(1/2)*(2*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1
/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-EllipticF(1/6*3^(1/2)*2^(1/2)*((2*
x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2)))/x/a/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/sqrt(-2*a*x^2 + 3*a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x}}{\sqrt{3\,a-2\,a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(3*a - 2*a*x^2)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)/(3*a - 2*a*x^2)^(1/2), x)
```

sympy [C] time = 0.86, size = 51, normalized size = 0.76

$$\frac{\sqrt{3}\,\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), 2*x**2*exp_po
lar(2*I*pi)/3)/(6*sqrt(a)*gamma(7/4))
```

3.640 $\int \frac{1}{\sqrt{cx}\,\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=63

$$\frac{2^{3/4}\sqrt{3-2x^2}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a\left(3-2x^2\right)}}$$

```
[Out] 1/3*2^(3/4)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2+3)
^(1/2)*3^(3/4)/c^(1/2)/(a*(-2*x^2+3))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {329, 224, 221}

$$\frac{2^{3/4}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (2^(3/4)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]],
 -1])/(3^(1/4)*Sqrt[c]*Sqrt[a*(3 - 2*x^2)])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$
$$= \frac{\left(2\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx, x, \sqrt{cx}\right)}{\sqrt{3}\,c\sqrt{a\left(3-2x^2\right)}}$$
$$= \frac{2^{3/4}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle| -1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a\left(3-2x^2\right)}}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.89

$$\frac{2x\sqrt{3-2x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{2x^2}{3}\right)}{\sqrt{3}\sqrt{a\left(3-2x^2\right)}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (2*x*Sqrt[3 - 2*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (2*x^2)/3])/(Sqrt[3]*
Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}}{2\,acx^3-3\,acx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(2*a*c*x^3 - 3*a*c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)), x)
```

maple [B] time = 0.02, size = 117, normalized size = 1.86

$$-\frac{\sqrt{-\left(2x^2-3\right)a}\,\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-\sqrt{2}\,\sqrt{3}\,x}\;\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right.}}{6}\right.}{6\sqrt{cx}\,\left(2x^2-3\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] -1/6*(-(2*x^2-3)*a)^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-
2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-2^(1/2)*3^(1/2)*x)^(1/2)*Elli
pticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2
*2^(1/2))/(c*x)^(1/2)/a/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,ax^2 + 3\,a}\,\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x}\,\sqrt{3\,a-2\,a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(3*a - 2*a*x^2)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(3*a - 2*a*x^2)^(1/2)), x)
```

sympy [A] time = 1.04, size = 51, normalized size = 0.81

$$\frac{\sqrt{3}\,\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\,\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**2*exp_polar(2*I*p
i)/3)/(6*sqrt(a)*sqrt(c)*gamma(5/4))
```

3.641 $\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=107

$$\frac{2\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}$$

```
[Out] 2/3*2^(1/4)*EllipticE(1/6*(3-x*6^(1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)^(1/2)*
(-2*x^2+3)^(1/2)*3^(1/4)/c^2/x^(1/2)/(-2*a*x^2+3*a)^(1/2)-2/3*(-2*a*x^2+3*a
)^(1/2)/a/c/(c*x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {325, 320, 319, 318, 424}

$$\frac{2\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(3*a*c*Sqrt[c*x]) + (2*2^(1/4)*Sqrt[c*x]*Sqrt[3 -
2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(3^(3/4)*c^2*Sqrt
[x]*Sqrt[3*a - 2*a*x^2])
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx &= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}-\frac{2\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{3c^2}\\
&= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}-\frac{\left(2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{3c^2\sqrt{x}}\\
&= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}-\frac{\left(2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{3c^2\sqrt{x}\sqrt{3a-2ax^2}}\\
&= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}+\frac{\left(2\sqrt[4]{\frac{2}{3}}\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-\sqrt{\frac{2}{3}}\,x}}{\sqrt{2}}\right)}{c^2\sqrt{x}\sqrt{3a-2ax^2}}\\
&= -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}+\frac{2\sqrt[4]{2}\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 51, normalized size = 0.48

$$-\frac{2x\sqrt{3-2x^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};\frac{2x^2}{3}\right)}{\sqrt{a\left(9-6x^2\right)}\,(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*Sqrt[3*a - 2*a*x^2]),x]
[Out] (-2*x*Sqrt[3 - 2*x^2]*Hypergeometric2F1[-1/4, 1/2, 3/4, (2*x^2)/3])/((c*x)^
(3/2)*Sqrt[a*(9 - 6*x^2)])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}}{2\,ac^2x^4-3\,ac^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(2*a*c^2*x^4 - 3*a*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2)), x)

maple [B] time = 0.02, size = 228, normalized size = 2.13

$$-\frac{\sqrt{-\left(2x^{2}-3\right)a}\,\left(24x^{2}+2\sqrt{2}\,\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-\sqrt{2}\,\sqrt{3}\,x}\,\mathrm{EllipticE}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] -1/18*(-(2*x^2-3)*a)^(1/2)*(2*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2
))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-2^(1/2)*3
^(1/2)*x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2
)*3^(1/2))^(1/2),1/2*2^(1/2))-2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2
))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-2^(1/2)*3
^(1/2)*x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2
)*3^(1/2))^(1/2),1/2*2^(1/2))+24*x^2-36)/c/(c*x)^(1/2)/a/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,ax^{2}+3\,a}\,(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{3/2}\,\sqrt{3\,a-2\,a\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(1/2)), x)
```

sympy [C] time = 1.58, size = 54, normalized size = 0.50

$$\frac{\sqrt{3}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\right|\frac{2x^{2}e^{2i\pi}}{3}\right)}{6\sqrt{a}\,c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), 2*x**2*exp_polar(2*I*pi)/3)/
(6*sqrt(a)*c**(3/2)*sqrt(x)*gamma(3/4))
```

3.642 $\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx$

Optimal. Leaf size=98

$$\frac{2\,2^{3/4}\sqrt{3-2x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{9\sqrt[4]{3}\,c^{5/2}\sqrt{a\left(3-2x^2\right)}}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}$$

```
[Out] 2/27*2^(3/4)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2+3
)^(1/2)*3^(3/4)/c^(5/2)/(a*(-2*x^2+3))^(1/2)-2/9*(-2*a*x^2+3*a)^(1/2)/a/c/(
c*x)^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {325, 329, 224, 221}

$$\frac{2\,2^{3/4}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{9\sqrt[4]{3}\,c^{5/2}\sqrt{a\left(3-2x^2\right)}}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (-2*Sqrt[3*a - 2*a*x^2])/(9*a*c*(c*x)^(3/2)) + (2*2^(3/4)*Sqrt[3 - 2*x^2]*E
llipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(9*3^(1/4)*c^(5/2)*S
qrt[a*(3 - 2*x^2)])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx = -\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}} + \frac{2\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx}{9c^2}$$

$$= -\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}} + \frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{9c^3}$$

$$= -\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}} + \frac{\left(4\sqrt{3-2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx, x, \sqrt{cx}\right)}{9\sqrt{3}\,c^3\sqrt{a\left(3-2x^2\right)}}$$

$$= -\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}} + \frac{2\,2^{3/4}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{9\sqrt[4]{3}\,c^{5/2}\sqrt{a\left(3-2x^2\right)}}$$

Mathematica [C] time = 0.02, size = 53, normalized size = 0.54

$$-\frac{2x\sqrt{3-2x^2}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};\frac{2x^2}{3}\right)}{3\sqrt{a\left(9-6x^2\right)}\,(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*Sqrt[3*a - 2*a*x^2]),x]
```

```
[Out] (-2*x*Sqrt[3 - 2*x^2]*Hypergeometric2F1[-3/4, 1/2, 1/4, (2*x^2)/3])/(3*(c*x
)^(5/2)*Sqrt[a*(9 - 6*x^2)])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}}{2\,ac^3x^5-3\,ac^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(2*a*c^3*x^5 - 3*a*c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2)), x)
```

maple [A] time = 0.03, size = 132, normalized size = 1.35

$$-\frac{\sqrt{-\left(2x^2-3\right)a}\,\left(12x^2+\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-\sqrt{2}\,\sqrt{3}\,x}\,x\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{}}{}\right.\right.}{27\sqrt{cx}\,\left(2x^2-3\right)a\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] -1/27*(-(2*x^2-3)*a)^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(
(-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-2^(1/2)*3^(1/2)*x)^(1/2)*El
lipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1
/2*2^(1/2))*x+12*x^2-18)/x/a/c^2/(c*x)^(1/2)/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,ax^2+3\,a}\,\left(cx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{5/2}\,\sqrt{3\,a-2\,a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(1/2)), x)
```

sympy [A] time = 3.77, size = 54, normalized size = 0.55

$$\frac{\sqrt{3}\,\Gamma\left(-\frac{3}{4}\right)\,{}_{2}F_{1}\left(\left.\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{1}{4} \end{matrix}\right|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\,c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(-2*a*x**2+3*a)**(1/2),x)
```

```
[Out] sqrt(3)*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), 2*x**2*exp_polar(2*I*pi)/3)/
(6*sqrt(a)*c**(5/2)*x**(3/2)*gamma(1/4))
```

3.643 $\int \frac{(cx)^{5/2}}{\left(3a-2ax^2\right)^{3/2}}\,dx$

Optimal. Leaf size=110

$$\frac{3\sqrt[4]{3}\,c^2\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{2\,2^{3/4}a\sqrt{x}\,\sqrt{3a-2ax^2}}+\frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}}$$

```
[Out] 1/2*c*(c*x)^(3/2)/a/(-2*a*x^2+3*a)^(1/2)+3/4*3^(1/4)*c^2*EllipticE(1/6*(3-x
*6^(1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)^(1/2)*(-2*x^2+3)^(1/2)*2^(1/4)/a/x^(
1/2)/(-2*a*x^2+3*a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {288, 320, 319, 318, 424}

$$\frac{3\sqrt[4]{3}\,c^2\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{2\,2^{3/4}a\sqrt{x}\,\sqrt{3a-2ax^2}}+\frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] (c*(c*x)^(3/2))/(2*a*Sqrt[3*a - 2*a*x^2]) + (3*3^(1/4)*c^2*Sqrt[c*x]*Sqrt[3
 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(2*2^(3/4)*a*S
qrt[x]*Sqrt[3*a - 2*a*x^2])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^{5/2}}{\left(3a-2ax^2\right)^{3/2}}\,dx &= \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} - \frac{\left(3c^2\right)\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{4a} \\
&= \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} - \frac{\left(3c^2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{4a\sqrt{x}} \\
&= \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} - \frac{\left(3c^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{4a\sqrt{x}\sqrt{3a-2ax^2}} \\
&= \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} + \frac{\left(3\left(\frac{3}{2}\right)^{3/4}c^2\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{\frac{2}{3}}x}}{\sqrt{2}}\right)}{2a\sqrt{x}\sqrt{3a-2ax^2}} \\
&= \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} + \frac{3\sqrt[4]{3}c^2\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{2\,2^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 59, normalized size = 0.54

$$
\frac{c(cx)^{3/2}\left(\sqrt{9-6x^2}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};\frac{2x^2}{3}\right)-3\right)}{3a\sqrt{a\left(3-2x^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] (c*(c*x)^(3/2)*(-3 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[3/4, 3/2, 7/4, (2*x^
2)/3]))/(3*a*Sqrt[a*(3 - 2*x^2)])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,c^2x^2}{4\,a^2x^4-12\,a^2x^2+9\,a^2}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c^2*x^2/(4*a^2*x^4 - 12*a^2*x^2 + 9
*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(cx)^{\frac{5}{2}}}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

maple [B] time = 0.05, size = 230, normalized size = 2.09

$$-\frac{\sqrt{cx}\,\sqrt{-\left(2x^{2}-3\right)a}\,\left(8x^{2}+2\sqrt{2}\,\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-\sqrt{2}\,\sqrt{3}\,x}\ \mathrm{Ellipt}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x)
```

```
[Out] -1/16*c^2/x*(c*x)^(1/2)*(-(2*x^2-3)*a)^(1/2)*(2*2^(1/2)*((2*x+2^(1/2)*3^(1/
2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3
^(1/2)*(-2^(1/2)*3^(1/2)*x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/
2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-2^(1/2)*((2*x+2^(1/2)*3^(1/
2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3
^(1/2)*(-2^(1/2)*3^(1/2)*x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/
2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))+8*x^2)/a^2/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(-2\,ax^{2}+3\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{5/2}}{\left(3\,a-2\,a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(3*a - 2*a*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(5/2)/(3*a - 2*a*x^2)^(3/2), x)
```

sympy [C] time = 7.34, size = 51, normalized size = 0.46

$$\frac{\sqrt{3}\,c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{2x^{2}e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(-2*a*x**2+3*a)**(3/2),x)
```

```
[Out] sqrt(3)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/2, 7/4), (11/4,), 2*x**2*exp_
polar(2*I*pi)/3)/(18*a**(3/2)*gamma(11/4))
```

3.644 $\int \frac{(cx)^{3/2}}{\left(3a-2ax^2\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}}-\frac{c^{3/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{2\sqrt[4]{6}\,a\sqrt{a\left(3-2x^2\right)}}$$

```
[Out] -1/12*c^(3/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2+
3)^(1/2)*6^(3/4)/a/(a*(-2*x^2+3))^(1/2)+1/2*c*(c*x)^(1/2)/a/(-2*a*x^2+3*a)^
(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {288, 329, 224, 221}

$$\frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}}-\frac{c^{3/2}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{2\sqrt[4]{6}\,a\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c*x])/(2*a*Sqrt[3*a - 2*a*x^2]) - (c^(3/2)*Sqrt[3 - 2*x^2]*Elliptic
F[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(2*6^(1/4)*a*Sqrt[a*(3 - 2*
x^2)])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\left(3a-2ax^2\right)^{3/2}}\,dx = \frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}} - \frac{c^2\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx}{4a}$$

$$= \frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}} - \frac{c\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2a}$$

$$= \frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}} - \frac{\left(c\sqrt{3-2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx, x, \sqrt{cx}\right)}{2\sqrt{3}\,a\sqrt{a\left(3-2x^2\right)}}$$

$$= \frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}} - \frac{c^{3/2}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{2\sqrt[4]{6}\,a\sqrt{a\left(3-2x^2\right)}}$$

Mathematica [C] time = 0.02, size = 59, normalized size = 0.63

$$-\frac{c\sqrt{cx}\left(\sqrt{9-6x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{2x^2}{3}\right)-3\right)}{6a\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(3*a - 2*a*x^2)^(3/2),x]
[Out] -1/6*(c*Sqrt[c*x]*(-3 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (2
*x^2)/3]))/(a*Sqrt[a*(3 - 2*x^2)])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}\,cx}{4\,a^2x^4-12\,a^2x^2+9\,a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c*x/(4*a^2*x^4 - 12*a^2*x^2 + 9*a^2
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x)^(3/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

maple [A] time = 0.04, size = 126, normalized size = 1.34

$$\frac{\sqrt{cx}\,\sqrt{-\left(2x^2-3\right)a}\,\left(-12x+\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-\sqrt{2}\,\sqrt{3}\,x}\,\mathrm{EllipticF}\left(\frac{\sqrt{}}{-}\right.\right.}{24\left(2x^2-3\right)a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x)
```

```
[Out] 1/24*c*(c*x)^(1/2)*(-(2*x^2-3)*a)^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(
1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-2^(1/2)*3^(1/2
)*x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(
1/2))^(1/2),1/2*2^(1/2))-12*x)/x/a^2/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{\left(3\,a-2\,a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(3*a - 2*a*x^2)^(3/2),x)
```

```
[Out] int((c*x)^(3/2)/(3*a - 2*a*x^2)^(3/2), x)
```

sympy [A] time = 2.10, size = 51, normalized size = 0.54

$$\frac{\sqrt{3}\,c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(-2*a*x**2+3*a)**(3/2),x)
```

```
[Out] sqrt(3)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 3/2), (9/4,), 2*x**2*exp_p
olar(2*I*pi)/3)/(18*a**(3/2)*gamma(9/4))
```

3.645 $\int \frac{\sqrt{cx}}{\left(3a-2ax^2\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}+\frac{\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{6^{3/4}a\sqrt{x}\,\sqrt{3a-2ax^2}}$$

```
[Out] 1/3*(c*x)^(3/2)/a/c/(-2*a*x^2+3*a)^(1/2)+1/6*EllipticE(1/6*(3-x*6^(1/2))^(1
/2)*6^(1/2),2^(1/2))*(c*x)^(1/2)*(-2*x^2+3)^(1/2)*6^(1/4)/a/x^(1/2)/(-2*a*x
^2+3*a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {290, 320, 319, 318, 424}

$$\frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}+\frac{\sqrt{3-2x^2}\,\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|2\right)}{6^{3/4}a\sqrt{x}\,\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] (c*x)^(3/2)/(3*a*c*Sqrt[3*a - 2*a*x^2]) + (Sqrt[c*x]*Sqrt[3 - 2*x^2]*Ellipt
icE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(6^(3/4)*a*Sqrt[x]*Sqrt[3*a -
2*a*x^2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx}}{\left(3a-2ax^2\right)^{3/2}}\,dx &= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}} - \frac{\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{6a}\\
&= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}} - \frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{6a\sqrt{x}}\\
&= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}} - \frac{\left(\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{6a\sqrt{x}\sqrt{3a-2ax^2}}\\
&= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}} + \frac{\left(\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\sqrt{\frac{2}{3}}x}}{\sqrt{2}}\right)}{2^{3/4}\sqrt[4]{3}\,a\sqrt{x}\sqrt{3a-2ax^2}}\\
&= \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}} + \frac{\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{6^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.57

$$\frac{2x\left(3-2x^2\right)^{3/2}\sqrt{cx}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};\frac{2x^2}{3}\right)}{9\sqrt{3}\left(a\left(3-2x^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(3*a - 2*a*x^2)^(3/2),x]
```

```
[Out] (2*x*Sqrt[c*x]*(3 - 2*x^2)^(3/2)*Hypergeometric2F1[3/4, 3/2, 7/4, (2*x^2)/3
])/(9*Sqrt[3]*(a*(3 - 2*x^2))^(3/2))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}}{4\,a^2x^4-12\,a^2x^2+9\,a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(4*a^2*x^4 - 12*a^2*x^2 + 9*a^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(c*x)/(-2*a*x^2 + 3*a)^(3/2), x)

maple [B] time = 0.02, size = 227, normalized size = 2.25

$$-\frac{\sqrt{cx}\,\sqrt{-\left(2x^2-3\right)a}\,\left(24x^2+2\sqrt{2}\,\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-\sqrt{2}\,\sqrt{3}\,x}\,\mathrm{Ellip}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x)

[Out] -1/72*(c*x)^(1/2)*(-(2*x^2-3)*a)^(1/2)*(2*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-2^(1/2)*3^(1/2)*x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))-2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-2^(1/2)*3^(1/2)*x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1/2*2^(1/2))+24*x^2)/a^2/x/(2*x^2-3)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x)/(-2*a*x^2 + 3*a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x}}{\left(3\,a-2\,a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(1/2)/(3*a - 2*a*x^2)^(3/2),x)

[Out] int((c*x)^(1/2)/(3*a - 2*a*x^2)^(3/2), x)

sympy [C] time = 1.27, size = 51, normalized size = 0.50

$$\frac{\sqrt{3}\,\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\right|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(1/2)/(-2*a*x**2+3*a)**(3/2),x)

[Out] sqrt(3)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2), (7/4,), 2*x**2*exp_polar(2*I*pi)/3)/(18*a**(3/2)*gamma(7/4))

3.646 $\int \frac{1}{\sqrt{cx}\left(3a-2ax^2\right)^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{\sqrt{3-2x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{3\sqrt[4]{6}\,a\sqrt{c}\sqrt{a\left(3-2x^2\right)}}+\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}$$

```
[Out] 1/18*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2+3)^(1/2)*
6^(3/4)/a/c^(1/2)/(a*(-2*x^2+3))^(1/2)+1/3*(c*x)^(1/2)/a/c/(-2*a*x^2+3*a)^(
1/2)
```

Rubi [A] time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {290, 329, 224, 221}

$$\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}+\frac{\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3\sqrt[4]{6}\,a\sqrt{c}\sqrt{a\left(3-2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c*x]/(3*a*c*Sqrt[3*a - 2*a*x^2]) + (Sqrt[3 - 2*x^2]*EllipticF[ArcSin[(
(2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3*6^(1/4)*a*Sqrt[c]*Sqrt[a*(3 - 2*x^
2)])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\,\left(3a-2ax^2\right)^{3/2}}\,dx = \frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}} + \frac{\int \frac{1}{\sqrt{cx}\,\sqrt{3a-2ax^2}}\,dx}{6a}$$

$$= \frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{3ac}$$

$$= \frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}} + \frac{\sqrt{3-2x^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx, x, \sqrt{cx}\right)}{3\sqrt{3}\,ac\sqrt{a\left(3-2x^2\right)}}$$

$$= \frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}} + \frac{\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3\sqrt[4]{6}\,a\sqrt{c}\,\sqrt{a\left(3-2x^2\right)}}$$

Mathematica [C] time = 0.03, size = 59, normalized size = 0.61

$$\frac{x\left(\sqrt{9-6x^2}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{2x^2}{3}\right)+3\right)}{9a\sqrt{a\left(3-2x^2\right)}\,\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] (x*(3 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (2*x^2)/3]))/(9*a*
Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)])
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}}{4\,a^2cx^5-12\,a^2cx^3+9\,a^2cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(4*a^2*c*x^5 - 12*a^2*c*x^3 + 9*a^2
*c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*sqrt(c*x)), x)
```

maple [A] time = 0.02, size = 122, normalized size = 1.27

$$-\frac{\sqrt{-\left(2x^2-3\right)a}\,\left(12x+\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-\sqrt{2}\,\sqrt{3}\,x}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}}{\right.}\right.}{36\sqrt{cx}\,\left(2x^2-3\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x)
```

```
[Out] -1/36*(-(2*x^2-3)*a)^(1/2)*(((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(
(-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-2^(1/2)*3^(1/2)*x)^(1/2)*El
lipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2),1
/2*2^(1/2))+12*x)/a^2/(c*x)^(1/2)/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*sqrt(c*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x}\,\left(3\,a-2\,a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(3*a - 2*a*x^2)^(3/2)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(3*a - 2*a*x^2)^(3/2)), x)
```

sympy [A] time = 1.79, size = 51, normalized size = 0.53

$$\frac{\sqrt{3}\,\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(-2*a*x**2+3*a)**(3/2),x)
```

```
[Out] sqrt(3)*sqrt(x)*gamma(1/4)*hyper((1/4, 3/2), (5/4,), 2*x**2*exp_polar(2*I*p
i)/3)/(18*a**(3/2)*sqrt(c)*gamma(5/4))
```

3.647 $\int \frac{1}{(cx)^{3/2}\left(3a-2ax^2\right)^{3/2}}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}+\frac{\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}ac^2\sqrt{x}\sqrt{3a-2ax^2}}+\frac{1}{3ac\sqrt{3a-2ax^2}\sqrt{cx}}$$

```
[Out] 1/3/a/c/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2)+1/3*2^(1/4)*EllipticE(1/6*(3-x*6^(
1/2))^(1/2)*6^(1/2),2^(1/2))*(c*x)^(1/2)*(-2*x^2+3)^(1/2)*3^(1/4)/a/c^2/x^(
1/2)/(-2*a*x^2+3*a)^(1/2)-1/3*(-2*a*x^2+3*a)^(1/2)/a^2/c/(c*x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {290, 325, 320, 319, 318, 424}

$$-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}+\frac{\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}ac^2\sqrt{x}\sqrt{3a-2ax^2}}+\frac{1}{3ac\sqrt{3a-2ax^2}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] 1/(3*a*c*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2]) - Sqrt[3*a - 2*a*x^2]/(3*a^2*c*Sqrt
[c*x]) + (2^(1/4)*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[
6]*x]/Sqrt[6]], 2])/(3^(3/4)*a*c^2*Sqrt[x]*Sqrt[3*a - 2*a*x^2])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 318

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[-2/(Sqrt[a]*(-(b/
a))^(3/4)), Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-
(b/a)]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-(b/a), 0] && GtQ[a, 0]
```

Rule 319

```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + (b*x^2)/
a]/Sqrt[a + b*x^2], Int[Sqrt[x]/Sqrt[1 + (b*x^2)/a], x], x] /; FreeQ[{a, b}
, x] && GtQ[-(b/a), 0] &&  !GtQ[a, 0]
```

Rule 320

```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[c*x]/
Sqrt[x], Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[
-(b/a), 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{3/2}\left(3a-2ax^2\right)^{3/2}}\,dx &= \frac{1}{3ac\sqrt{cx}\,\sqrt{3a-2ax^2}}+\frac{\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx}{2a}\\
&= \frac{1}{3ac\sqrt{cx}\,\sqrt{3a-2ax^2}}-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}-\frac{\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx}{3ac^2}\\
&= \frac{1}{3ac\sqrt{cx}\,\sqrt{3a-2ax^2}}-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}-\frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}\,dx}{3ac^2\sqrt{x}}\\
&= \frac{1}{3ac\sqrt{cx}\,\sqrt{3a-2ax^2}}-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}-\frac{\left(\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\int \frac{\sqrt{x}}{\sqrt{1-\frac{2x^2}{3}}}\,dx}{3ac^2\sqrt{x}\,\sqrt{3a-2ax^2}}\\
&= \frac{1}{3ac\sqrt{cx}\,\sqrt{3a-2ax^2}}-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}+\frac{\left(\sqrt[4]{\frac{2}{3}}\sqrt{cx}\sqrt{1-\frac{2x^2}{3}}\right)\text{Subst}\left(\int \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}\,dx,\right.}{ac^2\sqrt{x}\,\sqrt{3a-2ax^2}}\\
&= \frac{1}{3ac\sqrt{cx}\,\sqrt{3a-2ax^2}}-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}+\frac{\sqrt[4]{2}\sqrt{cx}\sqrt{3-2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}\,x}}{\sqrt{6}}\right)\middle|\,2\right)}{3^{3/4}ac^2\sqrt{x}\,\sqrt{3a-2ax^2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.41

$$
-\frac{2x\left(3-2x^2\right)^{3/2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};\frac{2x^2}{3}\right)}{3\sqrt{3}\left(a\left(3-2x^2\right)\right)^{3/2}(cx)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] (-2*x*(3 - 2*x^2)^(3/2)*Hypergeometric2F1[-1/4, 3/2, 3/4, (2*x^2)/3])/(3*Sq
rt[3]*(c*x)^(3/2)*(a*(3 - 2*x^2))^(3/2))
```

fricas [F] time = 1.21, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}}{4\,a^2c^2x^6-12\,a^2c^2x^4+9\,a^2c^2x^2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(4*a^2*c^2*x^6 - 12*a^2*c^2*x^4 + 9
*a^2*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(3/2)), x)
```

maple [B] time = 0.03, size = 228, normalized size = 1.63

$$-\frac{\sqrt{-\left(2x^2-3\right)a}\,\left(24x^2+2\sqrt{2}\,\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{3}\,\sqrt{-\sqrt{2}\,\sqrt{3}\,x}\ \mathrm{EllipticE}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x)
```

```
[Out] -1/36*(-(2*x^2-3)*a)^(1/2)*(2*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2
))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-2^(1/2)*3
^(1/2)*x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2
)*3^(1/2))^(1/2),1/2*2^(1/2))-2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2
))^(1/2)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*3^(1/2)*(-2^(1/2)*3
^(1/2)*x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2
)*3^(1/2))^(1/2),1/2*2^(1/2))+24*x^2-24)/a^2/c/(c*x)^(1/2)/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{(c\,x)}^{3/2}\,{\left(3\,a-2\,a\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(3/2)),x)
```

```
[Out] int(1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(3/2)), x)
```

sympy [C] time = 3.16, size = 54, normalized size = 0.39

$$\frac{\sqrt{3}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\,\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(-2*a*x**2+3*a)**(3/2),x)
```

```
[Out] sqrt(3)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), 2*x**2*exp_polar(2*I*pi)/3)/
(18*a**(3/2)*c**(3/2)*sqrt(x)*gamma(3/4))
```

3.648 $\int \frac{1}{(cx)^{5/2}\left(3a-2ax^2\right)^{3/2}}\,dx$

Optimal. Leaf size=132

$$\frac{5\,2^{3/4}\sqrt{3-2x^2}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{27\sqrt[4]{3}\,ac^{5/2}\sqrt{a\left(3-2x^2\right)}}-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{1}{3ac\sqrt{3a-2ax^2}\,(cx)^{3/2}}$$

```
[Out] 5/81*2^(3/4)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*(-2*x^2+3
)^(1/2)*3^(3/4)/a/c^(5/2)/(a*(-2*x^2+3))^(1/2)+1/3/a/c/(c*x)^(3/2)/(-2*a*x^
2+3*a)^(1/2)-5/27*(-2*a*x^2+3*a)^(1/2)/a^2/c/(c*x)^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {290, 325, 329, 224, 221}

$$-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{5\,2^{3/4}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{27\sqrt[4]{3}\,ac^{5/2}\sqrt{a\left(3-2x^2\right)}}+\frac{1}{3ac\sqrt{3a-2ax^2}\,(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] 1/(3*a*c*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2]) - (5*Sqrt[3*a - 2*a*x^2])/(27*a^2
*c*(c*x)^(3/2)) + (5*2^(3/4)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4)*
Sqrt[c*x])/Sqrt[c]], -1])/(27*3^(1/4)*a*c^(5/2)*Sqrt[a*(3 - 2*x^2)])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{5/2}\left(3a-2ax^2\right)^{3/2}}\,dx &= \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}}+\frac{5\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx}{6a}\\
&= \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}}-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{5\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx}{27ac^2}\\
&= \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}}-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{10\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{3a-\frac{2ax^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{27ac^3}\\
&= \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}}-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{\left(10\sqrt{3-2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{2x^4}{3c^2}}}\,dx,x,\right.}{27\sqrt{3}\,ac^3\sqrt{a\left(3-2x^2\right)}}\\
&= \frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}}-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{5\,2^{3/4}\sqrt{3-2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right.}{27\sqrt[4]{3}\,ac^{5/2}\sqrt{a\left(3-2x^2\right)}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.44

$$-\frac{2x\left(3-2x^2\right)^{3/2}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};\frac{2x^2}{3}\right)}{9\sqrt{3}\left(a\left(3-2x^2\right)\right)^{3/2}(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

```
[Out] (-2*x*(3 - 2*x^2)^(3/2)*Hypergeometric2F1[-3/4, 3/2, 1/4, (2*x^2)/3])/(9*Sq
rt[3]*(c*x)^(5/2)*(a*(3 - 2*x^2))^(3/2))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-2\,ax^2+3\,a}\,\sqrt{cx}}{4\,a^2c^3x^7-12\,a^2c^3x^5+9\,a^2c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)/(4*a^2*c^3*x^7 - 12*a^2*c^3*x^5 + 9
*a^2*c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(5/2)), x)
```

maple [A] time = 0.03, size = 133, normalized size = 1.01

$$-\frac{\sqrt{-\left(2x^2-3\right)a}\left(60x^2+5\sqrt{\left(2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{\left(-2x+\sqrt{2}\,\sqrt{3}\right)\sqrt{2}\,\sqrt{3}}\,\sqrt{-\sqrt{2}\,\sqrt{3}\,x}\,x\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}}{\right.}\right.}{162\sqrt{cx}\,\left(2x^2-3\right)a^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x)
[Out] -1/162*(-(2*x^2-3)*a)^(1/2)*(5*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2
)*((-2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2)*(-2^(1/2)*3^(1/2)*x)^(1/2)
*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+2^(1/2)*3^(1/2))*2^(1/2)*3^(1/2))^(1/2
),1/2*2^(1/2))*x+60*x^2-36)/x/a^2/c^2/(c*x)^(1/2)/(2*x^2-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{5/2}\,\left(3\,a-2\,a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(3/2)),x)
[Out] int(1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(3/2)), x)
```

sympy [A] time = 8.11, size = 54, normalized size = 0.41

$$\frac{\sqrt{3}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\right|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(-2*a*x**2+3*a)**(3/2),x)
[Out] sqrt(3)*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), 2*x**2*exp_polar(2*I*pi)/3)/
(18*a**(3/2)*c**(5/2)*x**(3/2)*gamma(1/4))
```

3.649 $\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=21

$$\frac{2\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{a}\sqrt{x}\right),-1\right)}{\sqrt{a}}$$

```
[Out] 2*EllipticF(a^(1/2)*x^(1/2),I)/a^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {329, 221}

$$\frac{2F\left(\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)\middle|-1\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (2*EllipticF[ArcSin[Sqrt[a]*Sqrt[x]], -1])/Sqrt[a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^4}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2F\left(\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)\middle|-1\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 24, normalized size = 1.14

$$2\sqrt{x}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] 2*Sqrt[x]*Hypergeometric2F1[1/4, 1/2, 5/4, a^2*x^2]
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\sqrt{x}}{a^2x^3-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*sqrt(x)/(a^2*x^3 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^{2} x^{2} + 1} \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(x)), x)
```

maple [B] time = 0.06, size = 66, normalized size = 3.14

$$-\frac{\sqrt{-a^{2} x^{2}+1}\, \sqrt{a x +1}\, \sqrt{-2 a x +2}\, \sqrt{-a x}\, \mathrm{EllipticF}\left(\sqrt{a x +1}, \frac{\sqrt{2}}{2}\right)}{\left(a^{2} x^{2}-1\right) a \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/x^(1/2)*(-a^2*x^2+1)^(1/2)*(a*x+1)^(1/2)*(-2*a*x+2)^(1/2)*(-a*x)^(1/2)*E
llipticF((a*x+1)^(1/2),1/2*2^(1/2))/a/(a^2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^{2} x^{2} + 1} \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{x}\, \sqrt{1-a^2\, x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [B] time = 0.80, size = 36, normalized size = 1.71

$$\frac{\sqrt{x} \Gamma\left(\frac{1}{4}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| {a^{2} x^{2} e^{2 i \pi}} \right)}}{2 \Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), a**2*x**2*exp_polar(2*I*pi))/(
2*gamma(5/4))
```

3.650 $\int \frac{1}{\sqrt{x}\sqrt{1+ax^2}}\,dx$

Optimal. Leaf size=67

$$\frac{\left(\sqrt{a}\,x+1\right)\sqrt{\frac{ax^2+1}{\left(\sqrt{a}\,x+1\right)^2}}\ \text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{a}\,\sqrt{x}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt{ax^2+1}}$$

```
[Out] (cos(2*arctan(a^(1/4)*x^(1/2)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/2)))*Ell
ipticF(sin(2*arctan(a^(1/4)*x^(1/2))),1/2*2^(1/2))*(1+x*a^(1/2))*((a*x^2+1)
/(1+x*a^(1/2))^2)^(1/2)/a^(1/4)/(a*x^2+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {329, 220}

$$\frac{\left(\sqrt{a}\,x+1\right)\sqrt{\frac{ax^2+1}{\left(\sqrt{a}\,x+1\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{a}\,\sqrt{x}\right)\middle|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt{ax^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[1 + a*x^2]),x]
```

```
[Out] ((1 + Sqrt[a]*x)*Sqrt[(1 + a*x^2)/(1 + Sqrt[a]*x)^2]*EllipticF[2*ArcTan[a^(
1/4)*Sqrt[x]], 1/2])/(a^(1/4)*Sqrt[1 + a*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{1+ax^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^4}}\,dx,x,\sqrt{x}\right)\\ &= \frac{\left(1+\sqrt{a}\,x\right)\sqrt{\frac{1+ax^2}{\left(1+\sqrt{a}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{a}\,\sqrt{x}\right)\middle|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt{1+ax^2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 23, normalized size = 0.34

$$2\sqrt{x}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 + a*x^2]),x]
```

```
[Out] 2*Sqrt[x]*Hypergeometric2F1[1/4, 1/2, 5/4, -(a*x^2)]
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ax^2+1}\sqrt{x}}{ax^3+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(a*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x^2 + 1)*sqrt(x)/(a*x^3 + x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2+1}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(a*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x^2 + 1)*sqrt(x)), x)
```

maple [A] time = 0.05, size = 73, normalized size = 1.09

$$-\frac{\sqrt{-\sqrt{-a}\,x+1}\,\sqrt{2}\,\sqrt{\sqrt{-a}\,x+1}\,\sqrt{\sqrt{-a}\,x}\,\operatorname{EllipticF}\left(\sqrt{-\sqrt{-a}\,x+1},\frac{\sqrt{2}}{2}\right)}{\sqrt{a\,x^2+1}\,\sqrt{-a}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(a*x^2+1)^(1/2),x)
```

```
[Out] -1/x^(1/2)/(a*x^2+1)^(1/2)*(-(-a)^(1/2)*x+1)^(1/2)*2^(1/2)*((-a)^(1/2)*x+1)
^(1/2)*((-a)^(1/2)*x)^(1/2)*EllipticF((-(-a)^(1/2)*x+1)^(1/2),1/2*2^(1/2))/
(-a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2+1}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(a*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x^2 + 1)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x}\,\sqrt{a\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x^2 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*(a*x^2 + 1)^(1/2)), x)
```

sympy [C] time = 0.75, size = 32, normalized size = 0.48

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|ax^2e^{i\pi}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(a*x**2+1)**(1/2),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), a*x**2*exp_polar(I*pi))/(2*gam
ma(5/4))
```

3.651 $\int x^m \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=50

$$\frac{x^{m+1}\left(a+bx^2\right)^{5/2} {}_2F_1\left(1,\frac{m+6}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*(b*x^2+a)^(5/2)*hypergeom([1, 3+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(1+m
)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{ax^{m+1}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{(m+1)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^(3/2),x]
```

```
[Out] (a*x^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)])/((1 + m)*Sqrt[1 + (b*x^2)/a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)^{3/2} dx &= \frac{\left(a\sqrt{a+bx^2}\right)\int x^m\left(1+\frac{bx^2}{a}\right)^{3/2} dx}{\sqrt{1+\frac{bx^2}{a}}}\\ &= \frac{ax^{1+m}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{1+\frac{bx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 1.32

$$\frac{ax^{m+1}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{(m+1)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^(3/2),x]
[Out] (a*x^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, 1 + (1 + m)
/2, -((b*x^2)/a)])/((1 + m)*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{3}{2}}x^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/2)*x^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{3}{2}}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/2)*x^m, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{3}{2}}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^(3/2),x)
[Out] int(x^m*(b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{3}{2}}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/2)*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,{\left(b\,x^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^(3/2),x)
[Out] int(x^m*(a + b*x^2)^(3/2), x)
```

sympy [C] time = 2.85, size = 54, normalized size = 1.08

$$\frac{a^{\frac{3}{2}}xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(3/2)*x*x**m*gamma(m/2 + 1/2)*hyper((-3/2, m/2 + 1/2), (m/2 + 3/2,), b*x
**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

3.652 $\int x^m \sqrt{a+bx^2}\,dx$

Optimal. Leaf size=50

$$\frac{x^{m+1}\left(a+bx^2\right)^{3/2} {}_2F_1\left(1,\frac{m+4}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*(b*x^2+a)^(3/2)*hypergeom([1, 2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(1+m
)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{(m+1)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)])/((1 + m)*Sqrt[1 + (b*x^2)/a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int x^m\sqrt{a+bx^2}\,dx &= \frac{\sqrt{a+bx^2}\int x^m\sqrt{1+\frac{bx^2}{a}}\,dx}{\sqrt{1+\frac{bx^2}{a}}}\\
&= \frac{x^{1+m}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{1+\frac{bx^2}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 65, normalized size = 1.30

$$\frac{x^{m+1}\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{(m+1)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, 1 + (1 + m)/2
, -((b*x^2)/a)])/((1 + m)*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^2 + a}\, x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*x^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^2 + a}\, x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^m, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \sqrt{b\,x^2 + a}\; x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^m*(b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^2 + a}\, x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \sqrt{b\,x^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^m*(a + b*x^2)^(1/2), x)
```

sympy [C] time = 1.17, size = 54, normalized size = 1.08

$$\frac{\sqrt{a}\, x x^m \Gamma\left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2} + \frac{1}{2} \\ \frac{m}{2} + \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), b*x*
*2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

3.653 $\int \frac{x^m}{\sqrt{a+bx^2}} dx$

Optimal. Leaf size=50

$$\frac{x^{m+1}\sqrt{a+bx^2}\, {}_2F_1\left(1,\frac{m+2}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*hypergeom([1, 1+1/2*m],[3/2+1/2*m],-b*x^2/a)*(b*x^2+a)^(1/2)/a/(1+m
)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\, {}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx^2}} dx &= \frac{\sqrt{1+\frac{bx^2}{a}} \int \frac{x^m}{\sqrt{1+\frac{bx^2}{a}}} dx}{\sqrt{a+bx^2}}\\ &= \frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\, {}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.30

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\, {}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, 1 + (1 + m
)/2, -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^2+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x^2 + a), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{b\,x^2+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^m/(b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^2+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\sqrt{b\,x^2+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^m/(a + b*x^2)^(1/2), x)
```

sympy [C] time = 1.01, size = 53, normalized size = 1.06

$$\frac{xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**(1/2),x)
```

```
[Out] x*x**m*gamma(m/2 + 1/2)*hyper((1/2, m/2 + 1/2), (m/2 + 3/2,), b*x**2*exp_po
lar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3/2))
```

3.654 $\int \frac{x^m}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)\sqrt{a+bx^2}}$$

```
[Out] x^(1+m)*hypergeom([1, 1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(1+m)/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.38, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/(a*(1 + m)*Sqrt[a + b*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^m}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2}}\\ &= \frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 1.42

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/2, (1 + m)/2, 1 + (1 + m
)/2, -((b*x^2)/a)])/(a*(1 + m)*Sqrt[a + b*x^2])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{bx^2 + a}\,x^m}{b^2x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^2 + a)^(3/2), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(b\,x^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^(3/2),x)
```

```
[Out] int(x^m/(b*x^2+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{{\left(b\,x^2 + a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^2)^(3/2),x)
```

```
[Out] int(x^m/(a + b*x^2)^(3/2), x)
```

sympy [C] time = 1.39, size = 53, normalized size = 1.10

$$\frac{xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**(3/2),x)
```

```
[Out] x*x**m*gamma(m/2 + 1/2)*hyper((3/2, m/2 + 1/2), (m/2 + 3/2,), b*x**2*exp_po
lar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 3/2))
```

3.655 $\int \frac{x^m}{\left(a+bx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=50

$$\frac{x^{m+1} \, {}_2F_1\left(1,\frac{m-2}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)\left(a+bx^2\right)^{3/2}}$$

[Out] x^(1+m)*hypergeom([1, -1+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(1+m)/(b*x^2+a)^(3/2)

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1} \, {}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^2(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^m/(a + b*x^2)^(5/2),x]

[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^2*(1 + m)*Sqrt[a + b*x^2])

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^m}{\left(a+bx^2\right)^{5/2}} \, dx &= \frac{\sqrt{1+\frac{bx^2}{a}} \int \frac{x^m}{\left(1+\frac{bx^2}{a}\right)^{5/2}} \, dx}{a^2\sqrt{a+bx^2}}\\
&= \frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}} \, {}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^2(1+m)\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 1.36

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1} \, {}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a^2(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[5/2, (1 + m)/2, 1 + (1 + m
)/2, -((b*x^2)/a)])/(a^2*(1 + m)*Sqrt[a + b*x^2])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^2 + a}\,x^m}{b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*x^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^2 + a)^(5/2), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(b\,x^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^2+a)^(5/2),x)
```

```
[Out] int(x^m/(b*x^2+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{{\left(b\,x^2 + a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^m/(a + b*x^2)^(5/2), x)
```

sympy [C] time = 2.94, size = 53, normalized size = 1.06

$$\frac{xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**(5/2),x)
```

```
[Out] x*x**m*gamma(m/2 + 1/2)*hyper((5/2, m/2 + 1/2), (m/2 + 3/2,), b*x**2*exp_po
lar(I*pi)/a)/(2*a**(5/2)*gamma(m/2 + 3/2))
```

3.656 $\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^{m+3}\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{m+4}{2};\frac{m+5}{2};-\frac{bx^2}{a}\right)}{a(m+3)}$$

```
[Out] x^(3+m)*hypergeom([1, 2+1/2*m],[5/2+1/2*m],-b*x^2/a)*(b*x^2+a)^(1/2)/a/(3+m
)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{x^{m+3}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+3}{2};\frac{m+5}{2};-\frac{bx^2}{a}\right)}{(m+3)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(3 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2,
 -((b*x^2)/a)])/((3 + m)*Sqrt[a + b*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^{2+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}\\ &= \frac{x^{3+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};-\frac{bx^2}{a}\right)}{(3+m)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.30

$$\frac{x^{m+3}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+3}{2};\frac{m+3}{2}+1;-\frac{bx^2}{a}\right)}{(m+3)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(3 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (3 + m)/2, 1 + (3 + m
)/2, -((b*x^2)/a)])/((3 + m)*Sqrt[a + b*x^2])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{m+2}}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m + 2)/sqrt(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m+2}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)/sqrt(b*x^2 + a), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^{m+2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m+2)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^(m+2)/(b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m+2}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 2)/sqrt(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m+2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 2)/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^(m + 2)/(a + b*x^2)^(1/2), x)
```

sympy [C] time = 2.64, size = 54, normalized size = 1.08

$$\frac{x^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] x**3*x**m*gamma(m/2 + 3/2)*hyper((1/2, m/2 + 3/2), (m/2 + 5/2,), b*x**2*exp
_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 5/2))
```

3.657 $\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^{m+2}\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{a(m+2)}$$

```
[Out] x^(2+m)*hypergeom([1, 3/2+1/2*m],[2+1/2*m],-b*x^2/a)*(b*x^2+a)^(1/2)/a/(2+m
)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{x^{m+2}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
 -((b*x^2)/a)])/((2 + m)*Sqrt[a + b*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^{1+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}\\ &= \frac{x^{2+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{(2+m)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.30

$$\frac{x^{m+2}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+2}{2}+1;-\frac{bx^2}{a}\right)}{(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (2 + m)/2, 1 + (2 + m
)/2, -((b*x^2)/a)])/((2 + m)*Sqrt[a + b*x^2])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{m+1}}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m + 1)/sqrt(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m+1}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)/sqrt(b*x^2 + a), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^{m+1}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m+1)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^(m+1)/(b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m+1}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 1)/sqrt(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m+1}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 1)/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^(m + 1)/(a + b*x^2)^(1/2), x)
```

sympy [C] time = 1.87, size = 48, normalized size = 0.96

$$\frac{x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\right|\frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_polar
(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 2))
```

3.658 $\int \frac{x^m}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^{m+1}\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*hypergeom([1, 1+1/2*m],[3/2+1/2*m],-b*x^2/a)*(b*x^2+a)^(1/2)/a/(1+m
)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^m}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}\\ &= \frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{(1+m)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 65, normalized size = 1.30

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, 1 + (1 + m
)/2, -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x^2 + a), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)*x^m,x)
```

```
[Out] int(1/(b*x^2+a)^(1/2)*x^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^m/(a + b*x^2)^(1/2), x)
```

sympy [C] time = 1.02, size = 53, normalized size = 1.06

$$\frac{xx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**2+a)**(1/2),x)
```

```
[Out] x*x**m*gamma(m/2 + 1/2)*hyper((1/2, m/2 + 1/2), (m/2 + 3/2,), b*x**2*exp_po
lar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3/2))
```

3.659 $\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=46

$$\frac{x^m\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)}{am}$$

```
[Out] x^m*hypergeom([1, 1/2+1/2*m],[1+1/2*m],-b*x^2/a)*(b*x^2+a)^(1/2)/a/m
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)}{m\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, -((b*x^2)/a
)])/(m*Sqrt[a + b*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^{-1+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}\\ &= \frac{x^m\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};-\frac{bx^2}{a}\right)}{m\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 1.24

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m}{2}+1;-\frac{bx^2}{a}\right)}{m\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, m/2, 1 + m/2, -((b*x^2)/a)]
)/(m*Sqrt[a + b*x^2])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{m-1}}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m - 1)/sqrt(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m-1}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)/sqrt(b*x^2 + a), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{x^{m-1}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-1)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^(m-1)/(b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m-1}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 1)/sqrt(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m-1}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^(m - 1)/(a + b*x^2)^(1/2), x)
```

sympy [C] time = 3.06, size = 41, normalized size = 0.89

$$\frac{x^m\Gamma\left(\frac{m}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] x**m*gamma(m/2)*hyper((1/2, m/2), (m/2 + 1,), b*x**2*exp_polar(I*pi)/a)/(2*
sqrt(a)*gamma(m/2 + 1))
```

3.660 $\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=51

$$-\frac{x^{m-1}\sqrt{a+bx^2}\,{}_2F_1\left(1,\frac{m}{2};\frac{m+1}{2};-\frac{bx^2}{a}\right)}{a(1-m)}$$

```
[Out] -x^(-1+m)*hypergeom([1, 1/2*m],[1/2+1/2*m],-b*x^2/a)*(b*x^2+a)^(1/2)/a/(1-m
)
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$-\frac{x^{m-1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};-\frac{bx^2}{a}\right)}{(1-m)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] -((x^(-1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m
)/2, -((b*x^2)/a)])/((1 - m)*Sqrt[a + b*x^2]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{x^{-2+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}\\ &= -\frac{x^{-1+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{bx^2}{a}\right)}{(1-m)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.27

$$\frac{x^{m-1}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{2};\frac{m-1}{2}+1;-\frac{bx^2}{a}\right)}{(m-1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(-1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (-1 + m)/2, 1 + (-1
+ m)/2, -((b*x^2)/a)])/((-1 + m)*Sqrt[a + b*x^2])
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{m-2}}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m - 2)/sqrt(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m-2}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)/sqrt(b*x^2 + a), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{x^{m-2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-2)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(x^(m-2)/(b*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m-2}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 2)/sqrt(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m-2}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 2)/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^(m - 2)/(a + b*x^2)^(1/2), x)
```

sympy [C] time = 8.06, size = 53, normalized size = 1.04

$$\frac{x^m\Gamma\left(\frac{m}{2}-\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}-\frac{1}{2}\\ \frac{m}{2}+\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,x\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] x**m*gamma(m/2 - 1/2)*hyper((1/2, m/2 - 1/2), (m/2 + 1/2,), b*x**2*exp_pola
r(I*pi)/a)/(2*sqrt(a)*x*gamma(m/2 + 1/2))
```

3.661 $\int \frac{x^{1+m}\left(a(2+m)+b(3+m)x^2\right)}{\sqrt{a+bx^2}} dx$

Optimal. Leaf size=17

$$x^{m+2}\sqrt{a+bx^2}$$

```
[Out] x^(2+m)*(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {449}

$$x^{m+2}\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1 + m)*(a*(2 + m) + b*(3 + m)*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] x^(2 + m)*Sqrt[a + b*x^2]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{1+m}\left(a(2+m)+b(3+m)x^2\right)}{\sqrt{a+bx^2}} dx = x^{2+m}\sqrt{a+bx^2}$$

Mathematica [C] time = 0.11, size = 104, normalized size = 6.12

$$\frac{x^{m+2}\sqrt{\frac{bx^2}{a}+1}\left(b(m+3)x^2\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};-\frac{bx^2}{a}\right)+a(m+4)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)\right)}{(m+4)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1 + m)*(a*(2 + m) + b*(3 + m)*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(2 + m)*Sqrt[1 + (b*x^2)/a]*(a*(4 + m)*Hypergeometric2F1[1/2, (2 + m)/2,
 (4 + m)/2, -((b*x^2)/a)] + b*(3 + m)*x^2*Hypergeometric2F1[1/2, (4 + m)/2,
 (6 + m)/2, -((b*x^2)/a)]))/((4 + m)*Sqrt[a + b*x^2])
```

fricas [A] time = 0.90, size = 16, normalized size = 0.94

$$\sqrt{bx^2+a}\,xx^{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(b*x^2 + a)*x*x^(m + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b(m+3)x^2+a(m+2)\right)x^{m+1}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*(m + 3)*x^2 + a*(m + 2))*x^(m + 1)/sqrt(b*x^2 + a), x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\sqrt{b\,x^2+a}\;x^{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m+1)*(a*(m+2)+b*(m+3)*x^2)/(b*x^2+a)^(1/2),x)
```

```
[Out] x^(m+2)*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.88, size = 16, normalized size = 0.94

$$\sqrt{bx^2+a}\,x^2x^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] sqrt(b*x^2 + a)*x^2*x^m
```

mupad [B] time = 5.19, size = 24, normalized size = 1.41

$$\frac{x^{m+1}\,\left(b\,x^3+a\,x\right)}{\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m + 1)*(a*(m + 2) + b*x^2*(m + 3)))/(a + b*x^2)^(1/2),x)
```

```
[Out] (x^(m + 1)*(a*x + b*x^3))/(a + b*x^2)^(1/2)
```

sympy [C] time = 10.06, size = 202, normalized size = 11.88

$$\frac{\sqrt{a}\,mx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{\sqrt{a}\,x^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\Gamma\left(\frac{m}{2}+2\right)}+\frac{bmx^4x^m\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*(a*(2+m)+b*(3+m)*x**2)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*m*x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), b*x**2
*exp_polar(I*pi)/a)/(2*gamma(m/2 + 2)) + sqrt(a)*x**2*x**m*gamma(m/2 + 1)*h
yper((1/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/gamma(m/2 + 2) +
 b*m*x**4*x**m*gamma(m/2 + 2)*hyper((1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_
polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3)) + 3*b*x**4*x**m*gamma(m/2 + 2)*hy
per((1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(
m/2 + 3))
```

3.662 $\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right)\,dx$

Optimal. Leaf size=17

$$x^{m+2}\sqrt{a+bx^2}$$

```
[Out] x^(2+m)*(b*x^2+a)^(1/2)
```

Rubi [C] time = 0.07, antiderivative size = 127, normalized size of antiderivative = 7.47, number of steps used = 5, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {365, 364}

$$\frac{ax^{m+2}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}+\frac{b(m+3)x^{m+4}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};-\frac{bx^2}{a}\right)}{(m+4)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(2 + m)*x^(1 + m))/Sqrt[a + b*x^2] + (b*(3 + m)*x^(3 + m))/Sqrt[a +
b*x^2],x]
```

```
[Out] (a*x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/
2, -((b*x^2)/a)])/Sqrt[a + b*x^2] + (b*(3 + m)*x^(4 + m)*Sqrt[1 + (b*x^2)/a
]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, -((b*x^2)/a)])/((4 + m)*Sqrt
[a + b*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right)dx &= (a(2+m))\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx+(b(3+m))\int \frac{x^{3+m}}{\sqrt{a+bx^2}}\,dx\\
&= \frac{\left(a(2+m)\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{1+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}+\frac{\left(b(3+m)\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{3+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}\\
&= \frac{ax^{2+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{4+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{4+m}{2};\frac{6+m}{2};-\frac{bx^2}{a}\right)}{(4+m)\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 104, normalized size = 6.12

$$\frac{x^{m+2}\sqrt{\frac{bx^2}{a}+1}\left(b(m+3)x^2\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};-\frac{bx^2}{a}\right)+a(m+4)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)\right)}{(m+4)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(2 + m)*x^(1 + m))/Sqrt[a + b*x^2] + (b*(3 + m)*x^(3 + m))/Sqr
t[a + b*x^2],x]
```

```
[Out] (x^(2 + m)*Sqrt[1 + (b*x^2)/a]*(a*(4 + m)*Hypergeometric2F1[1/2, (2 + m)/2,
 (4 + m)/2, -((b*x^2)/a)] + b*(3 + m)*x^2*Hypergeometric2F1[1/2, (4 + m)/2,
 (6 + m)/2, -((b*x^2)/a)]))/((4 + m)*Sqrt[a + b*x^2])
```

fricas [A] time = 0.71, size = 18, normalized size = 1.06

$$\frac{\sqrt{bx^2 + a}\, x^{m+3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x
, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*x^(m + 3)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b(m+3)x^{m+3}}{\sqrt{bx^2 + a}} + \frac{a(m+2)x^{m+1}}{\sqrt{bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(b*(m + 3)*x^(m + 3)/sqrt(b*x^2 + a) + a*(m + 2)*x^(m + 1)/sqrt(b*
x^2 + a), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(m+2)\, a\, x^{m+1}}{\sqrt{b\, x^2 + a}} + \frac{(m+3)\, b\, x^{m+3}}{\sqrt{b\, x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*(m+2)*x^(m+1)/(b*x^2+a)^(1/2)+b*(m+3)*x^(m+3)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(a*(m+2)*x^(m+1)/(b*x^2+a)^(1/2)+b*(m+3)*x^(m+3)/(b*x^2+a)^(1/2),x)
```

maxima [A] time = 1.89, size = 16, normalized size = 0.94

$$\sqrt{bx^2 + a}\, x^2 x^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x
, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)*x^2*x^m
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{a\, x^{m+1}\, (m+2)}{\sqrt{b\, x^2 + a}} + \frac{b\, x^{m+3}\, (m+3)}{\sqrt{b\, x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(m + 1)*(m + 2))/(a + b*x^2)^(1/2) + (b*x^(m + 3)*(m + 3))/(a + b*
x^2)^(1/2),x)
```

```
[Out] int((a*x^(m + 1)*(m + 2))/(a + b*x^2)^(1/2) + (b*x^(m + 3)*(m + 3))/(a + b*
x^2)^(1/2), x)
```

sympy [C] time = 5.29, size = 105, normalized size = 6.18

$$\frac{\sqrt{a}\,x^2x^m\,(m+2)\,\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{bx^4x^m\,(m+3)\,\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x**(1+m)/(b*x**2+a)**(1/2)+b*(3+m)*x**(3+m)/(b*x**2+a)**(
1/2),x)
```

```
[Out] sqrt(a)*x**2*x**m*(m + 2)*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,),
b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 2)) + b*x**4*x**m*(m + 3)*gamma(m/
2 + 2)*hyper((1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(
a)*gamma(m/2 + 3))
```

3.663 $\int \frac{x^{-1+m}\left(am+b(-1+m)x^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^2}}$$

```
[Out] x^m/(b*x^2+a)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {449}

$$\frac{x^m}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*(a*m + b*(-1 + m)*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] x^m/Sqrt[a + b*x^2]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{-1+m}\left(am+b(-1+m)x^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{x^m}{\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.11, size = 103, normalized size = 6.87

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\left(b(m-1)x^2\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)+a(m+2)\,{}_2F_1\left(\frac{3}{2},\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)\right)}{a(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(a*m + b*(-1 + m)*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*(a*(2 + m)*Hypergeometric2F1[3/2, m/2, (2 + m)/2,
-((b*x^2)/a)] + b*(-1 + m)*x^2*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2,
 -((b*x^2)/a)]))/(a*(2 + m)*Sqrt[a + b*x^2])
```

fricas [A] time = 0.68, size = 16, normalized size = 1.07

$$\frac{xx^{m-1}}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] x*x^(m - 1)/sqrt(b*x^2 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b(m-1)x^2 + am\right)x^{m-1}}{\left(bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((b*(m - 1)*x^2 + a*m)*x^(m - 1)/(b*x^2 + a)^(3/2), x)
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$\frac{x^m}{\sqrt{b\,x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-1)*(a*m+b*(m-1)*x^2)/(b*x^2+a)^(3/2),x)
[Out] 1/(b*x^2+a)^(1/2)*x^m
```

maxima [A] time = 1.91, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] x^m/sqrt(b*x^2 + a)
```

mupad [B] time = 5.43, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{b\,x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m - 1)*(a*m + b*x^2*(m - 1)))/(a + b*x^2)^(3/2),x)
[Out] x^m/(a + b*x^2)^(1/2)
```

sympy [C] time = 52.96, size = 97, normalized size = 6.47

$$\frac{mx^m\Gamma\left(\frac{m}{2}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+1\right)} + \frac{bx^2x^m\left(m-1\right)\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*(a*m+b*(-1+m)*x**2)/(b*x**2+a)**(3/2),x)
[Out] m*x**m*gamma(m/2)*hyper((3/2, m/2), (m/2 + 1,), b*x**2*exp_polar(I*pi)/a)/(
2*sqrt(a)*gamma(m/2 + 1)) + b*x**2*x**m*(m - 1)*gamma(m/2 + 1)*hyper((3/2,
m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 2))
```

3.664 $\int \left(-\frac{bx^{1+m}}{\left(a+bx^2\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right) dx$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^2}}$$

```
[Out] x^m/(b*x^2+a)^(1/2)
```

Rubi [C] time = 0.07, antiderivative size = 123, normalized size of antiderivative = 8.20, number of steps used = 5, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {365, 364}

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}-\frac{bx^{m+2}\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{a(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[-((b*x^(1 + m))/(a + b*x^2)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b*x^2],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, -((b*x^2)/a
)])/Sqrt[a + b*x^2] - (b*x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/
2, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*(2 + m)*Sqrt[a + b*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(-\frac{bx^{1+m}}{\left(a+bx^2\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right) dx &= -\left(b\int \frac{x^{1+m}}{\left(a+bx^2\right)^{3/2}}\,dx\right)+m\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx\\
&= -\frac{\left(b\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{1+m}}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2}}+\frac{\left(m\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{-1+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}\\
&= \frac{x^m\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}-\frac{bx^{2+m}\sqrt{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};-\right.}{a(2+m)\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 103, normalized size = 6.87

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\left(b(m-1)x^2\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)+a(m+2)\,{}_2F_1\left(\frac{3}{2},\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)\right)}{a(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((b*x^(1 + m))/(a + b*x^2)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b*x^
2],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*(a*(2 + m)*Hypergeometric2F1[3/2, m/2, (2 + m)/2,
-((b*x^2)/a)] + b*(-1 + m)*x^2*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2,
 -((b*x^2)/a)]))/(a*(2 + m)*Sqrt[a + b*x^2])
```

fricas [A] time = 0.95, size = 26, normalized size = 1.73

$$\frac{\sqrt{bx^2 + a}\, x^{m+1}}{bx^3 + ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorith
m="fricas")
```

```
[Out] sqrt(b*x^2 + a)*x^(m + 1)/(b*x^3 + a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{mx^{m-1}}{\sqrt{bx^2 + a}} - \frac{bx^{m+1}}{\left(bx^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(m*x^(m - 1)/sqrt(b*x^2 + a) - b*x^(m + 1)/(b*x^2 + a)^(3/2), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int -\frac{b\, x^{m+1}}{\left(b\, x^2 + a\right)^{\frac{3}{2}}} + \frac{m\, x^{m-1}}{\sqrt{b\, x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-b*x^(m+1)/(b*x^2+a)^(3/2)+m*x^(m-1)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(-b*x^(m+1)/(b*x^2+a)^(3/2)+m*x^(m-1)/(b*x^2+a)^(1/2),x)
```

maxima [A] time = 1.92, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorith
m="maxima")
```

```
[Out] x^m/sqrt(b*x^2 + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$-\int \frac{b\, x^{m+1}}{\left(b\, x^2 + a\right)^{3/2}} - \frac{m\, x^{m-1}}{\sqrt{b\, x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^(m - 1))/(a + b*x^2)^(1/2) - (b*x^(m + 1))/(a + b*x^2)^(3/2),x)
```

```
[Out] -int((b*x^(m + 1))/(a + b*x^2)^(3/2) - (m*x^(m - 1))/(a + b*x^2)^(1/2), x)
```

sympy [C] time = 5.59, size = 94, normalized size = 6.27

$$\frac{mx^m\Gamma\left(\frac{m}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+1\right)}-\frac{bx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x**(1+m)/(b*x**2+a)**(3/2)+m*x**(-1+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] m*x**m*gamma(m/2)*hyper((1/2, m/2), (m/2 + 1,), b*x**2*exp_polar(I*pi)/a)/(
2*sqrt(a)*gamma(m/2 + 1)) - b*x**2*x**m*gamma(m/2 + 1)*hyper((3/2, m/2 + 1)
, (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 2))
```

3.665 $\int x^7 \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=80

$$-\frac{3a^3\left(a+bx^2\right)^{4/3}}{8b^4}+\frac{9a^2\left(a+bx^2\right)^{7/3}}{14b^4}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^4}-\frac{9a\left(a+bx^2\right)^{10/3}}{20b^4}$$

```
[Out] -3/8*a^3*(b*x^2+a)^(4/3)/b^4+9/14*a^2*(b*x^2+a)^(7/3)/b^4-9/20*a*(b*x^2+a)^
(10/3)/b^4+3/26*(b*x^2+a)^(13/3)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{7/3}}{14b^4}-\frac{3a^3\left(a+bx^2\right)^{4/3}}{8b^4}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^4}-\frac{9a\left(a+bx^2\right)^{10/3}}{20b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(4/3))/(8*b^4) + (9*a^2*(a + b*x^2)^(7/3))/(14*b^4) - (
9*a*(a + b*x^2)^(10/3))/(20*b^4) + (3*(a + b*x^2)^(13/3))/(26*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7 \sqrt[3]{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3 \sqrt[3]{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3\sqrt[3]{a+bx}}{b^3}+\frac{3a^2(a+bx)^{4/3}}{b^3}-\frac{3a(a+bx)^{7/3}}{b^3}+\frac{(a+bx)^{10/3}}{b^3}\right)dx,x,x^2\right)\\ &= -\frac{3a^3\left(a+bx^2\right)^{4/3}}{8b^4}+\frac{9a^2\left(a+bx^2\right)^{7/3}}{14b^4}-\frac{9a\left(a+bx^2\right)^{10/3}}{20b^4}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{3\left(a+bx^2\right)^{4/3}\left(-81a^3+108a^2bx^2-126ab^2x^4+140b^3x^6\right)}{3640b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3)*(-81*a^3 + 108*a^2*b*x^2 - 126*a*b^2*x^4 + 140*b^3*x^6
))/(3640*b^4)
```

fricas [A] time = 0.49, size = 57, normalized size = 0.71

$$\frac{3\left(140\,b^4x^8+14\,ab^3x^6-18\,a^2b^2x^4+27\,a^3bx^2-81\,a^4\right)\left(bx^2+a\right)^{\frac{1}{3}}}{3640\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/3640*(140*b^4*x^8 + 14*a*b^3*x^6 - 18*a^2*b^2*x^4 + 27*a^3*b*x^2 - 81*a^4
)*(b*x^2 + a)^(1/3)/b^4
```

giac [A] time = 0.92, size = 57, normalized size = 0.71

$$\frac{3\left(140\left(bx^2+a\right)^{\frac{13}{3}}-546\left(bx^2+a\right)^{\frac{10}{3}}a+780\left(bx^2+a\right)^{\frac{7}{3}}a^2-455\left(bx^2+a\right)^{\frac{4}{3}}a^3\right)}{3640\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/3640*(140*(b*x^2 + a)^(13/3) - 546*(b*x^2 + a)^(10/3)*a + 780*(b*x^2 + a)
^(7/3)*a^2 - 455*(b*x^2 + a)^(4/3)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{3\left(b\,x^2+a\right)^{\frac{4}{3}}\left(-140b^3x^6+126a\,b^2x^4-108a^2b\,x^2+81a^3\right)}{3640b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(1/3),x)
```

```
[Out] -3/3640*(b*x^2+a)^(4/3)*(-140*b^3*x^6+126*a*b^2*x^4-108*a^2*b*x^2+81*a^3)/b
^4
```

maxima [A] time = 1.37, size = 64, normalized size = 0.80

$$\frac{3\left(bx^2+a\right)^{\frac{13}{3}}}{26\,b^4}-\frac{9\left(bx^2+a\right)^{\frac{10}{3}}a}{20\,b^4}+\frac{9\left(bx^2+a\right)^{\frac{7}{3}}a^2}{14\,b^4}-\frac{3\left(bx^2+a\right)^{\frac{4}{3}}a^3}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/26*(b*x^2 + a)^(13/3)/b^4 - 9/20*(b*x^2 + a)^(10/3)*a/b^4 + 9/14*(b*x^2 +
 a)^(7/3)*a^2/b^4 - 3/8*(b*x^2 + a)^(4/3)*a^3/b^4
```

mupad [B] time = 5.25, size = 55, normalized size = 0.69

$$\left(b\,x^2+a\right)^{1/3}\,\left(\frac{3\,x^8}{26}-\frac{243\,a^4}{3640\,b^4}+\frac{3\,a\,x^6}{260\,b}-\frac{27\,a^2\,x^4}{1820\,b^2}+\frac{81\,a^3\,x^2}{3640\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(1/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((3*x^8)/26 - (243*a^4)/(3640*b^4) + (3*a*x^6)/(260*b) -
(27*a^2*x^4)/(1820*b^2) + (81*a^3*x^2)/(3640*b^3))
```

sympy [B] time = 2.85, size = 1795, normalized size = 22.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(1/3),x)
```

```
[Out] -243*a**(73/3)*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x*
*2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8
+ 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 243*a**(73/3)/(3640*a*
*20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7
*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x
**12) - 1377*a**(70/3)*b*x**2*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 2184
0*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a
**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 1458*a*
*(70/3)*b*x**2/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*
x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**
10 + 3640*a**14*b**10*x**12) - 3213*a**(67/3)*b**2*x**4*(1 + b*x**2/a)**(1/
3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800
*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a*
*14*b**10*x**12) + 3645*a**(67/3)*b**2*x**4/(3640*a**20*b**4 + 21840*a**19*
b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**
8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) - 3927*a**(64/3)*
b**3*x**6*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 +
54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 218
40*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 4860*a**(64/3)*b**3*x**6/(3
640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**1
7*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b
**10*x**12) - 2163*a**(61/3)*b**4*x**8*(1 + b*x**2/a)**(1/3)/(3640*a**20*b*
*4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6
+ 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12)
+ 3645*a**(61/3)*b**4*x**8/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600
*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a*
*15*b**9*x**10 + 3640*a**14*b**10*x**12) + 1827*a**(58/3)*b**5*x**10*(1 + b
*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6
*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x*
*10 + 3640*a**14*b**10*x**12) + 1458*a**(58/3)*b**5*x**10/(3640*a**20*b**4
+ 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 5
4600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 6
573*a**(55/3)*b**6*x**12*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**
19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*
b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 243*a**(55/3
)*b**6*x**12/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x*
*4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10
 + 3640*a**14*b**10*x**12) + 8787*a**(52/3)*b**7*x**14*(1 + b*x**2/a)**(1/3
)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*
a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**
14*b**10*x**12) + 6498*a**(49/3)*b**8*x**16*(1 + b*x**2/a)**(1/3)/(3640*a**
20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*
x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x*
*12) + 2562*a**(46/3)*b**9*x**18*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 2
1840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 5460
0*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 420*
a**(43/3)*b**10*x**20*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*
b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**
8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12)
```

3.666 $\int x^5 \sqrt[3]{a+bx^2}\, dx$

Optimal. Leaf size=59

$$\frac{3a^2\left(a+bx^2\right)^{4/3}}{8b^3}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^3}-\frac{3a\left(a+bx^2\right)^{7/3}}{7b^3}$$

```
[Out] 3/8*a^2*(b*x^2+a)^(4/3)/b^3-3/7*a*(b*x^2+a)^(7/3)/b^3+3/20*(b*x^2+a)^(10/3)
/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{4/3}}{8b^3}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^3}-\frac{3a\left(a+bx^2\right)^{7/3}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(4/3))/(8*b^3) - (3*a*(a + b*x^2)^(7/3))/(7*b^3) + (3*(a
 + b*x^2)^(10/3))/(20*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \sqrt[3]{a+bx^2}\, dx &= \frac{1}{2}\text{Subst}\left(\int x^2 \sqrt[3]{a+bx}\, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2\sqrt[3]{a+bx}}{b^2}-\frac{2a(a+bx)^{4/3}}{b^2}+\frac{(a+bx)^{7/3}}{b^2}\right) dx, x, x^2\right)\\
&= \frac{3a^2\left(a+bx^2\right)^{4/3}}{8b^3}-\frac{3a\left(a+bx^2\right)^{7/3}}{7b^3}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{4/3}\left(9a^2-12abx^2+14b^2x^4\right)}{280b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3)*(9*a^2 - 12*a*b*x^2 + 14*b^2*x^4))/(280*b^3)
```

fricas [A] time = 0.74, size = 46, normalized size = 0.78

$$\frac{3\left(14\,b^3x^6+2\,ab^2x^4-3\,a^2bx^2+9\,a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}}{280\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/280*(14*b^3*x^6 + 2*a*b^2*x^4 - 3*a^2*b*x^2 + 9*a^3)*(b*x^2 + a)^(1/3)/b^
3
```

giac [A] time = 1.03, size = 43, normalized size = 0.73

$$\frac{3\left(14\left(bx^2+a\right)^{\frac{10}{3}}-40\left(bx^2+a\right)^{\frac{7}{3}}a+35\left(bx^2+a\right)^{\frac{4}{3}}a^2\right)}{280\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/280*(14*(b*x^2 + a)^(10/3) - 40*(b*x^2 + a)^(7/3)*a + 35*(b*x^2 + a)^(4/3
)*a^2)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{3\left(b\,x^2+a\right)^{\frac{4}{3}}\left(14b^2x^4-12ab\,x^2+9a^2\right)}{280b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(1/3),x)
```

```
[Out] 3/280*(b*x^2+a)^(4/3)*(14*b^2*x^4-12*a*b*x^2+9*a^2)/b^3
```

maxima [A] time = 1.34, size = 47, normalized size = 0.80

$$\frac{3\left(bx^2+a\right)^{\frac{10}{3}}}{20\,b^3}-\frac{3\left(bx^2+a\right)^{\frac{7}{3}}a}{7\,b^3}+\frac{3\left(bx^2+a\right)^{\frac{4}{3}}a^2}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/20*(b*x^2 + a)^(10/3)/b^3 - 3/7*(b*x^2 + a)^(7/3)*a/b^3 + 3/8*(b*x^2 + a)
^(4/3)*a^2/b^3
```

mupad [B] time = 4.80, size = 44, normalized size = 0.75

$$\left(b\,x^2+a\right)^{1/3}\,\left(\frac{3\,x^6}{20}+\frac{27\,a^3}{280\,b^3}+\frac{3\,a\,x^4}{140\,b}-\frac{9\,a^2\,x^2}{280\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(1/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((3*x^6)/20 + (27*a^3)/(280*b^3) + (3*a*x^4)/(140*b) - (9
*a^2*x^2)/(280*b^2))
```

sympy [B] time = 1.88, size = 700, normalized size = 11.86

$$\frac{27a^{\frac{34}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}-\frac{27a^{\frac{34}{3}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}+\frac{}{280a^8b^3+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(1/3),x)
```

```
[Out] 27*a**(34/3)*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 84
0*a**6*b**5*x**4 + 280*a**5*b**6*x**6) - 27*a**(34/3)/(280*a**8*b**3 + 840*
a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 72*a**(31/3)*b*
x**2*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b
**5*x**4 + 280*a**5*b**6*x**6) - 81*a**(31/3)*b*x**2/(280*a**8*b**3 + 840*a
**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 60*a**(28/3)*b**
2*x**4*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6
*b**5*x**4 + 280*a**5*b**6*x**6) - 81*a**(28/3)*b**2*x**4/(280*a**8*b**3 +
840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 60*a**(25/3
)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840
*a**6*b**5*x**4 + 280*a**5*b**6*x**6) - 27*a**(25/3)*b**3*x**6/(280*a**8*b*
*3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 135*a*
*(22/3)*b**4*x**8*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2
 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 132*a**(19/3)*b**5*x**10*(1 +
 b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4
+ 280*a**5*b**6*x**6) + 42*a**(16/3)*b**6*x**12*(1 + b*x**2/a)**(1/3)/(280*
a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6)
```

3.667 $\int x^3 \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=38

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b^2}-\frac{3a\left(a+bx^2\right)^{4/3}}{8b^2}$$

```
[Out] -3/8*a*(b*x^2+a)^(4/3)/b^2+3/14*(b*x^2+a)^(7/3)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b^2}-\frac{3a\left(a+bx^2\right)^{4/3}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(4/3))/(8*b^2) + (3*(a + b*x^2)^(7/3))/(14*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3 \sqrt[3]{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\sqrt[3]{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a\sqrt[3]{a+bx}}{b}+\frac{(a+bx)^{4/3}}{b}\right)dx,x,x^2\right)\\ &= -\frac{3a\left(a+bx^2\right)^{4/3}}{8b^2}+\frac{3\left(a+bx^2\right)^{7/3}}{14b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{3\left(a+bx^2\right)^{4/3}\left(4bx^2-3a\right)}{56b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3)*(-3*a + 4*b*x^2))/(56*b^2)
```

fricas [A] time = 0.78, size = 34, normalized size = 0.89

$$\frac{3\left(4\,b^{2}x^{4}+abx^{2}-3\,a^{2}\right)\left(bx^{2}+a\right)^{\frac{1}{3}}}{56\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] 3/56*(4*b^2*x^4 + a*b*x^2 - 3*a^2)*(b*x^2 + a)^(1/3)/b^2
```

giac [A] time = 1.20, size = 29, normalized size = 0.76

$$\frac{3\left(4\left(bx^{2}+a\right)^{\frac{7}{3}}-7\left(bx^{2}+a\right)^{\frac{4}{3}}a\right)}{56\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] 3/56*(4*(b*x^2 + a)^(7/3) - 7*(b*x^2 + a)^(4/3)*a)/b^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-4b\,x^{2}+3a\right)}{56b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(1/3),x)
[Out] -3/56*(b*x^2+a)^(4/3)*(-4*b*x^2+3*a)/b^2
```

maxima [A] time = 1.32, size = 30, normalized size = 0.79

$$\frac{3\left(bx^{2}+a\right)^{\frac{7}{3}}}{14\,b^{2}}-\frac{3\left(bx^{2}+a\right)^{\frac{4}{3}}a}{8\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] 3/14*(b*x^2 + a)^(7/3)/b^2 - 3/8*(b*x^2 + a)^(4/3)*a/b^2
```

mupad [B] time = 4.72, size = 33, normalized size = 0.87

$$\left(b\,x^{2}+a\right)^{1/3}\left(\frac{3\,x^{4}}{14}-\frac{9\,a^{2}}{56\,b^{2}}+\frac{3\,a\,x^{2}}{56\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(1/3),x)
[Out] (a + b*x^2)^(1/3)*((3*x^4)/14 - (9*a^2)/(56*b^2) + (3*a*x^2)/(56*b))
```

sympy [B] time = 1.22, size = 223, normalized size = 5.87

$$-\frac{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^{2}}{a}}}{56a^{2}b^{2}+56ab^{3}x^{2}}+\frac{9a^{\frac{13}{3}}}{56a^{2}b^{2}+56ab^{3}x^{2}}-\frac{6a^{\frac{10}{3}}bx^{2}\sqrt[3]{1+\frac{bx^{2}}{a}}}{56a^{2}b^{2}+56ab^{3}x^{2}}+\frac{9a^{\frac{10}{3}}bx^{2}}{56a^{2}b^{2}+56ab^{3}x^{2}}+\frac{15a^{\frac{7}{3}}b^{2}x^{4}\sqrt[3]{1+\frac{bx^{2}}{a}}}{56a^{2}b^{2}+56ab^{3}x^{2}}+\frac{12a^{\frac{4}{3}}b^{3}}{56a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(1/3),x)
```

```
[Out] -9*a**(13/3)*(1 + b*x**2/a)**(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 9*a**(
13/3)/(56*a**2*b**2 + 56*a*b**3*x**2) - 6*a**(10/3)*b*x**2*(1 + b*x**2/a)**
(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 9*a**(10/3)*b*x**2/(56*a**2*b**2 +
56*a*b**3*x**2) + 15*a**(7/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(56*a**2*b**2
 + 56*a*b**3*x**2) + 12*a**(4/3)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(56*a**2*b
**2 + 56*a*b**3*x**2)
```

3.668 $\int x\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=18

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b}$$

```
[Out] 3/8*(b*x^2+a)^(4/3)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(4/3))/(8*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt[3]{a+bx^2}\,dx=\frac{3\left(a+bx^2\right)^{4/3}}{8b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(4/3))/(8*b)
```

fricas [A] time = 0.89, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{4}{3}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] 3/8*(b*x^2 + a)^(4/3)/b
```

giac [A] time = 1.12, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{4}{3}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{3\left(b\,x^2+a\right)^{\frac{4}{3}}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(1/3),x)
```

```
[Out] 3/8*(b*x^2+a)^(4/3)/b
```

maxima [A] time = 1.37, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{4}{3}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b
```

mupad [B] time = 4.66, size = 14, normalized size = 0.78

$$\frac{3\left(b\,x^2+a\right)^{4/3}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(1/3),x)
```

```
[Out] (3*(a + b*x^2)^(4/3))/(8*b)
```

sympy [A] time = 0.20, size = 42, normalized size = 2.33

$$\begin{cases} \frac{3a\sqrt[3]{a+bx^2}}{8b} + \frac{3x^2\sqrt[3]{a+bx^2}}{8} & \text{for}\ b \neq 0 \\ \frac{\sqrt[3]{a}\,x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(1/3),x)
```

```
[Out] Piecewise((3*a*(a + b*x**2)**(1/3)/(8*b) + 3*x**2*(a + b*x**2)**(1/3)/8, Ne
(b, 0)), (a**(1/3)*x**2/2, True))
```

3.669 $\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx$

Optimal. Leaf size=101

$$\frac{3}{2}\sqrt[3]{a+bx^2}+\frac{3}{4}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\log(x)$$

```
[Out] 3/2*(b*x^2+a)^(1/3)-1/2*a^(1/3)*ln(x)+3/4*a^(1/3)*ln(a^(1/3)-(b*x^2+a)^(1/3
))-1/2*a^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1
/2)
```

Rubi [A] time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 57, 617, 204, 31}

$$\frac{3}{2}\sqrt[3]{a+bx^2}+\frac{3}{4}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x,x]
```

```
[Out] (3*(a + b*x^2)^(1/3))/2 - (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x^2)^
(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(1/3)*Log[x])/2 + (3*a^(1/3)*Log[a^(1/3)
- (a + b*x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{2}\sqrt[3]{a+bx^2} + \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\sqrt[3]{a+bx^2} - \frac{1}{2}\sqrt[3]{a}\log(x) - \frac{1}{4}\left(3\sqrt[3]{a}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right) - \frac{1}{4}\left(3a^{2/3}\right)\operatorname{Subst}\Bigg(\\
&= \frac{3}{2}\sqrt[3]{a+bx^2} - \frac{1}{2}\sqrt[3]{a}\log(x) + \frac{3}{4}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right) + \frac{1}{2}\left(3\sqrt[3]{a}\right)\operatorname{Subst}\Bigg(\int \frac{1}{-3-x^2}\,dx, x,\\
&= \frac{3}{2}\sqrt[3]{a+bx^2} - \frac{1}{2}\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{1}{2}\sqrt[3]{a}\log(x) + \frac{3}{4}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 126, normalized size = 1.25

$$
\frac{1}{4}\Bigg(-\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)+6\sqrt[3]{a+bx^2}+2\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-2\sqrt{3}\sqrt[3]{a}\tan^{-1}\Bigg(
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x,x]
```

```
[Out] (6*(a + b*x^2)^(1/3) - 2*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/
a^(1/3))/Sqrt[3]] + 2*a^(1/3)*Log[a^(1/3) - (a + b*x^2)^(1/3)] - a^(1/3)*Lo
g[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/4
```

fricas [A] time = 0.93, size = 102, normalized size = 1.01

$$
-\frac{1}{2}\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)-\frac{1}{4}a^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{1}{3}}\log\Bigg(\left(bx^2+a\right)^{\frac{1}{3}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/3) + sqrt
(3)*a)/a) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
 a^(2/3)) + 1/2*a^(1/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/2*(b*x^2 + a)^
(1/3)
```

giac [A] time = 2.41, size = 98, normalized size = 0.97

$$-\frac{1}{2}\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{4}\,a^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}\,a^{\frac{1}{3}}\log\left(\left|\left(bx^2+a\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/2*a^(1/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/2*(b*x^2 + a)
^(1/3)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^2+a\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x,x)
```

maxima [A] time = 3.03, size = 97, normalized size = 0.96

$$-\frac{1}{2}\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{4}\,a^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}\,a^{\frac{1}{3}}\log\left(\left(bx^2+a\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/2*a^(1/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/2*(b*x^2 + a)^(1/3
)
```

mupad [B] time = 4.74, size = 115, normalized size = 1.14

$$\frac{a^{1/3}\ln\left(\frac{9\,a\,\left(b\,x^2+a\right)^{1/3}}{4}-\frac{9\,a^{4/3}}{4}\right)}{2}+\frac{3\left(b\,x^2+a\right)^{1/3}}{2}-\frac{a^{1/3}\ln\left(\frac{9\,a^{4/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2}+\frac{9\,a\,\left(b\,x^2+a\right)^{1/3}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2}+a^{1/3}\ln\left(\frac{9\,a\,(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/x,x)
```

```
[Out] (a^(1/3)*log((9*a*(a + b*x^2)^(1/3))/4 - (9*a^(4/3))/4))/2 + (3*(a + b*x^2)
^(1/3))/2 - (a^(1/3)*log((9*a^(4/3)*((3^(1/2)*1i)/2 + 1/2))/2 + (9*a*(a + b
*x^2)^(1/3))/2)*((3^(1/2)*1i)/2 + 1/2))/2 + a^(1/3)*log((9*a*(a + b*x^2)^(1
/3))/2 - 9*a^(4/3)*((3^(1/2)*1i)/4 - 1/4))*((3^(1/2)*1i)/4 - 1/4)
```

sympy [C] time = 1.07, size = 46, normalized size = 0.46

$$-\frac{\sqrt[3]{b}\,x^{\frac{2}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x,x)
```

```
[Out] -b**(1/3)*x**(2/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), a*exp_polar(I*pi
)/(b*x**2))/(2*gamma(2/3))
```

3.670 $\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx$

Optimal. Leaf size=107

$$\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}-\frac{\sqrt[3]{a+bx^2}}{2x^2}$$

```
[Out] -1/2*(b*x^2+a)^(1/3)/x^2-1/6*b*ln(x)/a^(2/3)+1/4*b*ln(a^(1/3)-(b*x^2+a)^(1/
3))/a^(2/3)-1/6*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a
^(2/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 47, 57, 617, 204, 31}

$$\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}-\frac{\sqrt[3]{a+bx^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x^3,x]
```

```
[Out] -(a + b*x^2)^(1/3)/(2*x^2) - (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqr
t[3]*a^(1/3))])/(2*Sqrt[3]*a^(2/3)) - (b*Log[x])/(6*a^(2/3)) + (b*Log[a^(1/
3) - (a + b*x^2)^(1/3)])/(4*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{2x^2} + \frac{1}{6}b\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{2x^2} - \frac{b\log(x)}{6a^{2/3}} - \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^{2/3}} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a\right.}}{4\sqrt[3]{a}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2x^2} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}} + \frac{b\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2a^{2/3}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2x^2} - \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt{3}\,a^{2/3}} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.35

$$\frac{3b\left(a+bx^2\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{bx^2}{a}+1\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x^3,x]
```

```
[Out] (3*b*(a + b*x^2)^(4/3)*Hypergeometric2F1[4/3, 2, 7/3, 1 + (b*x^2)/a])/(8*a^
2)
```

fricas [A] time = 0.78, size = 155, normalized size = 1.45

$$-\frac{2\sqrt{3}\left(a^2\right)^{\frac{1}{6}}abx^2\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right)+\left(a^2\right)^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}\left(a\right.\right.}{12\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(3)*(a^2)^(1/6)*a*b*x^2*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^
(1/3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + (a^2)^(2/3)*b*x^2
*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2/3)) -
 2*(a^2)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x^2 + a)
^(1/3)*a^2)/(a^2*x^2)
```

giac [A] time = 2.48, size = 115, normalized size = 1.07

$$-\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2b^2\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{6\left(bx^2+a\right)^{\frac{1}{3}}b}{x^2}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/12*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(2/3) + b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(2/3) - 2*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3) + 6*(b*
x^2 + a)^(1/3)*b/x^2)/b
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x^3,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x^3,x)
```

maxima [A] time = 2.91, size = 103, normalized size = 0.96

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{6a^{\frac{2}{3}}}-\frac{b\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}}+\frac{b\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\left(bx^2+\right.}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(2/3) - 1/12*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(2/3) + 1/6*b*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(2/3) - 1/2*(b*x^2 +
a)^(1/3)/x^2
```

mupad [B] time = 4.88, size = 125, normalized size = 1.17

$$\frac{b\,\ln\left(\frac{3\,b\,\left(b\,x^2+a\right)^{1/3}}{2}-\frac{3\,a^{1/3}\,b}{2}\right)}{6\,a^{2/3}}-\frac{\left(b\,x^2+a\right)^{1/3}}{2\,x^2}-\frac{\ln\left(\frac{3\,a^{1/3}\,\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{4}+\frac{3\,b\,\left(b\,x^2+a\right)^{1/3}}{2}\right)\,\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{12\,a^{2/3}}-\frac{\ln\left(\frac{3\,a^{1/3}\,\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/x^3,x)
```

```
[Out] (b*log((3*b*(a + b*x^2)^(1/3))/2 - (3*a^(1/3)*b)/2))/(6*a^(2/3)) - (a + b*x
^2)^(1/3)/(2*x^2) - (log((3*a^(1/3)*(b - 3^(1/2)*b*1i))/4 + (3*b*(a + b*x^2
)^(1/3))/2)*(b - 3^(1/2)*b*1i))/(12*a^(2/3)) - (log((3*a^(1/3)*(b + 3^(1/2)
*b*1i))/4 + (3*b*(a + b*x^2)^(1/3))/2)*(b + 3^(1/2)*b*1i))/(12*a^(2/3))
```

sympy [C] time = 1.20, size = 42, normalized size = 0.39

$$-\frac{\sqrt[3]{b}\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x**3,x)
```

```
[Out] -b**(1/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(4/3)*gamma(5/3))
```

3.671 $\int \frac{\sqrt[3]{a+bx^2}}{x^5}\,dx$

Optimal. Leaf size=135

$$-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}}+\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{5/3}}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b\sqrt[3]{a+bx^2}}{12ax^2}-\frac{\sqrt[3]{a+bx^2}}{4x^4}$$

```
[Out] -1/4*(b*x^2+a)^(1/3)/x^4-1/12*b*(b*x^2+a)^(1/3)/a/x^2+1/18*b^2*ln(x)/a^(5/3
)-1/12*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(5/3)+1/18*b^2*arctan(1/3*(a^(1/3)
+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {266, 47, 51, 57, 617, 204, 31}

$$-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}}+\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{5/3}}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b\sqrt[3]{a+bx^2}}{12ax^2}-\frac{\sqrt[3]{a+bx^2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x^5,x]
```

```
[Out] -(a + b*x^2)^(1/3)/(4*x^4) - (b*(a + b*x^2)^(1/3))/(12*a*x^2) + (b^2*ArcTan
[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(5/3)) +
(b^2*Log[x])/(18*a^(5/3)) - (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(12*a^(5
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{4x^4}+\frac{1}{12}b\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{4x^4}-\frac{b\sqrt[3]{a+bx^2}}{12ax^2}-\frac{b^2\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)}{18a}\\
&= -\frac{\sqrt[3]{a+bx^2}}{4x^4}-\frac{b\sqrt[3]{a+bx^2}}{12ax^2}+\frac{b^2\log(x)}{18a^{5/3}}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{12a^{5/3}}+\frac{b^2\operatorname{Subst}\left(\int \frac{}{a^{2/}}\right.}{}\\
&= -\frac{\sqrt[3]{a+bx^2}}{4x^4}-\frac{b\sqrt[3]{a+bx^2}}{12ax^2}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}}-\frac{b^2\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{6a^{5/3}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{4x^4}-\frac{b\sqrt[3]{a+bx^2}}{12ax^2}+\frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6\sqrt{3}\,a^{5/3}}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.29

$$
-\frac{3b^2\left(a+bx^2\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},3;\frac{7}{3};\frac{bx^2}{a}+1\right)}{8a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x^5,x]
```

```
[Out] (-3*b^2*(a + b*x^2)^(4/3)*Hypergeometric2F1[4/3, 3, 7/3, 1 + (b*x^2)/a])/(8
*a^3)
```

fricas [A] time = 0.69, size = 199, normalized size = 1.47

$$\frac{2\sqrt{3}\,ab^2x^4\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a-\left(-\right.\right.}{36a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="fricas")
```

```
[Out] 1/36*(2*sqrt(3)*a*b^2*x^4*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(
1/3)*a - 2*sqrt(3)*(b*x^2 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2)
 + (-a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^2 +
 a)^(1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a -
(-a^2)^(2/3)) - 3*(a^2*b*x^2 + 3*a^3)*(b*x^2 + a)^(1/3))/(a^3*x^4)
```

giac [A] time = 2.37, size = 140, normalized size = 1.04

$$\frac{\frac{2\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^3\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2b^3\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\left(\left(bx^2+a\right)^{\frac{4}{3}}b^3+2\left(bx^2+a\right)^{\frac{1}{3}}ab^3\right)}{ab^2x^4}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="giac")
```

```
[Out] 1/36*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(5/3) + b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2
/3))/a^(5/3) - 2*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*((b*
x^2 + a)^(4/3)*b^3 + 2*(b*x^2 + a)^(1/3)*a*b^3)/(a*b^2*x^4))/b
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^2+a\right)^{\frac{1}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x^5,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x^5,x)
```

maxima [A] time = 2.91, size = 155, normalized size = 1.15

$$\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{18a^{\frac{5}{3}}}+\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{36a^{\frac{5}{3}}}-\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{18a^{\frac{5}{3}}}-\frac{(bx}{12\left(\left(b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(5/3) + 1/36*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
```

```
(2/3))/a^(5/3) - 1/18*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(5/3) - 1/12*(
(b*x^2 + a)^(4/3)*b^2 + 2*(b*x^2 + a)^(1/3)*a*b^2)/((b*x^2 + a)^2*a - 2*(b*
x^2 + a)*a^2 + a^3)
```

mupad [B] time = 5.11, size = 217, normalized size = 1.61

$$\frac{b^2\ln\left(\frac{b^2}{2(-a)^{2/3}}-\frac{b^2\left(bx^2+a\right)^{1/3}}{2a}\right)}{18(-a)^{5/3}}-\frac{\ln\left(\frac{b^2+\sqrt{3}\,b^2\,1\mathrm{i}}{4(-a)^{2/3}}+\frac{b^2\left(bx^2+a\right)^{1/3}}{2a}\right)\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{36(-a)^{5/3}}-\frac{\frac{b^2\left(bx^2+a\right)^{1/3}}{3}+\frac{b^2\left(bx^2+a\right)^{4/3}}{6a}}{2\left(bx^2+a\right)^2-4a\left(bx^2+a\right)+2a^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/x^5,x)
```

```
[Out] (b^2*log(b^2/(2*(-a)^(2/3)) - (b^2*(a + b*x^2)^(1/3))/(2*a)))/(18*(-a)^(5/3
)) - (log((3^(1/2)*b^2*1i + b^2)/(4*(-a)^(2/3)) + (b^2*(a + b*x^2)^(1/3))/(
2*a))*(3^(1/2)*b^2*1i + b^2))/(36*(-a)^(5/3)) - ((b^2*(a + b*x^2)^(1/3))/3
+ (b^2*(a + b*x^2)^(4/3))/(6*a))/(2*(a + b*x^2)^2 - 4*a*(a + b*x^2) + 2*a^2
) + (b^2*log((b^2*(a + b*x^2)^(1/3))/(2*a) - (b^2*((3^(1/2)*1i)/2 - 1/2))/(
2*(-a)^(2/3)))*((3^(1/2)*1i)/2 - 1/2))/(18*(-a)^(5/3))
```

sympy [C] time = 1.39, size = 42, normalized size = 0.31

$$-\frac{\sqrt[3]{b}\,\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{10}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x**5,x)
```

```
[Out] -b**(1/3)*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(10/3)*gamma(8/3))
```

3.672 $\int x^4 \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=314

$$-\frac{54\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-\right.}{935b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] -54/935*a^2*x*(b*x^2+a)^(1/3)/b^2+6/187*a*x^3*(b*x^2+a)^(1/3)/b+3/17*x^5*(b
*x^2+a)^(1/3)-54/935*3^(3/4)*a^3*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x
^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I
-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^3/x/(-a^(1/
3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2
)
```

Rubi [A] time = 0.28, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 236, 219}

$$-\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2}-\frac{54\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right.\right.}{935b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(1/3),x]
```

```
[Out] (-54*a^2*x*(a + b*x^2)^(1/3))/(935*b^2) + (6*a*x^3*(a + b*x^2)^(1/3))/(187*
b) + (3*x^5*(a + b*x^2)^(1/3))/17 - (54*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(a^(1
/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b
*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSi
n[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(935*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
+ b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^4\sqrt[3]{a+bx^2}\,dx &= \frac{3}{17}x^5\sqrt[3]{a+bx^2}+\frac{1}{17}(2a)\int\frac{x^4}{\left(a+bx^2\right)^{2/3}}\,dx\\
&= \frac{6ax^3\sqrt[3]{a+bx^2}}{187b}+\frac{3}{17}x^5\sqrt[3]{a+bx^2}-\frac{\left(18a^2\right)\int\frac{x^2}{\left(a+bx^2\right)^{2/3}}\,dx}{187b}\\
&= -\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2}+\frac{6ax^3\sqrt[3]{a+bx^2}}{187b}+\frac{3}{17}x^5\sqrt[3]{a+bx^2}+\frac{\left(54a^3\right)\int\frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{935b^2}\\
&= -\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2}+\frac{6ax^3\sqrt[3]{a+bx^2}}{187b}+\frac{3}{17}x^5\sqrt[3]{a+bx^2}+\frac{\left(81a^3\sqrt{bx^2}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{-a+x^3}}\,dx,x,\right.}{935b^3x}\\
&= -\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2}+\frac{6ax^3\sqrt[3]{a+bx^2}}{187b}+\frac{3}{17}x^5\sqrt[3]{a+bx^2}-\frac{54\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 94, normalized size = 0.30

$$\frac{3x\sqrt[3]{a+bx^2}\left(\sqrt[3]{\frac{bx^2}{a}+1}\left(-9a^2+2abx^2+11b^2x^4\right)+9a^2\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{187b^2\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*((1 + (b*x^2)/a)^(1/3)*(-9*a^2 + 2*a*b*x^2 + 11*b^2*
x^4) + 9*a^2*Hypergeometric2F1[-1/3, 1/2, 3/2, -((b*x^2)/a)]))/(187*b^2*(1
+ (b*x^2)/a)^(1/3))
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{1}{3}}x^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{1}{3}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)*x^4, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \left(b\,x^2 + a\right)^{\frac{1}{3}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/3),x)
[Out] int(x^4*(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{1}{3}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(b\,x^2 + a\right)^{1/3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(1/3),x)
[Out] int(x^4*(a + b*x^2)^(1/3), x)
```

sympy [A] time = 0.93, size = 29, normalized size = 0.09

$$\frac{\sqrt[3]{a}\,x^5 {}_{2}F_{1}\left(\left.\begin{matrix} -\frac{1}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/3),x)
[Out] a**(1/3)*x**5*hyper((-1/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

3.673 $\int x^2 \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=290

$$\frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{55b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+$$

```
[Out] 6/55*a*x*(b*x^2+a)^(1/3)/b+3/11*x^3*(b*x^2+a)^(1/3)+6/55*3^(3/4)*a^2*(a^(1/
3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x
^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)
^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/
2*6^(1/2)-1/2*2^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a
)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 236, 219}

$$\frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{55b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3}{11}x^3\sqrt[3]{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(1/3),x]
```

```
[Out] (6*a*x*(a + b*x^2)^(1/3))/(55*b) + (3*x^3*(a + b*x^2)^(1/3))/11 + (6*3^(3/4
)*Sqrt[2 - Sqrt[3]]*a^2*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/
3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x
^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(55*b^2*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sqrt[3]{a+bx^2}\,dx &= \frac{3}{11}x^3\sqrt[3]{a+bx^2} + \frac{1}{11}(2a)\int \frac{x^2}{\left(a+bx^2\right)^{2/3}}\,dx \\
&= \frac{6ax\sqrt[3]{a+bx^2}}{55b} + \frac{3}{11}x^3\sqrt[3]{a+bx^2} - \frac{\left(6a^2\right)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{55b} \\
&= \frac{6ax\sqrt[3]{a+bx^2}}{55b} + \frac{3}{11}x^3\sqrt[3]{a+bx^2} - \frac{\left(9a^2\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{55b^2x} \\
&= \frac{6ax\sqrt[3]{a+bx^2}}{55b} + \frac{3}{11}x^3\sqrt[3]{a+bx^2} + \frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a\cdots}}}}{55b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{\cdots}\right.}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}
\end{aligned}
$$

Mathematica [C] time = 0.05, size = 62, normalized size = 0.21

$$
\frac{3x\sqrt[3]{a+bx^2}\left(-\frac{a\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{\frac{bx^2}{a}+1}}+a+bx^2\right)}{11b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*(a + b*x^2 - (a*Hypergeometric2F1[-1/3, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(1/3)))/(11*b)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$
\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{3}}x^2, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \left(bx^2+a\right)^{\frac{1}{3}}x^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)*x^2, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \left(b\,x^{2}+a\right)^{\frac{1}{3}} x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/3),x)
[Out] int(x^2*(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2}+a\right)^{\frac{1}{3}} x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{2}\,\left(b\,x^{2}+a\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(1/3),x)
[Out] int(x^2*(a + b*x^2)^(1/3), x)
```

sympy [A] time = 0.84, size = 29, normalized size = 0.10

$$\frac{\sqrt[3]{a}\,x^{3}{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/3),x)
[Out] a**(1/3)*x**3*hyper((-1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

3.674 $\int \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=266

$$\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx}}\right.\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] 3/5*x*(b*x^2+a)^(1/3)-2/5*3^(3/4)*a*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(
b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),
2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+
a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b/x/(-a^(1
/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/
2)
```

Rubi [A] time = 0.12, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 236, 219}

$$\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3))/5 - (2*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a +
b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)
)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3))], -7 + 4*Sqrt[3]])/(5*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
```

```
, x]
```

Rubi steps

$$\int \sqrt[3]{a+bx^2}\,dx = \frac{3}{5}x\sqrt[3]{a+bx^2} + \frac{1}{5}(2a)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{3}{5}x\sqrt[3]{a+bx^2} + \frac{\left(3a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{5bx}$$

$$= \frac{3}{5}x\sqrt[3]{a+bx^2} - \frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)}{\left(1-\sqrt{3}\right)}\right.\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.17

$$\frac{x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3),x]
[Out] (x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/2, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(1/3)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3), x)
```

mupad [B] time = 4.59, size = 37, normalized size = 0.14

$$\frac{x\left(b\,x^2+a\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3),x)
```

```
[Out] (x*(a + b*x^2)^(1/3)*hypergeom([-1/3, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(1/3)
```

sympy [A] time = 0.78, size = 26, normalized size = 0.10

$$\sqrt[3]{a}\,x\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*x*hyper((-1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

3.675 $\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=260

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{\sqrt[3]{a+bx^2}}{x}$$

```
[Out] -(b*x^2+a)^(1/3)/x-2/3*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3
)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2)
)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/x/(-a^(1/3)*(a^
(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 236, 219}

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{\sqrt[3]{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x^2,x]
```

```
[Out] -((a + b*x^2)^(1/3)/x) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))
*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*
Sqrt[3]])/(3^(1/4)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx = -\frac{\sqrt[3]{a+bx^2}}{x} + \frac{1}{3}(2b)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= -\frac{\sqrt[3]{a+bx^2}}{x} + \frac{\sqrt{bx^2}\ \mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{x}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{x} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}\right.\right.}{\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.19

$$-\frac{\sqrt[3]{a+bx^2}\ {}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x^2,x]
```

```
[Out] -(((a + b*x^2)^(1/3)*Hypergeometric2F1[-1/2, -1/3, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(1/3)))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/x^2, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x^2,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/x^2, x)
```

mupad [B] time = 4.78, size = 40, normalized size = 0.15

$$-\frac{3\left(b\,x^2+a\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{1}{6};\ \frac{7}{6};\ -\frac{a}{b\,x^2}\right)}{x\left(\frac{a}{b\,x^2}+1\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/x^2,x)
```

```
[Out] -(3*(a + b*x^2)^(1/3)*hypergeom([-1/3, 1/6], 7/6, -a/(b*x^2)))/(x*(a/(b*x^2
) + 1)^(1/3))
```

sympy [A] time = 0.83, size = 29, normalized size = 0.11

$$-\frac{\sqrt[3]{a}\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x**2,x)
```

```
[Out] -a**(1/3)*hyper((-1/2, -1/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

3.676 $\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx$

Optimal. Leaf size=290

$$\frac{2\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}\,ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{2b}{}$$

```
[Out] -1/3*(b*x^2+a)^(1/3)/x^3-2/9*b*(b*x^2+a)^(1/3)/a/x+2/27*b*(a^(1/3)-(b*x^2+a
)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x
^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1
/2*2^(1/2))*3^(3/4)/a/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 325, 236, 219}

$$\frac{2\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{2b\sqrt[3]{a+}}{9ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x^4,x]
```

```
[Out] -(a + b*x^2)^(1/3)/(3*x^3) - (2*b*(a + b*x^2)^(1/3))/(9*a*x) + (2*Sqrt[2 -
Sqrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2
)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]
*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a*x*Sqrt[-((a^
(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx &= -\frac{\sqrt[3]{a+bx^2}}{3x^3}+\frac{1}{9}(2b)\int \frac{1}{x^2\left(a+bx^2\right)^{2/3}}\,dx\\
&= -\frac{\sqrt[3]{a+bx^2}}{3x^3}-\frac{2b\sqrt[3]{a+bx^2}}{9ax}-\frac{\left(2b^2\right)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{27a}\\
&= -\frac{\sqrt[3]{a+bx^2}}{3x^3}-\frac{2b\sqrt[3]{a+bx^2}}{9ax}-\frac{\left(b\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{9ax}\\
&= -\frac{\sqrt[3]{a+bx^2}}{3x^3}-\frac{2b\sqrt[3]{a+bx^2}}{9ax}+\frac{2\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin\right.}{9\sqrt[4]{3}\,ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.18

$$-\frac{\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x^4,x]
```

```
[Out] -1/3*((a + b*x^2)^(1/3)*Hypergeometric2F1[-3/2, -1/3, -1/2, -((b*x^2)/a)])/
(x^3*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^4,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)/x^4, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x^4,x)
[Out] int((b*x^2+a)^(1/3)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^4,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{1/3}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/x^4,x)
[Out] int((a + b*x^2)^(1/3)/x^4, x)
```

sympy [A] time = 0.95, size = 34, normalized size = 0.12

$$-\frac{\sqrt[3]{a}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{1}{3} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x**4,x)
[Out] -a**(1/3)*hyper((-3/2, -1/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

3.677 $\int x^7 \left(a+bx^2\right)^{2/3} dx$

Optimal. Leaf size=80

$$-\frac{3a^3\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{9a^2\left(a+bx^2\right)^{8/3}}{16b^4}+\frac{3\left(a+bx^2\right)^{14/3}}{28b^4}-\frac{9a\left(a+bx^2\right)^{11/3}}{22b^4}$$

```
[Out] -3/10*a^3*(b*x^2+a)^(5/3)/b^4+9/16*a^2*(b*x^2+a)^(8/3)/b^4-9/22*a*(b*x^2+a)
^(11/3)/b^4+3/28*(b*x^2+a)^(14/3)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{8/3}}{16b^4}-\frac{3a^3\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{14/3}}{28b^4}-\frac{9a\left(a+bx^2\right)^{11/3}}{22b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(5/3))/(10*b^4) + (9*a^2*(a + b*x^2)^(8/3))/(16*b^4) -
(9*a*(a + b*x^2)^(11/3))/(22*b^4) + (3*(a + b*x^2)^(14/3))/(28*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^{2/3} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^3(a+bx)^{2/3} dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a^3(a+bx)^{2/3}}{b^3}+\frac{3a^2(a+bx)^{5/3}}{b^3}-\frac{3a(a+bx)^{8/3}}{b^3}+\frac{(a+bx)^{11/3}}{b^3}\right) dx, x, x^2\right)\\ &= -\frac{3a^3\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{9a^2\left(a+bx^2\right)^{8/3}}{16b^4}-\frac{9a\left(a+bx^2\right)^{11/3}}{22b^4}+\frac{3\left(a+bx^2\right)^{14/3}}{28b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{3\left(a+bx^2\right)^{5/3}\left(-81a^3+135a^2bx^2-180ab^2x^4+220b^3x^6\right)}{6160b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(5/3)*(-81*a^3 + 135*a^2*b*x^2 - 180*a*b^2*x^4 + 220*b^3*x^6
))/(6160*b^4)
```

fricas [A] time = 0.88, size = 57, normalized size = 0.71

$$\frac{3\left(220\,b^4x^8+40\,ab^3x^6-45\,a^2b^2x^4+54\,a^3bx^2-81\,a^4\right)\left(bx^2+a\right)^{\frac{2}{3}}}{6160\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/6160*(220*b^4*x^8 + 40*a*b^3*x^6 - 45*a^2*b^2*x^4 + 54*a^3*b*x^2 - 81*a^4
)*(b*x^2 + a)^(2/3)/b^4
```

giac [A] time = 0.98, size = 57, normalized size = 0.71

$$\frac{3\left(220\left(bx^2+a\right)^{\frac{14}{3}}-840\left(bx^2+a\right)^{\frac{11}{3}}a+1155\left(bx^2+a\right)^{\frac{8}{3}}a^2-616\left(bx^2+a\right)^{\frac{5}{3}}a^3\right)}{6160\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/6160*(220*(b*x^2 + a)^(14/3) - 840*(b*x^2 + a)^(11/3)*a + 1155*(b*x^2 + a
)^(8/3)*a^2 - 616*(b*x^2 + a)^(5/3)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{3\left(b\,x^2+a\right)^{\frac{5}{3}}\left(-220b^3x^6+180a\,b^2x^4-135a^2b\,x^2+81a^3\right)}{6160b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(2/3),x)
```

```
[Out] -3/6160*(b*x^2+a)^(5/3)*(-220*b^3*x^6+180*a*b^2*x^4-135*a^2*b*x^2+81*a^3)/b
^4
```

maxima [A] time = 1.30, size = 64, normalized size = 0.80

$$\frac{3\left(bx^2+a\right)^{\frac{14}{3}}}{28\,b^4}-\frac{9\left(bx^2+a\right)^{\frac{11}{3}}a}{22\,b^4}+\frac{9\left(bx^2+a\right)^{\frac{8}{3}}a^2}{16\,b^4}-\frac{3\left(bx^2+a\right)^{\frac{5}{3}}a^3}{10\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/28*(b*x^2 + a)^(14/3)/b^4 - 9/22*(b*x^2 + a)^(11/3)*a/b^4 + 9/16*(b*x^2 +
 a)^(8/3)*a^2/b^4 - 3/10*(b*x^2 + a)^(5/3)*a^3/b^4
```

mupad [B] time = 4.63, size = 55, normalized size = 0.69

$${\left(b\,x^2+a\right)}^{2/3}\,\left(\frac{3\,x^8}{28}-\frac{243\,a^4}{6160\,b^4}+\frac{3\,a\,x^6}{154\,b}-\frac{27\,a^2\,x^4}{1232\,b^2}+\frac{81\,a^3\,x^2}{3080\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(2/3),x)
```

```
[Out] (a + b*x^2)^(2/3)*((3*x^8)/28 - (243*a^4)/(6160*b^4) + (3*a*x^6)/(154*b) -
(27*a^2*x^4)/(1232*b^2) + (81*a^3*x^2)/(3080*b^3))
```

sympy [B] time = 2.94, size = 1795, normalized size = 22.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(2/3),x)
```

```
[Out] -243*a**(74/3)*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x*
*2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8
 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 243*a**(74/3)/(6160*a
**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b*
*7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10
*x**12) - 1296*a**(71/3)*b*x**2*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36
960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 9240
0*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 1458
*a**(71/3)*b*x**2/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b*
*6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9
*x**10 + 6160*a**14*b**10*x**12) - 2808*a**(68/3)*b**2*x**4*(1 + b*x**2/a)*
*(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 1
23200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 61
60*a**14*b**10*x**12) + 3645*a**(68/3)*b**2*x**4/(6160*a**20*b**4 + 36960*a
**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**
16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) - 3120*a**(
65/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x
**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**
8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 4860*a**(65/3)*b**3*
x**6/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123
200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160
*a**14*b**10*x**12) - 1050*a**(62/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(6160*
a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b
**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**1
0*x**12) + 3645*a**(62/3)*b**4*x**8/(6160*a**20*b**4 + 36960*a**19*b**5*x**
2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8
+ 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 4032*a**(59/3)*b**5*x*
*10*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*
a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a*
*15*b**9*x**10 + 6160*a**14*b**10*x**12) + 1458*a**(59/3)*b**5*x**10/(6160*
a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b
**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**1
0*x**12) + 11004*a**(56/3)*b**6*x**12*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**
4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6
+ 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12)
+ 243*a**(56/3)*b**6*x**12/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400
*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a
**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 14352*a**(53/3)*b**7*x**14*(1 +
 b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b*
*6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9
*x**10 + 6160*a**14*b**10*x**12) + 10485*a**(50/3)*b**8*x**16*(1 + b*x**2/a
)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 +
 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 +
6160*a**14*b**10*x**12) + 4080*a**(47/3)*b**9*x**18*(1 + b*x**2/a)**(2/3)/(
6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a*
*17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14
*b**10*x**12) + 660*a**(44/3)*b**10*x**20*(1 + b*x**2/a)**(2/3)/(6160*a**20
*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x
**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**
12)
```

3.678 $\int x^5 \left(a+bx^2\right)^{2/3} \, dx$

Optimal. Leaf size=59

$$\frac{3a^2\left(a+bx^2\right)^{5/3}}{10b^3}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^3}-\frac{3a\left(a+bx^2\right)^{8/3}}{8b^3}$$

```
[Out] 3/10*a^2*(b*x^2+a)^(5/3)/b^3-3/8*a*(b*x^2+a)^(8/3)/b^3+3/22*(b*x^2+a)^(11/3
)/b^3
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{5/3}}{10b^3}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^3}-\frac{3a\left(a+bx^2\right)^{8/3}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(5/3))/(10*b^3) - (3*a*(a + b*x^2)^(8/3))/(8*b^3) + (3*(
a + b*x^2)^(11/3))/(22*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5\left(a+bx^2\right)^{2/3} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^{2/3} \, dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2(a+bx)^{2/3}}{b^2}-\frac{2a(a+bx)^{5/3}}{b^2}+\frac{(a+bx)^{8/3}}{b^2}\right) dx,x,x^2\right)\\
&= \frac{3a^2\left(a+bx^2\right)^{5/3}}{10b^3}-\frac{3a\left(a+bx^2\right)^{8/3}}{8b^3}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{5/3}\left(9a^2-15abx^2+20b^2x^4\right)}{440b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(5/3)*(9*a^2 - 15*a*b*x^2 + 20*b^2*x^4))/(440*b^3)
```

fricas [A] time = 0.81, size = 46, normalized size = 0.78

$$\frac{3\left(20\,b^{3}x^{6}+5\,ab^{2}x^{4}-6\,a^{2}bx^{2}+9\,a^{3}\right)\left(bx^{2}+a\right)^{\frac{2}{3}}}{440\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/440*(20*b^3*x^6 + 5*a*b^2*x^4 - 6*a^2*b*x^2 + 9*a^3)*(b*x^2 + a)^(2/3)/b^
3
```

giac [A] time = 0.97, size = 43, normalized size = 0.73

$$\frac{3\left(20\left(bx^{2}+a\right)^{\frac{11}{3}}-55\left(bx^{2}+a\right)^{\frac{8}{3}}a+44\left(bx^{2}+a\right)^{\frac{5}{3}}a^{2}\right)}{440\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/440*(20*(b*x^2 + a)^(11/3) - 55*(b*x^2 + a)^(8/3)*a + 44*(b*x^2 + a)^(5/3
)*a^2)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{3}}\left(20b^{2}x^{4}-15ab\,x^{2}+9a^{2}\right)}{440b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(2/3),x)
```

```
[Out] 3/440*(b*x^2+a)^(5/3)*(20*b^2*x^4-15*a*b*x^2+9*a^2)/b^3
```

maxima [A] time = 1.35, size = 47, normalized size = 0.80

$$\frac{3\left(bx^{2}+a\right)^{\frac{11}{3}}}{22\,b^{3}}-\frac{3\left(bx^{2}+a\right)^{\frac{8}{3}}a}{8\,b^{3}}+\frac{3\left(bx^{2}+a\right)^{\frac{5}{3}}a^{2}}{10\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/22*(b*x^2 + a)^(11/3)/b^3 - 3/8*(b*x^2 + a)^(8/3)*a/b^3 + 3/10*(b*x^2 + a
)^(5/3)*a^2/b^3
```

mupad [B] time = 4.66, size = 44, normalized size = 0.75

$$\left(b\,x^{2}+a\right)^{2/3}\left(\frac{3\,x^{6}}{22}+\frac{27\,a^{3}}{440\,b^{3}}+\frac{3\,a\,x^{4}}{88\,b}-\frac{9\,a^{2}\,x^{2}}{220\,b^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(2/3),x)
```

```
[Out] (a + b*x^2)^(2/3)*((3*x^6)/22 + (27*a^3)/(440*b^3) + (3*a*x^4)/(88*b) - (9*
a^2*x^2)/(220*b^2))
```

sympy [B] time = 2.00, size = 700, normalized size = 11.86

$$\frac{27a^{\frac{35}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}-\frac{27a^{\frac{35}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}+\frac{}{440a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(2/3),x)
```

```
[Out] 27*a**(35/3)*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1
320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 27*a**(35/3)/(440*a**8*b**3 + 13
20*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 63*a**(32/3
)*b*x**2*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320*
a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 81*a**(32/3)*b*x**2/(440*a**8*b**3 +
 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 42*a**(2
9/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 +
 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 81*a**(29/3)*b**2*x**4/(440*a*
*8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) +
 78*a**(26/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b*
*4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 27*a**(26/3)*b**3*x**
6/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**
6*x**6) + 207*a**(23/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 13
20*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 195*a**(20/
3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 +
1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 60*a**(17/3)*b**6*x**12*(1 + b*
x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 +
 440*a**5*b**6*x**6)
```

3.679 $\int x^3 \left(a+bx^2\right)^{2/3} \, dx$

Optimal. Leaf size=38

$$\frac{3\left(a+bx^2\right)^{8/3}}{16b^2}-\frac{3a\left(a+bx^2\right)^{5/3}}{10b^2}$$

```
[Out] -3/10*a*(b*x^2+a)^(5/3)/b^2+3/16*(b*x^2+a)^(8/3)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3\left(a+bx^2\right)^{8/3}}{16b^2}-\frac{3a\left(a+bx^2\right)^{5/3}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(5/3))/(10*b^2) + (3*(a + b*x^2)^(8/3))/(16*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+bx^2\right)^{2/3} \, dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^{2/3} \, dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a(a+bx)^{2/3}}{b}+\frac{(a+bx)^{5/3}}{b}\right) dx,x,x^2\right)\\
&= -\frac{3a\left(a+bx^2\right)^{5/3}}{10b^2}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{3\left(a+bx^2\right)^{5/3}\left(5bx^2-3a\right)}{80b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(5/3)*(-3*a + 5*b*x^2))/(80*b^2)
```

fricas [A] time = 0.89, size = 35, normalized size = 0.92

$$\frac{3\left(5\,b^2x^4+2\,abx^2-3\,a^2\right)\left(bx^2+a\right)^{\frac{2}{3}}}{80\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] 3/80*(5*b^2*x^4 + 2*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3)/b^2
```

giac [A] time = 1.08, size = 29, normalized size = 0.76

$$\frac{3\left(5\left(bx^2+a\right)^{\frac{8}{3}}-8\left(bx^2+a\right)^{\frac{5}{3}}a\right)}{80\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] 3/80*(5*(b*x^2 + a)^(8/3) - 8*(b*x^2 + a)^(5/3)*a)/b^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{3\left(b\,x^2+a\right)^{\frac{5}{3}}\left(-5b\,x^2+3a\right)}{80b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(2/3),x)
[Out] -3/80*(b*x^2+a)^(5/3)*(-5*b*x^2+3*a)/b^2
```

maxima [A] time = 1.35, size = 30, normalized size = 0.79

$$\frac{3\left(bx^2+a\right)^{\frac{8}{3}}}{16\,b^2}-\frac{3\left(bx^2+a\right)^{\frac{5}{3}}a}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] 3/16*(b*x^2 + a)^(8/3)/b^2 - 3/10*(b*x^2 + a)^(5/3)*a/b^2
```

mupad [B] time = 4.71, size = 33, normalized size = 0.87

$$\left(b\,x^2+a\right)^{2/3}\left(\frac{3\,x^4}{16}-\frac{9\,a^2}{80\,b^2}+\frac{3\,a\,x^2}{40\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(2/3),x)
[Out] (a + b*x^2)^(2/3)*((3*x^4)/16 - (9*a^2)/(80*b^2) + (3*a*x^2)/(40*b))
```

sympy [A] time = 0.75, size = 66, normalized size = 1.74

$$\begin{cases} -\frac{9a^2\left(a+bx^2\right)^{\frac{2}{3}}}{80b^2}+\frac{3ax^2\left(a+bx^2\right)^{\frac{2}{3}}}{40b}+\frac{3x^4\left(a+bx^2\right)^{\frac{2}{3}}}{16} & \text{for } b\neq 0 \\ \frac{a^{\frac{2}{3}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(2/3),x)
```

```
[Out] Piecewise((-9*a**2*(a + b*x**2)**(2/3)/(80*b**2) + 3*a*x**2*(a + b*x**2)**(
2/3)/(40*b) + 3*x**4*(a + b*x**2)**(2/3)/16, Ne(b, 0)), (a**(2/3)*x**4/4, T
rue))
```

3.680 $\int x\left(a+bx^2\right)^{2/3}\,dx$

Optimal. Leaf size=18

$$\frac{3\left(a+bx^2\right)^{5/3}}{10b}$$

```
[Out] 3/10*(b*x^2+a)^(5/3)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\left(a+bx^2\right)^{5/3}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(5/3))/(10*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{2/3}\,dx=\frac{3\left(a+bx^2\right)^{5/3}}{10b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{3\left(a+bx^2\right)^{5/3}}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(5/3))/(10*b)
```

fricas [A] time = 0.83, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{5}{3}}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/10*(b*x^2 + a)^(5/3)/b
```

giac [A] time = 1.06, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{5}{3}}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/10*(b*x^2 + a)^(5/3)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{3 \left(b \,x^{2}+a \right)^{\frac{5}{3}}}{10 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(2/3),x)
```

```
[Out] 3/10*(b*x^2+a)^(5/3)/b
```

maxima [A] time = 1.33, size = 14, normalized size = 0.78

$$\frac{3 \, {\left(b x^{2} + a\right)}^{\frac{5}{3}}}{10 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x^2 + a)^(5/3)/b
```

mupad [B] time = 4.58, size = 14, normalized size = 0.78

$$\frac{3\,{\left(b\,x^2+a\right)}^{5/3}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(2/3),x)
```

```
[Out] (3*(a + b*x^2)^(5/3))/(10*b)
```

sympy [A] time = 0.37, size = 42, normalized size = 2.33

$$\begin{cases} \frac{3 a \left(a + b x^{2}\right)^{\frac{2}{3}}}{10 b} + \frac{3 x^{2} \left(a + b x^{2}\right)^{\frac{2}{3}}}{10} & \text{for}\: b \neq 0 \\\frac{a^{\frac{2}{3}} x^{2}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(2/3),x)
```

```
[Out] Piecewise((3*a*(a + b*x**2)**(2/3)/(10*b) + 3*x**2*(a + b*x**2)**(2/3)/10,
Ne(b, 0)), (a**(2/3)*x**2/2, True))
```

3.681 $\int \frac{\left(a+bx^2\right)^{2/3}}{x} \, dx$

Optimal. Leaf size=101

$$\frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\frac{1}{2}\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{4}\left(a+bx^2\right)^{2/3}$$

```
[Out] 3/4*(b*x^2+a)^(2/3)-1/2*a^(2/3)*ln(x)+3/4*a^(2/3)*ln(a^(1/3)-(b*x^2+a)^(1/3
))+1/2*a^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1
/2)
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 50, 55, 617, 204, 31}

$$\frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\frac{1}{2}\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{4}\left(a+bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x^2)^(2/3))/4 + (Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x^2)^
(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(2/3)*Log[x])/2 + (3*a^(2/3)*Log[a^(1/3)
- (a + b*x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{2/3}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{4}\left(a+bx^2\right)^{2/3}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)\\
&= \frac{3}{4}\left(a+bx^2\right)^{2/3}-\frac{1}{2}a^{2/3}\log(x)-\frac{1}{4}\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)+\frac{1}{4}(3a)\,\text{Subst}\\
&= \frac{3}{4}\left(a+bx^2\right)^{2/3}-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.\\
&= \frac{3}{4}\left(a+bx^2\right)^{2/3}+\frac{1}{2}\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 93, normalized size = 0.92

$$\frac{1}{4}\left(3\left(a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\left(a+bx^2\right)^{2/3}\right)+2\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-2a^{2/3}\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x,x]
```

```
[Out] (2*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 2*
a^(2/3)*Log[x] + 3*((a + b*x^2)^(2/3) + a^(2/3)*Log[a^(1/3) - (a + b*x^2)^(
1/3)]))/4
```

fricas [A] time = 0.78, size = 122, normalized size = 1.21

$$\frac{1}{2}\sqrt{3}\left(a^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,a+2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{1}{3}}}{3\,a}\right)-\frac{1}{4}\left(a^2\right)^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)+\frac{1}{2}\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)
*(a^2)^(1/3))/a) - 1/4*(a^2)^(1/3)*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a
+ (b*x^2 + a)^(1/3)*(a^2)^(2/3)) + 1/2*(a^2)^(1/3)*log((b*x^2 + a)^(1/3)*a
- (a^2)^(2/3)) + 3/4*(b*x^2 + a)^(2/3)
```

giac [A] time = 2.36, size = 98, normalized size = 0.97

$$\frac{1}{2}\sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{1}{4}a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{2}{3}}\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3)) - 1/4*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2
/3)) + 1/2*a^(2/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/4*(b*x^2 + a)^
(2/3)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x,x)
```

maxima [A] time = 2.97, size = 97, normalized size = 0.96

$$\frac{1}{2}\sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{1}{4}a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3)) - 1/4*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2
/3)) + 1/2*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/4*(b*x^2 + a)^(2/3)
```

mupad [B] time = 4.67, size = 125, normalized size = 1.24

$$\frac{3\left(b\,x^2+a\right)^{2/3}}{4}+\frac{a^{2/3}\,\ln\left(\frac{9\,a^2\,\left(b\,x^2+a\right)^{1/3}}{4}-\frac{9\,a^{7/3}}{4}\right)}{2}-\frac{a^{2/3}\,\ln\left(\frac{9\,a^2\,\left(b\,x^2+a\right)^{1/3}}{4}-\frac{9\,a^{7/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{4}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2}+a^{2/3}\,\ln\left(\frac{9\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3)/x,x)
```

```
[Out] (3*(a + b*x^2)^(2/3))/4 + (a^(2/3)*log((9*a^2*(a + b*x^2)^(1/3))/4 - (9*a^(
7/3))/4))/2 - (a^(2/3)*log((9*a^2*(a + b*x^2)^(1/3))/4 - (9*a^(7/3)*((3^(1/
2)*1i)/2 + 1/2)^2)/4)*((3^(1/2)*1i)/2 + 1/2))/2 + a^(2/3)*log((9*a^2*(a + b
*x^2)^(1/3))/4 - 9*a^(7/3)*((3^(1/2)*1i)/4 - 1/4)^2)*((3^(1/2)*1i)/4 - 1/4)
```

sympy [C] time = 1.12, size = 46, normalized size = 0.46

$$-\frac{b^{\frac{2}{3}}x^{\frac{4}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x,x)
```

```
[Out] -b**(2/3)*x**(4/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), a*exp_polar(I*pi
)/(b*x**2))/(2*gamma(1/3))
```

3.682 $\int \frac{\left(a+bx^2\right)^{2/3}}{x^3}\,dx$

Optimal. Leaf size=104

$$-\frac{\left(a+bx^2\right)^{2/3}}{2x^2}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}+\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}$$

```
[Out] -1/2*(b*x^2+a)^(2/3)/x^2-1/3*b*ln(x)/a^(1/3)+1/2*b*ln(a^(1/3)-(b*x^2+a)^(1/
3))/a^(1/3)+1/3*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a
^(1/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 47, 55, 617, 204, 31}

$$-\frac{\left(a+bx^2\right)^{2/3}}{2x^2}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}+\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x^3,x]
```

```
[Out] -(a + b*x^2)^(2/3)/(2*x^2) + (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)) - (b*Log[x])/(3*a^(1/3)) + (b*Log[a^(1/3)
 - (a + b*x^2)^(1/3)])/(2*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{2/3}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2x^2}+\frac{1}{3}b\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2x^2}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,x,\sqrt[3]{a+bx^2}\right)-\frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{}}\right.}{}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2x^2}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{b\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2x^2}+\frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.36

$$\frac{3b\left(a+bx^2\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};\frac{bx^2}{a}+1\right)}{10a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x^3,x]
```

```
[Out] (3*b*(a + b*x^2)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + (b*x^2)/a])/(10*a
^2)
```

fricas [A] time = 1.01, size = 290, normalized size = 2.79

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abx^2\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^2+3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x^2}\right)-a^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2\right.\right.}{6ax^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x^2*sqrt(-1/a^(2/3))*log((2*b*x^2 + 3*sqrt(1/3)*(2*(b
*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) -
 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - a^(2/3)*b*x^2*log((b*x^2 + a)^(2
/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x^2*log((b*x^2 + a
)^(1/3) - a^(1/3)) - 3*(b*x^2 + a)^(2/3)*a)/(a*x^2), 1/6*(6*sqrt(1/3)*a^(2/
3)*b*x^2*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3
)*b*x^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^
(2/3)*b*x^2*log((b*x^2 + a)^(1/3) - a^(1/3)) - 3*(b*x^2 + a)^(2/3)*a)/(a*x^
2)]
```

giac [A] time = 2.37, size = 116, normalized size = 1.12

$$\frac{\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{b^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{2\,b^2\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}-\frac{3\,(bx^2+a)^{\frac{2}{3}}b}{x^2}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(1/3) - b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/
3))/a^(1/3) + 2*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(1/3) - 3*(b*x^
2 + a)^(2/3)*b/x^2)/b
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x^3,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x^3,x)
```

maxima [A] time = 2.97, size = 103, normalized size = 0.99

$$\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}}-\frac{b\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}}+\frac{b\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}-\frac{\left(bx^2+a\right.}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(1/3) - 1/6*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(1/3) + 1/3*b*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(1/3) - 1/2*(b*x^2 + a)
^(2/3)/x^2
```

mupad [B] time = 4.89, size = 136, normalized size = 1.31

$$\frac{b\,\ln\left(a^{1/3}\,b^2-b^2\,\left(b\,x^2+a\right)^{1/3}\right)}{3\,a^{1/3}}-\frac{\left(b\,x^2+a\right)^{2/3}}{2\,x^2}-\frac{\ln\left(\frac{a^{1/3}\,{\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}^2}{4}-b^2\,\left(b\,x^2+a\right)^{1/3}\right)\,\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{6\,a^{1/3}}-\ln\left(\frac{a^{1/3}\,(b+\sqrt{}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3)/x^3,x)
```

```
[Out] (b*log(a^(1/3)*b^2 - b^2*(a + b*x^2)^(1/3)))/(3*a^(1/3)) - (a + b*x^2)^(2/3
)/(2*x^2) - (log((a^(1/3)*(b - 3^(1/2)*b*1i)^2)/4 - b^2*(a + b*x^2)^(1/3))*
(b - 3^(1/2)*b*1i))/(6*a^(1/3)) - (log((a^(1/3)*(b + 3^(1/2)*b*1i)^2)/4 - b
^2*(a + b*x^2)^(1/3))*(b + 3^(1/2)*b*1i))/(6*a^(1/3))
```

sympy [C] time = 1.22, size = 42, normalized size = 0.40

$$-\frac{b^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x**3,x)
```

```
[Out] -b**(2/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(2/3)*gamma(4/3))
```

3.683 $\int \frac{(a+bx^2)^{2/3}}{x^5}\,dx$

Optimal. Leaf size=135

$$-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}-\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{4/3}}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b\left(a+bx^2\right)^{2/3}}{6ax^2}-\frac{\left(a+bx^2\right)^{2/3}}{4x^4}$$

```
[Out] -1/4*(b*x^2+a)^(2/3)/x^4-1/6*b*(b*x^2+a)^(2/3)/a/x^2+1/18*b^2*ln(x)/a^(4/3)
-1/12*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(4/3)-1/18*b^2*arctan(1/3*(a^(1/3)+
2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {266, 47, 51, 55, 617, 204, 31}

$$-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}-\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{4/3}}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b\left(a+bx^2\right)^{2/3}}{6ax^2}-\frac{\left(a+bx^2\right)^{2/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x^5,x]
```

```
[Out] -(a + b*x^2)^(2/3)/(4*x^4) - (b*(a + b*x^2)^(2/3))/(6*a*x^2) - (b^2*ArcTan[
(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(4/3)) + (
b^2*Log[x])/(18*a^(4/3)) - (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(12*a^(4/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{2/3}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4}+\frac{1}{6}b\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4}-\frac{b\left(a+bx^2\right)^{2/3}}{6ax^2}-\frac{b^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^2\right)}{18a}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4}-\frac{b\left(a+bx^2\right)^{2/3}}{6ax^2}+\frac{b^2\log(x)}{18a^{4/3}}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}-\frac{b^2\operatorname{Subst}}{}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4}-\frac{b\left(a+bx^2\right)^{2/3}}{6ax^2}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\right.}{6a}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4}-\frac{b\left(a+bx^2\right)^{2/3}}{6ax^2}-\frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6\sqrt{3}\,a^{4/3}}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.29

$$-\frac{3b^2\left(a+bx^2\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},3;\frac{8}{3};\frac{bx^2}{a}+1\right)}{10a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x^5,x]
```

```
[Out] (-3*b^2*(a + b*x^2)^(5/3)*Hypergeometric2F1[5/3, 3, 8/3, 1 + (b*x^2)/a])/(1
0*a^3)
```

fricas [A] time = 0.96, size = 380, normalized size = 2.81

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab^2x^4\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,bx^2-3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\left(bx^2+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3\,a}{x^2}\right)+(-a)^{\frac{2}{3}}\,b^2x^4\log\Big(\cdots}{36\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="fricas")
```

```
[Out] [1/36*(3*sqrt(1/3)*a*b^2*x^4*sqrt((-a)^(1/3)/a)*log((2*b*x^2 - 3*sqrt(1/3)*
(2*(b*x^2 + a)^(2/3)*(-a)^(2/3) - (b*x^2 + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt(
(-a)^(1/3)/a) - 3*(b*x^2 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^2) + (-a)^(2/3)*b^2
*x^4*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2
*(-a)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3) + (-a)^(1/3)) - 3*(2*a*b*x^2 + 3*
a^2)*(b*x^2 + a)^(2/3))/(a^2*x^4), -1/36*(6*sqrt(1/3)*a*b^2*x^4*sqrt(-(-a)^
(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/
3)/a)) - (-a)^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^
(1/3) + (-a)^(2/3)) + 2*(-a)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3) + (-a)^(1/
3)) + 3*(2*a*b*x^2 + 3*a^2)*(b*x^2 + a)^(2/3))/(a^2*x^4)]
```

giac [A] time = 2.55, size = 141, normalized size = 1.04

$$-\frac{\frac{2\sqrt{3}\,b^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^3\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2\,b^3\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{3\left(2\left(bx^2+a\right)^{\frac{5}{3}}b^3+\left(bx^2+a\right)^{\frac{2}{3}}ab^3\cdots\right.}{ab^2x^4}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="giac")
```

```
[Out] -1/36*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(4/3) - b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(4/3) + 2*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 3*(2*
(b*x^2 + a)^(5/3)*b^3 + (b*x^2 + a)^(2/3)*a*b^3)/(a*b^2*x^4))/b
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{2}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x^5,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x^5,x)
```

maxima [A] time = 3.02, size = 155, normalized size = 1.15

$$-\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{18\,a^{\frac{4}{3}}}+\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{36\,a^{\frac{4}{3}}}-\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{18\,a^{\frac{4}{3}}}-\frac{2\cdots}{12\big(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(4/3) + 1/36*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3))/a^(4/3) - 1/18*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(4/3) - 1/12*
(2*(b*x^2 + a)^(5/3)*b^2 + (b*x^2 + a)^(2/3)*a*b^2)/((b*x^2 + a)^2*a - 2*(b
*x^2 + a)*a^2 + a^3)
```

mupad [B] time = 5.16, size = 212, normalized size = 1.57

$$\frac{(-1)^{1/3}\,b^2\,\ln\left(\left(b\,x^2+a\right)^{1/3}-(-1)^{2/3}\,a^{1/3}\right)}{18\,a^{4/3}}-\frac{\frac{b^2\,\left(b\,x^2+a\right)^{2/3}}{6}+\frac{b^2\,\left(b\,x^2+a\right)^{5/3}}{3\,a}}{2\,\left(b\,x^2+a\right)^2-4\,a\,\left(b\,x^2+a\right)+2\,a^2}+\frac{(-1)^{1/3}\,b^2\,\ln\left(\frac{b^4\,\left(b\,x^2+a\right)^{1/3}}{36\,a^2}-\frac{(-1)^{2/3}}{}\right.}{18\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3)/x^5,x)
```

```
[Out] ((-1)^(1/3)*b^2*log((a + b*x^2)^(1/3) - (-1)^(2/3)*a^(1/3)))/(18*a^(4/3)) -
 ((b^2*(a + b*x^2)^(2/3))/6 + (b^2*(a + b*x^2)^(5/3))/(3*a))/(2*(a + b*x^2)
^2 - 4*a*(a + b*x^2) + 2*a^2) + ((-1)^(1/3)*b^2*log((b^4*(a + b*x^2)^(1/3))
/(36*a^2) - ((-1)^(2/3)*b^4*((3^(1/2)*1i)/2 - 1/2)^2)/(36*a^(5/3)))*((3^(1/
2)*1i)/2 - 1/2))/(18*a^(4/3)) - ((-1)^(1/3)*b^2*log((b^4*(a + b*x^2)^(1/3))
/(36*a^2) - ((-1)^(2/3)*b^4*((3^(1/2)*1i)/2 + 1/2)^2)/(36*a^(5/3)))*((3^(1/
2)*1i)/2 + 1/2))/(18*a^(4/3))
```

sympy [C] time = 1.41, size = 42, normalized size = 0.31

$$-\frac{b^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x**5,x)
```

```
[Out] -b**(2/3)*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(8/3)*gamma(7/3))
```

3.684 $\int x^4 \left(a+bx^2\right)^{2/3} dx$

Optimal. Leaf size=601

$$-\frac{108\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{1729b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] -108/1729*a^2*x*(b*x^2+a)^(2/3)/b^2+12/247*a*x^3*(b*x^2+a)^(2/3)/b+3/19*x^5
*(b*x^2+a)^(2/3)-324/1729*a^3*x/b^2/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-
108/1729*3^(3/4)*a^(10/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(
1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1
/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)
^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/
3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+162/1729*3^(1/4)*a^(10/
3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)
))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*
(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^
(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(
-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {279, 321, 235, 304, 219, 1879}

$$-\frac{108a^2x\left(a+bx^2\right)^{2/3}}{1729b^2}-\frac{324a^3x}{1729b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{108\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots\right.}}}{1729b^3x\sqrt{-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(2/3),x]
```

```
[Out] (-108*a^2*x*(a + b*x^2)^(2/3))/(1729*b^2) + (12*a*x^3*(a + b*x^2)^(2/3))/(2
47*b) + (3*x^5*(a + b*x^2)^(2/3))/19 - (324*a^3*x)/(1729*b^2*((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))) + (162*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(a
^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a
+ b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (
a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) -
 (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (10
8*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^
(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b^3
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^{2/3}dx=\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}+\frac{1}{19}(4a)\int\frac{x^4}{\sqrt[3]{a+bx^2}}dx$$
$$=\frac{12ax^3\left(a+bx^2\right)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}-\frac{\left(36a^2\right)\int\frac{x^2}{\sqrt[3]{a+bx^2}}dx}{247b}$$
$$=-\frac{108a^2x\left(a+bx^2\right)^{2/3}}{1729b^2}+\frac{12ax^3\left(a+bx^2\right)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}+\frac{\left(108a^3\right)\int\frac{1}{\sqrt[3]{a+bx^2}}dx}{1729b^2}$$
$$=-\frac{108a^2x\left(a+bx^2\right)^{2/3}}{1729b^2}+\frac{12ax^3\left(a+bx^2\right)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}+\frac{\left(162a^3\sqrt{bx^2}\right)\text{Subst}}{}$$
$$=-\frac{108a^2x\left(a+bx^2\right)^{2/3}}{1729b^2}+\frac{12ax^3\left(a+bx^2\right)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}-\frac{\left(162a^3\sqrt{bx^2}\right)\text{Subst}}{}$$
$$=-\frac{108a^2x\left(a+bx^2\right)^{2/3}}{1729b^2}+\frac{12ax^3\left(a+bx^2\right)^{2/3}}{247b}+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}-\frac{324a^3}{1729b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{}\right.}$$

Mathematica [C] time = 0.05, size = 94, normalized size = 0.16

$$\frac{3x\left(a+bx^2\right)^{2/3}\left(\left(\frac{bx^2}{a}+1\right)^{2/3}\left(-9a^2+4abx^2+13b^2x^4\right)+9a^2\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{247b^2\left(\frac{bx^2}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(2/3),x]
[Out] (3*x*(a + b*x^2)^(2/3)*((1 + (b*x^2)/a)^(2/3)*(-9*a^2 + 4*a*b*x^2 + 13*b^2*
x^4) + 9*a^2*Hypergeometric2F1[-2/3, 1/2, 3/2, -((b*x^2)/a)]))/(247*b^2*(1
+ (b*x^2)/a)^(2/3))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{2}{3}}x^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3)*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{2}{3}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(2/3)*x^4, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \left(b\,x^{2}+a\right)^{\frac{2}{3}} x^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(2/3),x)
```

```
[Out] int(x^4*(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2}+a\right)^{\frac{2}{3}} x^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(2/3)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{4}\,\left(b\,x^{2}+a\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(2/3),x)
```

```
[Out] int(x^4*(a + b*x^2)^(2/3), x)
```

sympy [A] time = 1.00, size = 29, normalized size = 0.05

$$\frac{a^{\frac{2}{3}} x^{5} {}_{2}F_{1}\left(\left.\begin{matrix} -\frac{2}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\right| \frac{bx^{2} e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(2/3),x)
```

```
[Out] a**(2/3)*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

3.685 $\int x^2\left(a+bx^2\right)^{2/3}\,dx$

Optimal. Leaf size=577

$$\frac{12\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)\ 18}{91b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] 12/91*a*x*(b*x^2+a)^(2/3)/b+3/13*x^3*(b*x^2+a)^(2/3)+36/91*a^2*x/b/(-(b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+12/91*3^(3/4)*a^(7/3)*(a^(1/3)-(b*x^2+a)^(1/
3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(
b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b^2/x/(-a^(
1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)-18/91*3^(1/4)*a^(7/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(
1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1
/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a
^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^2/x/(-a^(1/3)*(a^(
1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {279, 321, 235, 304, 219, 1879}

$$\frac{12\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|-7+4\sqrt{3}\right)\ 18\sqrt[4]{3}}{91b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(2/3),x]
```

```
[Out] (12*a*x*(a + b*x^2)^(2/3))/(91*b) + (3*x^3*(a + b*x^2)^(2/3))/13 + (36*a^2*
x)/(91*b*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (18*3^(1/4)*Sqrt[2
+ Sqrt[3]]*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
 + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b^2*x*Sqrt[-(
(a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))^2)]) + (12*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*S
qrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(91*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt
[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^2 \left(a+bx^2\right)^{2/3} dx = \frac{3}{13}x^3 \left(a+bx^2\right)^{2/3} + \frac{1}{13}(4a)\int \frac{x^2}{\sqrt[3]{a+bx^2}}dx$$
$$= \frac{12ax\left(a+bx^2\right)^{2/3}}{91b} + \frac{3}{13}x^3\left(a+bx^2\right)^{2/3} - \frac{\left(12a^2\right)\int \frac{1}{\sqrt[3]{a+bx^2}}dx}{91b}$$
$$= \frac{12ax\left(a+bx^2\right)^{2/3}}{91b} + \frac{3}{13}x^3\left(a+bx^2\right)^{2/3} - \frac{\left(18a^2\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}dx, x, \sqrt[3]{a+bx^2}\right.}{91b^2x}$$
$$= \frac{12ax\left(a+bx^2\right)^{2/3}}{91b} + \frac{3}{13}x^3\left(a+bx^2\right)^{2/3} + \frac{\left(18a^2\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}dx, x, \sqrt[3]{a+\right.}{91b^2x}$$
$$= \frac{12ax\left(a+bx^2\right)^{2/3}}{91b} + \frac{3}{13}x^3\left(a+bx^2\right)^{2/3} + \frac{36a^2x}{91b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{18\sqrt[4]{3}\sqrt{2-}}{}$$

Mathematica [C] time = 0.05, size = 62, normalized size = 0.11

$$\frac{3x\left(a+bx^2\right)^{2/3}\left(-\frac{a\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{2/3}}+a+bx^2\right)}{13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(2/3)*(a + b*x^2 - (a*Hypergeometric2F1[-2/3, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(2/3)))/(13*b)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{2}{3}}x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{2}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(2/3)*x^2, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{2}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(2/3),x)
```

```
[Out] int(x^2*(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{2}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(2/3)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(b\,x^2+a\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(2/3),x)
```

```
[Out] int(x^2*(a + b*x^2)^(2/3), x)
```

sympy [A] time = 0.91, size = 29, normalized size = 0.05

$$\frac{a^{\frac{2}{3}}x^3{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(2/3),x)
```

```
[Out] a**(2/3)*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

3.686 $\int \left(a+bx^2\right)^{2/3}\,dx$

Optimal. Leaf size=550

$$-\frac{4\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)\ 6}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+-$$

```
[Out] 3/7*x*(b*x^2+a)^(2/3)-12/7*a*x/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-4/7*3
^(3/4)*a^(4/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3
)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2
)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+
a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+6/7*3^(1/4)*a^(4/3)*(a^(1/3)-(b*x^2+
a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3
)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*
x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+
1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {195, 235, 304, 219, 1879}

$$-\frac{4\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|-7+4\sqrt{3}\right)\ 6\sqrt[4]{3}\sqrt{}}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(2/3))/7 - (12*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))) + (6*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a + b*x^2)^(1/
3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 +
 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (4*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1
/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b
*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSi
n[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*
x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^2\right)^{2/3}\,dx &= \frac{3}{7}x\left(a+bx^2\right)^{2/3}+\frac{1}{7}(4a)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx\\
&= \frac{3}{7}x\left(a+bx^2\right)^{2/3}+\frac{\left(6a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{7bx}\\
&= \frac{3}{7}x\left(a+bx^2\right)^{2/3}-\frac{\left(6a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{7bx}+\frac{\left(6\sqrt{2\left(2+\sqrt{3}\right)}\,a^{4/3}\sqrt{}\right.}{}\\
&= \frac{3}{7}x\left(a+bx^2\right)^{2/3}-\frac{12ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{}}{7b}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.08

$$\frac{x\left(a+bx^2\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3),x]
[Out] (x*(a + b*x^2)^(2/3)*Hypergeometric2F1[-2/3, 1/2, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(2/3)
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(2/3), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(b\,x^2 + a\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3),x)
[Out] int((b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(2/3), x)
```

mupad [B] time = 5.17, size = 37, normalized size = 0.07

$$\frac{x\left(b\,x^2 + a\right)^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3),x)
[Out] (x*(a + b*x^2)^(2/3)*hypergeom([-2/3, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(2/3)
```

sympy [A] time = 0.81, size = 26, normalized size = 0.05

$$a^{\frac{2}{3}}x{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3),x)
```

```
[Out] a**(2/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

3.687 $\int \frac{\left(a+bx^2\right)^{2/3}}{x^2}\,dx$

Optimal. Leaf size=538

$$-\frac{4\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{2\sqrt[4]{3}\cdots}{\cdots}$$

```
[Out] -(b*x^2+a)^(2/3)/x-4*b*x/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-4/3*a^(1/3)
*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))
/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^
(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)
))^2)^(1/2)*3^(3/4)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+2*3^(1/4)*a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))*
EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*
(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/
3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2
))/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/
2)))^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 538, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 235, 304, 219, 1879}

$$-\frac{4\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{2\sqrt[4]{3}\sqrt{2+\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x^2,x]
```

```
[Out] -((a + b*x^2)^(2/3)/x) - (4*b*x)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
) + (2*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
 + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[
3]])/(x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))^2)]) - (4*Sqrt[2]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1
/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(
1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7
+ 4*Sqrt[3]])/(3^(1/4)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{2/3}}{x^2}\,dx &= -\frac{\left(a+bx^2\right)^{2/3}}{x}+\frac{1}{3}(4b)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{x}+\frac{\left(2\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{x}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{x}-\frac{\left(2\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{x}+\frac{\left(2\sqrt{2\left(2+\sqrt{3}\right)}\sqrt[3]{a}\sqrt{bx\cdots}\right.}{\cdots}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{x}-\frac{4bx}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}+\frac{2\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{\cdots}}{\left(\left(1-\cdots\right.\right.}}}{x\sqrt{-\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.09

$$
-\frac{\left(a+bx^2\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(\frac{bx^2}{a}+1\right)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x^2,x]
```

```
[Out] -(((a + b*x^2)^(2/3)*Hypergeometric2F1[-2/3, -1/2, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(2/3)))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{2}{3}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(2/3)/x^2, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{2}{3}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x^2,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{2}{3}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(2/3)/x^2, x)
```

mupad [B] time = 5.47, size = 40, normalized size = 0.07

$$\frac{3\left(b\,x^2+a\right)^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},-\frac{1}{6};\ \frac{5}{6};\ -\frac{a}{b\,x^2}\right)}{x\left(\frac{a}{b\,x^2}+1\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3)/x^2,x)
```

```
[Out] (3*(a + b*x^2)^(2/3)*hypergeom([-2/3, -1/6], 5/6, -a/(b*x^2)))/(x*(a/(b*x^2
) + 1)^(2/3))
```

sympy [A] time = 0.89, size = 29, normalized size = 0.05

$$-\frac{a^{\frac{2}{3}} {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x**2,x)
```

```
[Out] -a**(2/3)*hyper((-2/3, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

3.688 $\int \frac{(a+bx^2)^{2/3}}{x^4}\,dx$

Optimal. Leaf size=575

$$-\frac{4\sqrt{2}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}\,a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{2\sqrt{2+}}{}$$

```
[Out] -1/3*(b*x^2+a)^(2/3)/x^3-4/9*b*(b*x^2+a)^(2/3)/a/x-4/9*b^2*x/a/(-(b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))-4/27*b*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*
x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*
I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(
b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/x/(-a^(1/3)*(a
^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+2/9
*b*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)
))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*
(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^
(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(2/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2
+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 575, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {277, 325, 235, 304, 219, 1879}

$$-\frac{4\sqrt{2}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{2\sqrt{2+\sqrt{3}}\,b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x^4,x]
```

```
[Out] -(a + b*x^2)^(2/3)/(3*x^3) - (4*b*(a + b*x^2)^(2/3))/(9*a*x) - (4*b^2*x)/(9
*a*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (2*Sqrt[2 + Sqrt[3]]*b*(a
^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a
+ b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (
a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(3/4)*a^(2/3)*x*Sqrt[-((a^(1/3)*(
a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2
)]) - (4*Sqrt[2]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
 + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a^(2/3
)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{2/3}}{x^4}\,dx &= -\frac{\left(a+bx^2\right)^{2/3}}{3x^3}+\frac{1}{9}(4b)\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx \\
&= -\frac{\left(a+bx^2\right)^{2/3}}{3x^3}-\frac{4b\left(a+bx^2\right)^{2/3}}{9ax}+\frac{\left(4b^2\right)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{27a} \\
&= -\frac{\left(a+bx^2\right)^{2/3}}{3x^3}-\frac{4b\left(a+bx^2\right)^{2/3}}{9ax}+\frac{\left(2b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{9ax} \\
&= -\frac{\left(a+bx^2\right)^{2/3}}{3x^3}-\frac{4b\left(a+bx^2\right)^{2/3}}{9ax}-\frac{\left(2b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{9ax}+\frac{(2 \cdots}{\cdots} \\
&= -\frac{\left(a+bx^2\right)^{2/3}}{3x^3}-\frac{4b\left(a+bx^2\right)^{2/3}}{9ax}-\frac{4b^2x}{9a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{2\sqrt{2+\sqrt{3}}\,b\left(\sqrt[3]{a}-\cdots\right.}{\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.09

$$
-\frac{\left(a+bx^2\right)^{2/3}\,{}_2F_1\left(-\frac{3}{2},-\frac{2}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(\frac{bx^2}{a}+1\right)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x^4,x]
[Out] -1/3*((a + b*x^2)^(2/3)*Hypergeometric2F1[-3/2, -2/3, -1/2, -((b*x^2)/a)])/
(x^3*(1 + (b*x^2)/a)^(2/3))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{x^4},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^4,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(bx^2+a\right)^{\frac{2}{3}}}{x^4}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^4,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(2/3)/x^4, x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{2}{3}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x^4,x)
[Out] int((b*x^2+a)^(2/3)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{2}{3}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^4,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(2/3)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{2/3}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3)/x^4,x)
[Out] int((a + b*x^2)^(2/3)/x^4, x)
```

sympy [A] time = 0.98, size = 34, normalized size = 0.06

$$-\frac{a^{\frac{2}{3}}\,{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{3}{2},-\frac{2}{3}\\-\frac{1}{2}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x**4,x)
[Out] -a**(2/3)*hyper((-3/2, -2/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

3.689 $\int x^7 \left(a+bx^2\right)^{4/3} dx$

Optimal. Leaf size=80

$$-\frac{3a^3\left(a+bx^2\right)^{7/3}}{14b^4}+\frac{9a^2\left(a+bx^2\right)^{10/3}}{20b^4}+\frac{3\left(a+bx^2\right)^{16/3}}{32b^4}-\frac{9a\left(a+bx^2\right)^{13/3}}{26b^4}$$

```
[Out] -3/14*a^3*(b*x^2+a)^(7/3)/b^4+9/20*a^2*(b*x^2+a)^(10/3)/b^4-9/26*a*(b*x^2+a
)^(13/3)/b^4+3/32*(b*x^2+a)^(16/3)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{10/3}}{20b^4}-\frac{3a^3\left(a+bx^2\right)^{7/3}}{14b^4}+\frac{3\left(a+bx^2\right)^{16/3}}{32b^4}-\frac{9a\left(a+bx^2\right)^{13/3}}{26b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(7/3))/(14*b^4) + (9*a^2*(a + b*x^2)^(10/3))/(20*b^4) -
 (9*a*(a + b*x^2)^(13/3))/(26*b^4) + (3*(a + b*x^2)^(16/3))/(32*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^{4/3}dx&=\frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{4/3}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{4/3}}{b^3}+\frac{3a^2(a+bx)^{7/3}}{b^3}-\frac{3a(a+bx)^{10/3}}{b^3}+\frac{(a+bx)^{13/3}}{b^3}\right)dx,x,x\right.\\&=-\frac{3a^3\left(a+bx^2\right)^{7/3}}{14b^4}+\frac{9a^2\left(a+bx^2\right)^{10/3}}{20b^4}-\frac{9a\left(a+bx^2\right)^{13/3}}{26b^4}+\frac{3\left(a+bx^2\right)^{16/3}}{32b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{3\left(a+bx^2\right)^{7/3}\left(-81a^3+189a^2bx^2-315ab^2x^4+455b^3x^6\right)}{14560b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3)*(-81*a^3 + 189*a^2*b*x^2 - 315*a*b^2*x^4 + 455*b^3*x^6
))/(14560*b^4)
```

fricas [A] time = 0.54, size = 68, normalized size = 0.85

$$\frac{3\left(455\,b^{5}x^{10}+595\,ab^{4}x^{8}+14\,a^{2}b^{3}x^{6}-18\,a^{3}b^{2}x^{4}+27\,a^{4}bx^{2}-81\,a^{5}\right)\left(bx^{2}+a\right)^{\frac{1}{3}}}{14560\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/14560*(455*b^5*x^10 + 595*a*b^4*x^8 + 14*a^2*b^3*x^6 - 18*a^3*b^2*x^4 + 2
7*a^4*b*x^2 - 81*a^5)*(b*x^2 + a)^(1/3)/b^4
```

giac [A] time = 0.65, size = 57, normalized size = 0.71

$$\frac{3\left(455\left(bx^{2}+a\right)^{\frac{16}{3}}-1680\left(bx^{2}+a\right)^{\frac{13}{3}}a+2184\left(bx^{2}+a\right)^{\frac{10}{3}}a^{2}-1040\left(bx^{2}+a\right)^{\frac{7}{3}}a^{3}\right)}{14560\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/14560*(455*(b*x^2 + a)^(16/3) - 1680*(b*x^2 + a)^(13/3)*a + 2184*(b*x^2 +
 a)^(10/3)*a^2 - 1040*(b*x^2 + a)^(7/3)*a^3)/b^4
```

maple [A] time = 0.00, size = 47, normalized size = 0.59

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(-455b^{3}x^{6}+315a\,b^{2}x^{4}-189a^{2}b\,x^{2}+81a^{3}\right)}{14560b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(4/3),x)
```

```
[Out] -3/14560*(b*x^2+a)^(7/3)*(-455*b^3*x^6+315*a*b^2*x^4-189*a^2*b*x^2+81*a^3)/
b^4
```

maxima [A] time = 1.38, size = 64, normalized size = 0.80

$$\frac{3\left(bx^{2}+a\right)^{\frac{16}{3}}}{32\,b^{4}}-\frac{9\left(bx^{2}+a\right)^{\frac{13}{3}}a}{26\,b^{4}}+\frac{9\left(bx^{2}+a\right)^{\frac{10}{3}}a^{2}}{20\,b^{4}}-\frac{3\left(bx^{2}+a\right)^{\frac{7}{3}}a^{3}}{14\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/32*(b*x^2 + a)^(16/3)/b^4 - 9/26*(b*x^2 + a)^(13/3)*a/b^4 + 9/20*(b*x^2 +
 a)^(10/3)*a^2/b^4 - 3/14*(b*x^2 + a)^(7/3)*a^3/b^4
```

mupad [B] time = 5.19, size = 64, normalized size = 0.80

$${\left(b\,x^2+a\right)}^{1/3}\,\left(\frac{51\,a\,x^8}{416}+\frac{3\,b\,x^{10}}{32}-\frac{243\,a^5}{14560\,b^4}+\frac{3\,a^2\,x^6}{1040\,b}-\frac{27\,a^3\,x^4}{7280\,b^2}+\frac{81\,a^4\,x^2}{14560\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(4/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((51*a*x^8)/416 + (3*b*x^10)/32 - (243*a^5)/(14560*b^4) +
 (3*a^2*x^6)/(1040*b) - (27*a^3*x^4)/(7280*b^2) + (81*a^4*x^2)/(14560*b^3))
```

sympy [A] time = 6.58, size = 136, normalized size = 1.70

$$\begin{cases} -\frac{243a^5\sqrt[3]{a+bx^2}}{14560b^4} + \frac{81a^4x^2\sqrt[3]{a+bx^2}}{14560b^3} - \frac{27a^3x^4\sqrt[3]{a+bx^2}}{7280b^2} + \frac{3a^2x^6\sqrt[3]{a+bx^2}}{1040b} + \frac{51ax^8\sqrt[3]{a+bx^2}}{416} + \frac{3bx^{10}\sqrt[3]{a+bx^2}}{32} & \text{for } b \neq 0 \\ \frac{a^{\frac{4}{3}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((-243*a**5*(a + b*x**2)**(1/3)/(14560*b**4) + 81*a**4*x**2*(a + b
*x**2)**(1/3)/(14560*b**3) - 27*a**3*x**4*(a + b*x**2)**(1/3)/(7280*b**2) +
 3*a**2*x**6*(a + b*x**2)**(1/3)/(1040*b) + 51*a*x**8*(a + b*x**2)**(1/3)/4
16 + 3*b*x**10*(a + b*x**2)**(1/3)/32, Ne(b, 0)), (a**(4/3)*x**8/8, True))
```

3.690 $\int x^5 \left(a+bx^2\right)^{4/3} dx$

Optimal. Leaf size=59

$$\frac{3a^2\left(a+bx^2\right)^{7/3}}{14b^3}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^3}-\frac{3a\left(a+bx^2\right)^{10/3}}{10b^3}$$

```
[Out] 3/14*a^2*(b*x^2+a)^(7/3)/b^3-3/10*a*(b*x^2+a)^(10/3)/b^3+3/26*(b*x^2+a)^(13
/3)/b^3
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{7/3}}{14b^3}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^3}-\frac{3a\left(a+bx^2\right)^{10/3}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(7/3))/(14*b^3) - (3*a*(a + b*x^2)^(10/3))/(10*b^3) + (3
*(a + b*x^2)^(13/3))/(26*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^2\right)^{4/3} dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^{4/3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2(a+bx)^{4/3}}{b^2}-\frac{2a(a+bx)^{7/3}}{b^2}+\frac{(a+bx)^{10/3}}{b^2}\right)dx,x,x^2\right)\\
&= \frac{3a^2\left(a+bx^2\right)^{7/3}}{14b^3}-\frac{3a\left(a+bx^2\right)^{10/3}}{10b^3}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{7/3}\left(9a^2-21abx^2+35b^2x^4\right)}{910b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3)*(9*a^2 - 21*a*b*x^2 + 35*b^2*x^4))/(910*b^3)
```

fricas [A] time = 0.91, size = 57, normalized size = 0.97

$$\frac{3\left(35\,b^{4}x^{8}+49\,ab^{3}x^{6}+2\,a^{2}b^{2}x^{4}-3\,a^{3}bx^{2}+9\,a^{4}\right)\left(bx^{2}+a\right)^{\frac{1}{3}}}{910\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/910*(35*b^4*x^8 + 49*a*b^3*x^6 + 2*a^2*b^2*x^4 - 3*a^3*b*x^2 + 9*a^4)*(b*
x^2 + a)^(1/3)/b^3
```

giac [A] time = 0.65, size = 43, normalized size = 0.73

$$\frac{3\left(35\left(bx^{2}+a\right)^{\frac{13}{3}}-91\left(bx^{2}+a\right)^{\frac{10}{3}}a+65\left(bx^{2}+a\right)^{\frac{7}{3}}a^{2}\right)}{910\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/910*(35*(b*x^2 + a)^(13/3) - 91*(b*x^2 + a)^(10/3)*a + 65*(b*x^2 + a)^(7/
3)*a^2)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(35b^{2}x^{4}-21ab\,x^{2}+9a^{2}\right)}{910b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(4/3),x)
```

```
[Out] 3/910*(b*x^2+a)^(7/3)*(35*b^2*x^4-21*a*b*x^2+9*a^2)/b^3
```

maxima [A] time = 1.33, size = 47, normalized size = 0.80

$$\frac{3\left(bx^{2}+a\right)^{\frac{13}{3}}}{26\,b^{3}}-\frac{3\left(bx^{2}+a\right)^{\frac{10}{3}}a}{10\,b^{3}}+\frac{3\left(bx^{2}+a\right)^{\frac{7}{3}}a^{2}}{14\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/26*(b*x^2 + a)^(13/3)/b^3 - 3/10*(b*x^2 + a)^(10/3)*a/b^3 + 3/14*(b*x^2 +
 a)^(7/3)*a^2/b^3
```

mupad [B] time = 5.10, size = 53, normalized size = 0.90

$$\left(b\,x^{2}+a\right)^{1/3}\left(\frac{21\,a\,x^{6}}{130}+\frac{3\,b\,x^{8}}{26}+\frac{27\,a^{4}}{910\,b^{3}}+\frac{3\,a^{2}\,x^{4}}{455\,b}-\frac{9\,a^{3}\,x^{2}}{910\,b^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(4/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((21*a*x^6)/130 + (3*b*x^8)/26 + (27*a^4)/(910*b^3) + (3*
a^2*x^4)/(455*b) - (9*a^3*x^2)/(910*b^2))
```

sympy [A] time = 3.98, size = 112, normalized size = 1.90

$$\begin{cases} \frac{27a^4\sqrt[3]{a+bx^2}}{910b^3} - \frac{9a^3x^2\sqrt[3]{a+bx^2}}{910b^2} + \frac{3a^2x^4\sqrt[3]{a+bx^2}}{455b} + \frac{21ax^6\sqrt[3]{a+bx^2}}{130} + \frac{3bx^8\sqrt[3]{a+bx^2}}{26} & \text{for } b \neq 0 \\ \frac{a^{\frac{4}{3}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((27*a**4*(a + b*x**2)**(1/3)/(910*b**3) - 9*a**3*x**2*(a + b*x**2
)**(1/3)/(910*b**2) + 3*a**2*x**4*(a + b*x**2)**(1/3)/(455*b) + 21*a*x**6*(
a + b*x**2)**(1/3)/130 + 3*b*x**8*(a + b*x**2)**(1/3)/26, Ne(b, 0)), (a**(4
/3)*x**6/6, True))
```

3.691 $\int x^3 \left(a+bx^2\right)^{4/3} dx$

Optimal. Leaf size=38

$$\frac{3\left(a+bx^2\right)^{10/3}}{20b^2}-\frac{3a\left(a+bx^2\right)^{7/3}}{14b^2}$$

```
[Out] -3/14*a*(b*x^2+a)^(7/3)/b^2+3/20*(b*x^2+a)^(10/3)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3\left(a+bx^2\right)^{10/3}}{20b^2}-\frac{3a\left(a+bx^2\right)^{7/3}}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(7/3))/(14*b^2) + (3*(a + b*x^2)^(10/3))/(20*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+bx^2\right)^{4/3} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^{4/3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a(a+bx)^{4/3}}{b}+\frac{(a+bx)^{7/3}}{b}\right)dx,x,x^2\right)\\
&= -\frac{3a\left(a+bx^2\right)^{7/3}}{14b^2}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{3\left(a+bx^2\right)^{7/3}\left(7bx^2-3a\right)}{140b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3)*(-3*a + 7*b*x^2))/(140*b^2)
```

fricas [A] time = 0.93, size = 45, normalized size = 1.18

$$\frac{3\left(7\,b^{3}x^{6}+11\,ab^{2}x^{4}+a^{2}bx^{2}-3\,a^{3}\right)\left(bx^{2}+a\right)^{\frac{1}{3}}}{140\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/140*(7*b^3*x^6 + 11*a*b^2*x^4 + a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(1/3)/b^2
```

giac [A] time = 0.69, size = 29, normalized size = 0.76

$$\frac{3\left(7\left(bx^{2}+a\right)^{\frac{10}{3}}-10\left(bx^{2}+a\right)^{\frac{7}{3}}a\right)}{140\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/140*(7*(b*x^2 + a)^(10/3) - 10*(b*x^2 + a)^(7/3)*a)/b^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(-7b\,x^{2}+3a\right)}{140b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(4/3),x)
```

```
[Out] -3/140*(b*x^2+a)^(7/3)*(-7*b*x^2+3*a)/b^2
```

maxima [A] time = 1.32, size = 30, normalized size = 0.79

$$\frac{3\left(bx^{2}+a\right)^{\frac{10}{3}}}{20\,b^{2}}-\frac{3\left(bx^{2}+a\right)^{\frac{7}{3}}a}{14\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/20*(b*x^2 + a)^(10/3)/b^2 - 3/14*(b*x^2 + a)^(7/3)*a/b^2
```

mupad [B] time = 5.05, size = 42, normalized size = 1.11

$$\left(b\,x^{2}+a\right)^{1/3}\left(\frac{33\,a\,x^{4}}{140}+\frac{3\,b\,x^{6}}{20}-\frac{9\,a^{3}}{140\,b^{2}}+\frac{3\,a^{2}\,x^{2}}{140\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(4/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((33*a*x^4)/140 + (3*b*x^6)/20 - (9*a^3)/(140*b^2) + (3*a
^2*x^2)/(140*b))
```

sympy [A] time = 2.54, size = 88, normalized size = 2.32

$$\begin{cases} -\frac{9a^{3}\sqrt[3]{a+bx^{2}}}{140b^{2}}+\frac{3a^{2}x^{2}\sqrt[3]{a+bx^{2}}}{140b}+\frac{33ax^{4}\sqrt[3]{a+bx^{2}}}{140}+\frac{3bx^{6}\sqrt[3]{a+bx^{2}}}{20} & \text{for}\ b\neq 0 \\ \frac{a^{\frac{4}{3}}x^{4}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((-9*a**3*(a + b*x**2)**(1/3)/(140*b**2) + 3*a**2*x**2*(a + b*x**2
)**(1/3)/(140*b) + 33*a*x**4*(a + b*x**2)**(1/3)/140 + 3*b*x**6*(a + b*x**2
)**(1/3)/20, Ne(b, 0)), (a**(4/3)*x**4/4, True))
```

3.692 $\int x\left(a+bx^2\right)^{4/3}\,dx$

Optimal. Leaf size=18

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b}$$

```
[Out] 3/14*(b*x^2+a)^(7/3)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3))/(14*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{4/3}\,dx=\frac{3\left(a+bx^2\right)^{7/3}}{14b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3))/(14*b)
```

fricas [B] time = 0.65, size = 32, normalized size = 1.78

$$\frac{3\left(b^2x^4+2\,abx^2+a^2\right)\left(bx^2+a\right)^{\frac{1}{3}}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/14*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b*x^2 + a)^(1/3)/b
```

giac [A] time = 0.63, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{7}{3}}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/14*(b*x^2 + a)^(7/3)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{3}}}{14b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(4/3),x)
```

```
[Out] 3/14*(b*x^2+a)^(7/3)/b
```

maxima [A] time = 1.33, size = 14, normalized size = 0.78

$$\frac{3\left(bx^{2}+a\right)^{\frac{7}{3}}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x^2 + a)^(7/3)/b
```

mupad [B] time = 4.99, size = 14, normalized size = 0.78

$$\frac{3\left(b\,x^{2}+a\right)^{7/3}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(4/3),x)
```

```
[Out] (3*(a + b*x^2)^(7/3))/(14*b)
```

sympy [A] time = 1.37, size = 65, normalized size = 3.61

$$\begin{cases} \frac{3a^{2}\sqrt[3]{a+bx^{2}}}{14b}+\frac{3ax^{2}\sqrt[3]{a+bx^{2}}}{7}+\frac{3bx^{4}\sqrt[3]{a+bx^{2}}}{14} & \text{for}\ b\neq 0\\ \frac{a^{\frac{4}{3}}x^{2}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((3*a**2*(a + b*x**2)**(1/3)/(14*b) + 3*a*x**2*(a + b*x**2)**(1/3)
/7 + 3*b*x**4*(a + b*x**2)**(1/3)/14, Ne(b, 0)), (a**(4/3)*x**2/2, True))
```

3.693 $\int \frac{\left(a+bx^2\right)^{4/3}}{x}\,dx$

Optimal. Leaf size=117

$$\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}$$

```
[Out] 3/2*a*(b*x^2+a)^(1/3)+3/8*(b*x^2+a)^(4/3)-1/2*a^(4/3)*ln(x)+3/4*a^(4/3)*ln(
a^(1/3)-(b*x^2+a)^(1/3))-1/2*a^(4/3)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))
/a^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 57, 617, 204, 31}

$$\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x,x]
```

```
[Out] (3*a*(a + b*x^2)^(1/3))/2 + (3*(a + b*x^2)^(4/3))/8 - (Sqrt[3]*a^(4/3)*ArcT
an[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(4/3)*Log[x])
/2 + (3*a^(4/3)*Log[a^(1/3) - (a + b*x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{4/3}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x}\,dx,x,x^2\right)\\
&= \frac{3}{8}\left(a+bx^2\right)^{4/3}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx,x,x^2\right)\\
&= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx,x,x^2\right)\\
&= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}-\frac{1}{2}a^{4/3}\log(x)-\frac{1}{4}\left(3a^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right.\\
&= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\frac{1}{2}\left(3a^{4/3}\right)\text{S}\\
&= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}-\frac{1}{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{4}a^{4/3}\log
\end{aligned}$$

Mathematica [A] time = 0.05, size = 144, normalized size = 1.23

$$\frac{1}{8}\left(4a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-2a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)-4\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x,x]
```

```
[Out] (15*a*(a + b*x^2)^(1/3) + 3*b*x^2*(a + b*x^2)^(1/3) - 4*Sqrt[3]*a^(4/3)*Arc
Tan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[a^(1/3) -
(a + b*x^2)^(1/3)] - 2*a^(4/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a
 + b*x^2)^(2/3)])/8
```

fricas [A] time = 0.98, size = 111, normalized size = 0.95

$$-\frac{1}{2}\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)-\frac{1}{4}a^{\frac{4}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{4}{3}}\log\left(\left(bx^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*a^(4/3)*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/3) + sqrt
(3)*a)/a) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
```

```
a^(2/3)) + 1/2*a^(4/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/8*(b*x^2 + 5*a
)*(b*x^2 + a)^(1/3)
```

giac [A] time = 1.46, size = 110, normalized size = 0.94

$$-\frac{1}{2}\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{1}{4}a^{\frac{4}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{4}{3}}\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/2*a^(4/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/8*(b*x^2 + a)
^(4/3) + 3/2*(b*x^2 + a)^(1/3)*a
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{4}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x,x)
```

maxima [A] time = 3.06, size = 109, normalized size = 0.93

$$-\frac{1}{2}\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{1}{4}a^{\frac{4}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{4}{3}}\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/2*a^(4/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/8*(b*x^2 + a)^(4/3
) + 3/2*(b*x^2 + a)^(1/3)*a
```

mupad [B] time = 5.00, size = 133, normalized size = 1.14

$$\frac{3\,a\,\left(b\,x^2+a\right)^{1/3}}{2}+\frac{3\,\left(b\,x^2+a\right)^{4/3}}{8}+\frac{a^{4/3}\,\ln\left(\frac{9\,a^2\,\left(b\,x^2+a\right)^{1/3}}{2}-\frac{9\,a^{7/3}}{2}\right)}{2}-\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2}+\frac{9\,a^2\,\left(b\,x^2+a\right)^{1/3}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/x,x)
```

```
[Out] (3*a*(a + b*x^2)^(1/3))/2 + (3*(a + b*x^2)^(4/3))/8 + (a^(4/3)*log((9*a^2*(
a + b*x^2)^(1/3))/2 - (9*a^(7/3))/2))/2 - (a^(4/3)*log((9*a^(7/3)*((3^(1/2)
*1i)/2 + 1/2))/2 + (9*a^2*(a + b*x^2)^(1/3))/2)*((3^(1/2)*1i)/2 + 1/2))/2 +
```

```
 a^(4/3)*log(9*a^(7/3)*((3^(1/2)*1i)/4 - 1/4) - (9*a^2*(a + b*x^2)^(1/3))/2
)*((3^(1/2)*1i)/4 - 1/4)
```

sympy [C] time = 1.30, size = 49, normalized size = 0.42

$$-\frac{b^{\frac{4}{3}}x^{\frac{8}{3}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{4}{3},-\frac{4}{3}\\-\frac{1}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x,x)
```

```
[Out] -b**(4/3)*x**(8/3)*gamma(-4/3)*hyper((-4/3, -4/3), (-1/3,), a*exp_polar(I*p
i)/(b*x**2))/(2*gamma(-1/3))
```

3.694 $\int \frac{(a+bx^2)^{4/3}}{x^3}\,dx$

Optimal. Leaf size=116

$$-\frac{(a+bx^2)^{4/3}}{2x^2}+2b\sqrt[3]{a+bx^2}+\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{2\sqrt[3]{a}\,b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)$$

```
[Out] 2*b*(b*x^2+a)^(1/3)-1/2*(b*x^2+a)^(4/3)/x^2-2/3*a^(1/3)*b*ln(x)+a^(1/3)*b*l
n(a^(1/3)-(b*x^2+a)^(1/3))-2/3*a^(1/3)*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1
/3))/a^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {266, 47, 50, 57, 617, 204, 31}

$$-\frac{(a+bx^2)^{4/3}}{2x^2}+2b\sqrt[3]{a+bx^2}+\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{2\sqrt[3]{a}\,b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x^3,x]
```

```
[Out] 2*b*(a + b*x^2)^(1/3) - (a + b*x^2)^(4/3)/(2*x^2) - (2*a^(1/3)*b*ArcTan[(a^
(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (2*a^(1/3)*b*Log
[x])/3 + a^(1/3)*b*Log[a^(1/3) - (a + b*x^2)^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{4/3}}{x^3}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{4/3}}{2x^2}+\frac{1}{3}(2b)\,\mathrm{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx,x,x^2\right)\\
&= 2b\sqrt[3]{a+bx^2}-\frac{\left(a+bx^2\right)^{4/3}}{2x^2}+\frac{1}{3}(2ab)\,\mathrm{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx,x,x^2\right)\\
&= 2b\sqrt[3]{a+bx^2}-\frac{\left(a+bx^2\right)^{4/3}}{2x^2}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)-\left(\sqrt[3]{a}\,b\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)\\
&= 2b\sqrt[3]{a+bx^2}-\frac{\left(a+bx^2\right)^{4/3}}{2x^2}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\left(2\sqrt[3]{a}\,b\right)\mathrm{Subs}\\
&= 2b\sqrt[3]{a+bx^2}-\frac{\left(a+bx^2\right)^{4/3}}{2x^2}-\frac{2\sqrt[3]{a}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}\right.-
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.32

$$\frac{3b\left(a+bx^2\right)^{7/3}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{bx^2}{a}+1\right)}{14a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x^3,x]
```

```
[Out] (3*b*(a + b*x^2)^(7/3)*Hypergeometric2F1[2, 7/3, 10/3, 1 + (b*x^2)/a])/(14*
a^2)
```

fricas [A] time = 0.98, size = 129, normalized size = 1.11

$$-\frac{4\sqrt{3}\,a^{\frac{1}{3}}bx^2\arctan\left(\frac{2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+2\,a^{\frac{1}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4\,a^{\frac{1}{3}}bx^2\log\left(\left(bx^2\right.\right.}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*sqrt(3)*a^(1/3)*b*x^2*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/
3) + sqrt(3)*a)/a) + 2*a^(1/3)*b*x^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1
/3)*a^(1/3) + a^(2/3)) - 4*a^(1/3)*b*x^2*log((b*x^2 + a)^(1/3) - a^(1/3)) -
 3*(3*b*x^2 - a)*(b*x^2 + a)^(1/3))/x^2
```

giac [A] time = 1.42, size = 131, normalized size = 1.13

$$-\frac{4\sqrt{3}\,a^{\frac{1}{3}}b^{2}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)+2\,a^{\frac{1}{3}}b^{2}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4\,a^{\frac{1}{3}}b^{2}\log\left(\left|\left(bx^{2}+a\right)^{\frac{1}{3}}\right.\right.}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/6*(4*sqrt(3)*a^(1/3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/
3))/a^(1/3)) + 2*a^(1/3)*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1
/3) + a^(2/3)) - 4*a^(1/3)*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) - 9*(b
*x^2 + a)^(1/3)*b^2 + 3*(b*x^2 + a)^(1/3)*a*b/x^2)/b
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x^3,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x^3,x)
```

maxima [A] time = 2.90, size = 116, normalized size = 1.00

$$-\frac{2}{3}\sqrt{3}\,a^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{3}\,a^{\frac{1}{3}}b\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{2}{3}\,a^{\frac{1}{3}}b\log\left(\left(bx^{2}+a\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*a^(1/3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a
^(1/3)) - 1/3*a^(1/3)*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
 a^(2/3)) + 2/3*a^(1/3)*b*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/2*(b*x^2 + a
)^(1/3)*b - 1/2*(b*x^2 + a)^(1/3)*a/x^2
```

mupad [B] time = 5.42, size = 141, normalized size = 1.22

$$\frac{3\,b\,\left(b\,x^{2}+a\right)^{1/3}}{2}-\frac{a\,\left(b\,x^{2}+a\right)^{1/3}}{2\,x^{2}}+\frac{2\,a^{1/3}\,b\,\ln\left(6\,a^{4/3}\,b-6\,a\,b\,\left(b\,x^{2}+a\right)^{1/3}\right)}{3}+\frac{a^{1/3}\,b\,\ln\left(6\,a\,b\,\left(b\,x^{2}+a\right)^{1/3}-3\,a^{4/3}\,b\,\left(\right.\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/x^3,x)
```

```
[Out] (3*b*(a + b*x^2)^(1/3))/2 - (a*(a + b*x^2)^(1/3))/(2*x^2) + (2*a^(1/3)*b*lo
g(6*a^(4/3)*b - 6*a*b*(a + b*x^2)^(1/3)))/3 + (a^(1/3)*b*log(6*a*b*(a + b*x
^2)^(1/3) - 3*a^(4/3)*b*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/3 - (a^(1/3)*b*
log(3*a^(4/3)*b*(3^(1/2)*1i + 1) + 6*a*b*(a + b*x^2)^(1/3))*(3^(1/2)*1i + 1
))/3
```

sympy [C] time = 1.41, size = 46, normalized size = 0.40

$$-\frac{b^{\frac{4}{3}}x^{\frac{2}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x**3,x)
```

```
[Out] -b**(4/3)*x**(2/3)*gamma(-1/3)*hyper((-4/3, -1/3), (2/3,), a*exp_polar(I*pi
)/(b*x**2))/(2*gamma(2/3))
```

3.695 $\int \frac{(a+bx^2)^{4/3}}{x^5} \, dx$

Optimal. Leaf size=132

$$\frac{b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{6a^{2/3}} - \frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}} - \frac{b^2 \log(x)}{9a^{2/3}} - \frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{(a+bx^2)^{4/3}}{4x^4}$$

```
[Out] -1/3*b*(b*x^2+a)^(1/3)/x^2-1/4*(b*x^2+a)^(4/3)/x^4-1/9*b^2*ln(x)/a^(2/3)+1/
6*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(2/3)-1/9*b^2*arctan(1/3*(a^(1/3)+2*(b*
x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a^(2/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 47, 57, 617, 204, 31}

$$\frac{b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{6a^{2/3}} - \frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}} - \frac{b^2 \log(x)}{9a^{2/3}} - \frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{(a+bx^2)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x^5,x]
```

```
[Out] -(b*(a + b*x^2)^(1/3))/(3*x^2) - (a + b*x^2)^(4/3)/(4*x^4) - (b^2*ArcTan[(a
^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) - (b^
2*Log[x])/(9*a^(2/3)) + (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{4/3}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{4/3}}{4x^4} + \frac{1}{3}b\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{\left(a+bx^2\right)^{4/3}}{4x^4} + \frac{1}{9}b^2\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{\left(a+bx^2\right)^{4/3}}{4x^4} - \frac{b^2\log(x)}{9a^{2/3}} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{6a^{2/3}} - \frac{b^2\operatorname{Subs}}{}\\
&= -\frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{\left(a+bx^2\right)^{4/3}}{4x^4} - \frac{b^2\log(x)}{9a^{2/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{2/3}} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\right.}{3a}\\
&= -\frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{\left(a+bx^2\right)^{4/3}}{4x^4} - \frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}} - \frac{b^2\log(x)}{9a^{2/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right.}{6a^{2/3}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.30

$$
-\frac{3b^2\left(a+bx^2\right)^{7/3}\,{}_2F_1\left(\frac{7}{3},3;\frac{10}{3};\frac{bx^2}{a}+1\right)}{14a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x^5,x]
```

```
[Out] (-3*b^2*(a + b*x^2)^(7/3)*Hypergeometric2F1[7/3, 3, 10/3, 1 + (b*x^2)/a])/(
14*a^3)
```

fricas [A] time = 0.87, size = 174, normalized size = 1.32

$$
-\frac{4\sqrt{3}\left(a^2\right)^{\frac{1}{6}}ab^2x^4\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right) + 2\left(a^2\right)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^2+\right.\right.}{36a^2x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="fricas")
```

[Out] -1/36*(4*sqrt(3)*(a^2)^(1/6)*a*b^2*x^4*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2
)^(1/3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + 2*(a^2)^(2/3)*b
^2*x^4*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2
/3)) - 4*(a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*
a^2*b*x^2 + 3*a^3)*(b*x^2 + a)^(1/3))/(a^2*x^4)

giac [A] time = 1.16, size = 139, normalized size = 1.05

$$-\frac{\frac{4\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{2b^3\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{4b^3\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{3\left(7\left(bx^2+a\right)^{\frac{4}{3}}b^3-4\left(bx^2+a\right)^{\frac{1}{3}}ab^3\right)}{b^2x^4}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="giac")

[Out] -1/36*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(2/3) + 2*b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3))/a^(2/3) - 4*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3) + 3*(
7*(b*x^2 + a)^(4/3)*b^3 - 4*(b*x^2 + a)^(1/3)*a*b^3)/(b^2*x^4))/b

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{4}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(4/3)/x^5,x)

[Out] int((b*x^2+a)^(4/3)/x^5,x)

maxima [A] time = 3.08, size = 152, normalized size = 1.15

$$-\frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{2}{3}}}-\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{2}{3}}}+\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}-\frac{7\left(bx^2\right.}{12\left(\left(bx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="maxima")

[Out] -1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(2/3) - 1/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(2/3) + 1/9*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(2/3) - 1/12*(7
*(b*x^2 + a)^(4/3)*b^2 - 4*(b*x^2 + a)^(1/3)*a*b^2)/((b*x^2 + a)^2 - 2*(b*x
^2 + a)*a + a^2)

mupad [B] time = 5.49, size = 191, normalized size = 1.45

$$\frac{b^2\ln\left(b^2\left(bx^2+a\right)^{1/3}-a^{1/3}b^2\right)}{9a^{2/3}}-\frac{\ln\left(\frac{a^{1/3}\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{2}+b^2\left(bx^2+a\right)^{1/3}\right)\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{18a^{2/3}}-\frac{\frac{7b^2\left(bx^2+a\right)^{4/3}}{6}-\frac{2ab^2\left(b\right.}{}}{2\left(bx^2+a\right)^2-4a\left(bx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/x^5,x)
```

```
[Out] (b^2*log(b^2*(a + b*x^2)^(1/3) - a^(1/3)*b^2))/(9*a^(2/3)) - (log((a^(1/3)*
(3^(1/2)*b^2*1i + b^2))/2 + b^2*(a + b*x^2)^(1/3))*(3^(1/2)*b^2*1i + b^2))/
(18*a^(2/3)) - ((7*b^2*(a + b*x^2)^(4/3))/6 - (2*a*b^2*(a + b*x^2)^(1/3))/3
)/(2*(a + b*x^2)^2 - 4*a*(a + b*x^2) + 2*a^2) + (b^2*log(b^2*(a + b*x^2)^(1
/3) - a^(1/3)*b^2*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(2/3
))
```

sympy [C] time = 1.54, size = 42, normalized size = 0.32

$$-\frac{b^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x**5,x)
```

```
[Out] -b**(4/3)*gamma(2/3)*hyper((-4/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(4/3)*gamma(5/3))
```

3.696 $\int x^4 \left(a + bx^2\right)^{4/3} \, dx$

Optimal. Leaf size=335

$$-\frac{432\ 3^{3/4}\sqrt{2-\sqrt{3}}\ a^4 \left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\ \sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right), 4\sqrt{3} -}{21505b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] -432/21505*a^3*x*(b*x^2+a)^(1/3)/b^2+48/4301*a^2*x^3*(b*x^2+a)^(1/3)/b+24/3
91*a*x^5*(b*x^2+a)^(1/3)+3/23*x^5*(b*x^2+a)^(4/3)-432/21505*3^(3/4)*a^4*(a^
(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(
b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2
+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*
(1/2*6^(1/2)-1/2*2^(1/2))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^
2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 236, 219}

$$-\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2}-\frac{432\ 3^{3/4}\sqrt{2-\sqrt{3}}\ a^4\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\ \sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right.\right.}{21505b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(4/3),x]
```

```
[Out] (-432*a^3*x*(a + b*x^2)^(1/3))/(21505*b^2) + (48*a^2*x^3*(a + b*x^2)^(1/3))
/(4301*b) + (24*a*x^5*(a + b*x^2)^(1/3))/391 + (3*x^5*(a + b*x^2)^(4/3))/23
 - (432*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^4*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(21505*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int x^4\left(a+bx^2\right)^{4/3}\,dx &= \frac{3}{23}x^5\left(a+bx^2\right)^{4/3}+\frac{1}{23}(8a)\int x^4\sqrt[3]{a+bx^2}\,dx\\
&= \frac{24}{391}ax^5\sqrt[3]{a+bx^2}+\frac{3}{23}x^5\left(a+bx^2\right)^{4/3}+\frac{1}{391}\left(16a^2\right)\int\frac{x^4}{\left(a+bx^2\right)^{2/3}}\,dx\\
&= \frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b}+\frac{24}{391}ax^5\sqrt[3]{a+bx^2}+\frac{3}{23}x^5\left(a+bx^2\right)^{4/3}-\frac{\left(144a^3\right)\int\frac{x^2}{\left(a+bx^2\right)^{2/3}}\,dx}{4301b}\\
&= -\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2}+\frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b}+\frac{24}{391}ax^5\sqrt[3]{a+bx^2}+\frac{3}{23}x^5\left(a+bx^2\right)^{4/3}+\frac{(43\cdots}{\cdots}\\
&= -\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2}+\frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b}+\frac{24}{391}ax^5\sqrt[3]{a+bx^2}+\frac{3}{23}x^5\left(a+bx^2\right)^{4/3}+\frac{(64\cdots}{\cdots}\\
&= -\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2}+\frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b}+\frac{24}{391}ax^5\sqrt[3]{a+bx^2}+\frac{3}{23}x^5\left(a+bx^2\right)^{4/3}-\frac{432\cdots}{\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.08, size = 79, normalized size = 0.24

$$
\frac{3x\sqrt[3]{a+bx^2}\left(\frac{9a^3\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{\frac{bx^2}{a}+1}}-\left(9a-17bx^2\right)\left(a+bx^2\right)^2\right)}{391b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*(-((9*a - 17*b*x^2)*(a + b*x^2)^2) + (9*a^3*Hypergeo
metric2F1[-4/3, 1/2, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/3)))/(391*b^2)
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\left(bx^6+ax^4\right)\left(bx^2+a\right)^{\frac{1}{3}},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] integral((b*x^6 + a*x^4)*(b*x^2 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{4}{3}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)*x^4, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{4}{3}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(4/3),x)
[Out] int(x^4*(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{4}{3}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\left(b\,x^2+a\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(4/3),x)
[Out] int(x^4*(a + b*x^2)^(4/3), x)
```

sympy [A] time = 1.27, size = 29, normalized size = 0.09

$$\frac{a^{\frac{4}{3}}x^5{}_2F_1\left(\left.\begin{matrix}-\frac{4}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(4/3),x)
[Out] a**(4/3)*x**5*hyper((-4/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

3.697 $\int x^2 \left(a+bx^2\right)^{4/3} dx$

Optimal. Leaf size=311

$$\frac{48\, 3^{3/4}\sqrt{2-\sqrt{3}}\, a^3 \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-\right.}{935b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] 48/935*a^2*x*(b*x^2+a)^(1/3)/b+24/187*a*x^3*(b*x^2+a)^(1/3)+3/17*x^3*(b*x^2
+a)^(4/3)+48/935*3^(3/4)*a^3*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a
)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3
^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3
)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^2/x/(-a^(1/3)*(
a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 236, 219}

$$\frac{48\, 3^{3/4}\sqrt{2-\sqrt{3}}\, a^3 \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\, F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|-7+4\sqrt{3}\right)}{935b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{48}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(4/3),x]
```

```
[Out] (48*a^2*x*(a + b*x^2)^(1/3))/(935*b) + (24*a*x^3*(a + b*x^2)^(1/3))/187 + (
3*x^3*(a + b*x^2)^(4/3))/17 + (48*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(a^(1/3) -
(a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^
(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1
+ Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))], -7 + 4*Sqrt[3]])/(935*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^
2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1 + n*p)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+bx^2\right)^{4/3}dx &= \frac{3}{17}x^3\left(a+bx^2\right)^{4/3}+\frac{1}{17}(8a)\int x^2\sqrt[3]{a+bx^2}\,dx\\
&= \frac{24}{187}ax^3\sqrt[3]{a+bx^2}+\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}+\frac{1}{187}\left(16a^2\right)\int\frac{x^2}{\left(a+bx^2\right)^{2/3}}dx\\
&= \frac{48a^2x\sqrt[3]{a+bx^2}}{935b}+\frac{24}{187}ax^3\sqrt[3]{a+bx^2}+\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}-\frac{\left(48a^3\right)\int\frac{1}{\left(a+bx^2\right)^{2/3}}dx}{935b}\\
&= \frac{48a^2x\sqrt[3]{a+bx^2}}{935b}+\frac{24}{187}ax^3\sqrt[3]{a+bx^2}+\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}-\frac{\left(72a^3\sqrt{bx^2}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{-a+x^3}}\right.}{935b^2x}\\
&= \frac{48a^2x\sqrt[3]{a+bx^2}}{935b}+\frac{24}{187}ax^3\sqrt[3]{a+bx^2}+\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}+\frac{48\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\left(\sqrt[3]{a}-\sqrt[3]{a}\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 67, normalized size = 0.22

$$\frac{3x\sqrt[3]{a+bx^2}\left(\left(a+bx^2\right)^2-\frac{a^2\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{\frac{bx^2}{a}+1}}\right)}{17b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*((a + b*x^2)^2 - (a^2*Hypergeometric2F1[-4/3, 1/2, 3
/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/3)))/(17*b)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^4+ax^2\right)\left(bx^2+a\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a*x^2)*(b*x^2 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{4}{3}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)*x^2, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(b\,x^2 + a\right)^{\frac{4}{3}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(4/3),x)
[Out] int(x^2*(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{4}{3}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(b\,x^2 + a\right)^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(4/3),x)
[Out] int(x^2*(a + b*x^2)^(4/3), x)
```

sympy [A] time = 1.14, size = 29, normalized size = 0.09

$$\frac{a^{\frac{4}{3}} x^3 {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(4/3),x)
[Out] a**(4/3)*x**3*hyper((-4/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

3.698 $\int \left(a+bx^2\right)^{4/3} dx$

Optimal. Leaf size=285

$$-\frac{16\, 3^{3/4}\sqrt{2-\sqrt{3}}\, a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{55bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] 24/55*a*x*(b*x^2+a)^(1/3)+3/11*x*(b*x^2+a)^(4/3)-16/55*3^(3/4)*a^2*(a^(1/3)
-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(
1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*
6^(1/2)-1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1
/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {195, 236, 219}

$$-\frac{16\, 3^{3/4}\sqrt{2-\sqrt{3}}\, a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{55bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{24}{55}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3),x]
```

```
[Out] (24*a*x*(a + b*x^2)^(1/3))/55 + (3*x*(a + b*x^2)^(4/3))/11 - (16*3^(3/4)*Sq
rt[2 - Sqrt[3]]*a^2*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(
a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1
- Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(55*b*x*Sqrt[-((
a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))^2)])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^2\right)^{4/3} dx &= \frac{3}{11}x\left(a+bx^2\right)^{4/3}+\frac{1}{11}(8a)\int \sqrt[3]{a+bx^2}\,dx\\
&= \frac{24}{55}ax\sqrt[3]{a+bx^2}+\frac{3}{11}x\left(a+bx^2\right)^{4/3}+\frac{1}{55}\left(16a^2\right)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx\\
&= \frac{24}{55}ax\sqrt[3]{a+bx^2}+\frac{3}{11}x\left(a+bx^2\right)^{4/3}+\frac{\left(24a^2\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{55bx}\\
&= \frac{24}{55}ax\sqrt[3]{a+bx^2}+\frac{3}{11}x\left(a+bx^2\right)^{4/3}-\frac{16\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+b}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}}}{55bx\sqrt{-\frac{\sqrt[3]{a}}{\left(\left(1-\right.\right.}}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.16

$$\frac{ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3),x]
```

```
[Out] (a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/2, 3/2, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^(1/3)
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{4}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3),x)
```

```
[Out] int((b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3), x)
```

mupad [B] time = 5.16, size = 37, normalized size = 0.13

$$\frac{x\left(b\,x^2+a\right)^{4/3}\,{}_2\mathrm{F}_1\left(-\frac{4}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3),x)
```

```
[Out] (x*(a + b*x^2)^(4/3)*hypergeom([-4/3, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(4/3)
```

sympy [A] time = 0.99, size = 26, normalized size = 0.09

$$a^{\frac{4}{3}} x {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*x*hyper((-4/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

3.699 $\int \frac{(a+bx^2)^{4/3}}{x^2}\,dx$

Optimal. Leaf size=280

$$-\frac{16\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{5\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-$$

```
[Out] 8/5*b*x*(b*x^2+a)^(1/3)-(b*x^2+a)^(4/3)/x-16/15*a*(a^(1/3)-(b*x^2+a)^(1/3))
*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)
*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2
/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/
2))*3^(3/4)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)
*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 236, 219}

$$-\frac{16\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x^2,x]
```

```
[Out] (8*b*x*(a + b*x^2)^(1/3))/5 - (a + b*x^2)^(4/3)/x - (16*Sqrt[2 - Sqrt[3]]*a
*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) +
(a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*3^(1/4)*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{4/3}}{x^2}\,dx &= -\frac{\left(a+bx^2\right)^{4/3}}{x}+\frac{1}{3}(8b)\int \sqrt[3]{a+bx^2}\,dx\\
&= \frac{8}{5}bx\sqrt[3]{a+bx^2}-\frac{\left(a+bx^2\right)^{4/3}}{x}+\frac{1}{15}(16ab)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx\\
&= \frac{8}{5}bx\sqrt[3]{a+bx^2}-\frac{\left(a+bx^2\right)^{4/3}}{x}+\frac{\left(8a\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{5x}\\
&= \frac{8}{5}bx\sqrt[3]{a+bx^2}-\frac{\left(a+bx^2\right)^{4/3}}{x}-\frac{16\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\ldots}{5\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\ldots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+\ldots}\right.}}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.18

$$
-\frac{a\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{4}{3},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[3]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x^2,x]
```

```
[Out] -((a*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/2, 1/2, -((b*x^2)/a)])/(x
*(1 + (b*x^2)/a)^(1/3)))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{4}{3}}}{x^2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/x^2, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x^2,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{4}{3}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/x^2, x)
```

mupad [B] time = 5.61, size = 40, normalized size = 0.14

$$\frac{3\left(b\,x^2+a\right)^{4/3}\,{}_2\mathrm{F}_1\left(-\frac{4}{3},-\frac{5}{6};\ \frac{1}{6};\ -\frac{a}{b\,x^2}\right)}{5\,x\left(\frac{a}{b\,x^2}+1\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/x^2,x)
```

```
[Out] (3*(a + b*x^2)^(4/3)*hypergeom([-4/3, -5/6], 1/6, -a/(b*x^2)))/(5*x*(a/(b*x
^2) + 1)^(4/3))
```

sympy [A] time = 1.09, size = 29, normalized size = 0.10

$$-\frac{a^{\frac{4}{3}}\,{}_2F_1\left(\left.\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\right| \frac{bx^{2}e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x**2,x)
```

```
[Out] -a**(4/3)*hyper((-4/3, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

3.700 $\int \frac{(a+bx^2)^{4/3}}{x^4}\,dx$

Optimal. Leaf size=284

$$-\frac{16\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\;\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-8b$$

```
[Out] -8/9*b*(b*x^2+a)^(1/3)/x-1/3*(b*x^2+a)^(4/3)/x^3-16/27*b*(a^(1/3)-(b*x^2+a)
^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+
a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^
2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/
2*2^(1/2))*3^(3/4)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+
a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {277, 236, 219}

$$-\frac{16\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\;F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{\left(a+bx^2\right.}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x^4,x]
```

```
[Out] (-8*b*(a + b*x^2)^(1/3))/(9*x) - (a + b*x^2)^(4/3)/(3*x^3) - (16*Sqrt[2 - S
qrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)
^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*
EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*x*Sqrt[-((a^(1/
3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{4/3}}{x^4}\,dx &= -\frac{\left(a+bx^2\right)^{4/3}}{3x^3}+\frac{1}{9}(8b)\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx\\
&= -\frac{8b\sqrt[3]{a+bx^2}}{9x}-\frac{\left(a+bx^2\right)^{4/3}}{3x^3}+\frac{1}{27}\left(16b^2\right)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx\\
&= -\frac{8b\sqrt[3]{a+bx^2}}{9x}-\frac{\left(a+bx^2\right)^{4/3}}{3x^3}+\frac{\left(8b\sqrt{bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{9x}\\
&= -\frac{8b\sqrt[3]{a+bx^2}}{9x}-\frac{\left(a+bx^2\right)^{4/3}}{3x^3}-\frac{16\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^{2}}}}{9\sqrt[4]{3}\,x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{\right.}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\right.}}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.18

$$-\frac{a\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x^4,x]
```

```
[Out] -1/3*(a*(a + b*x^2)^(1/3)*Hypergeometric2F1[-3/2, -4/3, -1/2, -((b*x^2)/a)]
)/(x^3*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(bx^2+a\right)^{\frac{4}{3}}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/x^4, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x^4,x)
[Out] int((b*x^2+a)^(4/3)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{4}{3}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^4,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{4/3}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/x^4,x)
[Out] int((a + b*x^2)^(4/3)/x^4, x)
```

sympy [A] time = 1.09, size = 34, normalized size = 0.12

$$-\frac{a^{\frac{4}{3}}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},-\frac{4}{3}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x**4,x)
[Out] -a**(4/3)*hyper((-3/2, -4/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

3.701 $\int x\left(-1+x^2\right)^{7/3}\,dx$

Optimal. Leaf size=13

$$\frac{3}{20}\left(x^2-1\right)^{10/3}$$

```
[Out] 3/20*(x^2-1)^(10/3)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{3}{20}\left(x^2-1\right)^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(-1 + x^2)^(7/3),x]
[Out] (3*(-1 + x^2)^(10/3))/20
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(-1+x^2\right)^{7/3}\,dx=\frac{3}{20}\left(-1+x^2\right)^{10/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{20}\left(x^2-1\right)^{10/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-1 + x^2)^(7/3),x]
[Out] (3*(-1 + x^2)^(10/3))/20
```

fricas [B] time = 0.96, size = 24, normalized size = 1.85

$$\frac{3}{20}\left(x^6-3\,x^4+3\,x^2-1\right)\left(x^2-1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(7/3),x, algorithm="fricas")
[Out] 3/20*(x^6 - 3*x^4 + 3*x^2 - 1)*(x^2 - 1)^(1/3)
```

giac [A] time = 0.56, size = 9, normalized size = 0.69

$$\frac{3}{20}\left(x^2-1\right)^{\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(7/3),x, algorithm="giac")
[Out] 3/20*(x^2 - 1)^(10/3)
```

maple [A] time = 0.00, size = 16, normalized size = 1.23

$$\frac{3(x+1)(x-1)\left(x^2-1\right)^{\frac{7}{3}}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2-1)^(7/3),x)
[Out] 3/20*(x+1)*(x-1)*(x^2-1)^(7/3)
```

maxima [A] time = 1.32, size = 9, normalized size = 0.69

$$\frac{3}{20}\left(x^2-1\right)^{\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(7/3),x, algorithm="maxima")
[Out] 3/20*(x^2 - 1)^(10/3)
```

mupad [B] time = 5.13, size = 25, normalized size = 1.92

$$\left(x^2-1\right)^{1/3}\left(\frac{3x^6}{20}-\frac{9x^4}{20}+\frac{9x^2}{20}-\frac{3}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 - 1)^(7/3),x)
[Out] (x^2 - 1)^(1/3)*((9*x^2)/20 - (9*x^4)/20 + (3*x^6)/20 - 3/20)
```

sympy [B] time = 2.91, size = 56, normalized size = 4.31

$$\frac{3x^6\sqrt[3]{x^2-1}}{20}-\frac{9x^4\sqrt[3]{x^2-1}}{20}+\frac{9x^2\sqrt[3]{x^2-1}}{20}-\frac{3\sqrt[3]{x^2-1}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2-1)**(7/3),x)
[Out] 3*x**6*(x**2 - 1)**(1/3)/20 - 9*x**4*(x**2 - 1)**(1/3)/20 + 9*x**2*(x**2 -
1)**(1/3)/20 - 3*(x**2 - 1)**(1/3)/20
```

3.702 $\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{3a^3\left(a+bx^2\right)^{2/3}}{4b^4}+\frac{9a^2\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^4}-\frac{9a\left(a+bx^2\right)^{8/3}}{16b^4}$$

```
[Out] -3/4*a^3*(b*x^2+a)^(2/3)/b^4+9/10*a^2*(b*x^2+a)^(5/3)/b^4-9/16*a*(b*x^2+a)^
(8/3)/b^4+3/22*(b*x^2+a)^(11/3)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{5/3}}{10b^4}-\frac{3a^3\left(a+bx^2\right)^{2/3}}{4b^4}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^4}-\frac{9a\left(a+bx^2\right)^{8/3}}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(2/3))/(4*b^4) + (9*a^2*(a + b*x^2)^(5/3))/(10*b^4) - (
9*a*(a + b*x^2)^(8/3))/(16*b^4) + (3*(a + b*x^2)^(11/3))/(22*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^3\sqrt[3]{a+bx}}+\frac{3a^2(a+bx)^{2/3}}{b^3}-\frac{3a(a+bx)^{5/3}}{b^3}+\frac{(a+bx)^{8/3}}{b^3}\right)dx, x, x^2\right)\\
&= -\frac{3a^3\left(a+bx^2\right)^{2/3}}{4b^4}+\frac{9a^2\left(a+bx^2\right)^{5/3}}{10b^4}-\frac{9a\left(a+bx^2\right)^{8/3}}{16b^4}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{3\left(a+bx^2\right)^{2/3}\left(-81a^3+54a^2bx^2-45ab^2x^4+40b^3x^6\right)}{880b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(2/3)*(-81*a^3 + 54*a^2*b*x^2 - 45*a*b^2*x^4 + 40*b^3*x^6))/
(880*b^4)
```

fricas [A] time = 0.84, size = 46, normalized size = 0.58

$$\frac{3\left(40\,b^{3}x^{6}-45\,ab^{2}x^{4}+54\,a^{2}bx^{2}-81\,a^{3}\right)\left(bx^{2}+a\right)^{\frac{2}{3}}}{880\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/880*(40*b^3*x^6 - 45*a*b^2*x^4 + 54*a^2*b*x^2 - 81*a^3)*(b*x^2 + a)^(2/3)
/b^4
```

giac [A] time = 0.58, size = 61, normalized size = 0.76

$$-\frac{3\left(bx^{2}+a\right)^{\frac{2}{3}}a^{3}}{4\,b^{4}}+\frac{3\left(40\left(bx^{2}+a\right)^{\frac{11}{3}}-165\left(bx^{2}+a\right)^{\frac{8}{3}}a+264\left(bx^{2}+a\right)^{\frac{5}{3}}a^{2}\right)}{880\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] -3/4*(b*x^2 + a)^(2/3)*a^3/b^4 + 3/880*(40*(b*x^2 + a)^(11/3) - 165*(b*x^2
+ a)^(8/3)*a + 264*(b*x^2 + a)^(5/3)*a^2)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{2}{3}}\left(-40b^{3}x^{6}+45a\,b^{2}x^{4}-54a^{2}b\,x^{2}+81a^{3}\right)}{880b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(1/3),x)
```

```
[Out] -3/880*(b*x^2+a)^(2/3)*(-40*b^3*x^6+45*a*b^2*x^4-54*a^2*b*x^2+81*a^3)/b^4
```

maxima [A] time = 1.36, size = 64, normalized size = 0.80

$$\frac{3\left(bx^{2}+a\right)^{\frac{11}{3}}}{22\,b^{4}}-\frac{9\left(bx^{2}+a\right)^{\frac{8}{3}}a}{16\,b^{4}}+\frac{9\left(bx^{2}+a\right)^{\frac{5}{3}}a^{2}}{10\,b^{4}}-\frac{3\left(bx^{2}+a\right)^{\frac{2}{3}}a^{3}}{4\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/22*(b*x^2 + a)^(11/3)/b^4 - 9/16*(b*x^2 + a)^(8/3)*a/b^4 + 9/10*(b*x^2 +
a)^(5/3)*a^2/b^4 - 3/4*(b*x^2 + a)^(2/3)*a^3/b^4
```

mupad [B] time = 5.32, size = 48, normalized size = 0.60

$$-\left(b\,x^{2}+a\right)^{2/3}\left(\frac{243\,a^{3}}{880\,b^{4}}-\frac{3\,x^{6}}{22\,b}+\frac{27\,a\,x^{4}}{176\,b^{2}}-\frac{81\,a^{2}\,x^{2}}{440\,b^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^(1/3),x)
```

```
[Out] -(a + b*x^2)^(2/3)*((243*a^3)/(880*b^4) - (3*x^6)/(22*b) + (27*a*x^4)/(176*
b^2) - (81*a^2*x^2)/(440*b^3))
```

sympy [B] time = 2.77, size = 1690, normalized size = 21.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(1/3),x)
```

```
[Out] -243*a**(71/3)*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2
 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 +
5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 243*a**(71/3)/(880*a**20*b
**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6
+ 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) -
1296*a**(68/3)*b*x**2*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b*
*5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*
x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 1458*a**(68/3)*b*x*
*2/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a
**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*
b**10*x**12) - 2808*a**(65/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(880*a**20*b*
*4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 +
 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 3
645*a**(65/3)*b**2*x**4/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**1
8*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b*
*9*x**10 + 880*a**14*b**10*x**12) - 3120*a**(62/3)*b**3*x**6*(1 + b*x**2/a)
**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17
600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a
**14*b**10*x**12) + 4860*a**(62/3)*b**3*x**6/(880*a**20*b**4 + 5280*a**19*b
**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8
*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) - 1710*a**(59/3)*b**
4*x**8*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200
*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**
15*b**9*x**10 + 880*a**14*b**10*x**12) + 3645*a**(59/3)*b**4*x**8/(880*a**2
0*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x*
*6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12)
 + 72*a**(56/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**
19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*
b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 1458*a**(56/3)
*b**5*x**10/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4
+ 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 8
80*a**14*b**10*x**12) + 1104*a**(53/3)*b**6*x**12*(1 + b*x**2/a)**(2/3)/(88
0*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b
**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*
x**12) + 243*a**(53/3)*b**6*x**12/(880*a**20*b**4 + 5280*a**19*b**5*x**2 +
13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 528
0*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 1152*a**(50/3)*b**7*x**14*(1
+ b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**
6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x*
*10 + 880*a**14*b**10*x**12) + 585*a**(47/3)*b**8*x**16*(1 + b*x**2/a)**(2/
3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a
**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*
b**10*x**12) + 120*a**(44/3)*b**9*x**18*(1 + b*x**2/a)**(2/3)/(880*a**20*b*
*4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 +
 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12)
```

3.703 $\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=59

$$\frac{3a^2\left(a+bx^2\right)^{2/3}}{4b^3}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^3}-\frac{3a\left(a+bx^2\right)^{5/3}}{5b^3}$$

```
[Out] 3/4*a^2*(b*x^2+a)^(2/3)/b^3-3/5*a*(b*x^2+a)^(5/3)/b^3+3/16*(b*x^2+a)^(8/3)/
b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{2/3}}{4b^3}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^3}-\frac{3a\left(a+bx^2\right)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(2/3))/(4*b^3) - (3*a*(a + b*x^2)^(5/3))/(5*b^3) + (3*(a
 + b*x^2)^(8/3))/(16*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt[3]{a+bx}}-\frac{2a(a+bx)^{2/3}}{b^2}+\frac{(a+bx)^{5/3}}{b^2}\right)dx,x,x^2\right)\\
&= \frac{3a^2\left(a+bx^2\right)^{2/3}}{4b^3}-\frac{3a\left(a+bx^2\right)^{5/3}}{5b^3}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{2/3}\left(9a^2-6abx^2+5b^2x^4\right)}{80b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(2/3)*(9*a^2 - 6*a*b*x^2 + 5*b^2*x^4))/(80*b^3)
```

fricas [A] time = 0.91, size = 35, normalized size = 0.59

$$\frac{3\left(5\,b^2x^4-6\,abx^2+9\,a^2\right)\left(bx^2+a\right)^{\frac{2}{3}}}{80\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] 3/80*(5*b^2*x^4 - 6*a*b*x^2 + 9*a^2)*(b*x^2 + a)^(2/3)/b^3
```

giac [A] time = 0.57, size = 47, normalized size = 0.80

$$\frac{3\left(bx^2+a\right)^{\frac{2}{3}}a^2}{4\,b^3}+\frac{3\left(5\left(bx^2+a\right)^{\frac{8}{3}}-16\left(bx^2+a\right)^{\frac{5}{3}}a\right)}{80\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] 3/4*(b*x^2 + a)^(2/3)*a^2/b^3 + 3/80*(5*(b*x^2 + a)^(8/3) - 16*(b*x^2 + a)^
(5/3)*a)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{3\left(b\,x^2+a\right)^{\frac{2}{3}}\left(5b^2x^4-6ab\,x^2+9a^2\right)}{80b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(1/3),x)
[Out] 3/80*(b*x^2+a)^(2/3)*(5*b^2*x^4-6*a*b*x^2+9*a^2)/b^3
```

maxima [A] time = 1.30, size = 47, normalized size = 0.80

$$\frac{3\left(bx^2+a\right)^{\frac{8}{3}}}{16\,b^3}-\frac{3\left(bx^2+a\right)^{\frac{5}{3}}a}{5\,b^3}+\frac{3\left(bx^2+a\right)^{\frac{2}{3}}a^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] 3/16*(b*x^2 + a)^(8/3)/b^3 - 3/5*(b*x^2 + a)^(5/3)*a/b^3 + 3/4*(b*x^2 + a)^
(2/3)*a^2/b^3
```

mupad [B] time = 5.21, size = 36, normalized size = 0.61

$$\left(b\,x^2+a\right)^{2/3}\,\left(\frac{27\,a^2}{80\,b^3}+\frac{3\,x^4}{16\,b}-\frac{9\,a\,x^2}{40\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(1/3),x)
[Out] (a + b*x^2)^(2/3)*((27*a^2)/(80*b^3) + (3*x^4)/(16*b) - (9*a*x^2)/(40*b^2))
```

sympy [B] time = 1.78, size = 631, normalized size = 10.69

$$\frac{27a^{\frac{32}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}-\frac{27a^{\frac{32}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}+\frac{63a^{\frac{29}{3}}}{80a^8b^3+240a^7b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(1/3),x)
```

```
[Out] 27*a**(32/3)*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240
*a**6*b**5*x**4 + 80*a**5*b**6*x**6) - 27*a**(32/3)/(80*a**8*b**3 + 240*a**
7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 63*a**(29/3)*b*x**2
*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x
**4 + 80*a**5*b**6*x**6) - 81*a**(29/3)*b*x**2/(80*a**8*b**3 + 240*a**7*b**
4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 42*a**(26/3)*b**2*x**4*(
1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**
4 + 80*a**5*b**6*x**6) - 81*a**(26/3)*b**2*x**4/(80*a**8*b**3 + 240*a**7*b*
*4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 18*a**(23/3)*b**3*x**6*
(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x*
*4 + 80*a**5*b**6*x**6) - 27*a**(23/3)*b**3*x**6/(80*a**8*b**3 + 240*a**7*b
**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 27*a**(20/3)*b**4*x**8
*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x
**4 + 80*a**5*b**6*x**6) + 15*a**(17/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(8
0*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6)
```

3.704 $\int \frac{x^3}{\sqrt[3]{a+bx^2}} dx$

Optimal. Leaf size=38

$$\frac{3\left(a+bx^2\right)^{5/3}}{10b^2}-\frac{3a\left(a+bx^2\right)^{2/3}}{4b^2}$$

```
[Out] -3/4*a*(b*x^2+a)^(2/3)/b^2+3/10*(b*x^2+a)^(5/3)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3\left(a+bx^2\right)^{5/3}}{10b^2}-\frac{3a\left(a+bx^2\right)^{2/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(2/3))/(4*b^2) + (3*(a + b*x^2)^(5/3))/(10*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt[3]{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b}\right)dx,x,x^2\right)\\
&= -\frac{3a\left(a+bx^2\right)^{2/3}}{4b^2}+\frac{3\left(a+bx^2\right)^{5/3}}{10b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{3\left(a+bx^2\right)^{2/3}\left(2bx^2-3a\right)}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(2/3)*(-3*a + 2*b*x^2))/(20*b^2)
```

fricas [A] time = 0.83, size = 24, normalized size = 0.63

$$\frac{3\left(2\,bx^2-3\,a\right)\left(bx^2+a\right)^{\frac{2}{3}}}{20\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] 3/20*(2*b*x^2 - 3*a)*(b*x^2 + a)^(2/3)/b^2
```

giac [A] time = 0.57, size = 30, normalized size = 0.79

$$\frac{3\left(bx^2+a\right)^{\frac{5}{3}}}{10\,b^2}-\frac{3\left(bx^2+a\right)^{\frac{2}{3}}a}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] 3/10*(b*x^2 + a)^(5/3)/b^2 - 3/4*(b*x^2 + a)^(2/3)*a/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{3\left(b\,x^2+a\right)^{\frac{2}{3}}\left(-2b\,x^2+3a\right)}{20b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/3),x)
[Out] -3/20*(b*x^2+a)^(2/3)*(-2*b*x^2+3*a)/b^2
```

maxima [A] time = 1.40, size = 30, normalized size = 0.79

$$\frac{3\left(bx^2+a\right)^{\frac{5}{3}}}{10\,b^2}-\frac{3\left(bx^2+a\right)^{\frac{2}{3}}a}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] 3/10*(b*x^2 + a)^(5/3)/b^2 - 3/4*(b*x^2 + a)^(2/3)*a/b^2
```

mupad [B] time = 5.01, size = 24, normalized size = 0.63

$$-\frac{3\left(b\,x^2+a\right)^{2/3}\left(3\,a-2\,b\,x^2\right)}{20\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(1/3),x)
[Out] -(3*(a + b*x^2)^(2/3)*(3*a - 2*b*x^2))/(20*b^2)
```

sympy [B] time = 1.14, size = 178, normalized size = 4.68

$$-\frac{9a^{\frac{11}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2}+\frac{9a^{\frac{11}{3}}}{20a^2b^2+20ab^3x^2}-\frac{3a^{\frac{8}{3}}bx^2\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2}+\frac{9a^{\frac{8}{3}}bx^2}{20a^2b^2+20ab^3x^2}+\frac{6a^{\frac{5}{3}}b^2x^4\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/3),x)
```

```
[Out] -9*a**(11/3)*(1 + b*x**2/a)**(2/3)/(20*a**2*b**2 + 20*a*b**3*x**2) + 9*a**(
11/3)/(20*a**2*b**2 + 20*a*b**3*x**2) - 3*a**(8/3)*b*x**2*(1 + b*x**2/a)**(
2/3)/(20*a**2*b**2 + 20*a*b**3*x**2) + 9*a**(8/3)*b*x**2/(20*a**2*b**2 + 20
*a*b**3*x**2) + 6*a**(5/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(20*a**2*b**2 +
20*a*b**3*x**2)
```

3.705 $\int \frac{x}{\sqrt[3]{a+bx^2}} \, dx$

Optimal. Leaf size=18

$$\frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

```
[Out] 3/4*(b*x^2+a)^(2/3)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(2/3))/(4*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{a+bx^2}} \, dx = \frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(2/3))/(4*b)
```

fricas [A] time = 0.92, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{2}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] 3/4*(b*x^2 + a)^(2/3)/b
```

giac [A] time = 0.57, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{2}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] 3/4*(b*x^2 + a)^(2/3)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{3\left(b\,x^{2}+a\right)^{\frac{2}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/3),x)
[Out] 3/4*(b*x^2+a)^(2/3)/b
```

maxima [A] time = 1.30, size = 14, normalized size = 0.78

$$\frac{3\left(bx^{2}+a\right)^{\frac{2}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] 3/4*(b*x^2 + a)^(2/3)/b
```

mupad [B] time = 4.67, size = 14, normalized size = 0.78

$$\frac{3\left(b\,x^{2}+a\right)^{2/3}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(1/3),x)
[Out] (3*(a + b*x^2)^(2/3))/(4*b)
```

sympy [A] time = 0.40, size = 24, normalized size = 1.33

$$\begin{cases} \frac{3\left(a+bx^{2}\right)^{\frac{2}{3}}}{4b} & \text{for}\: b \neq 0 \\ \frac{x^{2}}{2\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/3),x)
[Out] Piecewise((3*(a + b*x**2)**(2/3)/(4*b), Ne(b, 0)), (x**2/(2*a**(1/3)), True
))
```

3.706 $\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

```
[Out] -1/2*ln(x)/a^(1/3)+3/4*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(1/3)+1/2*arctan(1/3*(
a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)
```

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 55, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(2*a^(1
/3)) - Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(4*a^(1/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)$$
$$= -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^2}\right) - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+\right.}{4\sqrt[3]{a}}$$
$$= -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{a}}$$
$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt[3]{a}} - \frac{\log(x)}{2\sqrt[3]{a}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.81

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-2\log(x)}{4\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[x] +
 3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(4*a^(1/3))
```

fricas [A] time = 0.83, size = 235, normalized size = 2.73

$$\left[\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^2+\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^2}\right)-a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a\right.}{4\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*a*sqrt(-1/a^(2/3))*log((2*b*x^2 + sqrt(3)*(2*(b*x^2 + a)^(2/3
)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^2 + a)
^(1/3)*a^(2/3) + 3*a)/x^2) - a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1
/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)))/a, 1/4
*(2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3)) - a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))
 + 2*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)))/a]
```

giac [A] time = 1.08, size = 87, normalized size = 1.01

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{1}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4a^{\frac{1}{3}}}+\frac{\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
1/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
1/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(1/3)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/x/(b*x^2+a)^(1/3),x)
```

maxima [A] time = 2.98, size = 86, normalized size = 1.00

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{1}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4a^{\frac{1}{3}}}+\frac{\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
1/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
1/3) + 1/2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(1/3)
```

mupad [B] time = 4.83, size = 106, normalized size = 1.23

$$\frac{\ln\left(\frac{9\left(b\,x^2+a\right)^{1/3}}{4}-\frac{9\,a^{1/3}}{4}\right)}{2\,a^{1/3}}+\frac{\ln\left(\frac{9\left(b\,x^2+a\right)^{1/3}}{4}-\frac{9\,a^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4\,a^{1/3}}-\frac{\ln\left(\frac{9\left(b\,x^2+a\right)^{1/3}}{4}-\frac{9\,a^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\right.}{4\,a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(1/3)),x)
```

```
[Out] log((9*(a + b*x^2)^(1/3))/4 - (9*a^(1/3))/4)/(2*a^(1/3)) + (log((9*(a + b*x
^2)^(1/3))/4 - (9*a^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/(4*a^(1
/3)) - (log((9*(a + b*x^2)^(1/3))/4 - (9*a^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3
^(1/2)*1i + 1))/(4*a^(1/3))
```

sympy [C] time = 1.02, size = 41, normalized size = 0.48

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^{2}}\right)}{2\sqrt[3]{b}\,x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(1/3),x)
```

```
[Out] -gamma(1/3)*hyper((1/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1/
3)*x**(2/3)*gamma(4/3))
```

3.707 $\int \frac{1}{x^3 \sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=110

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{\left(a+bx^2\right)^{2/3}}{2ax^2}$$

```
[Out] -1/2*(b*x^2+a)^(2/3)/a/x^2+1/6*b*ln(x)/a^(4/3)-1/4*b*ln(a^(1/3)-(b*x^2+a)^(
1/3))/a^(4/3)-1/6*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))
/a^(4/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 55, 617, 204, 31}

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{\left(a+bx^2\right)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(1/3)),x]
```

```
[Out] -(a + b*x^2)^(2/3)/(2*a*x^2) - (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(S
qrt[3]*a^(1/3))])/(2*Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(
1/3) - (a + b*x^2)^(1/3)])/(4*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt[3]{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2ax^2}-\frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^2\right)}{6a}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2ax^2}+\frac{b\log(x)}{6a^{4/3}}+\frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}-\frac{b\operatorname{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,d\right.}{4a}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2ax^2}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}+\frac{b\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right.}{2a^{4/3}}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.34

$$
\frac{3b\left(a+bx^2\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{bx^2}{a}+1\right)}{4a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(1/3)),x]
```

```
[Out] (3*b*(a + b*x^2)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, 1 + (b*x^2)/a])/(4*a^
2)
```

fricas [A] time = 0.91, size = 344, normalized size = 3.13

$$
\left[\frac{3\sqrt{\frac{1}{3}}\,abx^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2bx^2-3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\left(bx^2+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x^2}\right)+(-a)^{\frac{2}{3}}bx^2\log\left(\left(b\right.\right.}{12a^2x^2}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(1/3)*a*b*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x^2 - 3*sqrt(1/3)*(2
*(b*x^2 + a)^(2/3)*(-a)^(2/3) - (b*x^2 + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-
a)^(1/3)/a) - 3*(b*x^2 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^2) + (-a)^(2/3)*b*x^2
*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a
)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3) + (-a)^(1/3)) - 6*(b*x^2 + a)^(2/3)*a)/
(a^2*x^2), -1/12*(6*sqrt(1/3)*a*b*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*
(2*(b*x^2 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b*x^2*
log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-a)
^(2/3)*b*x^2*log((b*x^2 + a)^(1/3) + (-a)^(1/3)) + 6*(b*x^2 + a)^(2/3)*a)/(
a^2*x^2)]
```

giac [A] time = 1.14, size = 119, normalized size = 1.08

$$-\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2b^2\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{6\left(bx^2+a\right)^{\frac{2}{3}}b}{ax^2}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] -1/12*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(4/3) - b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(4/3) + 2*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 6*(b*
x^2 + a)^(2/3)*b/(a*x^2))/b
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/x^3/(b*x^2+a)^(1/3),x)
```

maxima [A] time = 2.95, size = 118, normalized size = 1.07

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{6a^{\frac{4}{3}}}-\frac{\left(bx^2+a\right)^{\frac{2}{3}}b}{2\left(\left(bx^2+a\right)a-a^2\right)}+\frac{b\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{12a^{\frac{4}{3}}}-\frac{b\log\left(\left(bx^2+a\right.\right.}{6a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(4/3) - 1/2*(b*x^2 + a)^(2/3)*b/((b*x^2 + a)*a - a^2) + 1/12*b*log((b*x^2
 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/6*b*log((b*x
^2 + a)^(1/3) - a^(1/3))/a^(4/3)
```

mupad [B] time = 5.01, size = 138, normalized size = 1.25

$$-\frac{b\,\ln\left(\left(b\,x^2+a\right)^{1/3}-a^{1/3}\right)}{6\,a^{4/3}}-\frac{\left(b\,x^2+a\right)^{2/3}}{2\,a\,x^2}+\frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{16\,a^{5/3}}-\frac{b^2\,\left(b\,x^2+a\right)^{1/3}}{4\,a^2}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{12\,a^{4/3}}+\frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{16\,a^{5/3}}-\frac{b^2\,\left(b\,x^2+a\right.}{4\,a^2}\right.}{12\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(1/3)),x)
```

```
[Out] (log((b - 3^(1/2)*b*1i)^2/(16*a^(5/3)) - (b^2*(a + b*x^2)^(1/3))/(4*a^2))*(
b - 3^(1/2)*b*1i))/(12*a^(4/3)) - (a + b*x^2)^(2/3)/(2*a*x^2) - (b*log((a +
 b*x^2)^(1/3) - a^(1/3)))/(6*a^(4/3)) + (log((b + 3^(1/2)*b*1i)^2/(16*a^(5/
3)) - (b^2*(a + b*x^2)^(1/3))/(4*a^2))*(b + 3^(1/2)*b*1i))/(12*a^(4/3))
```

sympy [C] time = 1.19, size = 41, normalized size = 0.37

$$-\frac{\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\sqrt[3]{b}\,x^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/3),x)
```

```
[Out] -gamma(4/3)*hyper((1/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1/
3)*x**(8/3)*gamma(7/3))
```

3.708 $\int \frac{1}{x^5 \sqrt[3]{a+bx^2}} \, dx$

Optimal. Leaf size=138

$$\frac{b^2 \log \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{7/3}}+\frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{b^2 \log(x)}{9a^{7/3}}+\frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2}-\frac{\left(a+bx^2\right)^{2/3}}{4ax^4}$$

```
[Out] -1/4*(b*x^2+a)^(2/3)/a/x^4+1/3*b*(b*x^2+a)^(2/3)/a^2/x^2-1/9*b^2*ln(x)/a^(7
/3)+1/6*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(7/3)+1/9*b^2*arctan(1/3*(a^(1/3)
+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 55, 617, 204, 31}

$$\frac{b^2 \log \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{7/3}}+\frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{b^2 \log(x)}{9a^{7/3}}+\frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2}-\frac{\left(a+bx^2\right)^{2/3}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^(1/3)),x]
```

```
[Out] -(a + b*x^2)^(2/3)/(4*a*x^4) + (b*(a + b*x^2)^(2/3))/(3*a^2*x^2) + (b^2*Arc
Tan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3))
 - (b^2*Log[x])/(9*a^(7/3)) + (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a^(
7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt[3]{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} - \frac{b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{3a}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} + \frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{9a^2}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} + \frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2} - \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{6a^{7/3}} + \frac{b^2\,\text{Su}}{}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} + \frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{7/3}} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{-3-}\right.}{}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} + \frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2} + \frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+\right.}{6a^{7/3}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.28

$$-\frac{3b^2\left(a+bx^2\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};\frac{bx^2}{a}+1\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^(1/3)),x]
```

```
[Out] (-3*b^2*(a + b*x^2)^(2/3)*Hypergeometric2F1[2/3, 3, 5/3, 1 + (b*x^2)/a])/(4
*a^3)
```

fricas [A] time = 0.80, size = 326, normalized size = 2.36

$$\left[\frac{6\sqrt{\frac{1}{3}}\,ab^2x^4\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\,\log\left(\frac{2\,bx^2+3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^2}\right)-2\,a^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}\right.}{36\,a^3x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/36*(6*sqrt(1/3)*a*b^2*x^4*sqrt(-1/a^(2/3))*log((2*b*x^2 + 3*sqrt(1/3)*(2
*(b*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)
) - 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - 2*a^(2/3)*b^2*x^4*log((b*x^2
+ a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^4*log((
b*x^2 + a)^(1/3) - a^(1/3)) + 3*(4*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3))/(a^3
*x^4), 1/36*(12*sqrt(1/3)*a^(2/3)*b^2*x^4*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(
1/3) + a^(1/3))/a^(1/3)) - 2*a^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3) + (b*x^2
 + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3) -
a^(1/3)) + 3*(4*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3))/(a^3*x^4)]
```

giac [A] time = 1.13, size = 142, normalized size = 1.03

$$\frac{\frac{4\sqrt{3}\,b^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}}-\frac{2\,b^3\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}}+\frac{4\,b^3\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{7}{3}}}+\frac{3\left(4\left(bx^2+a\right)^{\frac{5}{3}}b^3-7\left(bx^2+a\right)^{\frac{2}{3}}ab^3\right)}{a^2b^2x^4}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/36*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(7/3) - 2*b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(7/3) + 4*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(7/3) + 3*(4
*(b*x^2 + a)^(5/3)*b^3 - 7*(b*x^2 + a)^(2/3)*a*b^3)/(a^2*b^2*x^4))/b
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{3}} x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/x^5/(b*x^2+a)^(1/3),x)
```

maxima [A] time = 2.97, size = 158, normalized size = 1.14

$$\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}}-\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{7}{3}}}+\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{7}{3}}}+\frac{4\,(bx^2}{12\,((bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(7/3) - 1/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(7/3) + 1/9*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(7/3) + 1/12*(4*
(b*x^2 + a)^(5/3)*b^2 - 7*(b*x^2 + a)^(2/3)*a*b^2)/((b*x^2 + a)^2*a^2 - 2*(
b*x^2 + a)*a^3 + a^4)
```

mupad [B] time = 5.06, size = 201, normalized size = 1.46

$$\frac{b^2\,\ln\left(\left(b\,x^2+a\right)^{1/3}-a^{1/3}\right)}{9\,a^{7/3}}-\frac{\ln\left(\frac{b^4\,\left(b\,x^2+a\right)^{1/3}}{9\,a^4}-\frac{\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)^2}{36\,a^{11/3}}\right)\,\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{18\,a^{7/3}}-\frac{\frac{7\,b^2\,\left(b\,x^2+a\right)^{2/3}}{6\,a}-\frac{2\,b^2\,\left(b\,x^2+a\right)^{5/3}}{3\,a^2}}{2\,\left(b\,x^2+a\right)^2-4\,a\,\left(b\,x^2+a\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^(1/3)),x)
```

```
[Out] (b^2*log((a + b*x^2)^(1/3) - a^(1/3)))/(9*a^(7/3)) - (log((b^4*(a + b*x^2)^
(1/3))/(9*a^4) - (3^(1/2)*b^2*1i + b^2)^2/(36*a^(11/3)))*(3^(1/2)*b^2*1i +
b^2))/(18*a^(7/3)) - ((7*b^2*(a + b*x^2)^(2/3))/(6*a) - (2*b^2*(a + b*x^2)^
(5/3))/(3*a^2))/(2*(a + b*x^2)^2 - 4*a*(a + b*x^2) + 2*a^2) + (b^2*log((b^4
*(a + b*x^2)^(1/3))/(9*a^4) - (b^4*((3^(1/2)*1i)/2 - 1/2)^2)/(9*a^(11/3)))*
((3^(1/2)*1i)/2 - 1/2))/(9*a^(7/3))
```

sympy [C] time = 1.37, size = 41, normalized size = 0.30

$$-\frac{\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\sqrt[3]{b}\,x^{\frac{14}{3}}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(1/3),x)
```

```
[Out] -gamma(7/3)*hyper((1/3, 7/3), (10/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1
/3)*x**(14/3)*gamma(10/3))
```

3.709 $\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=580

$$-\frac{27\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{91b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+81$$

[Out] -27/91*a*x*(b*x^2+a)^(2/3)/b^2+3/13*x^3*(b*x^2+a)^(2/3)/b-81/91*a^2*x/b^2/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-27/91*3^(3/4)*a^(7/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+81/182*3^(1/4)*a^(7/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A] time = 0.37, antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 235, 304, 219, 1879}

$$-\frac{81a^2x}{91b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{27\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{}}{\left(1-\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{91b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b*x^2)^(1/3),x]

[Out] (-27*a*x*(a + b*x^2)^(2/3))/(91*b^2) + (3*x^3*(a + b*x^2)^(2/3))/(13*b) - (81*a^2*x)/(91*b^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (81*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(182*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (27*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx &= \frac{3x^3\left(a+bx^2\right)^{2/3}}{13b}-\frac{(9a)\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx}{13b}\\
&= -\frac{27ax\left(a+bx^2\right)^{2/3}}{91b^2}+\frac{3x^3\left(a+bx^2\right)^{2/3}}{13b}+\frac{\left(27a^2\right)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{91b^2}\\
&= -\frac{27ax\left(a+bx^2\right)^{2/3}}{91b^2}+\frac{3x^3\left(a+bx^2\right)^{2/3}}{13b}+\frac{\left(81a^2\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{182b^3x}\\
&= -\frac{27ax\left(a+bx^2\right)^{2/3}}{91b^2}+\frac{3x^3\left(a+bx^2\right)^{2/3}}{13b}-\frac{\left(81a^2\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right.}{182b^3x}\\
&= -\frac{27ax\left(a+bx^2\right)^{2/3}}{91b^2}+\frac{3x^3\left(a+bx^2\right)^{2/3}}{13b}-\frac{81a^2x}{91b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{}}}{}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 79, normalized size = 0.14

$$\frac{3\left(9a^2x\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-9a^2x-2abx^3+7b^2x^5\right)}{91b^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(1/3),x]
[Out] (3*(-9*a^2*x - 2*a*b*x^3 + 7*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(1/3)*Hyperg
eometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)]))/(91*b^2*(a + b*x^2)^(1/3))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\left(bx^2+a\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] integral(x^4/(b*x^2 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate(x^4/(b*x^2 + a)^(1/3), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/3),x)
[Out] int(x^4/(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate(x^4/(b*x^2 + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(1/3),x)
```

```
[Out] int(x^4/(a + b*x^2)^(1/3), x)
```

sympy [A] time = 0.82, size = 27, normalized size = 0.05

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/3),x)
```

```
[Out] x**5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/3))
```

3.710 $\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=556

$$\frac{3\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{7b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{9\sqrt[4]{3}\ \cdots}{\cdots}$$

```
[Out] 3/7*x*(b*x^2+a)^(2/3)/b+9/7*a*x/b/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+3/
7*3^(3/4)*a^(4/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(
1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(
1/2)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a
^(1/3)*(1-3^(1/2)))^2)^(1/2)/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b
*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-9/14*3^(1/4)*a^(4/3)*(a^(1/3)-(
b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a
)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/
3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^
(1/2)+1/2*2^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1
/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 235, 304, 219, 1879}

$$\frac{3\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{7b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{9\sqrt[4]{3}\sqrt{2+\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(2/3))/(7*b) + (9*a*x)/(7*b*((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))) - (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a + b*x^2
)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(14*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (3*Sqrt[2]*3^(3/4)*a^(4/
3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3)
+ (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3)
 - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx &= \frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{(3a)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{7b}\\
&= \frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{\left(9a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{14b^2x}\\
&= \frac{3x\left(a+bx^2\right)^{2/3}}{7b}+\frac{\left(9a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{14b^2x}-\frac{\left(9\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\,a^{4/3}\right.}{}\\
&= \frac{3x\left(a+bx^2\right)^{2/3}}{7b}+\frac{9ax}{7b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{1}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.11

$$\frac{3x\left(-a\sqrt[3]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{7b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(1/3),x]
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2,
-((b*x^2)/a)]))/(7*b*(a + b*x^2)^(1/3))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\left(bx^2+a\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] integral(x^2/(b*x^2 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate(x^2/(b*x^2 + a)^(1/3), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/3),x)
[Out] int(x^2/(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate(x^2/(b*x^2 + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(1/3),x)
[Out] int(x^2/(a + b*x^2)^(1/3), x)
```

sympy [A] time = 0.78, size = 27, normalized size = 0.05

$$\frac{x^{3}{}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/3),x)
[Out] x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/3))
```

3.711 $\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=529

$$-\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3\sqrt[4]{3}}{}$$

```
[Out] -3*x/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-3^(3/4)*a^(1/3)*(a^(1/3)-(b*x^2
+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(b*x^2+a)^
(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/
(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^
2)^(1/2)+3/2*3^(1/4)*a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a
)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3
^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3
)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^
(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {235, 304, 219, 1879}

$$-\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1/3),x]
```

```
[Out] (-3*x)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3*3^(1/4)*Sqrt[2 + Sq
rt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b
*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*Sqrt[-((a^(1/3
)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
)^2)]) - (Sqrt[2]*3^(3/4)*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/
3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
- (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^
2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = \frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2bx}$$

$$= -\frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2bx} + \frac{\left(3\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt[3]{a}\sqrt{bx^2}\right)\text{Subst}\left(\int \cdots\right.}{bx}$$

$$= -\frac{3x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}} + \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\cdots}}}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+\cdots}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a\cdots}\right)}}}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.09

$$\frac{x\sqrt[3]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1/3),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(1/3)
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(-1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2 + a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-1/3), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2 + a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2 + a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-1/3), x)
```

mupad [B] time = 4.63, size = 37, normalized size = 0.07

$$\frac{x\,\left(\frac{b\,x^2}{a}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/3),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(1/3)*hypergeom([1/3, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(1/3)
```

sympy [A] time = 0.75, size = 24, normalized size = 0.05

$$\frac{x {}_{2}F_{1}\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{bx^{2} e^{i \pi}}{a}} \right)}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/3),x)
```

```
[Out] x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/3)
```

3.712 $\int \frac{1}{x^2 \sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=546

$$-\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}\,a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{\sqrt[4]{3}\sqrt{2+}}{}$$

```
[Out] -(b*x^2+a)^(2/3)/a/x-b*x/a/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-1/3*(a^(1
/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*
(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^
(1/2)*3^(3/4)/a^(2/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+1/2*3^(1/4)*(a^(1/3)-(b*x^2+a)^(1/3))*Elli
pticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3
^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/
(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/a
^(2/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3
^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 235, 304, 219, 1879}

$$-\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(1/3)),x]
```

```
[Out] -((a + b*x^2)^(2/3)/(a*x)) - (b*x)/(a*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^
(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])
/(2*a^(2/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (Sqrt[2]*(a^(1/3) - (a + b*x^2)^(1/3))
*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*
Sqrt[3]])/(3^(1/4)*a^(2/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx &= -\frac{\left(a+bx^2\right)^{2/3}}{ax} + \frac{b\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{3a} \\
&= -\frac{\left(a+bx^2\right)^{2/3}}{ax} + \frac{\sqrt{bx^2}\ \text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2ax} \\
&= -\frac{\left(a+bx^2\right)^{2/3}}{ax} - \frac{\sqrt{bx^2}\ \text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2ax} + \frac{\left(\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt{bx^2}\right)\text{Subs}}{a} \\
&= -\frac{\left(a+bx^2\right)^{2/3}}{ax} - \frac{bx}{a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}}{((1-\sqrt{}}}}{2a^{2/3}x}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.09

$$
-\frac{\sqrt[3]{\frac{bx^2}{a}+1}\ {}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[3]{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(1/3)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-1/2, 1/3, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(1/3)))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{bx^4+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)/(b*x^4 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*x^2), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*x^2), x)
```

mupad [B] time = 4.83, size = 40, normalized size = 0.07

$$-\frac{3\left(\frac{a}{b\,x^2}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{5}{6};\ \frac{11}{6};\ -\frac{a}{b\,x^2}\right)}{5\,x\,\left(b\,x^2+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(1/3)),x)
```

```
[Out] -(3*(a/(b*x^2) + 1)^(1/3)*hypergeom([1/3, 5/6], 11/6, -a/(b*x^2)))/(5*x*(a
+ b*x^2)^(1/3))
```

sympy [A] time = 0.82, size = 27, normalized size = 0.05

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{1}{3}\\ \frac{1}{2}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[3]{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/3),x)
```

```
[Out] -hyper((-1/2, 1/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(1/3)*x)
```

3.713 $\int \frac{1}{x^4 \sqrt[3]{a+bx^2}} \, dx$

Optimal. Leaf size=578

$$\frac{5\sqrt{2}\, b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\, \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right), 4\sqrt{3}-7\right)}{9\sqrt[4]{3}\, a^{5/3} x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}} - \frac{5\sqrt{2+\sqrt{}}}{}$$

```
[Out] -1/3*(b*x^2+a)^(2/3)/a/x^3+5/9*b*(b*x^2+a)^(2/3)/a^2/x+5/9*b^2*x/a^2/(-(b*x
^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+5/27*b*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF
((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2
))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3
))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/x/(-a^(1
/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/
2)-5/18*b*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+
3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a
^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2
)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(5/3)/x/(-a^(1/3)*(a^(1/3)
-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 235, 304, 219, 1879}

$$\frac{5b^2 x}{9a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{5b\left(a+bx^2\right)^{2/3}}{9a^2 x}+\frac{5\sqrt{2}\, b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\, F\left(\sin^{-1}\right.}{9\sqrt[4]{3}\, a^{5/3} x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(1/3)),x]
```

```
[Out] -(a + b*x^2)^(2/3)/(3*a*x^3) + (5*b*(a + b*x^2)^(2/3))/(9*a^2*x) + (5*b^2*x
)/(9*a^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (5*Sqrt[2 + Sqrt[3]
]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3)
 + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Ellipt
icE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(6*3^(3/4)*a^(5/3)*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1
/3))^2)]) + (5*Sqrt[2]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1
/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))
/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*
a^(5/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(
1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt[3]{a+bx^2}}\,dx &= -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3}-\frac{(5b)\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx}{9a}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3}+\frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x}-\frac{\left(5b^2\right)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{27a^2}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3}+\frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x}-\frac{\left(5b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{18a^2x}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3}+\frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x}+\frac{\left(5b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{18a^2x}-\frac{\left(5\sqrt{\right.}{}}{}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{3ax^3}+\frac{5b\left(a+bx^2\right)^{2/3}}{9a^2x}+\frac{5b^2x}{9a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{5\sqrt{2+\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{\right.}{}}{}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.09

$$-\frac{\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(1/3)),x]
```

```
[Out] -1/3*((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-3/2, 1/3, -1/2, -((b*x^2)/a)
])/(x^3*(a + b*x^2)^(1/3))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{bx^6+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)/(b*x^6 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*x^4), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{1}{x^4\,{\left(b\,x^2+a\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(1/3)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)^(1/3)), x)
```

sympy [A] time = 0.94, size = 32, normalized size = 0.06

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{3} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/3),x)
```

```
[Out] -hyper((-3/2, 1/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/3)*x**3)
```

3.714 $\int \frac{x^7}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=80

$$-\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4}+\frac{9a^2\left(a+bx^2\right)^{4/3}}{8b^4}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^4}-\frac{9a\left(a+bx^2\right)^{7/3}}{14b^4}$$

```
[Out] -3/2*a^3*(b*x^2+a)^(1/3)/b^4+9/8*a^2*(b*x^2+a)^(4/3)/b^4-9/14*a*(b*x^2+a)^(
7/3)/b^4+3/20*(b*x^2+a)^(10/3)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{4/3}}{8b^4}-\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^4}-\frac{9a\left(a+bx^2\right)^{7/3}}{14b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(1/3))/(2*b^4) + (9*a^2*(a + b*x^2)^(4/3))/(8*b^4) - (9
*a*(a + b*x^2)^(7/3))/(14*b^4) + (3*(a + b*x^2)^(10/3))/(20*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a+bx^2\right)^{2/3}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{(a+bx)^{2/3}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{2/3}}+\frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{b^3}+\frac{(a+bx)^{7/3}}{b^3}\right)dx,x,x^2\right)\\
&= -\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4}+\frac{9a^2\left(a+bx^2\right)^{4/3}}{8b^4}-\frac{9a\left(a+bx^2\right)^{7/3}}{14b^4}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^4}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.62

$$\frac{3\sqrt[3]{a+bx^2}\left(-81a^3+27a^2bx^2-18ab^2x^4+14b^3x^6\right)}{280b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3)*(-81*a^3 + 27*a^2*b*x^2 - 18*a*b^2*x^4 + 14*b^3*x^6))/
(280*b^4)
```

fricas [A] time = 0.82, size = 46, normalized size = 0.58

$$\frac{3\left(14\,b^{3}x^{6}-18\,ab^{2}x^{4}+27\,a^{2}bx^{2}-81\,a^{3}\right)\left(bx^{2}+a\right)^{\frac{1}{3}}}{280\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/280*(14*b^3*x^6 - 18*a*b^2*x^4 + 27*a^2*b*x^2 - 81*a^3)*(b*x^2 + a)^(1/3)
/b^4
```

giac [A] time = 0.58, size = 61, normalized size = 0.76

$$-\frac{3\left(bx^{2}+a\right)^{\frac{1}{3}}a^{3}}{2\,b^{4}}+\frac{3\left(14\left(bx^{2}+a\right)^{\frac{10}{3}}-60\left(bx^{2}+a\right)^{\frac{7}{3}}a+105\left(bx^{2}+a\right)^{\frac{4}{3}}a^{2}\right)}{280\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] -3/2*(b*x^2 + a)^(1/3)*a^3/b^4 + 3/280*(14*(b*x^2 + a)^(10/3) - 60*(b*x^2 +
 a)^(7/3)*a + 105*(b*x^2 + a)^(4/3)*a^2)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-14b^{3}x^{6}+18a\,b^{2}x^{4}-27a^{2}b\,x^{2}+81a^{3}\right)}{280b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(2/3),x)
```

```
[Out] -3/280*(b*x^2+a)^(1/3)*(-14*b^3*x^6+18*a*b^2*x^4-27*a^2*b*x^2+81*a^3)/b^4
```

maxima [A] time = 1.34, size = 64, normalized size = 0.80

$$\frac{3\left(bx^{2}+a\right)^{\frac{10}{3}}}{20\,b^{4}}-\frac{9\left(bx^{2}+a\right)^{\frac{7}{3}}a}{14\,b^{4}}+\frac{9\left(bx^{2}+a\right)^{\frac{4}{3}}a^{2}}{8\,b^{4}}-\frac{3\left(bx^{2}+a\right)^{\frac{1}{3}}a^{3}}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/20*(b*x^2 + a)^(10/3)/b^4 - 9/14*(b*x^2 + a)^(7/3)*a/b^4 + 9/8*(b*x^2 + a
)^(4/3)*a^2/b^4 - 3/2*(b*x^2 + a)^(1/3)*a^3/b^4
```

mupad [B] time = 4.75, size = 48, normalized size = 0.60

$$-{\left(b\,x^2+a\right)}^{1/3}\,\left(\frac{243\,a^3}{280\,b^4}-\frac{3\,x^6}{20\,b}+\frac{27\,a\,x^4}{140\,b^2}-\frac{81\,a^2\,x^2}{280\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^(2/3),x)
```

```
[Out] -(a + b*x^2)^(1/3)*((243*a^3)/(280*b^4) - (3*x^6)/(20*b) + (27*a*x^4)/(140*
b^2) - (81*a^2*x^2)/(280*b^3))
```

sympy [B] time = 2.74, size = 1690, normalized size = 21.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(2/3),x)
```

```
[Out] -243*a**(70/3)*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2
 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 168
0*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 243*a**(70/3)/(280*a**20*b**4
 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 420
0*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) - 1377*a
**(67/3)*b*x**2*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**
2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 16
80*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 1458*a**(67/3)*b*x**2/(280*a
**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x
**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12)
 - 3213*a**(64/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a*
*19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b*
*8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 3645*a**(64/3)*b
**2*x**4/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 56
00*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**
14*b**10*x**12) - 3927*a**(61/3)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(280*a**20
*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6
+ 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 4
860*a**(61/3)*b**3*x**6/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18
*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*
x**10 + 280*a**14*b**10*x**12) - 2583*a**(58/3)*b**4*x**8*(1 + b*x**2/a)**(
1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a
**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b
**10*x**12) + 3645*a**(58/3)*b**4*x**8/(280*a**20*b**4 + 1680*a**19*b**5*x*
*2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1
680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) - 693*a**(55/3)*b**5*x**10*(1
 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**
6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**1
0 + 280*a**14*b**10*x**12) + 1458*a**(55/3)*b**5*x**10/(280*a**20*b**4 + 16
80*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**
16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 273*a**(52/
3)*b**6*x**12*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2
+ 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680
*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 243*a**(52/3)*b**6*x**12/(280*
a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*
x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12
) + 387*a**(49/3)*b**7*x**14*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a
**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b
**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 198*a**(46/3)*b
**8*x**16*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 42
00*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**
15*b**9*x**10 + 280*a**14*b**10*x**12) + 42*a**(43/3)*b**9*x**18*(1 + b*x**
2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 +
 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*
a**14*b**10*x**12)
```

3.715 $\int \frac{x^5}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=59

$$\frac{3a^2\sqrt[3]{a+bx^2}}{2b^3}+\frac{3\left(a+bx^2\right)^{7/3}}{14b^3}-\frac{3a\left(a+bx^2\right)^{4/3}}{4b^3}$$

```
[Out] 3/2*a^2*(b*x^2+a)^(1/3)/b^3-3/4*a*(b*x^2+a)^(4/3)/b^3+3/14*(b*x^2+a)^(7/3)/
b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\sqrt[3]{a+bx^2}}{2b^3}+\frac{3\left(a+bx^2\right)^{7/3}}{14b^3}-\frac{3a\left(a+bx^2\right)^{4/3}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(1/3))/(2*b^3) - (3*a*(a + b*x^2)^(4/3))/(4*b^3) + (3*(a
 + b*x^2)^(7/3))/(14*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^2\right)^{2/3}}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{2/3}}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{2/3}}-\frac{2a\sqrt[3]{a+bx}}{b^2}+\frac{(a+bx)^{4/3}}{b^2}\right)dx,x,x^2\right)\\&=\frac{3a^2\sqrt[3]{a+bx^2}}{2b^3}-\frac{3a\left(a+bx^2\right)^{4/3}}{4b^3}+\frac{3\left(a+bx^2\right)^{7/3}}{14b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{3\sqrt[3]{a+bx^2}\left(9a^2-3abx^2+2b^2x^4\right)}{28b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3)*(9*a^2 - 3*a*b*x^2 + 2*b^2*x^4))/(28*b^3)
```

fricas [A] time = 1.00, size = 35, normalized size = 0.59

$$\frac{3\left(2\,b^{2}x^{4}-3\,abx^{2}+9\,a^{2}\right)\left(bx^{2}+a\right)^{\frac{1}{3}}}{28\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/28*(2*b^2*x^4 - 3*a*b*x^2 + 9*a^2)*(b*x^2 + a)^(1/3)/b^3
```

giac [A] time = 0.57, size = 47, normalized size = 0.80

$$\frac{3\left(bx^{2}+a\right)^{\frac{1}{3}}a^{2}}{2\,b^{3}}+\frac{3\left(2\left(bx^{2}+a\right)^{\frac{7}{3}}-7\left(bx^{2}+a\right)^{\frac{4}{3}}a\right)}{28\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/2*(b*x^2 + a)^(1/3)*a^2/b^3 + 3/28*(2*(b*x^2 + a)^(7/3) - 7*(b*x^2 + a)^(
4/3)*a)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(2b^{2}x^{4}-3ab\,x^{2}+9a^{2}\right)}{28b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(2/3),x)
```

```
[Out] 3/28*(b*x^2+a)^(1/3)*(2*b^2*x^4-3*a*b*x^2+9*a^2)/b^3
```

maxima [A] time = 1.33, size = 47, normalized size = 0.80

$$\frac{3\left(bx^{2}+a\right)^{\frac{7}{3}}}{14\,b^{3}}-\frac{3\left(bx^{2}+a\right)^{\frac{4}{3}}a}{4\,b^{3}}+\frac{3\left(bx^{2}+a\right)^{\frac{1}{3}}a^{2}}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x^2 + a)^(7/3)/b^3 - 3/4*(b*x^2 + a)^(4/3)*a/b^3 + 3/2*(b*x^2 + a)^
(1/3)*a^2/b^3
```

mupad [B] time = 4.77, size = 36, normalized size = 0.61

$$\left(b\,x^2+a\right)^{1/3}\,\left(\frac{27\,a^2}{28\,b^3}+\frac{3\,x^4}{14\,b}-\frac{9\,a\,x^2}{28\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(2/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((27*a^2)/(28*b^3) + (3*x^4)/(14*b) - (9*a*x^2)/(28*b^2))
```

sympy [B] time = 1.80, size = 631, normalized size = 10.69

$$\frac{27a^{\frac{31}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6}-\frac{27a^{\frac{31}{3}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6}+\frac{72a^{\frac{28}{3}}bx^2\sqrt[3]{}}{28a^8b^3+84a^7b^4x^2+84}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(2/3),x)
```

```
[Out] 27*a**(31/3)*(1 + b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a
**6*b**5*x**4 + 28*a**5*b**6*x**6) - 27*a**(31/3)/(28*a**8*b**3 + 84*a**7*b
**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 72*a**(28/3)*b*x**2*(1
+ b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 +
28*a**5*b**6*x**6) - 81*a**(28/3)*b*x**2/(28*a**8*b**3 + 84*a**7*b**4*x**2
+ 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 60*a**(25/3)*b**2*x**4*(1 + b*x*
*2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**
5*b**6*x**6) - 81*a**(25/3)*b**2*x**4/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 8
4*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 18*a**(22/3)*b**3*x**6*(1 + b*x**2/
a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b
**6*x**6) - 27*a**(22/3)*b**3*x**6/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a
**6*b**5*x**4 + 28*a**5*b**6*x**6) + 9*a**(19/3)*b**4*x**8*(1 + b*x**2/a)**
(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*
x**6) + 6*a**(16/3)*b**5*x**10*(1 + b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**
7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6)
```

3.716 $\int \frac{x^3}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=38

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b^2}-\frac{3a\sqrt[3]{a+bx^2}}{2b^2}$$

```
[Out] -3/2*a*(b*x^2+a)^(1/3)/b^2+3/8*(b*x^2+a)^(4/3)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b^2}-\frac{3a\sqrt[3]{a+bx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(2/3),x]
[Out] (-3*a*(a + b*x^2)^(1/3))/(2*b^2) + (3*(a + b*x^2)^(4/3))/(8*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^{2/3}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{2/3}}+\frac{\sqrt[3]{a+bx}}{b}\right)dx, x, x^2\right)\\
&= -\frac{3a\sqrt[3]{a+bx^2}}{2b^2}+\frac{3\left(a+bx^2\right)^{4/3}}{8b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$\frac{3\left(bx^2-3a\right)\sqrt[3]{a+bx^2}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(2/3),x]
[Out] (3*(-3*a + b*x^2)*(a + b*x^2)^(1/3))/(8*b^2)
```

fricas [A] time = 0.51, size = 23, normalized size = 0.61

$$\frac{3\left(bx^2+a\right)^{\frac{1}{3}}\left(bx^2-3a\right)}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] 3/8*(b*x^2 + a)^(1/3)*(b*x^2 - 3*a)/b^2
```

giac [A] time = 0.59, size = 30, normalized size = 0.79

$$\frac{3\left(bx^2+a\right)^{\frac{4}{3}}}{8b^2}-\frac{3\left(bx^2+a\right)^{\frac{1}{3}}a}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] 3/8*(b*x^2 + a)^(4/3)/b^2 - 3/2*(b*x^2 + a)^(1/3)*a/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{3\left(bx^2+a\right)^{\frac{1}{3}}\left(-bx^2+3a\right)}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(2/3),x)
[Out] -3/8*(b*x^2+a)^(1/3)*(-b*x^2+3*a)/b^2
```

maxima [A] time = 1.34, size = 30, normalized size = 0.79

$$\frac{3\left(bx^2+a\right)^{\frac{4}{3}}}{8b^2}-\frac{3\left(bx^2+a\right)^{\frac{1}{3}}a}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] 3/8*(b*x^2 + a)^(4/3)/b^2 - 3/2*(b*x^2 + a)^(1/3)*a/b^2
```

mupad [B] time = 4.79, size = 24, normalized size = 0.63

$$-\frac{3\left(bx^2+a\right)^{1/3}\left(3a-bx^2\right)}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(2/3),x)
[Out] -(3*(a + b*x^2)^(1/3)*(3*a - b*x^2))/(8*b^2)
```

sympy [B] time = 1.14, size = 178, normalized size = 4.68

$$-\frac{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}+\frac{9a^{\frac{10}{3}}}{8a^2b^2+8ab^3x^2}-\frac{6a^{\frac{7}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}+\frac{9a^{\frac{7}{3}}bx^2}{8a^2b^2+8ab^3x^2}+\frac{3a^{\frac{4}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(2/3),x)
```

```
[Out] -9*a**(10/3)*(1 + b*x**2/a)**(1/3)/(8*a**2*b**2 + 8*a*b**3*x**2) + 9*a**(10
/3)/(8*a**2*b**2 + 8*a*b**3*x**2) - 6*a**(7/3)*b*x**2*(1 + b*x**2/a)**(1/3)
/(8*a**2*b**2 + 8*a*b**3*x**2) + 9*a**(7/3)*b*x**2/(8*a**2*b**2 + 8*a*b**3*
x**2) + 3*a**(4/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(8*a**2*b**2 + 8*a*b**3*
x**2)
```

3.717 $\int \frac{x}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=18

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

[Out] 3/2*(b*x^2+a)^(1/3)/b

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x^2)^(2/3),x]

[Out] (3*(a + b*x^2)^(1/3))/(2*b)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx^2}}{2b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x^2)^(2/3),x]

[Out] (3*(a + b*x^2)^(1/3))/(2*b)

fricas [A] time = 0.76, size = 14, normalized size = 0.78

$$\frac{3\,(bx^2+a)^{\frac{1}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^2+a)^(2/3),x, algorithm="fricas")

[Out] 3/2*(b*x^2 + a)^(1/3)/b

giac [A] time = 0.57, size = 14, normalized size = 0.78

$$\frac{3\,(bx^2+a)^{\frac{1}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/2*(b*x^2 + a)^(1/3)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(2/3),x)
```

```
[Out] 3/2*(b*x^2+a)^(1/3)/b
```

maxima [A] time = 1.24, size = 14, normalized size = 0.78

$$\frac{3\left(bx^{2}+a\right)^{\frac{1}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/2*(b*x^2 + a)^(1/3)/b
```

mupad [B] time = 4.69, size = 14, normalized size = 0.78

$$\frac{3\left(b\,x^{2}+a\right)^{1/3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(2/3),x)
```

```
[Out] (3*(a + b*x^2)^(1/3))/(2*b)
```

sympy [A] time = 0.41, size = 24, normalized size = 1.33

$$\begin{cases} \frac{3\sqrt[3]{a+bx^{2}}}{2b} & \text{for}\ b\neq 0 \\ \frac{x^{2}}{2a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(2/3),x)
```

```
[Out] Piecewise((3*(a + b*x**2)**(1/3)/(2*b), Ne(b, 0)), (x**2/(2*a**(2/3)), True
))
```

3.718 $\int \frac{1}{x\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=86

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

```
[Out] -1/2*ln(x)/a^(2/3)+3/4*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(2/3)-1/2*arctan(1/3*(
a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(2/3)
```

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 57, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(2/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(2*a^(
2/3)) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(4*a^(2/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^{2/3}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= -\frac{\log(x)}{2a^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}\\
&= -\frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2a^{2/3}}\\
&= -\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2a^{2/3}} - \frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 101, normalized size = 1.17

$$
-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{4a^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(2/3)),x]
```

```
[Out] -1/4*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log
[a^(1/3) - (a + b*x^2)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (
a + b*x^2)^(2/3)])/a^(2/3)
```

fricas [B] time = 0.60, size = 123, normalized size = 1.43

$$
-\frac{2\sqrt{3}\left(a^2\right)^{\frac{1}{6}}a\arctan\left(\frac{\sqrt{3}\left(a^2\right)^{\frac{1}{6}}\left(\left(a^2\right)^{\frac{1}{3}}a+2\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right)+\left(a^2\right)^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)-}{4a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*sqrt(3)*(a^2)^(1/6)*((a^2)^(1/3)*a
 + 2*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + (a^2)^(2/3)*log((b*x^2 + a)^(2/3
)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2/3)) - 2*(a^2)^(2/3)*log((b
*x^2 + a)^(1/3)*a - (a^2)^(2/3)))/a^2
```

giac [A] time = 1.13, size = 87, normalized size = 1.01

$$
-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{2}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4a^{\frac{2}{3}}}+\frac{\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2a^{\frac{2}{3}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(2/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(2/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{2}{3}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x/(b*x^2+a)^(2/3),x)
```

maxima [A] time = 3.09, size = 86, normalized size = 1.00

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{2}{3}}}-\frac{\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{2}{3}}}+\frac{\log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(2/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(2/3) + 1/2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(2/3)
```

mupad [B] time = 4.84, size = 102, normalized size = 1.19

$$\frac{\ln\left(\frac{9\left(b\,x^2+a\right)^{1/3}}{2}-\frac{9\,a^{1/3}}{2}\right)}{2\,a^{2/3}}+\frac{\ln\left(\frac{9\,a^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}-\frac{9\left(b\,x^2+a\right)^{1/3}}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4\,a^{2/3}}-\frac{\ln\left(\frac{9\,a^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}+\frac{9\left(b\,x^2+a\right)^{1/3}}{2}\right)\left(1+\sqrt{3}\,1\right.}{4\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(2/3)),x)
```

```
[Out] log((9*(a + b*x^2)^(1/3))/2 - (9*a^(1/3))/2)/(2*a^(2/3)) + (log((9*a^(1/3)*
(3^(1/2)*1i - 1))/4 - (9*(a + b*x^2)^(1/3))/2)*(3^(1/2)*1i - 1))/(4*a^(2/3)
) - (log((9*a^(1/3)*(3^(1/2)*1i + 1))/4 + (9*(a + b*x^2)^(1/3))/2)*(3^(1/2)
*1i + 1))/(4*a^(2/3))
```

sympy [C] time = 1.06, size = 41, normalized size = 0.48

$$-\frac{\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2b^{\frac{2}{3}}x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(2/3),x)
```

```
[Out] -gamma(2/3)*hyper((2/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2/
3)*x**(4/3)*gamma(5/3))
```

3.719 $\int \frac{1}{x^3\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=107

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}+\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{\sqrt[3]{a+bx^2}}{2ax^2}$$

```
[Out] -1/2*(b*x^2+a)^(1/3)/a/x^2+1/3*b*ln(x)/a^(5/3)-1/2*b*ln(a^(1/3)-(b*x^2+a)^(
1/3))/a^(5/3)+1/3*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))
/a^(5/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 57, 617, 204, 31}

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}+\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{\sqrt[3]{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(2/3)),x]
```

```
[Out] -(a + b*x^2)^(1/3)/(2*a*x^2) + (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (b*Log[x])/(3*a^(5/3)) - (b*Log[a^(1/
3) - (a + b*x^2)^(1/3)])/(2*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)^{2/3}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{2ax^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)}{3a}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2ax^2} + \frac{b\log(x)}{3a^{5/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2a^{5/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,\right.}{2a^{4/3}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2ax^2} + \frac{b\log(x)}{3a^{5/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}} - \frac{b\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{a^{5/3}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2ax^2} + \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}} + \frac{b\log(x)}{3a^{5/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.35

$$\frac{3b\sqrt[3]{a+bx^2}\;{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{bx^2}{a}+1\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*b*(a + b*x^2)^(1/3)*Hypergeometric2F1[1/3, 2, 4/3, 1 + (b*x^2)/a])/(2*a^
2)
```

fricas [B] time = 0.63, size = 182, normalized size = 1.70

$$\frac{2\sqrt{3}\,abx^2\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\,\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3\,a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}\right.}{6\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*a*b*x^2*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3
)*a - 2*sqrt(3)*(b*x^2 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) +
```

```
(-a^2)^(2/3)*b*x^2*log((b*x^2 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^2 + a)^(
1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3)*a - (-a^2)^
(2/3)) - 3*(b*x^2 + a)^(1/3)*a^2)/(a^3*x^2)
```

giac [A] time = 1.11, size = 118, normalized size = 1.10

$$\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2b^2\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\left(bx^2+a\right)^{\frac{1}{3}}b}{ax^2}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(5/3) + b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/
3))/a^(5/3) - 2*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*(b*x^
2 + a)^(1/3)*b/(a*x^2))/b
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{2}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x^3/(b*x^2+a)^(2/3),x)
```

maxima [A] time = 2.96, size = 118, normalized size = 1.10

$$\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{5}{3}}}-\frac{\left(bx^2+a\right)^{\frac{1}{3}}b}{2\left(\left(bx^2+a\right)a-a^2\right)}+\frac{b\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{5}{3}}}-\frac{b\log\left(\left(bx^2+\right.\right.}{3a^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(5/3) - 1/2*(b*x^2 + a)^(1/3)*b/((b*x^2 + a)*a - a^2) + 1/6*b*log((b*x^2 +
 a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) - 1/3*b*log((b*x^2
 + a)^(1/3) - a^(1/3))/a^(5/3)
```

mupad [B] time = 5.05, size = 130, normalized size = 1.21

$$\frac{\ln\left(\frac{3\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{2\,a^{2/3}}+\frac{3\,b\left(b\,x^2+a\right)^{1/3}}{a}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{5/3}}+\frac{\ln\left(\frac{3\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{2\,a^{2/3}}+\frac{3\,b\left(b\,x^2+a\right)^{1/3}}{a}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{5/3}}-\frac{b\,\ln\left(\left(b\,x^2+a\right)^{1/3}\right.}{3\,a^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(2/3)),x)
```

```
[Out] (log((3*(b - 3^(1/2)*b*1i))/(2*a^(2/3)) + (3*b*(a + b*x^2)^(1/3))/a)*(b - 3
^(1/2)*b*1i))/(6*a^(5/3)) + (log((3*(b + 3^(1/2)*b*1i))/(2*a^(2/3)) + (3*b*
```

```
(a + b*x^2)^(1/3))/a)*(b + 3^(1/2)*b*1i))/(6*a^(5/3)) - (b*log((a + b*x^2)^
(1/3) - a^(1/3)))/(3*a^(5/3)) - (a + b*x^2)^(1/3)/(2*a*x^2)
```

sympy [C] time = 1.24, size = 41, normalized size = 0.38

$$-\frac{\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3}, \frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}}x^{\frac{10}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(2/3),x)
```

```
[Out] -gamma(5/3)*hyper((2/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2/
3)*x**(10/3)*gamma(8/3))
```

3.720 $\int \frac{1}{x^5(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=138

$$\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}-\frac{\sqrt[3]{a+bx^2}}{4ax^4}$$

```
[Out] -1/4*(b*x^2+a)^(1/3)/a/x^4+5/12*b*(b*x^2+a)^(1/3)/a^2/x^2-5/18*b^2*ln(x)/a^
(8/3)+5/12*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(8/3)-5/18*b^2*arctan(1/3*(a^(
1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a^(8/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 57, 617, 204, 31}

$$\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}-\frac{\sqrt[3]{a+bx^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^(2/3)),x]
```

```
[Out] -(a + b*x^2)^(1/3)/(4*a*x^4) + (5*b*(a + b*x^2)^(1/3))/(12*a^2*x^2) - (5*b^
2*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(
8/3)) - (5*b^2*Log[x])/(18*a^(8/3)) + (5*b^2*Log[a^(1/3) - (a + b*x^2)^(1/3
)])/(12*a^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\left(a+bx^2\right)^{2/3}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(a+bx)^{2/3}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{4ax^4}-\frac{(5b)\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}}\,dx,x,x^2\right)}{12a}\\
&= -\frac{\sqrt[3]{a+bx^2}}{4ax^4}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}+\frac{\left(5b^2\right)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx,x,x^2\right)}{18a^2}\\
&= -\frac{\sqrt[3]{a+bx^2}}{4ax^4}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}-\frac{5b^2\log(x)}{18a^{8/3}}-\frac{\left(5b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)}{12a^{8/3}}-\frac{\left(5b^2\right.}{}\\
&= -\frac{\sqrt[3]{a+bx^2}}{4ax^4}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{8/3}}+\frac{\left(5b^2\right)\operatorname{Subst}\left(\int\right.}{}\\
&= -\frac{\sqrt[3]{a+bx^2}}{4ax^4}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}-\frac{5b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+\right.}}{12a^{8/3}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.28

$$
-\frac{3b^2\sqrt[3]{a+bx^2}\,{}_2F_1\left(\frac{1}{3},3;\frac{4}{3};\frac{bx^2}{a}+1\right)}{2a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*b^2*(a + b*x^2)^(1/3)*Hypergeometric2F1[1/3, 3, 4/3, 1 + (b*x^2)/a])/(2
*a^3)
```

fricas [A] time = 1.01, size = 174, normalized size = 1.26

$$
-\frac{10\sqrt{3}\left(a^2\right)^{\frac{1}{6}}ab^2x^4\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right)+5\left(a^2\right)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right.\right.}{36a^4x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/36*(10*sqrt(3)*(a^2)^(1/6)*a*b^2*x^4*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^
2)^(1/3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + 5*(a^2)^(2/3)*
b^2*x^4*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(
2/3)) - 10*(a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) - 3*(
5*a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(1/3))/(a^4*x^4)
```

giac [A] time = 1.13, size = 142, normalized size = 1.03

$$-\frac{\frac{10\sqrt{3}\,b^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{8}{3}}}+\frac{5\,b^3\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{8}{3}}}-\frac{10\,b^3\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{8}{3}}}-\frac{3\left(5\left(bx^2+a\right)^{\frac{4}{3}}b^3-8\left(bx^2+a\right)^{\frac{1}{3}}\cdots\right)}{a^2b^2x^4}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] -1/36*(10*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^
(1/3))/a^(8/3) + 5*b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
a^(2/3))/a^(8/3) - 10*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(8/3) - 3
*(5*(b*x^2 + a)^(4/3)*b^3 - 8*(b*x^2 + a)^(1/3)*a*b^3)/(a^2*b^2*x^4))/b
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{2}{3}} x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x^5/(b*x^2+a)^(2/3),x)
```

maxima [A] time = 2.98, size = 158, normalized size = 1.14

$$-\frac{5\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{18\,a^{\frac{8}{3}}}-\frac{5\,b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{36\,a^{\frac{8}{3}}}+\frac{5\,b^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{18\,a^{\frac{8}{3}}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -5/18*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(8/3) - 5/36*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3))/a^(8/3) + 5/18*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(8/3) + 1/12*
(5*(b*x^2 + a)^(4/3)*b^2 - 8*(b*x^2 + a)^(1/3)*a*b^2)/((b*x^2 + a)^2*a^2 -
2*(b*x^2 + a)*a^3 + a^4)
```

mupad [B] time = 5.14, size = 193, normalized size = 1.40

$$\frac{5\,b^2\,\ln\left(\left(b\,x^2+a\right)^{1/3}-a^{1/3}\right)}{18\,a^{8/3}}-\frac{\frac{4\,b^2\,\left(b\,x^2+a\right)^{1/3}}{3\,a}-\frac{5\,b^2\,\left(b\,x^2+a\right)^{4/3}}{6\,a^2}}{2\,\left(b\,x^2+a\right)^2-4\,a\,\left(b\,x^2+a\right)+2\,a^2}+\frac{5\,b^2\,\ln\left(\frac{5\,b^2\,\left(b\,x^2+a\right)^{1/3}}{2\,a^2}-\frac{5\,b^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,a^{5/3}}\right)\left(-\frac{1}{2}\cdots\right.}{18\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^(2/3)),x)
```

```
[Out] (5*b^2*log((a + b*x^2)^(1/3) - a^(1/3)))/(18*a^(8/3)) - ((4*b^2*(a + b*x^2)
^(1/3))/(3*a) - (5*b^2*(a + b*x^2)^(4/3))/(6*a^2))/(2*(a + b*x^2)^2 - 4*a*(
a + b*x^2) + 2*a^2) + (5*b^2*log((5*b^2*(a + b*x^2)^(1/3))/(2*a^2) - (5*b^2
*((3^(1/2)*1i)/2 - 1/2))/(2*a^(5/3)))*((3^(1/2)*1i)/2 - 1/2))/(18*a^(8/3))
- (5*b^2*log((5*b^2*(a + b*x^2)^(1/3))/(2*a^2) + (5*b^2*((3^(1/2)*1i)/2 + 1
/2))/(2*a^(5/3)))*((3^(1/2)*1i)/2 + 1/2))/(18*a^(8/3))
```

sympy [C] time = 1.45, size = 41, normalized size = 0.30

$$-\frac{\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{8}{3}\\\frac{11}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}}x^{\frac{16}{3}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(2/3),x)
```

```
[Out] -gamma(8/3)*hyper((2/3, 8/3), (11/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2
/3)*x**(16/3)*gamma(11/3))
```

3.721 $\int \frac{x^4}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=293

$$-\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-\right.}{55b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] -27/55*a*x*(b*x^2+a)^(1/3)/b^2+3/11*x^3*(b*x^2+a)^(1/3)/b-27/55*3^(3/4)*a^2
*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))
/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b
*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(
b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 236, 219}

$$-\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{55b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(2/3),x]
```

```
[Out] (-27*a*x*(a + b*x^2)^(1/3))/(55*b^2) + (3*x^3*(a + b*x^2)^(1/3))/(11*b) - (
27*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^2*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(55
*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+bx^2\right)^{2/3}}\,dx &= \frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{(9a)\int \frac{x^2}{\left(a+bx^2\right)^{2/3}}\,dx}{11b}\\ &= -\frac{27ax\sqrt[3]{a+bx^2}}{55b^2}+\frac{3x^3\sqrt[3]{a+bx^2}}{11b}+\frac{\left(27a^2\right)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{55b^2}\\ &= -\frac{27ax\sqrt[3]{a+bx^2}}{55b^2}+\frac{3x^3\sqrt[3]{a+bx^2}}{11b}+\frac{\left(81a^2\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{110b^3x}\\ &= -\frac{27ax\sqrt[3]{a+bx^2}}{55b^2}+\frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}\right.}}}{55b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{\cdots}\right.}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}\end{aligned}$$

Mathematica [C] time = 0.03, size = 79, normalized size = 0.27

$$\frac{3\left(9a^2x\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)-9a^2x-4abx^3+5b^2x^5\right)}{55b^2\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(-9*a^2*x - 4*a*b*x^3 + 5*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(2/3)*Hyperg
eometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)]))/(55*b^2*(a + b*x^2)^(2/3))
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^4}{\left(bx^2+a\right)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(x^4/(b*x^2 + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate(x^4/(b*x^2 + a)^(2/3), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(2/3),x)
[Out] int(x^4/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate(x^4/(b*x^2 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(2/3),x)
[Out] int(x^4/(a + b*x^2)^(2/3), x)
```

sympy [A] time = 0.85, size = 27, normalized size = 0.09

$$\frac{x^5{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(2/3),x)
[Out] x**5*hyper((2/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(2/3))
```

3.722 $\int \frac{x^2}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=269

$$\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3}{}$$

```
[Out] 3/5*x*(b*x^2+a)^(1/3)/b+3/5*3^(3/4)*a*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((
-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^
2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^2/x/(-
a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 236, 219}

$$\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{3x\sqrt[3]{a}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3))/(5*b) + (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (
a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(
2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{2/3}} dx = \frac{3x\sqrt[3]{a+bx^2}}{5b} - \frac{(3a)\int \frac{1}{\left(a+bx^2\right)^{2/3}} dx}{5b}$$

$$= \frac{3x\sqrt[3]{a+bx^2}}{5b} - \frac{\left(9a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}} dx, x, \sqrt[3]{a+bx^2}\right)}{10b^2x}$$

$$= \frac{3x\sqrt[3]{a+bx^2}}{5b} + \frac{3\, 3^{3/4}\sqrt{2-\sqrt{3}}\, a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\, F\left(\sin^{-1}\left(\cdots\right.\right.}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.23

$$\frac{3x\left(-a\left(\frac{bx^2}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{5b\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2,
-((b*x^2)/a)]))/(5*b*(a + b*x^2)^(2/3))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\left(bx^2+a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*x^2 + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(2/3), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(b\,x^{2}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(2/3),x)
```

```
[Out] int(x^2/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(bx^{2}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{2}}{{\left(b\,x^{2}+a\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(2/3),x)
```

```
[Out] int(x^2/(a + b*x^2)^(2/3), x)
```

sympy [A] time = 0.78, size = 27, normalized size = 0.10

$$\frac{x^{3}{}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|{\frac{bx^{2}e^{i\pi}}{a}}\right)}{3a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(2/3),x)
```

```
[Out] x**3*hyper((2/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(2/3))
```

3.723 $\int \frac{1}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=246

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

```
[Out] -3^(3/4)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3
^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^
(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)
))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3
))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {236, 219}

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-2/3),x]
```

```
[Out] -((3^(3/4)*Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
+ b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))^2)]))
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx=\frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{2bx}$$

$$=-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)\right|-}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.19

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-2/3),x]
[Out] (x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(2/3)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(-2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-2/3), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(2/3),x)
[Out] int(1/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-2/3), x)
```

mupad [B] time = 5.37, size = 37, normalized size = 0.15

$$\frac{x\left(\frac{b\,x^2}{a}+1\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{2}{3};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(2/3),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(2/3)*hypergeom([1/2, 2/3], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(2/3)
```

sympy [A] time = 0.76, size = 24, normalized size = 0.10

$$\frac{x{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(2/3),x)
```

```
[Out] x*hyper((1/2, 2/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(2/3)
```

3.724 $\int \frac{1}{x^2(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=265

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}\,ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{\sqrt[3]{a+bx^2}}{ax}$$

```
[Out] -(b*x^2+a)^(1/3)/a/x+1/3*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1
/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/
2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^
(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a/x/(-a^(1/3)
*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 236, 219}

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{\sqrt[3]{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(2/3)),x]
```

```
[Out] -((a + b*x^2)^(1/3)/(a*x)) + (Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(3^(1/4)*a*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a+bx^2\right)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx^2}}{ax}-\frac{b\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{3a}\\
&= -\frac{\sqrt[3]{a+bx^2}}{ax}-\frac{\sqrt{bx^2}\ \mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2ax}\\
&= -\frac{\sqrt[3]{a+bx^2}}{ax}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)}{\left(1-\sqrt{3}\right)}\right)\right)}{\sqrt[4]{3}\,ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.18

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(2/3)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/2, 2/3, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(2/3)))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{bx^4+ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)/(b*x^4 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*x^2), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{2}{3}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2 + a\right)^{\frac{2}{3}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*x^2), x)
```

mupad [B] time = 5.46, size = 40, normalized size = 0.15

$$-\frac{3\left(\frac{a}{b\,x^2}+1\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{2}{3},\frac{7}{6};\ \frac{13}{6};\ -\frac{a}{b\,x^2}\right)}{7\,x\,\left(b\,x^2+a\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(2/3)),x)
```

```
[Out] -(3*(a/(b*x^2) + 1)^(2/3)*hypergeom([2/3, 7/6], 13/6, -a/(b*x^2)))/(7*x*(a
+ b*x^2)^(2/3))
```

sympy [A] time = 0.90, size = 27, normalized size = 0.10

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{2}{3} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{2}{3}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(2/3),x)
```

```
[Out] -hyper((-1/2, 2/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(2/3)*x)
```

3.725 $\int \frac{1}{x^4\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=293

$$-\frac{7\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}\,a^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{7}{\cdots}$$

```
[Out] -1/3*(b*x^2+a)^(1/3)/a/x^3+7/9*b*(b*x^2+a)^(1/3)/a^2/x-7/27*b*(a^(1/3)-(b*x
^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(
1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+
(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/
2)-1/2*2^(1/2))*3^(3/4)/a^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+
a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 236, 219}

$$\frac{7b\sqrt[3]{a+bx^2}}{9a^2x}-\frac{7\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\cdots\right)}{9\sqrt[4]{3}\,a^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(2/3)),x]
```

```
[Out] -(a + b*x^2)^(1/3)/(3*a*x^3) + (7*b*(a + b*x^2)^(1/3))/(9*a^2*x) - (7*Sqrt[
2 - Sqrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b
*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a^2*x*Sqrt
[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx^2}}{3ax^3}-\frac{(7b)\int \frac{1}{x^2\left(a+bx^2\right)^{2/3}}\,dx}{9a}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{3ax^3}+\frac{7b\sqrt[3]{a+bx^2}}{9a^2x}+\frac{\left(7b^2\right)\int \frac{1}{\left(a+bx^2\right)^{2/3}}\,dx}{27a^2}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{3ax^3}+\frac{7b\sqrt[3]{a+bx^2}}{9a^2x}+\frac{\left(7b\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{18a^2x}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{3ax^3}+\frac{7b\sqrt[3]{a+bx^2}}{9a^2x}-\frac{7\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F}{9\sqrt[4]{3}\,a^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}\right.}}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.17

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(2/3)),x]
```

```
[Out] -1/3*((1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-3/2, 2/3, -1/2, -((b*x^2)/a)
])/(x^3*(a + b*x^2)^(2/3))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{bx^6+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)/(b*x^6 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*x^4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{2}{3}} x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{2}{3}} x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{4}\left(b\,x^{2}+a\right)^{2/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(2/3)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)^(2/3)), x)
```

sympy [A] time = 1.01, size = 32, normalized size = 0.11

$$-\frac{{}_{2}F_{1}\left(\left.\begin{matrix} -\frac{3}{2}, \frac{2}{3} \\ -\frac{1}{2} \end{matrix}\right| \frac{bx^{2}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(2/3),x)
```

```
[Out] -hyper((-3/2, 2/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(2/3)*x**3)
```

3.726 $\int \frac{x^7}{\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=80

$$\frac{3a^3}{2b^4\sqrt[3]{a+bx^2}}+\frac{9a^2\left(a+bx^2\right)^{2/3}}{4b^4}-\frac{9a\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^4}$$

```
[Out] 3/2*a^3/b^4/(b*x^2+a)^(1/3)+9/4*a^2*(b*x^2+a)^(2/3)/b^4-9/10*a*(b*x^2+a)^(5
/3)/b^4+3/16*(b*x^2+a)^(8/3)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^3}{2b^4\sqrt[3]{a+bx^2}}+\frac{9a^2\left(a+bx^2\right)^{2/3}}{4b^4}-\frac{9a\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a^3)/(2*b^4*(a + b*x^2)^(1/3)) + (9*a^2*(a + b*x^2)^(2/3))/(4*b^4) - (9*
a*(a + b*x^2)^(5/3))/(10*b^4) + (3*(a + b*x^2)^(8/3))/(16*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a+bx^2\right)^{4/3}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{(a+bx)^{4/3}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{4/3}}+\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{(a+bx)^{5/3}}{b^3}\right)dx,x,x^2\right)\\
&= \frac{3a^3}{2b^4\sqrt[3]{a+bx^2}}+\frac{9a^2\left(a+bx^2\right)^{2/3}}{4b^4}-\frac{9a\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^4}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.62

$$\frac{3\left(81a^3+27a^2bx^2-9ab^2x^4+5b^3x^6\right)}{80b^4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(81*a^3 + 27*a^2*b*x^2 - 9*a*b^2*x^4 + 5*b^3*x^6))/(80*b^4*(a + b*x^2)^(
1/3))
```

fricas [A] time = 0.76, size = 58, normalized size = 0.72

$$\frac{3\left(5\,b^{3}x^{6}-9\,ab^{2}x^{4}+27\,a^{2}bx^{2}+81\,a^{3}\right)\left(bx^{2}+a\right)^{\frac{2}{3}}}{80\left(b^{5}x^{2}+ab^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/80*(5*b^3*x^6 - 9*a*b^2*x^4 + 27*a^2*b*x^2 + 81*a^3)*(b*x^2 + a)^(2/3)/(b
^5*x^2 + a*b^4)
```

giac [A] time = 0.59, size = 70, normalized size = 0.88

$$\frac{3\,a^{3}}{2\left(bx^{2}+a\right)^{\frac{1}{3}}b^{4}}+\frac{3\left(5\left(bx^{2}+a\right)^{\frac{8}{3}}b^{28}-24\left(bx^{2}+a\right)^{\frac{5}{3}}ab^{28}+60\left(bx^{2}+a\right)^{\frac{2}{3}}a^{2}b^{28}\right)}{80\,b^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/2*a^3/((b*x^2 + a)^(1/3)*b^4) + 3/80*(5*(b*x^2 + a)^(8/3)*b^28 - 24*(b*x^
2 + a)^(5/3)*a*b^28 + 60*(b*x^2 + a)^(2/3)*a^2*b^28)/b^32
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$\frac{\frac{3}{16}b^{3}x^{6}-\frac{27}{80}a\,b^{2}x^{4}+\frac{81}{80}a^{2}b\,x^{2}+\frac{243}{80}a^{3}}{\left(b\,x^{2}+a\right)^{\frac{1}{3}}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(4/3),x)
```

```
[Out] 3/80/(b*x^2+a)^(1/3)*(5*b^3*x^6-9*a*b^2*x^4+27*a^2*b*x^2+81*a^3)/b^4
```

maxima [A] time = 1.34, size = 64, normalized size = 0.80

$$\frac{3\left(bx^{2}+a\right)^{\frac{8}{3}}}{16\,b^{4}}-\frac{9\left(bx^{2}+a\right)^{\frac{5}{3}}a}{10\,b^{4}}+\frac{9\left(bx^{2}+a\right)^{\frac{2}{3}}a^{2}}{4\,b^{4}}+\frac{3\,a^{3}}{2\left(bx^{2}+a\right)^{\frac{1}{3}}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/16*(b*x^2 + a)^(8/3)/b^4 - 9/10*(b*x^2 + a)^(5/3)*a/b^4 + 9/4*(b*x^2 + a)
^(2/3)*a^2/b^4 + 3/2*a^3/((b*x^2 + a)^(1/3)*b^4)
```

mupad [B] time = 5.44, size = 55, normalized size = 0.69

$$\frac{180\,a^{2}\,\left(b\,x^{2}+a\right)-72\,a\left(b\,x^{2}+a\right)^{2}+15\left(b\,x^{2}+a\right)^{3}+120\,a^{3}}{80\,b^{4}\left(b\,x^{2}+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^(4/3),x)
```

```
[Out] (180*a^2*(a + b*x^2) - 72*a*(a + b*x^2)^2 + 15*(a + b*x^2)^3 + 120*a^3)/(80
*b^4*(a + b*x^2)^(1/3))
```

sympy [B] time = 2.84, size = 1584, normalized size = 19.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(4/3),x)
```

```
[Out] 243*a**(68/3)*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**2 +
1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a*
*15*b**9*x**10 + 80*a**14*b**10*x**12) - 243*a**(68/3)/(80*a**20*b**4 + 480
*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16
*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 1296*a**(65/3)*
b*x**2*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a*
*18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**
9*x**10 + 80*a**14*b**10*x**12) - 1458*a**(65/3)*b*x**2/(80*a**20*b**4 + 48
0*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**1
6*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 2808*a**(62/3)
*b**2*x**4*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 120
0*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15
*b**9*x**10 + 80*a**14*b**10*x**12) - 3645*a**(62/3)*b**2*x**4/(80*a**20*b*
*4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 12
00*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 3120*a*
*(59/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**
2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 48
0*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 4860*a**(59/3)*b**3*x**6/(80*a
**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x*
*6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) +
1830*a**(56/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b
**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x*
*8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 3645*a**(56/3)*b**4*x**
8/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*
b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x*
*12) + 528*a**(53/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*
a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*
b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 1458*a**(53/3)*b
**5*x**10/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 160
0*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*
b**10*x**12) + 96*a**(50/3)*b**6*x**12*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4
 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200
*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 243*a**(5
0/3)*b**6*x**12/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4
 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*
a**14*b**10*x**12) + 48*a**(47/3)*b**7*x**14*(1 + b*x**2/a)**(2/3)/(80*a**2
0*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6
+ 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 15*
a**(44/3)*b**8*x**16*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*
x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 +
 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12)
```

3.727 $\int \frac{x^5}{\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=59

$$-\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}}-\frac{3a\left(a+bx^2\right)^{2/3}}{2b^3}+\frac{3\left(a+bx^2\right)^{5/3}}{10b^3}$$

```
[Out] -3/2*a^2/b^3/(b*x^2+a)^(1/3)-3/2*a*(b*x^2+a)^(2/3)/b^3+3/10*(b*x^2+a)^(5/3)
/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}}-\frac{3a\left(a+bx^2\right)^{2/3}}{2b^3}+\frac{3\left(a+bx^2\right)^{5/3}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*a^2)/(2*b^3*(a + b*x^2)^(1/3)) - (3*a*(a + b*x^2)^(2/3))/(2*b^3) + (3*(
a + b*x^2)^(5/3))/(10*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^{4/3}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^{4/3}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{4/3}}-\frac{2a}{b^2\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b^2}\right)dx,x,x^2\right)\\
&= -\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}}-\frac{3a\left(a+bx^2\right)^{2/3}}{2b^3}+\frac{3\left(a+bx^2\right)^{5/3}}{10b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.64

$$\frac{3\left(-9a^2-3abx^2+b^2x^4\right)}{10b^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(4/3),x]
[Out] (3*(-9*a^2 - 3*a*b*x^2 + b^2*x^4))/(10*b^3*(a + b*x^2)^(1/3))
```

fricas [A] time = 0.83, size = 46, normalized size = 0.78

$$\frac{3\left(b^{2}x^{4}-3\,abx^{2}-9\,a^{2}\right)\left(bx^{2}+a\right)^{\frac{2}{3}}}{10\left(b^{4}x^{2}+ab^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] 3/10*(b^2*x^4 - 3*a*b*x^2 - 9*a^2)*(b*x^2 + a)^(2/3)/(b^4*x^2 + a*b^3)
```

giac [A] time = 0.58, size = 52, normalized size = 0.88

$$-\frac{3\,a^{2}}{2\left(bx^{2}+a\right)^{\frac{1}{3}}b^{3}}+\frac{3\left(\left(bx^{2}+a\right)^{\frac{5}{3}}b^{12}-5\left(bx^{2}+a\right)^{\frac{2}{3}}ab^{12}\right)}{10\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] -3/2*a^2/((b*x^2 + a)^(1/3)*b^3) + 3/10*((b*x^2 + a)^(5/3)*b^12 - 5*(b*x^2
+ a)^(2/3)*a*b^12)/b^15
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$-\frac{3\left(-b^{2}x^{4}+3ab\,x^{2}+9a^{2}\right)}{10\left(b\,x^{2}+a\right)^{\frac{1}{3}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(4/3),x)
[Out] -3/10/(b*x^2+a)^(1/3)*(-b^2*x^4+3*a*b*x^2+9*a^2)/b^3
```

maxima [A] time = 1.28, size = 47, normalized size = 0.80

$$\frac{3\left(bx^{2}+a\right)^{\frac{5}{3}}}{10\,b^{3}}-\frac{3\left(bx^{2}+a\right)^{\frac{2}{3}}a}{2\,b^{3}}-\frac{3\,a^{2}}{2\left(bx^{2}+a\right)^{\frac{1}{3}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] 3/10*(b*x^2 + a)^(5/3)/b^3 - 3/2*(b*x^2 + a)^(2/3)*a/b^3 - 3/2*a^2/((b*x^2
+ a)^(1/3)*b^3)
```

mupad [B] time = 5.36, size = 41, normalized size = 0.69

$$-\frac{15\,a\,\left(b\,x^{2}+a\right)-3\,\left(b\,x^{2}+a\right)^{2}+15\,a^{2}}{10\,b^{3}\,\left(b\,x^{2}+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(4/3),x)
```

```
[Out] -(15*a*(a + b*x^2) - 3*(a + b*x^2)^2 + 15*a^2)/(10*b^3*(a + b*x^2)^(1/3))
```

sympy [B] time = 1.82, size = 561, normalized size = 9.51

$$-\frac{27a^{\frac{29}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}+\frac{27a^{\frac{29}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}-\frac{63a^{\frac{26}{3}}bx}{10a^8b^3+30a^7b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(4/3),x)
```

```
[Out] -27*a**(29/3)*(1 + b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*
a**6*b**5*x**4 + 10*a**5*b**6*x**6) + 27*a**(29/3)/(10*a**8*b**3 + 30*a**7*
b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 63*a**(26/3)*b*x**2*(1
 + b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 +
 10*a**5*b**6*x**6) + 81*a**(26/3)*b*x**2/(10*a**8*b**3 + 30*a**7*b**4*x**2
 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 42*a**(23/3)*b**2*x**4*(1 + b*x
**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a*
*5*b**6*x**6) + 81*a**(23/3)*b**2*x**4/(10*a**8*b**3 + 30*a**7*b**4*x**2 +
30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 3*a**(20/3)*b**3*x**6*(1 + b*x**2/
a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b
**6*x**6) + 27*a**(20/3)*b**3*x**6/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a
**6*b**5*x**4 + 10*a**5*b**6*x**6) + 3*a**(17/3)*b**4*x**8*(1 + b*x**2/a)**
(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*
x**6)
```

3.728 $\int \frac{x^3}{\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=38

$$\frac{3a}{2b^2\sqrt[3]{a+bx^2}}+\frac{3\left(a+bx^2\right)^{2/3}}{4b^2}$$

```
[Out] 3/2*a/b^2/(b*x^2+a)^(1/3)+3/4*(b*x^2+a)^(2/3)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a}{2b^2\sqrt[3]{a+bx^2}}+\frac{3\left(a+bx^2\right)^{2/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a)/(2*b^2*(a + b*x^2)^(1/3)) + (3*(a + b*x^2)^(2/3))/(4*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^{4/3}}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x}{(a+bx)^{4/3}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{4/3}}+\frac{1}{b\sqrt[3]{a+bx}}\right)dx,x,x^2\right)\\
&= \frac{3a}{2b^2\sqrt[3]{a+bx^2}}+\frac{3\left(a+bx^2\right)^{2/3}}{4b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$\frac{3\left(3a+bx^2\right)}{4b^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(3*a + b*x^2))/(4*b^2*(a + b*x^2)^(1/3))
```

fricas [A] time = 0.86, size = 35, normalized size = 0.92

$$\frac{3\left(bx^2+3\,a\right)\left(bx^2+a\right)^{\frac{2}{3}}}{4\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] 3/4*(b*x^2 + 3*a)*(b*x^2 + a)^(2/3)/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.57, size = 34, normalized size = 0.89

$$\frac{3\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{b}+\frac{2\,a}{\left(bx^2+a\right)^{\frac{1}{3}}b}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] 3/4*((b*x^2 + a)^(2/3)/b + 2*a/((b*x^2 + a)^(1/3)*b))/b
```

maple [A] time = 0.00, size = 24, normalized size = 0.63

$$\frac{\frac{3b\,x^2}{4}+\frac{9a}{4}}{\left(b\,x^2+a\right)^{\frac{1}{3}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(4/3),x)
[Out] 3/4/(b*x^2+a)^(1/3)*(b*x^2+3*a)/b^2
```

maxima [A] time = 1.35, size = 30, normalized size = 0.79

$$\frac{3\left(bx^2+a\right)^{\frac{2}{3}}}{4\,b^2}+\frac{3\,a}{2\left(bx^2+a\right)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] 3/4*(b*x^2 + a)^(2/3)/b^2 + 3/2*a/((b*x^2 + a)^(1/3)*b^2)
```

mupad [B] time = 5.59, size = 24, normalized size = 0.63

$$\frac{3\,b\,x^2+9\,a}{4\,b^2\left(b\,x^2+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(4/3),x)
[Out] (9*a + 3*b*x^2)/(4*b^2*(a + b*x^2)^(1/3))
```

sympy [A] time = 0.70, size = 46, normalized size = 1.21

$$\begin{cases} \frac{9a}{4b^2\sqrt[3]{a+bx^2}} + \frac{3x^2}{4b\sqrt[3]{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((9*a/(4*b**2*(a + b*x**2)**(1/3)) + 3*x**2/(4*b*(a + b*x**2)**(1/
3)), Ne(b, 0)), (x**4/(4*a**(4/3)), True))
```

3.729 $\int \frac{x}{(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=18

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

```
[Out] -3/2/b/(b*x^2+a)^(1/3)
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(4/3),x]
[Out] -3/(2*b*(a + b*x^2)^(1/3))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^{4/3}}\,dx = -\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(4/3),x]
[Out] -3/(2*b*(a + b*x^2)^(1/3))
```

fricas [A] time = 0.70, size = 24, normalized size = 1.33

$$-\frac{3\left(bx^2+a\right)^{\frac{2}{3}}}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] -3/2*(b*x^2 + a)^(2/3)/(b^2*x^2 + a*b)
```

giac [A] time = 0.58, size = 14, normalized size = 0.78

$$-\frac{3}{2\left(bx^2+a\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] -3/2/((b*x^2 + a)^(1/3)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{3}{2\left(b\,x^{2}+a\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(4/3),x)
[Out] -3/2/b/(b*x^2+a)^(1/3)
```

maxima [A] time = 1.32, size = 14, normalized size = 0.78

$$-\frac{3}{2\left(bx^{2}+a\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] -3/2/((b*x^2 + a)^(1/3)*b)
```

mupad [B] time = 5.39, size = 14, normalized size = 0.78

$$-\frac{3}{2\,b\,\left(b\,x^2+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(4/3),x)
[Out] -3/(2*b*(a + b*x^2)^(1/3))
```

sympy [A] time = 0.68, size = 26, normalized size = 1.44

$$\begin{cases} - \frac{3}{2 b \sqrt[3]{a + b x^{2}}} & \text{for}\: b \neq 0 \\ \frac{x^{2}}{2 a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(4/3),x)
[Out] Piecewise((-3/(2*b*(a + b*x**2)**(1/3)), Ne(b, 0)), (x**2/(2*a**(4/3)), Tru
e))
```

3.730 $\int \frac{1}{x(a+bx^2)^{4/3}} \, dx$

Optimal. Leaf size=104

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{2a\sqrt[3]{a+bx^2}}$$

```
[Out] 3/2/a/(b*x^2+a)^(1/3)-1/2*ln(x)/a^(4/3)+3/4*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(
4/3)+1/2*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^
(4/3)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 55, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{2a\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*(a + b*x^2)^(1/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))
/(Sqrt[3]*a^(1/3))])/(2*a^(4/3)) - Log[x]/(2*a^(4/3)) + (3*Log[a^(1/3) - (a
 + b*x^2)^(1/3)])/(4*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^{4/3}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{4/3}}\,dx, x, x^2\right)$$

$$= \frac{3}{2a\sqrt[3]{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{2a}$$

$$= \frac{3}{2a\sqrt[3]{a+bx^2}} - \frac{\log(x)}{2a^{4/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^{4/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x\right.}{4a}$$

$$= \frac{3}{2a\sqrt[3]{a+bx^2}} - \frac{\log(x)}{2a^{4/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2a^{4/3}}$$

$$= \frac{3}{2a\sqrt[3]{a+bx^2}} + \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2a^{4/3}} - \frac{\log(x)}{2a^{4/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.35

$$\frac{3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{bx^2}{a}+1\right)}{2a\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^2)^(4/3)),x]

[Out] (3*Hypergeometric2F1[-1/3, 1, 2/3, 1 + (b*x^2)/a])/(2*a*(a + b*x^2)^(1/3))

fricas [A] time = 1.01, size = 327, normalized size = 3.14

$$\left[\frac{\sqrt{3}\left(abx^2+a^2\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^2+\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x^2}\right)-\left(bx^2+a\right)a^{\frac{2}{3}}\log\left(\left(bx^2+a\right.\right.}{4\left(a^2bx^2+a^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*(a*b*x^2 + a^2)*sqrt(-1/a^(2/3))*log((2*b*x^2 + sqrt(3)*(2*(b
*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) -
 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - (b*x^2 + a)*a^(2/3)*log((b*x^2 +
 a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b*x^2 + a)*a^(2/3)*lo
g((b*x^2 + a)^(1/3) - a^(1/3)) + 6*(b*x^2 + a)^(2/3)*a)/(a^2*b*x^2 + a^3),
-1/4*((b*x^2 + a)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3)
 + a^(2/3)) - 2*(b*x^2 + a)*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) - 2*sq
rt(3)*(a*b*x^2 + a^2)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^
(1/3))/a^(1/3) - 6*(b*x^2 + a)^(2/3)*a)/(a^2*b*x^2 + a^3)]
```

giac [A] time = 1.10, size = 101, normalized size = 0.97

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{4}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4a^{\frac{4}{3}}}+\frac{\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2a^{\frac{4}{3}}}+\frac{3}{2\left(bx^2+a\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
4/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
4/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 3/2/((b*x^2 + a)
^(1/3)*a)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{4}{3}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x/(b*x^2+a)^(4/3),x)
```

maxima [A] time = 3.02, size = 100, normalized size = 0.96

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{4}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4a^{\frac{4}{3}}}+\frac{\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2a^{\frac{4}{3}}}+\frac{3}{2\left(bx^2+a\right)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
4/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
4/3) + 1/2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(4/3) + 3/2/((b*x^2 + a)^(1/3
)*a)
```

mupad [B] time = 5.59, size = 123, normalized size = 1.18

$$\frac{\ln\left(18\,a\,\left(b\,x^2+a\right)^{1/3}-18\,a^{4/3}\right)}{2\,a^{4/3}}+\frac{3}{2\,a\,\left(b\,x^2+a\right)^{1/3}}+\frac{\ln\left(18\,a\,\left(b\,x^2+a\right)^{1/3}-\frac{9\,a^{4/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{4\,a^{4/3}}-\ln\left(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(4/3)),x)
```

```
[Out] log(18*a*(a + b*x^2)^(1/3) - 18*a^(4/3))/(2*a^(4/3)) + 3/(2*a*(a + b*x^2)^(
1/3)) + (log(18*a*(a + b*x^2)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1)^2)/2)*(3^
(1/2)*1i - 1))/(4*a^(4/3)) - (log(18*a*(a + b*x^2)^(1/3) - (9*a^(4/3)*(3^(1
/2)*1i + 1)^2)/2)*(3^(1/2)*1i + 1))/(4*a^(4/3))
```

sympy [C] time = 1.13, size = 41, normalized size = 0.39

$$-\frac{\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}}x^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(4/3),x)
```

```
[Out] -gamma(4/3)*hyper((4/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(4/
3)*x**(8/3)*gamma(7/3))
```

3.731 $\int \frac{1}{x^3(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=123

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{a^{7/3}}-\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}-\frac{2b}{a^2\sqrt[3]{a+bx^2}}-\frac{1}{2ax^2\sqrt[3]{a+bx^2}}$$

```
[Out] -2*b/a^2/(b*x^2+a)^(1/3)-1/2/a/x^2/(b*x^2+a)^(1/3)+2/3*b*ln(x)/a^(7/3)-b*ln
(a^(1/3)-(b*x^2+a)^(1/3))/a^(7/3)-2/3*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/
3))/a^(1/3)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 55, 617, 204, 31}

$$-\frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{a^{7/3}}-\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}+\frac{3}{2ax^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*x^2*(a + b*x^2)^(1/3)) - (2*(a + b*x^2)^(2/3))/(a^2*x^2) - (2*b*ArcT
an[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) +
(2*b*Log[x])/(3*a^(7/3)) - (b*Log[a^(1/3) - (a + b*x^2)^(1/3)])/a^(7/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)^{4/3}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{4/3}}\,dx,x,x^2\right)\\
&= \frac{3}{2ax^2\sqrt[3]{a+bx^2}}+\frac{2\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx,x,x^2\right)}{a}\\
&= \frac{3}{2ax^2\sqrt[3]{a+bx^2}}-\frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^2\right)}{3a^2}\\
&= \frac{3}{2ax^2\sqrt[3]{a+bx^2}}-\frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2}+\frac{2b\log(x)}{3a^{7/3}}+\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)}{a^{7/3}}-\frac{b\,\text{Sub}}{}\\
&= \frac{3}{2ax^2\sqrt[3]{a+bx^2}}-\frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2}+\frac{2b\log(x)}{3a^{7/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{a^{7/3}}+\frac{(2b)\,\text{Subst}\left(\int -\right.}{}\\
&= \frac{3}{2ax^2\sqrt[3]{a+bx^2}}-\frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2}-\frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+\right.}{a^{7/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.30

$$-\frac{3b\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{bx^2}{a}+1\right)}{2a^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(4/3)),x]
```

```
[Out] (-3*b*Hypergeometric2F1[-1/3, 2, 2/3, 1 + (b*x^2)/a])/(2*a^2*(a + b*x^2)^(1
/3))
```

fricas [B] time = 0.98, size = 453, normalized size = 3.68

$$\left[\frac{6\sqrt{\frac{1}{3}}\left(ab^2x^4+a^2bx^2\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2bx^2-3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\left(bx^2+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x^2}\right)+2\left(b^2x^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*sqrt(1/3)*(a*b^2*x^4 + a^2*b*x^2)*sqrt((-a)^(1/3)/a)*log((2*b*x^2 -
 3*sqrt(1/3)*(2*(b*x^2 + a)^(2/3)*(-a)^(2/3) - (b*x^2 + a)^(1/3)*a + (-a)^(
1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^2 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^2) + 2
*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(
-a)^(1/3) + (-a)^(2/3)) - 4*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^
(1/3) + (-a)^(1/3)) - 3*(4*a*b*x^2 + a^2)*(b*x^2 + a)^(2/3))/(a^3*b*x^4 + a
^4*x^2), -1/6*(12*sqrt(1/3)*(a*b^2*x^4 + a^2*b*x^2)*sqrt(-(-a)^(1/3)/a)*arc
tan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - 2*(
b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a
)^(1/3) + (-a)^(2/3)) + 4*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^(1
/3) + (-a)^(1/3)) + 3*(4*a*b*x^2 + a^2)*(b*x^2 + a)^(2/3))/(a^3*b*x^4 + a^4
*x^2)]
```

giac [A] time = 1.11, size = 134, normalized size = 1.09

$$-\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}}+\frac{b\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{7}{3}}}-\frac{2\,b\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{7}{3}}}-\frac{\cdots}{2\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(7/3) + 1/3*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(7/3) - 2/3*b*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(7/3) - 1/2*(4*(b
*x^2 + a)*b - 3*a*b)/(((b*x^2 + a)^(4/3) - (b*x^2 + a)^(1/3)*a)*a^2)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{4}{3}}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x^3/(b*x^2+a)^(4/3),x)
```

maxima [A] time = 3.01, size = 136, normalized size = 1.11

$$-\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}}-\frac{4\left(bx^2+a\right)b-3\,ab}{2\left(\left(bx^2+a\right)^{\frac{4}{3}}a^2-\left(bx^2+a\right)^{\frac{1}{3}}a^3\right)}+\frac{b\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right.}{3\,a^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(7/3) - 1/2*(4*(b*x^2 + a)*b - 3*a*b)/((b*x^2 + a)^(4/3)*a^2 - (b*x^2 + a
```

```
)^(1/3)*a^3) + 1/3*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(7/3) - 2/3*b*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(7/3)
```

mupad [B] time = 5.65, size = 178, normalized size = 1.45

$$-\frac{\frac{3\,b}{a}-\frac{4\,b\,\left(b\,x^2+a\right)}{a^2}}{2\,a\,{\left(b\,x^2+a\right)}^{1/3}-2\,{\left(b\,x^2+a\right)}^{4/3}}-\frac{2\,b\,\ln\left(4\,a^{7/3}\,b^2-4\,a^2\,b^2\,{\left(b\,x^2+a\right)}^{1/3}\right)}{3\,a^{7/3}}+\frac{\ln\left(a^{7/3}\,{\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}^2-4\,a^2\,b^2\,\left(b\,x^2\right.\right.}{3\,a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(4/3)),x)
```

```
[Out] (log(a^(7/3)*(b - 3^(1/2)*b*1i)^2 - 4*a^2*b^2*(a + b*x^2)^(1/3))*(b - 3^(1/
2)*b*1i))/(3*a^(7/3)) - (2*b*log(4*a^(7/3)*b^2 - 4*a^2*b^2*(a + b*x^2)^(1/3
)))/(3*a^(7/3)) - ((3*b)/a - (4*b*(a + b*x^2))/a^2)/(2*a*(a + b*x^2)^(1/3)
- 2*(a + b*x^2)^(4/3)) + (log(a^(7/3)*(b + 3^(1/2)*b*1i)^2 - 4*a^2*b^2*(a +
 b*x^2)^(1/3))*(b + 3^(1/2)*b*1i))/(3*a^(7/3))
```

sympy [C] time = 1.35, size = 41, normalized size = 0.33

$$-\frac{\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{4}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}}x^{\frac{14}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(4/3),x)
```

```
[Out] -gamma(7/3)*hyper((4/3, 7/3), (10/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(4
/3)*x**(14/3)*gamma(10/3))
```

3.732 $\int \frac{1}{x^5\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=159

$$\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{10/3}}+\frac{7b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2}{3a^3\sqrt[3]{a+bx^2}}+\frac{7b}{12a^2x^2\sqrt[3]{a+bx^2}}-\frac{1}{4ax^4\sqrt[3]{a+bx^2}}$$

```
[Out] 7/3*b^2/a^3/(b*x^2+a)^(1/3)-1/4/a/x^4/(b*x^2+a)^(1/3)+7/12*b/a^2/x^2/(b*x^2
+a)^(1/3)-7/9*b^2*ln(x)/a^(10/3)+7/6*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(10/
3)+7/9*b^2*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))/a^(1/3)*3^(1/2))/a^(10/3)
*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 55, 617, 204, 31}

$$\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{10/3}}+\frac{7b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2}-\frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4}+\frac{3}{2ax^4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*x^4*(a + b*x^2)^(1/3)) - (7*(a + b*x^2)^(2/3))/(4*a^2*x^4) + (7*b*(a
 + b*x^2)^(2/3))/(3*a^3*x^2) + (7*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3)
)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (7*b^2*Log[x])/(9*a^(10/3)) +
(7*b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(a+bx^2\right)^{4/3}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{4/3}}\,dx, x, x^2\right)\\
&= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} + \frac{7\,\text{Subst}\left(\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{2a}\\
&= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} - \frac{(7b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{3a^2}\\
&= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} + \frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2} + \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{9a^3}\\
&= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} + \frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2} - \frac{7b^2\log(x)}{9a^{10/3}} - \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx\right.}{6a^{10/3}}\\
&= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} + \frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2} - \frac{7b^2\log(x)}{9a^{10/3}} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx\right.}}{6a^{10/3}}\\
&= \frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} + \frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2} + \frac{7b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{10/3}} - \frac{7b^2\log(x)}{9a^{10/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.25

$$\frac{3b^2\,{}_2F_1\left(-\frac{1}{3},3;\frac{2}{3};\frac{bx^2}{a}+1\right)}{2a^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^(4/3)),x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-1/3, 3, 2/3, 1 + (b*x^2)/a])/(2*a^3*(a + b*x^2)^(
1/3))
```

fricas [A] time = 0.81, size = 437, normalized size = 2.75

$$\left[\frac{42\sqrt{\frac{1}{3}}\left(ab^3x^6+a^2b^2x^4\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^2+3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x^2}\right)-14\left(b^3x^6+a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/36*(42*sqrt(1/3)*(a*b^3*x^6 + a^2*b^2*x^4)*sqrt(-1/a^(2/3))*log((2*b*x^2
 + 3*sqrt(1/3)*(2*(b*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3)
)*sqrt(-1/a^(2/3)) - 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - 14*(b^3*x^6
+ a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3)) + 28*(b^3*x^6 + a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3))
+ 3*(28*a*b^2*x^4 + 7*a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(2/3))/(a^4*b*x^6 + a^
5*x^4), -1/36*(14*(b^3*x^6 + a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*
x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) - 28*(b^3*x^6 + a*b^2*x^4)*a^(2/3)*log((b
*x^2 + a)^(1/3) - a^(1/3)) - 84*sqrt(1/3)*(a*b^3*x^6 + a^2*b^2*x^4)*arctan(
sqrt(1/3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 3*(28*a*b^2*x^
4 + 7*a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(2/3))/(a^4*b*x^6 + a^5*x^4)]
```

giac [A] time = 1.10, size = 154, normalized size = 0.97

$$\frac{7\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{10}{3}}}-\frac{7b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{10}{3}}}+\frac{7b^2\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9a^{\frac{10}{3}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 7/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(10/3) - 7/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(10/3) + 7/9*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(10/3) +
3/2*b^2/((b*x^2 + a)^(1/3)*a^3) + 1/12*(10*(b*x^2 + a)^(5/3)*b^2 - 13*(b*x^
2 + a)^(2/3)*a*b^2)/(a^3*b^2*x^4)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{4}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x^5/(b*x^2+a)^(4/3),x)
```

maxima [A] time = 2.97, size = 176, normalized size = 1.11

$$\frac{7\,\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{10}{3}}}+\frac{28\,\left(bx^2+a\right)^2b^2-49\,\left(bx^2+a\right)ab^2+18\,a^2b^2}{12\,\left(\left(bx^2+a\right)^{\frac{7}{3}}a^3-2\,\left(bx^2+a\right)^{\frac{4}{3}}a^4+\left(bx^2+a\right)^{\frac{1}{3}}a^5\right)}-\frac{7\,b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(\right.\right.}{18\,a^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 7/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(10/3) + 1/12*(28*(b*x^2 + a)^2*b^2 - 49*(b*x^2 + a)*a*b^2 + 18*a^2*b^2)
/((b*x^2 + a)^(7/3)*a^3 - 2*(b*x^2 + a)^(4/3)*a^4 + (b*x^2 + a)^(1/3)*a^5)
- 7/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
10/3) + 7/9*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(10/3)
```

mupad [B] time = 5.67, size = 224, normalized size = 1.41

$$\frac{\frac{3\,b^2}{a}-\frac{49\,b^2\,\left(b\,x^2+a\right)}{6\,a^2}+\frac{14\,b^2\,\left(b\,x^2+a\right)^2}{3\,a^3}}{2\,\left(b\,x^2+a\right)^{7/3}-4\,a\,\left(b\,x^2+a\right)^{4/3}+2\,a^2\,\left(b\,x^2+a\right)^{1/3}}+\frac{7\,b^2\,\ln\left(147\,a^3\,b^4\,\left(b\,x^2+a\right)^{1/3}-147\,a^{10/3}\,b^4\right)}{9\,a^{10/3}}+\frac{7\,b^2\,\ln\left(147\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^(4/3)),x)
```

```
[Out] ((3*b^2)/a - (49*b^2*(a + b*x^2))/(6*a^2) + (14*b^2*(a + b*x^2)^2)/(3*a^3))
/(2*(a + b*x^2)^(7/3) - 4*a*(a + b*x^2)^(4/3) + 2*a^2*(a + b*x^2)^(1/3)) +
(7*b^2*log(147*a^3*b^4*(a + b*x^2)^(1/3) - 147*a^(10/3)*b^4))/(9*a^(10/3))
+ (7*b^2*log(147*a^3*b^4*(a + b*x^2)^(1/3) - 147*a^(10/3)*b^4*((3^(1/2)*1i)
/2 - 1/2)^2)*((3^(1/2)*1i)/2 - 1/2))/(9*a^(10/3)) - (7*b^2*log(147*a^3*b^4*
(a + b*x^2)^(1/3) - 147*a^(10/3)*b^4*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1/2)*1i
)/2 + 1/2))/(9*a^(10/3))
```

sympy [C] time = 1.62, size = 41, normalized size = 0.26

$$-\frac{\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{10}{3}\\\frac{13}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}}x^{\frac{20}{3}}\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(4/3),x)
```

```
[Out] -gamma(10/3)*hyper((4/3, 10/3), (13/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**
(4/3)*x**(20/3)*gamma(13/3))
```

3.733 $\int \frac{x^4}{(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=577

$$\frac{27\, 3^{3/4} a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{7\sqrt{2}\,b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}} - 81\sqrt[4]{\cdots}$$

```
[Out] -3/2*x^3/b/(b*x^2+a)^(1/3)+27/14*x*(b*x^2+a)^(2/3)/b^2+81/14*a*x/b^2/(-(b*x
^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+27/14*3^(3/4)*a^(4/3)*(a^(1/3)-(b*x^2+a)^(
1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^
(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+
a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b^3/x*2^(1/2)/(-a
^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^
(1/2)-81/28*3^(1/4)*a^(4/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)
^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^
(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^3/x/(-a^(1/3)*(a
^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {288, 321, 235, 304, 219, 1879}

$$\frac{27\, 3^{3/4} a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{7\sqrt{2}\,b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}} - 81\sqrt[4]{3}\sqrt{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*x^3)/(2*b*(a + b*x^2)^(1/3)) + (27*x*(a + b*x^2)^(2/3))/(14*b^2) + (81*
a*x)/(14*b^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (81*3^(1/4)*Sqr
t[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3
)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^
2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/(
(1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(28*b^3*x*Sqr
t[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))^2)]) + (27*3^(3/4)*a^(4/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[
(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^
(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
+ b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3
]])/(7*Sqrt[2]*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{4/3}}\,dx = -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{9\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx}{2b}$$

$$= -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{27x\left(a+bx^2\right)^{2/3}}{14b^2} - \frac{(27a)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{14b^2}$$

$$= -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{27x\left(a+bx^2\right)^{2/3}}{14b^2} - \frac{\left(81a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{28b^3x}$$

$$= -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{27x\left(a+bx^2\right)^{2/3}}{14b^2} + \frac{\left(81a\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{28b^3x} -$$

$$= -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{27x\left(a+bx^2\right)^{2/3}}{14b^2} + \frac{81ax}{14b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\cdots}{\cdots}$$

Mathematica [C] time = 0.03, size = 65, normalized size = 0.11

$$\frac{3x\left(-9a\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+9a+2bx^2\right)}{14b^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*x*(9*a + 2*b*x^2 - 9*a*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2,
 3/2, -((b*x^2)/a)]))/(14*b^2*(a + b*x^2)^(1/3))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}x^4}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(4/3), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(4/3),x)
```

```
[Out] int(x^4/(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(4/3),x)
```

```
[Out] int(x^4/(a + b*x^2)^(4/3), x)
```

sympy [A] time = 0.88, size = 27, normalized size = 0.05

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{4}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(4/3),x)
```

```
[Out] x**5*hyper((4/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(4/3))
```

3.734 $\int \frac{x^2}{\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=553

$$-\frac{3\,3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{\sqrt{2}\,b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{9\sqrt[4]{3}}{}$$

```
[Out] -3/2*x/b/(b*x^2+a)^(1/3)-9/2*x/b/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-3/2
*3^(3/4)*a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1
/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^
(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1
-3^(1/2)))^2)^(1/2)/b^2/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+9/4*3^(1/4)*a^(1/3)*(a^(1/3)-(b*
x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)
+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1
/2)+1/2*2^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3
)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {288, 235, 304, 219, 1879}

$$-\frac{3\,3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}\,b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*x)/(2*b*(a + b*x^2)^(1/3)) - (9*x)/(2*b*((1 - Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)
^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1
- Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*
a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(4*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((
1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (3*3^(3/4)*a^(1/3)*(a^(1/3
) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x
^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (
a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^2\right)^{4/3}}\,dx &= -\frac{3x}{2b\sqrt[3]{a+bx^2}}+\frac{3\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{2b}\\
&= -\frac{3x}{2b\sqrt[3]{a+bx^2}}+\frac{\left(9\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{4b^2x}\\
&= -\frac{3x}{2b\sqrt[3]{a+bx^2}}-\frac{\left(9\sqrt{bx^2}\right)\mathrm{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a+bx^2}\right)}{4b^2x}+\frac{\left(9\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt[3]{a}\sqrt{b\cdots}\right.}{\cdots}\\
&= -\frac{3x}{2b\sqrt[3]{a+bx^2}}-\frac{9x}{2b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\cdots}}{4b\cdots}\end{aligned}$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.10

$$\frac{3x\left(\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-1\right)}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(4/3),x]
[Out] (3*x*(-1 + (1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)
/a)]))/(2*b*(a + b*x^2)^(1/3))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}x^2}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate(x^2/(b*x^2 + a)^(4/3), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(4/3),x)
[Out] int(x^2/(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate(x^2/(b*x^2 + a)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(b\,x^2+a\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(4/3),x)
```

```
[Out] int(x^2/(a + b*x^2)^(4/3), x)
```

sympy [A] time = 0.88, size = 27, normalized size = 0.05

$$\frac{x^{3}{}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(4/3),x)
```

```
[Out] x**3*hyper((4/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(4/3))
```

3.735 $\int \frac{1}{\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=552

$$\frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{\sqrt{2}\,a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{3\sqrt[4]{3}\sqrt{2+}}{}$$

```
[Out] 3/2*x/a/(b*x^2+a)^(1/3)+3/2*x/a/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1/2*
3^(3/4)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^
(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(
1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)/a^(2/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-3/4*3^(1/4)*(a^(1/3)-(b*x^2+a)^(1/3
))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/
3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^
(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(
1/2))/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(
1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {199, 235, 304, 219, 1879}

$$\frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}\,a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\ (}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-4/3),x]
```

```
[Out] (3*x)/(2*a*(a + b*x^2)^(1/3)) + (3*x)/(2*a*((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*S
qrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(4*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 -
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (3^(3/4)*(a^(1/3) - (a + b*x^
2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((
1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))]
, -7 + 4*Sqrt[3]])/(Sqrt[2]*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x
^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{4/3}}\,dx = \frac{3x}{2a\sqrt[3]{a+bx^2}} - \frac{\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{2a}$$

$$= \frac{3x}{2a\sqrt[3]{a+bx^2}} - \frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4abx}$$

$$= \frac{3x}{2a\sqrt[3]{a+bx^2}} + \frac{\left(3\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4abx} - \frac{\left(3\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt{bx^2}\right)\text{S}}{}$$

$$= \frac{3x}{2a\sqrt[3]{a+bx^2}} + \frac{3x}{2a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{}}{\left((1-\right.}}}{4a^{2/3}bx}$$

Mathematica [C] time = 0.01, size = 58, normalized size = 0.11

$$\frac{3x-x\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{2a\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-4/3),x]
```

```
[Out] (3*x - x*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a
)])/(2*a*(a + b*x^2)^(1/3))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-4/3), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-4/3), x)
```

mupad [B] time = 5.42, size = 37, normalized size = 0.07

$$\frac{x\left(\frac{b\,x^2}{a}+1\right)^{4/3}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{4}{3};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(4/3),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(4/3)*hypergeom([1/2, 4/3], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(4/3)
```

sympy [A] time = 0.82, size = 24, normalized size = 0.04

$$\frac{x {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{4}{3} \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(4/3),x)
```

```
[Out] x*hyper((1/2, 4/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(4/3)
```

3.736 $\int \frac{1}{x^2(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=571

$$-\frac{5\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{\sqrt{2}\sqrt[4]{3}\,a^{5/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}+\frac{5\sqrt[4]{3}\sqrt{2+\cdots}}{\cdots}$$

[Out] 3/2/a/x/(b*x^2+a)^(1/3)-5/2*(b*x^2+a)^(2/3)/a^2/x-5/2*b*x/a^2/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-5/6*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+5/4*3^(1/4)*(a^(1/3)-(b*x^2+a)^(1/3))*EllipticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/a^(5/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A] time = 0.35, antiderivative size = 571, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {290, 325, 235, 304, 219, 1879}

$$-\frac{5bx}{2a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}-\frac{5\left(a+bx^2\right)^{2/3}}{2a^2x}-\frac{5\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1+\cdots}{(1-\cdots}\right.\right.}{\sqrt{2}\sqrt[4]{3}\,a^{5/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*x^2)^(4/3)),x]

[Out] 3/(2*a*x*(a + b*x^2)^(1/3)) - (5*(a + b*x^2)^(2/3))/(2*a^2*x) - (5*b*x)/(2*a^2*((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (5*3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*a^(5/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (5*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*3^(1/4)*a^(5/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^{4/3}}\,dx = \frac{3}{2ax\sqrt[3]{a+bx^2}} + \frac{5\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx}{2a}$$
$$= \frac{3}{2ax\sqrt[3]{a+bx^2}} - \frac{5\left(a+bx^2\right)^{2/3}}{2a^2x} + \frac{(5b)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{6a^2}$$
$$= \frac{3}{2ax\sqrt[3]{a+bx^2}} - \frac{5\left(a+bx^2\right)^{2/3}}{2a^2x} + \frac{\left(5\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^2x}$$
$$= \frac{3}{2ax\sqrt[3]{a+bx^2}} - \frac{5\left(a+bx^2\right)^{2/3}}{2a^2x} - \frac{\left(5\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^2x} + \frac{(}{}$$
$$= \frac{3}{2ax\sqrt[3]{a+bx^2}} - \frac{5\left(a+bx^2\right)^{2/3}}{2a^2x} - \frac{5bx}{2a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} + \frac{5\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\left(\right.}{}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.09

$$-\frac{\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{2},\frac{4}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{ax\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(4/3)),x]
[Out] -(((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-1/2, 4/3, 1/2, -((b*x^2)/a)])/(
a*x*(a + b*x^2)^(1/3)))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{b^2x^6+2\,abx^4+a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(2/3)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{4}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(4/3)*x^2), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{4}{3}} x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{4}{3}} x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(4/3)*x^2), x)
```

mupad [B] time = 5.57, size = 40, normalized size = 0.07

$$-\frac{3\left(\frac{a}{b\,x^2}+1\right)^{4/3}{}_2\mathrm{F}_1\left(\frac{4}{3},\frac{11}{6};\ \frac{17}{6};\ -\frac{a}{b\,x^2}\right)}{11\,x\,\left(b\,x^2+a\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(4/3)),x)
```

```
[Out] -(3*(a/(b*x^2) + 1)^(4/3)*hypergeom([4/3, 11/6], 17/6, -a/(b*x^2)))/(11*x*(
a + b*x^2)^(4/3))
```

sympy [A] time = 1.02, size = 27, normalized size = 0.05

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{4}{3}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(4/3),x)
```

```
[Out] -hyper((-1/2, 4/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(4/3)*x)
```

3.737 $\int \frac{1}{x^4\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=599

$$\frac{55b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),4\sqrt{3}-7\right)}{9\sqrt{2}\sqrt[4]{3}\,a^{8/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{55\sqrt{2+\sqrt{3}}\cdots}{\cdots}$$

```
[Out] 3/2/a/x^3/(b*x^2+a)^(1/3)-11/6*(b*x^2+a)^(2/3)/a^2/x^3+55/18*b*(b*x^2+a)^(2
/3)/a^3/x+55/18*b^2*x/a^3/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+55/54*b*(a
^(1/3)-(b*x^2+a)^(1/3))*EllipticF((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-
(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^
2+a)^(1/3)+(b*x^2+a)^(2/3))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*3^(3/4)/a^(8/3)/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-55/36*b*(a^(1/3)-(b*x^2+a)^(1/3))*Ellip
ticE((-(b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^
(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(
-(b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^
(1/4)/a^(8/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(-(b*x^2+a)^(1/3)+a^(1/
3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 599, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {290, 325, 235, 304, 219, 1879}

$$\frac{55b^2x}{18a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{55b\left(a+bx^2\right)^{2/3}}{18a^3x}-\frac{11\left(a+bx^2\right)^{2/3}}{6a^2x^3}+\frac{55b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots\right)^2}}\cdots}{9\sqrt{2}\sqrt[4]{3}\,a^{8/3}x\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*x^3*(a + b*x^2)^(1/3)) - (11*(a + b*x^2)^(2/3))/(6*a^2*x^3) + (55*b*
(a + b*x^2)^(2/3))/(18*a^3*x) + (55*b^2*x)/(18*a^3*((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))) - (55*Sqrt[2 + Sqrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3)
)*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt
[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4
*Sqrt[3]])/(12*3^(3/4)*a^(8/3)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/
3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (55*b*(a^(1/3) - (a
 + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2
/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(9*Sqrt[2]*3^(1/4)*a^(8/3)*x*Sqrt[-((a^(1/3)*(a^(
1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{4/3}}\,dx = \frac{3}{2ax^3\sqrt[3]{a+bx^2}} + \frac{11\int \frac{1}{x^4\sqrt[3]{a+bx^2}}\,dx}{2a}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11\left(a+bx^2\right)^{2/3}}{6a^2x^3} - \frac{(55b)\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx}{18a^2}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11\left(a+bx^2\right)^{2/3}}{6a^2x^3} + \frac{55b\left(a+bx^2\right)^{2/3}}{18a^3x} - \frac{\left(55b^2\right)\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx}{54a^3}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11\left(a+bx^2\right)^{2/3}}{6a^2x^3} + \frac{55b\left(a+bx^2\right)^{2/3}}{18a^3x} - \frac{\left(55b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx\right.}{36a^3x}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11\left(a+bx^2\right)^{2/3}}{6a^2x^3} + \frac{55b\left(a+bx^2\right)^{2/3}}{18a^3x} + \frac{\left(55b\sqrt{bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-a+x^3}}\right.}{36a^3x}$$

$$= \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11\left(a+bx^2\right)^{2/3}}{6a^2x^3} + \frac{55b\left(a+bx^2\right)^{2/3}}{18a^3x} + \frac{55b^2x}{18a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right.}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.09

$$-\frac{\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{2},\frac{4}{3};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3ax^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(4/3)),x]
```

```
[Out] -1/3*((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-3/2, 4/3, -1/2, -((b*x^2)/a)
])/(a*x^3*(a + b*x^2)^(1/3))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{b^2x^8+2\,abx^6+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(2/3)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{4}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(4/3)*x^4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{4}{3}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{4}{3}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(4/3)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^{4/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(4/3)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)^(4/3)), x)
```

sympy [A] time = 1.15, size = 32, normalized size = 0.05

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{4}{3} \\ -\frac{1}{2} \end{matrix}\middle| {\frac{bx^2 e^{i\pi}}{a}} \right)}{3a^{\frac{4}{3}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(4/3),x)
```

```
[Out] -hyper((-3/2, 4/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(4/3)*x**3)
```

3.738 $\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=195

$$-\frac{5a^3c^{13/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{108b^{8/3}}-\frac{5a^3c^{13/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{54\sqrt{3}\,b^{8/3}}-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}+\frac{(cx)^{16/3}\sqrt[3]{a+}}{6c}$$

```
[Out] -5/108*a^2*c^3*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b^2+1/36*a*c*(c*x)^(10/3)*(b*x^2
+a)^(1/3)/b+1/6*(c*x)^(16/3)*(b*x^2+a)^(1/3)/c-5/108*a^3*c^(13/3)*ln(b^(1/3
)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(8/3)-5/162*a^3*c^(13/3)*arctan(1/
3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))/b^(8/3)*3^(1/2
)
```

Rubi [A] time = 0.39, antiderivative size = 275, normalized size of antiderivative = 1.41, number of steps used = 12, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {279, 321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}-\frac{5a^3c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{162b^{8/3}}+\frac{5a^3c^{13/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{324b^{8/3}}-\frac{5a^3c^{13/3}\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(13/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (-5*a^2*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(108*b^2) + (a*c*(c*x)^(10/3)*(a
 + b*x^2)^(1/3))/(36*b) + ((c*x)^(16/3)*(a + b*x^2)^(1/3))/(6*c) - (5*a^3*c
^(13/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[
3]*c^(2/3))])/(54*Sqrt[3]*b^(8/3)) - (5*a^3*c^(13/3)*Log[c^(2/3) - (b^(1/3)
*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(162*b^(8/3)) + (5*a^3*c^(13/3)*Log[c^(4/
3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3)
)/(a + b*x^2)^(1/3)])/(324*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{1}{9}a\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}dx$$

$$= \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} - \frac{\left(5a^2c^2\right)\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}dx}{54b}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{\left(5a^3c^4\right)\int \frac{\sqrt[3]{c}}{(a+bx}}{81b^2}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{\left(5a^3c^3\right)\text{Subst}}{}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{\left(5a^3c^3\right)\text{Subst}}{}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{\left(5a^3c^3\right)\text{Subst}}{}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{\left(5a^3c^{11/3}\right)\text{Subs}}{}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} - \frac{5a^3c^{13/3}\log\left(c^2\right.}{162b}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} - \frac{5a^3c^{13/3}\log\left(c^2\right.}{162b}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} - \frac{5a^3c^{13/3}\tan^{-1}\left(\right.}{54\sqrt{3}}$$

Mathematica [C] time = 0.07, size = 102, normalized size = 0.52

$$\frac{c^3(cx)^{4/3}\sqrt[3]{a+bx^2}\left(\sqrt[3]{\frac{bx^2}{a}+1}\left(-5a^2+abx^2+6b^2x^4\right)+5a^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)\right)}{36b^2\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3)*((1 + (b*x^2)/a)^(1/3)*(-5*a^2 + a*b*x^2
 + 6*b^2*x^4) + 5*a^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^2)/a)]))/(36
*b^2*(1 + (b*x^2)/a)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{13}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(13/3), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{13}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)*(b*x^2+a)^(1/3),x)
```

```
[Out] int((c*x)^(13/3)*(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{13}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(13/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{13/3}\,\left(b\,x^2+a\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)*(a + b*x^2)^(1/3),x)
```

```
[Out] int((c*x)^(13/3)*(a + b*x^2)^(1/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] Timed out
```

3.739 $\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=164

$$\frac{a^2c^{7/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{12b^{5/3}}+\frac{a^2c^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{6\sqrt{3}\,b^{5/3}}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}+\frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}$$

```
[Out] 1/12*a*c*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b+1/4*(c*x)^(10/3)*(b*x^2+a)^(1/3)/c+1
/12*a^2*c^(7/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(5/3)+1/1
8*a^2*c^(7/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*
3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 244, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {279, 321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{a^2c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{5/3}}-\frac{a^2c^{7/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{36b^{5/3}}+\frac{a^2c^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{6\sqrt{3}\,b^{5/3}}+\frac{(cx)^{10/3}\sqrt[3]{a}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (a*c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(12*b) + ((c*x)^(10/3)*(a + b*x^2)^(1/3
))/(4*c) + (a^2*c^(7/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^
2)^(1/3))/(Sqrt[3]*c^(2/3))])/(6*Sqrt[3]*b^(5/3)) + (a^2*c^(7/3)*Log[c^(2/3
) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(18*b^(5/3)) - (a^2*c^(7/3)*L
og[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*
x)^(2/3))/(a + b*x^2)^(1/3)])/(36*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} + \frac{1}{6}a\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} - \frac{\left(a^2c^2\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{9b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} - \frac{\left(a^2c\right)\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{3b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} - \frac{\left(a^2c\right)\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{6b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} - \frac{\left(a^2c\right)\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} - \frac{\left(a^2c^{5/3}\right)\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{4/3}} + \ldots$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} + \frac{a^2c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{5/3}} + \frac{\left(a^2c^{5/3}\right)\text{Subs}\ldots}{\ldots}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} + \frac{a^2c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{5/3}} - \frac{a^2c^{7/3}\log\left(c^{4/\ldots}\right.}{\ldots}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} + \frac{a^2c^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{6\sqrt{3}\,b^{5/3}} + \frac{a^2c^{7/3}\log\left(c^{2/3}\ldots\right.}{18b^{5\ldots}}$$

Mathematica [C] time = 0.05, size = 85, normalized size = 0.52

$$\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}\left(\left(a+bx^2\right)\sqrt[3]{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)\right)}{4b\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2)^(1/3)*((a + b*x^2)*(1 + (b*x^2)/a)^(1/3) - a*Hyp
ergeometric2F1[-1/3, 2/3, 5/3, -((b*x^2)/a)]))/(4*b*(1 + (b*x^2)/a)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}} (cx)^{\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(7/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{7}{3}} \left(b\,x^2+a\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)*(b*x^2+a)^(1/3),x)
```

```
[Out] int((c*x)^(7/3)*(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}} (cx)^{\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(7/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{7/3}\,\left(b\,x^2+a\right)^{1/3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)*(a + b*x^2)^(1/3),x)
```

```
[Out] int((c*x)^(7/3)*(a + b*x^2)^(1/3), x)
```

sympy [C] time = 33.29, size = 46, normalized size = 0.28

$$\frac{\sqrt[3]{a}\,c^{\frac{7}{3}}x^{\frac{10}{3}}\Gamma\left(\frac{5}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix} -\frac{1}{3}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\right| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**(7/3)*x**(10/3)*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(8/3))
```

3.740 $\int \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=133

$$-\frac{a\sqrt[3]{c}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4b^{2/3}}-\frac{a\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{2/3}}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}$$

```
[Out] 1/2*(c*x)^(4/3)*(b*x^2+a)^(1/3)/c-1/4*a*c^(1/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2
/3)*(b*x^2+a)^(1/3))/b^(2/3)-1/6*a*c^(1/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2
/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 211, normalized size of antiderivative = 1.59, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {279, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{a\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}}+\frac{a\sqrt[3]{c}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{12b^{2/3}}-\frac{a\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{2\sqrt{3}\,b^{2/3}}+\frac{(cx)^{4/3}\sqrt[3]{a+}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] ((c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*c) - (a*c^(1/3)*ArcTan[(c^(2/3) + (2*b^(
1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(2*Sqrt[3]*b^(2/3)
) - (a*c^(1/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(6*b
^(2/3)) + (a*c^(1/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3)
+ (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)]])/(12*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{1}{3}a\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\,\mathrm{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\,\mathrm{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\,\mathrm{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6\sqrt[3]{b}\sqrt[3]{c}} - \frac{a\,\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(c}{\sqrt[3]{a}}\right.}{6\sqrt[3]{b}\sqrt[3]{c}}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} - \frac{a\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4\sqrt[3]{b}\sqrt[3]{c}}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} - \frac{a\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}} + \frac{a\sqrt[3]{c}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right.}{12b^{2/3}}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} - \frac{a\sqrt[3]{c}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{2/3}} - \frac{a\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}} + \frac{a\sqrt[3]{c}\log\left(c\right.}{}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.42

$$\frac{3x\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)}{4\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(c*x)^(1/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x
^2)/a)])/(4*(1 + (b*x^2)/a)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}} (cx)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{1}{3}} \left(b\,x^2+a\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)*(b*x^2+a)^(1/3),x)
[Out] int((c*x)^(1/3)*(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}} (cx)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{1/3}\,\left(b\,x^2+a\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)*(a + b*x^2)^(1/3),x)
[Out] int((c*x)^(1/3)*(a + b*x^2)^(1/3), x)
```

sympy [C] time = 1.52, size = 46, normalized size = 0.35

$$\frac{\sqrt[3]{a}\,\sqrt[3]{c}\,x^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/3)*(b*x**2+a)**(1/3),x)
[Out] a**(1/3)*c**(1/3)*x**(4/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(5/3))
```

3.741 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx$

Optimal. Leaf size=131

$$-\frac{3\sqrt[3]{b}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4c^{5/3}}-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2c^{5/3}}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

```
[Out] -3/2*(b*x^2+a)^(1/3)/c/(c*x)^(2/3)-3/4*b^(1/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/
3)*(b*x^2+a)^(1/3))/c^(5/3)-1/2*b^(1/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)
/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))*3^(1/2)/c^(5/3)
```

Rubi [A] time = 0.27, antiderivative size = 208, normalized size of antiderivative = 1.59, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {277, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{4c^{5/3}}-\frac{\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}}-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{2c^{5/3}}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(5/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*c*(c*x)^(2/3)) - (Sqrt[3]*b^(1/3)*ArcTan[(c^(2/3)
 + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(2*c^(5/3
)) - (b^(1/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(2*c^
(5/3)) + (b^(1/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (
b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(4*c^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{b\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{c^2}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{b^{2/3}\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{7/3}} - \frac{b^{2/3}\,\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/}}{\sqrt[3]{a+b}}\right.}{2c^{7/3}}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} - \frac{\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}} - \frac{\left(3b^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4c^{7/3}} + \cdots$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} - \frac{\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}} + \frac{\sqrt[3]{b}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4c^{5/3}} + \frac{\left(3\sqrt[3]{b}\right.}{}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} - \frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2c^{5/3}} - \frac{\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}} + \frac{\sqrt[3]{b}\log\left(c^{4/3}+\frac{b^2}{(a}\right.}{4}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.43

$$-\frac{3x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{bx^2}{a}\right)}{2(cx)^{5/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(5/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, -((b*x^2)/a)])/(
2*(c*x)^(5/3)*(1 + (b*x^2)/a)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{1}{3}}}{(cx)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(5/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2 + a\right)^{\frac{1}{3}}}{(cx)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(5/3),x)
[Out] int((b*x^2+a)^(1/3)/(c*x)^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{1}{3}}}{(cx)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2 + a\right)^{1/3}}{(c\,x)^{5/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(5/3),x)
[Out] int((a + b*x^2)^(1/3)/(c*x)^(5/3), x)
```

sympy [C] time = 3.05, size = 49, normalized size = 0.37

$$\frac{\sqrt[3]{a}\,\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2c^{\frac{5}{3}} x^{\frac{2}{3}} \Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(5/3),x)
[Out] a**(1/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(5/3)*x**(2/3)*gamma(2/3))
```

3.742 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx$

Optimal. Leaf size=28

$$-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{8/3}}$$

```
[Out] -3/8*(b*x^2+a)^(4/3)/a/c/(c*x)^(8/3)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {264}

$$-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(8/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx = -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{8/3}}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{3x\left(a+bx^2\right)^{4/3}}{8a(cx)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(4/3))/(8*a*(c*x)^(11/3))
```

fricas [A] time = 1.26, size = 25, normalized size = 0.89

$$-\frac{3\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{1}{3}}}{8\,ac^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="fricas")
```

```
[Out] -3/8*(b*x^2 + a)^(4/3)*(c*x)^(1/3)/(a*c^4*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(11/3), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.75

$$-\frac{3\left(b\,x^2+a\right)^{\frac{4}{3}}x}{8\,(cx)^{\frac{11}{3}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(11/3),x)
[Out] -3/8*x*(b*x^2+a)^(4/3)/a/(c*x)^(11/3)
```

maxima [A] time = 1.45, size = 35, normalized size = 1.25

$$-\frac{3\left(bc^{\frac{1}{3}}x^3+ac^{\frac{1}{3}}x\right)\left(bx^2+a\right)^{\frac{1}{3}}}{8\,ac^4x^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="maxima")
[Out] -3/8*(b*c^(1/3)*x^3 + a*c^(1/3)*x)*(b*x^2 + a)^(1/3)/(a*c^4*x^(11/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(b\,x^2+a\right)^{1/3}}{(c\,x)^{11/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(11/3),x)
[Out] int((a + b*x^2)^(1/3)/(c*x)^(11/3), x)
```

sympy [B] time = 56.59, size = 78, normalized size = 2.79

$$\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{4}{3}\right)}{2c^{\frac{11}{3}}x^2\Gamma\left(-\frac{1}{3}\right)}+\frac{b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{4}{3}\right)}{2ac^{\frac{11}{3}}\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(11/3),x)
[Out] b**(1/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(2*c**(11/3)*x**2*gamma(-1/3))
 + b**(4/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(2*a*c**(11/3)*gamma(-1/3))
```

3.743 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}} \, dx$

Optimal. Leaf size=57

$$\frac{9\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{14/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}$$

```
[Out] -3/8*(b*x^2+a)^(4/3)/a/c/(c*x)^(14/3)+9/56*(b*x^2+a)^(7/3)/a^2/c/(c*x)^(14/
3)
```

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {273, 264}

$$\frac{9\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{14/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(17/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(14/3)) + (9*(a + b*x^2)^(7/3))/(56*a^2
*c*(c*x)^(14/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}} \, dx &= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}-\frac{3\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{17/3}} \, dx}{4a}\\ &= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}+\frac{9\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{14/3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.72

$$\frac{3\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\left(3bx^2-4a\right)}{56a^2c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(17/3),x]
```

```
[Out] (3*(c*x)^(1/3)*(a + b*x^2)^(4/3)*(-4*a + 3*b*x^2))/(56*a^2*c^6*x^5)
```

fricas [A] time = 0.76, size = 46, normalized size = 0.81

$$\frac{3\left(3\,b^{2}x^{4}-abx^{2}-4\,a^{2}\right)\left(bx^{2}+a\right)^{\frac{1}{3}}\left(cx\right)^{\frac{1}{3}}}{56\,a^{2}c^{6}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="fricas")
```

```
[Out] 3/56*(3*b^2*x^4 - a*b*x^2 - 4*a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^2*c^6*x
^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(17/3), x)
```

maple [A] time = 0.01, size = 31, normalized size = 0.54

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-3b\,x^{2}+4a\right)x}{56\left(cx\right)^{\frac{17}{3}}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(17/3),x)
```

```
[Out] -3/56*x*(b*x^2+a)^(4/3)*(-3*b*x^2+4*a)/a^2/(c*x)^(17/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(17/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^2+a\right)}^{1/3}}{{\left(c\,x\right)}^{17/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(17/3),x)
```

```
[Out] int((a + b*x^2)^(1/3)/(c*x)^(17/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(17/3),x)
```

```
[Out] Timed out
```

3.744 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}} dx$

Optimal. Leaf size=85

$$-\frac{27\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{20/3}}+\frac{9\left(a+bx^2\right)^{7/3}}{28a^2c(cx)^{20/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}$$

```
[Out] -3/8*(b*x^2+a)^(4/3)/a/c/(c*x)^(20/3)+9/28*(b*x^2+a)^(7/3)/a^2/c/(c*x)^(20/
3)-27/280*(b*x^2+a)^(10/3)/a^3/c/(c*x)^(20/3)
```

Rubi [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{27\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{20/3}}+\frac{9\left(a+bx^2\right)^{7/3}}{28a^2c(cx)^{20/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(23/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(7/3))/(28*a^2
*c*(c*x)^(20/3)) - (27*(a + b*x^2)^(10/3))/(280*a^3*c*(c*x)^(20/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}} dx &= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}-\frac{3\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{23/3}} dx}{2a}\\
&= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}+\frac{9\left(a+bx^2\right)^{7/3}}{28a^2c(cx)^{20/3}}+\frac{9\int \frac{\left(a+bx^2\right)^{7/3}}{(cx)^{23/3}} dx}{14a^2}\\
&= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}+\frac{9\left(a+bx^2\right)^{7/3}}{28a^2c(cx)^{20/3}}-\frac{27\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{20/3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.61

$$-\frac{3\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\left(14a^2-12abx^2+9b^2x^4\right)}{280a^3c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(23/3),x]
```

```
[Out] (-3*(c*x)^(1/3)*(a + b*x^2)^(4/3)*(14*a^2 - 12*a*b*x^2 + 9*b^2*x^4))/(280*a
^3*c^8*x^7)
```

fricas [A] time = 1.25, size = 57, normalized size = 0.67

$$-\frac{3\left(9\,b^{3}x^{6}-3\,ab^{2}x^{4}+2\,a^{2}bx^{2}+14\,a^{3}\right)\left(bx^{2}+a\right)^{\frac{1}{3}}\left(cx\right)^{\frac{1}{3}}}{280\,a^{3}c^{8}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="fricas")
```

```
[Out] -3/280*(9*b^3*x^6 - 3*a*b^2*x^4 + 2*a^2*b*x^2 + 14*a^3)*(b*x^2 + a)^(1/3)*(
c*x)^(1/3)/(a^3*c^8*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^{2}+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(23/3), x)
```

maple [A] time = 0.01, size = 42, normalized size = 0.49

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(9b^{2}x^{4}-12ab\,x^{2}+14a^{2}\right)x}{280\left(cx\right)^{\frac{23}{3}}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(23/3),x)
```

```
[Out] -3/280*x*(b*x^2+a)^(4/3)*(9*b^2*x^4-12*a*b*x^2+14*a^2)/a^3/(c*x)^(23/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^{2}+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(23/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(b\,x^2+a\right)}^{1/3}}{{\left(c\,x\right)}^{23/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(23/3),x)
```

```
[Out] int((a + b*x^2)^(1/3)/(c*x)^(23/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(23/3),x)
```

```
[Out] Timed out
```

3.745 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx$

Optimal. Leaf size=113

$$\frac{243\left(a+bx^2\right)^{13/3}}{3640a^4c(cx)^{26/3}}-\frac{81\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{26/3}}+\frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}$$

```
[Out] -3/8*(b*x^2+a)^(4/3)/a/c/(c*x)^(26/3)+27/56*(b*x^2+a)^(7/3)/a^2/c/(c*x)^(26
/3)-81/280*(b*x^2+a)^(10/3)/a^3/c/(c*x)^(26/3)+243/3640*(b*x^2+a)^(13/3)/a^
4/c/(c*x)^(26/3)
```

Rubi [A] time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{243\left(a+bx^2\right)^{13/3}}{3640a^4c(cx)^{26/3}}-\frac{81\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{26/3}}+\frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(29/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(26/3)) + (27*(a + b*x^2)^(7/3))/(56*a^
2*c*(c*x)^(26/3)) - (81*(a + b*x^2)^(10/3))/(280*a^3*c*(c*x)^(26/3)) + (243
*(a + b*x^2)^(13/3))/(3640*a^4*c*(c*x)^(26/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx &= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}-\frac{9\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{29/3}}\,dx}{4a}\\
&= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}+\frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}}+\frac{27\int \frac{\left(a+bx^2\right)^{7/3}}{(cx)^{29/3}}\,dx}{14a^2}\\
&= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}+\frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}}-\frac{81\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{26/3}}-\frac{81\int \frac{\left(a+bx^2\right)^{10/3}}{(cx)^{29/3}}\,dx}{140a^3}\\
&= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}+\frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}}-\frac{81\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{26/3}}+\frac{243\left(a+bx^2\right)^{13/3}}{3640a^4c(cx)^{26/3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.56

$$\frac{3\left(a+bx^2\right)^{4/3}\left(-140a^3+126a^2bx^2-108ab^2x^4+81b^3x^6\right)}{3640a^4c^9x^8(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(29/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3)*(-140*a^3 + 126*a^2*b*x^2 - 108*a*b^2*x^4 + 81*b^3*x^6
))/(3640*a^4*c^9*x^8*(c*x)^(2/3))
```

fricas [A] time = 1.21, size = 68, normalized size = 0.60

$$\frac{3\left(81\,b^4x^8-27\,ab^3x^6+18\,a^2b^2x^4-14\,a^3bx^2-140\,a^4\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{3640\,a^4c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="fricas")
```

```
[Out] 3/3640*(81*b^4*x^8 - 27*a*b^3*x^6 + 18*a^2*b^2*x^4 - 14*a^3*b*x^2 - 140*a^4
)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^4*c^10*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{29}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(29/3), x)
```

maple [A] time = 0.01, size = 53, normalized size = 0.47

$$-\frac{3\left(b\,x^2+a\right)^{\frac{4}{3}}\left(-81b^3x^6+108a\,b^2x^4-126a^2b\,x^2+140a^3\right)x}{3640\,(cx)^{\frac{29}{3}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(29/3),x)
```

```
[Out] -3/3640*x*(b*x^2+a)^(4/3)*(-81*b^3*x^6+108*a*b^2*x^4-126*a^2*b*x^2+140*a^3)
/a^4/(c*x)^(29/3)
```

maxima [A] time = 1.47, size = 64, normalized size = 0.57

$$\frac{3\left(81\,b^4x^9-27\,ab^3x^7+18\,a^2b^2x^5-14\,a^3bx^3-140\,a^4x\right)\left(bx^2+a\right)^{\frac{1}{3}}}{3640\,a^4c^{\frac{29}{3}}x^{\frac{29}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="maxima")
```

```
[Out] 3/3640*(81*b^4*x^9 - 27*a*b^3*x^7 + 18*a^2*b^2*x^5 - 14*a^3*b*x^3 - 140*a^4
*x)*(b*x^2 + a)^(1/3)/(a^4*c^(29/3)*x^(29/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{1/3}}{{\left(c\,x\right)}^{29/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(29/3),x)
```

```
[Out] int((a + b*x^2)^(1/3)/(c*x)^(29/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(29/3),x)
```

```
[Out] Timed out
```

3.746 $\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=451

$$\frac{7a^2c^{7/3}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{135\sqrt[4]{3}\,b^2\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] -14/135*a^2*c^3*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b^2+2/45*a*c*(c*x)^(7/3)*(b*x^2
+a)^(1/3)/b+1/5*(c*x)^(13/3)*(b*x^2+a)^(1/3)/c+7/405*a^2*c^(7/3)*(c*x)^(1/3
)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b
^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2
/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3
^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)
^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/
3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/
4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3
)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1
/2))/(b*x^2+a)^(1/3))^2)^(1/2)*3^(3/4)/b^2/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b
^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^
(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {279, 321, 329, 241, 225}

$$-\frac{14a^2c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{135b^2}+\frac{7a^2c^{7/3}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}\,(c}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(c}{\sqrt[3]{bx^2+a}}}\right.\right.}{135\sqrt[4]{3}\,b^2\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(10/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (-14*a^2*c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(135*b^2) + (2*a*c*(c*x)^(7/3)*
(a + b*x^2)^(1/3))/(45*b) + ((c*x)^(13/3)*(a + b*x^2)^(1/3))/(5*c) + (7*a^2
*c^(7/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a
+ b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (
b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b
^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 -
 Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(135*3^(1/4)*b
^2*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)
^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/
(a + b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{1}{15}(2a)\int \frac{(cx)^{10/3}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} - \frac{\left(14a^2c^2\right)\int \frac{(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{135b}$$

$$= -\frac{14a^2c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{135b^2} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{\left(14a^3c^4\right)\int \frac{1}{(cx)^{2/3}(a\cdots}}{405b^2}$$

$$= -\frac{14a^2c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{135b^2} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{\left(14a^3c^3\right)\mathrm{Subst}\left(\int\cdots\right.}{1\cdots}$$

$$= -\frac{14a^2c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{135b^2} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{\left(14a^3c^3\right)\mathrm{Subst}\left(\int\cdots\right.}{135b^2\sqrt{\frac{\cdots}{a\cdots}}}$$

$$= -\frac{14a^2c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{135b^2} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{7a^2c^{7/3}\sqrt[3]{cx}\,\sqrt[3]{a+b\cdots}}{\cdots}$$

Mathematica [C] time = 0.06, size = 103, normalized size = 0.23

$$\frac{c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(\sqrt[3]{\frac{bx^2}{a}+1}\left(-7a^2+2abx^2+9b^2x^4\right)+7a^2\,{}_2F_1\left(-\frac{1}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)\right)}{45b^2\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(10/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*((1 + (b*x^2)/a)^(1/3)*(-7*a^2 + 2*a*b*x
^2 + 9*b^2*x^4) + 7*a^2*Hypergeometric2F1[-1/3, 1/6, 7/6, -((b*x^2)/a)]))/(
45*b^2*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}c^3x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)*c^3*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{10}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(10/3), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{10}{3}} \left(b\,x^{2}+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(10/3)*(b*x^2+a)^(1/3),x)
[Out] int((c*x)^(10/3)*(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2}+a\right)^{\frac{1}{3}} (cx)^{\frac{10}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(10/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{10/3}\,\left(b\,x^2+a\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(10/3)*(a + b*x^2)^(1/3),x)
[Out] int((c*x)^(10/3)*(a + b*x^2)^(1/3), x)
```

sympy [C] time = 127.90, size = 46, normalized size = 0.10

$$\frac{\sqrt[3]{a}\,c^{\frac{10}{3}}\,x^{\frac{13}{3}}\,\Gamma\left(\frac{13}{6}\right)\,{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{13}{6} \\ \frac{19}{6} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{2\Gamma\left(\frac{19}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(10/3)*(b*x**2+a)**(1/3),x)
[Out] a**(1/3)*c**(10/3)*x**(13/3)*gamma(13/6)*hyper((-1/3, 13/6), (19/6,), b*x**
2*exp_polar(I*pi)/a)/(2*gamma(19/6))
```

3.747 $\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=418

$$-\frac{a\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\mathrm{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\sqrt[4]{3}\,b\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] 2/9*a*c*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b+1/3*(c*x)^(7/3)*(b*x^2+a)^(1/3)/c-1/2
7*a*c^(1/3)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2
+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^
(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^
(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)
*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-
3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2
+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/
(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/
3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)*3^(3/4)/b/(-b^(1/3)*(c
*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^
(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {279, 321, 329, 241, 225}

$$-\frac{a\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\sqrt[4]{3}\,b\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{(cx)^{7/}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(4/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (2*a*c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(9*b) + ((c*x)^(7/3)*(a + b*x^2)^(1/3
))/(3*c) - (a*c^(1/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*
x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x
^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1
 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^
(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - (
(1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(
9*3^(1/4)*b*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a
 + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x
)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
```

```
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}+\frac{1}{9}(2a)\int \frac{(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{2ac\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b}+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}-\frac{\left(2a^2c^2\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{27b}$$

$$= \frac{2ac\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b}+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}-\frac{\left(2a^2c\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx,x,\sqrt[3]{cx}\right)}{9b}$$

$$= \frac{2ac\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b}+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}-\frac{\left(2a^2c\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx,x,\frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{9b\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a+bx^2}}$$

$$= \frac{2ac\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b}+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}-\frac{a\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx}{(a+bx^2}}{\left(c^{2/3}-\frac{(1+}{}\right.}}}{9\sqrt[4]{3}\,b\sqrt{-\frac{\sqrt[3]{}}{\sqrt[3]{a+}}}}$$

Mathematica [C] time = 0.04, size = 85, normalized size = 0.20

$$\frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(\left(a+bx^2\right)\sqrt[3]{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)\right)}{3b\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(4/3)*(a + b*x^2)^(1/3),x]
[Out] (c*(c*x)^(1/3)*(a + b*x^2)^(1/3)*((a + b*x^2)*(1 + (b*x^2)/a)^(1/3) - a*Hyp
ergeometric2F1[-1/3, 1/6, 7/6, -((b*x^2)/a)]))/(3*b*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}cx,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)*c*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(4/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{4}{3}} \left(b\,x^{2}+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(4/3)*(b*x^2+a)^(1/3),x)
```

```
[Out] int((c*x)^(4/3)*(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2}+a\right)^{\frac{1}{3}} (cx)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{4/3}\,\left(b\,x^2+a\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(4/3)*(a + b*x^2)^(1/3),x)
```

```
[Out] int((c*x)^(4/3)*(a + b*x^2)^(1/3), x)
```

sympy [C] time = 6.72, size = 46, normalized size = 0.11

$$\frac{\sqrt[3]{a}\,c^{\frac{4}{3}}x^{\frac{7}{3}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{3},\frac{7}{6}\\\frac{13}{6}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(4/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**(4/3)*x**(7/3)*gamma(7/6)*hyper((-1/3, 7/6), (13/6,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(13/6))
```

3.748 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx$

Optimal. Leaf size=381

$$\frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\;\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,c^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\sqrt[3]{c\cdots}$$

```
[Out] (c*x)^(1/3)*(b*x^2+a)^(1/3)/c+1/3*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1
/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/
(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3)
)^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/
3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b
^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2
/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3
)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)
^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)*
3^(3/4)/c^(5/3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a
)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)
^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {279, 329, 241, 225}

$$\frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\;F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,c^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{\sqrt[3]{cx}\,\sqrt[3]{a+b\cdots}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(2/3),x]
```

```
[Out] ((c*x)^(1/3)*(a + b*x^2)^(1/3))/c + ((c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3)
 - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^
(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))
/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*Ellip
ticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3
))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 +
 Sqrt[3])/4])/(3^(1/4)*c^(5/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1
/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sq
rt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
```

```
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx = \frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{c} + \frac{1}{3}(2a)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{c} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c}$$

$$= \frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{c} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{c\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a+bx^2}}$$

$$= \frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{c} + \frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\left(\ldots\right.}{c^{2/3}-\left(\ldots\right.}\right.\right.}{\sqrt[4]{3}\,c^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.14

$$\frac{3x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)}{(cx)^{2/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/6, 7/6, -((b*x^2)/a)])/((c
*x)^(2/3)*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(2/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(2/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{1/3}}{{\left(c\,x\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(2/3),x)
```

```
[Out] int((a + b*x^2)^(1/3)/(c*x)^(2/3), x)
```

sympy [C] time = 1.31, size = 46, normalized size = 0.12

$$\frac{\sqrt[3]{a}\,\sqrt[3]{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(2/3),x)
```

```
[Out] a**(1/3)*x**(1/3)*gamma(1/6)*hyper((-1/3, 1/6), (7/6,), b*x**2*exp_polar(I*
pi)/a)/(2*c**(2/3)*gamma(7/6))
```

3.749 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx$

Optimal. Leaf size=391

$$\frac{3^{3/4}b\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5ac^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] -3/5*(b*x^2+a)^(1/3)/c/(c*x)^(5/3)+1/5*3^(3/4)*b*(c*x)^(1/3)*(b*x^2+a)^(1/3
)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/
3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/
(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a
)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*Elliptic
F((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b
^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^
(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^
(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(
1/3))^2)^(1/2)/a/c^(11/3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3
)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))
/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {277, 329, 241, 225}

$$\frac{3^{3/4}b\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5ac^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{3\sqrt[3]{a+}}{5c(cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(8/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(5*c*(c*x)^(5/3)) + (3^(3/4)*b*(c*x)^(1/3)*(a + b*x^
2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3)
+ (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(
a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2
)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))
/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x
^2)^(1/3))], (2 + Sqrt[3])/4])/(5*a*c^(11/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c
^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2
/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
```

```
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx &= -\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}+\frac{(2b)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{5c^2}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}+\frac{(6b)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{5c^3}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}+\frac{(6b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{5c^3\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}+\frac{3^{3/4}b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/\cdots}}{c^{2/\cdots}}\right.\right.}{5ac^{11/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.14

$$
-\frac{3x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{3};\frac{1}{6};-\frac{bx^2}{a}\right)}{5(cx)^{8/3}\sqrt[3]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(8/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-5/6, -1/3, 1/6, -((b*x^2)/a)])/(
5*(c*x)^(8/3)*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(8/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(8/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(8/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(8/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{1/3}}{{\left(c\,x\right)}^{8/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(8/3),x)
```

```
[Out] int((a + b*x^2)^(1/3)/(c*x)^(8/3), x)
```

sympy [C] time = 12.23, size = 32, normalized size = 0.08

$$-\frac{\sqrt[3]{b}\,{}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{c^{\frac{8}{3}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(8/3),x)
```

```
[Out] -b**(1/3)*hyper((-1/3, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(c**(8/3)*
x)
```

3.750 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx$

Optimal. Leaf size=422

$$-\frac{3\,3^{3/4}b^2\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\right.\right.}{55a^2c^{17/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] -3/11*(b*x^2+a)^(1/3)/c/(c*x)^(11/3)-6/55*b*(b*x^2+a)^(1/3)/a/c^3/(c*x)^(5/
3)-3/55*3^(3/4)*b^2*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3
)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3
))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^
(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*
x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(
2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2)
)/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x
)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/
3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/a^2/c^(17/3)/(
-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a
)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {277, 325, 329, 241, 225}

$$-\frac{3\,3^{3/4}b^2\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55a^2c^{17/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{6b}{55}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(14/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(11*c*(c*x)^(11/3)) - (6*b*(a + b*x^2)^(1/3))/(55*a*
c^3*(c*x)^(5/3)) - (3*3^(3/4)*b^2*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) -
(b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/
3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c
^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*Elliptic
F[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/
(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sq
rt[3])/4])/(55*a^2*c^(17/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)
*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[
3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx &= -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}+\frac{(2b)\int \frac{1}{(cx)^{8/3}\left(a+bx^2\right)^{2/3}}\,dx}{11c^2}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}-\frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}}-\frac{\left(6b^2\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{55ac^4}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}-\frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}}-\frac{\left(18b^2\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx,x,\sqrt[3]{cx}\right)}{55ac^5}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}-\frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}}-\frac{\left(18b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx,x,\frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{55ac^5\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}-\frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}}-\frac{3\,3^{3/4}b^2\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)\cdots}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\cdots}{55a^2c^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\cdots\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\cdots\right)}{\cdots}\right)}}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.13

$$
-\frac{3x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{11}{6},-\frac{1}{3};-\frac{5}{6};-\frac{bx^2}{a}\right)}{11(cx)^{14/3}\sqrt[3]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(14/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-11/6, -1/3, -5/6, -((b*x^2)/a)])
/(11*(c*x)^(14/3)*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{c^5x^5},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(14/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(c^5*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{14}{3}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(14/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(14/3), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{\left(c x\right)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(14/3),x)
[Out] int((b*x^2+a)^(1/3)/(c*x)^(14/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(14/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(14/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{1/3}}{\left(c\,x\right)^{14/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(14/3),x)
[Out] int((a + b*x^2)^(1/3)/(c*x)^(14/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(14/3),x)
[Out] Timed out
```

3.751 $\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=58

$$\frac{3(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{\frac{bx^2}{a}+1}}$$

```
[Out] 3/5*(c*x)^(5/3)*(b*x^2+a)^(1/3)*hypergeom([-1/3, 5/6],[11/6],-b*x^2/a)/c/(1
+b*x^2/a)^(1/3)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{3(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(2/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(c*x)^(5/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 5/6, 11/6, -((b*x^
2)/a)])/(5*c*(1 + (b*x^2)/a)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx &= \frac{\sqrt[3]{a+bx^2}\int (cx)^{2/3}\sqrt[3]{1+\frac{bx^2}{a}}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}\\ &= \frac{3(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{1+\frac{bx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 0.97

$$\frac{3x(cx)^{2/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(2/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 5/6, 11/6, -((b*
x^2)/a)])/(5*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{2}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2/3)*(b*x^2+a)^(1/3),x)
```

```
[Out] int((c*x)^(2/3)*(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^{2/3}\,\left(b\,x^2+a\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2/3)*(a + b*x^2)^(1/3),x)
```

```
[Out] int((c*x)^(2/3)*(a + b*x^2)^(1/3), x)
```

sympy [C] time = 1.90, size = 46, normalized size = 0.79

$$\frac{\sqrt[3]{a}\,c^{\frac{2}{3}}x^{\frac{5}{3}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(2/3)*(b*x**2+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**(2/3)*x**(5/3)*gamma(5/6)*hyper((-1/3, 5/6), (11/6,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(11/6))
```

3.752 $\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}\,dx$

Optimal. Leaf size=58

$$\frac{3(cx)^{2/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{\frac{bx^2}{a}+1}}$$

```
[Out] 3/2*(c*x)^(2/3)*(b*x^2+a)^(1/3)*hypergeom([-1/3, 1/3],[4/3],-b*x^2/a)/c/(1+
b*x^2/a)^(1/3)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{3(cx)^{2/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(1/3),x]
```

```
[Out] (3*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^2
)/a)])/(2*c*(1 + (b*x^2)/a)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}\,dx &= \frac{\sqrt[3]{a+bx^2}\int \frac{\sqrt[3]{1+\frac{bx^2}{a}}}{\sqrt[3]{cx}}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}\\
&= \frac{3(cx)^{2/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{1+\frac{bx^2}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 0.97

$$\frac{3x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2\sqrt[3]{cx}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^2)/a)])/(2*
(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}}{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(1/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(1/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{{\left(b\,x^2+a\right)}^{1/3}}{{\left(c\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(1/3),x)
```

```
[Out] int((a + b*x^2)^(1/3)/(c*x)^(1/3), x)
```

sympy **[C]** time = 1.09, size = 46, normalized size = 0.79

$$\frac{\sqrt[3]{a}\,x^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[3]{c}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(1/3),x)
```

```
[Out] a**(1/3)*x**(2/3)*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**2*exp_polar(I*
pi)/a)/(2*c**(1/3)*gamma(4/3))
```

3.753 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\,\sqrt[3]{\frac{bx^2}{a}+1}}$$

```
[Out] -3*(b*x^2+a)^(1/3)*hypergeom([-1/3, -1/6],[5/6],-b*x^2/a)/c/(c*x)^(1/3)/(1+
b*x^2/a)^(1/3)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$-\frac{3\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\,\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, -1/6, 5/6, -((b*x^2)/a)])/(c*
(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx &= \frac{\sqrt[3]{a+bx^2}\int \frac{\sqrt[3]{1+\frac{bx^2}{a}}}{(cx)^{4/3}}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}\\ &= -\frac{3\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\,\sqrt[3]{1+\frac{bx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.96

$$-\frac{3x\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{(cx)^{4/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(4/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, -1/6, 5/6, -((b*x^2)/a)])/(
(c*x)^(4/3)*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}}{c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(4/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(4/3),x)
```

```
[Out] int((b*x^2+a)^(1/3)/(c*x)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(b\,x^2+a\right)^{1/3}}{(c\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(4/3),x)
```

```
[Out] int((a + b*x^2)^(1/3)/(c*x)^(4/3), x)
```

sympy [C] time = 2.02, size = 49, normalized size = 0.88

$$\frac{\sqrt[3]{a}\,\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{4}{3}}\sqrt[3]{x}\,\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(4/3),x)
```

```
[Out] a**(1/3)*gamma(-1/6)*hyper((-1/3, -1/6), (5/6,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(4/3)*x**(1/3)*gamma(5/6))
```

3.754 $\int (cx)^{13/3} \left(a+bx^2\right)^{4/3} dx$

Optimal. Leaf size=223

$$-\frac{5a^4c^{13/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{324b^{8/3}}-\frac{5a^4c^{13/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{162\sqrt{3}b^{8/3}}-\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{}}{108b}$$

```
[Out] -5/324*a^3*c^3*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b^2+1/108*a^2*c*(c*x)^(10/3)*(b*
x^2+a)^(1/3)/b+1/18*a*(c*x)^(16/3)*(b*x^2+a)^(1/3)/c+1/8*(c*x)^(16/3)*(b*x^
2+a)^(4/3)/c-5/324*a^4*c^(13/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1
/3))/b^(8/3)-5/486*a^4*c^(13/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)
/(b*x^2+a)^(1/3))*3^(1/2))/b^(8/3)*3^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 303, normalized size of antiderivative = 1.36, number of steps used = 13, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {279, 321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}-\frac{5a^4c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{486b^{8/3}}+\frac{5a^4c^{13/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{972b^{8/3}}-\frac{5a^4c^{13/3}\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(13/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (-5*a^3*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(324*b^2) + (a^2*c*(c*x)^(10/3)*
(a + b*x^2)^(1/3))/(108*b) + (a*(c*x)^(16/3)*(a + b*x^2)^(1/3))/(18*c) + ((
c*x)^(16/3)*(a + b*x^2)^(4/3))/(8*c) - (5*a^4*c^(13/3)*ArcTan[(c^(2/3) + (2
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(162*Sqrt[3]*b
^(8/3)) - (5*a^4*c^(13/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(
1/3)])/(486*b^(8/3)) + (5*a^4*c^(13/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/
(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(972*
b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int (cx)^{13/3}\left(a+bx^2\right)^{4/3}dx = \frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c} + \frac{1}{3}a\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx$$

$$= \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c} + \frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c} + \frac{1}{27}a^2\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}dx$$

$$= \frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c} + \frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c} - \frac{\left(5a^3c^2\right)\int \frac{(c}{(a+l}}{162b}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c} + \frac{(cx)^{16/3}\left(a\right.}{8}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c} + \frac{(cx)^{16/3}\left(a\right.}{8}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c} + \frac{(cx)^{16/3}\left(a\right.}{8}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c} + \frac{(cx)^{16/3}\left(a\right.}{8}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c} + \frac{(cx)^{16/3}\left(a\right.}{8}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c} + \frac{(cx)^{16/3}\left(a\right.}{8}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c} + \frac{(cx)^{16/3}\left(a\right.}{8}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c} + \frac{(cx)^{16/3}\left(a\right.}{8}$$

Mathematica [C] time = 0.08, size = 102, normalized size = 0.46

$$\frac{c^3(cx)^{4/3}\sqrt[3]{a+bx^2}\left(5a^3\,{}_2F_1\left(-\frac{4}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)-\left(5a-9bx^2\right)\left(a+bx^2\right)^2\sqrt[3]{\frac{bx^2}{a}+1}\right)}{72b^2\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3)*(-((5*a - 9*b*x^2)*(a + b*x^2)^2*(1 + (b
*x^2)/a)^(1/3)) + 5*a^3*Hypergeometric2F1[-4/3, 2/3, 5/3, -((b*x^2)/a)]))/(
72*b^2*(1 + (b*x^2)/a)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{13}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(13/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{13}{3}}\left(b\,x^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)*(b*x^2+a)^(4/3),x)
[Out] int((c*x)^(13/3)*(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{13}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(13/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{13/3}\left(b\,x^2+a\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)*(a + b*x^2)^(4/3),x)
[Out] int((c*x)^(13/3)*(a + b*x^2)^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/3)*(b*x**2+a)**(4/3),x)
[Out] Timed out
```

3.755 $\int (cx)^{7/3} \left(a+bx^2\right)^{4/3} dx$

Optimal. Leaf size=192

$$\frac{a^3c^{7/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{27b^{5/3}}+\frac{2a^3c^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{27\sqrt{3}\,b^{5/3}}+\frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}$$

```
[Out] 1/27*a^2*c*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b+1/9*a*(c*x)^(10/3)*(b*x^2+a)^(1/3)
/c+1/6*(c*x)^(10/3)*(b*x^2+a)^(4/3)/c+1/27*a^3*c^(7/3)*ln(b^(1/3)*(c*x)^(2/
3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(5/3)+2/81*a^3*c^(7/3)*arctan(1/3*(1+2*b^(1/3
)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 272, normalized size of antiderivative = 1.42, number of steps used = 12, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {279, 321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{2a^3c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{81b^{5/3}}-\frac{a^3c^{7/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{81b^{5/3}}+\frac{2a^3c^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{27\sqrt{3}\,b^{5/3}}+\frac{a^2c(cx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (a^2*c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(27*b) + (a*(c*x)^(10/3)*(a + b*x^2)^
(1/3))/(9*c) + ((c*x)^(10/3)*(a + b*x^2)^(4/3))/(6*c) + (2*a^3*c^(7/3)*ArcT
an[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))]
)/(27*Sqrt[3]*b^(5/3)) + (2*a^3*c^(7/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))
/(a + b*x^2)^(1/3)])/(81*b^(5/3)) - (a^3*c^(7/3)*Log[c^(4/3) + (b^(2/3)*(c*
x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3)])/(81*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int (cx)^{7/3} \left(a+bx^2\right)^{4/3} dx = \frac{(cx)^{10/3} \left(a+bx^2\right)^{4/3}}{6c} + \frac{1}{9}(4a) \int (cx)^{7/3} \sqrt[3]{a+bx^2} \, dx$$

$$= \frac{a(cx)^{10/3} \sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3} \left(a+bx^2\right)^{4/3}}{6c} + \frac{1}{27}\left(2a^2\right) \int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}} dx$$

$$= \frac{a^2c(cx)^{4/3} \sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3} \sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3} \left(a+bx^2\right)^{4/3}}{6c} - \frac{\left(4a^3c^2\right) \int \frac{\sqrt[3]{c}}{\left(a+bx^2\right.}}{81b}$$

$$= \frac{a^2c(cx)^{4/3} \sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3} \sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3} \left(a+bx^2\right)^{4/3}}{6c} - \frac{\left(4a^3c\right) \text{Subst}\left(\int\right.}{}$$

$$= \frac{a^2c(cx)^{4/3} \sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3} \sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3} \left(a+bx^2\right)^{4/3}}{6c} - \frac{\left(2a^3c\right) \text{Subst}\left(\int\right.}{}$$

$$= \frac{a^2c(cx)^{4/3} \sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3} \sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3} \left(a+bx^2\right)^{4/3}}{6c} - \frac{\left(2a^3c\right) \text{Subst}\left(\int\right.}{}$$

$$= \frac{a^2c(cx)^{4/3} \sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3} \sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3} \left(a+bx^2\right)^{4/3}}{6c} - \frac{\left(2a^3c^{5/3}\right) \text{Subst}}{}$$

$$= \frac{a^2c(cx)^{4/3} \sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3} \sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3} \left(a+bx^2\right)^{4/3}}{6c} + \frac{2a^3c^{7/3} \log\left(c^{2/3}\right.}{81b^5}$$

$$= \frac{a^2c(cx)^{4/3} \sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3} \sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3} \left(a+bx^2\right)^{4/3}}{6c} + \frac{2a^3c^{7/3} \log\left(c^{2/3}\right.}{81b^5}$$

$$= \frac{a^2c(cx)^{4/3} \sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3} \sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3} \left(a+bx^2\right)^{4/3}}{6c} + \frac{2a^3c^{7/3} \tan^{-1}\left(\right.}{27\sqrt{3}\,b}$$

Mathematica [C] time = 0.07, size = 89, normalized size = 0.46

$$\frac{c(cx)^{4/3} \sqrt[3]{a+bx^2} \left(\left(a+bx^2\right)^2 \sqrt[3]{\frac{bx^2}{a}+1} - a^2 \, _2F_1\left(-\frac{4}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^2}{a}\right)\right)}{6b\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2)^(1/3)*((a + b*x^2)^2*(1 + (b*x^2)/a)^(1/3) - a^2
*Hypergeometric2F1[-4/3, 2/3, 5/3, -((b*x^2)/a)]))/(6*b*(1 + (b*x^2)/a)^(1/
3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(7/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{7}{3}} \left(b\,x^2+a\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(7/3)*(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(7/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{7/3}\,\left(b\,x^2+a\right)^{4/3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)*(a + b*x^2)^(4/3),x)
```

```
[Out] int((c*x)^(7/3)*(a + b*x^2)^(4/3), x)
```

sympy [C] time = 64.86, size = 46, normalized size = 0.24

$$\frac{a^{\frac{4}{3}} c^{\frac{7}{3}} x^{\frac{10}{3}} \Gamma\left(\frac{5}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| {\frac{bx^2 e^{i\pi}}{a}} \right)}{2\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*c**(7/3)*x**(10/3)*gamma(5/3)*hyper((-4/3, 5/3), (8/3,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(8/3))
```

3.756 $\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\,dx$

Optimal. Leaf size=163

$$-\frac{a^2\sqrt[3]{c}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{6b^{2/3}}-\frac{a^2\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}}+\frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}$$

```
[Out] 1/3*a*(c*x)^(4/3)*(b*x^2+a)^(1/3)/c+1/4*(c*x)^(4/3)*(b*x^2+a)^(4/3)/c-1/6*a
^2*c^(1/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(2/3)-1/9*a^2*
c^(1/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2
))/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 243, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {279, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{a^2\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{9b^{2/3}}+\frac{a^2\sqrt[3]{c}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{18b^{2/3}}-\frac{a^2\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{3\sqrt{3}\,b^{2/3}}+\frac{a(cx)^{4/3}\ldots}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (a*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(3*c) + ((c*x)^(4/3)*(a + b*x^2)^(4/3))/(
4*c) - (a^2*c^(1/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(
1/3))/(Sqrt[3]*c^(2/3))])/(3*Sqrt[3]*b^(2/3)) - (a^2*c^(1/3)*Log[c^(2/3) -
(b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(9*b^(2/3)) + (a^2*c^(1/3)*Log[c^
(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2
/3))/(a + b*x^2)^(1/3)])/(18*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}dx = \frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{1}{3}(2a)\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}dx$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{1}{9}\left(2a^2\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}dx$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}dx,x,\sqrt[3]{cx}\right)}{3c}$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{a^2\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}dx,x,(cx)^{2/3}\right)}{3c}$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{a^2\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}dx,x,\frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c}$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{a^2\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}dx,x,\frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{9\sqrt[3]{b}\sqrt[3]{c}}-\frac{a^2\text{Su}}{}$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}-\frac{a^2\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{9b^{2/3}}-\frac{a^2\text{Subst}\left(\int\right.}{}$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}-\frac{a^2\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{9b^{2/3}}+\frac{a^2\sqrt[3]{c}\log\left(c\right.}{}$$

$$= \frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}-\frac{a^2\sqrt[3]{c}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}-\frac{a^2\sqrt[3]{c}\log\left(c^2\right.}{9b}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.35

$$\frac{3ax\sqrt[3]{cx}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)}{4\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a*x*(c*x)^(1/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 2/3, 5/3, -((b
*x^2)/a)])/(4*(1 + (b*x^2)/a)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{4}{3}} (cx)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(1/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{1}{3}} \left(b\,x^2 + a\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(1/3)*(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{4}{3}} (cx)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{1/3}\,\left(b\,x^2 + a\right)^{4/3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)*(a + b*x^2)^(4/3),x)
```

```
[Out] int((c*x)^(1/3)*(a + b*x^2)^(4/3), x)
```

sympy [C] time = 7.09, size = 46, normalized size = 0.28

$$\frac{a^{\frac{4}{3}}\sqrt[3]{c}\,x^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix} -\frac{4}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\right| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*c**(1/3)*x**(4/3)*gamma(2/3)*hyper((-4/3, 2/3), (5/3,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(5/3))
```

3.757 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}} dx$

Optimal. Leaf size=153

$$-\frac{a\sqrt[3]{b}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{c^{5/3}}-\frac{2a\sqrt[3]{b}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}}+\frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3}-\frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}}$$

```
[Out] 2*b*(c*x)^(4/3)*(b*x^2+a)^(1/3)/c^3-3/2*(b*x^2+a)^(4/3)/c/(c*x)^(2/3)-a*b^(
1/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/c^(5/3)-2/3*a*b^(1/3)*
arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))/c^(5/
3)*3^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 233, normalized size of antiderivative = 1.52, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {277, 279, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{a\sqrt[3]{b}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{3c^{5/3}}+\frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3}-\frac{2a\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{5/3}}-\frac{2a\sqrt[3]{b}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{\sqrt{3}c^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(5/3),x]
```

```
[Out] (2*b*(c*x)^(4/3)*(a + b*x^2)^(1/3))/c^3 - (3*(a + b*x^2)^(4/3))/(2*c*(c*x)^
(2/3)) - (2*a*b^(1/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)
^(1/3))/(Sqrt[3]*c^(2/3))])/(Sqrt[3]*c^(5/3)) - (2*a*b^(1/3)*Log[c^(2/3) -
(b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(3*c^(5/3)) + (a*b^(1/3)*Log[c^(4
/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3
))/(a + b*x^2)^(1/3)])/(3*c^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{5/3}}\,dx = -\frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(4b)\int \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\,dx}{c^2}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(4ab)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{3c^2}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(4ab)\,\mathrm{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c^3}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{c^3}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{c^3}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{\left(2ab^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{7/3}} - \frac{\left(2ab^{2/3}\right)}{}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} - \frac{2a\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{5/3}} - \frac{\left(ab^{2/3}\right)\mathrm{Subst}\left(\int \frac{}{1+}\right)}{}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} - \frac{2a\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{5/3}} + \frac{a\sqrt[3]{b}\log\left(c^{4/3}+\frac{b^{2/3}(}{(a+b}\right.}{3c}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} - \frac{2a\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}} - \frac{2a\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(}{\sqrt[3]{a}}\right.}{3c^{5/3}}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.37

$$-\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{1}{3};\frac{2}{3};-\frac{bx^2}{a}\right)}{2(cx)^{5/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(5/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/3, 2/3, -((b*x^2)/a)])
/(2*(c*x)^(5/3)*(1 + (b*x^2)/a)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(5/3), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(5/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{4/3}}{{\left(c\,x\right)}^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(5/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(5/3), x)
```

sympy [C] time = 7.23, size = 49, normalized size = 0.32

$$\frac{a^{\frac{4}{3}}\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(5/3),x)
```

```
[Out] a**(4/3)*gamma(-1/3)*hyper((-4/3, -1/3), (2/3,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(5/3)*x**(2/3)*gamma(2/3))
```

3.758 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx$

Optimal. Leaf size=157

$$-\frac{3b^{4/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4c^{11/3}}-\frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2c^{11/3}}-\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}}$$

```
[Out] -3/2*b*(b*x^2+a)^(1/3)/c^3/(c*x)^(2/3)-3/8*(b*x^2+a)^(4/3)/c/(c*x)^(8/3)-3/
4*b^(4/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/c^(11/3)-1/2*b^(4
/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))*3
^(1/2)/c^(11/3)
```

Rubi [A] time = 0.29, antiderivative size = 234, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {277, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}}+\frac{b^{4/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{4c^{11/3}}-\frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{2c^{11/3}}-\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*b*(a + b*x^2)^(1/3))/(2*c^3*(c*x)^(2/3)) - (3*(a + b*x^2)^(4/3))/(8*c*(
c*x)^(8/3)) - (Sqrt[3]*b^(4/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a
 + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(2*c^(11/3)) - (b^(4/3)*Log[c^(2/3) -
(b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(2*c^(11/3)) + (b^(4/3)*Log[c^(4/
3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3)
)/(a + b*x^2)^(1/3)])/(4*c^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{11/3}}\,dx = -\frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} + \frac{b\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx}{c^2}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} + \frac{b^2\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{c^4}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} + \frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c^5}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} + \frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c^5}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} + \frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^5}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} + \frac{b^{5/3}\operatorname{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{13/3}} - \frac{b^{5/3}\operatorname{Subst}\left(\int \frac{1}{1+\cdots}\right.}{\cdots}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} - \frac{b^{4/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}} - \frac{\left(3b^{5/3}\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}\cdots}{c^{4/3}}}\right.}{4c^{13/3}}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} - \frac{b^{4/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}} + \frac{b^{4/3}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\cdots\right.}{4c^{11/3}}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} - \frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2c^{11/3}} - \frac{b^{4/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.36

$$-\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{4}{3};-\frac{1}{3};-\frac{bx^2}{a}\right)}{8(cx)^{11/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -4/3, -1/3, -((b*x^2)/a)]
)/(8*(c*x)^(11/3)*(1 + (b*x^2)/a)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(11/3), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(11/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(11/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(11/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{4/3}}{(c\,x)^{11/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(11/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(11/3), x)
```

sympy [C] time = 56.37, size = 53, normalized size = 0.34

$$\frac{a^{\frac{4}{3}}\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{4}{3}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{11}{3}}x^{\frac{8}{3}}\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(11/3),x)
```

```
[Out] a**(4/3)*gamma(-4/3)*hyper((-4/3, -4/3), (-1/3,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(11/3)*x**(8/3)*gamma(-1/3))
```

3.759 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}} \, dx$

Optimal. Leaf size=28

$$-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{14/3}}$$

[Out] -3/14*(b*x^2+a)^(7/3)/a/c/(c*x)^(14/3)

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(4/3)/(c*x)^(17/3),x]

[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(14/3))

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{17/3}} \, dx = -\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{14/3}}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{3x\left(a+bx^2\right)^{7/3}}{14a(cx)^{17/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(17/3),x]

[Out] (-3*x*(a + b*x^2)^(7/3))/(14*a*(c*x)^(17/3))

fricas [A] time = 1.17, size = 43, normalized size = 1.54

$$-\frac{3\left(b^2x^4+2\,abx^2+a^2\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{14\,ac^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="fricas")

[Out] -3/14*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a*c^6*x^5)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(17/3), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.75

$$-\frac{3\left(b\,x^2+a\right)^{\frac{7}{3}}x}{14\,(cx)^{\frac{17}{3}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(17/3),x)
```

```
[Out] -3/14*x*(b*x^2+a)^(7/3)/a/(c*x)^(17/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(17/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(b\,x^2+a\right)^{4/3}}{(c\,x)^{17/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(17/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(17/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(17/3),x)
```

```
[Out] Timed out
```

3.760 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx$

Optimal. Leaf size=57

$$\frac{9(a+bx^2)^{10/3}}{140a^2c(cx)^{20/3}}-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}$$

```
[Out] -3/14*(b*x^2+a)^(7/3)/a/c/(c*x)^(20/3)+9/140*(b*x^2+a)^(10/3)/a^2/c/(c*x)^(
20/3)
```

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{9(a+bx^2)^{10/3}}{140a^2c(cx)^{20/3}}-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(23/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(10/3))/(140*
a^2*c*(c*x)^(20/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx &= -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}-\frac{3\int \frac{(a+bx^2)^{7/3}}{(cx)^{23/3}}\,dx}{7a}\\ &= -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}+\frac{9(a+bx^2)^{10/3}}{140a^2c(cx)^{20/3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.72

$$\frac{3\sqrt[3]{cx}\,(a+bx^2)^{7/3}(3bx^2-7a)}{140a^2c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(23/3),x]
```

```
[Out] (3*(c*x)^(1/3)*(a + b*x^2)^(7/3)*(-7*a + 3*b*x^2))/(140*a^2*c^8*x^7)
```

fricas [A] time = 1.32, size = 57, normalized size = 1.00

$$\frac{3\left(3\,b^3x^6 - ab^2x^4 - 11\,a^2bx^2 - 7\,a^3\right)\left(bx^2 + a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{140\,a^2c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="fricas")
```

```
[Out] 3/140*(3*b^3*x^6 - a*b^2*x^4 - 11*a^2*b*x^2 - 7*a^3)*(b*x^2 + a)^(1/3)*(c*x
)^(1/3)/(a^2*c^8*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{4}{3}}}{(cx)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(23/3), x)
```

maple [A] time = 0.01, size = 31, normalized size = 0.54

$$-\frac{3\left(b\,x^2 + a\right)^{\frac{7}{3}}\left(-3b\,x^2 + 7a\right)x}{140\,(cx)^{\frac{23}{3}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(23/3),x)
```

```
[Out] -3/140*x*(b*x^2+a)^(7/3)*(-3*b*x^2+7*a)/a^2/(c*x)^(23/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{4}{3}}}{(cx)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(23/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2 + a\right)^{4/3}}{(c\,x)^{23/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(23/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(23/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(23/3),x)
```

```
[Out] Timed out
```

3.761 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx$

Optimal. Leaf size=85

$$-\frac{27\left(a+bx^2\right)^{13/3}}{910a^3c(cx)^{26/3}}+\frac{9\left(a+bx^2\right)^{10/3}}{70a^2c(cx)^{26/3}}-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}}$$

```
[Out] -3/14*(b*x^2+a)^(7/3)/a/c/(c*x)^(26/3)+9/70*(b*x^2+a)^(10/3)/a^2/c/(c*x)^(2
6/3)-27/910*(b*x^2+a)^(13/3)/a^3/c/(c*x)^(26/3)
```

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{27\left(a+bx^2\right)^{13/3}}{910a^3c(cx)^{26/3}}+\frac{9\left(a+bx^2\right)^{10/3}}{70a^2c(cx)^{26/3}}-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(29/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(26/3)) + (9*(a + b*x^2)^(10/3))/(70*a
^2*c*(c*x)^(26/3)) - (27*(a + b*x^2)^(13/3))/(910*a^3*c*(c*x)^(26/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{29/3}}\,dx &= -\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}}-\frac{6\int \frac{\left(a+bx^2\right)^{7/3}}{(cx)^{29/3}}\,dx}{7a}\\ &= -\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}}+\frac{9\left(a+bx^2\right)^{10/3}}{70a^2c(cx)^{26/3}}+\frac{9\int \frac{\left(a+bx^2\right)^{10/3}}{(cx)^{29/3}}\,dx}{35a^2}\\ &= -\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}}+\frac{9\left(a+bx^2\right)^{10/3}}{70a^2c(cx)^{26/3}}-\frac{27\left(a+bx^2\right)^{13/3}}{910a^3c(cx)^{26/3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.61

$$-\frac{3\left(a+bx^2\right)^{7/3}\left(35a^2-21abx^2+9b^2x^4\right)}{910a^3c^9x^8(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(29/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3)*(35*a^2 - 21*a*b*x^2 + 9*b^2*x^4))/(910*a^3*c^9*x^8*(
c*x)^(2/3))
```

fricas [A] time = 0.72, size = 68, normalized size = 0.80

$$-\frac{3\left(9\,b^{4}x^{8}-3\,ab^{3}x^{6}+2\,a^{2}b^{2}x^{4}+49\,a^{3}bx^{2}+35\,a^{4}\right)\left(bx^{2}+a\right)^{\frac{1}{3}}\left(cx\right)^{\frac{1}{3}}}{910\,a^{3}c^{10}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="fricas")
```

```
[Out] -3/910*(9*b^4*x^8 - 3*a*b^3*x^6 + 2*a^2*b^2*x^4 + 49*a^3*b*x^2 + 35*a^4)*(b
*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^3*c^10*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{29}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(29/3), x)
```

maple [A] time = 0.00, size = 42, normalized size = 0.49

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(9b^{2}x^{4}-21ab\,x^{2}+35a^{2}\right)x}{910\left(cx\right)^{\frac{29}{3}}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(29/3),x)
```

```
[Out] -3/910*x*(b*x^2+a)^(7/3)*(9*b^2*x^4-21*a*b*x^2+35*a^2)/a^3/(c*x)^(29/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{29}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(29/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{4/3}}{{\left(c\,x\right)}^{29/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(29/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(29/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(29/3),x)
```

```
[Out] Timed out
```

3.762 $\int (cx)^{10/3} \left(a+bx^2\right)^{4/3} \, dx$

Optimal. Leaf size=479

$$\frac{8a^3c^{7/3}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \mathrm{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\right.\right.}{405\sqrt[4]{3}\,b^2\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] -16/405*a^3*c^3*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b^2+16/945*a^2*c*(c*x)^(7/3)*(b
*x^2+a)^(1/3)/b+8/105*a*(c*x)^(13/3)*(b*x^2+a)^(1/3)/c+1/7*(c*x)^(13/3)*(b*
x^2+a)^(4/3)/c+8/1215*a^3*c^(7/3)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1
/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/
(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3)
)^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/
3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b
^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2
/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3
)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)
^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)*
3^(3/4)/b^2/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1
/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/
3))^2)^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {279, 321, 329, 241, 225}

$$-\frac{16a^3c^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{405b^2}+\frac{8a^3c^{7/3}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{bx}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{bx}}}\right.\right.}{405\sqrt[4]{3}\,b^2\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(10/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (-16*a^3*c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(405*b^2) + (16*a^2*c*(c*x)^(7/
3)*(a + b*x^2)^(1/3))/(945*b) + (8*a*(c*x)^(13/3)*(a + b*x^2)^(1/3))/(105*c
) + ((c*x)^(13/3)*(a + b*x^2)^(4/3))/(7*c) + (8*a^3*c^(7/3)*(c*x)^(1/3)*(a
+ b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^
(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2
/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^
(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a
 + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(405*3^(1/4)*b^2*Sqrt[-((b^(1/3)*(c*x)
^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1
/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int (cx)^{10/3}\left(a+bx^2\right)^{4/3}dx = \frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}+\frac{1}{21}(8a)\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx$$

$$=\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}+\frac{1}{315}\left(16a^2\right)\int \frac{(cx)^{10/3}}{\left(a+bx^2\right)^{2/3}}dx$$

$$=\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}-\frac{\left(16a^3c^2\right)\int}{40}$$

$$=-\frac{16a^3c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{405b^2}+\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}}{}$$

$$=-\frac{16a^3c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{405b^2}+\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}}{}$$

$$=-\frac{16a^3c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{405b^2}+\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}}{}$$

$$=-\frac{16a^3c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{405b^2}+\frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b}+\frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}+\frac{(cx)^{13/3}}{}$$

Mathematica [C] time = 0.07, size = 102, normalized size = 0.21

$$\frac{c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(7a^3\,{}_2F_1\left(-\frac{4}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)-\left(7a-15bx^2\right)\left(a+bx^2\right)^2\sqrt[3]{\frac{bx^2}{a}+1}\right)}{105b^2\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(10/3)*(a + b*x^2)^(4/3),x]
[Out] (c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(-((7*a - 15*b*x^2)*(a + b*x^2)^2*(1 + (
b*x^2)/a)^(1/3)) + 7*a^3*Hypergeometric2F1[-4/3, 1/6, 7/6, -((b*x^2)/a)]))/
(105*b^2*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 1.36, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bc^3x^5+ac^3x^3\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] integral((b*c^3*x^5 + a*c^3*x^3)*(b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{10}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(10/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{10}{3}} \left(b\,x^2+a\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(10/3)*(b*x^2+a)^(4/3),x)
[Out] int((c*x)^(10/3)*(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{10}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(10/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{10/3}\,\left(b\,x^2+a\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(10/3)*(a + b*x^2)^(4/3),x)
[Out] int((c*x)^(10/3)*(a + b*x^2)^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(10/3)*(b*x**2+a)**(4/3),x)
[Out] Timed out
```

3.763 $\int (cx)^{4/3}\left(a+bx^2\right)^{4/3}\,dx$

Optimal. Leaf size=448

$$-\frac{8a^2\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\cdots\right.\right.}{135\sqrt[4]{3}\,b\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] 16/135*a^2*c*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b+8/45*a*(c*x)^(7/3)*(b*x^2+a)^(1/
3)/c+1/5*(c*x)^(7/3)*(b*x^2+a)^(4/3)/c-8/405*a^2*c^(1/3)*(c*x)^(1/3)*(b*x^2
+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(
c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3
^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/
(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*
EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c
^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2
)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3
)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*
x^2+a)^(1/3))^2)^(1/2)*3^(3/4)/b/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*
x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3
^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {279, 321, 329, 241, 225}

$$-\frac{8a^2\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{135\sqrt[4]{3}\,b\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(4/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (16*a^2*c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(135*b) + (8*a*(c*x)^(7/3)*(a + b*
x^2)^(1/3))/(45*c) + ((c*x)^(7/3)*(a + b*x^2)^(4/3))/(5*c) - (8*a^2*c^(1/3)
*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)
^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*
c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(
c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3]
)*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)
*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(135*3^(1/4)*b*Sqrt[-(
(b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/(
(a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2
)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int (cx)^{4/3} \left(a+bx^2\right)^{4/3} dx &= \frac{(cx)^{7/3} \left(a+bx^2\right)^{4/3}}{5c} + \frac{1}{15}(8a) \int (cx)^{4/3} \sqrt[3]{a+bx^2}\, dx \\
&= \frac{8a(cx)^{7/3} \sqrt[3]{a+bx^2}}{45c} + \frac{(cx)^{7/3} \left(a+bx^2\right)^{4/3}}{5c} + \frac{1}{135}\left(16a^2\right) \int \frac{(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}} dx \\
&= \frac{16a^2 c \sqrt[3]{cx} \sqrt[3]{a+bx^2}}{135b} + \frac{8a(cx)^{7/3} \sqrt[3]{a+bx^2}}{45c} + \frac{(cx)^{7/3} \left(a+bx^2\right)^{4/3}}{5c} - \frac{\left(16a^3c^2\right) \int \frac{}{(cx)^{2/}} \cdots}{405 \cdots} \\
&= \frac{16a^2 c \sqrt[3]{cx} \sqrt[3]{a+bx^2}}{135b} + \frac{8a(cx)^{7/3} \sqrt[3]{a+bx^2}}{45c} + \frac{(cx)^{7/3} \left(a+bx^2\right)^{4/3}}{5c} - \frac{\left(16a^3c\right) \text{Subst} \cdots}{\cdots} \\
&= \frac{16a^2 c \sqrt[3]{cx} \sqrt[3]{a+bx^2}}{135b} + \frac{8a(cx)^{7/3} \sqrt[3]{a+bx^2}}{45c} + \frac{(cx)^{7/3} \left(a+bx^2\right)^{4/3}}{5c} - \frac{\left(16a^3c\right) \text{Subst} \cdots}{135b \sqrt{\cdots}} \\
&= \frac{16a^2 c \sqrt[3]{cx} \sqrt[3]{a+bx^2}}{135b} + \frac{8a(cx)^{7/3} \sqrt[3]{a+bx^2}}{45c} + \frac{(cx)^{7/3} \left(a+bx^2\right)^{4/3}}{5c} - \frac{8a^2 \sqrt[3]{c} \sqrt[3]{cx} \sqrt[3]{a} \cdots}{\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.05, size = 89, normalized size = 0.20

$$
\frac{c \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left(\left(a+bx^2\right)^2 \sqrt[3]{\frac{bx^2}{a}+1} - a^2 {}_2F_1\left(-\frac{4}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)\right)}{5b \sqrt[3]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(4/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (c*(c*x)^(1/3)*(a + b*x^2)^(1/3)*((a + b*x^2)^2*(1 + (b*x^2)/a)^(1/3) - a^2
*Hypergeometric2F1[-4/3, 1/6, 7/6, -((b*x^2)/a)]))/(5*b*(1 + (b*x^2)/a)^(1/
3))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$
\text{integral}\left(\left(bcx^3 + acx\right)\left(bx^2+a\right)^{\frac{1}{3}} (cx)^{\frac{1}{3}}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*c*x^3 + a*c*x)*(b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{4}{3}}\, dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(4/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{4}{3}} \left(b\,x^{2}+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(4/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(4/3)*(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2}+a\right)^{\frac{4}{3}} (cx)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{4/3}\,\left(b\,x^2+a\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(4/3)*(a + b*x^2)^(4/3),x)
```

```
[Out] int((c*x)^(4/3)*(a + b*x^2)^(4/3), x)
```

sympy [C] time = 14.95, size = 46, normalized size = 0.10

$$\frac{a^{\frac{4}{3}} c^{\frac{4}{3}} x^{\frac{7}{3}} \Gamma\left(\frac{7}{6}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, \frac{7}{6} \\ \frac{13}{6} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{2 \Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(4/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*c**(4/3)*x**(7/3)*gamma(7/6)*hyper((-4/3, 7/6), (13/6,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(13/6))
```

3.764 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}}\,dx$

Optimal. Leaf size=414

$$\frac{8a\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\sqrt[4]{3}\,c^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] 8/9*a*(c*x)^(1/3)*(b*x^2+a)^(1/3)/c+1/3*(c*x)^(1/3)*(b*x^2+a)^(4/3)/c+8/27*
a*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))
*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1
/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x
)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2
))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(
b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))
^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^
(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(
2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)*3^(3/4)/c^(5/3)/(-b^(1/3)*(c*x)^
(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3
)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {279, 329, 241, 225}

$$\frac{8a\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\sqrt[4]{3}\,c^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\sqrt[3]{cx}\,($$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(2/3),x]
```

```
[Out] (8*a*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(9*c) + ((c*x)^(1/3)*(a + b*x^2)^(4/3))
/(3*c) + (8*a*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3)
)/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3
) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[
3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) -
((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqr
t[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4
)*c^(5/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^
(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{2/3}}\,dx &= \frac{\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}}{3c}+\frac{1}{9}(8a)\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx\\
&= \frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9c}+\frac{\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}}{3c}+\frac{1}{27}\left(16a^2\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx\\
&= \frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9c}+\frac{\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}}{3c}+\frac{\left(16a^2\right)\operatorname{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx,x,\sqrt[3]{cx}\right)}{9c}\\
&= \frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9c}+\frac{\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}}{3c}+\frac{\left(16a^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx,x,\frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{9c\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}\\
&= \frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9c}+\frac{\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}}{3c}+\frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\sqrt[3]{\ldots}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\ldots}{\sqrt[3]{a+bx^2}\ldots}\right.}}}{9\sqrt[4]{3}\,c^{5/3}\sqrt{-\frac{\sqrt[3]{b}(c\ldots}{\sqrt[3]{a+bx^2}\left(\ldots\right.}}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.13

$$\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{6};\frac{7}{6};-\frac{bx^2}{a}\right)}{(cx)^{2/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(2/3),x]
```

```
[Out] (3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/6, 7/6, -((b*x^2)/a)])/(
(c*x)^(2/3)*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{1}{3}}}{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(2/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(2/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{4/3}}{(c\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(2/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(2/3), x)
```

sympy [C] time = 5.04, size = 46, normalized size = 0.11

$$\frac{a^{\frac{4}{3}}\sqrt[3]{x}\,\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{4}{3},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(2/3),x)
```

```
[Out] a**(4/3)*x**(1/3)*gamma(1/6)*hyper((-4/3, 1/6), (7/6,), b*x**2*exp_polar(I*
pi)/a)/(2*c**(2/3)*gamma(7/6))
```

3.765 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx$

Optimal. Leaf size=414

$$\frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{5\sqrt[4]{3}\,c^{11/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] 8/5*b*(c*x)^(1/3)*(b*x^2+a)^(1/3)/c^3-3/5*(b*x^2+a)^(4/3)/c/(c*x)^(5/3)+8/1
5*b*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3
))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^
(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c
*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1
/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))
/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3
))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a
)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)
^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)*3^(3/4)/c^(11/3)/(-b^(1/3)*(c*
x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(
2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {277, 279, 329, 241, 225}

$$\frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}\,c^{11/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{8b\sqrt[3]{cx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(8/3),x]
```

```
[Out] (8*b*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(5*c^3) - (3*(a + b*x^2)^(4/3))/(5*c*(c
*x)^(5/3)) + (8*b*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(
2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^
(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + S
qrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3
) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 +
 Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(5*3^
(1/4)*c^(11/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))
/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(
c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{8/3}}\,dx &= -\frac{3\left(a+bx^2\right)^{4/3}}{5c(cx)^{5/3}}+\frac{(8b)\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx}{5c^2} \\
&= \frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{5c^3}-\frac{3\left(a+bx^2\right)^{4/3}}{5c(cx)^{5/3}}+\frac{(16ab)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{15c^2} \\
&= \frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{5c^3}-\frac{3\left(a+bx^2\right)^{4/3}}{5c(cx)^{5/3}}+\frac{(16ab)\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx,x,\sqrt[3]{cx}\right)}{5c^3} \\
&= \frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{5c^3}-\frac{3\left(a+bx^2\right)^{4/3}}{5c(cx)^{5/3}}+\frac{(16ab)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx,x,\frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{5c^3\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a+bx^2}} \\
&= \frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{5c^3}-\frac{3\left(a+bx^2\right)^{4/3}}{5c(cx)^{5/3}}+\frac{8b\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\cdots}{\cdots}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(\cdots}{\sqrt[3]{a+bx^2}}\cdots\right.}}\cdots}{5\sqrt[4]{3}\,c^{11/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx\cdots}{\sqrt[3]{a+bx^2}\left(\cdots\right.}}\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.14

$$
-\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{5}{6};\frac{1}{6};-\frac{bx^2}{a}\right)}{5(cx)^{8/3}\sqrt[3]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(8/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -5/6, 1/6, -((b*x^2)/a)])
/(5*(c*x)^(8/3)*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{1}{3}}}{c^3x^3},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{8}{3}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(8/3), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(8/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(8/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(8/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{4/3}}{{\left(c\,x\right)}^{8/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(8/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(8/3), x)
```

sympy [C] time = 15.93, size = 32, normalized size = 0.08

$$\frac{b^{\frac{4}{3}} x {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| {\frac{a e^{i \pi}}{b x^{2}}} \right)}{c^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(8/3),x)
```

```
[Out] b**(4/3)*x*hyper((-4/3, -1/2), (1/2,), a*exp_polar(I*pi)/(b*x**2))/c**(8/3)
```

3.766 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx$

Optimal. Leaf size=419

$$\frac{8\,3^{3/4}b^2\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \mathrm{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\right.\right.}{55ac^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] -24/55*b*(b*x^2+a)^(1/3)/c^3/(c*x)^(5/3)-3/11*(b*x^2+a)^(4/3)/c/(c*x)^(11/3
)+8/55*3^(3/4)*b^2*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)
/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3)
)^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(
2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x
)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2
/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))
/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)
^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3
)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/a/c^(17/3)/(-b^
(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(
1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {277, 329, 241, 225}

$$\frac{8\,3^{3/4}b^2\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55ac^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(14/3),x]
```

```
[Out] (-24*b*(a + b*x^2)^(1/3))/(55*c^3*(c*x)^(5/3)) - (3*(a + b*x^2)^(4/3))/(11*
c*(c*x)^(11/3)) + (8*3^(3/4)*b^2*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (
b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3
))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^
(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF
[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(
c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqr
t[3])/4])/(55*a*c^(17/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c
*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{14/3}}\,dx &= -\frac{3\left(a+bx^2\right)^{4/3}}{11c(cx)^{11/3}}+\frac{(8b)\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx}{11c^2}\\
&= -\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{11c(cx)^{11/3}}+\frac{\left(16b^2\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{55c^4}\\
&= -\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{11c(cx)^{11/3}}+\frac{\left(48b^2\right)\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx,x,\sqrt[3]{cx}\right)}{55c^5}\\
&= -\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{11c(cx)^{11/3}}+\frac{\left(48b^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx,x,\frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{55c^5\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}\\
&= -\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{11c(cx)^{11/3}}+\frac{8\,3^{3/4}b^2\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\cdots}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\cdots}{\sqrt[3]{a+bx^2}}\cdots\right.\cdots}}\cdots}{55ac^{17/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\cdots}{\sqrt[3]{a+bx^2}\left(c^{2/3}\cdots\right.\cdots}}\cdots}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.14

$$-\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{11}{6},-\frac{4}{3};-\frac{5}{6};-\frac{bx^2}{a}\right)}{11(cx)^{14/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(14/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-11/6, -4/3, -5/6, -((b*x^2)/a)
])/(11*(c*x)^(14/3)*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{1}{3}}}{c^5x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(14/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c^5*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(14/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(14/3), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(14/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(14/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(14/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(14/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^{4/3}}{(c\,x)^{14/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(14/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(14/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(14/3),x)
```

```
[Out] Timed out
```

3.767 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx$

Optimal. Leaf size=450

$$-\frac{24\ 3^{3/4} b^3 \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}(2\cdots\right.}{935a^2c^{23/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] -24/187*b*(b*x^2+a)^(1/3)/c^3/(c*x)^(11/3)-48/935*b^2*(b*x^2+a)^(1/3)/a/c^5
/(c*x)^(5/3)-3/17*(b*x^2+a)^(4/3)/c/(c*x)^(17/3)-24/935*3^(3/4)*b^3*(c*x)^(
1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3
)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)
^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(
1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2
+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^
(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
,1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(
1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3
^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/a^2/c^(23/3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/
3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c
*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {277, 325, 329, 241, 225}

$$-\frac{24\ 3^{3/4} b^3 \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\cdots}{935a^2c^{23/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(20/3),x]
```

```
[Out] (-24*b*(a + b*x^2)^(1/3))/(187*c^3*(c*x)^(11/3)) - (48*b^2*(a + b*x^2)^(1/3
))/(935*a*c^5*(c*x)^(5/3)) - (3*(a + b*x^2)^(4/3))/(17*c*(c*x)^(17/3)) - (2
4*3^(3/4)*b^3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3)
)/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3
) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[
3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) -
((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqr
t[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(935*a^2*
c^(23/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a +
b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(
2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{20/3}}\,dx = -\frac{3\left(a+bx^2\right)^{4/3}}{17c(cx)^{17/3}} + \frac{(8b)\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx}{17c^2}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{17c(cx)^{17/3}} + \frac{\left(16b^2\right)\int \frac{1}{(cx)^{8/3}\left(a+bx^2\right)^{2/3}}\,dx}{187c^4}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{17c(cx)^{17/3}} - \frac{\left(48b^3\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{935ac^6}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{17c(cx)^{17/3}} - \frac{\left(144b^3\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{c\right.}}{935ac^7}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{17c(cx)^{17/3}} - \frac{\left(144b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{c}}{\sqrt[6]{a+b}}\right.}{935ac^7\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}$$

$$= -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{17c(cx)^{17/3}} - \frac{24\,3^{3/4}b^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx}{\sqrt[3]{a+b}}\right.}{}$$

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Mathematica [C] time = 0.01, size = 57, normalized size = 0.13

$$-\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{17}{6},-\frac{4}{3};-\frac{11}{6};-\frac{bx^2}{a}\right)}{17(cx)^{20/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(20/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-17/6, -4/3, -11/6, -((b*x^2)/a
)])/(17*(c*x)^(20/3)*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{1}{3}}}{c^7x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(20/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c^7*x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{20}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(20/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(20/3), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{20}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(20/3),x)
[Out] int((b*x^2+a)^(4/3)/(c*x)^(20/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{20}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(20/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(20/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{4/3}}{{\left(c\,x\right)}^{20/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(20/3),x)
[Out] int((a + b*x^2)^(4/3)/(c*x)^(20/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(20/3),x)
[Out] Timed out
```

3.768 $\int (cx)^{2/3} \left(a+bx^2\right)^{4/3} \, dx$

Optimal. Leaf size=59

$$\frac{3a(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{\frac{bx^2}{a}+1}}$$

```
[Out] 3/5*a*(c*x)^(5/3)*(b*x^2+a)^(1/3)*hypergeom([-4/3, 5/6],[11/6],-b*x^2/a)/c/
(1+b*x^2/a)^(1/3)
```

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{3a(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(2/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a*(c*x)^(5/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 5/6, 11/6, -((b*
x^2)/a)])/(5*c*(1 + (b*x^2)/a)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^{2/3}\left(a+bx^2\right)^{4/3}\,dx &= \frac{\left(a\sqrt[3]{a+bx^2}\right)\int (cx)^{2/3}\left(1+\frac{bx^2}{a}\right)^{4/3}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}\\ &= \frac{3a(cx)^{5/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\sqrt[3]{1+\frac{bx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 0.97

$$\frac{3ax(cx)^{2/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(2/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a*x*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 5/6, 11/6, -((
b*x^2)/a)])/(5*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int(cx)^{\frac{2}{3}}\left(b\,x^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(2/3)*(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int(c\,x)^{2/3}\,\left(b\,x^2+a\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2/3)*(a + b*x^2)^(4/3),x)
```

```
[Out] int((c*x)^(2/3)*(a + b*x^2)^(4/3), x)
```

sympy [C] time = 8.79, size = 46, normalized size = 0.78

$$\frac{a^{\frac{4}{3}}c^{\frac{2}{3}}x^{\frac{5}{3}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3}, \frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(2/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*c**(2/3)*x**(5/3)*gamma(5/6)*hyper((-4/3, 5/6), (11/6,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(11/6))
```

3.769 $\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}}\,dx$

Optimal. Leaf size=59

$$\frac{3a(cx)^{2/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{\frac{bx^2}{a}+1}}$$

```
[Out] 3/2*a*(c*x)^(2/3)*(b*x^2+a)^(1/3)*hypergeom([-4/3, 1/3],[4/3],-b*x^2/a)/c/(
1+b*x^2/a)^(1/3)
```

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{3a(cx)^{2/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(1/3),x]
```

```
[Out] (3*a*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x
^2)/a)])/(2*c*(1 + (b*x^2)/a)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}}\,dx &= \frac{\left(a\sqrt[3]{a+bx^2}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}}{\sqrt[3]{cx}}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}\\ &= \frac{3a(cx)^{2/3}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\sqrt[3]{1+\frac{bx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 0.97

$$\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2\sqrt[3]{cx}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(1/3),x]
```

```
[Out] (3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^2)/a)])/(
2*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{2}{3}}}{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(2/3)/(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(1/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(1/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{{\left(b\,x^2+a\right)}^{4/3}}{{\left(c\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(1/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(1/3), x)
```

sympy [C] time = 5.11, size = 46, normalized size = 0.78

$$\frac{a^{\frac{4}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[3]{c}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(1/3),x)
```

```
[Out] a**(4/3)*x**(2/3)*gamma(1/3)*hyper((-4/3, 1/3), (4/3,), b*x**2*exp_polar(I*
pi)/a)/(2*c**(1/3)*gamma(4/3))
```

3.770 $\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{4/3}}\,dx$

Optimal. Leaf size=57

$$-\frac{3a\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\sqrt[3]{\frac{bx^2}{a}+1}}$$

```
[Out] -3*a*(b*x^2+a)^(1/3)*hypergeom([-4/3, -1/6],[5/6],-b*x^2/a)/c/(c*x)^(1/3)/(
1+b*x^2/a)^(1/3)
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$-\frac{3a\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(4/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/6, 5/6, -((b*x^2)/a)])/(
c*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{4/3}}\,dx &= \frac{\left(a\sqrt[3]{a+bx^2}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}}{(cx)^{4/3}}\,dx}{\sqrt[3]{1+\frac{bx^2}{a}}}\\ &= -\frac{3a\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\sqrt[3]{1+\frac{bx^2}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 0.96

$$-\frac{3ax\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{4}{3},-\frac{1}{6};\frac{5}{6};-\frac{bx^2}{a}\right)}{(cx)^{4/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(4/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/6, 5/6, -((b*x^2)/a)])
/((c*x)^(4/3)*(1 + (b*x^2)/a)^(1/3))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{2}{3}}}{c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(4/3)*(c*x)^(2/3)/(c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(4/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(4/3),x)
```

```
[Out] int((b*x^2+a)^(4/3)/(c*x)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^2+a\right)}^{4/3}}{{\left(c\,x\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(4/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(4/3), x)
```

sympy [C] time = 5.07, size = 49, normalized size = 0.86

$$\frac{a^{\frac{4}{3}}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3}, -\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{4}{3}}\sqrt[3]{x}\,\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(4/3),x)
```

```
[Out] a**(4/3)*gamma(-1/6)*hyper((-4/3, -1/6), (5/6,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(4/3)*x**(1/3)*gamma(5/6))
```

3.771 $\int \frac{(cx)^{19/3}}{\left(a+bx^2\right)^{2/3}} dx$

Optimal. Leaf size=198

$$\frac{10a^3c^{19/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{27b^{11/3}}+\frac{20a^3c^{19/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{27\sqrt{3}\,b^{11/3}}+\frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3}-\frac{2ac^3(cx)^{10/3}\sqrt[3]{}}{9b^2}$$

```
[Out] 10/27*a^2*c^5*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b^3-2/9*a*c^3*(c*x)^(10/3)*(b*x^2
+a)^(1/3)/b^2+1/6*c*(c*x)^(16/3)*(b*x^2+a)^(1/3)/b+10/27*a^3*c^(19/3)*ln(b^
(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(11/3)+20/81*a^3*c^(19/3)*arct
an(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))/b^(11/3)*
3^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 278, normalized size of antiderivative = 1.40, number of steps used = 12, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3}+\frac{20a^3c^{19/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{81b^{11/3}}-\frac{10a^3c^{19/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{81b^{11/3}}+\frac{20a^3c^{19/3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(19/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (10*a^2*c^5*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(27*b^3) - (2*a*c^3*(c*x)^(10/3)
*(a + b*x^2)^(1/3))/(9*b^2) + (c*(c*x)^(16/3)*(a + b*x^2)^(1/3))/(6*b) + (2
0*a^3*c^(19/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))
/(Sqrt[3]*c^(2/3))])/(27*Sqrt[3]*b^(11/3)) + (20*a^3*c^(19/3)*Log[c^(2/3) -
 (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(81*b^(11/3)) - (10*a^3*c^(19/3)
*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(
c*x)^(2/3))/(a + b*x^2)^(1/3)])/(81*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(cx)^{19/3}}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(8ac^2\right)\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{9b}$$

$$= -\frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{\left(20a^2c^4\right)\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{27b^2}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(40a^3c^6\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}}{81b^3}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(40a^3c^5\right)\mathrm{Subst}\left(\int\right.}{27}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(20a^3c^5\right)\mathrm{Subst}\left(\int\right.}{2}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(20a^3c^5\right)\mathrm{Subst}\left(\int\right.}{27}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(20a^3c^{17/3}\right)\mathrm{Subst}\left(\right.}{81}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{20a^3c^{19/3}\log\left(c^{2/3}\right.}{81b^{11/3}}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{20a^3c^{19/3}\log\left(c^{2/3}\right.}{81b^{11/3}}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{20a^3c^{19/3}\tan^{-1}\left(\right.}{27\sqrt{3}\,b^{11/3}}$$

Mathematica [C] time = 0.03, size = 87, normalized size = 0.44

$$\frac{c^5(cx)^{4/3}\left(-20a^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^2}{bx^2+a}\right)+20a^3+8a^2bx^2-3ab^2x^4+9b^3x^6\right)}{54b^3\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(19/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c^5*(c*x)^(4/3)*(20*a^3 + 8*a^2*b*x^2 - 3*a*b^2*x^4 + 9*b^3*x^6 - 20*a^3*H
ypergeometric2F1[2/3, 1, 5/3, (b*x^2)/(a + b*x^2)]))/(54*b^3*(a + b*x^2)^(2
/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{19}{3}}}{(bx^2 + a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(19/3)/(b*x^2 + a)^(2/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{19}{3}}}{(b\,x^2 + a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(19/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(19/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{19}{3}}}{(bx^2 + a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(19/3)/(b*x^2 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{19/3}}{(b\,x^2 + a)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(19/3)/(a + b*x^2)^(2/3),x)
```

```
[Out] int((c*x)^(19/3)/(a + b*x^2)^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(19/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

3.772 $\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=167

$$-\frac{5a^2c^{13/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{12b^{8/3}}-\frac{5a^2c^{13/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{6\sqrt{3}\,b^{8/3}}-\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2}+\frac{c(cx)^{10/3}\sqrt[3]{a+bx}}{4b}$$

```
[Out] -5/12*a*c^3*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b^2+1/4*c*(c*x)^(10/3)*(b*x^2+a)^(1
/3)/b-5/12*a^2*c^(13/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(
8/3)-5/18*a^2*c^(13/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a
)^(1/3))*3^(1/2))/b^(8/3)*3^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 247, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{5a^2c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{8/3}}+\frac{5a^2c^{13/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{36b^{8/3}}-\frac{5a^2c^{13/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{6\sqrt{3}\,b^{8/3}}-\frac{5ac^3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(13/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (-5*a*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(12*b^2) + (c*(c*x)^(10/3)*(a + b*
x^2)^(1/3))/(4*b) - (5*a^2*c^(13/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3
))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(6*Sqrt[3]*b^(8/3)) - (5*a^2*c^(1
3/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(18*b^(8/3)) +
 (5*a^2*c^(13/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b
^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(36*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{\left(5ac^2\right)\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{6b}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^4\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{9b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^3\right)\mathrm{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{3b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^3\right)\mathrm{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{6b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^3\right)\mathrm{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^{11/3}\right)\mathrm{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{7/3}} - \cdots$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{5a^2c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{8/3}} - \frac{\left(5a^2c^{11/3}\right)\mathrm{Su}\cdots}{}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{5a^2c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{8/3}} + \frac{5a^2c^{13/3}\log\Big(\cdots}{}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{5a^2c^{13/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{6\sqrt{3}\,b^{8/3}} - \frac{5a^2c^{13/3}\log\Big(c\cdots}{18\cdots}$$

Mathematica [C] time = 0.02, size = 76, normalized size = 0.46

$$\frac{c^3(cx)^{4/3}\left(5a^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^2}{bx^2+a}\right)-5a^2-2abx^2+3b^2x^4\right)}{12b^2\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c^3*(c*x)^(4/3)*(-5*a^2 - 2*a*b*x^2 + 3*b^2*x^4 + 5*a^2*Hypergeometric2F1[
2/3, 1, 5/3, (b*x^2)/(a + b*x^2)]))/(12*b^2*(a + b*x^2)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{13}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate((c*x)^(13/3)/(b*x^2 + a)^(2/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{13}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)/(b*x^2+a)^(2/3),x)
[Out] int((c*x)^(13/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{13}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate((c*x)^(13/3)/(b*x^2 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{13/3}}{(b\,x^2+a)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)/(a + b*x^2)^(2/3),x)
[Out] int((c*x)^(13/3)/(a + b*x^2)^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/3)/(b*x**2+a)**(2/3),x)
[Out] Timed out
```

3.773 $\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=131

$$\frac{ac^{7/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{2b^{5/3}}+\frac{ac^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{5/3}}+\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}$$

```
[Out] 1/2*c*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b+1/2*a*c^(7/3)*ln(b^(1/3)*(c*x)^(2/3)-c^
(2/3)*(b*x^2+a)^(1/3))/b^(5/3)+1/3*a*c^(7/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^
(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 209, normalized size of antiderivative = 1.60, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}}-\frac{ac^{7/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{6b^{5/3}}+\frac{ac^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{\sqrt{3}\,b^{5/3}}+\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*b) + (a*c^(7/3)*ArcTan[(c^(2/3) + (2*b
^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(Sqrt[3]*b^(5/3)
) + (a*c^(7/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(3*b
^(5/3)) - (a*c^(7/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3)
+ (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(6*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{\left(2ac^2\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{3b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{(2ac)\,\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{(ac)\,\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{(ac)\,\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{\left(ac^{5/3}\right)\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{4/3}} + \frac{\left(ac^{5/3}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,d\right.}{3b^{4/3}}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} + \frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}} + \frac{\left(ac^{5/3}\right)\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right.}{2b^{4/3}}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} + \frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}} - \frac{ac^{7/3}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{5/3}} -$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} + \frac{ac^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{5/3}} + \frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}} - \frac{ac^{7/3}\log\left(c^{4/3}\right.}{}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.44

$$\frac{c(cx)^{4/3}\left(-a\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^2}{bx^2+a}\right)+a+bx^2\right)}{2b\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2 - a*Hypergeometric2F1[2/3, 1, 5/3, (b*x^2)/(a + b
*x^2)]))/(2*b*(a + b*x^2)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate((c*x)^(7/3)/(b*x^2 + a)^(2/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{3}}}{\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)/(b*x^2+a)^(2/3),x)
[Out] int((c*x)^(7/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate((c*x)^(7/3)/(b*x^2 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{7/3}}{\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)/(a + b*x^2)^(2/3),x)
[Out] int((c*x)^(7/3)/(a + b*x^2)^(2/3), x)
```

sympy [C] time = 30.01, size = 44, normalized size = 0.34

$$\frac{c^{\frac{7}{3}}x^{\frac{10}{3}}\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/3)/(b*x**2+a)**(2/3),x)
[Out] c**(7/3)*x**(10/3)*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(2/3)*gamma(8/3))
```

3.774 $\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=106

$$-\frac{3\sqrt[3]{c}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4b^{2/3}}-\frac{\sqrt{3}\,\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2b^{2/3}}$$

```
[Out] -3/4*c^(1/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(2/3)-1/2*c^
(1/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))
*3^(1/2)/b^(2/3)
```

Rubi [A] time = 0.24, antiderivative size = 183, normalized size of antiderivative = 1.73, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}}+\frac{\sqrt[3]{c}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{4b^{2/3}}-\frac{\sqrt{3}\,\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{2b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] -(Sqrt[3]*c^(1/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3))/(Sqrt[3]*c^(2/3))])/(2*b^(2/3)) - (c^(1/3)*Log[c^(2/3) - (b^(1/3)*(c*x)
^(2/3))/(a + b*x^2)^(1/3)])/(2*b^(2/3)) + (c^(1/3)*Log[c^(4/3) + (b^(2/3)*(
c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(
1/3)])/(4*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx &= \frac{3\,\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c} \\
&= \frac{3\,\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c} \\
&= \frac{3\,\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2\sqrt[3]{b}\,\sqrt[3]{c}} - \frac{\text{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}\,x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2\sqrt[3]{b}\,\sqrt[3]{c}} \\
&= -\frac{\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}\,x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4\sqrt[3]{b}\,\sqrt[3]{c}} + \cdots \\
&= -\frac{\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}} + \frac{\sqrt[3]{c}\,\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4b^{2/3}} + \cdots \\
&= -\frac{\sqrt{3}\,\sqrt[3]{c}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2b^{2/3}} - \frac{\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}} + \cdots
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.42

$$
\frac{3x\sqrt[3]{cx}\ {}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^2}{bx^2+a}\right)}{4\left(a+bx^2\right)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(c*x)^(1/3)*Hypergeometric2F1[2/3, 1, 5/3, (b*x^2)/(a + b*x^2)])/(4*(a
 + b*x^2)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(cx)^{\frac{1}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="giac")

[Out] integrate((c*x)^(1/3)/(b*x^2 + a)^(2/3), x)

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{3}}}{\left(b\,x^{2}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(1/3)/(b*x^2+a)^(2/3),x)

[Out] int((c*x)^(1/3)/(b*x^2+a)^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{3}}}{\left(bx^{2}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")

[Out] integrate((c*x)^(1/3)/(b*x^2 + a)^(2/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{1/3}}{\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(1/3)/(a + b*x^2)^(2/3),x)

[Out] int((c*x)^(1/3)/(a + b*x^2)^(2/3), x)

sympy [C] time = 1.52, size = 44, normalized size = 0.42

$$\frac{\sqrt[3]{c}\,x^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(1/3)/(b*x**2+a)**(2/3),x)

[Out] c**(1/3)*x**(4/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**2*exp_polar(I*pi)/a)/(2*a**(2/3)*gamma(5/3))

3.775 $\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=28

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

```
[Out] -3/2*(b*x^2+a)^(1/3)/a/c/(c*x)^(2/3)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(2/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx=-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{3x\sqrt[3]{a+bx^2}}{2a(cx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3))/(2*a*(c*x)^(5/3))
```

fricas [A] time = 1.14, size = 25, normalized size = 0.89

$$-\frac{3\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{2\,ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -3/2*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a*c^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(5/3)), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.75

$$-\frac{3\left(b\,x^2+a\right)^{\frac{1}{3}}x}{2\,(cx)^{\frac{5}{3}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x)
[Out] -3/2*x*(b*x^2+a)^(1/3)/a/(c*x)^(5/3)
```

maxima [A] time = 1.47, size = 35, normalized size = 1.25

$$-\frac{3\left(bc^{\frac{1}{3}}x^3+ac^{\frac{1}{3}}x\right)}{2\left(bx^2+a\right)^{\frac{2}{3}}ac^2x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] -3/2*(b*c^(1/3)*x^3 + a*c^(1/3)*x)/((b*x^2 + a)^(2/3)*a*c^2*x^(5/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,x)^{5/3}\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x)
[Out] int(1/((c*x)^(5/3)*(a + b*x^2)^(2/3)), x)
```

sympy [A] time = 4.17, size = 36, normalized size = 1.29

$$\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{1}{3}\right)}{2ac^{\frac{5}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/3)/(b*x**2+a)**(2/3),x)
[Out] b**(1/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-1/3)/(2*a*c**(5/3)*gamma(2/3))
```

3.776 $\int \frac{1}{(cx)^{11/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=57

$$\frac{9\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{8/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}$$

```
[Out] -3/2*(b*x^2+a)^(1/3)/a/c/(c*x)^(8/3)+9/8*(b*x^2+a)^(4/3)/a^2/c/(c*x)^(8/3)
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{9\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{8/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(8/3)) + (9*(a + b*x^2)^(4/3))/(8*a^2*c
*(c*x)^(8/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{11/3}\left(a+bx^2\right)^{2/3}}\,dx &= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}-\frac{3\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx}{a}\\ &= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}+\frac{9\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{8/3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.60

$$-\frac{3x\left(a-3bx^2\right)\sqrt[3]{a+bx^2}}{8a^2(cx)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*x*(a - 3*b*x^2)*(a + b*x^2)^(1/3))/(8*a^2*(c*x)^(11/3))
```

fricas [A] time = 1.14, size = 35, normalized size = 0.61

$$\frac{3\left(3\,bx^2-a\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{8\,a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] 3/8*(3*b*x^2 - a)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^2*c^4*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(11/3)), x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.51

$$-\frac{3\left(b\,x^2+a\right)^{\frac{1}{3}}\left(-3b\,x^2+a\right)x}{8\,(cx)^{\frac{11}{3}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x)
[Out] -3/8*x*(b*x^2+a)^(1/3)*(-3*b*x^2+a)/a^2/(c*x)^(11/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(11/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{(c\,x)^{11/3}\,\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x)
[Out] int(1/((c*x)^(11/3)*(a + b*x^2)^(2/3)), x)
```

sympy [A] time = 109.45, size = 78, normalized size = 1.37

$$-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{4}{3}\right)}{6ac^{\frac{11}{3}}x^2\Gamma\left(\frac{2}{3}\right)}+\frac{b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{4}{3}\right)}{2a^2c^{\frac{11}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] -b**(1/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(6*a*c**(11/3)*x**2*gamma(2/3
)) + b**(4/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(2*a**2*c**(11/3)*gamma(2
/3))
```

3.777 $\int \frac{1}{(cx)^{17/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=85

$$-\frac{27\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{14/3}}+\frac{9\left(a+bx^2\right)^{4/3}}{4a^2c(cx)^{14/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}$$

```
[Out] -3/2*(b*x^2+a)^(1/3)/a/c/(c*x)^(14/3)+9/4*(b*x^2+a)^(4/3)/a^2/c/(c*x)^(14/3
)-27/28*(b*x^2+a)^(7/3)/a^3/c/(c*x)^(14/3)
```

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{27\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{14/3}}+\frac{9\left(a+bx^2\right)^{4/3}}{4a^2c(cx)^{14/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(14/3)) + (9*(a + b*x^2)^(4/3))/(4*a^2*
c*(c*x)^(14/3)) - (27*(a + b*x^2)^(7/3))/(28*a^3*c*(c*x)^(14/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{17/3}\left(a+bx^2\right)^{2/3}}\,dx &= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}-\frac{6\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx}{a}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}+\frac{9\left(a+bx^2\right)^{4/3}}{4a^2c(cx)^{14/3}}+\frac{9\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{17/3}}\,dx}{2a^2}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}+\frac{9\left(a+bx^2\right)^{4/3}}{4a^2c(cx)^{14/3}}-\frac{27\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{14/3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.61

$$-\frac{3\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(2a^2-3abx^2+9b^2x^4\right)}{28a^3c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(2*a^2 - 3*a*b*x^2 + 9*b^2*x^4))/(28*a^3*
c^6*x^5)
```

fricas [A] time = 0.87, size = 46, normalized size = 0.54

$$-\frac{3\left(9\,b^{2}x^{4}-3\,abx^{2}+2\,a^{2}\right)\left(bx^{2}+a\right)^{\frac{1}{3}}\left(cx\right)^{\frac{1}{3}}}{28\,a^{3}c^{6}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -3/28*(9*b^2*x^4 - 3*a*b*x^2 + 2*a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^3*c^
6*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^{2}+a\right)^{\frac{2}{3}}\left(cx\right)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(17/3)), x)
```

maple [A] time = 0.01, size = 42, normalized size = 0.49

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(9b^{2}x^{4}-3ab\,x^{2}+2a^{2}\right)x}{28\left(cx\right)^{\frac{17}{3}}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] -3/28*x*(b*x^2+a)^(1/3)*(9*b^2*x^4-3*a*b*x^2+2*a^2)/a^3/(c*x)^(17/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^{2}+a\right)^{\frac{2}{3}}\left(cx\right)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(17/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{\left(c\,x\right)}^{17/3}\,{\left(b\,x^{2}+a\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x)
```

```
[Out] int(1/((c*x)^(17/3)*(a + b*x^2)^(2/3)), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(17/3)/(b*x**2+a)**(2/3),x)
[Out] Timed out
```

3.778 $\int \frac{1}{(cx)^{23/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=113

$$\frac{243\left(a+bx^2\right)^{10/3}}{280a^4c(cx)^{20/3}}-\frac{81\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{20/3}}+\frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}$$

```
[Out] -3/2*(b*x^2+a)^(1/3)/a/c/(c*x)^(20/3)+27/8*(b*x^2+a)^(4/3)/a^2/c/(c*x)^(20/
3)-81/28*(b*x^2+a)^(7/3)/a^3/c/(c*x)^(20/3)+243/280*(b*x^2+a)^(10/3)/a^4/c/
(c*x)^(20/3)
```

Rubi [A] time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{243\left(a+bx^2\right)^{10/3}}{280a^4c(cx)^{20/3}}-\frac{81\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{20/3}}+\frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(20/3)) + (27*(a + b*x^2)^(4/3))/(8*a^2
*c*(c*x)^(20/3)) - (81*(a + b*x^2)^(7/3))/(28*a^3*c*(c*x)^(20/3)) + (243*(a
 + b*x^2)^(10/3))/(280*a^4*c*(c*x)^(20/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{23/3}\left(a+bx^2\right)^{2/3}}\,dx &= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}-\frac{9\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx}{a}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}+\frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}}+\frac{27\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{23/3}}\,dx}{2a^2}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}+\frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}}-\frac{81\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{20/3}}-\frac{81\int \frac{\left(a+bx^2\right)^{7/3}}{(cx)^{23/3}}\,dx}{14a^3}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}+\frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}}-\frac{81\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{20/3}}+\frac{243\left(a+bx^2\right)^{10/3}}{280a^4c(cx)^{20/3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 0.56

$$\frac{3\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(-14a^3+18a^2bx^2-27ab^2x^4+81b^3x^6\right)}{280a^4c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(-14*a^3 + 18*a^2*b*x^2 - 27*a*b^2*x^4 + 8
1*b^3*x^6))/(280*a^4*c^8*x^7)
```

fricas [A] time = 0.83, size = 57, normalized size = 0.50

$$\frac{3\left(81\,b^3x^6-27\,ab^2x^4+18\,a^2bx^2-14\,a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{280\,a^4c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/280*(81*b^3*x^6 - 27*a*b^2*x^4 + 18*a^2*b*x^2 - 14*a^3)*(b*x^2 + a)^(1/3)
*(c*x)^(1/3)/(a^4*c^8*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(23/3)), x)
```

maple [A] time = 0.00, size = 53, normalized size = 0.47

$$-\frac{3\left(b\,x^2+a\right)^{\frac{1}{3}}\left(-81b^3x^6+27a\,b^2x^4-18a^2b\,x^2+14a^3\right)x}{280\,(cx)^{\frac{23}{3}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] -3/280*x*(b*x^2+a)^(1/3)*(-81*b^3*x^6+27*a*b^2*x^4-18*a^2*b*x^2+14*a^3)/a^4
/(c*x)^(23/3)
```

maxima [A] time = 1.54, size = 64, normalized size = 0.57

$$\frac{3\left(81\,b^4x^9+54\,ab^3x^7-9\,a^2b^2x^5+4\,a^3bx^3-14\,a^4x\right)}{280\left(bx^2+a\right)^{\frac{2}{3}}a^4c^{\frac{23}{3}}x^{\frac{23}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/280*(81*b^4*x^9 + 54*a*b^3*x^7 - 9*a^2*b^2*x^5 + 4*a^3*b*x^3 - 14*a^4*x)/
((b*x^2 + a)^(2/3)*a^4*c^(23/3)*x^(23/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{23/3}\,\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x)
```

```
[Out] int(1/((c*x)^(23/3)*(a + b*x^2)^(2/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(23/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

3.779 $\int \frac{(cx)^{10/3}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=421

$$\frac{7ac^{7/3}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{18\sqrt[4]{3}\,b^2\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] -7/9*a*c^3*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b^2+1/3*c*(c*x)^(7/3)*(b*x^2+a)^(1/3
)/b+7/54*a*c^(7/3)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)
/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3)
)^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(
2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x
)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2
/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))
/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)
^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3
)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)*3^(3/4)/b^2/(-b
^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^
(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {321, 329, 241, 225}

$$-\frac{7ac^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b^2}+\frac{7ac^{7/3}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\right.}{18\sqrt[4]{3}\,b^2\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(10/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (-7*a*c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3))/(9*b^2) + (c*(c*x)^(7/3)*(a + b*x^
2)^(1/3))/(3*b) + (7*a*c^(7/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^
(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))
/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2
/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[A
rcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^
(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[
3])/4])/(18*3^(1/4)*b^2*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*
x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*
b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{10/3}}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b} - \frac{\left(7ac^2\right)\int \frac{(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{9b}$$

$$= -\frac{7ac^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b} + \frac{\left(7a^2c^4\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{27b^2}$$

$$= -\frac{7ac^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b} + \frac{\left(7a^2c^3\right)\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{9b^2}$$

$$= -\frac{7ac^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b} + \frac{\left(7a^2c^3\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{9b^2\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a+bx^2}}$$

$$= -\frac{7ac^3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b} + \frac{7ac^{7/3}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}\cdots}{(a+b\cdots}}{\left(c^{2/3}-\cdots\right.}}}{18\sqrt[4]{3}\,b^2\sqrt{\frac{-\cdots}{\sqrt[3]{\cdots}}}}$$

Mathematica [C] time = 0.03, size = 87, normalized size = 0.21

$$\frac{c^3 \sqrt[3]{cx} \left(7a^2 \left(\frac{bx^2}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{6},\frac{2}{3};\frac{7}{6};-\frac{bx^2}{a}\right)-7a^2-4abx^2+3b^2x^4\right)}{9b^2\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(10/3)/(a + b*x^2)^(2/3),x]
[Out] (c^3*(c*x)^(1/3)*(-7*a^2 - 4*a*b*x^2 + 3*b^2*x^4 + 7*a^2*(1 + (b*x^2)/a)^(2
/3)*Hypergeometric2F1[1/6, 2/3, 7/6, -((b*x^2)/a)]))/(9*b^2*(a + b*x^2)^(2/
3))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(cx)^{\frac{1}{3}}c^3x^3}{\left(bx^2+a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] integral((c*x)^(1/3)*c^3*x^3/(b*x^2 + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{10}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate((c*x)^(10/3)/(b*x^2 + a)^(2/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{10}{3}}}{\left(b\,x^2+a\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(10/3)/(b*x^2+a)^(2/3),x)
[Out] int((c*x)^(10/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{10}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(10/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate((c*x)^(10/3)/(b*x^2 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x)^{10/3}}{\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(10/3)/(a + b*x^2)^(2/3),x)
```

```
[Out] int((c*x)^(10/3)/(a + b*x^2)^(2/3), x)
```

sympy [C] time = 126.93, size = 44, normalized size = 0.10

$$\frac{c^{\frac{10}{3}}x^{\frac{13}{3}}\Gamma\left(\frac{13}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{19}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(10/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] c**(10/3)*x**(13/3)*gamma(13/6)*hyper((2/3, 13/6), (19/6,), b*x**2*exp_pola
r(I*pi)/a)/(2*a**(2/3)*gamma(19/6))
```

3.780 $\int \frac{(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=388

$$\frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{b}-\frac{\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(c}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(c}{\sqrt[3]{a+bx^2}}}\right.\right.}{2\sqrt[4]{3}\,b\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] c*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b-1/6*c^(1/3)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^
(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1
-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^
2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/
3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-
(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3
)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2)
)*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)
/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))
^2)^(1/2)*3^(3/4)/b/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x
^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^
2+a)^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {321, 329, 241, 225}

$$\frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{b}-\frac{\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\left(\right.\right.}{2\sqrt[4]{3}\,b\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(4/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/b - (c^(1/3)*(c*x)^(1/3)*(a + b*x^2)^(1/3
)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2
/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x
^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)
)^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b
*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3))], (2 + Sqrt[3])/4])/(2*3^(1/4)*b*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) -
 (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((
1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
```

```
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{b} - \frac{\left(ac^2\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{3b}$$

$$= \frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{b} - \frac{(ac)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{b}$$

$$= \frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{b} - \frac{(ac)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{b\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a+bx^2}}$$

$$= \frac{c\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{b} - \frac{\sqrt[3]{c}\,\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3} \ldots}{c^{2/3} \ldots}\right.\right.}{2\sqrt[4]{3}\,b\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.17

$$\frac{c\sqrt[3]{cx}\left(-a\left(\frac{bx^2}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{6},\frac{2}{3};\frac{7}{6};-\frac{bx^2}{a}\right)+a+bx^2\right)}{b\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(4/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c*(c*x)^(1/3)*(a + b*x^2 - a*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/6,
2/3, 7/6, -((b*x^2)/a)]))/(b*(a + b*x^2)^(2/3))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^{\frac{1}{3}}cx}{\left(bx^2+a\right)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(1/3)*c*x/(b*x^2 + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{4}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(4/3)/(b*x^2 + a)^(2/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{4}{3}}}{\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{4}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(4/3)/(b*x^2 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x\right)}^{4/3}}{{\left(b\,x^2+a\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(4/3)/(a + b*x^2)^(2/3),x)
```

```
[Out] int((c*x)^(4/3)/(a + b*x^2)^(2/3), x)
```

sympy [C] time = 5.44, size = 44, normalized size = 0.11

$$\frac{c^{\frac{4}{3}}x^{\frac{7}{3}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{6}\\\frac{13}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(4/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] c**(4/3)*x**(7/3)*gamma(7/6)*hyper((2/3, 7/6), (13/6,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(2/3)*gamma(13/6))
```

3.781 $\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=364

$$\frac{3^{3/4}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2ac^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] 1/2*3^(3/4)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2
+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^
(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^
(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)
*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-
3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2
+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/
(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/
3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/a/c^(5/3)/(-b^(1/3)*(c
*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^
(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {329, 241, 225}

$$\frac{3^{3/4}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2ac^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(2/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3^(3/4)*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a
+ b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (
b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b
^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 -
 Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(2*a*c^(5/3)*S
qrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
```

```
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx = \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c}$$

$$= \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx, x, \frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{c\sqrt{\frac{a}{a+bx^2}}\,\sqrt{a+bx^2}}$$

$$= \frac{3^{3/4}\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(c\ldots}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(c\ldots}{\sqrt[3]{a+bx^2}}}\right.\right.}{2ac^{5/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.15

$$\frac{3x\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{6},\frac{2}{3};\frac{7}{6};-\frac{bx^2}{a}\right)}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(2/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/6, 2/3, 7/6, -((b*x^2)/a)])/
((c*x)^(2/3)*(a + b*x^2)^(2/3))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{bcx^3+acx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(b*c*x^3 + a*c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{2}{3}}\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{2/3}\,\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(2/3)*(a + b*x^2)^(2/3)),x)
```

```
[Out] int(1/((c*x)^(2/3)*(a + b*x^2)^(2/3)), x)
```

sympy [C] time = 1.79, size = 31, normalized size = 0.09

$$-\frac{{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{b^{\frac{2}{3}}c^{\frac{2}{3}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(2/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] -hyper((1/2, 2/3), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(2/3)*c**(2/3)*x
)
```

3.782 $\int \frac{1}{(cx)^{8/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=394

$$-\frac{3\,3^{3/4} b \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{10a^2c^{11/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] -3/5*(b*x^2+a)^(1/3)/a/c/(c*x)^(5/3)-3/10*3^(3/4)*b*(c*x)^(1/3)*(b*x^2+a)^(
1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^
(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2
))/(b*x^2+a)^(1/3))^2)^(1/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^
2+a)^(1/3))*(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*Ellip
ticF((1-(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3
)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4
*2^(1/2))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*
x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a
)^(1/3))^2)^(1/2)/a^2/c^(11/3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)
^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(
1/2))/(b*x^2+a)^(1/3))^2)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {325, 329, 241, 225}

$$-\frac{3\,3^{3/4} b \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{10a^2c^{11/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(8/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(5*a*c*(c*x)^(5/3)) - (3*3^(3/4)*b*(c*x)^(1/3)*(a +
b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4
/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3
))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b
*x^2)^(1/3))^2]*EllipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2
/3))/(a + b*x^2)^(1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a +
 b*x^2)^(1/3))], (2 + Sqrt[3])/4])/(10*a^2*c^(11/3)*Sqrt[-((b^(1/3)*(c*x)^(
2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3
)*(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
```

```
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{8/3}\left(a+bx^2\right)^{2/3}}\,dx &= -\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}-\frac{(3b)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{5ac^2}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}-\frac{(9b)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx,x,\sqrt[3]{cx}\right)}{5ac^3}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}-\frac{(9b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx,x,\frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{5ac^3\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}-\frac{3\,3^{3/4}b\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F}{10a^2c^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx}{\sqrt[3]{a+b}}\right.}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(}{\sqrt[3]{a+bx^2}}\right.}}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.14

$$
-\frac{3x\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{5}{6},\frac{2}{3};\frac{1}{6};-\frac{bx^2}{a}\right)}{5(cx)^{8/3}\left(a+bx^2\right)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(8/3)*(a + b*x^2)^(2/3)),x]
[Out] (-3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-5/6, 2/3, 1/6, -((b*x^2)/a)]
)/(5*(c*x)^(8/3)*(a + b*x^2)^(2/3))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{bc^3x^5+ac^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(b*c^3*x^5 + a*c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(8/3)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{8}{3}}\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x)
[Out] int(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(8/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{8/3}\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(8/3)*(a + b*x^2)^(2/3)),x)
```

[Out] int(1/((c*x)^(8/3)*(a + b*x^2)^(2/3)), x)

sympy [C] time = 25.88, size = 48, normalized size = 0.12

$$\frac{\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6}, \frac{2}{3}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}c^{\frac{8}{3}}x^{\frac{5}{3}}\Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(8/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] gamma(-5/6)*hyper((-5/6, 2/3), (1/6,), b*x**2*exp_polar(I*pi)/a)/(2*a**(2/3
)*c**(8/3)*x**(5/3)*gamma(1/6))
```

3.783 $\int \frac{1}{(cx)^{14/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=425

$$\frac{27\,3^{3/4} b^2 \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right),\frac{1}{4}\left(2+\sqrt{\vphantom{3}}\right.\right.}{110a^3c^{17/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

```
[Out] -3/11*(b*x^2+a)^(1/3)/a/c/(c*x)^(11/3)+27/55*b*(b*x^2+a)^(1/3)/a^2/c^3/(c*x
)^(5/3)+27/110*3^(3/4)*b^2*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*
x)^(2/3)/(b*x^2+a)^(1/3))*((c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+
a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1
/2)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))*(c^(2/3)-b^(1
/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))*EllipticF((1-(c^(2/3)-b^(1/3)*
(c*x)^(2/3)*(1-3^(1/2))/(b*x^2+a)^(1/3))^2/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+
3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((c^(4/3)+b^(2/
3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))
/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)^(1/2)/a^3/c^(
17/3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(
b*x^2+a)^(1/3)/(c^(2/3)-b^(1/3)*(c*x)^(2/3)*(1+3^(1/2))/(b*x^2+a)^(1/3))^2)
^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {325, 329, 241, 225}

$$\frac{27\,3^{3/4} b^2 \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{110a^3c^{17/3}\sqrt{-\frac{\sqrt[3]{b}\,(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}+\frac{27}{55}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(14/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(11*a*c*(c*x)^(11/3)) + (27*b*(a + b*x^2)^(1/3))/(55
*a^2*c^3*(c*x)^(5/3)) + (27*3^(3/4)*b^2*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(c^(2
/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*
x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2]*El
lipticF[ArcCos[(c^(2/3) - ((1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(
1/3))/(c^(2/3) - ((1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))], (
2 + Sqrt[3])/4])/(110*a^3*c^(17/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (
b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)))/((a + b*x^2)^(1/3)*(c^(2/3) - ((1
+ Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))^2))])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{14/3}\left(a+bx^2\right)^{2/3}}\,dx &= -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}-\frac{(9b)\int \frac{1}{(cx)^{8/3}\left(a+bx^2\right)^{2/3}}\,dx}{11ac^2}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}+\frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}}+\frac{\left(27b^2\right)\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx}{55a^2c^4}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}+\frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}}+\frac{\left(81b^2\right)\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx,x,\sqrt[3]{cx}\right)}{55a^2c^5}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}+\frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}}+\frac{\left(81b^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{bx^6}{c^2}}}\,dx,x,\frac{\sqrt[3]{cx}}{\sqrt[6]{a+bx^2}}\right)}{55a^2c^5\sqrt{\frac{a}{a+bx^2}}\sqrt{a+bx^2}}\\
&= -\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}+\frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}}+\frac{27\,3^{3/4}b^2\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\cdots}{\left(c^2\cdots\right.}}}{110a^3c^{17/3}\sqrt{\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.13

$$-\frac{3x\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{11}{6},\frac{2}{3};-\frac{5}{6};-\frac{bx^2}{a}\right)}{11(cx)^{14/3}\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(14/3)*(a + b*x^2)^(2/3)),x]
[Out] (-3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-11/6, 2/3, -5/6, -((b*x^2)/a
)])/(11*(c*x)^(14/3)*(a + b*x^2)^(2/3))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{bc^5x^7+ac^5x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(b*c^5*x^7 + a*c^5*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(14/3)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int\frac{1}{(cx)^{\frac{14}{3}}\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x)
[Out] int(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(14/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{1}{(c\,x)^{14/3}\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(14/3)*(a + b*x^2)^(2/3)),x)
```

```
[Out] int(1/((c*x)^(14/3)*(a + b*x^2)^(2/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(14/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

3.784 $\int \frac{(cx)^{2/3}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3(cx)^{5/3}\left(\frac{bx^2}{a}+1\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\left(a+bx^2\right)^{2/3}}$$

```
[Out] 3/5*(c*x)^(5/3)*(1+b*x^2/a)^(2/3)*hypergeom([2/3, 5/6],[11/6],-b*x^2/a)/c/(
b*x^2+a)^(2/3)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{3(cx)^{5/3}\left(\frac{bx^2}{a}+1\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(2/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(c*x)^(5/3)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -((b
*x^2)/a)])/(5*c*(a + b*x^2)^(2/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(cx)^{2/3}}{\left(a+bx^2\right)^{2/3}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{(cx)^{2/3}}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\left(a+bx^2\right)^{2/3}}\\ &= \frac{3(cx)^{5/3}\left(1+\frac{bx^2}{a}\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5c\left(a+bx^2\right)^{2/3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 0.97

$$\frac{3x(cx)^{2/3}\left(\frac{bx^2}{a}+1\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-\frac{bx^2}{a}\right)}{5\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(2/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(c*x)^(2/3)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -(
(b*x^2)/a)])/(5*(a + b*x^2)^(2/3))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^{\frac{2}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{2}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{2}{3}}}{\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{2}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x\right)}^{2/3}}{{\left(b\,x^2+a\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(2/3)/(a + b*x^2)^(2/3),x)

[Out] int((c*x)^(2/3)/(a + b*x^2)^(2/3), x)

sympy [C] time = 1.59, size = 44, normalized size = 0.76

$$\frac{c^{\frac{2}{3}}x^{\frac{5}{3}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(2/3)/(b*x**2+a)**(2/3),x)

[Out] c**(2/3)*x**(5/3)*gamma(5/6)*hyper((2/3, 5/6), (11/6,), b*x**2*exp_polar(I*pi)/a)/(2*a**(2/3)*gamma(11/6))

3.785 $\int \frac{1}{\sqrt[3]{cx}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3(cx)^{2/3}\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\left(a+bx^2\right)^{2/3}}$$

```
[Out] 3/2*(c*x)^(2/3)*(1+b*x^2/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^2/a)/c/(b
*x^2+a)^(2/3)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{3(cx)^{2/3}\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(1/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*(c*x)^(2/3)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*
x^2)/a)])/(2*c*(a + b*x^2)^(2/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{cx}\left(a+bx^2\right)^{2/3}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{1}{\sqrt[3]{cx}\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\left(a+bx^2\right)^{2/3}}\\ &= \frac{3(cx)^{2/3}\left(1+\frac{bx^2}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2c\left(a+bx^2\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 0.97

$$\frac{3x\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^2}{a}\right)}{2\sqrt[3]{cx}\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(1/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^2)/a)])/
(2*(c*x)^(1/3)*(a + b*x^2)^(2/3))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}}{bcx^3+acx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(b*c*x^3 + a*c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(1/3)), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{1}{3}}\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(c\,x)^{1/3}\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/3)*(a + b*x^2)^(2/3)),x)
```

```
[Out] int(1/((c*x)^(1/3)*(a + b*x^2)^(2/3)), x)
```

sympy [C] time = 1.38, size = 46, normalized size = 0.79

$$\frac{\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}}\sqrt[3]{c}\,x^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] gamma(-1/3)*hyper((1/3, 2/3), (4/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2/
3)*c**(1/3)*x**(2/3)*gamma(2/3))
```

3.786 $\int \frac{1}{(cx)^{4/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\,\left(a+bx^2\right)^{2/3}}$$

```
[Out] -3*(1+b*x^2/a)^(2/3)*hypergeom([-1/6, 2/3],[5/6],-b*x^2/a)/c/(c*x)^(1/3)/(b
*x^2+a)^(2/3)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$-\frac{3\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\,\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(4/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -((b*x^2)/a)])/
(c*(c*x)^(1/3)*(a + b*x^2)^(2/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{4/3}\left(a+bx^2\right)^{2/3}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{1}{(cx)^{4/3}\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\left(a+bx^2\right)^{2/3}}\\
&= -\frac{3\left(1+\frac{bx^2}{a}\right)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\,\left(a+bx^2\right)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.96

$$-\frac{3x\left(\frac{bx^2}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-\frac{bx^2}{a}\right)}{(cx)^{4/3}\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(4/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -((b*x^2)/a)]
)/((c*x)^(4/3)*(a + b*x^2)^(2/3))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}}{bc^2x^4+ac^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(b*c^2*x^4 + a*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(4/3)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{4}{3}}\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(4/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(c\,x)^{4/3}\,\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(4/3)*(a + b*x^2)^(2/3)),x)
```

```
[Out] int(1/((c*x)^(4/3)*(a + b*x^2)^(2/3)), x)
```

sympy [C] time = 3.07, size = 48, normalized size = 0.86

$$\frac{\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{2}{3} \\ \frac{5}{6} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}c^{\frac{4}{3}}\sqrt[3]{x}\,\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(4/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] gamma(-1/6)*hyper((-1/6, 2/3), (5/6,), b*x**2*exp_polar(I*pi)/a)/(2*a**(2/3
)*c**(4/3)*x**(1/3)*gamma(5/6))
```

3.787 $\int x^4 \sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=121

$$\frac{8a^{7/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{77b^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2}+\frac{2}{11}x^5\sqrt[4]{a+bx^2}+\frac{2ax^3\sqrt[4]{a+bx^2}}{77b}$$

```
[Out] -4/77*a^2*x*(b*x^2+a)^(1/4)/b^2+2/77*a*x^3*(b*x^2+a)^(1/4)/b+2/11*x^5*(b*x^
2+a)^(1/4)+8/77*a^(7/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)
))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b
^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 233, 231}

$$-\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2}+\frac{8a^{7/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a+bx^2\right)^{3/4}}+\frac{2}{11}x^5\sqrt[4]{a+bx^2}+\frac{2ax^3\sqrt[4]{a+bx^2}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(1/4),x]
```

```
[Out] (-4*a^2*x*(a + b*x^2)^(1/4))/(77*b^2) + (2*a*x^3*(a + b*x^2)^(1/4))/(77*b)
+ (2*x^5*(a + b*x^2)^(1/4))/11 + (8*a^(7/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF
[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(5/2)*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\sqrt[4]{a+bx^2}\,dx = \frac{2}{11}x^5\sqrt[4]{a+bx^2} + \frac{1}{11}a\int \frac{x^4}{\left(a+bx^2\right)^{3/4}}\,dx$$

$$= \frac{2ax^3\sqrt[4]{a+bx^2}}{77b} + \frac{2}{11}x^5\sqrt[4]{a+bx^2} - \frac{\left(6a^2\right)\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx}{77b}$$

$$= -\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2} + \frac{2ax^3\sqrt[4]{a+bx^2}}{77b} + \frac{2}{11}x^5\sqrt[4]{a+bx^2} + \frac{\left(4a^3\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{77b^2}$$

$$= -\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2} + \frac{2ax^3\sqrt[4]{a+bx^2}}{77b} + \frac{2}{11}x^5\sqrt[4]{a+bx^2} + \frac{\left(4a^3\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{77b^2\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2} + \frac{2ax^3\sqrt[4]{a+bx^2}}{77b} + \frac{2}{11}x^5\sqrt[4]{a+bx^2} + \frac{8a^{7/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.05, size = 93, normalized size = 0.77

$$\frac{2x\sqrt[4]{a+bx^2}\left(\sqrt[4]{\frac{bx^2}{a}+1}\left(-6a^2+abx^2+7b^2x^4\right)+6a^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{77b^2\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(1/4),x]
```

```
[Out] (2*x*(a + b*x^2)^(1/4)*((1 + (b*x^2)/a)^(1/4)*(-6*a^2 + a*b*x^2 + 7*b^2*x^4
) + 6*a^2*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x^2)/a)]))/(77*b^2*(1 + (b
*x^2)/a)^(1/4))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{1}{4}}x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{4}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*x^4, x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{1}{4}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/4),x)
[Out] int(x^4*(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{4}} x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(b\,x^2+a\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(1/4),x)
[Out] int(x^4*(a + b*x^2)^(1/4), x)
```

sympy [C] time = 0.97, size = 29, normalized size = 0.24

$$\frac{\sqrt[4]{a}\,x^5\,{}_2F_1\left(\left.\begin{matrix} -\frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/4),x)
[Out] a**(1/4)*x**5*hyper((-1/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

3.788 $\int x^2 \sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=97

$$-\frac{4a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{21b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{2ax\sqrt[4]{a+bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}$$

```
[Out] 2/21*a*x*(b*x^2+a)^(1/4)/b+2/7*x^3*(b*x^2+a)^(1/4)-4/21*a^(5/2)*(1+b*x^2/a)
^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2
)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(
b*x^2+a)^(3/4)
```

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 233, 231}

$$-\frac{4a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}+\frac{2ax\sqrt[4]{a+bx^2}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(1/4),x]
```

```
[Out] (2*a*x*(a + b*x^2)^(1/4))/(21*b) + (2*x^3*(a + b*x^2)^(1/4))/7 - (4*a^(5/2)
*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*b^(
3/2)*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt[4]{a+bx^2}\,dx &= \frac{2}{7}x^3\sqrt[4]{a+bx^2}+\frac{1}{7}a\int\frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx\\
&= \frac{2ax\sqrt[4]{a+bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}-\frac{\left(2a^2\right)\int\frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{21b}\\
&= \frac{2ax\sqrt[4]{a+bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}-\frac{\left(2a^2\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int\frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{21b\left(a+bx^2\right)^{3/4}}\\
&= \frac{2ax\sqrt[4]{a+bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}-\frac{4a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 62, normalized size = 0.64

$$\frac{2x\sqrt[4]{a+bx^2}\left(-\frac{a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[4]{\frac{bx^2}{a}+1}}+a+bx^2\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(1/4),x]
[Out] (2*x*(a + b*x^2)^(1/4)*(a + b*x^2 - (a*Hypergeometric2F1[-1/4, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(1/4)))/(7*b)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{1}{4}}x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{1}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)*x^2, x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int\left(b\,x^2+a\right)^{\frac{1}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/4),x)
```

```
[Out] int(x^2*(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{1}{4}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(b\,x^2 + a\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(1/4),x)
```

```
[Out] int(x^2*(a + b*x^2)^(1/4), x)
```

sympy [C] time = 0.86, size = 29, normalized size = 0.30

$$\frac{\sqrt[4]{a}\,x^3\,{}_2F_1\left(\left.\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**3*hyper((-1/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

3.789 $\int \sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=75

$$\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}$$

```
[Out] 2/3*x*(b*x^2+a)^(1/4)+2/3*a^(3/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*
arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^2+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 233, 231}

$$\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4),x]
```

```
[Out] (2*x*(a + b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcT
an[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a + b*x^2)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \sqrt[4]{a+bx^2}\,dx = \frac{2}{3}x\sqrt[4]{a+bx^2} + \frac{1}{3}a\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx$$
$$= \frac{2}{3}x\sqrt[4]{a+bx^2} + \frac{\left(a\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{3\left(a+bx^2\right)^{3/4}}$$
$$= \frac{2}{3}x\sqrt[4]{a+bx^2} + \frac{2a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.61

$$\frac{x\sqrt[4]{a+bx^2}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4),x]
[Out] (x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/4)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4),x)
[Out] int((b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4), x)
```

mupad [B] time = 4.66, size = 37, normalized size = 0.49

$$\frac{x\left(b\,x^2+a\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4),x)
[Out] (x*(a + b*x^2)^(1/4)*hypergeom([-1/4, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(1/4)
```

sympy [C] time = 0.81, size = 26, normalized size = 0.35

$$\sqrt[4]{a}\,x\,{}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4),x)
[Out] a**(1/4)*x*hyper((-1/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

3.790 $\int \frac{\sqrt[4]{a+bx^2}}{x^2} dx$

Optimal. Leaf size=72

$$\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{x}$$

```
[Out] -(b*x^2+a)^(1/4)/x+(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)
^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2
)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 233, 231}

$$\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/x^2,x]
```

```
[Out] -((a + b*x^2)^(1/4)/x) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[A
rcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a + b*x^2)^(3/4)
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx = -\frac{\sqrt[4]{a+bx^2}}{x} + \frac{1}{2}b\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{a+bx^2}}{x} + \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{2\left(a+bx^2\right)^{3/4}}$$
$$= -\frac{\sqrt[4]{a+bx^2}}{x} + \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.68

$$-\frac{\sqrt[4]{a+bx^2}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/x^2,x]
[Out] -(((a + b*x^2)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(1/4)))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/x^2, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/x^2,x)
```

[Out] int((b*x^2+a)^(1/4)/x^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/4)/x^2,x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(1/4)/x^2, x)

mupad [B] time = 4.81, size = 40, normalized size = 0.56

$$-\frac{2\left(b\,x^2+a\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{a}{b\,x^2}\right)}{x\left(\frac{a}{b\,x^2}+1\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^(1/4)/x^2,x)

[Out] -(2*(a + b*x^2)^(1/4)*hypergeom([-1/4, 1/4], 5/4, -a/(b*x^2)))/(x*(a/(b*x^2
) + 1)^(1/4))

sympy [C] time = 0.86, size = 29, normalized size = 0.40

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(1/4)/x**2,x)

[Out] -a**(1/4)*hyper((-1/2, -1/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/x

3.791 $\int \frac{\sqrt[4]{a+bx^2}}{x^4} dx$

Optimal. Leaf size=99

$$-\frac{b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{6\sqrt{a}\left(a+bx^2\right)^{3/4}}-\frac{b\sqrt[4]{a+bx^2}}{6ax}-\frac{\sqrt[4]{a+bx^2}}{3x^3}$$

```
[Out] -1/3*(b*x^2+a)^(1/4)/x^3-1/6*b*(b*x^2+a)^(1/4)/a/x-1/6*b^(3/2)*(1+b*x^2/a)^
(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)
/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^2+a)^
(3/4)/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 325, 233, 231}

$$-\frac{b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\left(a+bx^2\right)^{3/4}}-\frac{b\sqrt[4]{a+bx^2}}{6ax}-\frac{\sqrt[4]{a+bx^2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/x^4,x]
```

```
[Out] -(a + b*x^2)^(1/4)/(3*x^3) - (b*(a + b*x^2)^(1/4))/(6*a*x) - (b^(3/2)*(1 +
(b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*(a
 + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{x^4}\,dx = -\frac{\sqrt[4]{a+bx^2}}{3x^3} + \frac{1}{6}b\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{a+bx^2}}{3x^3} - \frac{b\sqrt[4]{a+bx^2}}{6ax} - \frac{b^2\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{12a}$$
$$= -\frac{\sqrt[4]{a+bx^2}}{3x^3} - \frac{b\sqrt[4]{a+bx^2}}{6ax} - \frac{\left(b^2\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{12a\left(a+bx^2\right)^{3/4}}$$
$$= -\frac{\sqrt[4]{a+bx^2}}{3x^3} - \frac{b\sqrt[4]{a+bx^2}}{6ax} - \frac{b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.52

$$-\frac{\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/x^4,x]
[Out] -1/3*((a + b*x^2)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, -((b*x^2)/a)])/
(x^3*(1 + (b*x^2)/a)^(1/4))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^4,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^4,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/x^4, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/x^4,x)
```

```
[Out] int((b*x^2+a)^(1/4)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{1/4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/x^4,x)
```

```
[Out] int((a + b*x^2)^(1/4)/x^4, x)
```

sympy [C] time = 0.98, size = 34, normalized size = 0.34

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/x**4,x)
```

```
[Out] -a**(1/4)*hyper((-3/2, -1/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

3.792 $\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx$

Optimal. Leaf size=123

$$\frac{b^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{12a^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{b^2\sqrt[4]{a+bx^2}}{12a^2x}-\frac{\sqrt[4]{a+bx^2}}{5x^5}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}$$

```
[Out] -1/5*(b*x^2+a)^(1/4)/x^5-1/30*b*(b*x^2+a)^(1/4)/a/x^3+1/12*b^2*(b*x^2+a)^(1
/4)/a^2/x+1/12*b^(5/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2))
)^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^
(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 325, 233, 231}

$$\frac{b^2\sqrt[4]{a+bx^2}}{12a^2x}+\frac{b^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a+bx^2\right)^{3/4}}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}-\frac{\sqrt[4]{a+bx^2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/x^6,x]
```

```
[Out] -(a + b*x^2)^(1/4)/(5*x^5) - (b*(a + b*x^2)^(1/4))/(30*a*x^3) + (b^2*(a + b
*x^2)^(1/4))/(12*a^2*x) + (b^(5/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(12*a^(3/2)*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx = -\frac{\sqrt[4]{a+bx^2}}{5x^5} + \frac{1}{10}b\int \frac{1}{x^4\left(a+bx^2\right)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{a+bx^2}}{5x^5} - \frac{b\sqrt[4]{a+bx^2}}{30ax^3} - \frac{b^2\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx}{12a}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{5x^5} - \frac{b\sqrt[4]{a+bx^2}}{30ax^3} + \frac{b^2\sqrt[4]{a+bx^2}}{12a^2x} + \frac{b^3\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{24a^2}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{5x^5} - \frac{b\sqrt[4]{a+bx^2}}{30ax^3} + \frac{b^2\sqrt[4]{a+bx^2}}{12a^2x} + \frac{\left(b^3\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{24a^2\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{5x^5} - \frac{b\sqrt[4]{a+bx^2}}{30ax^3} + \frac{b^2\sqrt[4]{a+bx^2}}{12a^2x} + \frac{b^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.41

$$-\frac{\sqrt[4]{a+bx^2}\;{}_2F_1\left(-\frac{5}{2},-\frac{1}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/x^6,x]
```

```
[Out] -1/5*((a + b*x^2)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, -((b*x^2)/a)])/
(x^5*(1 + (b*x^2)/a)^(1/4))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/x^6, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/x^6,x)
```

```
[Out] int((b*x^2+a)^(1/4)/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2+a)^{\frac{1}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^{1/4}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/x^6,x)
```

```
[Out] int((a + b*x^2)^(1/4)/x^6, x)
```

sympy [C] time = 1.10, size = 34, normalized size = 0.28

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{2},-\frac{1}{4}\\-\frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/x**6,x)
```

```
[Out] -a**(1/4)*hyper((-5/2, -1/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

3.793 $\int x^4 \sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=126

$$\frac{8a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{77b^{5/2}\left(a-bx^2\right)^{3/4}}-\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2}+\frac{2}{11}x^5\sqrt[4]{a-bx^2}-\frac{2ax^3\sqrt[4]{a-bx^2}}{77b}$$

```
[Out] -4/77*a^2*x*(-b*x^2+a)^(1/4)/b^2-2/77*a*x^3*(-b*x^2+a)^(1/4)/b+2/11*x^5*(-b
*x^2+a)^(1/4)+8/77*a^(7/2)*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1
/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(
x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(-b*x^2+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {279, 321, 233, 232}

$$-\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2}+\frac{8a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a-bx^2\right)^{3/4}}+\frac{2}{11}x^5\sqrt[4]{a-bx^2}-\frac{2ax^3\sqrt[4]{a-bx^2}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a - b*x^2)^(1/4),x]
```

```
[Out] (-4*a^2*x*(a - b*x^2)^(1/4))/(77*b^2) - (2*a*x^3*(a - b*x^2)^(1/4))/(77*b)
+ (2*x^5*(a - b*x^2)^(1/4))/11 + (8*a^(7/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(5/2)*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule steps

$$\int x^4 \sqrt[4]{a-bx^2}\,dx = \frac{2}{11}x^5\sqrt[4]{a-bx^2} + \frac{1}{11}a\int \frac{x^4}{\left(a-bx^2\right)^{3/4}}\,dx$$

$$= -\frac{2ax^3\sqrt[4]{a-bx^2}}{77b} + \frac{2}{11}x^5\sqrt[4]{a-bx^2} + \frac{\left(6a^2\right)\int \frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx}{77b}$$

$$= -\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2} - \frac{2ax^3\sqrt[4]{a-bx^2}}{77b} + \frac{2}{11}x^5\sqrt[4]{a-bx^2} + \frac{\left(4a^3\right)\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{77b^2}$$

$$= -\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2} - \frac{2ax^3\sqrt[4]{a-bx^2}}{77b} + \frac{2}{11}x^5\sqrt[4]{a-bx^2} + \frac{\left(4a^3\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{77b^2\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2} - \frac{2ax^3\sqrt[4]{a-bx^2}}{77b} + \frac{2}{11}x^5\sqrt[4]{a-bx^2} + \frac{8a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.06, size = 95, normalized size = 0.75

$$-\frac{2x\sqrt[4]{a-bx^2}\left(\sqrt[4]{1-\frac{bx^2}{a}}\left(6a^2+abx^2-7b^2x^4\right)-6a^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{77b^2\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a - b*x^2)^(1/4),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*((1 - (b*x^2)/a)^(1/4)*(6*a^2 + a*b*x^2 - 7*b^2*x^4
) - 6*a^2*Hypergeometric2F1[-1/4, 1/2, 3/2, (b*x^2)/a]))/(77*b^2*(1 - (b*x^
2)/a)^(1/4))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-bx^2+a\right)^{\frac{1}{4}}x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2+a\right)^{\frac{1}{4}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*x^4, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \left(-b\,x^2+a\right)^{\frac{1}{4}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-b*x^2+a)^(1/4),x)
```

```
[Out] int(x^4*(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2+a\right)^{\frac{1}{4}} x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(a-b\,x^2\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a - b*x^2)^(1/4),x)
```

```
[Out] int(x^4*(a - b*x^2)^(1/4), x)
```

sympy [C] time = 0.98, size = 31, normalized size = 0.25

$$\frac{\sqrt[4]{a}\,x^5\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**5*hyper((-1/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

3.794 $\int x^2 \sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=101

$$\frac{4a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{21b^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{2ax\sqrt[4]{a-bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a-bx^2}$$

```
[Out] -2/21*a*x*(-b*x^2+a)^(1/4)/b+2/7*x^3*(-b*x^2+a)^(1/4)+4/21*a^(5/2)*(1-b*x^2
/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(
1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2
)/(-b*x^2+a)^(3/4)
```

Rubi [A] time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {279, 321, 233, 232}

$$\frac{4a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a-bx^2\right)^{3/4}}+\frac{2}{7}x^3\sqrt[4]{a-bx^2}-\frac{2ax\sqrt[4]{a-bx^2}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a - b*x^2)^(1/4),x]
```

```
[Out] (-2*a*x*(a - b*x^2)^(1/4))/(21*b) + (2*x^3*(a - b*x^2)^(1/4))/7 + (4*a^(5/2
)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*b^
(3/2)*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2\sqrt[4]{a-bx^2}\,dx = \frac{2}{7}x^3\sqrt[4]{a-bx^2} + \frac{1}{7}a\int \frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx$$

$$= -\frac{2ax\sqrt[4]{a-bx^2}}{21b} + \frac{2}{7}x^3\sqrt[4]{a-bx^2} + \frac{\left(2a^2\right)\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{21b}$$

$$= -\frac{2ax\sqrt[4]{a-bx^2}}{21b} + \frac{2}{7}x^3\sqrt[4]{a-bx^2} + \frac{\left(2a^2\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{21b\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2ax\sqrt[4]{a-bx^2}}{21b} + \frac{2}{7}x^3\sqrt[4]{a-bx^2} + \frac{4a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.06, size = 64, normalized size = 0.63

$$\frac{2x\sqrt[4]{a-bx^2}\left(\frac{a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}-a+bx^2\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a - b*x^2)^(1/4),x]
```

```
[Out] (2*x*(a - b*x^2)^(1/4)*(-a + b*x^2 + (a*Hypergeometric2F1[-1/4, 1/2, 3/2, (
b*x^2)/a])/(1 - (b*x^2)/a)^(1/4)))/(7*b)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-bx^2+a\right)^{\frac{1}{4}}x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2+a\right)^{\frac{1}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*x^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(-b\,x^2+a\right)^{\frac{1}{4}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-b*x^2+a)^(1/4),x)
[Out] int(x^2*(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2+a\right)^{\frac{1}{4}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a-b\,x^2\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a - b*x^2)^(1/4),x)
[Out] int(x^2*(a - b*x^2)^(1/4), x)
```

sympy [C] time = 0.90, size = 31, normalized size = 0.31

$$\frac{\sqrt[4]{a}\,x^3\,{}_2F_1\left(\left.\begin{matrix} -\frac{1}{4},\frac{3}{2} \\ \frac{5}{2} \end{matrix}\right| \frac{bx^2e^{2i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-b*x**2+a)**(1/4),x)
[Out] a**(1/4)*x***3*hyper((-1/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3
```

3.795 $\int \sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=78

$$\frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}$$

```
[Out] 2/3*x*(-b*x^2+a)^(1/4)+2/3*a^(3/2)*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1
/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2
*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {195, 233, 232}

$$\frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4),x]
```

```
[Out] (2*x*(a - b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcS
in[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \sqrt[4]{a-bx^2}\,dx = \frac{2}{3}x\sqrt[4]{a-bx^2} + \frac{1}{3}a\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx$$

$$= \frac{2}{3}x\sqrt[4]{a-bx^2} + \frac{\left(a\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{3\left(a-bx^2\right)^{3/4}}$$

$$= \frac{2}{3}x\sqrt[4]{a-bx^2} + \frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.60

$$\frac{x\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4),x]
[Out] (x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, (b*x^2)/a])/(1 - (b*
x^2)/a)^(1/4)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(-bx^2+a\right)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(-b\,x^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4),x)
[Out] int((-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2 + a\right)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4), x)
```

mupad [B] time = 4.80, size = 38, normalized size = 0.49

$$\frac{x\left(a - b\,x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1 - \frac{b\,x^2}{a}\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4),x)
```

```
[Out] (x*(a - b*x^2)^(1/4)*hypergeom([-1/4, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/a
)^(1/4)
```

sympy [C] time = 0.83, size = 27, normalized size = 0.35

$$\sqrt[4]{a}\,x\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{4}, \frac{1}{2}\\ \frac{3}{2}\end{matrix}\right| \frac{bx^2e^{2i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*x*hyper((-1/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

3.796 $\int \frac{\sqrt[4]{a-bx^2}}{x^2} \, dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{x}$$

```
[Out] -(-b*x^2+a)^(1/4)/x-(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2
)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/
2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/(-b*x^2+a)^(3/4)
```

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {277, 233, 232}

$$-\frac{\sqrt[4]{a-bx^2}}{x}-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/x^2,x]
```

```
[Out] -((a - b*x^2)^(1/4)/x) - (Sqrt[a]*Sqrt[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[A
rcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a - b*x^2)^(3/4)
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx = -\frac{\sqrt[4]{a-bx^2}}{x} - \frac{1}{2}b\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{a-bx^2}}{x} - \frac{\left(b\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{2\left(a-bx^2\right)^{3/4}}$$
$$= -\frac{\sqrt[4]{a-bx^2}}{x} - \frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.66

$$-\frac{\sqrt[4]{a-bx^2}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{1}{2};\frac{bx^2}{a}\right)}{x\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/x^2,x]
```

```
[Out] -(((a - b*x^2)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, (b*x^2)/a])/(x*(1 -
 (b*x^2)/a)^(1/4)))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/x^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^2+a\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/x^2,x)
```

[Out] int((-b*x^2+a)^(1/4)/x^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2 + a\right)^{\frac{1}{4}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+a)^(1/4)/x^2,x, algorithm="maxima")

[Out] integrate((-b*x^2 + a)^(1/4)/x^2, x)

mupad [B] time = 4.98, size = 41, normalized size = 0.54

$$-\frac{2\left(a-b\,x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{a}{b\,x^2}\right)}{x\left(1-\frac{a}{b\,x^2}\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b*x^2)^(1/4)/x^2,x)

[Out] -(2*(a - b*x^2)^(1/4)*hypergeom([-1/4, 1/4], 5/4, a/(b*x^2)))/(x*(1 - a/(b*x^2))^(1/4))

sympy [C] time = 0.92, size = 31, normalized size = 0.41

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2}, -\frac{1}{4}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x**2+a)**(1/4)/x**2,x)

[Out] -a**(1/4)*hyper((-1/2, -1/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/x

3.797 $\int \frac{\sqrt[4]{a-bx^2}}{x^4} dx$

Optimal. Leaf size=103

$$-\frac{b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{6\sqrt{a}\left(a-bx^2\right)^{3/4}}+\frac{b\sqrt[4]{a-bx^2}}{6ax}-\frac{\sqrt[4]{a-bx^2}}{3x^3}$$

```
[Out] -1/3*(-b*x^2+a)^(1/4)/x^3+1/6*b*(-b*x^2+a)^(1/4)/a/x-1/6*b^(3/2)*(1-b*x^2/a
)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/
2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+
a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {277, 325, 233, 232}

$$-\frac{b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\left(a-bx^2\right)^{3/4}}+\frac{b\sqrt[4]{a-bx^2}}{6ax}-\frac{\sqrt[4]{a-bx^2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/x^4,x]
```

```
[Out] -(a - b*x^2)^(1/4)/(3*x^3) + (b*(a - b*x^2)^(1/4))/(6*a*x) - (b^(3/2)*(1 -
(b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*(a
 - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx = -\frac{\sqrt[4]{a-bx^2}}{3x^3} - \frac{1}{6}b\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{a-bx^2}}{3x^3} + \frac{b\sqrt[4]{a-bx^2}}{6ax} - \frac{b^2\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{12a}$$
$$= -\frac{\sqrt[4]{a-bx^2}}{3x^3} + \frac{b\sqrt[4]{a-bx^2}}{6ax} - \frac{\left(b^2\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{12a\left(a-bx^2\right)^{3/4}}$$
$$= -\frac{\sqrt[4]{a-bx^2}}{3x^3} + \frac{b\sqrt[4]{a-bx^2}}{6ax} - \frac{b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.50

$$-\frac{\sqrt[4]{a-bx^2}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/x^4,x]
[Out] -1/3*((a - b*x^2)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, (b*x^2)/a])/(x^
3*(1 - (b*x^2)/a)^(1/4))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^4,x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(1/4)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^4,x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4)/x^4, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^2+a\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/x^4,x)
[Out] int((-b*x^2+a)^(1/4)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^4,x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^2\right)^{1/4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/x^4,x)
[Out] int((a - b*x^2)^(1/4)/x^4, x)
```

sympy [C] time = 1.01, size = 36, normalized size = 0.35

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},-\frac{1}{4}\\-\frac{1}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/x**4,x)
[Out] -a**(1/4)*hyper((-3/2, -1/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*x**3)
```

3.798 $\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx$

Optimal. Leaf size=128

$$-\frac{b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{12a^{3/2}\left(a-bx^2\right)^{3/4}}+\frac{b^2\sqrt[4]{a-bx^2}}{12a^2x}-\frac{\sqrt[4]{a-bx^2}}{5x^5}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}$$

```
[Out] -1/5*(-b*x^2+a)^(1/4)/x^5+1/30*b*(-b*x^2+a)^(1/4)/a/x^3+1/12*b^2*(-b*x^2+a)
^(1/4)/a^2/x-1/12*b^(5/2)*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/
2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x
*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(-b*x^2+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {277, 325, 233, 232}

$$\frac{b^2\sqrt[4]{a-bx^2}}{12a^2x}-\frac{b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a-bx^2\right)^{3/4}}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}-\frac{\sqrt[4]{a-bx^2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/x^6,x]
```

```
[Out] -(a - b*x^2)^(1/4)/(5*x^5) + (b*(a - b*x^2)^(1/4))/(30*a*x^3) + (b^2*(a - b
*x^2)^(1/4))/(12*a^2*x) - (b^(5/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(12*a^(3/2)*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{x^6} dx = -\frac{\sqrt[4]{a-bx^2}}{5x^5} - \frac{1}{10} b \int \frac{1}{x^4 (a-bx^2)^{3/4}} dx$$

$$= -\frac{\sqrt[4]{a-bx^2}}{5x^5} + \frac{b\sqrt[4]{a-bx^2}}{30ax^3} - \frac{b^2 \int \frac{1}{x^2(a-bx^2)^{3/4}} dx}{12a}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{5x^5} + \frac{b\sqrt[4]{a-bx^2}}{30ax^3} + \frac{b^2\sqrt[4]{a-bx^2}}{12a^2x} - \frac{b^3 \int \frac{1}{(a-bx^2)^{3/4}} dx}{24a^2}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{5x^5} + \frac{b\sqrt[4]{a-bx^2}}{30ax^3} + \frac{b^2\sqrt[4]{a-bx^2}}{12a^2x} - \frac{\left(b^3 \left(1-\frac{bx^2}{a}\right)^{3/4}\right) \int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}} dx}{24a^2 (a-bx^2)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{5x^5} + \frac{b\sqrt[4]{a-bx^2}}{30ax^3} + \frac{b^2\sqrt[4]{a-bx^2}}{12a^2x} - \frac{b^{5/2} \left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2} (a-bx^2)^{3/4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.41

$$-\frac{\sqrt[4]{a-bx^2}\, {}_2F_1\left(-\frac{5}{2},-\frac{1}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5x^5 \sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/x^6,x]
```

```
[Out] -1/5*((a - b*x^2)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, (b*x^2)/a])/(x^
5*(1 - (b*x^2)/a)^(1/4))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(-bx^2+a)^{\frac{1}{4}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^6,x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-bx^2+a)^{\frac{1}{4}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^6,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/x^6, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/x^6,x)
```

```
[Out] int((-b*x^2+a)^(1/4)/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2 + a\right)^{\frac{1}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a - b\,x^2\right)^{1/4}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/x^6,x)
```

```
[Out] int((a - b*x^2)^(1/4)/x^6, x)
```

sympy [C] time = 1.13, size = 36, normalized size = 0.28

$$-\frac{\sqrt[4]{a}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, -\frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a} \right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/x**6,x)
```

```
[Out] -a**(1/4)*hyper((-5/2, -1/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*x**5)
```

3.799 $\int x^4 \left(a+bx^2\right)^{3/4} dx$

Optimal. Leaf size=143

$$-\frac{8a^{7/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a+bx^2}}+\frac{8a^3x}{65b^2\sqrt[4]{a+bx^2}}-\frac{4a^2x\left(a+bx^2\right)^{3/4}}{65b^2}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}+\frac{2ax^3\left(a+bx^2\right)^{3/4}}{39b}$$

```
[Out] 8/65*a^3*x/b^2/(b*x^2+a)^(1/4)-4/65*a^2*x*(b*x^2+a)^(3/4)/b^2+2/39*a*x^3*(b
*x^2+a)^(3/4)/b+2/13*x^5*(b*x^2+a)^(3/4)-8/65*a^(7/2)*(1+b*x^2/a)^(1/4)*(co
s(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2))
)*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^2+a)^(
1/4)
```

Rubi [A] time = 0.05, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {279, 321, 229, 227, 196}

$$-\frac{4a^2x\left(a+bx^2\right)^{3/4}}{65b^2}+\frac{8a^3x}{65b^2\sqrt[4]{a+bx^2}}-\frac{8a^{7/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a+bx^2}}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}+\frac{2ax^3\left(a+bx^2\right)^{3/4}}{39b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(3/4),x]
```

```
[Out] (8*a^3*x)/(65*b^2*(a + b*x^2)^(1/4)) - (4*a^2*x*(a + b*x^2)^(3/4))/(65*b^2)
 + (2*a*x^3*(a + b*x^2)^(3/4))/(39*b) + (2*x^5*(a + b*x^2)^(3/4))/13 - (8*a
^(7/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
65*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^{3/4}\,dx = \frac{2}{13}x^5\left(a+bx^2\right)^{3/4}+\frac{1}{13}(3a)\int\frac{x^4}{\sqrt[4]{a+bx^2}}\,dx$$
$$=\frac{2ax^3\left(a+bx^2\right)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}-\frac{\left(2a^2\right)\int\frac{x^2}{\sqrt[4]{a+bx^2}}\,dx}{13b}$$
$$=-\frac{4a^2x\left(a+bx^2\right)^{3/4}}{65b^2}+\frac{2ax^3\left(a+bx^2\right)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}+\frac{\left(4a^3\right)\int\frac{1}{\sqrt[4]{a+bx^2}}\,dx}{65b^2}$$
$$=-\frac{4a^2x\left(a+bx^2\right)^{3/4}}{65b^2}+\frac{2ax^3\left(a+bx^2\right)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}+\frac{\left(4a^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int\frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,d}{65b^2\sqrt[4]{a+bx^2}}$$
$$=\frac{8a^3x}{65b^2\sqrt[4]{a+bx^2}}-\frac{4a^2x\left(a+bx^2\right)^{3/4}}{65b^2}+\frac{2ax^3\left(a+bx^2\right)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}-\frac{\left(4a^3\sqrt[4]{1+\right.}{65b}$$
$$=\frac{8a^3x}{65b^2\sqrt[4]{a+bx^2}}-\frac{4a^2x\left(a+bx^2\right)^{3/4}}{65b^2}+\frac{2ax^3\left(a+bx^2\right)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}-\frac{8a^{7/2}\sqrt[4]{1+}{6}$$

Mathematica [C] time = 0.06, size = 93, normalized size = 0.65

$$\frac{2x\left(a+bx^2\right)^{3/4}\left(\left(\frac{bx^2}{a}+1\right)^{3/4}\left(-2a^2+abx^2+3b^2x^4\right)+2a^2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{39b^2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(3/4),x]
```

```
[Out] (2*x*(a + b*x^2)^(3/4)*((1 + (b*x^2)/a)^(3/4)*(-2*a^2 + a*b*x^2 + 3*b^2*x^4
) + 2*a^2*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a)]))/(39*b^2*(1 + (b
*x^2)/a)^(3/4))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{3}{4}}x^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{3}{4}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/4)*x^4, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(b\,x^{2}+a\right)^{\frac{3}{4}} x^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(3/4),x)
[Out] int(x^4*(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2}+a\right)^{\frac{3}{4}} x^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/4)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{4}\,\left(b\,x^{2}+a\right)^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(3/4),x)
[Out] int(x^4*(a + b*x^2)^(3/4), x)
```

sympy [C] time = 1.20, size = 29, normalized size = 0.20

$$\frac{a^{\frac{3}{4}} x^{5} {{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(3/4),x)
[Out] a**(3/4)*x**5*hyper((-3/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

3.800 $\int x^2 \left(a+bx^2\right)^{3/4} dx$

Optimal. Leaf size=119

$$\frac{4a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a+bx^2}}-\frac{4a^2x}{15b\sqrt[4]{a+bx^2}}+\frac{2ax\left(a+bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}$$

```
[Out] -4/15*a^2*x/b/(b*x^2+a)^(1/4)+2/15*a*x*(b*x^2+a)^(3/4)/b+2/9*x^3*(b*x^2+a)^
(3/4)+4/15*a^(5/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)
^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2
)/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.04, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{279, 321, 229, 227, 196\}$

$$\frac{4a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a+bx^2}}-\frac{4a^2x}{15b\sqrt[4]{a+bx^2}}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}+\frac{2ax\left(a+bx^2\right)^{3/4}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(3/4),x]
```

```
[Out] (-4*a^2*x)/(15*b*(a + b*x^2)^(1/4)) + (2*a*x*(a + b*x^2)^(3/4))/(15*b) + (2
*x^3*(a + b*x^2)^(3/4))/9 + (4*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcT
an[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^(3/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+bx^2\right)^{3/4}dx &= \frac{2}{9}x^3\left(a+bx^2\right)^{3/4}+\frac{1}{3}a\int\frac{x^2}{\sqrt[4]{a+bx^2}}dx\\
&= \frac{2ax\left(a+bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}-\frac{\left(2a^2\right)\int\frac{1}{\sqrt[4]{a+bx^2}}dx}{15b}\\
&= \frac{2ax\left(a+bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}-\frac{\left(2a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int\frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}dx}{15b\sqrt[4]{a+bx^2}}\\
&= -\frac{4a^2x}{15b\sqrt[4]{a+bx^2}}+\frac{2ax\left(a+bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}+\frac{\left(2a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int\frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}dx}{15b\sqrt[4]{a+bx^2}}\\
&= -\frac{4a^2x}{15b\sqrt[4]{a+bx^2}}+\frac{2ax\left(a+bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}+\frac{4a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.\right.}{15b^{3/2}\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.06, size = 62, normalized size = 0.52

$$\frac{2x\left(a+bx^2\right)^{3/4}\left(-\frac{a\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}+a+bx^2\right)}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(3/4),x]
```

```
[Out] (2*x*(a + b*x^2)^(3/4)*(a + b*x^2 - (a*Hypergeometric2F1[-3/4, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(3/4)))/(9*b)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{3}{4}}x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{3}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/4)*x^2, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \left(b\,x^{2}+a\right)^{\frac{3}{4}} x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(3/4),x)
```

```
[Out] int(x^2*(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2}+a\right)^{\frac{3}{4}} x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2}\,\left(b\,x^{2}+a\right)^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(3/4),x)
```

```
[Out] int(x^2*(a + b*x^2)^(3/4), x)
```

sympy [C] time = 1.06, size = 29, normalized size = 0.24

$$\frac{a^{\frac{3}{4}}x^{3}{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/4)*x**3*hyper((-3/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

3.801 $\int \left(a+bx^2\right)^{3/4}\,dx$

Optimal. Leaf size=92

$$-\frac{6a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\,\sqrt[4]{a+bx^2}}+\frac{6ax}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}$$

```
[Out] 6/5*a*x/(b*x^2+a)^(1/4)+2/5*x*(b*x^2+a)^(3/4)-6/5*a^(3/2)*(1+b*x^2/a)^(1/4)
*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1
/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^2+a)^(1/4)
/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {195, 229, 227, 196}

$$-\frac{6a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\,\sqrt[4]{a+bx^2}}+\frac{6ax}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4),x]
```

```
[Out] (6*a*x)/(5*(a + b*x^2)^(1/4)) + (2*x*(a + b*x^2)^(3/4))/5 - (6*a^(3/2)*(1 +
 (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(
a + b*x^2)^(1/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx^2\right)^{3/4} dx &= \frac{2}{5}x\left(a+bx^2\right)^{3/4}+\frac{1}{5}(3a)\int \frac{1}{\sqrt[4]{a+bx^2}}dx \\
&= \frac{2}{5}x\left(a+bx^2\right)^{3/4}+\frac{\left(3a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}dx}{5\sqrt[4]{a+bx^2}} \\
&= \frac{6ax}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}-\frac{\left(3a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}dx}{5\sqrt[4]{a+bx^2}} \\
&= \frac{6ax}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}-\frac{6a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.50

$$
\frac{x\left(a+bx^2\right)^{3/4}{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/4),x]
[Out] (x*(a + b*x^2)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(3/4)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{3}{4}},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \left(bx^2+a\right)^{\frac{3}{4}} dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$
\int \left(b\,x^2+a\right)^{\frac{3}{4}} dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4),x)
```

```
[Out] int((b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4), x)
```

mupad [B] time = 4.78, size = 37, normalized size = 0.40

$$\frac{x\left(b\,x^2+a\right)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/4),x)
```

```
[Out] (x*(a + b*x^2)^(3/4)*hypergeom([-3/4, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(3/4)
```

sympy [C] time = 0.94, size = 26, normalized size = 0.28

$$a^{\frac{3}{4}} x {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{bx^{2} e^{i\pi}}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/4)*x*hyper((-3/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

3.802 $\int \frac{(a+bx^2)^{3/4}}{x^2}\,dx$

Optimal. Leaf size=88

$$\frac{3bx}{\sqrt[4]{a+bx^2}} - \frac{\left(a+bx^2\right)^{3/4}}{x} - \frac{3\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a+bx^2}}$$

```
[Out] 3*b*x/(b*x^2+a)^(1/4)-(b*x^2+a)^(3/4)/x-3*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan
(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(
sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 229, 227, 196}

$$\frac{3bx}{\sqrt[4]{a+bx^2}} - \frac{\left(a+bx^2\right)^{3/4}}{x} - \frac{3\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4)/x^2,x]
```

```
[Out] (3*b*x)/(a + b*x^2)^(1/4) - (a + b*x^2)^(3/4)/x - (3*Sqrt[a]*Sqrt[b]*(1 + (
b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a + b*x^2)^(1
/4)
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/4}}{x^2}\,dx = -\frac{(a+bx^2)^{3/4}}{x} + \frac{1}{2}(3b)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx$$
$$= -\frac{(a+bx^2)^{3/4}}{x} + \frac{\left(3b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{2\sqrt[4]{a+bx^2}}$$
$$= \frac{3bx}{\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{x} - \frac{\left(3b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{2\sqrt[4]{a+bx^2}}$$
$$= \frac{3bx}{\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{x} - \frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.56

$$-\frac{(a+bx^2)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/4)/x^2,x]
[Out] -(((a + b*x^2)^(3/4)*Hypergeometric2F1[-3/4, -1/2, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(3/4)))
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(bx^2+a)^{\frac{3}{4}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2+a)^{\frac{3}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/4)/x^2, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{3}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4)/x^2,x)
```

```
[Out] int((b*x^2+a)^(3/4)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2+a)^{\frac{3}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/x^2, x)
```

mupad [B] time = 5.03, size = 40, normalized size = 0.45

$$\frac{2\left(b\,x^2+a\right)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},-\frac{1}{4};\;\frac{3}{4};\;-\frac{a}{b\,x^2}\right)}{x\left(\frac{a}{b\,x^2}+1\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/4)/x^2,x)
```

```
[Out] (2*(a + b*x^2)^(3/4)*hypergeom([-3/4, -1/4], 3/4, -a/(b*x^2)))/(x*(a/(b*x^2
) + 1)^(3/4))
```

sympy [C] time = 1.03, size = 29, normalized size = 0.33

$$-\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4)/x**2,x)
```

```
[Out] -a**(3/4)*hyper((-3/4, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

3.803 $\int \frac{(a+bx^2)^{3/4}}{x^4}\,dx$

Optimal. Leaf size=121

$$-\frac{b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,\sqrt[4]{a+bx^2}}+\frac{b^2x}{2a\sqrt[4]{a+bx^2}}-\frac{b\left(a+bx^2\right)^{3/4}}{2ax}-\frac{\left(a+bx^2\right)^{3/4}}{3x^3}$$

```
[Out] 1/2*b^2*x/a/(b*x^2+a)^(1/4)-1/3*(b*x^2+a)^(3/4)/x^3-1/2*b*(b*x^2+a)^(3/4)/a
/x-1/2*b^(3/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/
2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^
(1/2))),2^(1/2))/(b*x^2+a)^(1/4)/a^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 325, 229, 227, 196}

$$\frac{b^2x}{2a\sqrt[4]{a+bx^2}}-\frac{b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,\sqrt[4]{a+bx^2}}-\frac{b\left(a+bx^2\right)^{3/4}}{2ax}-\frac{\left(a+bx^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4)/x^4,x]
```

```
[Out] (b^2*x)/(2*a*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(3*x^3) - (b*(a + b*x^2
)^(3/4))/(2*a*x) - (b^(3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]
*x)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/4}}{x^4}\,dx &= -\frac{\left(a+bx^2\right)^{3/4}}{3x^3}+\frac{1}{2}b\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx \\
&= -\frac{\left(a+bx^2\right)^{3/4}}{3x^3}-\frac{b\left(a+bx^2\right)^{3/4}}{2ax}+\frac{b^2\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4a} \\
&= -\frac{\left(a+bx^2\right)^{3/4}}{3x^3}-\frac{b\left(a+bx^2\right)^{3/4}}{2ax}+\frac{\left(b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{4a\sqrt[4]{a+bx^2}} \\
&= \frac{b^2x}{2a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{3x^3}-\frac{b\left(a+bx^2\right)^{3/4}}{2ax}-\frac{\left(b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{4a\sqrt[4]{a+bx^2}} \\
&= \frac{b^2x}{2a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{3x^3}-\frac{b\left(a+bx^2\right)^{3/4}}{2ax}-\frac{b^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.42

$$
-\frac{\left(a+bx^2\right)^{3/4}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(\frac{bx^2}{a}+1\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/4)/x^4,x]
[Out] -1/3*((a + b*x^2)^(3/4)*Hypergeometric2F1[-3/2, -3/4, -1/2, -((b*x^2)/a)])/
(x^3*(1 + (b*x^2)/a)^(3/4))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{x^4},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^4,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(bx^2+a\right)^{\frac{3}{4}}}{x^4}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^4,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/4)/x^4, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{3}{4}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4)/x^4,x)
[Out] int((b*x^2+a)^(3/4)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{3}{4}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^4,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/4)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^{2}+a\right)^{3/4}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/4)/x^4,x)
[Out] int((a + b*x^2)^(3/4)/x^4, x)
```

sympy [C] time = 1.09, size = 34, normalized size = 0.28

$$-\frac{a^{\frac{3}{4}}{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},-\frac{3}{4}\\ -\frac{1}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4)/x**4,x)
[Out] -a**(3/4)*hyper((-3/2, -3/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

3.804 $\int \frac{(a+bx^2)^{3/4}}{x^6} dx$

Optimal. Leaf size=145

$$\frac{3b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a+bx^2}}-\frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}}+\frac{3b^2\left(a+bx^2\right)^{3/4}}{20a^2x}-\frac{\left(a+bx^2\right)^{3/4}}{5x^5}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}$$

```
[Out] -3/20*b^3*x/a^2/(b*x^2+a)^(1/4)-1/5*(b*x^2+a)^(3/4)/x^5-1/10*b*(b*x^2+a)^(3
/4)/a/x^3+3/20*b^2*(b*x^2+a)^(3/4)/a^2/x+3/20*b^(5/2)*(1+b*x^2/a)^(1/4)*(co
s(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2))
)*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^2+a)^(
1/4)
```

Rubi [A] time = 0.05, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 325, 229, 227, 196}

$$-\frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}}+\frac{3b^2\left(a+bx^2\right)^{3/4}}{20a^2x}+\frac{3b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a+bx^2}}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}-\frac{\left(a+bx^2\right)^{3/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/4)/x^6,x]
```

```
[Out] (-3*b^3*x)/(20*a^2*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(5*x^5) - (b*(a +
 b*x^2)^(3/4))/(10*a*x^3) + (3*b^2*(a + b*x^2)^(3/4))/(20*a^2*x) + (3*b^(5/
2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a
^(3/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/4}}{x^6}\,dx &= -\frac{\left(a+bx^2\right)^{3/4}}{5x^5}+\frac{1}{10}(3b)\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx\\
&= -\frac{\left(a+bx^2\right)^{3/4}}{5x^5}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}-\frac{\left(3b^2\right)\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx}{20a}\\
&= -\frac{\left(a+bx^2\right)^{3/4}}{5x^5}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}+\frac{3b^2\left(a+bx^2\right)^{3/4}}{20a^2x}-\frac{\left(3b^3\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{40a^2}\\
&= -\frac{\left(a+bx^2\right)^{3/4}}{5x^5}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}+\frac{3b^2\left(a+bx^2\right)^{3/4}}{20a^2x}-\frac{\left(3b^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{40a^2\sqrt[4]{a+bx^2}}\\
&= -\frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{5x^5}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}+\frac{3b^2\left(a+bx^2\right)^{3/4}}{20a^2x}+\frac{\left(3b^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int -}{40a^2\sqrt[4]{a+b}}\\
&= -\frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{5x^5}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}+\frac{3b^2\left(a+bx^2\right)^{3/4}}{20a^2x}+\frac{3b^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\right.}{20a^{3/2}\sqrt[4]{a}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.35

$$-\frac{\left(a+bx^2\right)^{3/4}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/4)/x^6,x]
```

```
[Out] -1/5*((a + b*x^2)^(3/4)*Hypergeometric2F1[-5/2, -3/4, -3/2, -((b*x^2)/a)])/
(x^5*(1 + (b*x^2)/a)^(3/4))
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/x^6, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{3}{4}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/4)/x^6,x)
```

```
[Out] int((b*x^2+a)^(3/4)/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{3}{4}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/4)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{3/4}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/4)/x^6,x)
```

```
[Out] int((a + b*x^2)^(3/4)/x^6, x)
```

sympy [C] time = 1.23, size = 34, normalized size = 0.23

$$-\frac{a^{\frac{3}{4}}{}_{2}F_{1}\left(\begin{matrix}-\frac{5}{2},-\frac{3}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{5x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/4)/x**6,x)
```

```
[Out] -a**(3/4)*hyper((-5/2, -3/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

3.805 $\int x^4 \left(a-bx^2\right)^{3/4} \, dx$

Optimal. Leaf size=126

$$\frac{8a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a-bx^2}}-\frac{4a^2x\left(a-bx^2\right)^{3/4}}{65b^2}+\frac{2}{13}x^5\left(a-bx^2\right)^{3/4}-\frac{2ax^3\left(a-bx^2\right)^{3/4}}{39b}$$

```
[Out] -4/65*a^2*x*(-b*x^2+a)^(3/4)/b^2-2/39*a*x^3*(-b*x^2+a)^(3/4)/b+2/13*x^5*(-b
*x^2+a)^(3/4)+8/65*a^(7/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1
/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(
x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.04, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {279, 321, 229, 228}

$$-\frac{4a^2x\left(a-bx^2\right)^{3/4}}{65b^2}+\frac{8a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a-bx^2}}+\frac{2}{13}x^5\left(a-bx^2\right)^{3/4}-\frac{2ax^3\left(a-bx^2\right)^{3/4}}{39b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a - b*x^2)^(3/4),x]
```

```
[Out] (-4*a^2*x*(a - b*x^2)^(3/4))/(65*b^2) - (2*a*x^3*(a - b*x^2)^(3/4))/(39*b)
+ (2*x^5*(a - b*x^2)^(3/4))/13 + (8*a^(7/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(65*b^(5/2)*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4 \left(a-bx^2\right)^{3/4} dx = \frac{2}{13}x^5\left(a-bx^2\right)^{3/4} + \frac{1}{13}(3a)\int \frac{x^4}{\sqrt[4]{a-bx^2}} dx$$

$$= -\frac{2ax^3\left(a-bx^2\right)^{3/4}}{39b} + \frac{2}{13}x^5\left(a-bx^2\right)^{3/4} + \frac{\left(2a^2\right)\int \frac{x^2}{\sqrt[4]{a-bx^2}} dx}{13b}$$

$$= -\frac{4a^2x\left(a-bx^2\right)^{3/4}}{65b^2} - \frac{2ax^3\left(a-bx^2\right)^{3/4}}{39b} + \frac{2}{13}x^5\left(a-bx^2\right)^{3/4} + \frac{\left(4a^3\right)\int \frac{1}{\sqrt[4]{a-bx^2}} dx}{65b^2}$$

$$= -\frac{4a^2x\left(a-bx^2\right)^{3/4}}{65b^2} - \frac{2ax^3\left(a-bx^2\right)^{3/4}}{39b} + \frac{2}{13}x^5\left(a-bx^2\right)^{3/4} + \frac{\left(4a^3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}} dx}{65b^2\sqrt[4]{a-bx^2}}$$

$$= -\frac{4a^2x\left(a-bx^2\right)^{3/4}}{65b^2} - \frac{2ax^3\left(a-bx^2\right)^{3/4}}{39b} + \frac{2}{13}x^5\left(a-bx^2\right)^{3/4} + \frac{8a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\right.\right.}{65b^{5/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.06, size = 95, normalized size = 0.75

$$-\frac{2x\left(a-bx^2\right)^{3/4}\left(\left(1-\frac{bx^2}{a}\right)^{3/4}\left(2a^2+abx^2-3b^2x^4\right)-2a^2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)\right)}{39b^2\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a - b*x^2)^(3/4),x]
```

```
[Out] (-2*x*(a - b*x^2)^(3/4)*((1 - (b*x^2)/a)^(3/4)*(2*a^2 + a*b*x^2 - 3*b^2*x^4
) - 2*a^2*Hypergeometric2F1[-3/4, 1/2, 3/2, (b*x^2)/a]))/(39*b^2*(1 - (b*x^
2)/a)^(3/4))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-bx^2+a\right)^{\frac{3}{4}}x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2+a\right)^{\frac{3}{4}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(3/4)*x^4, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(-b\,x^2+a\right)^{\frac{3}{4}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-b*x^2+a)^(3/4),x)
```

```
[Out] int(x^4*(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2+a\right)^{\frac{3}{4}} x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(3/4)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(a-b\,x^2\right)^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a - b*x^2)^(3/4),x)
```

```
[Out] int(x^4*(a - b*x^2)^(3/4), x)
```

sympy [C] time = 1.20, size = 31, normalized size = 0.25

$$\frac{a^{\frac{3}{4}} x^5 {}_2F_1\left(\left.\begin{matrix}-\frac{3}{4}, \frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{bx^2 e^{2i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/4)*x**5*hyper((-3/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

3.806 $\int x^2 \left(a-bx^2\right)^{3/4} dx$

Optimal. Leaf size=101

$$\frac{4a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a-bx^2}}-\frac{2ax\left(a-bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}$$

```
[Out] -2/15*a*x*(-b*x^2+a)^(3/4)/b+2/9*x^3*(-b*x^2+a)^(3/4)+4/15*a^(5/2)*(1-b*x^2
/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(
1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2
)/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {279, 321, 229, 228}

$$\frac{4a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a-bx^2}}+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}-\frac{2ax\left(a-bx^2\right)^{3/4}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a - b*x^2)^(3/4),x]
```

```
[Out] (-2*a*x*(a - b*x^2)^(3/4))/(15*b) + (2*x^3*(a - b*x^2)^(3/4))/9 + (4*a^(5/2
)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^
(3/2)*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2\left(a-bx^2\right)^{3/4}dx = \frac{2}{9}x^3\left(a-bx^2\right)^{3/4}+\frac{1}{3}a\int\frac{x^2}{\sqrt[4]{a-bx^2}}dx$$
$$= -\frac{2ax\left(a-bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}+\frac{\left(2a^2\right)\int\frac{1}{\sqrt[4]{a-bx^2}}dx}{15b}$$
$$= -\frac{2ax\left(a-bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}+\frac{\left(2a^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int\frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{15b\sqrt[4]{a-bx^2}}$$
$$= -\frac{2ax\left(a-bx^2\right)^{3/4}}{15b}+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}+\frac{4a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.06, size = 64, normalized size = 0.63

$$\frac{2x\left(a-bx^2\right)^{3/4}\left(\frac{a\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/4}}-a+bx^2\right)}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a - b*x^2)^(3/4),x]
[Out] (2*x*(a - b*x^2)^(3/4)*(-a + b*x^2 + (a*Hypergeometric2F1[-3/4, 1/2, 3/2, (
b*x^2)/a])/(1 - (b*x^2)/a)^(3/4)))/(9*b)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(-bx^2+a\right)^{\frac{3}{4}}x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-bx^2+a\right)^{\frac{3}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(3/4)*x^2, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int\left(-b\,x^2+a\right)^{\frac{3}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-b*x^2+a)^(3/4),x)
```

```
[Out] int(x^2*(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2 + a\right)^{\frac{3}{4}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(3/4)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a - b\,x^2\right)^{3/4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a - b*x^2)^(3/4),x)
[Out] int(x^2*(a - b*x^2)^(3/4), x)
```

sympy [C] time = 1.08, size = 31, normalized size = 0.31

$$\frac{a^{\frac{3}{4}} x^3 {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-b*x**2+a)**(3/4),x)
[Out] a**(3/4)*x**3*hyper((-3/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3
```

3.807 $\int \left(a-bx^2\right)^{3/4}\,dx$

Optimal. Leaf size=78

$$\frac{6a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\,\sqrt[4]{a-bx^2}}+\frac{2}{5}x\left(a-bx^2\right)^{3/4}$$

```
[Out] 2/5*x*(-b*x^2+a)^(3/4)+6/5*a^(3/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1
/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2
*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+a)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {195, 229, 228}

$$\frac{6a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\,\sqrt[4]{a-bx^2}}+\frac{2}{5}x\left(a-bx^2\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/4),x]
```

```
[Out] (2*x*(a - b*x^2)^(3/4))/5 + (6*a^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcS
in[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(a - b*x^2)^(1/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \left(a-bx^2\right)^{3/4} dx &= \frac{2}{5}x\left(a-bx^2\right)^{3/4}+\frac{1}{5}(3a)\int \frac{1}{\sqrt[4]{a-bx^2}}dx \\ &= \frac{2}{5}x\left(a-bx^2\right)^{3/4}+\frac{\left(3a\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{5\sqrt[4]{a-bx^2}} \\ &= \frac{2}{5}x\left(a-bx^2\right)^{3/4}+\frac{6a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a-bx^2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.60

$$\frac{x\left(a-bx^2\right)^{3/4}{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/4),x]
```

```
[Out] (x*(a - b*x^2)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, (b*x^2)/a])/(1 - (b*
x^2)/a)^(3/4)
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-bx^2+a\right)^{\frac{3}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2+a\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \left(-b\,x^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/4),x)
```

```
[Out] int((-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2+a\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(3/4), x)
```

mupad [B] time = 4.85, size = 38, normalized size = 0.49

$$\frac{x\left(a-b\,x^2\right)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(3/4),x)
```

```
[Out] (x*(a - b*x^2)^(3/4)*hypergeom([-3/4, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/a
)^(3/4)
```

sympy [C] time = 0.97, size = 27, normalized size = 0.35

$$a^{\frac{3}{4}}x{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(3/4),x)
```

```
[Out] a**(3/4)*x*hyper((-3/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

3.808 $\int \frac{(a-bx^2)^{3/4}}{x^2}\,dx$

Optimal. Leaf size=76

$$-\frac{(a-bx^2)^{3/4}}{x}-\frac{3\sqrt{a}\,\sqrt{b}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a-bx^2}}$$

```
[Out] -(-b*x^2+a)^(3/4)/x-3*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(
1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {277, 229, 228}

$$-\frac{(a-bx^2)^{3/4}}{x}-\frac{3\sqrt{a}\,\sqrt{b}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/4)/x^2,x]
```

```
[Out] -((a - b*x^2)^(3/4)/x) - (3*Sqrt[a]*Sqrt[b]*(1 - (b*x^2)/a)^(1/4)*EllipticE
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a - b*x^2)^(1/4)
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a-bx^2)^{3/4}}{x^2}\,dx = -\frac{(a-bx^2)^{3/4}}{x} - \frac{1}{2}(3b)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx$$
$$= -\frac{(a-bx^2)^{3/4}}{x} - \frac{\left(3b\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{2\sqrt[4]{a-bx^2}}$$
$$= -\frac{(a-bx^2)^{3/4}}{x} - \frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.66

$$-\frac{(a-bx^2)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{2};\frac{bx^2}{a}\right)}{x\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/4)/x^2,x]
[Out] -(((a - b*x^2)^(3/4)*Hypergeometric2F1[-3/4, -1/2, 1/2, (b*x^2)/a])/(x*(1 -
 (b*x^2)/a)^(3/4)))
```

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(-bx^2+a)^{\frac{3}{4}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^2,x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-bx^2+a)^{\frac{3}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^2,x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(3/4)/x^2, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(-b\,x^2+a)^{\frac{3}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/4)/x^2,x)
```

[Out] int((-b*x^2+a)^(3/4)/x^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+a)^(3/4)/x^2,x, algorithm="maxima")

[Out] integrate((-b*x^2 + a)^(3/4)/x^2, x)

mupad [B] time = 5.17, size = 41, normalized size = 0.54

$$\frac{2\left(a-b\,x^2\right)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},-\frac{1}{4};\ \frac{3}{4};\ \frac{a}{b\,x^2}\right)}{x\left(1-\frac{a}{b\,x^2}\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b*x^2)^(3/4)/x^2,x)

[Out] (2*(a - b*x^2)^(3/4)*hypergeom([-3/4, -1/4], 3/4, a/(b*x^2)))/(x*(1 - a/(b*x^2))^(3/4))

sympy [C] time = 1.05, size = 31, normalized size = 0.41

$$-\frac{a^{\frac{3}{4}}{}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x**2+a)**(3/4)/x**2,x)

[Out] -a**(3/4)*hyper((-3/4, -1/2), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/x

3.809 $\int \frac{\left(a-bx^2\right)^{3/4}}{x^4}\,dx$

Optimal. Leaf size=103

$$\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,\sqrt[4]{a-bx^2}}+\frac{b\left(a-bx^2\right)^{3/4}}{2ax}-\frac{\left(a-bx^2\right)^{3/4}}{3x^3}$$

```
[Out] -1/3*(-b*x^2+a)^(3/4)/x^3+1/2*b*(-b*x^2+a)^(3/4)/a/x+1/2*b^(3/2)*(1-b*x^2/a
)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/
2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+
a)^(1/4)/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {277, 325, 229, 228}

$$\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,\sqrt[4]{a-bx^2}}+\frac{b\left(a-bx^2\right)^{3/4}}{2ax}-\frac{\left(a-bx^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/4)/x^4,x]
```

```
[Out] -(a - b*x^2)^(3/4)/(3*x^3) + (b*(a - b*x^2)^(3/4))/(2*a*x) + (b^(3/2)*(1 -
(b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*(a
 - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx^2)^{3/4}}{x^4}\,dx &= -\frac{(a-bx^2)^{3/4}}{3x^3}-\frac{1}{2}b\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx\\ &= -\frac{(a-bx^2)^{3/4}}{3x^3}+\frac{b(a-bx^2)^{3/4}}{2ax}+\frac{b^2\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{4a}\\ &= -\frac{(a-bx^2)^{3/4}}{3x^3}+\frac{b(a-bx^2)^{3/4}}{2ax}+\frac{\left(b^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{4a\sqrt[4]{a-bx^2}}\\ &= -\frac{(a-bx^2)^{3/4}}{3x^3}+\frac{b(a-bx^2)^{3/4}}{2ax}+\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a-bx^2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.50

$$-\frac{(a-bx^2)^{3/4}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3x^3\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/4)/x^4,x]
[Out] -1/3*((a - b*x^2)^(3/4)*Hypergeometric2F1[-3/2, -3/4, -1/2, (b*x^2)/a])/(x^
3*(1 - (b*x^2)/a)^(3/4))
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(-bx^2+a)^{\frac{3}{4}}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^4,x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-bx^2+a)^{\frac{3}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^4,x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(3/4)/x^4, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(-bx^2+a)^{\frac{3}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/4)/x^4,x)
```

```
[Out] int((-b*x^2+a)^(3/4)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(3/4)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^2\right)^{3/4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(3/4)/x^4,x)
```

```
[Out] int((a - b*x^2)^(3/4)/x^4, x)
```

sympy [C] time = 1.12, size = 36, normalized size = 0.35

$$-\frac{a^{\frac{3}{4}}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},-\frac{3}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(3/4)/x**4,x)
```

```
[Out] -a**(3/4)*hyper((-3/2, -3/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*x**3)
```

3.810 $\int \frac{\left(a-bx^2\right)^{3/4}}{x^6}\,dx$

Optimal. Leaf size=128

$$\frac{3b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a-bx^2}}+\frac{3b^2\left(a-bx^2\right)^{3/4}}{20a^2x}-\frac{\left(a-bx^2\right)^{3/4}}{5x^5}+\frac{b\left(a-bx^2\right)^{3/4}}{10ax^3}$$

```
[Out] -1/5*(-b*x^2+a)^(3/4)/x^5+1/10*b*(-b*x^2+a)^(3/4)/a/x^3+3/20*b^2*(-b*x^2+a)
^(3/4)/a^2/x+3/20*b^(5/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/
2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x
*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {277, 325, 229, 228}

$$\frac{3b^2\left(a-bx^2\right)^{3/4}}{20a^2x}+\frac{3b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a-bx^2}}+\frac{b\left(a-bx^2\right)^{3/4}}{10ax^3}-\frac{\left(a-bx^2\right)^{3/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/4)/x^6,x]
```

```
[Out] -(a - b*x^2)^(3/4)/(5*x^5) + (b*(a - b*x^2)^(3/4))/(10*a*x^3) + (3*b^2*(a -
 b*x^2)^(3/4))/(20*a^2*x) + (3*b^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcS
in[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(3/2)*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\left(a-bx^2\right)^{3/4}}{x^6}\,dx = -\frac{\left(a-bx^2\right)^{3/4}}{5x^5} - \frac{1}{10}(3b)\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{5x^5} + \frac{b\left(a-bx^2\right)^{3/4}}{10ax^3} - \frac{\left(3b^2\right)\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{20a}$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{5x^5} + \frac{b\left(a-bx^2\right)^{3/4}}{10ax^3} + \frac{3b^2\left(a-bx^2\right)^{3/4}}{20a^2x} + \frac{\left(3b^3\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{40a^2}$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{5x^5} + \frac{b\left(a-bx^2\right)^{3/4}}{10ax^3} + \frac{3b^2\left(a-bx^2\right)^{3/4}}{20a^2x} + \frac{\left(3b^3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{40a^2\sqrt[4]{a-bx^2}}$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{5x^5} + \frac{b\left(a-bx^2\right)^{3/4}}{10ax^3} + \frac{3b^2\left(a-bx^2\right)^{3/4}}{20a^2x} + \frac{3b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.41

$$-\frac{\left(a-bx^2\right)^{3/4}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5x^5\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/4)/x^6,x]
```

```
[Out] -1/5*((a - b*x^2)^(3/4)*Hypergeometric2F1[-5/2, -3/4, -3/2, (b*x^2)/a])/(x^
5*(1 - (b*x^2)/a)^(3/4))
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^6,x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^6,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(3/4)/x^6, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^2+a\right)^{\frac{3}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/4)/x^6,x)
[Out] int((-b*x^2+a)^(3/4)/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/4)/x^6,x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(3/4)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^2\right)^{3/4}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(3/4)/x^6,x)
[Out] int((a - b*x^2)^(3/4)/x^6, x)
```

sympy [C] time = 1.28, size = 36, normalized size = 0.28

$$-\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{2}, -\frac{3}{4}\\ -\frac{3}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(3/4)/x**6,x)
[Out] -a**(3/4)*hyper((-5/2, -3/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*x**5)
```

3.811 $\int \left(a+bx^2\right)^{5/4} dx$

Optimal. Leaf size=92

$$\frac{10a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{10}{21}ax\sqrt[4]{a+bx^2}+\frac{2}{7}x\left(a+bx^2\right)^{5/4}$$

```
[Out] 10/21*a*x*(b*x^2+a)^(1/4)+2/7*x*(b*x^2+a)^(5/4)+10/21*a^(5/2)*(1+b*x^2/a)^(
3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/
a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^2+a)^(
3/4)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 233, 231}

$$\frac{10a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{10}{21}ax\sqrt[4]{a+bx^2}+\frac{2}{7}x\left(a+bx^2\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/4),x]
```

```
[Out] (10*a*x*(a + b*x^2)^(1/4))/21 + (2*x*(a + b*x^2)^(5/4))/7 + (10*a^(5/2)*(1
+ (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*Sqrt[b]
*(a + b*x^2)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^2\right)^{5/4} dx &= \frac{2}{7}x\left(a+bx^2\right)^{5/4}+\frac{1}{7}(5a)\int \sqrt[4]{a+bx^2}\,dx \\
&= \frac{10}{21}ax\sqrt[4]{a+bx^2}+\frac{2}{7}x\left(a+bx^2\right)^{5/4}+\frac{1}{21}\left(5a^2\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}}dx \\
&= \frac{10}{21}ax\sqrt[4]{a+bx^2}+\frac{2}{7}x\left(a+bx^2\right)^{5/4}+\frac{\left(5a^2\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}dx}{21\left(a+bx^2\right)^{3/4}} \\
&= \frac{10}{21}ax\sqrt[4]{a+bx^2}+\frac{2}{7}x\left(a+bx^2\right)^{5/4}+\frac{10a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.51

$$\frac{ax\sqrt[4]{a+bx^2}\;{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/4),x]
```

```
[Out] (a*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^(1/4)
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{5}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/4),x)
```

[Out] int((b*x^2+a)^(5/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/4),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(5/4), x)

mupad [B] time = 4.86, size = 37, normalized size = 0.40

$$\frac{x\left(b\,x^2+a\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^(5/4),x)

[Out] (x*(a + b*x^2)^(5/4)*hypergeom([-5/4, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(5/4)

sympy [C] time = 1.16, size = 26, normalized size = 0.28

$$a^{\frac{5}{4}}x\,{}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**(5/4),x)

[Out] a**(5/4)*x*hyper((-5/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)

3.812 $\int \left(a-bx^2\right)^{5/4}\,dx$

Optimal. Leaf size=96

$$\frac{10a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{21\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}$$

```
[Out] 10/21*a*x*(-b*x^2+a)^(1/4)+2/7*x*(-b*x^2+a)^(5/4)+10/21*a^(5/2)*(1-b*x^2/a)
^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2
)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+a
)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {195, 233, 232}

$$\frac{10a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/4),x]
```

```
[Out] (10*a*x*(a - b*x^2)^(1/4))/21 + (2*x*(a - b*x^2)^(5/4))/7 + (10*a^(5/2)*(1
- (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*Sqrt[b]
*(a - b*x^2)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \left(a-bx^2\right)^{5/4}\,dx &= \frac{2}{7}x\left(a-bx^2\right)^{5/4}+\frac{1}{7}(5a)\int \sqrt[4]{a-bx^2}\,dx\\
&= \frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}+\frac{1}{21}\left(5a^2\right)\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx\\
&= \frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}+\frac{\left(5a^2\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{21\left(a-bx^2\right)^{3/4}}\\
&= \frac{10}{21}ax\sqrt[4]{a-bx^2}+\frac{2}{7}x\left(a-bx^2\right)^{5/4}+\frac{10a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\left(a-bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.50

$$
\frac{ax\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(5/4),x]
[Out] (a*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(1/4)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\left(-bx^2+a\right)^{\frac{5}{4}},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(5/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \left(-bx^2+a\right)^{\frac{5}{4}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$
\int \left(-b\,x^2+a\right)^{\frac{5}{4}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/4),x)
```

```
[Out] int((-b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2 + a\right)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/4), x)
```

mupad [B] time = 4.84, size = 38, normalized size = 0.40

$$\frac{x\left(a-b\,x^2\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(5/4),x)
```

```
[Out] (x*(a - b*x^2)^(5/4)*hypergeom([-5/4, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/a
)^(5/4)
```

sympy [C] time = 1.19, size = 27, normalized size = 0.28

$$a^{\frac{5}{4}} x {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/4),x)
```

```
[Out] a**(5/4)*x*hyper((-5/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

3.813 $\int \left(a+bx^2\right)^{7/4}\,dx$

Optimal. Leaf size=111

$$-\frac{14a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\,\sqrt[4]{a+bx^2}}+\frac{14a^2x}{15\sqrt[4]{a+bx^2}}+\frac{14}{45}ax\left(a+bx^2\right)^{3/4}+\frac{2}{9}x\left(a+bx^2\right)^{7/4}$$

```
[Out] 14/15*a^2*x/(b*x^2+a)^(1/4)+14/45*a*x*(b*x^2+a)^(3/4)+2/9*x*(b*x^2+a)^(7/4)
-14/15*a^(5/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/
2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^
(1/2))),2^(1/2))/(b*x^2+a)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {195, 229, 227, 196}

$$\frac{14a^2x}{15\sqrt[4]{a+bx^2}}-\frac{14a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\,\sqrt[4]{a+bx^2}}+\frac{14}{45}ax\left(a+bx^2\right)^{3/4}+\frac{2}{9}x\left(a+bx^2\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(7/4),x]
```

```
[Out] (14*a^2*x)/(15*(a + b*x^2)^(1/4)) + (14*a*x*(a + b*x^2)^(3/4))/45 + (2*x*(a
 + b*x^2)^(7/4))/9 - (14*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sq
rt[b]*x)/Sqrt[a]]/2, 2])/(15*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx^2\right)^{7/4}\,dx &= \frac{2}{9}x\left(a+bx^2\right)^{7/4}+\frac{1}{9}(7a)\int \left(a+bx^2\right)^{3/4}\,dx\\
&= \frac{14}{45}ax\left(a+bx^2\right)^{3/4}+\frac{2}{9}x\left(a+bx^2\right)^{7/4}+\frac{1}{15}\left(7a^2\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx\\
&= \frac{14}{45}ax\left(a+bx^2\right)^{3/4}+\frac{2}{9}x\left(a+bx^2\right)^{7/4}+\frac{\left(7a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{15\sqrt[4]{a+bx^2}}\\
&= \frac{14a^2x}{15\sqrt[4]{a+bx^2}}+\frac{14}{45}ax\left(a+bx^2\right)^{3/4}+\frac{2}{9}x\left(a+bx^2\right)^{7/4}-\frac{\left(7a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{15\sqrt[4]{a+bx^2}}\\
&= \frac{14a^2x}{15\sqrt[4]{a+bx^2}}+\frac{14}{45}ax\left(a+bx^2\right)^{3/4}+\frac{2}{9}x\left(a+bx^2\right)^{7/4}-\frac{14a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.42

$$
\frac{ax\left(a+bx^2\right)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(7/4),x]
[Out] (a*x*(a + b*x^2)^(3/4)*Hypergeometric2F1[-7/4, 1/2, 3/2, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^(3/4)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{7}{4}},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(7/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \left(bx^2+a\right)^{\frac{7}{4}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(7/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$
\int \left(b\,x^2+a\right)^{\frac{7}{4}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(7/4),x)
[Out] int((b*x^2+a)^(7/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(7/4),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(7/4), x)
```

mupad [B] time = 4.82, size = 37, normalized size = 0.33

$$\frac{x\,\left(b\,x^2+a\right)^{7/4}\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(7/4),x)
[Out] (x*(a + b*x^2)^(7/4)*hypergeom([-7/4, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(7/4)
```

sympy [C] time = 1.54, size = 26, normalized size = 0.23

$$a^{\frac{7}{4}} x {}_{2}F_{1}\left(\begin{matrix} -\frac{7}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(7/4),x)
[Out] a**(7/4)*x*hyper((-7/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

3.814 $\int \left(a-bx^2\right)^{7/4}\,dx$

Optimal. Leaf size=96

$$\frac{14a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\,\sqrt[4]{a-bx^2}}+\frac{14}{45}ax\left(a-bx^2\right)^{3/4}+\frac{2}{9}x\left(a-bx^2\right)^{7/4}$$

```
[Out] 14/45*a*x*(-b*x^2+a)^(3/4)+2/9*x*(-b*x^2+a)^(7/4)+14/15*a^(5/2)*(1-b*x^2/a)
^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2
)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+a
)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {195, 229, 228}

$$\frac{14a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\,\sqrt[4]{a-bx^2}}+\frac{14}{45}ax\left(a-bx^2\right)^{3/4}+\frac{2}{9}x\left(a-bx^2\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(7/4),x]
```

```
[Out] (14*a*x*(a - b*x^2)^(3/4))/45 + (2*x*(a - b*x^2)^(7/4))/9 + (14*a^(5/2)*(1
- (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*Sqrt[b]
*(a - b*x^2)^(1/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \left(a-bx^2\right)^{7/4} dx = \frac{2}{9}x\left(a-bx^2\right)^{7/4} + \frac{1}{9}(7a)\int \left(a-bx^2\right)^{3/4} dx$$
$$= \frac{14}{45}ax\left(a-bx^2\right)^{3/4} + \frac{2}{9}x\left(a-bx^2\right)^{7/4} + \frac{1}{15}\left(7a^2\right)\int \frac{1}{\sqrt[4]{a-bx^2}}dx$$
$$= \frac{14}{45}ax\left(a-bx^2\right)^{3/4} + \frac{2}{9}x\left(a-bx^2\right)^{7/4} + \frac{\left(7a^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{15\sqrt[4]{a-bx^2}}$$
$$= \frac{14}{45}ax\left(a-bx^2\right)^{3/4} + \frac{2}{9}x\left(a-bx^2\right)^{7/4} + \frac{14a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.50

$$\frac{ax\left(a-bx^2\right)^{3/4} {}_2F_1\left(-\frac{7}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(7/4),x]
```

```
[Out] (a*x*(a - b*x^2)^(3/4)*Hypergeometric2F1[-7/4, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(3/4)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(-bx^2+a\right)^{\frac{7}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(7/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2+a\right)^{\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(7/4), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \left(-b\,x^2+a\right)^{\frac{7}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(7/4),x)
```

```
[Out] int((-b*x^2+a)^(7/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2 + a\right)^{\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(7/4), x)
```

mupad [B] time = 4.82, size = 38, normalized size = 0.40

$$\frac{x\left(a - b\,x^2\right)^{7/4}\,{}_2\mathrm{F}_1\left(-\frac{7}{4}, \frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1 - \frac{b\,x^2}{a}\right)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(7/4),x)
```

```
[Out] (x*(a - b*x^2)^(7/4)*hypergeom([-7/4, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/a
)^(7/4)
```

sympy [C] time = 1.54, size = 27, normalized size = 0.28

$$a^{\frac{7}{4}} x {}_{2}F_{1}\left(\left.\begin{matrix} -\frac{7}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\right| \frac{bx^{2} e^{2 i \pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(7/4),x)
```

```
[Out] a**(7/4)*x*hyper((-7/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

3.815 $\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=146

$$\frac{16a^{7/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a+bx^2}}-\frac{16a^3x}{39b^3\sqrt[4]{a+bx^2}}+\frac{8a^2x\left(a+bx^2\right)^{3/4}}{39b^3}-\frac{20ax^3\left(a+bx^2\right)^{3/4}}{117b^2}+\frac{2x^5\left(a+bx^2\right)^{3/4}}{13b}$$

```
[Out] -16/39*a^3*x/b^3/(b*x^2+a)^(1/4)+8/39*a^2*x*(b*x^2+a)^(3/4)/b^3-20/117*a*x^
3*(b*x^2+a)^(3/4)/b^2+2/13*x^5*(b*x^2+a)^(3/4)/b+16/39*a^(7/2)*(1+b*x^2/a)^
(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)
/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(b
*x^2+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {321, 229, 227, 196}

$$\frac{8a^2x\left(a+bx^2\right)^{3/4}}{39b^3}-\frac{16a^3x}{39b^3\sqrt[4]{a+bx^2}}+\frac{16a^{7/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a+bx^2}}-\frac{20ax^3\left(a+bx^2\right)^{3/4}}{117b^2}+\frac{2x^5\left(a+bx^2\right)^{3/4}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(1/4),x]
```

```
[Out] (-16*a^3*x)/(39*b^3*(a + b*x^2)^(1/4)) + (8*a^2*x*(a + b*x^2)^(3/4))/(39*b^
3) - (20*a*x^3*(a + b*x^2)^(3/4))/(117*b^2) + (2*x^5*(a + b*x^2)^(3/4))/(13
*b) + (16*a^(7/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a
]]/2, 2])/(39*b^(7/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx &= \frac{2x^5\left(a+bx^2\right)^{3/4}}{13b} - \frac{(10a)\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx}{13b} \\
&= -\frac{20ax^3\left(a+bx^2\right)^{3/4}}{117b^2} + \frac{2x^5\left(a+bx^2\right)^{3/4}}{13b} + \frac{\left(20a^2\right)\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx}{39b^2} \\
&= \frac{8a^2x\left(a+bx^2\right)^{3/4}}{39b^3} - \frac{20ax^3\left(a+bx^2\right)^{3/4}}{117b^2} + \frac{2x^5\left(a+bx^2\right)^{3/4}}{13b} - \frac{\left(8a^3\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{39b^3} \\
&= \frac{8a^2x\left(a+bx^2\right)^{3/4}}{39b^3} - \frac{20ax^3\left(a+bx^2\right)^{3/4}}{117b^2} + \frac{2x^5\left(a+bx^2\right)^{3/4}}{13b} - \frac{\left(8a^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{39b^3\sqrt[4]{a+bx^2}} \\
&= -\frac{16a^3x}{39b^3\sqrt[4]{a+bx^2}} + \frac{8a^2x\left(a+bx^2\right)^{3/4}}{39b^3} - \frac{20ax^3\left(a+bx^2\right)^{3/4}}{117b^2} + \frac{2x^5\left(a+bx^2\right)^{3/4}}{13b} + \frac{\left(8a^3\sqrt[4]{1+\frac{bx^2}{a}} \cdots\right.}{39b^3\sqrt[4]{\cdots}} \\
&= -\frac{16a^3x}{39b^3\sqrt[4]{a+bx^2}} + \frac{8a^2x\left(a+bx^2\right)^{3/4}}{39b^3} - \frac{20ax^3\left(a+bx^2\right)^{3/4}}{117b^2} + \frac{2x^5\left(a+bx^2\right)^{3/4}}{13b} + \frac{16a^{7/2}\sqrt[4]{1+\frac{bx^2}{a}} \cdots}{39b \cdots}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 90, normalized size = 0.62

$$
\frac{2\left(-12a^3x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+12a^3x+2a^2bx^3-ab^2x^5+9b^3x^7\right)}{117b^3\sqrt[4]{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(1/4),x]
```

```
[Out] (2*(12*a^3*x + 2*a^2*b*x^3 - a*b^2*x^5 + 9*b^3*x^7 - 12*a^3*x*(1 + (b*x^2)/
a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(117*b^3*(a + b*x
^2)^(1/4))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\frac{x^6}{\left(bx^2+a\right)^{\frac{1}{4}}}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(b*x^2 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^6}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(b\,x^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(1/4),x)
[Out] int(x^6/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(bx^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(x^6/(b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{6}}{\left(b\,x^{2}+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(1/4),x)
[Out] int(x^6/(a + b*x^2)^(1/4), x)
```

sympy [C] time = 0.97, size = 27, normalized size = 0.18

$$\frac{x^{7}{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4}, \frac{7}{2}\\ \frac{9}{2}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{7\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(1/4),x)
[Out] x**7*hyper((1/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(1/4))
```

3.816 $\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{8a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx^2}}+\frac{8a^2x}{15b^2\sqrt[4]{a+bx^2}}-\frac{4ax\left(a+bx^2\right)^{3/4}}{15b^2}+\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}$$

```
[Out] 8/15*a^2*x/b^2/(b*x^2+a)^(1/4)-4/15*a*x*(b*x^2+a)^(3/4)/b^2+2/9*x^3*(b*x^2+
a)^(3/4)/b-8/15*a^(5/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)
))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b
^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {321, 229, 227, 196}

$$\frac{8a^2x}{15b^2\sqrt[4]{a+bx^2}}-\frac{8a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx^2}}-\frac{4ax\left(a+bx^2\right)^{3/4}}{15b^2}+\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(1/4),x]
```

```
[Out] (8*a^2*x)/(15*b^2*(a + b*x^2)^(1/4)) - (4*a*x*(a + b*x^2)^(3/4))/(15*b^2) +
 (2*x^3*(a + b*x^2)^(3/4))/(9*b) - (8*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*Ellipti
cE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx = \frac{2x^3\left(a+bx^2\right)^{3/4}}{9b} - \frac{(2a)\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx}{3b}$$
$$= -\frac{4ax\left(a+bx^2\right)^{3/4}}{15b^2} + \frac{2x^3\left(a+bx^2\right)^{3/4}}{9b} + \frac{\left(4a^2\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{15b^2}$$
$$= -\frac{4ax\left(a+bx^2\right)^{3/4}}{15b^2} + \frac{2x^3\left(a+bx^2\right)^{3/4}}{9b} + \frac{\left(4a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{15b^2\sqrt[4]{a+bx^2}}$$
$$= \frac{8a^2x}{15b^2\sqrt[4]{a+bx^2}} - \frac{4ax\left(a+bx^2\right)^{3/4}}{15b^2} + \frac{2x^3\left(a+bx^2\right)^{3/4}}{9b} - \frac{\left(4a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{15b^2\sqrt[4]{a+bx^2}}$$
$$= \frac{8a^2x}{15b^2\sqrt[4]{a+bx^2}} - \frac{4ax\left(a+bx^2\right)^{3/4}}{15b^2} + \frac{2x^3\left(a+bx^2\right)^{3/4}}{9b} - \frac{8a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.02, size = 79, normalized size = 0.65

$$\frac{2\left(6a^2x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-6a^2x-abx^3+5b^2x^5\right)}{45b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(1/4),x]
```

```
[Out] (2*(-6*a^2*x - a*b*x^3 + 5*b^2*x^5 + 6*a^2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeo
metric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(45*b^2*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^4}{\left(bx^2+a\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(b*x^2 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/4),x)
```

```
[Out] int(x^4/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(1/4),x)
```

```
[Out] int(x^4/(a + b*x^2)^(1/4), x)
```

sympy [C] time = 0.86, size = 27, normalized size = 0.22

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/4),x)
```

```
[Out] x**5*hyper((1/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/4))
```

3.817 $\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=98

$$\frac{4a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx^2}}-\frac{4ax}{5b\sqrt[4]{a+bx^2}}+\frac{2x\left(a+bx^2\right)^{3/4}}{5b}$$

```
[Out] -4/5*a*x/b/(b*x^2+a)^(1/4)+2/5*x*(b*x^2+a)^(3/4)/b+4/5*a^(3/2)*(1+b*x^2/a)^
(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)
/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(b
*x^2+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = $\{321, 229, 227, 196\}$

$$\frac{4a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx^2}}-\frac{4ax}{5b\sqrt[4]{a+bx^2}}+\frac{2x\left(a+bx^2\right)^{3/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(1/4),x]
```

```
[Out] (-4*a*x)/(5*b*(a + b*x^2)^(1/4)) + (2*x*(a + b*x^2)^(3/4))/(5*b) + (4*a^(3/
2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^
(3/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx = \frac{2x\left(a+bx^2\right)^{3/4}}{5b} - \frac{(2a)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{5b}$$

$$= \frac{2x\left(a+bx^2\right)^{3/4}}{5b} - \frac{\left(2a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{5b\sqrt[4]{a+bx^2}}$$

$$= -\frac{4ax}{5b\sqrt[4]{a+bx^2}} + \frac{2x\left(a+bx^2\right)^{3/4}}{5b} + \frac{\left(2a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{5b\sqrt[4]{a+bx^2}}$$

$$= -\frac{4ax}{5b\sqrt[4]{a+bx^2}} + \frac{2x\left(a+bx^2\right)^{3/4}}{5b} + \frac{4a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.63

$$\frac{2x\left(-a\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{5b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(1/4),x]
[Out] (2*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2,
-((b*x^2)/a)]))/(5*b*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\left(bx^2+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(x^2/(b*x^2 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(x^2/(b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/4),x)
```

```
[Out] int(x^2/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(1/4),x)
```

```
[Out] int(x^2/(a + b*x^2)^(1/4), x)
```

sympy [C] time = 0.80, size = 27, normalized size = 0.28

$$\frac{x^3 {}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/4),x)
```

```
[Out] x**3*hyper((1/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/4))
```

3.818 $\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=71

$$\frac{2x}{\sqrt[4]{a+bx^2}}-\frac{2\sqrt{a}\,\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

```
[Out] 2*x/(b*x^2+a)^(1/4)-2*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(
1/2)/a^(1/2))),2^(1/2))*a^(1/2)/(b*x^2+a)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {229, 227, 196}

$$\frac{2x}{\sqrt[4]{a+bx^2}}-\frac{2\sqrt{a}\,\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1/4),x]
```

```
[Out] (2*x)/(a + b*x^2)^(1/4) - (2*Sqrt[a]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}$$
$$= \frac{2x}{\sqrt[4]{a+bx^2}} - \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{\sqrt[4]{a+bx^2}}$$
$$= \frac{2x}{\sqrt[4]{a+bx^2}} - \frac{2\sqrt{a}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.65

$$\frac{x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1/4),x]
[Out] (x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(1/4)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(-1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-1/4), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-1/4), x)
```

mupad [B] time = 4.85, size = 37, normalized size = 0.52

$$\frac{x\left(\frac{b\,x^2}{a}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/4),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(1/4)*hypergeom([1/4, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(1/4)
```

sympy [C] time = 0.78, size = 24, normalized size = 0.34

$$\frac{x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/4),x)
```

```
[Out] x*hyper((1/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/4)
```

3.819 $\int \frac{1}{x^2 \sqrt[4]{a+bx^2}} dx$

Optimal. Leaf size=93

$$\frac{bx}{a\sqrt[4]{a+bx^2}} - \frac{\left(a+bx^2\right)^{3/4}}{ax} - \frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}}$$

```
[Out] b*x/a/(b*x^2+a)^(1/4)-(b*x^2+a)^(3/4)/a/x-(1+b*x^2/a)^(1/4)*(cos(1/2*arctan
(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(
sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/(b*x^2+a)^(1/4)/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 229, 227, 196}

$$\frac{bx}{a\sqrt[4]{a+bx^2}} - \frac{\left(a+bx^2\right)^{3/4}}{ax} - \frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(1/4)),x]
```

```
[Out] (b*x)/(a*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(a*x) - (Sqrt[b]*(1 + (b*x^
2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^
2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx &= -\frac{\left(a+bx^2\right)^{3/4}}{ax}+\frac{b\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{2a}\\
&= -\frac{\left(a+bx^2\right)^{3/4}}{ax}+\frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{2a\sqrt[4]{a+bx^2}}\\
&= \frac{bx}{a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{ax}-\frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{2a\sqrt[4]{a+bx^2}}\\
&= \frac{bx}{a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{ax}-\frac{\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.53

$$
-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[4]{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(1/4)),x]
[Out] -(((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(1/4)))
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{bx^4+ax^2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)/(b*x^4 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^2), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$
\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{4}}x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^2), x)
```

mupad [B] time = 5.09, size = 40, normalized size = 0.43

$$-\frac{2\left(\frac{a}{b\,x^2}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{a}{b\,x^2}\right)}{3\,x\,\left(b\,x^2+a\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(1/4)),x)
```

```
[Out] -(2*(a/(b*x^2) + 1)^(1/4)*hypergeom([1/4, 3/4], 7/4, -a/(b*x^2)))/(3*x*(a +
 b*x^2)^(1/4))
```

sympy [C] time = 0.85, size = 27, normalized size = 0.29

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[4]{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((-1/2, 1/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(1/4)*x)
```

3.820 $\int \frac{1}{x^4 \sqrt[4]{a+bx^2}} \, dx$

Optimal. Leaf size=124

$$\frac{b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a+bx^2}}-\frac{b^2x}{2a^2\sqrt[4]{a+bx^2}}+\frac{b\left(a+bx^2\right)^{3/4}}{2a^2x}-\frac{\left(a+bx^2\right)^{3/4}}{3ax^3}$$

```
[Out] -1/2*b^2*x/a^2/(b*x^2+a)^(1/4)-1/3*(b*x^2+a)^(3/4)/a/x^3+1/2*b*(b*x^2+a)^(3
/4)/a^2/x+1/2*b^(3/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(
1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 229, 227, 196}

$$-\frac{b^2x}{2a^2\sqrt[4]{a+bx^2}}+\frac{b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a+bx^2}}+\frac{b\left(a+bx^2\right)^{3/4}}{2a^2x}-\frac{\left(a+bx^2\right)^{3/4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(1/4)),x]
```

```
[Out] -(b^2*x)/(2*a^2*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(3*a*x^3) + (b*(a +
b*x^2)^(3/4))/(2*a^2*x) + (b^(3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(3/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx = -\frac{\left(a+bx^2\right)^{3/4}}{3ax^3} - \frac{b\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx}{2a}$$
$$= -\frac{\left(a+bx^2\right)^{3/4}}{3ax^3} + \frac{b\left(a+bx^2\right)^{3/4}}{2a^2x} - \frac{b^2\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4a^2}$$
$$= -\frac{\left(a+bx^2\right)^{3/4}}{3ax^3} + \frac{b\left(a+bx^2\right)^{3/4}}{2a^2x} - \frac{\left(b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{4a^2\sqrt[4]{a+bx^2}}$$
$$= -\frac{b^2x}{2a^2\sqrt[4]{a+bx^2}} - \frac{\left(a+bx^2\right)^{3/4}}{3ax^3} + \frac{b\left(a+bx^2\right)^{3/4}}{2a^2x} + \frac{\left(b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{4a^2\sqrt[4]{a+bx^2}}$$
$$= -\frac{b^2x}{2a^2\sqrt[4]{a+bx^2}} - \frac{\left(a+bx^2\right)^{3/4}}{3ax^3} + \frac{b\left(a+bx^2\right)^{3/4}}{2a^2x} + \frac{b^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.41

$$-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(1/4)),x]
```

```
[Out] -1/3*((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, -((b*x^2)/a)
])/(x^3*(a + b*x^2)^(1/4))
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{bx^6+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b*x^6 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^4), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{4}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/4),x)
[Out] int(1/x^4/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\left(b\,x^2+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(1/4)),x)
[Out] int(1/(x^4*(a + b*x^2)^(1/4)), x)
```

sympy [C] time = 1.00, size = 32, normalized size = 0.26

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, \frac{1}{4}\\ -\frac{1}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[4]{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/4),x)
[Out] -hyper((-3/2, 1/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/4)*x**3)
```

3.821 $\int \frac{1}{x^6 \sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=148

$$-\frac{7b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a+bx^2}}+\frac{7b^3x}{20a^3\sqrt[4]{a+bx^2}}-\frac{7b^2\left(a+bx^2\right)^{3/4}}{20a^3x}+\frac{7b\left(a+bx^2\right)^{3/4}}{30a^2x^3}-\frac{\left(a+bx^2\right)^{3/4}}{5ax^5}$$

```
[Out] 7/20*b^3*x/a^3/(b*x^2+a)^(1/4)-1/5*(b*x^2+a)^(3/4)/a/x^5+7/30*b*(b*x^2+a)^(
3/4)/a^2/x^3-7/20*b^2*(b*x^2+a)^(3/4)/a^3/x-7/20*b^(5/2)*(1+b*x^2/a)^(1/4)*
(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/
2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(b*x^2+a
)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {325, 229, 227, 196}

$$\frac{7b^3x}{20a^3\sqrt[4]{a+bx^2}}-\frac{7b^2\left(a+bx^2\right)^{3/4}}{20a^3x}-\frac{7b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a+bx^2}}+\frac{7b\left(a+bx^2\right)^{3/4}}{30a^2x^3}-\frac{\left(a+bx^2\right)^{3/4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(1/4)),x]
```

```
[Out] (7*b^3*x)/(20*a^3*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(5*a*x^5) + (7*b*(
a + b*x^2)^(3/4))/(30*a^2*x^3) - (7*b^2*(a + b*x^2)^(3/4))/(20*a^3*x) - (7*
b^(5/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/
(20*a^(5/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\sqrt[4]{a+bx^2}}\,dx &= -\frac{\left(a+bx^2\right)^{3/4}}{5ax^5}-\frac{(7b)\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx}{10a} \\
&= -\frac{\left(a+bx^2\right)^{3/4}}{5ax^5}+\frac{7b\left(a+bx^2\right)^{3/4}}{30a^2x^3}+\frac{\left(7b^2\right)\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx}{20a^2} \\
&= -\frac{\left(a+bx^2\right)^{3/4}}{5ax^5}+\frac{7b\left(a+bx^2\right)^{3/4}}{30a^2x^3}-\frac{7b^2\left(a+bx^2\right)^{3/4}}{20a^3x}+\frac{\left(7b^3\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{40a^3} \\
&= -\frac{\left(a+bx^2\right)^{3/4}}{5ax^5}+\frac{7b\left(a+bx^2\right)^{3/4}}{30a^2x^3}-\frac{7b^2\left(a+bx^2\right)^{3/4}}{20a^3x}+\frac{\left(7b^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{40a^3\sqrt[4]{a+bx^2}} \\
&= \frac{7b^3x}{20a^3\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{5ax^5}+\frac{7b\left(a+bx^2\right)^{3/4}}{30a^2x^3}-\frac{7b^2\left(a+bx^2\right)^{3/4}}{20a^3x}-\frac{\left(7b^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{\ldots}{\left(1+\frac{b\ldots}{}\right.}}{40a^3\sqrt[4]{a+bx^2}} \\
&= \frac{7b^3x}{20a^3\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{5ax^5}+\frac{7b\left(a+bx^2\right)^{3/4}}{30a^2x^3}-\frac{7b^2\left(a+bx^2\right)^{3/4}}{20a^3x}-\frac{7b^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan\ldots\right.}{20a^{5/2}\sqrt[4]{a+\ldots}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.34

$$
-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(1/4)),x]
```

```
[Out] -1/5*((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, -((b*x^2)/a)
])/(x^5*(a + b*x^2)^(1/4))
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{bx^8+ax^6},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b*x^8 + a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}x^6}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^6), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{4}} x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{1}{4}} x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{6}\left(b\,x^{2}+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^(1/4)),x)
```

```
[Out] int(1/(x^6*(a + b*x^2)^(1/4)), x)
```

sympy [C] time = 1.11, size = 32, normalized size = 0.22

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{5\sqrt[4]{a}\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((-5/2, 1/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/4)*x**5)
```

3.822 $\int \frac{x^6}{\sqrt[4]{a-bx^2}} dx$

Optimal. Leaf size=129

$$\frac{16a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a-bx^2}}-\frac{8a^2x\left(a-bx^2\right)^{3/4}}{39b^3}-\frac{20ax^3\left(a-bx^2\right)^{3/4}}{117b^2}-\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b}$$

```
[Out] -8/39*a^2*x*(-b*x^2+a)^(3/4)/b^3-20/117*a*x^3*(-b*x^2+a)^(3/4)/b^2-2/13*x^5
*(-b*x^2+a)^(3/4)/b+16/39*a^(7/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*
arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.04, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {321, 229, 228}

$$-\frac{8a^2x\left(a-bx^2\right)^{3/4}}{39b^3}+\frac{16a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a-bx^2}}-\frac{20ax^3\left(a-bx^2\right)^{3/4}}{117b^2}-\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a - b*x^2)^(1/4),x]
```

```
[Out] (-8*a^2*x*(a - b*x^2)^(3/4))/(39*b^3) - (20*a*x^3*(a - b*x^2)^(3/4))/(117*b
^2) - (2*x^5*(a - b*x^2)^(3/4))/(13*b) + (16*a^(7/2)*(1 - (b*x^2)/a)^(1/4)*
EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(39*b^(7/2)*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx = -\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b}+\frac{(10a)\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx}{13b}$$
$$= -\frac{20ax^3\left(a-bx^2\right)^{3/4}}{117b^2}-\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b}+\frac{\left(20a^2\right)\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx}{39b^2}$$
$$= -\frac{8a^2x\left(a-bx^2\right)^{3/4}}{39b^3}-\frac{20ax^3\left(a-bx^2\right)^{3/4}}{117b^2}-\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b}+\frac{\left(8a^3\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{39b^3}$$
$$= -\frac{8a^2x\left(a-bx^2\right)^{3/4}}{39b^3}-\frac{20ax^3\left(a-bx^2\right)^{3/4}}{117b^2}-\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b}+\frac{\left(8a^3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{39b^3\sqrt[4]{a-bx^2}}$$
$$= -\frac{8a^2x\left(a-bx^2\right)^{3/4}}{39b^3}-\frac{20ax^3\left(a-bx^2\right)^{3/4}}{117b^2}-\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b}+\frac{16a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{39b^{7/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.03, size = 89, normalized size = 0.69

$$\frac{2x\left(12a^3\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-12a^3+2a^2bx^2+ab^2x^4+9b^3x^6\right)}{117b^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a - b*x^2)^(1/4),x]
[Out] (2*x*(-12*a^3 + 2*a^2*b*x^2 + a*b^2*x^4 + 9*b^3*x^6 + 12*a^3*(1 - (b*x^2)/a
)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a]))/(117*b^3*(a - b*x^2)^
(1/4))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}x^6}{bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(3/4)*x^6/(b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(x^6/(-b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(x^6/(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a - b*x^2)^(1/4),x)
```

```
[Out] int(x^6/(a - b*x^2)^(1/4), x)
```

sympy [C] time = 1.01, size = 29, normalized size = 0.22

$$\frac{x^7 {}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{7}{2}\\ \frac{9}{2}\end{matrix}\right| \frac{bx^2e^{2i\pi}}{a}\right)}{7\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-b*x**2+a)**(1/4),x)
```

```
[Out] x**7*hyper((1/4, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(1/4))
```

3.823 $\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=104

$$\frac{8a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a-bx^2}}-\frac{4ax\left(a-bx^2\right)^{3/4}}{15b^2}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}$$

```
[Out] -4/15*a*x*(-b*x^2+a)^(3/4)/b^2-2/9*x^3*(-b*x^2+a)^(3/4)/b+8/15*a^(5/2)*(1-b
*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x
*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^
(5/2)/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {321, 229, 228}

$$\frac{8a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a-bx^2}}-\frac{4ax\left(a-bx^2\right)^{3/4}}{15b^2}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^2)^(1/4),x]
```

```
[Out] (-4*a*x*(a - b*x^2)^(3/4))/(15*b^2) - (2*x^3*(a - b*x^2)^(3/4))/(9*b) + (8*
a^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/
(15*b^(5/2)*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx = -\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b} + \frac{(2a)\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx}{3b}$$
$$= -\frac{4ax\left(a-bx^2\right)^{3/4}}{15b^2} - \frac{2x^3\left(a-bx^2\right)^{3/4}}{9b} + \frac{\left(4a^2\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{15b^2}$$
$$= -\frac{4ax\left(a-bx^2\right)^{3/4}}{15b^2} - \frac{2x^3\left(a-bx^2\right)^{3/4}}{9b} + \frac{\left(4a^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{15b^2\sqrt[4]{a-bx^2}}$$
$$= -\frac{4ax\left(a-bx^2\right)^{3/4}}{15b^2} - \frac{2x^3\left(a-bx^2\right)^{3/4}}{9b} + \frac{8a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.03, size = 79, normalized size = 0.76

$$\frac{2\left(6a^2x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-6a^2x+abx^3+5b^2x^5\right)}{45b^2\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^2)^(1/4),x]
[Out] (2*(-6*a^2*x + a*b*x^3 + 5*b^2*x^5 + 6*a^2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeo
metric2F1[1/4, 1/2, 3/2, (b*x^2)/a]))/(45*b^2*(a - b*x^2)^(1/4))
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}x^4}{bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(3/4)*x^4/(b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(x^4/(-b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-b*x^2+a)^(1/4),x)
[Out] int(x^4/(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(x^4/(-b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b*x^2)^(1/4),x)
[Out] int(x^4/(a - b*x^2)^(1/4), x)
```

sympy [C] time = 0.91, size = 29, normalized size = 0.28

$$\frac{x^5{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-b*x**2+a)**(1/4),x)
[Out] x**5*hyper((1/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(1/4))
```

3.824 $\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=81

$$\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}$$

```
[Out] -2/5*x*(-b*x^2+a)^(3/4)/b+4/5*a^(3/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(
1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {321, 229, 228}

$$\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^(1/4),x]
```

```
[Out] (-2*x*(a - b*x^2)^(3/4))/(5*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^(3/2)*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx = -\frac{2x\left(a-bx^2\right)^{3/4}}{5b} + \frac{(2a)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{5b}$$

$$= -\frac{2x\left(a-bx^2\right)^{3/4}}{5b} + \frac{\left(2a\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{5b\sqrt[4]{a-bx^2}}$$

$$= -\frac{2x\left(a-bx^2\right)^{3/4}}{5b} + \frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.02, size = 64, normalized size = 0.79

$$\frac{2x\left(a\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-a+bx^2\right)}{5b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^(1/4),x]
[Out] (2*x*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2,
 (b*x^2)/a]))/(5*b*(a - b*x^2)^(1/4))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}x^2}{bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(3/4)*x^2/(b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(x^2/(-b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(x^2/(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2)^(1/4),x)
```

```
[Out] int(x^2/(a - b*x^2)^(1/4), x)
```

sympy [C] time = 0.82, size = 29, normalized size = 0.36

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**(1/4),x)
```

```
[Out] x**3*hyper((1/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(1/4))
```

3.825 $\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{a}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

```
[Out] 2*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
csin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/
2))*a^(1/2)/(-b*x^2+a)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {229, 228}

$$\frac{2\sqrt{a}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-1/4),x]
```

```
[Out] (2*Sqrt[a]*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx &= \frac{\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a-bx^2}}\\ &= \frac{2\sqrt{a}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a-bx^2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.81

$$\frac{x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-1/4),x]
```

```
[Out] (x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])/(a -
b*x^2)^(1/4)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)/(b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(-1/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(-1/4), x)
```

mupad [B] time = 4.87, size = 38, normalized size = 0.66

$$\frac{x\left(1-\frac{b\,x^2}{a}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(a-b\,x^2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(1/4),x)
```

```
[Out] (x*(1 - (b*x^2)/a)^(1/4)*hypergeom([1/4, 1/2], 3/2, (b*x^2)/a))/(a - b*x^2)
^(1/4)
```

sympy [C]   time = 0.79, size = 26, normalized size = 0.45

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/4),x)
```

```
[Out] x*hyper((1/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(1/4)
```

3.826 $\int \frac{1}{x^2 \sqrt[4]{a-bx^2}} \, dx$

Optimal. Leaf size=79

$$-\frac{\left(a-bx^2\right)^{3/4}}{ax}-\frac{\sqrt{b}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt[4]{a-bx^2}}$$

```
[Out] -(-b*x^2+a)^(3/4)/a/x-(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(
1/2)/a^(1/2))),2^(1/2))*b^(1/2)/(-b*x^2+a)^(1/4)/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = $\{325, 229, 228\}$

$$-\frac{\left(a-bx^2\right)^{3/4}}{ax}-\frac{\sqrt{b}\,\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^(1/4)),x]
```

```
[Out] -((a - b*x^2)^(3/4)/(a*x)) - (Sqrt[b]*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSi
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx = -\frac{\left(a-bx^2\right)^{3/4}}{ax} - \frac{b\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{2a}$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{ax} - \frac{\left(b\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{2a\sqrt[4]{a-bx^2}}$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{ax} - \frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.63

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{bx^2}{a}\right)}{x\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^(1/4)),x]
[Out] -(((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (b*x^2)/a])/(x*(
a - b*x^2)^(1/4)))
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{bx^4-ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(3/4)/(b*x^4 - a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^2), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{4}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/x^2/(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2 + a\right)^{\frac{1}{4}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^2), x)
```

mupad [B] time = 5.08, size = 41, normalized size = 0.52

$$-\frac{2\left(1-\frac{a}{b\,x^2}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ \frac{a}{b\,x^2}\right)}{3\,x\,\left(a-b\,x^2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^(1/4)),x)
```

```
[Out] -(2*(1 - a/(b*x^2))^(1/4)*hypergeom([1/4, 3/4], 7/4, a/(b*x^2)))/(3*x*(a -
b*x^2)^(1/4))
```

sympy [C] time = 0.90, size = 29, normalized size = 0.37

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| {\frac{bx^{2} e^{2 i \pi}}{a}} \right)}{\sqrt[4]{a} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((-1/2, 1/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/(a**(1/4)*x)
```

3.827 $\int \frac{1}{x^4 \sqrt[4]{a-bx^2}} \, dx$

Optimal. Leaf size=106

$$-\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a-bx^2}}-\frac{b\left(a-bx^2\right)^{3/4}}{2a^2x}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}$$

```
[Out] -1/3*(-b*x^2+a)^(3/4)/a/x^3-1/2*b*(-b*x^2+a)^(3/4)/a^2/x-1/2*b^(3/2)*(1-b*x
^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b
^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3
/2)/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {325, 229, 228}

$$-\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a-bx^2}}-\frac{b\left(a-bx^2\right)^{3/4}}{2a^2x}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a - b*x^2)^(1/4)),x]
```

```
[Out] -(a - b*x^2)^(3/4)/(3*a*x^3) - (b*(a - b*x^2)^(3/4))/(2*a^2*x) - (b^(3/2)*(
1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(3/2
)*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx = -\frac{\left(a-bx^2\right)^{3/4}}{3ax^3} + \frac{b\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{2a}$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{3ax^3} - \frac{b\left(a-bx^2\right)^{3/4}}{2a^2x} - \frac{b^2\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{4a^2}$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{3ax^3} - \frac{b\left(a-bx^2\right)^{3/4}}{2a^2x} - \frac{\left(b^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{4a^2\sqrt[4]{a-bx^2}}$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{3ax^3} - \frac{b\left(a-bx^2\right)^{3/4}}{2a^2x} - \frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.49

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a - b*x^2)^(1/4)),x]
```

```
[Out] -1/3*((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (b*x^2)/a])/
(x^3*(a - b*x^2)^(1/4))
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{bx^6-ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)/(b*x^6 - a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^4), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{4}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-b*x^2+a)^(1/4),x)
[Out] int(1/x^4/(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a - b*x^2)^(1/4)),x)
[Out] int(1/(x^4*(a - b*x^2)^(1/4)), x)
```

sympy [C] time = 1.01, size = 34, normalized size = 0.32

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| {\frac{bx^2e^{2i\pi}}{a}} \right)}{3\sqrt[4]{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-b*x**2+a)**(1/4),x)
[Out] -hyper((-3/2, 1/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(1/4)*x**3)
```

3.828 $\int \frac{1}{x^6 \sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=131

$$-\frac{7b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a-bx^2}}-\frac{7b^2\left(a-bx^2\right)^{3/4}}{20a^3x}-\frac{7b\left(a-bx^2\right)^{3/4}}{30a^2x^3}-\frac{\left(a-bx^2\right)^{3/4}}{5ax^5}$$

```
[Out] -1/5*(-b*x^2+a)^(3/4)/a/x^5-7/30*b*(-b*x^2+a)^(3/4)/a^2/x^3-7/20*b^2*(-b*x^
2+a)^(3/4)/a^3/x-7/20*b^(5/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcs
in(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {325, 229, 228}

$$-\frac{7b^2\left(a-bx^2\right)^{3/4}}{20a^3x}-\frac{7b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a-bx^2}}-\frac{7b\left(a-bx^2\right)^{3/4}}{30a^2x^3}-\frac{\left(a-bx^2\right)^{3/4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a - b*x^2)^(1/4)),x]
```

```
[Out] -(a - b*x^2)^(3/4)/(5*a*x^5) - (7*b*(a - b*x^2)^(3/4))/(30*a^2*x^3) - (7*b^
2*(a - b*x^2)^(3/4))/(20*a^3*x) - (7*b^(5/2)*(1 - (b*x^2)/a)^(1/4)*Elliptic
E[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(5/2)*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[4]{a-bx^2}}\,dx = -\frac{\left(a-bx^2\right)^{3/4}}{5ax^5} + \frac{(7b)\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx}{10a}$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{5ax^5} - \frac{7b\left(a-bx^2\right)^{3/4}}{30a^2x^3} + \frac{\left(7b^2\right)\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{20a^2}$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{5ax^5} - \frac{7b\left(a-bx^2\right)^{3/4}}{30a^2x^3} - \frac{7b^2\left(a-bx^2\right)^{3/4}}{20a^3x} - \frac{\left(7b^3\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{40a^3}$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{5ax^5} - \frac{7b\left(a-bx^2\right)^{3/4}}{30a^2x^3} - \frac{7b^2\left(a-bx^2\right)^{3/4}}{20a^3x} - \frac{\left(7b^3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{40a^3\sqrt[4]{a-bx^2}}$$
$$= -\frac{\left(a-bx^2\right)^{3/4}}{5ax^5} - \frac{7b\left(a-bx^2\right)^{3/4}}{30a^2x^3} - \frac{7b^2\left(a-bx^2\right)^{3/4}}{20a^3x} - \frac{7b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.40

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a - b*x^2)^(1/4)),x]
[Out] -1/5*((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, (b*x^2)/a])/
(x^5*(a - b*x^2)^(1/4))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{bx^8-ax^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(3/4)/(b*x^8 - a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^6), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{4}}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/x^6/(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a - b*x^2)^(1/4)),x)
```

```
[Out] int(1/(x^6*(a - b*x^2)^(1/4)), x)
```

sympy [C] time = 1.13, size = 34, normalized size = 0.26

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{bx^{2}e^{2i\pi}}{a}} \right)}{5\sqrt[4]{a}\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((-5/2, 1/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(1/4)*x**5)
```

3.829 $\int \frac{x^6}{\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=124

$$-\frac{80a^{7/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{77b^{7/2}\left(a+bx^2\right)^{3/4}}+\frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3}-\frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2}+\frac{2x^5\sqrt[4]{a+bx^2}}{11b}$$

```
[Out] 40/77*a^2*x*(b*x^2+a)^(1/4)/b^3-20/77*a*x^3*(b*x^2+a)^(1/4)/b^2+2/11*x^5*(b
*x^2+a)^(1/4)/b-80/77*a^(7/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arct
an(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 233, 231}

$$\frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3}-\frac{80a^{7/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77b^{7/2}\left(a+bx^2\right)^{3/4}}-\frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2}+\frac{2x^5\sqrt[4]{a+bx^2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(3/4),x]
```

```
[Out] (40*a^2*x*(a + b*x^2)^(1/4))/(77*b^3) - (20*a*x^3*(a + b*x^2)^(1/4))/(77*b^
2) + (2*x^5*(a + b*x^2)^(1/4))/(11*b) - (80*a^(7/2)*(1 + (b*x^2)/a)^(3/4)*E
llipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(7/2)*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^{3/4}}\,dx = \frac{2x^5\sqrt[4]{a+bx^2}}{11b} - \frac{(10a)\int \frac{x^4}{\left(a+bx^2\right)^{3/4}}\,dx}{11b}$$

$$= -\frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5\sqrt[4]{a+bx^2}}{11b} + \frac{\left(60a^2\right)\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx}{77b^2}$$

$$= \frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5\sqrt[4]{a+bx^2}}{11b} - \frac{\left(40a^3\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{77b^3}$$

$$= \frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5\sqrt[4]{a+bx^2}}{11b} - \frac{\left(40a^3\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{77b^3\left(a+bx^2\right)^{3/4}}$$

$$= \frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5\sqrt[4]{a+bx^2}}{11b} - \frac{80a^{7/2}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right|}{77b^{7/2}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.04, size = 90, normalized size = 0.73

$$\frac{2\left(-20a^3x\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)+20a^3x+10a^2bx^3-3ab^2x^5+7b^3x^7\right)}{77b^3\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(3/4),x]
```

```
[Out] (2*(20*a^3*x + 10*a^2*b*x^3 - 3*a*b^2*x^5 + 7*b^3*x^7 - 20*a^3*x*(1 + (b*x^
2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)]))/(77*b^3*(a + b
*x^2)^(3/4))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\left(bx^2+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(b*x^2 + a)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(b\,x^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(3/4),x)
```

```
[Out] int(x^6/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(bx^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{{\left(b\,x^2+a\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(3/4),x)
```

```
[Out] int(x^6/(a + b*x^2)^(3/4), x)
```

sympy [C] time = 0.96, size = 27, normalized size = 0.22

$$\frac{x^{7} {{}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{7 a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(3/4),x)
```

```
[Out] x**7*hyper((3/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(3/4))
```

3.830 $\int \frac{x^4}{\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=100

$$\frac{8a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{7b^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{4ax\sqrt[4]{a+bx^2}}{7b^2}+\frac{2x^3\sqrt[4]{a+bx^2}}{7b}$$

```
[Out] -4/7*a*x*(b*x^2+a)^(1/4)/b^2+2/7*x^3*(b*x^2+a)^(1/4)/b+8/7*a^(5/2)*(1+b*x^2
/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(
1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2
)/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 233, 231}

$$\frac{8a^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{4ax\sqrt[4]{a+bx^2}}{7b^2}+\frac{2x^3\sqrt[4]{a+bx^2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(3/4),x]
```

```
[Out] (-4*a*x*(a + b*x^2)^(1/4))/(7*b^2) + (2*x^3*(a + b*x^2)^(1/4))/(7*b) + (8*a
^(5/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
7*b^(5/2)*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{3/4}}\,dx = \frac{2x^3\sqrt[4]{a+bx^2}}{7b} - \frac{(6a)\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx}{7b}$$

$$= -\frac{4ax\sqrt[4]{a+bx^2}}{7b^2} + \frac{2x^3\sqrt[4]{a+bx^2}}{7b} + \frac{\left(4a^2\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{7b^2}$$

$$= -\frac{4ax\sqrt[4]{a+bx^2}}{7b^2} + \frac{2x^3\sqrt[4]{a+bx^2}}{7b} + \frac{\left(4a^2\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{7b^2\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{4ax\sqrt[4]{a+bx^2}}{7b^2} + \frac{2x^3\sqrt[4]{a+bx^2}}{7b} + \frac{8a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.03, size = 78, normalized size = 0.78

$$\frac{2\left(2a^2x\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)-2a^2x-abx^3+b^2x^5\right)}{7b^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(3/4),x]
```

```
[Out] (2*(-2*a^2*x - a*b*x^3 + b^2*x^5 + 2*a^2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeome
tric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)]))/(7*b^2*(a + b*x^2)^(3/4))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\left(bx^2+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(b*x^2 + a)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(3/4),x)
```

```
[Out] int(x^4/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(3/4),x)
```

```
[Out] int(x^4/(a + b*x^2)^(3/4), x)
```

sympy [C] time = 0.85, size = 27, normalized size = 0.27

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(3/4),x)
```

```
[Out] x**5*hyper((3/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(3/4))
```

3.831 $\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=78

$$\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a+bx^2\right)^{3/4}}$$

```
[Out] 2/3*x*(b*x^2+a)^(1/4)/b-4/3*a^(3/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(
1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/
2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 233, 231}

$$\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(3/4),x]
```

```
[Out] (2*x*(a + b*x^2)^(1/4))/(3*b) - (4*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx = \frac{2x\sqrt[4]{a+bx^2}}{3b} - \frac{(2a)\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{3b}$$

$$= \frac{2x\sqrt[4]{a+bx^2}}{3b} - \frac{\left(2a\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{3b\left(a+bx^2\right)^{3/4}}$$

$$= \frac{2x\sqrt[4]{a+bx^2}}{3b} - \frac{4a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.79

$$\frac{2x\left(-a\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{3b\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(3/4),x]
```

```
[Out] (2*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2,
-((b*x^2)/a)]))/(3*b*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^2}{\left(bx^2+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*x^2 + a)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(3/4),x)
```

```
[Out] int(x^2/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(3/4),x)
```

```
[Out] int(x^2/(a + b*x^2)^(3/4), x)
```

sympy [C] time = 0.82, size = 27, normalized size = 0.35

$$\frac{x^3{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(3/4),x)
```

```
[Out] x**3*hyper((3/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(3/4))
```

3.832 $\int \frac{1}{\left(a+bx^2\right)^{3/4}} \, dx$

Optimal. Leaf size=56

$$\frac{2\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

```
[Out] 2*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
ctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/
2))*a^(1/2)/(b*x^2+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {233, 231}

$$\frac{2\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-3/4),x]
```

```
[Out] (2*Sqrt[a]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)^{3/4}} \, dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}} \, dx}{\left(a+bx^2\right)^{3/4}}\\ &= \frac{2\sqrt{a}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.82

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-3/4),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(3/4)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(-3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-3/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-3/4), x)
```

mupad [B] time = 4.88, size = 37, normalized size = 0.66

$$\frac{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(b\,x^2+a\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(3/4),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(3/4)*hypergeom([1/2, 3/4], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(3/4)
```

sympy [C] time = 0.78, size = 24, normalized size = 0.43

$$\frac{x{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/4),x)
```

```
[Out] x*hyper((1/2, 3/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(3/4)
```

3.833 $\int \frac{1}{x^2(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}$$

```
[Out] -(b*x^2+a)^(1/4)/a/x-(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^
2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1
/2)/a^(1/2))),2^(1/2))*b^(1/2)/(b*x^2+a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 233, 231}

$$-\frac{\sqrt[4]{a+bx^2}}{ax}-\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(3/4)),x]
```

```
[Out] -((a + b*x^2)^(1/4)/(a*x)) - (Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTa
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a+bx^2\right)^{3/4}} \, dx = -\frac{\sqrt[4]{a+bx^2}}{ax} - \frac{b\int \frac{1}{\left(a+bx^2\right)^{3/4}} \, dx}{2a}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{ax} - \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}} \, dx}{2a\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{ax} - \frac{\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.64

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(3/4)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(3/4)))
```

fricas [F] time = 1.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{bx^4+ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)/(b*x^4 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^2), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{3}{4}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^2), x)
```

mupad [B] time = 5.07, size = 40, normalized size = 0.53

$$-\frac{2\left(\frac{a}{b\,x^2}+1\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ -\frac{a}{b\,x^2}\right)}{5\,x\,\left(b\,x^2+a\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(3/4)),x)
```

```
[Out] -(2*(a/(b*x^2) + 1)^(3/4)*hypergeom([3/4, 5/4], 9/4, -a/(b*x^2)))/(5*x*(a +
 b*x^2)^(3/4))
```

sympy [C] time = 0.96, size = 27, normalized size = 0.36

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{3}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((-1/2, 3/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(3/4)*x)
```

3.834 $\int \frac{1}{x^4\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=102

$$\frac{5b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{6a^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{5b\sqrt[4]{a+bx^2}}{6a^2x}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}$$

```
[Out] -1/3*(b*x^2+a)^(1/4)/a/x^3+5/6*b*(b*x^2+a)^(1/4)/a^2/x+5/6*b^(3/2)*(1+b*x^2
/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(
1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2
)/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.03, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 233, 231}

$$\frac{5b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6a^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{5b\sqrt[4]{a+bx^2}}{6a^2x}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(3/4)),x]
```

```
[Out] -(a + b*x^2)^(1/4)/(3*a*x^3) + (5*b*(a + b*x^2)^(1/4))/(6*a^2*x) + (5*b^(3/
2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*a^
(3/2)*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^2}}{3ax^3}-\frac{(5b)\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx}{6a}$$
$$= -\frac{\sqrt[4]{a+bx^2}}{3ax^3}+\frac{5b\sqrt[4]{a+bx^2}}{6a^2x}+\frac{\left(5b^2\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{12a^2}$$
$$= -\frac{\sqrt[4]{a+bx^2}}{3ax^3}+\frac{5b\sqrt[4]{a+bx^2}}{6a^2x}+\frac{\left(5b^2\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{12a^2\left(a+bx^2\right)^{3/4}}$$
$$= -\frac{\sqrt[4]{a+bx^2}}{3ax^3}+\frac{5b\sqrt[4]{a+bx^2}}{6a^2x}+\frac{5b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6a^{3/2}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.50

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(3/4)),x]
```

```
[Out] -1/3*((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, -((b*x^2)/a)
])/(x^3*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{bx^6+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)/(b*x^6 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^4), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(3/4)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)^(3/4)), x)
```

sympy [C] time = 1.09, size = 32, normalized size = 0.31

$$-\frac{{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{3}{2},\frac{3}{4}\\-\frac{1}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{3}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((-3/2, 3/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(3/4)*x**3)
```

3.835 $\int \frac{1}{x^6(a+bx^2)^{3/4}} dx$

Optimal. Leaf size=126

$$-\frac{3b^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{4a^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x}+\frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3}-\frac{\sqrt[4]{a+bx^2}}{5ax^5}$$

```
[Out] -1/5*(b*x^2+a)^(1/4)/a/x^5+3/10*b*(b*x^2+a)^(1/4)/a^2/x^3-3/4*b^2*(b*x^2+a)
^(1/4)/a^3/x-3/4*b^(5/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2
)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*
b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.04, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 233, 231}

$$-\frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x}-\frac{3b^{5/2}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\left(a+bx^2\right)^{3/4}}+\frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3}-\frac{\sqrt[4]{a+bx^2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(3/4)),x]
```

```
[Out] -(a + b*x^2)^(1/4)/(5*a*x^5) + (3*b*(a + b*x^2)^(1/4))/(10*a^2*x^3) - (3*b^
2*(a + b*x^2)^(1/4))/(4*a^3*x) - (3*b^(5/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF
[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(4*a^(5/2)*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^2\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^2}}{5ax^5} - \frac{(9b)\int \frac{1}{x^4\left(a+bx^2\right)^{3/4}}\,dx}{10a}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{5ax^5} + \frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3} + \frac{\left(3b^2\right)\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx}{4a^2}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{5ax^5} + \frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3} - \frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x} - \frac{\left(3b^3\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{8a^3}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{5ax^5} + \frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3} - \frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x} - \frac{\left(3b^3\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{8a^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^2}}{5ax^5} + \frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3} - \frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x} - \frac{3b^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.40

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(3/4)),x]
[Out] -1/5*((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, -((b*x^2)/a)
])/(x^5*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{bx^8+ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)/(b*x^8 + a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^6), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{3}{4}} x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{3}{4}} x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{6}\left(b\,x^{2}+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^(3/4)),x)
```

```
[Out] int(1/(x^6*(a + b*x^2)^(3/4)), x)
```

sympy [C] time = 1.23, size = 32, normalized size = 0.25

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{5a^{\frac{3}{4}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((-5/2, 3/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(3/4)*x**5)
```

3.836 $\int \frac{x^6}{(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=129

$$\frac{80a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{77b^{7/2}\left(a-bx^2\right)^{3/4}}-\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3}-\frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}$$

```
[Out] -40/77*a^2*x*(-b*x^2+a)^(1/4)/b^3-20/77*a*x^3*(-b*x^2+a)^(1/4)/b^2-2/11*x^5
*(-b*x^2+a)^(1/4)/b+80/77*a^(7/2)*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*
arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(-b*x^2+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {321, 233, 232}

$$-\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3}+\frac{80a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77b^{7/2}\left(a-bx^2\right)^{3/4}}-\frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a - b*x^2)^(3/4),x]
```

```
[Out] (-40*a^2*x*(a - b*x^2)^(1/4))/(77*b^3) - (20*a*x^3*(a - b*x^2)^(1/4))/(77*b
^2) - (2*x^5*(a - b*x^2)^(1/4))/(11*b) + (80*a^(7/2)*(1 - (b*x^2)/a)^(3/4)*
EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^(7/2)*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\left(a-bx^2\right)^{3/4}}\,dx = -\frac{2x^5\sqrt[4]{a-bx^2}}{11b} + \frac{(10a)\int \frac{x^4}{\left(a-bx^2\right)^{3/4}}\,dx}{11b}$$

$$= -\frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2} - \frac{2x^5\sqrt[4]{a-bx^2}}{11b} + \frac{\left(60a^2\right)\int \frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx}{77b^2}$$

$$= -\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2} - \frac{2x^5\sqrt[4]{a-bx^2}}{11b} + \frac{\left(40a^3\right)\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{77b^3}$$

$$= -\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2} - \frac{2x^5\sqrt[4]{a-bx^2}}{11b} + \frac{\left(40a^3\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{77b^3\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2} - \frac{2x^5\sqrt[4]{a-bx^2}}{11b} + \frac{80a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.\right.}{77b^{7/2}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.04, size = 91, normalized size = 0.71

$$\frac{2\left(20a^3x\left(1-\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)-20a^3x+10a^2bx^3+3ab^2x^5+7b^3x^7\right)}{77b^3\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a - b*x^2)^(3/4),x]
```

```
[Out] (2*(-20*a^3*x + 10*a^2*b*x^3 + 3*a*b^2*x^5 + 7*b^3*x^7 + 20*a^3*x*(1 - (b*x
^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a]))/(77*b^3*(a - b*x
^2)^(3/4))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}x^6}{bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)*x^6/(b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(x^6/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(-bx^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{{\left(a-b\,x^2\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a - b*x^2)^(3/4),x)
```

```
[Out] int(x^6/(a - b*x^2)^(3/4), x)
```

sympy [C] time = 0.99, size = 29, normalized size = 0.22

$$\frac{x^{7} {{}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right)}}{7 a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-b*x**2+a)**(3/4),x)
```

```
[Out] x**7*hyper((3/4, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(3/4))
```

3.837 $\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=104

$$\frac{8a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{7b^{5/2}\left(a-bx^2\right)^{3/4}}-\frac{4ax\sqrt[4]{a-bx^2}}{7b^2}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}$$

```
[Out] -4/7*a*x*(-b*x^2+a)^(1/4)/b^2-2/7*x^3*(-b*x^2+a)^(1/4)/b+8/7*a^(5/2)*(1-b*x
^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b
^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5
/2)/(-b*x^2+a)^(3/4)
```

Rubi [A] time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = $\{321, 233, 232\}$

$$\frac{8a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a-bx^2\right)^{3/4}}-\frac{4ax\sqrt[4]{a-bx^2}}{7b^2}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^2)^(3/4),x]
```

```
[Out] (-4*a*x*(a - b*x^2)^(1/4))/(7*b^2) - (2*x^3*(a - b*x^2)^(1/4))/(7*b) + (8*a
^(5/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
7*b^(5/2)*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx = -\frac{2x^3\sqrt[4]{a-bx^2}}{7b}+\frac{(6a)\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx}{7b}$$
$$= -\frac{4ax\sqrt[4]{a-bx^2}}{7b^2}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}+\frac{(4a^2)\int \frac{1}{(a-bx^2)^{3/4}}\,dx}{7b^2}$$
$$= -\frac{4ax\sqrt[4]{a-bx^2}}{7b^2}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}+\frac{\left(4a^2\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{7b^2(a-bx^2)^{3/4}}$$
$$= -\frac{4ax\sqrt[4]{a-bx^2}}{7b^2}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}+\frac{8a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}(a-bx^2)^{3/4}}$$

Mathematica [C] time = 0.03, size = 77, normalized size = 0.74

$$\frac{2x\left(2a^2\left(1-\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)-2a^2+abx^2+b^2x^4\right)}{7b^2(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^2)^(3/4),x]
```

```
[Out] (2*x*(-2*a^2 + a*b*x^2 + b^2*x^4 + 2*a^2*(1 - (b*x^2)/a)^(3/4)*Hypergeometr
ic2F1[1/2, 3/4, 3/2, (b*x^2)/a]))/(7*b^2*(a - b*x^2)^(3/4))
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(-bx^2+a)^{\frac{1}{4}}x^4}{bx^2-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)*x^4/(b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-b\,x^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-b*x^2+a)^(3/4),x)
[Out] int(x^4/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(x^4/(-b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(a-b\,x^2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b*x^2)^(3/4),x)
[Out] int(x^4/(a - b*x^2)^(3/4), x)
```

sympy [C] time = 0.90, size = 29, normalized size = 0.28

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{5a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-b*x**2+a)**(3/4),x)
[Out] x**5*hyper((3/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(3/4))
```

3.838 $\int \frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=81

$$\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}$$

```
[Out] -2/3*x*(-b*x^2+a)^(1/4)/b+4/3*a^(3/2)*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(
1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(-b*x^2+a)^(3/4)
```

Rubi [A] time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {321, 233, 232}

$$\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^(3/4),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4))/(3*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx = -\frac{2x\sqrt[4]{a-bx^2}}{3b} + \frac{(2a)\int \frac{1}{(a-bx^2)^{3/4}}\,dx}{3b}$$

$$= -\frac{2x\sqrt[4]{a-bx^2}}{3b} + \frac{\left(2a\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{3b\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2x\sqrt[4]{a-bx^2}}{3b} + \frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.02, size = 64, normalized size = 0.79

$$\frac{2x\left(a\left(1-\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)-a+bx^2\right)}{3b\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^(3/4),x]
[Out] (2*x*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2,
 (b*x^2)/a]))/(3*b*(a - b*x^2)^(3/4))
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}x^2}{bx^2-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(1/4)*x^2/(b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(x^2/(-b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^(3/4),x)
[Out] int(x^2/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(x^2/(-b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a-b\,x^2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2)^(3/4),x)
[Out] int(x^2/(a - b*x^2)^(3/4), x)
```

sympy [C] time = 0.84, size = 29, normalized size = 0.36

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**(3/4),x)
[Out] x**3*hyper((3/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(3/4))
```

3.839 $\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

```
[Out] 2*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
csin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/
2))*a^(1/2)/(-b*x^2+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {233, 232}

$$\frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-3/4),x]
```

```
[Out] (2*Sqrt[a]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx &= \frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{\left(a-bx^2\right)^{3/4}}\\ &= \frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a-bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.81

$$\frac{x\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)}{\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-3/4),x]
[Out] (x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])/(a -
b*x^2)^(3/4)
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{bx^2-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(1/4)/(b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(-3/4), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(3/4),x)
[Out] int(1/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(-3/4), x)
```

mupad [B] time = 4.91, size = 38, normalized size = 0.66

$$\frac{x\,{\left(1-\frac{b\,x^2}{a}\right)}^{3/4}\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{{\left(a-b\,x^2\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(3/4),x)
```

[Out] (x*(1 - (b*x^2)/a)^(3/4)*hypergeom([1/2, 3/4], 3/2, (b*x^2)/a))/(a - b*x^2)
^(3/4)

sympy [C] time = 0.81, size = 26, normalized size = 0.45

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x**2+a)**(3/4),x)

[Out] x*hyper((1/2, 3/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(3/4)

3.840 $\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}$$

```
[Out] -(-b*x^2+a)^(1/4)/a/x+(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(
1/2)/a^(1/2))),2^(1/2))*b^(1/2)/(-b*x^2+a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {325, 233, 232}

$$\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^(3/4)),x]
```

```
[Out] -((a - b*x^2)^(1/4)/(a*x)) + (Sqrt[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSi
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^2}}{ax} + \frac{b\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{2a}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{ax} + \frac{\left(b\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{2a\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{ax} + \frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.64

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};\frac{bx^2}{a}\right)}{x\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^(3/4)),x]
[Out] -(((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (b*x^2)/a])/(x*(
a - b*x^2)^(3/4)))
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{bx^4-ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(1/4)/(b*x^4 - a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{3}{4}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(1/x^2/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^2), x)
```

mupad [B] time = 5.10, size = 41, normalized size = 0.53

$$-\frac{2\left(1-\frac{a}{b\,x^2}\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ \frac{a}{b\,x^2}\right)}{5\,x\,\left(a-b\,x^2\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^(3/4)),x)
```

```
[Out] -(2*(1 - a/(b*x^2))^(3/4)*hypergeom([3/4, 5/4], 9/4, a/(b*x^2)))/(5*x*(a -
b*x^2)^(3/4))
```

sympy [C] time = 0.98, size = 29, normalized size = 0.37

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{3}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((-1/2, 3/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/(a**(3/4)*x)
```

3.841 $\int \frac{1}{x^4\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=106

$$\frac{5b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{6a^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{5b\sqrt[4]{a-bx^2}}{6a^2x}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}$$

```
[Out] -1/3*(-b*x^2+a)^(1/4)/a/x^3-5/6*b*(-b*x^2+a)^(1/4)/a^2/x+5/6*b^(3/2)*(1-b*x
^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b
^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3
/2)/(-b*x^2+a)^(3/4)
```

Rubi [A] time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {325, 233, 232}

$$\frac{5b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6a^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{5b\sqrt[4]{a-bx^2}}{6a^2x}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a - b*x^2)^(3/4)),x]
```

```
[Out] -(a - b*x^2)^(1/4)/(3*a*x^3) - (5*b*(a - b*x^2)^(1/4))/(6*a^2*x) + (5*b^(3/
2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*a^
(3/2)*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a-bx^2\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^2}}{3ax^3}+\frac{(5b)\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}\,dx}{6a}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{3ax^3}-\frac{5b\sqrt[4]{a-bx^2}}{6a^2x}+\frac{\left(5b^2\right)\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{12a^2}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{3ax^3}-\frac{5b\sqrt[4]{a-bx^2}}{6a^2x}+\frac{\left(5b^2\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{12a^2\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^2}}{3ax^3}-\frac{5b\sqrt[4]{a-bx^2}}{6a^2x}+\frac{5b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6a^{3/2}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.49

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3x^3\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a - b*x^2)^(3/4)),x]
```

```
[Out] -1/3*((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (b*x^2)/a])/
(x^3*(a - b*x^2)^(3/4))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{bx^6-ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)/(b*x^6 - a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-b*x^2+a)^(3/4),x)
[Out] int(1/x^4/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\left(a-b\,x^2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a - b*x^2)^(3/4)),x)
[Out] int(1/(x^4*(a - b*x^2)^(3/4)), x)
```

sympy [C] time = 1.10, size = 34, normalized size = 0.32

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},\frac{3}{4}\\-\frac{1}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{3}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-b*x**2+a)**(3/4),x)
[Out] -hyper((-3/2, 3/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(3/4)*x**3)
```

3.842 $\int \frac{1}{x^6\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=131

$$\frac{3b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{4a^{5/2}\left(a-bx^2\right)^{3/4}}-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}$$

```
[Out] -1/5*(-b*x^2+a)^(1/4)/a/x^5-3/10*b*(-b*x^2+a)^(1/4)/a^2/x^3-3/4*b^2*(-b*x^2
+a)^(1/4)/a^3/x+3/4*b^(5/2)*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin
(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(-b*x^2+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {325, 233, 232}

$$-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}+\frac{3b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\left(a-bx^2\right)^{3/4}}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a - b*x^2)^(3/4)),x]
```

```
[Out] -(a - b*x^2)^(1/4)/(5*a*x^5) - (3*b*(a - b*x^2)^(1/4))/(10*a^2*x^3) - (3*b^
2*(a - b*x^2)^(1/4))/(4*a^3*x) + (3*b^(5/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(4*a^(5/2)*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6 \left(a-bx^2\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{a-bx^2}}{5ax^5}+\frac{(9b)\int \frac{1}{x^4\left(a-bx^2\right)^{3/4}}\,dx}{10a}\\
&= -\frac{\sqrt[4]{a-bx^2}}{5ax^5}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}+\frac{\left(3b^2\right)\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}\,dx}{4a^2}\\
&= -\frac{\sqrt[4]{a-bx^2}}{5ax^5}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}+\frac{\left(3b^3\right)\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{8a^3}\\
&= -\frac{\sqrt[4]{a-bx^2}}{5ax^5}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}+\frac{\left(3b^3\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{8a^3\left(a-bx^2\right)^{3/4}}\\
&= -\frac{\sqrt[4]{a-bx^2}}{5ax^5}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}+\frac{3b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\left(a-bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.40

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5x^5\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a - b*x^2)^(3/4)),x]
```

```
[Out] -1/5*((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, (b*x^2)/a])/
(x^5*(a - b*x^2)^(3/4))
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{bx^8-ax^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)/(b*x^8 - a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^6), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{3}{4}} x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-b*x^2+a)^(3/4),x)
[Out] int(1/x^6/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^{2}+a\right)^{\frac{3}{4}} x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{6}\left(a-b\,x^{2}\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a - b*x^2)^(3/4)),x)
[Out] int(1/(x^6*(a - b*x^2)^(3/4)), x)
```

sympy [C] time = 1.25, size = 34, normalized size = 0.26

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a}\right)}{5a^{\frac{3}{4}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-b*x**2+a)**(3/4),x)
[Out] -hyper((-5/2, 3/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(3/4)*x**5)
```

3.843 $\int \frac{x^6}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=124

$$-\frac{16a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^2}}+\frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}}-\frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}}+\frac{2x^5}{9b\sqrt[4]{a+bx^2}}$$

```
[Out] 8/3*a^2*x/b^3/(b*x^2+a)^(1/4)-4/9*a*x^3/b^2/(b*x^2+a)^(1/4)+2/9*x^5/b/(b*x^
2+a)^(1/4)-16/3*a^(5/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)
))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b
^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = $\{285, 197, 196\}$

$$\frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}}-\frac{16a^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^2}}-\frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}}+\frac{2x^5}{9b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(5/4),x]
```

```
[Out] (8*a^2*x)/(3*b^3*(a + b*x^2)^(1/4)) - (4*a*x^3)/(9*b^2*(a + b*x^2)^(1/4)) +
 (2*x^5)/(9*b*(a + b*x^2)^(1/4)) - (16*a^(5/2)*(1 + (b*x^2)/a)^(1/4)*Ellipt
icE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(7/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rubi steps

$$\int \frac{x^6}{(a+bx^2)^{5/4}}\,dx = \frac{2x^5}{9b\sqrt[4]{a+bx^2}} - \frac{(10a)\int \frac{x^4}{(a+bx^2)^{5/4}}\,dx}{9b}$$

$$= -\frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}} + \frac{(4a^2)\int \frac{x^2}{(a+bx^2)^{5/4}}\,dx}{3b^2}$$

$$= \frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}} - \frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}} - \frac{(8a^3)\int \frac{1}{(a+bx^2)^{5/4}}\,dx}{3b^3}$$

$$= \frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}} - \frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}} - \frac{\left(8a^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{3b^3\sqrt[4]{a+bx^2}}$$

$$= \frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}} - \frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}} - \frac{16a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.03, size = 78, normalized size = 0.63

$$\frac{2\left(12a^2x\sqrt[4]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-12a^2x-2abx^3+b^2x^5\right)}{9b^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*(-12*a^2*x - 2*a*b*x^3 + b^2*x^5 + 12*a^2*x*(1 + (b*x^2)/a)^(1/4)*Hyperg
eometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(9*b^3*(a + b*x^2)^(1/4))
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^2+a)^{\frac{3}{4}}x^6}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(b\,x^{2}+a\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(5/4),x)
[Out] int(x^6/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(bx^{2}+a\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(x^6/(b*x^2 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{{\left(b\,x^2+a\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(5/4),x)
[Out] int(x^6/(a + b*x^2)^(5/4), x)
```

sympy [C] time = 0.97, size = 27, normalized size = 0.22

$$\frac{x^{7} {{}_{2}F_{1}\left(\begin{matrix} \frac{5}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{7 a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(5/4),x)
[Out] x**7*hyper((5/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(5/4))
```

3.844 $\int \frac{x^4}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=100

$$\frac{24a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^2}}-\frac{12ax}{5b^2\sqrt[4]{a+bx^2}}+\frac{2x^3}{5b\sqrt[4]{a+bx^2}}$$

```
[Out] -12/5*a*x/b^2/(b*x^2+a)^(1/4)+2/5*x^3/b/(b*x^2+a)^(1/4)+24/5*a^(3/2)*(1+b*x
^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b
^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5
/2)/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {285, 197, 196}

$$\frac{24a^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^2}}-\frac{12ax}{5b^2\sqrt[4]{a+bx^2}}+\frac{2x^3}{5b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(5/4),x]
```

```
[Out] (-12*a*x)/(5*b^2*(a + b*x^2)^(1/4)) + (2*x^3)/(5*b*(a + b*x^2)^(1/4)) + (24
*a^(3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/(5*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{2x^3}{5b\sqrt[4]{a+bx^2}} - \frac{(6a)\int \frac{x^2}{\left(a+bx^2\right)^{5/4}}\,dx}{5b}$$

$$= -\frac{12ax}{5b^2\sqrt[4]{a+bx^2}} + \frac{2x^3}{5b\sqrt[4]{a+bx^2}} + \frac{\left(12a^2\right)\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx}{5b^2}$$

$$= -\frac{12ax}{5b^2\sqrt[4]{a+bx^2}} + \frac{2x^3}{5b\sqrt[4]{a+bx^2}} + \frac{\left(12a\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{5b^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{12ax}{5b^2\sqrt[4]{a+bx^2}} + \frac{2x^3}{5b\sqrt[4]{a+bx^2}} + \frac{24a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.02, size = 65, normalized size = 0.65

$$\frac{2\left(-6ax\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+6ax+bx^3\right)}{5b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(5/4),x]
[Out] (2*(6*a*x + b*x^3 - 6*a*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2,
 3/2, -((b*x^2)/a)]))/(5*b^2*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}x^4}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(x^4/(b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(5/4),x)
```

```
[Out] int(x^4/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{(b\,x^2+a)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(5/4),x)
```

```
[Out] int(x^4/(a + b*x^2)^(5/4), x)
```

sympy [C] time = 0.93, size = 27, normalized size = 0.27

$$\frac{x^5 {}_2F_1\left(\left.\begin{matrix}\frac{5}{4}, \frac{5}{2}\\ \frac{7}{2}\end{matrix}\right| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(5/4),x)
```

```
[Out] x**5*hyper((5/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/4))
```

3.845 $\int \frac{x^2}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=74

$$\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{4\sqrt{a}\,\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 2*x/b/(b*x^2+a)^(1/4)-4*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)
))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b
^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/b^(3/2)/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {285, 197, 196}

$$\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{4\sqrt{a}\,\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*x)/(b*(a + b*x^2)^(1/4)) - (4*Sqrt[a]*(1 + (b*x^2)/a)^(1/4)*EllipticE[Ar
cTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2)^{5/4}}\,dx = \frac{2x}{b\sqrt[4]{a+bx^2}} - \frac{(2a)\int \frac{1}{(a+bx^2)^{5/4}}\,dx}{b}$$
$$= \frac{2x}{b\sqrt[4]{a+bx^2}} - \frac{\left(2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{b\sqrt[4]{a+bx^2}}$$
$$= \frac{2x}{b\sqrt[4]{a+bx^2}} - \frac{4\sqrt{a}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.02, size = 53, normalized size = 0.72

$$\frac{2x\left(\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-1\right)}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*x*(-1 + (1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)
/a)]))/(b*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}x^2}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(5/4),x)
```

```
[Out] int(x^2/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(x^2/(b*x^2 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(5/4),x)
[Out] int(x^2/(a + b*x^2)^(5/4), x)
```

sympy [C] time = 0.92, size = 27, normalized size = 0.36

$$\frac{x^3 {}_2F_1\left(\left.\begin{matrix}\frac{5}{4}, \frac{3}{2}\\ \frac{5}{2}\end{matrix}\right| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(5/4),x)
[Out] x**3*hyper((5/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/4))
```

3.846 $\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=56

$$\frac{2\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

```
[Out] 2*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
ctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/
2))/(b*x^2+a)^(1/4)/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{197, 196\}$

$$\frac{2\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-5/4),x]
```

```
[Out] (2*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt
[a]*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx &= \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{a\sqrt[4]{a+bx^2}}\\
&= \frac{2\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.98

$$\frac{2x-x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{a\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-5/4),x]
```

```
[Out] (2*x - x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a
)])/(a*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-5/4), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-5/4), x)
```

mupad [B] time = 4.92, size = 37, normalized size = 0.66

$$\frac{x\left(\frac{b\,x^2}{a}+1\right)^{5/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(5/4),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(5/4)*hypergeom([1/2, 5/4], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(5/4)
```

sympy [C] time = 0.89, size = 24, normalized size = 0.43

$$\frac{x {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{5}{4} \\ \frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/4),x)
```

```
[Out] x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/4)
```

3.847 $\int \frac{1}{x^2(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=76

$$-\frac{3\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^2}}-\frac{1}{ax\sqrt[4]{a+bx^2}}$$

```
[Out] -1/a/x/(b*x^2+a)^(1/4)-3*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2
)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*
b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/a^(3/2)/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {286, 197, 196}

$$-\frac{3\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^2}}-\frac{1}{ax\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(5/4)),x]
```

```
[Out] -(1/(a*x*(a + b*x^2)^(1/4))) - (3*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[A
rcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^{5/4}}\,dx = -\frac{1}{ax\sqrt[4]{a+bx^2}} - \frac{(3b)\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx}{2a}$$

$$= -\frac{1}{ax\sqrt[4]{a+bx^2}} - \frac{\left(3b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{2a^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{1}{ax\sqrt[4]{a+bx^2}} - \frac{3\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.68

$$-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{2},\frac{5}{4};\frac{1}{2};-\frac{bx^2}{a}\right)}{ax\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(5/4)),x]
[Out] -(((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, -((b*x^2)/a)])/(
a*x*(a + b*x^2)^(1/4)))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{b^2x^6+2\,abx^4+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^2), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{5}{4}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^2), x)
```

mupad [B] time = 5.16, size = 40, normalized size = 0.53

$$-\frac{2\left(\frac{a}{b\,x^2}+1\right)^{5/4}\,{}_2\mathrm{F}_1\left(\frac{5}{4},\frac{7}{4};\ \frac{11}{4};\ -\frac{a}{b\,x^2}\right)}{7\,x\,\left(b\,x^2+a\right)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(5/4)),x)
```

```
[Out] -(2*(a/(b*x^2) + 1)^(5/4)*hypergeom([5/4, 7/4], 11/4, -a/(b*x^2)))/(7*x*(a
+ b*x^2)^(5/4))
```

sympy [C] time = 1.07, size = 27, normalized size = 0.36

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(5/4),x)
```

```
[Out] -hyper((-1/2, 5/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(5/4)*x)
```

3.848 $\int \frac{1}{x^4\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=102

$$\frac{7b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a+bx^2}}+\frac{7b}{6a^2x\sqrt[4]{a+bx^2}}-\frac{1}{3ax^3\sqrt[4]{a+bx^2}}$$

```
[Out] -1/3/a/x^3/(b*x^2+a)^(1/4)+7/6*b/a^2/x/(b*x^2+a)^(1/4)+7/2*b^(3/2)*(1+b*x^2
/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(
1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2
)/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.03, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {286, 197, 196}

$$\frac{7b^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a+bx^2}}+\frac{7b}{6a^2x\sqrt[4]{a+bx^2}}-\frac{1}{3ax^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(5/4)),x]
```

```
[Out] -1/(3*a*x^3*(a + b*x^2)^(1/4)) + (7*b)/(6*a^2*x*(a + b*x^2)^(1/4)) + (7*b^(
3/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*
a^(5/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{5/4}}\,dx = -\frac{1}{3ax^3\sqrt[4]{a+bx^2}} - \frac{(7b)\int \frac{1}{x^2\left(a+bx^2\right)^{5/4}}\,dx}{6a}$$

$$= -\frac{1}{3ax^3\sqrt[4]{a+bx^2}} + \frac{7b}{6a^2x\sqrt[4]{a+bx^2}} + \frac{\left(7b^2\right)\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx}{4a^2}$$

$$= -\frac{1}{3ax^3\sqrt[4]{a+bx^2}} + \frac{7b}{6a^2x\sqrt[4]{a+bx^2}} + \frac{\left(7b^2\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{4a^3\sqrt[4]{a+bx^2}}$$

$$= -\frac{1}{3ax^3\sqrt[4]{a+bx^2}} + \frac{7b}{6a^2x\sqrt[4]{a+bx^2}} + \frac{7b^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.53

$$-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{3}{2},\frac{5}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3ax^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(5/4)),x]
[Out] -1/3*((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, -((b*x^2)/a)
])/(a*x^3*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{b^2x^8+2\,abx^6+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^4), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{5}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/x^4/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^2+a)^{\frac{5}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,(b\,x^2+a)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(5/4)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)^(5/4)), x)
```

sympy [C] time = 1.23, size = 32, normalized size = 0.31

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{5}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(5/4),x)
```

```
[Out] -hyper((-3/2, 5/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/4)*x**3)
```

3.849 $\int \frac{1}{x^6\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=126

$$-\frac{77b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a+bx^2}}-\frac{77b^2}{60a^3x\sqrt[4]{a+bx^2}}+\frac{11b}{30a^2x^3\sqrt[4]{a+bx^2}}-\frac{1}{5ax^5\sqrt[4]{a+bx^2}}$$

```
[Out] -1/5/a/x^5/(b*x^2+a)^(1/4)+11/30*b/a^2/x^3/(b*x^2+a)^(1/4)-77/60*b^2/a^3/x/
(b*x^2+a)^(1/4)-77/20*b^(5/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arct
an(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.04, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {286, 197, 196}

$$-\frac{77b^2}{60a^3x\sqrt[4]{a+bx^2}}-\frac{77b^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a+bx^2}}+\frac{11b}{30a^2x^3\sqrt[4]{a+bx^2}}-\frac{1}{5ax^5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(5/4)),x]
```

```
[Out] -1/(5*a*x^5*(a + b*x^2)^(1/4)) + (11*b)/(30*a^2*x^3*(a + b*x^2)^(1/4)) - (7
7*b^2)/(60*a^3*x*(a + b*x^2)^(1/4)) - (77*b^(5/2)*(1 + (b*x^2)/a)^(1/4)*Ell
ipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(7/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^2\right)^{5/4}}\,dx = -\frac{1}{5ax^5\sqrt[4]{a+bx^2}} - \frac{(11b)\int \frac{1}{x^4\left(a+bx^2\right)^{5/4}}\,dx}{10a}$$

$$= -\frac{1}{5ax^5\sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3\sqrt[4]{a+bx^2}} + \frac{\left(77b^2\right)\int \frac{1}{x^2\left(a+bx^2\right)^{5/4}}\,dx}{60a^2}$$

$$= -\frac{1}{5ax^5\sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3\sqrt[4]{a+bx^2}} - \frac{77b^2}{60a^3x\sqrt[4]{a+bx^2}} - \frac{\left(77b^3\right)\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx}{40a^3}$$

$$= -\frac{1}{5ax^5\sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3\sqrt[4]{a+bx^2}} - \frac{77b^2}{60a^3x\sqrt[4]{a+bx^2}} - \frac{\left(77b^3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{40a^4\sqrt[4]{a+bx^2}}$$

$$= -\frac{1}{5ax^5\sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3\sqrt[4]{a+bx^2}} - \frac{77b^2}{60a^3x\sqrt[4]{a+bx^2}} - \frac{77b^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.\right.}{20a^{7/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.43

$$-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{2},\frac{5}{4};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5ax^5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(5/4)),x]
```

```
[Out] -1/5*((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 5/4, -3/2, -((b*x^2)/a)
])/(a*x^5*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{b^2x^{10}+2\,abx^8+a^2x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)/(b^2*x^10 + 2*a*b*x^8 + a^2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^6), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{5}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(b\,x^2+a\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^(5/4)),x)
```

```
[Out] int(1/(x^6*(a + b*x^2)^(5/4)), x)
```

sympy [C] time = 1.40, size = 32, normalized size = 0.25

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{5}{4} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{5a^{\frac{5}{4}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(5/4),x)
```

```
[Out] -hyper((-5/2, 5/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/4)*x**5)
```

3.850 $\int \frac{x^6}{\left(a-bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=124

$$-\frac{16a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a-bx^2}}+\frac{8ax\left(a-bx^2\right)^{3/4}}{3b^3}+\frac{20x^3\left(a-bx^2\right)^{3/4}}{9b^2}+\frac{2x^5}{b\sqrt[4]{a-bx^2}}$$

```
[Out] 2*x^5/b/(-b*x^2+a)^(1/4)+8/3*a*x*(-b*x^2+a)^(3/4)/b^3+20/9*x^3*(-b*x^2+a)^(
3/4)/b^2-16/3*a^(5/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(
1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {288, 321, 229, 228}

$$-\frac{16a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a-bx^2}}+\frac{20x^3\left(a-bx^2\right)^{3/4}}{9b^2}+\frac{8ax\left(a-bx^2\right)^{3/4}}{3b^3}+\frac{2x^5}{b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a - b*x^2)^(5/4),x]
```

```
[Out] (2*x^5)/(b*(a - b*x^2)^(1/4)) + (8*a*x*(a - b*x^2)^(3/4))/(3*b^3) + (20*x^3
*(a - b*x^2)^(3/4))/(9*b^2) - (16*a^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[A
rcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(7/2)*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6}{\left(a-bx^2\right)^{5/4}}\,dx &= \frac{2x^5}{b\sqrt[4]{a-bx^2}}-\frac{10\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx}{b}\\
&= \frac{2x^5}{b\sqrt[4]{a-bx^2}}+\frac{20x^3\left(a-bx^2\right)^{3/4}}{9b^2}-\frac{(20a)\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx}{3b^2}\\
&= \frac{2x^5}{b\sqrt[4]{a-bx^2}}+\frac{8ax\left(a-bx^2\right)^{3/4}}{3b^3}+\frac{20x^3\left(a-bx^2\right)^{3/4}}{9b^2}-\frac{\left(8a^2\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{3b^3}\\
&= \frac{2x^5}{b\sqrt[4]{a-bx^2}}+\frac{8ax\left(a-bx^2\right)^{3/4}}{3b^3}+\frac{20x^3\left(a-bx^2\right)^{3/4}}{9b^2}-\frac{\left(8a^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{3b^3\sqrt[4]{a-bx^2}}\\
&= \frac{2x^5}{b\sqrt[4]{a-bx^2}}+\frac{8ax\left(a-bx^2\right)^{3/4}}{3b^3}+\frac{20x^3\left(a-bx^2\right)^{3/4}}{9b^2}-\frac{16a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a-bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 78, normalized size = 0.63

$$
-\frac{2x\left(12a^2\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-12a^2+2abx^2+b^2x^4\right)}{9b^3\sqrt[4]{a-bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a - b*x^2)^(5/4),x]
```

```
[Out] (-2*x*(-12*a^2 + 2*a*b*x^2 + b^2*x^4 + 12*a^2*(1 - (b*x^2)/a)^(1/4)*Hyperge
ometric2F1[1/4, 1/2, 3/2, (b*x^2)/a]))/(9*b^3*(a - b*x^2)^(1/4))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}x^6}{b^2x^4-2\,abx^2+a^2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)*x^6/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^6}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$
\int \frac{x^6}{\left(-b\,x^2+a\right)^{\frac{5}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-b*x^2+a)^(5/4),x)
```

```
[Out] int(x^6/(-b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(a-b\,x^2\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a - b*x^2)^(5/4),x)
```

```
[Out] int(x^6/(a - b*x^2)^(5/4), x)
```

sympy [C] time = 0.99, size = 29, normalized size = 0.23

$$\frac{x^7\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{7a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-b*x**2+a)**(5/4),x)
```

```
[Out] x**7*hyper((5/4, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(5/4))
```

3.851 $\int \frac{x^4}{\left(a-bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=101

$$-\frac{24a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a-bx^2}}+\frac{12x\left(a-bx^2\right)^{3/4}}{5b^2}+\frac{2x^3}{b\sqrt[4]{a-bx^2}}$$

```
[Out] 2*x^3/b/(-b*x^2+a)^(1/4)+12/5*x*(-b*x^2+a)^(3/4)/b^2-24/5*a^(3/2)*(1-b*x^2/
a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1
/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)
/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {288, 321, 229, 228}

$$-\frac{24a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a-bx^2}}+\frac{12x\left(a-bx^2\right)^{3/4}}{5b^2}+\frac{2x^3}{b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^2)^(5/4),x]
```

```
[Out] (2*x^3)/(b*(a - b*x^2)^(1/4)) + (12*x*(a - b*x^2)^(3/4))/(5*b^2) - (24*a^(3
/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b
^(5/2)*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\left(a-bx^2\right)^{5/4}}\,dx &= \frac{2x^3}{b\sqrt[4]{a-bx^2}} - \frac{6\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx}{b} \\
&= \frac{2x^3}{b\sqrt[4]{a-bx^2}} + \frac{12x\left(a-bx^2\right)^{3/4}}{5b^2} - \frac{(12a)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{5b^2} \\
&= \frac{2x^3}{b\sqrt[4]{a-bx^2}} + \frac{12x\left(a-bx^2\right)^{3/4}}{5b^2} - \frac{\left(12a\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{5b^2\sqrt[4]{a-bx^2}} \\
&= \frac{2x^3}{b\sqrt[4]{a-bx^2}} + \frac{12x\left(a-bx^2\right)^{3/4}}{5b^2} - \frac{24a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a-bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.65

$$
-\frac{2\left(6ax\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-6ax+bx^3\right)}{5b^2\sqrt[4]{a-bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^2)^(5/4),x]
```

```
[Out] (-2*(-6*a*x + b*x^3 + 6*a*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/
2, 3/2, (b*x^2)/a]))/(5*b^2*(a - b*x^2)^(1/4))
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}x^4}{b^2x^4-2\,abx^2+a^2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)*x^4/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^4}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$
\int \frac{x^4}{\left(-b\,x^2+a\right)^{\frac{5}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-b*x^2+a)^(5/4),x)
```

```
[Out] int(x^4/(-b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-b*x^2 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(a-b\,x^2\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b*x^2)^(5/4),x)
```

```
[Out] int(x^4/(a - b*x^2)^(5/4), x)
```

sympy [C] time = 0.95, size = 29, normalized size = 0.29

$$\frac{x^5\,{}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{5a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-b*x**2+a)**(5/4),x)
```

```
[Out] x**5*hyper((5/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(5/4))
```

3.852 $\int \frac{x^2}{(a-bx^2)^{5/4}}\,dx$

Optimal. Leaf size=77

$$\frac{2x}{b\sqrt[4]{a-bx^2}}-\frac{4\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a-bx^2}}$$

```
[Out] 2*x/b/(-b*x^2+a)^(1/4)-4*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2
)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*
b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/b^(3/2)/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {288, 229, 228}

$$\frac{2x}{b\sqrt[4]{a-bx^2}}-\frac{4\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^(5/4),x]
```

```
[Out] (2*x)/(b*(a - b*x^2)^(1/4)) - (4*Sqrt[a]*(1 - (b*x^2)/a)^(1/4)*EllipticE[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{(a-bx^2)^{5/4}}\,dx = \frac{2x}{b\sqrt[4]{a-bx^2}} - \frac{2\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{b}$$
$$= \frac{2x}{b\sqrt[4]{a-bx^2}} - \frac{\left(2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{b\sqrt[4]{a-bx^2}}$$
$$= \frac{2x}{b\sqrt[4]{a-bx^2}} - \frac{4\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.73

$$\frac{2x-2x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^(5/4),x]
[Out] (2*x - 2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a
])/(b*(a - b*x^2)^(1/4))
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}x^2}{b^2x^4-2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)*x^2/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(x^2/(-b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^(5/4),x)
```

[Out] int(x^2/(-b*x^2+a)^(5/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-b*x^2+a)^(5/4),x, algorithm="maxima")

[Out] integrate(x^2/(-b*x^2 + a)^(5/4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a-b\,x^2)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a - b*x^2)^(5/4),x)

[Out] int(x^2/(a - b*x^2)^(5/4), x)

sympy [C] time = 0.93, size = 29, normalized size = 0.38

$$\frac{x^3 {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{3a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(-b*x**2+a)**(5/4),x)

[Out] x**3*hyper((5/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(5/4))

3.853 $\int \frac{1}{(a-bx^2)^{5/4}}\,dx$

Optimal. Leaf size=77

$$\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{2\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

```
[Out] 2*x/a/(-b*x^2+a)^(1/4)-2*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2
)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*
b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+a)^(1/4)/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {199, 229, 228}

$$\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{2\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-5/4),x]
```

```
[Out] (2*x)/(a*(a - b*x^2)^(1/4)) - (2*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*Sqrt[b]*(a - b*x^2)^(1/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^{5/4}}\,dx = \frac{2x}{a\sqrt[4]{a-bx^2}} - \frac{\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{a}$$
$$= \frac{2x}{a\sqrt[4]{a-bx^2}} - \frac{\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{a\sqrt[4]{a-bx^2}}$$
$$= \frac{2x}{a\sqrt[4]{a-bx^2}} - \frac{2\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.73

$$\frac{2x - x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{a\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-5/4),x]
[Out] (2*x - x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])
/(a*(a - b*x^2)^(1/4))
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{b^2x^4-2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(-5/4), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(5/4),x)
```

```
[Out] int(1/(-b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2 + a\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(-5/4), x)
```

mupad [B] time = 4.91, size = 38, normalized size = 0.49

$$\frac{x\left(1-\frac{b\,x^2}{a}\right)^{5/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(a-b\,x^2\right)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(5/4),x)
```

```
[Out] (x*(1 - (b*x^2)/a)^(5/4)*hypergeom([1/2, 5/4], 3/2, (b*x^2)/a))/(a - b*x^2)
^(5/4)
```

sympy [C] time = 0.89, size = 26, normalized size = 0.34

$$\frac{x {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, \frac{5}{4} \\ \frac{3}{2} \end{matrix}\middle| {\frac{bx^{2} e^{2 i \pi}}{a}} \right)}{a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(5/4),x)
```

```
[Out] x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(5/4)
```

3.854 $\int \frac{1}{x^2\left(a-bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=99

$$-\frac{3\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a-bx^2}}-\frac{3\left(a-bx^2\right)^{3/4}}{a^2x}+\frac{2}{ax\sqrt[4]{a-bx^2}}$$

```
[Out] 2/a/x/(-b*x^2+a)^(1/4)-3*(-b*x^2+a)^(3/4)/a^2/x-3*(1-b*x^2/a)^(1/4)*(cos(1/
2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*El
lipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/a^(3/2)/(-b*x^2
+a)^(1/4)
```

Rubi [A] time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {290, 325, 229, 228}

$$-\frac{3\left(a-bx^2\right)^{3/4}}{a^2x}-\frac{3\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a-bx^2}}+\frac{2}{ax\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*x*(a - b*x^2)^(1/4)) - (3*(a - b*x^2)^(3/4))/(a^2*x) - (3*Sqrt[b]*(1 -
 (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*(a
- b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a-bx^2\right)^{5/4}}\,dx &= \frac{2}{ax\sqrt[4]{a-bx^2}}+\frac{3\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{a}\\
&= \frac{2}{ax\sqrt[4]{a-bx^2}}-\frac{3\left(a-bx^2\right)^{3/4}}{a^2x}-\frac{(3b)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{2a^2}\\
&= \frac{2}{ax\sqrt[4]{a-bx^2}}-\frac{3\left(a-bx^2\right)^{3/4}}{a^2x}-\frac{\left(3b\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{2a^2\sqrt[4]{a-bx^2}}\\
&= \frac{2}{ax\sqrt[4]{a-bx^2}}-\frac{3\left(a-bx^2\right)^{3/4}}{a^2x}-\frac{3\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a-bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 53, normalized size = 0.54

$$
-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{2},\frac{5}{4};\frac{1}{2};\frac{bx^2}{a}\right)}{ax\sqrt[4]{a-bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^(5/4)),x]
[Out] -(((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, (b*x^2)/a])/(a*x
*(a - b*x^2)^(1/4)))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{b^2x^6-2\,abx^4+a^2x^2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)/(b^2*x^6 - 2*a*b*x^4 + a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\left(-bx^2+a\right)^{\frac{5}{4}}x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^2), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$
\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{5}{4}}\,x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^(5/4),x)
```

```
[Out] int(1/x^2/(-b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{5}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^2), x)
```

mupad [B] time = 5.10, size = 41, normalized size = 0.41

$$-\frac{2\left(1-\frac{a}{b\,x^2}\right)^{5/4}\,{}_2\mathrm{F}_1\left(\frac{5}{4},\frac{7}{4};\ \frac{11}{4};\ \frac{a}{b\,x^2}\right)}{7\,x\,\left(a-b\,x^2\right)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^(5/4)),x)
```

```
[Out] -(2*(1 - a/(b*x^2))^(5/4)*hypergeom([5/4, 7/4], 11/4, a/(b*x^2)))/(7*x*(a -
b*x^2)^(5/4))
```

sympy [C] time = 1.09, size = 29, normalized size = 0.29

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{5}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**(5/4),x)
```

```
[Out] -hyper((-1/2, 5/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/(a**(5/4)*x)
```

3.855 $\int \frac{1}{x^4 (a-bx^2)^{5/4}} \, dx$

Optimal. Leaf size=126

$$-\frac{7b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a-bx^2}}-\frac{7b\left(a-bx^2\right)^{3/4}}{2a^3x}-\frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3}+\frac{2}{ax^3\sqrt[4]{a-bx^2}}$$

```
[Out] 2/a/x^3/(-b*x^2+a)^(1/4)-7/3*(-b*x^2+a)^(3/4)/a^2/x^3-7/2*b*(-b*x^2+a)^(3/4
)/a^3/x-7/2*b^(3/2)*(1-b*x^2/a)^(1/4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2
)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/
2)/a^(1/2))),2^(1/2))/a^(5/2)/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {290, 325, 229, 228}

$$-\frac{7b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a-bx^2}}-\frac{7b\left(a-bx^2\right)^{3/4}}{2a^3x}-\frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3}+\frac{2}{ax^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a - b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*x^3*(a - b*x^2)^(1/4)) - (7*(a - b*x^2)^(3/4))/(3*a^2*x^3) - (7*b*(a -
 b*x^2)^(3/4))/(2*a^3*x) - (7*b^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSi
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(5/2)*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a-bx^2\right)^{5/4}}\,dx = \frac{2}{ax^3\sqrt[4]{a-bx^2}} + \frac{7\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx}{a}$$

$$= \frac{2}{ax^3\sqrt[4]{a-bx^2}} - \frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3} + \frac{(7b)\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{2a^2}$$

$$= \frac{2}{ax^3\sqrt[4]{a-bx^2}} - \frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3} - \frac{7b\left(a-bx^2\right)^{3/4}}{2a^3x} - \frac{\left(7b^2\right)\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{4a^3}$$

$$= \frac{2}{ax^3\sqrt[4]{a-bx^2}} - \frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3} - \frac{7b\left(a-bx^2\right)^{3/4}}{2a^3x} - \frac{\left(7b^2\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{4a^3\sqrt[4]{a-bx^2}}$$

$$= \frac{2}{ax^3\sqrt[4]{a-bx^2}} - \frac{7\left(a-bx^2\right)^{3/4}}{3a^2x^3} - \frac{7b\left(a-bx^2\right)^{3/4}}{2a^3x} - \frac{7b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.44

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{3}{2},\frac{5}{4};-\frac{1}{2};\frac{bx^2}{a}\right)}{3ax^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a - b*x^2)^(5/4)),x]
[Out] -1/3*((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, (b*x^2)/a])/
(a*x^3*(a - b*x^2)^(1/4))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{b^2x^8-2\,abx^6+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(3/4)/(b^2*x^8 - 2*a*b*x^6 + a^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{5}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^4), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{5}{4}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-b*x^2+a)^(5/4),x)
[Out] int(1/x^4/(-b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2 + a\right)^{\frac{5}{4}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 \left(a - b\, x^2\right)^{5/4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a - b*x^2)^(5/4)),x)
[Out] int(1/(x^4*(a - b*x^2)^(5/4)), x)
```

sympy [C] time = 1.25, size = 34, normalized size = 0.27

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{5}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{3a^{\frac{5}{4}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-b*x**2+a)**(5/4),x)
[Out] -hyper((-3/2, 5/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(5/4)*x**3)
```

3.856 $\int \frac{1}{x^6\left(a-bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=151

$$-\frac{77b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a-bx^2}}-\frac{77b^2\left(a-bx^2\right)^{3/4}}{20a^4x}-\frac{77b\left(a-bx^2\right)^{3/4}}{30a^3x^3}-\frac{11\left(a-bx^2\right)^{3/4}}{5a^2x^5}+\frac{2}{ax^5\sqrt[4]{a-bx^2}}$$

```
[Out] 2/a/x^5/(-b*x^2+a)^(1/4)-11/5*(-b*x^2+a)^(3/4)/a^2/x^5-77/30*b*(-b*x^2+a)^(
3/4)/a^3/x^3-77/20*b^2*(-b*x^2+a)^(3/4)/a^4/x-77/20*b^(5/2)*(1-b*x^2/a)^(1/
4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^
(1/2)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/(-b*x
^2+a)^(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {290, 325, 229, 228}

$$-\frac{77b^2\left(a-bx^2\right)^{3/4}}{20a^4x}-\frac{77b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a-bx^2}}-\frac{77b\left(a-bx^2\right)^{3/4}}{30a^3x^3}-\frac{11\left(a-bx^2\right)^{3/4}}{5a^2x^5}+\frac{2}{ax^5\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a - b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*x^5*(a - b*x^2)^(1/4)) - (11*(a - b*x^2)^(3/4))/(5*a^2*x^5) - (77*b*(a
 - b*x^2)^(3/4))/(30*a^3*x^3) - (77*b^2*(a - b*x^2)^(3/4))/(20*a^4*x) - (77
*b^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/(20*a^(7/2)*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\left(a-bx^2\right)^{5/4}}\,dx = \frac{2}{ax^5\sqrt[4]{a-bx^2}} + \frac{11\int \frac{1}{x^6\sqrt[4]{a-bx^2}}\,dx}{a}$$

$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}} - \frac{11\left(a-bx^2\right)^{3/4}}{5a^2x^5} + \frac{(77b)\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx}{10a^2}$$

$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}} - \frac{11\left(a-bx^2\right)^{3/4}}{5a^2x^5} - \frac{77b\left(a-bx^2\right)^{3/4}}{30a^3x^3} + \frac{\left(77b^2\right)\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx}{20a^3}$$

$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}} - \frac{11\left(a-bx^2\right)^{3/4}}{5a^2x^5} - \frac{77b\left(a-bx^2\right)^{3/4}}{30a^3x^3} - \frac{77b^2\left(a-bx^2\right)^{3/4}}{20a^4x} - \frac{\left(77b^3\right)\int \frac{1}{\sqrt[4]{a-b}}}{40a^4}$$

$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}} - \frac{11\left(a-bx^2\right)^{3/4}}{5a^2x^5} - \frac{77b\left(a-bx^2\right)^{3/4}}{30a^3x^3} - \frac{77b^2\left(a-bx^2\right)^{3/4}}{20a^4x} - \frac{\left(77b^3\sqrt[4]{1-\frac{bx}{a}}\right.}{40a^4\sqrt[4]{}}$$

$$= \frac{2}{ax^5\sqrt[4]{a-bx^2}} - \frac{11\left(a-bx^2\right)^{3/4}}{5a^2x^5} - \frac{77b\left(a-bx^2\right)^{3/4}}{30a^3x^3} - \frac{77b^2\left(a-bx^2\right)^{3/4}}{20a^4x} - \frac{77b^{5/2}\sqrt[4]{1-\frac{b}{a}}}{20a}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.36

$$-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{5}{2},\frac{5}{4};-\frac{3}{2};\frac{bx^2}{a}\right)}{5ax^5\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a - b*x^2)^(5/4)),x]
```

```
[Out] -1/5*((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 5/4, -3/2, (b*x^2)/a])/
(a*x^5*(a - b*x^2)^(1/4))
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}}{b^2x^{10}-2\,abx^8+a^2x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(3/4)/(b^2*x^10 - 2*a*b*x^8 + a^2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{5}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^6), x)

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{5}{4}} x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(-b*x^2+a)^(5/4),x)

[Out] int(1/x^6/(-b*x^2+a)^(5/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^{2}+a\right)^{\frac{5}{4}} x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(-b*x^2+a)^(5/4),x, algorithm="maxima")

[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^6), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{6}\left(a-b\,x^{2}\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6*(a - b*x^2)^(5/4)),x)

[Out] int(1/(x^6*(a - b*x^2)^(5/4)), x)

sympy [C] time = 1.44, size = 34, normalized size = 0.23

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{5}{4} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{bx^{2}e^{2i\pi}}{a}} \right)}{5a^{\frac{5}{4}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**6/(-b*x**2+a)**(5/4),x)

[Out] -hyper((-5/2, 5/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(5/4)*x**5)

3.857 $\int \frac{1}{\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=78

$$\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2x}{3a\left(a+bx^2\right)^{3/4}}$$

```
[Out] 2/3*x/a/(b*x^2+a)^(3/4)+2/3*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan
(x*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^2+a)^(3/4)/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {199, 233, 231}

$$\frac{2x}{3a\left(a+bx^2\right)^{3/4}}+\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-7/4),x]
```

```
[Out] (2*x)/(3*a*(a + b*x^2)^(3/4)) + (2*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a + b*x^2)^(3/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{7/4}}\,dx=\frac{2x}{3a\left(a+bx^2\right)^{3/4}}+\frac{\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{3a}$$
$$=\frac{2x}{3a\left(a+bx^2\right)^{3/4}}+\frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{3a\left(a+bx^2\right)^{3/4}}$$
$$=\frac{2x}{3a\left(a+bx^2\right)^{3/4}}+\frac{2\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,\sqrt{b}\,\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.71

$$\frac{x\left(\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)+2\right)}{3a\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-7/4),x]
[Out] (x*(2 + (1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)
]))/(3*a*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-7/4), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(7/4),x)
```

```
[Out] int(1/(b*x^2+a)^(7/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-7/4), x)
```

mupad [B] time = 4.88, size = 37, normalized size = 0.47

$$\frac{x\left(\frac{b\,x^2}{a}+1\right)^{7/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(7/4),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(7/4)*hypergeom([1/2, 7/4], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(7/4)
```

sympy [C] time = 1.04, size = 24, normalized size = 0.31

$$\frac{x{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(7/4),x)
```

```
[Out] x*hyper((1/2, 7/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/4)
```

3.858 $\int \frac{1}{(a+bx^2)^{9/4}}\,dx$

Optimal. Leaf size=78

$$\frac{6\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\,\sqrt[4]{a+bx^2}}+\frac{2x}{5a\left(a+bx^2\right)^{5/4}}$$

```
[Out] 2/5*x/a/(b*x^2+a)^(5/4)+6/5*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan
(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^2+a)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {199, 197, 196}

$$\frac{6\sqrt[4]{\frac{bx^2}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\,\sqrt[4]{a+bx^2}}+\frac{2x}{5a\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-9/4),x]
```

```
[Out] (2*x)/(5*a*(a + b*x^2)^(5/4)) + (6*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{9/4}}\,dx = \frac{2x}{5a\left(a+bx^2\right)^{5/4}} + \frac{3\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx}{5a}$$

$$= \frac{2x}{5a\left(a+bx^2\right)^{5/4}} + \frac{\left(3\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{5a^2\sqrt[4]{a+bx^2}}$$

$$= \frac{2x}{5a\left(a+bx^2\right)^{5/4}} + \frac{6\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.03, size = 72, normalized size = 0.92

$$\frac{-3x\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+8ax+6bx^3}{5a^2\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-9/4),x]
[Out] (8*a*x + 6*b*x^3 - 3*x*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[
1/4, 1/2, 3/2, -((b*x^2)/a)])/(5*a^2*(a + b*x^2)^(5/4))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-9/4), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(9/4),x)
```

```
[Out] int(1/(b*x^2+a)^(9/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-9/4), x)
```

mupad [B] time = 4.88, size = 37, normalized size = 0.47

$$\frac{x\left(\frac{b\,x^2}{a}+1\right)^{9/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(9/4),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(9/4)*hypergeom([1/2, 9/4], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(9/4)
```

sympy [C] time = 1.36, size = 24, normalized size = 0.31

$$\frac{x_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(9/4),x)
```

```
[Out] x*hyper((1/2, 9/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(9/4)
```

3.859 $\int \frac{1}{\left(a+bx^2\right)^{11/4}}\,dx$

Optimal. Leaf size=97

$$\frac{10\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{21a^{3/2}\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{10x}{21a^2\left(a+bx^2\right)^{3/4}}+\frac{2x}{7a\left(a+bx^2\right)^{7/4}}$$

```
[Out] 2/7*x/a/(b*x^2+a)^(7/4)+10/21*x/a^2/(b*x^2+a)^(3/4)+10/21*(1+b*x^2/a)^(3/4)
*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1
/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^2+
a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {199, 233, 231}

$$\frac{10x}{21a^2\left(a+bx^2\right)^{3/4}}+\frac{10\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2x}{7a\left(a+bx^2\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-11/4),x]
```

```
[Out] (2*x)/(7*a*(a + b*x^2)^(7/4)) + (10*x)/(21*a^2*(a + b*x^2)^(3/4)) + (10*(1
+ (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)
*Sqrt[b]*(a + b*x^2)^(3/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{11/4}}\,dx = \frac{2x}{7a\left(a+bx^2\right)^{7/4}} + \frac{5\int \frac{1}{\left(a+bx^2\right)^{7/4}}\,dx}{7a}$$

$$= \frac{2x}{7a\left(a+bx^2\right)^{7/4}} + \frac{10x}{21a^2\left(a+bx^2\right)^{3/4}} + \frac{5\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{21a^2}$$

$$= \frac{2x}{7a\left(a+bx^2\right)^{7/4}} + \frac{10x}{21a^2\left(a+bx^2\right)^{3/4}} + \frac{\left(5\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{21a^2\left(a+bx^2\right)^{3/4}}$$

$$= \frac{2x}{7a\left(a+bx^2\right)^{7/4}} + \frac{10x}{21a^2\left(a+bx^2\right)^{3/4}} + \frac{10\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.04, size = 75, normalized size = 0.77

$$\frac{5x\left(a+bx^2\right)\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^2}{a}\right)+2x\left(8a+5bx^2\right)}{21a^2\left(a+bx^2\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-11/4),x]
[Out] (2*x*(8*a + 5*b*x^2) + 5*x*(a + b*x^2)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric
2F1[1/2, 3/4, 3/2, -((b*x^2)/a)])/(21*a^2*(a + b*x^2)^(7/4))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-11/4), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(11/4),x)
[Out] int(1/(b*x^2+a)^(11/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(11/4),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(-11/4), x)
```

mupad [B] time = 4.89, size = 37, normalized size = 0.38

$$\frac{x\left(\frac{b\,x^2}{a}+1\right)^{11/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(11/4),x)
[Out] (x*((b*x^2)/a + 1)^(11/4)*hypergeom([1/2, 11/4], 3/2, -(b*x^2)/a))/(a + b*x
^2)^(11/4)
```

sympy [C] time = 1.81, size = 24, normalized size = 0.25

$$\frac{x{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(11/4),x)
[Out] x*hyper((1/2, 11/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(11/4)
```

3.860 $\int \frac{1}{\left(a-bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=81

$$\frac{2\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\,\sqrt{b}\,\left(a-bx^2\right)^{3/4}}+\frac{2x}{3a\left(a-bx^2\right)^{3/4}}$$

```
[Out] 2/3*x/a/(-b*x^2+a)^(3/4)+2/3*(1-b*x^2/a)^(3/4)*(cos(1/2*arcsin(x*b^(1/2)/a^
(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsi
n(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+a)^(3/4)/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {199, 233, 232}

$$\frac{2x}{3a\left(a-bx^2\right)^{3/4}}+\frac{2\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,\sqrt{b}\,\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-7/4),x]
```

```
[Out] (2*x)/(3*a*(a - b*x^2)^(3/4)) + (2*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(
Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^{7/4}}\,dx = \frac{2x}{3a\left(a-bx^2\right)^{3/4}}+\frac{\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{3a}$$

$$= \frac{2x}{3a\left(a-bx^2\right)^{3/4}}+\frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{3a\left(a-bx^2\right)^{3/4}}$$

$$= \frac{2x}{3a\left(a-bx^2\right)^{3/4}}+\frac{2\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.69

$$\frac{x\left(\left(1-\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)+2\right)}{3a\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-7/4),x]
[Out] (x*(2 + (1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a]))
/(3*a*(a - b*x^2)^(3/4))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{b^2x^4-2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/4),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(1/4)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/4),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(-7/4), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(7/4),x)
```

```
[Out] int(1/(-b*x^2+a)^(7/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(-7/4), x)
```

mupad [B] time = 4.87, size = 38, normalized size = 0.47

$$\frac{x\left(1-\frac{b\,x^2}{a}\right)^{7/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(a-b\,x^2\right)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(7/4),x)
```

```
[Out] (x*(1 - (b*x^2)/a)^(7/4)*hypergeom([1/2, 7/4], 3/2, (b*x^2)/a))/(a - b*x^2)
^(7/4)
```

sympy [C] time = 1.06, size = 26, normalized size = 0.32

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(7/4),x)
```

```
[Out] x*hyper((1/2, 7/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(7/4)
```

3.861 $\int \frac{1}{\left(a-bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=101

$$-\frac{6\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a-bx^2}}+\frac{6x}{5a^2\sqrt[4]{a-bx^2}}+\frac{2x}{5a\left(a-bx^2\right)^{5/4}}$$

```
[Out] 2/5*x/a/(-b*x^2+a)^(5/4)+6/5*x/a^2/(-b*x^2+a)^(1/4)-6/5*(1-b*x^2/a)^(1/4)*(
cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^(1/2
)))*EllipticE(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(-b*x^2+a
)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {199, 229, 228}

$$\frac{6x}{5a^2\sqrt[4]{a-bx^2}}-\frac{6\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a-bx^2}}+\frac{2x}{5a\left(a-bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-9/4),x]
```

```
[Out] (2*x)/(5*a*(a - b*x^2)^(5/4)) + (6*x)/(5*a^2*(a - b*x^2)^(1/4)) - (6*(1 - (
b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*Sqr
t[b]*(a - b*x^2)^(1/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{(a-bx^2)^{9/4}}\,dx = \frac{2x}{5a(a-bx^2)^{5/4}} + \frac{3\int \frac{1}{(a-bx^2)^{5/4}}\,dx}{5a}$$
$$= \frac{2x}{5a(a-bx^2)^{5/4}} + \frac{6x}{5a^2\sqrt[4]{a-bx^2}} - \frac{3\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx}{5a^2}$$
$$= \frac{2x}{5a(a-bx^2)^{5/4}} + \frac{6x}{5a^2\sqrt[4]{a-bx^2}} - \frac{\left(3\sqrt[4]{1-\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx}{5a^2\sqrt[4]{a-bx^2}}$$
$$= \frac{2x}{5a(a-bx^2)^{5/4}} + \frac{6x}{5a^2\sqrt[4]{a-bx^2}} - \frac{6\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\,\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.04, size = 74, normalized size = 0.73

$$\frac{-3x(a-bx^2)\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)+8ax-6bx^3}{5a^2(a-bx^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-9/4),x]
```

```
[Out] (8*a*x - 6*b*x^3 - 3*x*(a - b*x^2)*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[
1/4, 1/2, 3/2, (b*x^2)/a])/(5*a^2*(a - b*x^2)^(5/4))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(-bx^2+a)^{\frac{3}{4}}}{b^3x^6-3\,ab^2x^4+3\,a^2bx^2-a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-bx^2+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(-9/4), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{(-b\,x^2+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(9/4),x)
```

```
[Out] int(1/(-b*x^2+a)^(9/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(-9/4), x)
```

mupad [B] time = 4.86, size = 38, normalized size = 0.38

$$\frac{x\left(1-\frac{b\,x^2}{a}\right)^{9/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(a-b\,x^2\right)^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(9/4),x)
```

```
[Out] (x*(1 - (b*x^2)/a)^(9/4)*hypergeom([1/2, 9/4], 3/2, (b*x^2)/a))/(a - b*x^2)
^(9/4)
```

sympy [C] time = 1.36, size = 26, normalized size = 0.26

$$\frac{x{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(9/4),x)
```

```
[Out] x*hyper((1/2, 9/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(9/4)
```

3.862 $\int \frac{1}{(a-bx^2)^{11/4}}\,dx$

Optimal. Leaf size=101

$$\frac{10\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{21a^{3/2}\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{10x}{21a^2\left(a-bx^2\right)^{3/4}}+\frac{2x}{7a\left(a-bx^2\right)^{7/4}}$$

```
[Out] 2/7*x/a/(-b*x^2+a)^(7/4)+10/21*x/a^2/(-b*x^2+a)^(3/4)+10/21*(1-b*x^2/a)^(3/
4)*(cos(1/2*arcsin(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x*b^(1/2)/a^
(1/2)))*EllipticF(sin(1/2*arcsin(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(-b*x
^2+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {199, 233, 232}

$$\frac{10x}{21a^2\left(a-bx^2\right)^{3/4}}+\frac{10\left(1-\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{2x}{7a\left(a-bx^2\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-11/4),x]
```

```
[Out] (2*x)/(7*a*(a - b*x^2)^(7/4)) + (10*x)/(21*a^2*(a - b*x^2)^(3/4)) + (10*(1
- (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)
*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^{11/4}}\,dx = \frac{2x}{7a\left(a-bx^2\right)^{7/4}} + \frac{5\int \frac{1}{\left(a-bx^2\right)^{7/4}}\,dx}{7a}$$

$$= \frac{2x}{7a\left(a-bx^2\right)^{7/4}} + \frac{10x}{21a^2\left(a-bx^2\right)^{3/4}} + \frac{5\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx}{21a^2}$$

$$= \frac{2x}{7a\left(a-bx^2\right)^{7/4}} + \frac{10x}{21a^2\left(a-bx^2\right)^{3/4}} + \frac{\left(5\left(1-\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx}{21a^2\left(a-bx^2\right)^{3/4}}$$

$$= \frac{2x}{7a\left(a-bx^2\right)^{7/4}} + \frac{10x}{21a^2\left(a-bx^2\right)^{3/4}} + \frac{10\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.04, size = 77, normalized size = 0.76

$$\frac{5x\left(a-bx^2\right)\left(1-\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^2}{a}\right)+2x\left(8a-5bx^2\right)}{21a^2\left(a-bx^2\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-11/4),x]
```

```
[Out] (2*x*(8*a - 5*b*x^2) + 5*x*(a - b*x^2)*(1 - (b*x^2)/a)^(3/4)*Hypergeometric
2F1[1/2, 3/4, 3/2, (b*x^2)/a])/(21*a^2*(a - b*x^2)^(7/4))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{b^3x^6-3\,ab^2x^4+3\,a^2bx^2-a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(11/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(11/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(-11/4), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(11/4),x)
```

```
[Out] int(1/(-b*x^2+a)^(11/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(11/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(-11/4), x)
```

mupad [B] time = 4.81, size = 38, normalized size = 0.38

$$\frac{x\left(1-\frac{b\,x^2}{a}\right)^{11/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(a-b\,x^2\right)^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(11/4),x)
```

```
[Out] (x*(1 - (b*x^2)/a)^(11/4)*hypergeom([1/2, 11/4], 3/2, (b*x^2)/a))/(a - b*x^
2)^(11/4)
```

sympy [C] time = 1.84, size = 26, normalized size = 0.26

$$\frac{x{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\\frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(11/4),x)
```

```
[Out] x*hyper((1/2, 11/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(11/4)
```

3.863 $\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=99

$$\frac{32\left(3x^2+2\right)^{3/4}x}{1053}-\frac{128x}{1053\sqrt[4]{3x^2+2}}+\frac{2}{39}\left(3x^2+2\right)^{3/4}x^5-\frac{40\left(3x^2+2\right)^{3/4}x^3}{1053}+\frac{128\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{1053\sqrt{3}}$$

```
[Out] -128/1053*x/(3*x^2+2)^(1/4)+32/1053*x*(3*x^2+2)^(3/4)-40/1053*x^3*(3*x^2+2)
^(3/4)+2/39*x^5*(3*x^2+2)^(3/4)+128/3159*2^(1/4)*(cos(1/2*arctan(1/2*x*6^(1
/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arctan(1/2*
x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 227, 196}

$$\frac{2}{39}\left(3x^2+2\right)^{3/4}x^5-\frac{40\left(3x^2+2\right)^{3/4}x^3}{1053}+\frac{32\left(3x^2+2\right)^{3/4}x}{1053}-\frac{128x}{1053\sqrt[4]{3x^2+2}}+\frac{128\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{1053\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 + 3*x^2)^(1/4),x]
```

```
[Out] (-128*x)/(1053*(2 + 3*x^2)^(1/4)) + (32*x*(2 + 3*x^2)^(3/4))/1053 - (40*x^3
*(2 + 3*x^2)^(3/4))/1053 + (2*x^5*(2 + 3*x^2)^(3/4))/39 + (128*2^(1/4)*Elli
pticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(1053*Sqrt[3])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx = \frac{2}{39}x^5\left(2+3x^2\right)^{3/4} - \frac{20}{39}\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx$$
$$= -\frac{40x^3\left(2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(2+3x^2\right)^{3/4} + \frac{80}{351}\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx$$
$$= \frac{32x\left(2+3x^2\right)^{3/4}}{1053} - \frac{40x^3\left(2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(2+3x^2\right)^{3/4} - \frac{64\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx}{1053}$$
$$= -\frac{128x}{1053\sqrt[4]{2+3x^2}} + \frac{32x\left(2+3x^2\right)^{3/4}}{1053} - \frac{40x^3\left(2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(2+3x^2\right)^{3/4} + \frac{128\int \frac{1}{\left(2+3x^2\right)^{5/}}}{1053}$$
$$= -\frac{128x}{1053\sqrt[4]{2+3x^2}} + \frac{32x\left(2+3x^2\right)^{3/4}}{1053} - \frac{40x^3\left(2+3x^2\right)^{3/4}}{1053} + \frac{2}{39}x^5\left(2+3x^2\right)^{3/4} + \frac{128\sqrt[4]{2}\,E\left(\frac{1}{2}\,\mathrm{ta}\right.}{105}$$

Mathematica [C] time = 0.03, size = 54, normalized size = 0.55

$$\frac{2x\left(\left(3x^2+2\right)^{3/4}\left(27x^4-20x^2+16\right)-16\,2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)\right)}{1053}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 + 3*x^2)^(1/4),x]
[Out] (2*x*((2 + 3*x^2)^(3/4)*(16 - 20*x^2 + 27*x^4) - 16*2^(3/4)*Hypergeometric2
F1[1/4, 1/2, 3/2, (-3*x^2)/2]))/1053
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^6}{\left(3\,x^2+2\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(1/4),x, algorithm="fricas")
[Out] integral(x^6/(3*x^2 + 2)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate(x^6/(3*x^2 + 2)^(1/4), x)
```

maple [C] time = 0.30, size = 43, normalized size = 0.43

$$-\frac{32\,2^{\frac{3}{4}}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{1053} + \frac{2\left(27x^4-20x^2+16\right)\left(3x^2+2\right)^{\frac{3}{4}}x}{1053}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2+2)^(1/4),x)
```

```
[Out] 2/1053*x*(27*x^4-20*x^2+16)*(3*x^2+2)^(3/4)-32/1053*2^(3/4)*x*hypergeom([1/
4,1/2],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(3*x^2 + 2)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{(3\,x^2+2)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2 + 2)^(1/4),x)
```

```
[Out] int(x^6/(3*x^2 + 2)^(1/4), x)
```

sympy [C] time = 0.83, size = 27, normalized size = 0.27

$$\frac{2^{\frac{3}{4}} x^7 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{3x^2 e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**7*hyper((1/4, 7/2), (9/2,), 3*x**2*exp_polar(I*pi)/2)/14
```

3.864 $\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=81

$$-\frac{8}{135}\left(3x^2+2\right)^{3/4}x+\frac{32x}{135\sqrt[4]{3x^2+2}}+\frac{2}{27}\left(3x^2+2\right)^{3/4}x^3-\frac{32\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

```
[Out] 32/135*x/(3*x^2+2)^(1/4)-8/135*x*(3*x^2+2)^(3/4)+2/27*x^3*(3*x^2+2)^(3/4)-3
2/405*2^(1/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x
*6^(1/2)))*EllipticE(sin(1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 227, 196}

$$\frac{2}{27}\left(3x^2+2\right)^{3/4}x^3-\frac{8}{135}\left(3x^2+2\right)^{3/4}x+\frac{32x}{135\sqrt[4]{3x^2+2}}-\frac{32\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 + 3*x^2)^(1/4),x]
```

```
[Out] (32*x)/(135*(2 + 3*x^2)^(1/4)) - (8*x*(2 + 3*x^2)^(3/4))/135 + (2*x^3*(2 +
3*x^2)^(3/4))/27 - (32*2^(1/4)*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(135*Sq
rt[3])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx = \frac{2}{27}x^3\left(2+3x^2\right)^{3/4} - \frac{4}{9}\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx$$
$$= -\frac{8}{135}x\left(2+3x^2\right)^{3/4} + \frac{2}{27}x^3\left(2+3x^2\right)^{3/4} + \frac{16}{135}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx$$
$$= \frac{32x}{135\sqrt[4]{2+3x^2}} - \frac{8}{135}x\left(2+3x^2\right)^{3/4} + \frac{2}{27}x^3\left(2+3x^2\right)^{3/4} - \frac{32}{135}\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx$$
$$= \frac{32x}{135\sqrt[4]{2+3x^2}} - \frac{8}{135}x\left(2+3x^2\right)^{3/4} + \frac{2}{27}x^3\left(2+3x^2\right)^{3/4} - \frac{32\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.60

$$\frac{2}{135}x\left(4\,2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)+\left(3x^2+2\right)^{3/4}\left(5x^2-4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*((2 + 3*x^2)^(3/4)*(-4 + 5*x^2) + 4*2^(3/4)*Hypergeometric2F1[1/4, 1/2
, 3/2, (-3*x^2)/2]))/135
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\left(3\,x^2+2\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(3*x^2 + 2)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(3*x^2 + 2)^(1/4), x)
```

maple [C] time = 0.27, size = 38, normalized size = 0.47

$$\frac{8\,2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{135}+\frac{2\left(5x^2-4\right)\left(3x^2+2\right)^{\frac{3}{4}}x}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2+2)^(1/4),x)
```

```
[Out] 2/135*x*(5*x^2-4)*(3*x^2+2)^(3/4)+8/135*2^(3/4)*x*hypergeom([1/4,1/2],[3/2]
,-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(3*x^2 + 2)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(3\,x^2+2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2 + 2)^(1/4),x)
```

```
[Out] int(x^4/(3*x^2 + 2)^(1/4), x)
```

sympy [C] time = 0.75, size = 27, normalized size = 0.33

$$\frac{2^{\frac{3}{4}} x^5 {}_2F_1\left(\left.\begin{matrix} \frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\right| \frac{3x^2 e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**5*hyper((1/4, 5/2), (7/2,), 3*x**2*exp_polar(I*pi)/2)/10
```

3.865 $\int \frac{x^2}{\sqrt[4]{2+3x^2}} \, dx$

Optimal. Leaf size=63

$$\frac{2}{15}\left(3x^2+2\right)^{3/4}x-\frac{8x}{15\sqrt[4]{3x^2+2}}+\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}$$

```
[Out] -8/15*x/(3*x^2+2)^(1/4)+2/15*x*(3*x^2+2)^(3/4)+8/45*2^(1/4)*(cos(1/2*arctan
(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticE(sin(1/2*
arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 227, 196}

$$\frac{2}{15}\left(3x^2+2\right)^{3/4}x-\frac{8x}{15\sqrt[4]{3x^2+2}}+\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + 3*x^2)^(1/4),x]
```

```
[Out] (-8*x)/(15*(2 + 3*x^2)^(1/4)) + (2*x*(2 + 3*x^2)^(3/4))/15 + (8*2^(1/4)*Ell
ipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(15*Sqrt[3])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx&=\frac{2}{15}x\left(2+3x^2\right)^{3/4}-\frac{4}{15}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx\\&=-\frac{8x}{15\sqrt[4]{2+3x^2}}+\frac{2}{15}x\left(2+3x^2\right)^{3/4}+\frac{8}{15}\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx\\&=-\frac{8x}{15\sqrt[4]{2+3x^2}}+\frac{2}{15}x\left(2+3x^2\right)^{3/4}+\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.65

$$\frac{2}{15}x\left(\left(3x^2+2\right)^{3/4}-2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*((2 + 3*x^2)^(3/4) - 2^(3/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2)
/2]))/15
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\left(3\,x^2+2\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^2/(3*x^2 + 2)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(3*x^2 + 2)^(1/4), x)
```

maple [C] time = 0.27, size = 31, normalized size = 0.49

$$-\frac{2\,2^{\frac{3}{4}}x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{15}+\frac{2\left(3x^2+2\right)^{\frac{3}{4}}x}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+2)^(1/4),x)
```

```
[Out] 2/15*x*(3*x^2+2)^(3/4)-2/15*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(3*x^2 + 2)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(3\,x^2+2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2 + 2)^(1/4),x)
```

```
[Out] int(x^2/(3*x^2 + 2)^(1/4), x)
```

sympy [C] time = 0.70, size = 27, normalized size = 0.43

$$\frac{2^{\frac{3}{4}} x^3 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{3x^2 e^{i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**3*hyper((1/4, 3/2), (5/2,), 3*x**2*exp_polar(I*pi)/2)/6
```

3.866 $\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=43

$$\frac{2x}{\sqrt[4]{3x^2+2}}-\frac{2\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

```
[Out] 2*x/(3*x^2+2)^(1/4)-2/3*2^(1/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)/co
s(1/2*arctan(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arctan(1/2*x*6^(1/2))),2^(1/
2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {227, 196}

$$\frac{2x}{\sqrt[4]{3x^2+2}}-\frac{2\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)^(-1/4),x]
```

```
[Out] (2*x)/(2 + 3*x^2)^(1/4) - (2*2^(1/4)*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/S
qrt[3]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx &= \frac{2x}{\sqrt[4]{2+3x^2}}-2\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx\\ &= \frac{2x}{\sqrt[4]{2+3x^2}}-\frac{2\sqrt[4]{2}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 24, normalized size = 0.56

$$\frac{x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)^(-1/4),x]
[Out] (x*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2)/2])/2^(1/4)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{\left(3\,x^2+2\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4),x, algorithm="fricas")
[Out] integral((3*x^2 + 2)^(-1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)^(-1/4), x)
```

maple [C] time = 0.27, size = 18, normalized size = 0.42

$$\frac{2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+2)^(1/4),x)
[Out] 1/2*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(1/4),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)^(-1/4), x)
```

mupad [B] time = 0.09, size = 16, normalized size = 0.37

$$\frac{8^{1/4}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{3\,x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + 2)^(1/4),x)
[Out] (8^(1/4)*x*hypergeom([1/4, 1/2], 3/2, -(3*x^2)/2))/2
```

sympy [C] time = 0.67, size = 26, normalized size = 0.60

$$\frac{2^{\frac{3}{4}} x {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{3x^{2}e^{i\pi}}{2}} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x*hyper((1/4, 1/2), (3/2,), 3*x**2*exp_polar(I*pi)/2)/2
```

3.867 $\int \frac{1}{x^2 \sqrt[4]{2+3x^2}}\, dx$

Optimal. Leaf size=63

$$\frac{3x}{2\sqrt[4]{3x^2+2}} - \frac{\left(3x^2+2\right)^{3/4}}{2x} - \frac{\sqrt{3}\, E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\, x\right)\middle|2\right)}{2^{3/4}}$$

```
[Out] 3/2*x/(3*x^2+2)^(1/4)-1/2*(3*x^2+2)^(3/4)/x-1/2*2^(1/4)*(cos(1/2*arctan(1/2
*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arct
an(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 227, 196}

$$\frac{3x}{2\sqrt[4]{3x^2+2}} - \frac{\left(3x^2+2\right)^{3/4}}{2x} - \frac{\sqrt{3}\, E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\, x\right)\middle|2\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] (3*x)/(2*(2 + 3*x^2)^(1/4)) - (2 + 3*x^2)^(3/4)/(2*x) - (Sqrt[3]*EllipticE[
ArcTan[Sqrt[3/2]*x]/2, 2])/2^(3/4)
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx = -\frac{\left(2+3x^2\right)^{3/4}}{2x} + \frac{3}{4}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx$$
$$= \frac{3x}{2\sqrt[4]{2+3x^2}} - \frac{\left(2+3x^2\right)^{3/4}}{2x} - \frac{3}{2}\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx$$
$$= \frac{3x}{2\sqrt[4]{2+3x^2}} - \frac{\left(2+3x^2\right)^{3/4}}{2x} - \frac{\sqrt{3}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2^{3/4}}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.43

$$-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-\frac{3x^2}{2}\right)}{\sqrt[4]{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 + 3*x^2)^(1/4)),x]
[Out] -(Hypergeometric2F1[-1/2, 1/4, 1/2, (-3*x^2)/2]/(2^(1/4)*x))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,x^2+2\right)^{\frac{3}{4}}}{3\,x^4+2\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(1/4),x, algorithm="fricas")
[Out] integral((3*x^2 + 2)^(3/4)/(3*x^4 + 2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^2), x)
```

maple [C] time = 0.27, size = 33, normalized size = 0.52

$$\frac{3\,2^{\frac{3}{4}}x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{8} - \frac{\left(3x^2+2\right)^{\frac{3}{4}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2+2)^(1/4),x)
[Out] -1/2*(3*x^2+2)^(3/4)/x+3/8*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^2), x)
```

mupad [B] time = 5.02, size = 36, normalized size = 0.57

$$-\frac{2\,3^{3/4}\,\left(\frac{2}{x^2}+3\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{2}{3\,x^2}\right)}{9\,x\,\left(3\,x^2+2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^2 + 2)^(1/4)),x)
```

```
[Out] -(2*3^(3/4)*(2/x^2 + 3)^(1/4)*hypergeom([1/4, 3/4], 7/4, -2/(3*x^2)))/(9*x*
(3*x^2 + 2)^(1/4))
```

sympy [C] time = 0.74, size = 29, normalized size = 0.46

$$-\frac{2^{\frac{3}{4}}\,{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\right|\frac{3x^{2}e^{i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2+2)**(1/4),x)
```

```
[Out] -2**(3/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)/(2*x)
```

3.868 $\int \frac{1}{x^4 \sqrt[4]{2+3x^2}}\,dx$

Optimal. Leaf size=83

$$-\frac{9x}{8\sqrt[4]{3x^2+2}}+\frac{3\left(3x^2+2\right)^{3/4}}{8x}-\frac{\left(3x^2+2\right)^{3/4}}{6x^3}+\frac{3\sqrt{3}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\,2^{3/4}}$$

```
[Out] -9/8*x/(3*x^2+2)^(1/4)-1/6*(3*x^2+2)^(3/4)/x^3+3/8*(3*x^2+2)^(3/4)/x+3/8*2^
(1/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)
))*EllipticE(sin(1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 227, 196}

$$-\frac{9x}{8\sqrt[4]{3x^2+2}}+\frac{3\left(3x^2+2\right)^{3/4}}{8x}-\frac{\left(3x^2+2\right)^{3/4}}{6x^3}+\frac{3\sqrt{3}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] (-9*x)/(8*(2 + 3*x^2)^(1/4)) - (2 + 3*x^2)^(3/4)/(6*x^3) + (3*(2 + 3*x^2)^(
3/4))/(8*x) + (3*Sqrt[3]*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(4*2^(3/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[4]{2+3x^2}}\,dx = -\frac{\left(2+3x^2\right)^{3/4}}{6x^3} - \frac{3}{4}\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx$$
$$= -\frac{\left(2+3x^2\right)^{3/4}}{6x^3} + \frac{3\left(2+3x^2\right)^{3/4}}{8x} - \frac{9}{16}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx$$
$$= -\frac{9x}{8\sqrt[4]{2+3x^2}} - \frac{\left(2+3x^2\right)^{3/4}}{6x^3} + \frac{3\left(2+3x^2\right)^{3/4}}{8x} + \frac{9}{8}\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx$$
$$= -\frac{9x}{8\sqrt[4]{2+3x^2}} - \frac{\left(2+3x^2\right)^{3/4}}{6x^3} + \frac{3\left(2+3x^2\right)^{3/4}}{8x} + \frac{3\sqrt{3}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\,2^{3/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.35

$$-\frac{{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};-\frac{3x^2}{2}\right)}{3\sqrt[4]{2}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 + 3*x^2)^(1/4)),x]
[Out] -1/3*Hypergeometric2F1[-3/2, 1/4, -1/2, (-3*x^2)/2]/(2^(1/4)*x^3)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(3\,x^2+2\right)^{\frac{3}{4}}}{3\,x^6+2\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(1/4),x, algorithm="fricas")
[Out] integral((3*x^2 + 2)^(3/4)/(3*x^6 + 2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^4), x)
```

maple [C] time = 0.27, size = 45, normalized size = 0.54

$$-\frac{9\,2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{32} + \frac{27x^4+6x^2-8}{24\left(3x^2+2\right)^{\frac{1}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2+2)^(1/4),x)
```

```
[Out] 1/24*(27*x^4+6*x^2-8)/x^3/(3*x^2+2)^(1/4)-9/32*2^(3/4)*x*hypergeom([1/4,1/2
],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(3\,x^2+2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(3*x^2 + 2)^(1/4)),x)
```

```
[Out] int(1/(x^4*(3*x^2 + 2)^(1/4)), x)
```

sympy [C] time = 0.84, size = 32, normalized size = 0.39

$$-\frac{2^{\frac{3}{4}}{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\,\frac{3x^2e^{i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2+2)**(1/4),x)
```

```
[Out] -2**(3/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2)/(6*x**3)
```

3.869 $\int \frac{1}{x^6 \sqrt[4]{2+3x^2}} \, dx$

Optimal. Leaf size=101

$$\frac{189x}{160\sqrt[4]{3x^2+2}} - \frac{63\left(3x^2+2\right)^{3/4}}{160x} - \frac{\left(3x^2+2\right)^{3/4}}{10x^5} + \frac{7\left(3x^2+2\right)^{3/4}}{40x^3} - \frac{63\sqrt{3}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{80\ 2^{3/4}}$$

```
[Out] 189/160*x/(3*x^2+2)^(1/4)-1/10*(3*x^2+2)^(3/4)/x^5+7/40*(3*x^2+2)^(3/4)/x^3
-63/160*(3*x^2+2)^(3/4)/x-63/160*2^(1/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)
^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arctan(1/2*x*6^(1/2
))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 227, 196}

$$\frac{189x}{160\sqrt[4]{3x^2+2}} - \frac{63\left(3x^2+2\right)^{3/4}}{160x} + \frac{7\left(3x^2+2\right)^{3/4}}{40x^3} - \frac{\left(3x^2+2\right)^{3/4}}{10x^5} - \frac{63\sqrt{3}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{80\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(2 + 3*x^2)^(1/4)),x]
```

```
[Out] (189*x)/(160*(2 + 3*x^2)^(1/4)) - (2 + 3*x^2)^(3/4)/(10*x^5) + (7*(2 + 3*x^
2)^(3/4))/(40*x^3) - (63*(2 + 3*x^2)^(3/4))/(160*x) - (63*Sqrt[3]*EllipticE
[ArcTan[Sqrt[3/2]*x]/2, 2])/(80*2^(3/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[4]{2+3x^2}}\,dx = -\frac{\left(2+3x^2\right)^{3/4}}{10x^5} - \frac{21}{20}\int \frac{1}{x^4\sqrt[4]{2+3x^2}}\,dx$$
$$= -\frac{\left(2+3x^2\right)^{3/4}}{10x^5} + \frac{7\left(2+3x^2\right)^{3/4}}{40x^3} + \frac{63}{80}\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx$$
$$= -\frac{\left(2+3x^2\right)^{3/4}}{10x^5} + \frac{7\left(2+3x^2\right)^{3/4}}{40x^3} - \frac{63\left(2+3x^2\right)^{3/4}}{160x} + \frac{189}{320}\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx$$
$$= \frac{189x}{160\sqrt[4]{2+3x^2}} - \frac{\left(2+3x^2\right)^{3/4}}{10x^5} + \frac{7\left(2+3x^2\right)^{3/4}}{40x^3} - \frac{63\left(2+3x^2\right)^{3/4}}{160x} - \frac{189}{160}\int \frac{1}{\left(2+3x^2\right)^{5/4}}\,dx$$
$$= \frac{189x}{160\sqrt[4]{2+3x^2}} - \frac{\left(2+3x^2\right)^{3/4}}{10x^5} + \frac{7\left(2+3x^2\right)^{3/4}}{40x^3} - \frac{63\left(2+3x^2\right)^{3/4}}{160x} - \frac{63\sqrt{3}\,E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\right)}{80\,2^{3/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.29

$$-\frac{{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};-\frac{3x^2}{2}\right)}{5\sqrt[4]{2}\,x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(2 + 3*x^2)^(1/4)),x]
[Out] -1/5*Hypergeometric2F1[-5/2, 1/4, -3/2, (-3*x^2)/2]/(2^(1/4)*x^5)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(3\,x^2+2\right)^{\frac{3}{4}}}{3\,x^8+2\,x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(1/4),x, algorithm="fricas")
[Out] integral((3*x^2 + 2)^(3/4)/(3*x^8 + 2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^6), x)
```

maple [C] time = 0.27, size = 50, normalized size = 0.50

$$\frac{189\,2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{640} - \frac{189x^6+42x^4-8x^2+32}{160\left(3x^2+2\right)^{\frac{1}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(3*x^2+2)^(1/4),x)
```

```
[Out] -1/160*(189*x^6+42*x^4-8*x^2+32)/x^5/(3*x^2+2)^(1/4)+189/640*2^(3/4)*x*hype
rgeom([1/4,1/2],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(3\,x^2+2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(3*x^2 + 2)^(1/4)),x)
```

```
[Out] int(1/(x^6*(3*x^2 + 2)^(1/4)), x)
```

sympy [C] time = 0.95, size = 32, normalized size = 0.32

$$-\frac{2^{\frac{3}{4}}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{3x^{2}e^{i\pi}}{2}} \right)}{10x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(3*x**2+2)**(1/4),x)
```

```
[Out] -2**(3/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2)/(10*x**5)
```

3.870 $\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=83

$$-\frac{32\left(2-3x^2\right)^{3/4}x}{1053}-\frac{2}{39}\left(2-3x^2\right)^{3/4}x^5-\frac{40\left(2-3x^2\right)^{3/4}x^3}{1053}+\frac{128\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{1053\sqrt{3}}$$

```
[Out] -32/1053*x*(-3*x^2+2)^(3/4)-40/1053*x^3*(-3*x^2+2)^(3/4)-2/39*x^5*(-3*x^2+2
)^(3/4)+128/3159*2^(1/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*a
rcsin(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(
1/2)
```

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{321, 228\}$

$$-\frac{2}{39}\left(2-3x^2\right)^{3/4}x^5-\frac{40\left(2-3x^2\right)^{3/4}x^3}{1053}-\frac{32\left(2-3x^2\right)^{3/4}x}{1053}+\frac{128\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{1053\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-32*x*(2 - 3*x^2)^(3/4))/1053 - (40*x^3*(2 - 3*x^2)^(3/4))/1053 - (2*x^5*(
2 - 3*x^2)^(3/4))/39 + (128*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(1
053*Sqrt[3])
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx &= -\frac{2}{39}x^5\left(2-3x^2\right)^{3/4}+\frac{20}{39}\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx\\
&= -\frac{40x^3\left(2-3x^2\right)^{3/4}}{1053}-\frac{2}{39}x^5\left(2-3x^2\right)^{3/4}+\frac{80}{351}\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx\\
&= -\frac{32x\left(2-3x^2\right)^{3/4}}{1053}-\frac{40x^3\left(2-3x^2\right)^{3/4}}{1053}-\frac{2}{39}x^5\left(2-3x^2\right)^{3/4}+\frac{64\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx}{1053}\\
&= -\frac{32x\left(2-3x^2\right)^{3/4}}{1053}-\frac{40x^3\left(2-3x^2\right)^{3/4}}{1053}-\frac{2}{39}x^5\left(2-3x^2\right)^{3/4}+\frac{128\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{1053\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 54, normalized size = 0.65

$$-\frac{2x\left(\left(2-3x^2\right)^{3/4}\left(27x^4+20x^2+16\right)-16\,2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\right)}{1053}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*((2 - 3*x^2)^(3/4)*(16 + 20*x^2 + 27*x^4) - 16*2^(3/4)*Hypergeometric
2F1[1/4, 1/2, 3/2, (3*x^2)/2]))/1053
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{3}{4}}x^6}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)*x^6/(3*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-3*x^2 + 2)^(1/4), x)
```

maple [C] time = 0.28, size = 50, normalized size = 0.60

$$\frac{32\,2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{1053}+\frac{2\left(27x^4+20x^2+16\right)\left(3x^2-2\right)x}{1053\left(-3x^2+2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-3*x^2+2)^(1/4),x)
```

```
[Out] 2/1053*x*(27*x^4+20*x^2+16)*(3*x^2-2)/(-3*x^2+2)^(1/4)+32/1053*2^(3/4)*x*hy
pergeom([1/4,1/2],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-3*x^2 + 2)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(2-3\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(2 - 3*x^2)^(1/4),x)
```

```
[Out] int(x^6/(2 - 3*x^2)^(1/4), x)
```

sympy [C] time = 0.85, size = 29, normalized size = 0.35

$$\frac{2^{\frac{3}{4}}x^7{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x***7*hyper((1/4, 7/2), (9/2,), 3*x**2*exp_polar(2*I*pi)/2)/14
```

3.871 $\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{8}{135}\left(2-3x^2\right)^{3/4}x-\frac{2}{27}\left(2-3x^2\right)^{3/4}x^3+\frac{32\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

```
[Out] -8/135*x*(-3*x^2+2)^(3/4)-2/27*x^3*(-3*x^2+2)^(3/4)+32/405*2^(1/4)*(cos(1/2
*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticE(s
in(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 228}

$$-\frac{2}{27}\left(2-3x^2\right)^{3/4}x^3-\frac{8}{135}\left(2-3x^2\right)^{3/4}x+\frac{32\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-8*x*(2 - 3*x^2)^(3/4))/135 - (2*x^3*(2 - 3*x^2)^(3/4))/27 + (32*2^(1/4)*E
llipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(135*Sqrt[3])
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx &= -\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}+\frac{4}{9}\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx\\
&= -\frac{8}{135}x\left(2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}+\frac{16}{135}\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx\\
&= -\frac{8}{135}x\left(2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}+\frac{32\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.75

$$-\frac{2}{135}x\left(\left(2-3x^2\right)^{3/4}\left(5x^2+4\right)-4\,2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*((2 - 3*x^2)^(3/4)*(4 + 5*x^2) - 4*2^(3/4)*Hypergeometric2F1[1/4, 1/2
, 3/2, (3*x^2)/2]))/135
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{3}{4}}x^4}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)*x^4/(3*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-3*x^2 + 2)^(1/4), x)
```

maple [C] time = 0.28, size = 45, normalized size = 0.69

$$\frac{8\,2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{135}+\frac{2\left(5x^2+4\right)\left(3x^2-2\right)x}{135\left(-3x^2+2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2+2)^(1/4),x)
```

```
[Out] 2/135*x*(5*x^2+4)*(3*x^2-2)/(-3*x^2+2)^(1/4)+8/135*2^(3/4)*x*hypergeom([1/4
,1/2],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-3*x^2 + 2)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{{\left(2-3\,x^2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2 - 3*x^2)^(1/4),x)
```

```
[Out] int(x^4/(2 - 3*x^2)^(1/4), x)
```

sympy [C] time = 0.77, size = 29, normalized size = 0.45

$$\frac{2^{\frac{3}{4}}x^5{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2+2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**5*hyper((1/4, 5/2), (7/2,), 3*x**2*exp_polar(2*I*pi)/2)/10
```

3.872 $\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=47

$$\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}-\frac{2}{15}x\left(2-3x^2\right)^{3/4}$$

```
[Out] -2/15*x*(-3*x^2+2)^(3/4)+8/45*2^(1/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1
/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2)))
,2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 228}

$$\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}-\frac{2}{15}x\left(2-3x^2\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*(2 - 3*x^2)^(3/4))/15 + (8*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2
])/(15*Sqrt[3])
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx &= -\frac{2}{15}x\left(2-3x^2\right)^{3/4}+\frac{4}{15}\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx\\ &= -\frac{2}{15}x\left(2-3x^2\right)^{3/4}+\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.87

$$-\frac{2}{15}x\left(\left(2-3x^2\right)^{3/4}-2^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*((2 - 3*x^2)^(3/4) - 2^(3/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2)
/2]))/15
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{3}{4}}x^2}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)*x^2/(3*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-3*x^2 + 2)^(1/4), x)
```

maple [C] time = 0.29, size = 38, normalized size = 0.81

$$\frac{2\,2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{15}+\frac{2\left(3x^2-2\right)x}{15\left(-3x^2+2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(1/4),x)
```

```
[Out] 2/15*x*(3*x^2-2)/(-3*x^2+2)^(1/4)+2/15*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],
3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-3*x^2 + 2)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{{\left(2-3\,x^2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2 - 3*x^2)^(1/4),x)
```

[Out] int(x^2/(2 - 3*x^2)^(1/4), x)

sympy [C] time = 0.72, size = 29, normalized size = 0.62

$$\frac{2^{\frac{3}{4}}x^3{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(-3*x**2+2)**(1/4),x)

[Out] 2**(3/4)*x**3*hyper((1/4, 3/2), (5/2,), 3*x**2*exp_polar(2*I*pi)/2)/6

3.873 $\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{2\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

```
[Out] 2/3*2^(1/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6
^(1/2)))*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {228}

$$\frac{2\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x^2)^(-1/4),x]
```

```
[Out] (2*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx = \frac{2\sqrt[4]{2}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

Mathematica [C] time = 0.00, size = 24, normalized size = 0.86

$$\frac{x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x^2)^(-1/4),x]
```

```
[Out] (x*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2)/2])/2^(1/4)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{3}{4}}}{3\,x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)/(3*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate((-3*x^2 + 2)^(-1/4), x)
```

maple [C] time = 0.28, size = 18, normalized size = 0.64

$$\frac{2^{\frac{3}{4}} x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], \frac{3 x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/4),x)
[Out] 1/2*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4),x, algorithm="maxima")
[Out] integrate((-3*x^2 + 2)^(-1/4), x)
```

mupad [B] time = 0.09, size = 16, normalized size = 0.57

$$\frac{2^{3/4}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{3\,x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^2)^(1/4),x)
[Out] (2^(3/4)*x*hypergeom([1/4, 1/2], 3/2, (3*x^2)/2))/2
```

sympy [C] time = 0.69, size = 27, normalized size = 0.96

$$\frac{2^{\frac{3}{4}} x {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{3 x^{2} e^{2 i \pi}}{2}} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/4),x)
[Out] 2**(3/4)*x*hyper((1/4, 1/2), (3/2,), 3*x**2*exp_polar(2*I*pi)/2)/2
```

3.874 $\int \frac{1}{x^2 \sqrt[4]{2-3x^2}} \, dx$

Optimal. Leaf size=47

$$-\frac{\left(2-3x^2\right)^{3/4}}{2x}-\frac{\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2^{3/4}}$$

```
[Out] -1/2*(-3*x^2+2)^(3/4)/x-1/2*2^(1/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2
)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2
^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 228}

$$-\frac{\left(2-3x^2\right)^{3/4}}{2x}-\frac{\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -(2 - 3*x^2)^(3/4)/(2*x) - (Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/2^
(3/4)
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \sqrt[4]{2-3x^2}} \, dx &= -\frac{\left(2-3x^2\right)^{3/4}}{2x}-\frac{3}{4}\int \frac{1}{\sqrt[4]{2-3x^2}} \, dx \\
&= -\frac{\left(2-3x^2\right)^{3/4}}{2x}-\frac{\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.57

$$-\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{3x^2}{2}\right)}{\sqrt[4]{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1/4, 1/2, (3*x^2)/2]/(2^(1/4)*x))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{3}{4}}}{3\,x^4-2\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)/(3*x^4 - 2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^2), x)
```

maple [C] time = 0.28, size = 40, normalized size = 0.85

$$-\frac{3\,2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{8}+\frac{3x^2-2}{2\left(-3x^2+2\right)^{\frac{1}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2+2)^(1/4),x)
```

```
[Out] 1/2*(3*x^2-2)/x/(-3*x^2+2)^(1/4)-3/8*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],3/
2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^2), x)
```

mupad [B] time = 5.05, size = 36, normalized size = 0.77

$$-\frac{2\,3^{3/4}\,\left(3-\frac{2}{x^2}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ \frac{2}{3\,x^2}\right)}{9\,x\,\left(2-3\,x^2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(2 - 3*x^2)^(1/4)),x)
```

```
[Out] -(2*3^(3/4)*(3 - 2/x^2)^(1/4)*hypergeom([1/4, 3/4], 7/4, 2/(3*x^2)))/(9*x*(
2 - 3*x^2)^(1/4))
```

sympy [C] time = 0.77, size = 31, normalized size = 0.66

$$-\frac{2^{\frac{3}{4}}{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{3x^2e^{2i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2+2)**(1/4),x)
```

```
[Out] -2**(3/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(2*x)
```

3.875 $\int \frac{1}{x^4 \sqrt[4]{2-3x^2}} \, dx$

Optimal. Leaf size=67

$$-\frac{3\left(2-3x^2\right)^{3/4}}{8x}-\frac{\left(2-3x^2\right)^{3/4}}{6x^3}-\frac{3\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{4\,2^{3/4}}$$

```
[Out] -1/6*(-3*x^2+2)^(3/4)/x^3-3/8*(-3*x^2+2)^(3/4)/x-3/8*2^(1/4)*(cos(1/2*arcsi
n(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticE(sin(1/2
*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 228}

$$-\frac{3\left(2-3x^2\right)^{3/4}}{8x}-\frac{\left(2-3x^2\right)^{3/4}}{6x^3}-\frac{3\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -(2 - 3*x^2)^(3/4)/(6*x^3) - (3*(2 - 3*x^2)^(3/4))/(8*x) - (3*Sqrt[3]*Ellip
ticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(4*2^(3/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \sqrt[4]{2-3x^2}} \, dx &= -\frac{\left(2-3x^2\right)^{3/4}}{6x^3}+\frac{3}{4}\int \frac{1}{x^2 \sqrt[4]{2-3x^2}} \, dx \\
&= -\frac{\left(2-3x^2\right)^{3/4}}{6x^3}-\frac{3\left(2-3x^2\right)^{3/4}}{8x}-\frac{9}{16}\int \frac{1}{\sqrt[4]{2-3x^2}} \, dx \\
&= -\frac{\left(2-3x^2\right)^{3/4}}{6x^3}-\frac{3\left(2-3x^2\right)^{3/4}}{8x}-\frac{3\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{4\,2^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.43

$$-\frac{{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};\frac{3x^2}{2}\right)}{3\sqrt[4]{2}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 - 3*x^2)^(1/4)),x]
[Out] -1/3*Hypergeometric2F1[-3/2, 1/4, -1/2, (3*x^2)/2]/(2^(1/4)*x^3)
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{3}{4}}}{3\,x^6-2\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(3/4)/(3*x^6 - 2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^4), x)
```

maple [C] time = 0.28, size = 45, normalized size = 0.67

$$-\frac{9\,2^{\frac{3}{4}}x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{32}+\frac{27x^4-6x^2-8}{24\left(-3x^2+2\right)^{\frac{1}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2+2)^(1/4),x)
[Out] 1/24*(27*x^4-6*x^2-8)/x^3/(-3*x^2+2)^(1/4)-9/32*2^(3/4)*x*hypergeom([1/4,1/
2],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(2-3\,x^2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(2 - 3*x^2)^(1/4)),x)
```

```
[Out] int(1/(x^4*(2 - 3*x^2)^(1/4)), x)
```

sympy [C] time = 0.87, size = 34, normalized size = 0.51

$$-\frac{2^{\frac{3}{4}}{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{3x^2e^{2i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2+2)**(1/4),x)
```

```
[Out] -2**(3/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(6*x**3)
```

3.876 $\int \frac{1}{x^6 \sqrt[4]{2-3x^2}} \, dx$

Optimal. Leaf size=85

$$-\frac{63\left(2-3x^2\right)^{3/4}}{160x}-\frac{\left(2-3x^2\right)^{3/4}}{10x^5}-\frac{7\left(2-3x^2\right)^{3/4}}{40x^3}-\frac{63\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{80\;2^{3/4}}$$

```
[Out] -1/10*(-3*x^2+2)^(3/4)/x^5-7/40*(-3*x^2+2)^(3/4)/x^3-63/160*(-3*x^2+2)^(3/4
)/x-63/160*2^(1/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(
1/2*x*6^(1/2)))*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 228}

$$-\frac{63\left(2-3x^2\right)^{3/4}}{160x}-\frac{7\left(2-3x^2\right)^{3/4}}{40x^3}-\frac{\left(2-3x^2\right)^{3/4}}{10x^5}-\frac{63\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{80\;2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -(2 - 3*x^2)^(3/4)/(10*x^5) - (7*(2 - 3*x^2)^(3/4))/(40*x^3) - (63*(2 - 3*x
^2)^(3/4))/(160*x) - (63*Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(80*2
^(3/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6 \sqrt[4]{2-3x^2}} \, dx &= -\frac{\left(2-3x^2\right)^{3/4}}{10x^5}+\frac{21}{20}\int \frac{1}{x^4 \sqrt[4]{2-3x^2}} \, dx \\
&= -\frac{\left(2-3x^2\right)^{3/4}}{10x^5}-\frac{7\left(2-3x^2\right)^{3/4}}{40x^3}+\frac{63}{80}\int \frac{1}{x^2 \sqrt[4]{2-3x^2}} \, dx \\
&= -\frac{\left(2-3x^2\right)^{3/4}}{10x^5}-\frac{7\left(2-3x^2\right)^{3/4}}{40x^3}-\frac{63\left(2-3x^2\right)^{3/4}}{160x}-\frac{189}{320}\int \frac{1}{\sqrt[4]{2-3x^2}} \, dx \\
&= -\frac{\left(2-3x^2\right)^{3/4}}{10x^5}-\frac{7\left(2-3x^2\right)^{3/4}}{40x^3}-\frac{63\left(2-3x^2\right)^{3/4}}{160x}-\frac{63\sqrt{3}\,E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{80\;2^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 29, normalized size = 0.34

$$-\frac{{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};\frac{3x^2}{2}\right)}{5\sqrt[4]{2}x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(2 - 3*x^2)^(1/4)),x]
```

```
[Out] -1/5*Hypergeometric2F1[-5/2, 1/4, -3/2, (3*x^2)/2]/(2^(1/4)*x^5)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-3x^2+2\right)^{\frac{3}{4}}}{3x^8-2x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(3/4)/(3*x^8 - 2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^2+2\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^6), x)
```

maple [C] time = 0.28, size = 50, normalized size = 0.59

$$-\frac{189\,2^{\frac{3}{4}}x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{640}+\frac{189x^6-42x^4-8x^2-32}{160\left(-3x^2+2\right)^{\frac{1}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-3*x^2+2)^(1/4),x)
```

```
[Out] 1/160*(189*x^6-42*x^4-8*x^2-32)/x^5/(-3*x^2+2)^(1/4)-189/640*2^(3/4)*x*hype
rgeom([1/4,1/2],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^2+2\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(2-3x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(2 - 3*x^2)^(1/4)),x)
```

```
[Out] int(1/(x^6*(2 - 3*x^2)^(1/4)), x)
```

sympy [C] time = 0.97, size = 34, normalized size = 0.40

$$-\frac{2^{\frac{3}{4}}{}_2F_1\left(\begin{matrix} -\frac{5}{2},\frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{3x^2e^{2i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-3*x**2+2)**(1/4),x)
```

```
[Out] -2**(3/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2*exp_polar(2*I*pi)/2)/(10*x**5)
```

3.877 $\int \frac{x^6}{(2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=83

$$-\frac{320\ 2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right),2\right)}{2079\sqrt{3}}+\frac{160\sqrt[4]{3x^2+2}\,x}{2079}+\frac{2}{33}\sqrt[4]{3x^2+2}\,x^5-\frac{40}{693}\sqrt[4]{3x^2+2}\,x^3$$

```
[Out] 160/2079*x*(3*x^2+2)^(1/4)-40/693*x^3*(3*x^2+2)^(1/4)+2/33*x^5*(3*x^2+2)^(1
/4)-320/6237*2^(3/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcta
n(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = $\{321, 231\}$

$$\frac{2}{33}\sqrt[4]{3x^2+2}\,x^5-\frac{40}{693}\sqrt[4]{3x^2+2}\,x^3+\frac{160\sqrt[4]{3x^2+2}\,x}{2079}-\frac{320\ 2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2079\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (160*x*(2 + 3*x^2)^(1/4))/2079 - (40*x^3*(2 + 3*x^2)^(1/4))/693 + (2*x^5*(2
 + 3*x^2)^(1/4))/33 - (320*2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(20
79*Sqrt[3])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\left(2+3x^2\right)^{3/4}}\,dx &= \frac{2}{33}x^5\sqrt[4]{2+3x^2}-\frac{20}{33}\int \frac{x^4}{\left(2+3x^2\right)^{3/4}}\,dx\\
&= -\frac{40}{693}x^3\sqrt[4]{2+3x^2}+\frac{2}{33}x^5\sqrt[4]{2+3x^2}+\frac{80}{231}\int \frac{x^2}{\left(2+3x^2\right)^{3/4}}\,dx\\
&= \frac{160x\sqrt[4]{2+3x^2}}{2079}-\frac{40}{693}x^3\sqrt[4]{2+3x^2}+\frac{2}{33}x^5\sqrt[4]{2+3x^2}-\frac{320\int \frac{1}{\left(2+3x^2\right)^{3/4}}\,dx}{2079}\\
&= \frac{160x\sqrt[4]{2+3x^2}}{2079}-\frac{40}{693}x^3\sqrt[4]{2+3x^2}+\frac{2}{33}x^5\sqrt[4]{2+3x^2}-\frac{320\ 2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2079\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.65

$$\frac{2x\left(\sqrt[4]{3x^2+2}\left(63x^4-60x^2+80\right)-80\sqrt[4]{2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)\right)}{2079}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*((2 + 3*x^2)^(1/4)*(80 - 60*x^2 + 63*x^4) - 80*2^(1/4)*Hypergeometric2
F1[1/2, 3/4, 3/2, (-3*x^2)/2]))/2079
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^6}{\left(3\,x^2+2\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(3*x^2 + 2)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(3*x^2 + 2)^(3/4), x)
```

maple [C] time = 0.29, size = 43, normalized size = 0.52

$$-\frac{160\,2^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{2079}+\frac{2\left(63x^4-60x^2+80\right)\left(3x^2+2\right)^{\frac{1}{4}}x}{2079}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2+2)^(3/4),x)
```

```
[Out] 2/2079*x*(63*x^4-60*x^2+80)*(3*x^2+2)^(1/4)-160/2079*2^(1/4)*x*hypergeom([1
/2,3/4],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(3*x^2 + 2)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(3\,x^2+2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2 + 2)^(3/4),x)
```

```
[Out] int(x^6/(3*x^2 + 2)^(3/4), x)
```

sympy [C] time = 0.82, size = 27, normalized size = 0.33

$$\frac{\sqrt[4]{2}\,x^7\,{}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x***7*hyper((3/4, 7/2), (9/2,), 3*x**2*exp_polar(I*pi)/2)/14
```

3.878 $\int \frac{x^4}{\left(2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=65

$$\frac{16\ 2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right),2\right)}{63\sqrt{3}}-\frac{8}{63}\sqrt[4]{3x^2+2}\,x+\frac{2}{21}\sqrt[4]{3x^2+2}\,x^3$$

```
[Out] -8/63*x*(3*x^2+2)^(1/4)+2/21*x^3*(3*x^2+2)^(1/4)+16/189*2^(3/4)*(cos(1/2*ar
ctan(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticF(sin(
1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 231}

$$\frac{2}{21}\sqrt[4]{3x^2+2}\,x^3-\frac{8}{63}\sqrt[4]{3x^2+2}\,x+\frac{16\ 2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{63\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (-8*x*(2 + 3*x^2)^(1/4))/63 + (2*x^3*(2 + 3*x^2)^(1/4))/21 + (16*2^(3/4)*El
lipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(63*Sqrt[3])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(2+3x^2\right)^{3/4}}\,dx &= \frac{2}{21}x^3\sqrt[4]{2+3x^2}-\frac{4}{7}\int \frac{x^2}{\left(2+3x^2\right)^{3/4}}\,dx\\
&= -\frac{8}{63}x\sqrt[4]{2+3x^2}+\frac{2}{21}x^3\sqrt[4]{2+3x^2}+\frac{16}{63}\int \frac{1}{\left(2+3x^2\right)^{3/4}}\,dx\\
&= -\frac{8}{63}x\sqrt[4]{2+3x^2}+\frac{2}{21}x^3\sqrt[4]{2+3x^2}+\frac{16\ 2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{63\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.75

$$\frac{2}{63}x\left(4\sqrt[4]{2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)+\sqrt[4]{3x^2+2}\,\left(3x^2-4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*((-4 + 3*x^2)*(2 + 3*x^2)^(1/4) + 4*2^(1/4)*Hypergeometric2F1[1/2, 3/4
, 3/2, (-3*x^2)/2]))/63
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\left(3\,x^2+2\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(3*x^2 + 2)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(3*x^2 + 2)^(3/4), x)
```

maple [C] time = 0.29, size = 38, normalized size = 0.58

$$\frac{8\,2^{\frac{1}{4}}x\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{63}+\frac{2\left(3x^2-4\right)\left(3x^2+2\right)^{\frac{1}{4}}x}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2+2)^(3/4),x)
```

```
[Out] 2/63*x*(3*x^2-4)*(3*x^2+2)^(1/4)+8/63*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-
3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(3*x^2 + 2)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{{\left(3\,x^2+2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2 + 2)^(3/4),x)
```

```
[Out] int(x^4/(3*x^2 + 2)^(3/4), x)
```

sympy [C] time = 0.74, size = 27, normalized size = 0.42

$$\frac{\sqrt[4]{2}\,x^{5}{}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{3x^{2}e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**5*hyper((3/4, 5/2), (7/2,), 3*x**2*exp_polar(I*pi)/2)/10
```

3.879 $\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=47

$$\frac{2}{9}x\sqrt[4]{3x^2+2}-\frac{4\,2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}$$

```
[Out] 2/9*x*(3*x^2+2)^(1/4)-4/27*2^(3/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)
/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arctan(1/2*x*6^(1/2))),2^
(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = $\{321, 231\}$

$$\frac{2}{9}x\sqrt[4]{3x^2+2}-\frac{4\,2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(2 + 3*x^2)^(1/4))/9 - (4*2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])
/(9*Sqrt[3])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(2+3x^2\right)^{3/4}}\,dx&=\frac{2}{9}x\sqrt[4]{2+3x^2}-\frac{4}{9}\int \frac{1}{\left(2+3x^2\right)^{3/4}}\,dx\\&=\frac{2}{9}x\sqrt[4]{2+3x^2}-\frac{4\,2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{9\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.87

$$\frac{2}{9}x\left(\sqrt[4]{3x^2+2}-\sqrt[4]{2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*((2 + 3*x^2)^(1/4) - 2^(1/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2)
/2]))/9
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\left(3\,x^2+2\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^2/(3*x^2 + 2)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(3*x^2 + 2)^(3/4), x)
```

maple [C] time = 0.29, size = 31, normalized size = 0.66

$$-\frac{2\,2^{\frac{1}{4}}x\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{9}+\frac{2\left(3x^2+2\right)^{\frac{1}{4}}x}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+2)^(3/4),x)
```

```
[Out] 2/9*x*(3*x^2+2)^(1/4)-2/9*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(3*x^2 + 2)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{{\left(3\,x^2+2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2 + 2)^(3/4),x)
```

```
[Out] int(x^2/(3*x^2 + 2)^(3/4), x)
```

sympy [C] time = 0.71, size = 27, normalized size = 0.57

$$\frac{\sqrt[4]{2}\,x^3{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**3*hyper((3/4, 3/2), (5/2,), 3*x**2*exp_polar(I*pi)/2)/6
```

3.880 $\int \frac{1}{(2+3x^2)^{3/4}} \, dx$

Optimal. Leaf size=27

$$\frac{2^{3/4} \operatorname{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{\sqrt{3}}$$

```
[Out] 1/3*2^(3/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6
^(1/2)))*EllipticF(sin(1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {231}

$$\frac{2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)^(-3/4),x]
```

```
[Out] (2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(2+3x^2\right)^{3/4}} \, dx = \frac{2^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.89

$$\frac{x\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)^(-3/4),x]
```

```
[Out] (x*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2])/2^(3/4)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(3\,x^2+2\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral((3*x^2 + 2)^(-3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)^(-3/4), x)
```

maple [C] time = 0.27, size = 18, normalized size = 0.67

$$\frac{2^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+2)^(3/4),x)
[Out] 1/2*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)^(-3/4), x)
```

mupad [B] time = 0.08, size = 16, normalized size = 0.59

$$\frac{2^{1/4}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ -\frac{3\,x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + 2)^(3/4),x)
[Out] (2^(1/4)*x*hypergeom([1/2, 3/4], 3/2, -(3*x^2)/2))/2
```

sympy [C] time = 0.68, size = 26, normalized size = 0.96

$$\frac{\sqrt[4]{2}\,x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{3x^2e^{i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+2)**(3/4),x)
[Out] 2**(1/4)*x*hyper((1/2, 3/4), (3/2,), 3*x**2*exp_polar(I*pi)/2)/2
```

3.881 $\int \frac{1}{x^2(2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}}-\frac{\sqrt[4]{3x^2+2}}{2x}$$

```
[Out] -1/2*(3*x^2+2)^(1/4)/x-1/4*2^(3/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)
/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arctan(1/2*x*6^(1/2))),2^
(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 231}

$$-\frac{\sqrt[4]{3x^2+2}}{2x}-\frac{\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -(2 + 3*x^2)^(1/4)/(2*x) - (Sqrt[3]*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(2
*2^(1/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(2+3x^2\right)^{3/4}}\,dx&=-\frac{\sqrt[4]{2+3x^2}}{2x}-\frac{3}{4}\int \frac{1}{\left(2+3x^2\right)^{3/4}}\,dx\\&=-\frac{\sqrt[4]{2+3x^2}}{2x}-\frac{\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.55

$$-\frac{{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};-\frac{3x^2}{2}\right)}{2^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 + 3*x^2)^(3/4)),x]
[Out] -(Hypergeometric2F1[-1/2, 3/4, 1/2, (-3*x^2)/2]/(2^(3/4)*x))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(3\,x^2+2\right)^{\frac{1}{4}}}{3\,x^4+2\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral((3*x^2 + 2)^(1/4)/(3*x^4 + 2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^2), x)
```

maple [C] time = 0.27, size = 33, normalized size = 0.67

$$-\frac{3\,2^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{8}-\frac{\left(3x^2+2\right)^{\frac{1}{4}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2+2)^(3/4),x)
[Out] -1/2*(3*x^2+2)^(1/4)/x-3/8*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^2), x)
```

mupad [B] time = 4.99, size = 36, normalized size = 0.73

$$-\frac{2\,3^{1/4}\left(\frac{2}{x^2}+3\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ -\frac{2}{3\,x^2}\right)}{15\,x\left(3\,x^2+2\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^2 + 2)^(3/4)),x)
[Out] -(2*3^(1/4)*(2/x^2 + 3)^(3/4)*hypergeom([3/4, 5/4], 9/4, -2/(3*x^2)))/(15*x
*(3*x^2 + 2)^(3/4))
```

sympy [C] time = 0.82, size = 29, normalized size = 0.59

$$-\frac{\sqrt[4]{2}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{1}{2}\end{matrix}\middle|\frac{3x^{2}e^{i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2+2)**(3/4),x)
[Out] -2**(1/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)/(2*x)
```

3.882 $\int \frac{1}{x^4(2+3x^2)^{3/4}} \, dx$

Optimal. Leaf size=67

$$\frac{5\sqrt{3}\ \text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{8\sqrt[4]{2}}+\frac{5\sqrt[4]{3x^2+2}}{8x}-\frac{\sqrt[4]{3x^2+2}}{6x^3}$$

```
[Out] -1/6*(3*x^2+2)^(1/4)/x^3+5/8*(3*x^2+2)^(1/4)/x+5/16*2^(3/4)*(cos(1/2*arctan
(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x*6^(1/2)))*EllipticF(sin(1/2*
arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 231}

$$\frac{5\sqrt[4]{3x^2+2}}{8x}-\frac{\sqrt[4]{3x^2+2}}{6x^3}+\frac{5\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -(2 + 3*x^2)^(1/4)/(6*x^3) + (5*(2 + 3*x^2)^(1/4))/(8*x) + (5*Sqrt[3]*Ellip
ticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(8*2^(1/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(2+3x^2\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{2+3x^2}}{6x^3}-\frac{5}{4}\int \frac{1}{x^2\left(2+3x^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{2+3x^2}}{6x^3}+\frac{5\sqrt[4]{2+3x^2}}{8x}+\frac{15}{16}\int \frac{1}{\left(2+3x^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{2+3x^2}}{6x^3}+\frac{5\sqrt[4]{2+3x^2}}{8x}+\frac{5\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\sqrt[4]{2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.43

$$-\frac{{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};-\frac{3x^2}{2}\right)}{3\ 2^{3/4}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 + 3*x^2)^(3/4)),x]
[Out] -1/3*Hypergeometric2F1[-3/2, 3/4, -1/2, (-3*x^2)/2]/(2^(3/4)*x^3)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(3\,x^2+2\right)^{\frac{1}{4}}}{3\,x^6+2\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral((3*x^2 + 2)^(1/4)/(3*x^6 + 2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^4), x)
```

maple [C] time = 0.28, size = 45, normalized size = 0.67

$$\frac{15\,2^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{32}+\frac{45x^4+18x^2-8}{24\left(3x^2+2\right)^{\frac{3}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2+2)^(3/4),x)
[Out] 1/24*(45*x^4+18*x^2-8)/x^3/(3*x^2+2)^(3/4)+15/32*2^(1/4)*x*hypergeom([1/2,3
/4],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+2\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(3\,x^2+2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(3*x^2 + 2)^(3/4)),x)
```

```
[Out] int(1/(x^4*(3*x^2 + 2)^(3/4)), x)
```

sympy [C] time = 0.93, size = 32, normalized size = 0.48

$$-\frac{\sqrt[4]{2}\, {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{3x^{2}e^{i\pi}}{2}\right)}{6x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2+2)**(3/4),x)
[Out] -2**(1/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2)/(6*x**3)
```

3.883 $\int \frac{1}{x^6(2+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=85

$$-\frac{27\sqrt{3}\ \text{EllipticF}\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{32\sqrt[4]{2}}-\frac{27\sqrt[4]{3x^2+2}}{32x}-\frac{\sqrt[4]{3x^2+2}}{10x^5}+\frac{9\sqrt[4]{3x^2+2}}{40x^3}$$

```
[Out] -1/10*(3*x^2+2)^(1/4)/x^5+9/40*(3*x^2+2)^(1/4)/x^3-27/32*(3*x^2+2)^(1/4)/x-
27/64*2^(3/4)*(cos(1/2*arctan(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arctan(1/2*x
*6^(1/2)))*EllipticF(sin(1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 231}

$$-\frac{27\sqrt[4]{3x^2+2}}{32x}+\frac{9\sqrt[4]{3x^2+2}}{40x^3}-\frac{\sqrt[4]{3x^2+2}}{10x^5}-\frac{27\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(2 + 3*x^2)^(3/4)),x]
```

```
[Out] -(2 + 3*x^2)^(1/4)/(10*x^5) + (9*(2 + 3*x^2)^(1/4))/(40*x^3) - (27*(2 + 3*x
^2)^(1/4))/(32*x) - (27*Sqrt[3]*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(32*2^
(1/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(2+3x^2\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{2+3x^2}}{10x^5}-\frac{27}{20}\int \frac{1}{x^4\left(2+3x^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{2+3x^2}}{10x^5}+\frac{9\sqrt[4]{2+3x^2}}{40x^3}+\frac{27}{16}\int \frac{1}{x^2\left(2+3x^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{2+3x^2}}{10x^5}+\frac{9\sqrt[4]{2+3x^2}}{40x^3}-\frac{27\sqrt[4]{2+3x^2}}{32x}-\frac{81}{64}\int \frac{1}{\left(2+3x^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{2+3x^2}}{10x^5}+\frac{9\sqrt[4]{2+3x^2}}{40x^3}-\frac{27\sqrt[4]{2+3x^2}}{32x}-\frac{27\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.34

$$-\frac{{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};-\frac{3x^2}{2}\right)}{5\ 2^{3/4}x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(2 + 3*x^2)^(3/4)),x]
[Out] -1/5*Hypergeometric2F1[-5/2, 3/4, -3/2, (-3*x^2)/2]/(2^(3/4)*x^5)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3x^2+2\right)^{\frac{1}{4}}}{3x^8+2x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral((3*x^2 + 2)^(1/4)/(3*x^8 + 2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2+2\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^6), x)
```

maple [C] time = 0.27, size = 50, normalized size = 0.59

$$-\frac{81\,2^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{128}-\frac{405x^6+162x^4-24x^2+32}{160\left(3x^2+2\right)^{\frac{3}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(3*x^2+2)^(3/4),x)
[Out] -1/160*(405*x^6+162*x^4-24*x^2+32)/x^5/(3*x^2+2)^(3/4)-81/128*2^(1/4)*x*hyp
ergeom([1/2,3/4],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2+2\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(3x^2+2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(3*x^2 + 2)^(3/4)),x)
[Out] int(1/(x^6*(3*x^2 + 2)^(3/4)), x)
```

sympy [C] time = 1.08, size = 32, normalized size = 0.38

$$-\frac{\sqrt[4]{2}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(3*x**2+2)**(3/4),x)
[Out] -2**(1/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2)/(10*x**5)
```

3.884 $\int \frac{x^6}{\left(2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=83

$$\frac{320\ 2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right),2\right)}{2079\sqrt{3}}-\frac{160\sqrt[4]{2-3x^2}\,x}{2079}-\frac{2}{33}\sqrt[4]{2-3x^2}\,x^5-\frac{40}{693}\sqrt[4]{2-3x^2}\,x^3$$

```
[Out] -160/2079*x*(-3*x^2+2)^(1/4)-40/693*x^3*(-3*x^2+2)^(1/4)-2/33*x^5*(-3*x^2+2
)^(1/4)+320/6237*2^(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*a
rcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(
1/2)
```

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{321, 232\}$

$$-\frac{2}{33}\sqrt[4]{2-3x^2}\,x^5-\frac{40}{693}\sqrt[4]{2-3x^2}\,x^3-\frac{160\sqrt[4]{2-3x^2}\,x}{2079}+\frac{320\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2079\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 - 3*x^2)^(3/4),x]
```

```
[Out] (-160*x*(2 - 3*x^2)^(1/4))/2079 - (40*x^3*(2 - 3*x^2)^(1/4))/693 - (2*x^5*(
2 - 3*x^2)^(1/4))/33 + (320*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(2
079*Sqrt[3])
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\left(2-3x^2\right)^{3/4}}\,dx &= -\frac{2}{33}x^5\sqrt[4]{2-3x^2}+\frac{20}{33}\int \frac{x^4}{\left(2-3x^2\right)^{3/4}}\,dx\\
&= -\frac{40}{693}x^3\sqrt[4]{2-3x^2}-\frac{2}{33}x^5\sqrt[4]{2-3x^2}+\frac{80}{231}\int \frac{x^2}{\left(2-3x^2\right)^{3/4}}\,dx\\
&= -\frac{160x\sqrt[4]{2-3x^2}}{2079}-\frac{40}{693}x^3\sqrt[4]{2-3x^2}-\frac{2}{33}x^5\sqrt[4]{2-3x^2}+\frac{320\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx}{2079}\\
&= -\frac{160x\sqrt[4]{2-3x^2}}{2079}-\frac{40}{693}x^3\sqrt[4]{2-3x^2}-\frac{2}{33}x^5\sqrt[4]{2-3x^2}+\frac{320\ 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{2079\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.71

$$\frac{320\,2^{3/4}\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)-6x\sqrt[4]{2-3x^2}\left(63x^4+60x^2+80\right)}{6237}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 - 3*x^2)^(3/4),x]
[Out] (-6*x*(2 - 3*x^2)^(1/4)*(80 + 60*x^2 + 63*x^4) + 320*2^(3/4)*Sqrt[3]*Ellipt
icF[ArcSin[Sqrt[3/2]*x]/2, 2])/6237
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{1}{4}}x^6}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(1/4)*x^6/(3*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^6}{\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(x^6/(-3*x^2 + 2)^(3/4), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{x^6}{\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-3*x^2+2)^(3/4),x)
[Out] int(x^6/(-3*x^2+2)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^6}{\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate(x^6/(-3*x^2 + 2)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^6}{\left(2-3\,x^2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(2 - 3*x^2)^(3/4),x)
```

```
[Out] int(x^6/(2 - 3*x^2)^(3/4), x)
```

sympy [C] time = 0.84, size = 29, normalized size = 0.35

$$\frac{\sqrt[4]{2}\,x^7\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\,\frac{3x^2e^{2i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-3*x**2+2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**7*hyper((3/4, 7/2), (9/2,), 3*x**2*exp_polar(2*I*pi)/2)/14
```

3.885 $\int \frac{x^4}{\left(2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=65

$$\frac{16\, 2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{63\sqrt{3}}-\frac{8}{63}\sqrt[4]{2-3x^2}\,x-\frac{2}{21}\sqrt[4]{2-3x^2}\,x^3$$

```
[Out] -8/63*x*(-3*x^2+2)^(1/4)-2/21*x^3*(-3*x^2+2)^(1/4)+16/189*2^(3/4)*(cos(1/2*
arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(si
n(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 232}

$$-\frac{2}{21}\sqrt[4]{2-3x^2}\,x^3-\frac{8}{63}\sqrt[4]{2-3x^2}\,x+\frac{16\, 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{63\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 - 3*x^2)^(3/4),x]
```

```
[Out] (-8*x*(2 - 3*x^2)^(1/4))/63 - (2*x^3*(2 - 3*x^2)^(1/4))/21 + (16*2^(3/4)*El
lipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(63*Sqrt[3])
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(2-3x^2\right)^{3/4}}\,dx &= -\frac{2}{21}x^3\sqrt[4]{2-3x^2}+\frac{4}{7}\int \frac{x^2}{\left(2-3x^2\right)^{3/4}}\,dx\\ &= -\frac{8}{63}x\sqrt[4]{2-3x^2}-\frac{2}{21}x^3\sqrt[4]{2-3x^2}+\frac{16}{63}\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx\\ &= -\frac{8}{63}x\sqrt[4]{2-3x^2}-\frac{2}{21}x^3\sqrt[4]{2-3x^2}+\frac{16\, 2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{63\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.83

$$-\frac{2}{189}\left(3x\sqrt[4]{2-3x^2}\left(3x^2+4\right)-8\, 2^{3/4}\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 - 3*x^2)^(3/4),x]
[Out] (-2*(3*x*(2 - 3*x^2)^(1/4)*(4 + 3*x^2) - 8*2^(3/4)*Sqrt[3]*EllipticF[ArcSin
[Sqrt[3/2]*x]/2, 2]))/189
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{1}{4}}x^4}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(1/4)*x^4/(3*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(x^4/(-3*x^2 + 2)^(3/4), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2+2)^(3/4),x)
[Out] int(x^4/(-3*x^2+2)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate(x^4/(-3*x^2 + 2)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{{\left(2-3\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2 - 3*x^2)^(3/4),x)
```

```
[Out] int(x^4/(2 - 3*x^2)^(3/4), x)
```

sympy [C] time = 0.77, size = 29, normalized size = 0.45

$$\frac{\sqrt[4]{2}\,x^{5}{}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{3x^{2}e^{2i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2+2)**(3/4),x)
[Out] 2**(1/4)*x**5*hyper((3/4, 5/2), (7/2,), 3*x**2*exp_polar(2*I*pi)/2)/10
```

3.886 $\int \frac{x^2}{\left(2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=47

$$\frac{4\,2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right),2\right)}{9\sqrt{3}}-\frac{2}{9}x\sqrt[4]{2-3x^2}$$

```
[Out] -2/9*x*(-3*x^2+2)^(1/4)+4/27*2^(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/
2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),
2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 232}

$$\frac{4\,2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{9\sqrt{3}}-\frac{2}{9}x\sqrt[4]{2-3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - 3*x^2)^(3/4),x]
```

```
[Out] (-2*x*(2 - 3*x^2)^(1/4))/9 + (4*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2]
)/(9*Sqrt[3])
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(2-3x^2\right)^{3/4}}\,dx&=-\frac{2}{9}x\sqrt[4]{2-3x^2}+\frac{4}{9}\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx\\&=-\frac{2}{9}x\sqrt[4]{2-3x^2}+\frac{4\,2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{9\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$\frac{4\,2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right),2\right)}{9\sqrt{3}}-\frac{2}{9}x\sqrt[4]{2-3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - 3*x^2)^(3/4),x]
```

```
[Out] (-2*x*(2 - 3*x^2)^(1/4))/9 + (4*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2]
)/(9*Sqrt[3])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{1}{4}}x^2}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 + 2)^(1/4)*x^2/(3*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-3*x^2 + 2)^(3/4), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-3x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(3/4),x)
```

```
[Out] int(x^2/(-3*x^2+2)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-3*x^2 + 2)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{{\left(2-3\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2 - 3*x^2)^(3/4),x)
```

[Out] int(x^2/(2 - 3*x^2)^(3/4), x)

sympy [C] time = 0.73, size = 29, normalized size = 0.62

$$\frac{\sqrt[4]{2}\, x^{3} {}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{3x^{2}e^{2i\pi}}{2} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(-3*x**2+2)**(3/4),x)

[Out] 2**(1/4)*x**3*hyper((3/4, 3/2), (5/2,), 3*x**2*exp_polar(2*I*pi)/2)/6

3.887 $\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=27

$$\frac{2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right),2\right)}{\sqrt{3}}$$

```
[Out] 1/3*2^(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6
^(1/2)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {232}

$$\frac{2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x^2)^(-3/4),x]
```

```
[Out] (2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx=\frac{2^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right),2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x^2)^(-3/4),x]
```

```
[Out] (2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{1}{4}}}{3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(1/4)/(3*x^2 - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate((-3*x^2 + 2)^(-3/4), x)
```

maple [C] time = 0.29, size = 18, normalized size = 0.67

$$\frac{2^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(3/4),x)
[Out] 1/2*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate((-3*x^2 + 2)^(-3/4), x)
```

mupad [B] time = 4.76, size = 16, normalized size = 0.59

$$\frac{2^{1/4}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ \frac{3\,x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^2)^(3/4),x)
[Out] (2^(1/4)*x*hypergeom([1/2, 3/4], 3/2, (3*x^2)/2))/2
```

sympy [C] time = 0.69, size = 27, normalized size = 1.00

$$\frac{\sqrt[4]{2}\,x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\right|\frac{3x^2e^{2i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(3/4),x)
[Out] 2**(1/4)*x*hyper((1/2, 3/4), (3/2,), 3*x**2*exp_polar(2*I*pi)/2)/2
```

3.888 $\int \frac{1}{x^2(2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=49

$$\frac{\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{2x}$$

```
[Out] -1/2*(-3*x^2+2)^(1/4)/x+1/4*2^(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2
)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),2
^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 232}

$$\frac{\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 - 3*x^2)^(3/4)),x]
```

```
[Out] -(2 - 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(2
*2^(1/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(2-3x^2\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{2-3x^2}}{2x}+\frac{3}{4}\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx\\ &= -\frac{\sqrt[4]{2-3x^2}}{2x}+\frac{\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.55

$$-\frac{{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};\frac{3x^2}{2}\right)}{2^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 - 3*x^2)^(3/4)),x]
[Out] -(Hypergeometric2F1[-1/2, 3/4, 1/2, (3*x^2)/2]/(2^(3/4)*x))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-3\,x^{2}+2\right)^{\frac{1}{4}}}{3\,x^{4}-2\,x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(1/4)/(3*x^4 - 2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^{2}+2\right)^{\frac{3}{4}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^2), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^{2}+2\right)^{\frac{3}{4}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2+2)^(3/4),x)
[Out] int(1/x^2/(-3*x^2+2)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^{2}+2\right)^{\frac{3}{4}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^2), x)
```

mupad [B] time = 5.03, size = 36, normalized size = 0.73

$$-\frac{2\,3^{1/4}\left(3-\frac{2}{x^2}\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ \frac{2}{3\,x^2}\right)}{15\,x\left(2-3\,x^2\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(2 - 3*x^2)^(3/4)),x)
[Out] -(2*3^(1/4)*(3 - 2/x^2)^(3/4)*hypergeom([3/4, 5/4], 9/4, 2/(3*x^2)))/(15*x*
(2 - 3*x^2)^(3/4))
```

sympy [C] time = 0.84, size = 31, normalized size = 0.63

$$-\frac{\sqrt[4]{2}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2+2)**(3/4),x)
```

```
[Out] -2**(1/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(2*x)
```

3.889 $\int \frac{1}{x^4\left(2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=67

$$\frac{5\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right),2\right)}{8\sqrt[4]{2}}-\frac{5\sqrt[4]{2-3x^2}}{8x}-\frac{\sqrt[4]{2-3x^2}}{6x^3}$$

```
[Out] -1/6*(-3*x^2+2)^(1/4)/x^3-5/8*(-3*x^2+2)^(1/4)/x+5/16*2^(3/4)*(cos(1/2*arcs
in(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/
2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 232}

$$-\frac{5\sqrt[4]{2-3x^2}}{8x}-\frac{\sqrt[4]{2-3x^2}}{6x^3}+\frac{5\sqrt{3}\,F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{8\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 - 3*x^2)^(3/4)),x]
```

```
[Out] -(2 - 3*x^2)^(1/4)/(6*x^3) - (5*(2 - 3*x^2)^(1/4))/(8*x) + (5*Sqrt[3]*Ellip
ticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(8*2^(1/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(2-3x^2\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{2-3x^2}}{6x^3}+\frac{5}{4}\int \frac{1}{x^2\left(2-3x^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{2-3x^2}}{6x^3}-\frac{5\sqrt[4]{2-3x^2}}{8x}+\frac{15}{16}\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{2-3x^2}}{6x^3}-\frac{5\sqrt[4]{2-3x^2}}{8x}+\frac{5\sqrt{3}\,F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|2\right)}{8\sqrt[4]{2}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 29, normalized size = 0.43

$$-\frac{{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};\frac{3x^2}{2}\right)}{3\,2^{3/4}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 - 3*x^2)^(3/4)),x]
[Out] -1/3*Hypergeometric2F1[-3/2, 3/4, -1/2, (3*x^2)/2]/(2^(3/4)*x^3)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{1}{4}}}{3\,x^6-2\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(1/4)/(3*x^6 - 2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^2+2\right)^{\frac{3}{4}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2+2)^(3/4),x)
[Out] int(1/x^4/(-3*x^2+2)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(2-3\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(2 - 3*x^2)^(3/4)),x)
[Out] int(1/(x^4*(2 - 3*x^2)^(3/4)), x)
```

sympy [C] time = 0.95, size = 34, normalized size = 0.51

$$-\frac{\sqrt[4]{2}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{3x^{2}e^{2i\pi}}{2}\right)}{6x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2+2)**(3/4),x)
```

```
[Out] -2**(1/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(6*x**3)
```

3.890 $\int \frac{1}{x^6(2-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=85

$$\frac{27\sqrt{3}\ \mathrm{EllipticF}\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right),2\right)}{32\sqrt[4]{2}}-\frac{27\sqrt[4]{2-3x^2}}{32x}-\frac{\sqrt[4]{2-3x^2}}{10x^5}-\frac{9\sqrt[4]{2-3x^2}}{40x^3}$$

```
[Out] -1/10*(-3*x^2+2)^(1/4)/x^5-9/40*(-3*x^2+2)^(1/4)/x^3-27/32*(-3*x^2+2)^(1/4)
/x+27/64*2^(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/
2*x*6^(1/2)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {325, 232}

$$-\frac{27\sqrt[4]{2-3x^2}}{32x}-\frac{9\sqrt[4]{2-3x^2}}{40x^3}-\frac{\sqrt[4]{2-3x^2}}{10x^5}+\frac{27\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(2 - 3*x^2)^(3/4)),x]
```

```
[Out] -(2 - 3*x^2)^(1/4)/(10*x^5) - (9*(2 - 3*x^2)^(1/4))/(40*x^3) - (27*(2 - 3*x
^2)^(1/4))/(32*x) + (27*Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(32*2^
(1/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(2-3x^2\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{2-3x^2}}{10x^5}+\frac{27}{20}\int \frac{1}{x^4\left(2-3x^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{2-3x^2}}{10x^5}-\frac{9\sqrt[4]{2-3x^2}}{40x^3}+\frac{27}{16}\int \frac{1}{x^2\left(2-3x^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{2-3x^2}}{10x^5}-\frac{9\sqrt[4]{2-3x^2}}{40x^3}-\frac{27\sqrt[4]{2-3x^2}}{32x}+\frac{81}{64}\int \frac{1}{\left(2-3x^2\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{2-3x^2}}{10x^5}-\frac{9\sqrt[4]{2-3x^2}}{40x^3}-\frac{27\sqrt[4]{2-3x^2}}{32x}+\frac{27\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.34

$$-\frac{{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};\frac{3x^2}{2}\right)}{5\,2^{3/4}x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(2 - 3*x^2)^(3/4)),x]
[Out] -1/5*Hypergeometric2F1[-5/2, 3/4, -3/2, (3*x^2)/2]/(2^(3/4)*x^5)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-3\,x^2+2\right)^{\frac{1}{4}}}{3\,x^8-2\,x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(3/4),x, algorithm="fricas")
[Out] integral(-(-3*x^2 + 2)^(1/4)/(3*x^8 - 2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^6), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^2+2\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-3*x^2+2)^(3/4),x)
[Out] int(1/x^6/(-3*x^2+2)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2+2)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,{\left(2-3\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(2 - 3*x^2)^(3/4)),x)
```

```
[Out] int(1/(x^6*(2 - 3*x^2)^(3/4)), x)
```

sympy [C] time = 1.11, size = 34, normalized size = 0.40

$$-\frac{\sqrt[4]{2}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{3x^{2}e^{2i\pi}}{2}\right)}{10x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-3*x**2+2)**(3/4),x)
```

```
[Out] -2**(1/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2*exp_polar(2*I*pi)/2)/(10*x**5)
```

3.891 $\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=258

$$\frac{64\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{1053\sqrt{3}\,x}+\frac{32\left(3x^2-2\right)^{3/4}x}{1053}+\frac{128\sqrt[4]{3x^2-2}\,x}{1053\left(\sqrt{3x^2-2}+\right.}$$

```
[Out] 32/1053*x*(3*x^2-2)^(3/4)+40/1053*x^3*(3*x^2-2)^(3/4)+2/39*x^5*(3*x^2-2)^(3
/4)+128/1053*x*(3*x^2-2)^(1/4)/(2^(1/2)+(3*x^2-2)^(1/2))-128/3159*2^(1/4)*(
cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2
-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/
2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2
)/x*3^(1/2)+64/3159*2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^
(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2
*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(
1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {321, 230, 305, 220, 1196}

$$\frac{2}{39}\left(3x^2-2\right)^{3/4}x^5+\frac{40\left(3x^2-2\right)^{3/4}x^3}{1053}+\frac{32\left(3x^2-2\right)^{3/4}x}{1053}+\frac{128\sqrt[4]{3x^2-2}\,x}{1053\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{64\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2\right.}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (32*x*(-2 + 3*x^2)^(3/4))/1053 + (40*x^3*(-2 + 3*x^2)^(3/4))/1053 + (2*x^5*
(-2 + 3*x^2)^(3/4))/39 + (128*x*(-2 + 3*x^2)^(1/4))/(1053*(Sqrt[2] + Sqrt[-
2 + 3*x^2])) - (128*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[
2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2]
)/(1053*Sqrt[3]*x) + (64*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(
Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)],
 1/2])/(1053*Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx &= \frac{2}{39}x^5\left(-2+3x^2\right)^{3/4}+\frac{20}{39}\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx\\
&= \frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053}+\frac{2}{39}x^5\left(-2+3x^2\right)^{3/4}+\frac{80}{351}\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx\\
&= \frac{32x\left(-2+3x^2\right)^{3/4}}{1053}+\frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053}+\frac{2}{39}x^5\left(-2+3x^2\right)^{3/4}+\frac{64\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx}{1053}\\
&= \frac{32x\left(-2+3x^2\right)^{3/4}}{1053}+\frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053}+\frac{2}{39}x^5\left(-2+3x^2\right)^{3/4}+\frac{\left(64\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \cdots\right.}{105\cdots}\\
&= \frac{32x\left(-2+3x^2\right)^{3/4}}{1053}+\frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053}+\frac{2}{39}x^5\left(-2+3x^2\right)^{3/4}+\frac{\left(128\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\cdots}}\right.}{1053\sqrt{\cdots}}\\
&= \frac{32x\left(-2+3x^2\right)^{3/4}}{1053}+\frac{40x^3\left(-2+3x^2\right)^{3/4}}{1053}+\frac{2}{39}x^5\left(-2+3x^2\right)^{3/4}+\frac{128x\sqrt[4]{-2+3x^2}}{1053\left(\sqrt{2}+\sqrt{-2+3x^2}\cdots\right.}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 68, normalized size = 0.26

$$
\frac{2x\left(16\,2^{3/4}\sqrt[4]{2-3x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)+81x^6+6x^4+8x^2-32\right)}{1053\sqrt[4]{3x^2-2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-32 + 8*x^2 + 6*x^4 + 81*x^6 + 16*2^(3/4)*(2 - 3*x^2)^(1/4)*Hypergeom
etric2F1[1/4, 1/2, 3/2, (3*x^2)/2]))/(1053*(-2 + 3*x^2)^(1/4))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{x^6}{\left(3\,x^2-2\right)^{\frac{1}{4}}},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(3*x^2-2)^(1/4),x, algorithm="fricas")

[Out] integral(x^6/(3*x^2 - 2)^(1/4), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(3x^2-2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(3*x^2-2)^(1/4),x, algorithm="giac")

[Out] integrate(x^6/(3*x^2 - 2)^(1/4), x)

maple [C] time = 0.29, size = 65, normalized size = 0.25

$$\frac{32\,2^{\frac{3}{4}}\left(-\mathrm{signum}\left(\frac{3x^2}{2}-1\right)\right)^{\frac{1}{4}} x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{1053\mathrm{signum}\left(\frac{3x^2}{2}-1\right)^{\frac{1}{4}}}+\frac{2\left(27x^4+20x^2+16\right)\left(3x^2-2\right)^{\frac{3}{4}} x}{1053}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(3*x^2-2)^(1/4),x)

[Out] 2/1053*x*(27*x^4+20*x^2+16)*(3*x^2-2)^(3/4)+32/1053*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(3x^2-2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(3*x^2-2)^(1/4),x, algorithm="maxima")

[Out] integrate(x^6/(3*x^2 - 2)^(1/4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{(3x^2-2)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(3*x^2 - 2)^(1/4),x)

[Out] int(x^6/(3*x^2 - 2)^(1/4), x)

sympy [C] time = 0.85, size = 29, normalized size = 0.11

$$\frac{2^{\frac{3}{4}} x^7 e^{-\frac{i\pi}{4}} {}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\right|\frac{3x^2}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**7*exp(-I*pi/4)*hyper((1/4, 7/2), (9/2,), 3*x**2/2)/14
```

3.892 $\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=240

$$\frac{16\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\mathrm{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{135\sqrt{3}\,x}+\frac{8}{135}\left(3x^2-2\right)^{3/4}x+\frac{32\sqrt[4]{3x^2-2}\,x}{135\left(\sqrt{3x^2-2}+\right.}$$

```
[Out] 8/135*x*(3*x^2-2)^(3/4)+2/27*x^3*(3*x^2-2)^(3/4)+32/135*x*(3*x^2-2)^(1/4)/(
2^(1/2)+(3*x^2-2)^(1/2))-32/405*2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2
^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(
2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2
))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)+16/405*2^(1/4)*(cos(2*
arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1
/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1
/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^
(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 230, 305, 220, 1196}

$$\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3+\frac{8}{135}\left(3x^2-2\right)^{3/4}x+\frac{32\sqrt[4]{3x^2-2}\,x}{135\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{16\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan\right.}{135\sqrt{3}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (8*x*(-2 + 3*x^2)^(3/4))/135 + (2*x^3*(-2 + 3*x^2)^(3/4))/27 + (32*x*(-2 +
3*x^2)^(1/4))/(135*(Sqrt[2] + Sqrt[-2 + 3*x^2])) - (32*2^(1/4)*Sqrt[x^2/(Sq
rt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTa
n[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x) + (16*2^(1/4)*Sqrt[x^2
/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*A
rcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx &= \frac{2}{27}x^3\left(-2+3x^2\right)^{3/4}+\frac{4}{9}\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx\\
&= \frac{8}{135}x\left(-2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(-2+3x^2\right)^{3/4}+\frac{16}{135}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx\\
&= \frac{8}{135}x\left(-2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(-2+3x^2\right)^{3/4}+\frac{\left(16\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3
}{135x}\\
&= \frac{8}{135}x\left(-2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(-2+3x^2\right)^{3/4}+\frac{\left(32\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{135\sqrt{3}\,x}\\
&= \frac{8}{135}x\left(-2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(-2+3x^2\right)^{3/4}+\frac{32x\sqrt[4]{-2+3x^2}}{135\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}-\frac{32\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 63, normalized size = 0.26

$$
\frac{2x\left(4\,2^{3/4}\sqrt[4]{2-3x^2}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)+15x^4+2x^2-8\right)}{135\sqrt[4]{3x^2-2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-8 + 2*x^2 + 15*x^4 + 4*2^(3/4)*(2 - 3*x^2)^(1/4)*Hypergeometric2F1[1
/4, 1/2, 3/2, (3*x^2)/2]))/(135*(-2 + 3*x^2)^(1/4))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{x^4}{\left(3\,x^2-2\right)^{\frac{1}{4}}},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(3*x^2 - 2)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(3\,x^2-2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(1/4),x, algorithm="giac")
[Out] integrate(x^4/(3*x^2 - 2)^(1/4), x)
```

maple [C] time = 0.29, size = 60, normalized size = 0.25

$$\frac{8\,2^{\frac{3}{4}}\left(-\mathrm{signum}\left(\frac{3x^2}{2}-1\right)\right)^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{135\mathrm{signum}\left(\frac{3x^2}{2}-1\right)^{\frac{1}{4}}}+\frac{2\left(5x^2+4\right)\left(3x^2-2\right)^{\frac{3}{4}}x}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2-2)^(1/4),x)
[Out] 2/135*x*(5*x^2+4)*(3*x^2-2)^(3/4)+8/135*2^(3/4)/signum(3/2*x^2-1)^(1/4)*(-s
ignum(3/2*x^2-1))^(1/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(3\,x^2-2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(1/4),x, algorithm="maxima")
[Out] integrate(x^4/(3*x^2 - 2)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(3\,x^2-2)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2 - 2)^(1/4),x)
[Out] int(x^4/(3*x^2 - 2)^(1/4), x)
```

sympy [C] time = 0.78, size = 29, normalized size = 0.12

$$\frac{2^{\frac{3}{4}}x^5e^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2-2)**(1/4),x)
[Out] 2**(3/4)*x**5*exp(-I*pi/4)*hyper((1/4, 5/2), (7/2,), 3*x**2/2)/10
```

3.893 $\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=222

$$\frac{4\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{15\sqrt{3}\,x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x+\frac{8\sqrt[4]{3x^2-2}\,x}{15\left(\sqrt{3x^2-2}+\sqrt{}\right.}$$

```
[Out] 2/15*x*(3*x^2-2)^(3/4)+8/15*x*(3*x^2-2)^(1/4)/(2^(1/2)+(3*x^2-2)^(1/2))-8/4
5*2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan
(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2
^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/
2))^2)^(1/2)/x*3^(1/2)+4/45*2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/
4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*ar
ctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(
x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 230, 305, 220, 1196}

$$\frac{2}{15}\left(3x^2-2\right)^{3/4}x+\frac{8\sqrt[4]{3x^2-2}\,x}{15\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{4\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{15\sqrt{3}\,x}-\frac{8\sqrt[4]{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-2 + 3*x^2)^(3/4))/15 + (8*x*(-2 + 3*x^2)^(1/4))/(15*(Sqrt[2] + Sqrt[
-2 + 3*x^2])) - (8*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2
] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])
/(15*Sqrt[3]*x) + (4*2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt
[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2
])/(15*Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx &= \frac{2}{15}x\left(-2+3x^2\right)^{3/4}+\frac{4}{15}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx\\
&= \frac{2}{15}x\left(-2+3x^2\right)^{3/4}+\frac{\left(4\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{15x}\\
&= \frac{2}{15}x\left(-2+3x^2\right)^{3/4}+\frac{\left(8\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{15\sqrt{3}\,x}-\frac{\left(8\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1-\frac{x^2}{\sqrt{2}}}{\sqrt{1+\frac{x^4}{2}}}\right.}{15\sqrt{3}}\\
&= \frac{2}{15}x\left(-2+3x^2\right)^{3/4}+\frac{8x\sqrt[4]{-2+3x^2}}{15\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}-\frac{8\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(\right.}{15\sqrt{3}\,x}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.26

$$\frac{2x\left(2^{3/4}\sqrt[4]{2-3x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)+3x^2-2\right)}{15\sqrt[4]{3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-2 + 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-2 + 3*x^2 + 2^(3/4)*(2 - 3*x^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/
2, (3*x^2)/2]))/(15*(-2 + 3*x^2)^(1/4))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\left(3x^2-2\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^2/(3*x^2 - 2)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(3*x^2 - 2)^(1/4), x)
```

maple [C] time = 0.29, size = 53, normalized size = 0.24

$$\frac{2\,2^{\frac{3}{4}}\left(-\mathrm{signum}\left(\frac{3x^2}{2}-1\right)\right)^{\frac{1}{4}} x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{15\mathrm{signum}\left(\frac{3x^2}{2}-1\right)^{\frac{1}{4}}}+\frac{2\left(3x^2-2\right)^{\frac{3}{4}} x}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2)^(1/4),x)
```

```
[Out] 2/15*x*(3*x^2-2)^(3/4)+2/15*2^(3/4)/signum(3/2*x^2-1)^(1/4)*(-signum(3/2*x^
2-1))^(1/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(3*x^2 - 2)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(3\,x^2-2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2 - 2)^(1/4),x)
```

```
[Out] int(x^2/(3*x^2 - 2)^(1/4), x)
```

sympy [C] time = 0.73, size = 29, normalized size = 0.13

$$\frac{2^{\frac{3}{4}} x^3 e^{-\frac{i\pi}{4}}\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{3x^2}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**3*exp(-I*pi/4)*hyper((1/4, 3/2), (5/2,), 3*x**2/2)/6
```

3.894 $\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=199

$$\frac{\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{\sqrt{3}\,x}+\frac{2\sqrt[4]{3x^2-2}\,x}{\sqrt{3x^2-2}+\sqrt{2}}-\frac{2\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\ \left(}{}$$

```
[Out] 2*x*(3*x^2-2)^(1/4)/(2^(1/2)+(3*x^2-2)^(1/2))-2/3*2^(1/4)*(cos(2*arctan(1/2
*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4
)))*EllipticE(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1
/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)+1/3*
2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1
/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(
3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2)
)^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {230, 305, 220, 1196}

$$\frac{2\sqrt[4]{3x^2-2}\,x}{\sqrt{3x^2-2}+\sqrt{2}}+\frac{\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt{3}\,x}-\frac{2\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x^2)^(-1/4),x]
```

```
[Out] (2*x*(-2 + 3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 + 3*x^2]) - (2*2^(1/4)*Sqrt[x^2
/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*A
rcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x) + (2^(1/4)*Sqrt[x^2/(S
qrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcT
an[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
```

```
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx = \frac{\left(\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{x}$$

$$= \frac{\left(2\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{\sqrt{3}\,x} - \frac{\left(2\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1-\frac{x^2}{\sqrt{2}}}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right.}{\sqrt{3}\,x}$$

$$= \frac{2x\sqrt[4]{-2+3x^2}}{\sqrt{2}+\sqrt{-2+3x^2}} - \frac{2\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt{3}\,x}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.22

$$\frac{x\sqrt[4]{1-\frac{3x^2}{2}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{3x^2}{2}\right)}{\sqrt[4]{3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x^2)^(-1/4),x]
```

```
[Out] (x*(1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2)/2])/(-2 +
3*x^2)^(1/4)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(3\,x^2-2\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(-1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 - 2)^(-1/4), x)
```

maple [C] time = 0.29, size = 40, normalized size = 0.20

$$\frac{2^{\frac{3}{4}} \left(-\mathrm{signum}\left(\frac{3x^2}{2}-1\right)\right)^{\frac{1}{4}} x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{2\mathrm{signum}\left(\frac{3x^2}{2}-1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)^(1/4),x)
```

```
[Out] 1/2*2^(3/4)/signum(3/2*x^2-1)^(1/4)*(-signum(3/2*x^2-1))^(1/4)*x*hypergeom(
[1/4,1/2],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 - 2)^(-1/4), x)
```

mupad [B] time = 4.74, size = 34, normalized size = 0.17

$$\frac{2^{3/4}\,x\,\left(2-3\,x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{3\,x^2}{2}\right)}{2\,\left(3\,x^2-2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 2)^(1/4),x)
```

```
[Out] (2^(3/4)*x*(2 - 3*x^2)^(1/4)*hypergeom([1/4, 1/2], 3/2, (3*x^2)/2))/(2*(3*x
^2 - 2)^(1/4))
```

sympy [C] time = 0.69, size = 27, normalized size = 0.14

$$\frac{2^{\frac{3}{4}} x e^{-\frac{i\pi}{4}} {}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{3}{2}\end{matrix}\right|\frac{3x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x*exp(-I*pi/4)*hyper((1/4, 1/2), (3/2,), 3*x**2/2)/2
```

3.895 $\int \frac{1}{x^2 \sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=221

$$-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2\,2^{3/4}x}-\frac{3\sqrt[4]{3x^2-2}\,x}{2\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{\left(3x^2-2\right)^{3/4}}{2x}+$$

```
[Out] 1/2*(3*x^2-2)^(3/4)/x-3/2*x*(3*x^2-2)^(1/4)/(2^(1/2)+(3*x^2-2)^(1/2))+1/2*2
^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/
2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3
/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))
^2)^(1/2)/x*3^(1/2)-1/4*2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))
^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan
(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/
(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 230, 305, 220, 1196}

$$-\frac{3\sqrt[4]{3x^2-2}\,x}{2\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{\left(3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{2\,2^{3/4}x}+\frac{\sqrt{3}\sqrt{\frac{}{\left(\sqrt{}\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] (-2 + 3*x^2)^(3/4)/(2*x) - (3*x*(-2 + 3*x^2)^(1/4))/(2*(Sqrt[2] + Sqrt[-2 +
 3*x^2])) + (Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sq
rt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(3
/4)*x) - (Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[
-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2*2^(3/
4)*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx &= \frac{\left(-2+3x^2\right)^{3/4}}{2x}-\frac{3}{4}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx\\
&= \frac{\left(-2+3x^2\right)^{3/4}}{2x}-\frac{\left(\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{2x}\\
&= \frac{\left(-2+3x^2\right)^{3/4}}{2x}-\frac{\left(\sqrt{3}\sqrt{x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{2x}+\frac{\left(\sqrt{3}\sqrt{x^2}\right)\operatorname{Subst}\left(\int \frac{1-\cdots}{\sqrt{1\cdots}}\right.}{2x\cdots}\\
&= \frac{\left(-2+3x^2\right)^{3/4}}{2x}-\frac{3x\sqrt[4]{-2+3x^2}}{2\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\tan\cdots\right.}{2^{3/4}x\cdots}\end{aligned}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.21

$$-\frac{\sqrt[4]{1-\frac{3x^2}{2}}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{3x^2}{2}\right)}{x\sqrt[4]{3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] -(((1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (3*x^2)/2])/(x*(
-2 + 3*x^2)^(1/4)))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(3\,x^2-2\right)^{\frac{3}{4}}}{3\,x^4-2\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(3/4)/(3*x^4 - 2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^2), x)
```

maple [C] time = 0.30, size = 55, normalized size = 0.25

$$-\frac{3\,2^{\frac{3}{4}}\left(-\mathrm{signum}\left(\frac{3x^2}{2}-1\right)\right)^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{8\mathrm{signum}\left(\frac{3x^2}{2}-1\right)^{\frac{1}{4}}}+\frac{\left(3x^2-2\right)^{\frac{3}{4}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2-2)^(1/4),x)
```

```
[Out] 1/2*(3*x^2-2)^(3/4)/x-3/8*2^(3/4)/signum(3/2*x^2-1)^(1/4)*(-signum(3/2*x^2-
1))^(1/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^2), x)
```

mupad [B] time = 4.89, size = 36, normalized size = 0.16

$$-\frac{2\,3^{3/4}\,{\left(3-\frac{2}{x^2}\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ \frac{2}{3\,x^2}\right)}{9\,x\,{\left(3\,x^2-2\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^2 - 2)^(1/4)),x)
```

```
[Out] -(2*3^(3/4)*(3 - 2/x^2)^(1/4)*hypergeom([1/4, 3/4], 7/4, 2/(3*x^2)))/(9*x*(
3*x^2 - 2)^(1/4))
```

sympy [C] time = 0.76, size = 31, normalized size = 0.14

$$\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{1}{2}\end{matrix}\right|\frac{3x^2}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2/2)/(2*x)
```

3.896 $\int \frac{1}{x^4 \sqrt[4]{-2+3x^2}} \, dx$

Optimal. Leaf size=242

$$-\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{8\ 2^{3/4}x}-\frac{9\sqrt[4]{3x^2-2}\,x}{8\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{3\left(3x^2-2\right)^{3/4}}{8x}+$$

```
[Out] 1/6*(3*x^2-2)^(3/4)/x^3+3/8*(3*x^2-2)^(3/4)/x-9/8*x*(3*x^2-2)^(1/4)/(2^(1/2
)+(3*x^2-2)^(1/2))+3/8*2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^
2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(
1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(
2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)-3/16*2^(1/4)*(cos(2*arctan(1/2*
(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)
))*EllipticF(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/
2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 230, 305, 220, 1196}

$$-\frac{9\sqrt[4]{3x^2-2}\,x}{8\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{3\left(3x^2-2\right)^{3/4}}{8x}+\frac{\left(3x^2-2\right)^{3/4}}{6x^3}-\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)\right.}{8\ 2^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] (-2 + 3*x^2)^(3/4)/(6*x^3) + (3*(-2 + 3*x^2)^(3/4))/(8*x) - (9*x*(-2 + 3*x^
2)^(1/4))/(8*(Sqrt[2] + Sqrt[-2 + 3*x^2])) + (3*Sqrt[3]*Sqrt[x^2/(Sqrt[2] +
 Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 +
 3*x^2)^(1/4)/2^(1/4)], 1/2])/(4*2^(3/4)*x) - (3*Sqrt[3]*Sqrt[x^2/(Sqrt[2]
+ Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2
+ 3*x^2)^(1/4)/2^(1/4)], 1/2])/(8*2^(3/4)*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx &= \frac{\left(-2+3x^2\right)^{3/4}}{6x^3}+\frac{3}{4}\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx \\
&= \frac{\left(-2+3x^2\right)^{3/4}}{6x^3}+\frac{3\left(-2+3x^2\right)^{3/4}}{8x}-\frac{9}{16}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx \\
&= \frac{\left(-2+3x^2\right)^{3/4}}{6x^3}+\frac{3\left(-2+3x^2\right)^{3/4}}{8x}-\frac{\left(3\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{8x} \\
&= \frac{\left(-2+3x^2\right)^{3/4}}{6x^3}+\frac{3\left(-2+3x^2\right)^{3/4}}{8x}-\frac{\left(3\sqrt{3}\sqrt{x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{8x}+ - \\
&= \frac{\left(-2+3x^2\right)^{3/4}}{6x^3}+\frac{3\left(-2+3x^2\right)^{3/4}}{8x}-\frac{9x\sqrt[4]{-2+3x^2}}{8\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}+\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.20

$$-\frac{\sqrt[4]{1-\frac{3x^2}{2}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};\frac{3x^2}{2}\right)}{3x^3\sqrt[4]{3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] -1/3*((1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (3*x^2)/2])/
(x^3*(-2 + 3*x^2)^(1/4))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(3\,x^2-2\right)^{\frac{3}{4}}}{3\,x^6-2\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(3/4)/(3*x^6 - 2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^4), x)
```

maple [C] time = 0.29, size = 67, normalized size = 0.28

$$-\frac{9\,2^{\frac{3}{4}}\left(-\mathrm{signum}\left(\frac{3x^2}{2}-1\right)\right)^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{32\mathrm{signum}\left(\frac{3x^2}{2}-1\right)^{\frac{1}{4}}}+\frac{27x^4-6x^2-8}{24\left(3x^2-2\right)^{\frac{1}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2-2)^(1/4),x)
[Out] 1/24*(27*x^4-6*x^2-8)/x^3/(3*x^2-2)^(1/4)-9/32*2^(3/4)/signum(3/2*x^2-1)^(1
/4)*(-signum(3/2*x^2-1))^(1/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\left(3\,x^2-2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(3*x^2 - 2)^(1/4)),x)
[Out] int(1/(x^4*(3*x^2 - 2)^(1/4)), x)
```

sympy [C] time = 0.87, size = 34, normalized size = 0.14

$$\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\,\frac{3x^2}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2-2)**(1/4),x)
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2/2)/(6*x**3)
```

3.897 $\int \frac{1}{x^6 \sqrt[4]{-2+3x^2}}\,dx$

Optimal. Leaf size=260

$$-\frac{63\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{160\ 2^{3/4}x}-\frac{189\sqrt[4]{3x^2-2}\,x}{160\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{63\left(3x^2-\right.}{160x}$$

```
[Out] 1/10*(3*x^2-2)^(3/4)/x^5+7/40*(3*x^2-2)^(3/4)/x^3+63/160*(3*x^2-2)^(3/4)/x-
189/160*x*(3*x^2-2)^(1/4)/(2^(1/2)+(3*x^2-2)^(1/2))+63/160*2^(1/4)*(cos(2*a
rctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/
4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/
2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(
1/2)-63/320*2^(1/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/co
s(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(3*x^2-
2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*
x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 230, 305, 220, 1196}

$$-\frac{189\sqrt[4]{3x^2-2}\,x}{160\left(\sqrt{3x^2-2}+\sqrt{2}\right)}+\frac{63\left(3x^2-2\right)^{3/4}}{160x}+\frac{7\left(3x^2-2\right)^{3/4}}{40x^3}+\frac{\left(3x^2-2\right)^{3/4}}{10x^5}-\frac{63\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{\right.}}{160\ 2^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] (-2 + 3*x^2)^(3/4)/(10*x^5) + (7*(-2 + 3*x^2)^(3/4))/(40*x^3) + (63*(-2 + 3
*x^2)^(3/4))/(160*x) - (189*x*(-2 + 3*x^2)^(1/4))/(160*(Sqrt[2] + Sqrt[-2 +
 3*x^2])) + (63*Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] +
 Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(8
0*2^(3/4)*x) - (63*Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2
] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])
/(160*2^(3/4)*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\sqrt[4]{-2+3x^2}}\,dx &= \frac{\left(-2+3x^2\right)^{3/4}}{10x^5}+\frac{21}{20}\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx \\
&= \frac{\left(-2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(-2+3x^2\right)^{3/4}}{40x^3}+\frac{63}{80}\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx \\
&= \frac{\left(-2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(-2+3x^2\right)^{3/4}}{40x^3}+\frac{63\left(-2+3x^2\right)^{3/4}}{160x}-\frac{189}{320}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx \\
&= \frac{\left(-2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(-2+3x^2\right)^{3/4}}{40x^3}+\frac{63\left(-2+3x^2\right)^{3/4}}{160x}-\frac{\left(63\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x\right.}{160x} \\
&= \frac{\left(-2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(-2+3x^2\right)^{3/4}}{40x^3}+\frac{63\left(-2+3x^2\right)^{3/4}}{160x}-\frac{\left(63\sqrt{3}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x\right.}{160x} \\
&= \frac{\left(-2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(-2+3x^2\right)^{3/4}}{40x^3}+\frac{63\left(-2+3x^2\right)^{3/4}}{160x}-\frac{189x\sqrt[4]{-2+3x^2}}{160\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}+\frac{63\sqrt{3}}{}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.18

$$
-\frac{\sqrt[4]{1-\frac{3x^2}{2}}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};\frac{3x^2}{2}\right)}{5x^5\sqrt[4]{3x^2-2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-2 + 3*x^2)^(1/4)),x]
```

```
[Out] -1/5*((1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, (3*x^2)/2])/
(x^5*(-2 + 3*x^2)^(1/4))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\left(3\,x^2-2\right)^{\frac{3}{4}}}{3\,x^8-2\,x^6},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(1/4),x, algorithm="fricas")
[Out] integral((3*x^2 - 2)^(3/4)/(3*x^8 - 2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^{2}-2\right)^{\frac{1}{4}} x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^6), x)
```

maple [C] time = 0.29, size = 72, normalized size = 0.28

$$-\frac{189\,2^{\frac{3}{4}}\left(-\mathrm{signum}\left(\frac{3x^{2}}{2}-1\right)\right)^{\frac{1}{4}} x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^{2}}{2}\right)}{640\mathrm{signum}\left(\frac{3x^{2}}{2}-1\right)^{\frac{1}{4}}}+\frac{189x^{6}-42x^{4}-8x^{2}-32}{160\left(3x^{2}-2\right)^{\frac{1}{4}} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(3*x^2-2)^(1/4),x)
[Out] 1/160*(189*x^6-42*x^4-8*x^2-32)/x^5/(3*x^2-2)^(1/4)-189/640*2^(3/4)/signum(
3/2*x^2-1)^(1/4)*(-signum(3/2*x^2-1))^(1/4)*x*hypergeom([1/4,1/2],[3/2],3/2
*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^{2}-2\right)^{\frac{1}{4}} x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 - 2)^(1/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{6}\,{\left(3\,x^{2}-2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(3*x^2 - 2)^(1/4)),x)
[Out] int(1/(x^6*(3*x^2 - 2)^(1/4)), x)
```

sympy [C] time = 0.98, size = 34, normalized size = 0.13

$$\frac{2^{\frac{3}{4}} e^{\frac{3i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{5}{2},\frac{1}{4}\\-\frac{3}{2}\end{matrix}\middle|\,\frac{3x^{2}}{2}\right)}{10x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2/2)/(10*x**5)
```

3.898 $\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=260

$$\frac{64\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{1053\sqrt{3}x}-\frac{32\left(-3x^2-2\right)^{3/4}x}{1053}-\frac{128\sqrt[4]{}}{1053\left(\sqrt{-}\right.}$$

```
[Out] -32/1053*x*(-3*x^2-2)^(3/4)+40/1053*x^3*(-3*x^2-2)^(3/4)-2/39*x^5*(-3*x^2-2
)^(3/4)-128/1053*x*(-3*x^2-2)^(1/4)/(2^(1/2)+(-3*x^2-2)^(1/2))-128/3159*2^(
1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2
*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(
3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1
/2))^2)^(1/2)/x*3^(1/2)+64/3159*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*
2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(si
n(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^
(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 230, 305, 220, 1196}

$$-\frac{2}{39}\left(-3x^2-2\right)^{3/4}x^5+\frac{40\left(-3x^2-2\right)^{3/4}x^3}{1053}-\frac{32\left(-3x^2-2\right)^{3/4}x}{1053}-\frac{128\sqrt[4]{-3x^2-2}\,x}{1053\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}+\frac{64\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2+}\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (-32*x*(-2 - 3*x^2)^(3/4))/1053 + (40*x^3*(-2 - 3*x^2)^(3/4))/1053 - (2*x^5
*(-2 - 3*x^2)^(3/4))/39 - (128*x*(-2 - 3*x^2)^(1/4))/(1053*(Sqrt[2] + Sqrt[
-2 - 3*x^2])) - (128*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(S
qrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)],
1/2])/(1053*Sqrt[3]*x) + (64*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2]
)^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^
(1/4)], 1/2])/(1053*Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx = -\frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} - \frac{20}{39}\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx$$

$$= \frac{40x^3\left(-2-3x^2\right)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} + \frac{80}{351}\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx$$

$$= -\frac{32x\left(-2-3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2-3x^2\right)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} - \frac{64\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx}{1053}$$

$$= -\frac{32x\left(-2-3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2-3x^2\right)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} + \frac{\left(64\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int -\right.}{1053}$$

$$= -\frac{32x\left(-2-3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2-3x^2\right)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} + \frac{\left(128\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+}}\right.}{1053\sqrt{3}}$$

$$= -\frac{32x\left(-2-3x^2\right)^{3/4}}{1053} + \frac{40x^3\left(-2-3x^2\right)^{3/4}}{1053} - \frac{2}{39}x^5\left(-2-3x^2\right)^{3/4} - \frac{128x\sqrt[4]{-2-3x^2}}{1053\left(\sqrt{2}+\sqrt{-2-3x^2}\right)} -$$

Mathematica [C] time = 0.02, size = 68, normalized size = 0.26

$$\frac{2x\left(-16\,2^{3/4}\sqrt[4]{3x^2+2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)+81x^6-6x^4+8x^2+32\right)}{1053\sqrt[4]{-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(32 + 8*x^2 - 6*x^4 + 81*x^6 - 16*2^(3/4)*(2 + 3*x^2)^(1/4)*Hypergeome
tric2F1[1/4, 1/2, 3/2, (-3*x^2)/2]))/(1053*(-2 - 3*x^2)^(1/4))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\frac{3159\,x\text{integral}\left(\frac{256\left(-3\,x^2-2\right)^{\frac{3}{4}}}{3159\left(3\,x^4+2\,x^2\right)},x\right)-2\left(81\,x^6-60\,x^4+48\,x^2-64\right)\left(-3\,x^2-2\right)^{\frac{3}{4}}}{3159\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3159*(3159*x*integral(256/3159*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2
*(81*x^6 - 60*x^4 + 48*x^2 - 64)*(-3*x^2 - 2)^(3/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-3*x^2 - 2)^(1/4), x)
```

maple [C] time = 0.27, size = 53, normalized size = 0.20

$$\frac{32\,(-1)^{\frac{3}{4}}\,2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{1053}+\frac{2\left(27x^4-20x^2+16\right)\left(3x^2+2\right)x}{1053\left(-3x^2-2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-3*x^2-2)^(1/4),x)
```

```
[Out] 2/1053*x*(27*x^4-20*x^2+16)*(3*x^2+2)/(-3*x^2-2)^(1/4)+32/1053*(-1)^(3/4)*2
^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-3*x^2 - 2)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(-3\,x^2-2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(- 3*x^2 - 2)^(1/4),x)
```

```
[Out] int(x^6/(- 3*x^2 - 2)^(1/4), x)
```

sympy [C] time = 0.84, size = 34, normalized size = 0.13

$$\frac{2^{\frac{3}{4}}x^7e^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**7*exp(-I*pi/4)*hyper((1/4, 7/2), (9/2,), 3*x**2*exp_polar(I*pi)
/2)/14
```

3.899 $\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=242

$$-\frac{16\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{135\sqrt{3}\,x}+\frac{8}{135}\left(-3x^2-2\right)^{3/4}x+\frac{32}{135\left(\sqrt{\right.}}$$

```
[Out] 8/135*x*(-3*x^2-2)^(3/4)-2/27*x^3*(-3*x^2-2)^(3/4)+32/135*x*(-3*x^2-2)^(1/4
)/(2^(1/2)+(-3*x^2-2)^(1/2))+32/405*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1
/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*Elliptic
E(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2
-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)-16/405*2^(1/
4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(
-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/
4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2
))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 230, 305, 220, 1196}

$$-\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3+\frac{8}{135}\left(-3x^2-2\right)^{3/4}x+\frac{32\sqrt[4]{-3x^2-2}\,x}{135\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}-\frac{16\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\right.}{135\sqrt{3}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (8*x*(-2 - 3*x^2)^(3/4))/135 - (2*x^3*(-2 - 3*x^2)^(3/4))/27 + (32*x*(-2 -
3*x^2)^(1/4))/(135*(Sqrt[2] + Sqrt[-2 - 3*x^2])) + (32*2^(1/4)*Sqrt[-(x^2/(
Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*Ar
cTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x) - (16*2^(1/4)*Sqrt[
-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*Ellipti
cF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(135*Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx = -\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}-\frac{4}{9}\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx$$
$$= \frac{8}{135}x\left(-2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}+\frac{16}{135}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$$
$$= \frac{8}{135}x\left(-2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}-\frac{\left(16\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right.}{135x}$$
$$= \frac{8}{135}x\left(-2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}-\frac{\left(32\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{135\sqrt{3}\,x}+$$
$$= \frac{8}{135}x\left(-2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}+\frac{32x\sqrt[4]{-2-3x^2}}{135\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}+\frac{32\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right.}}}{}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 0.26

$$\frac{2x\left(4\,2^{3/4}\sqrt[4]{3x^2+2}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)+15x^4-2x^2-8\right)}{135\sqrt[4]{-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(-8 - 2*x^2 + 15*x^4 + 4*2^(3/4)*(2 + 3*x^2)^(1/4)*Hypergeometric2F1[1
/4, 1/2, 3/2, (-3*x^2)/2]))/(135*(-2 - 3*x^2)^(1/4))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\frac{405\,x\text{integral}\left(-\frac{64\left(-3\,x^2-2\right)^{\frac{3}{4}}}{405\left(3\,x^4+2\,x^2\right)},x\right)-2\left(15\,x^4-12\,x^2+16\right)\left(-3\,x^2-2\right)^{\frac{3}{4}}}{405\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/405*(405*x*integral(-64/405*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2*(1
5*x^4 - 12*x^2 + 16)*(-3*x^2 - 2)^(3/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-3*x^2 - 2)^(1/4), x)
```

maple [C] time = 0.27, size = 48, normalized size = 0.20

$$-\frac{8\,(-1)^{\frac{3}{4}}\,2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{135}+\frac{2\left(5x^2-4\right)\left(3x^2+2\right)x}{135\left(-3x^2-2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2-2)^(1/4),x)
```

```
[Out] 2/135*x*(5*x^2-4)*(3*x^2+2)/(-3*x^2-2)^(1/4)-8/135*(-1)^(3/4)*2^(3/4)*x*hyp
ergeom([1/4,1/2],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-3*x^2 - 2)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(-3\,x^2-2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(- 3*x^2 - 2)^(1/4),x)
```

```
[Out] int(x^4/(- 3*x^2 - 2)^(1/4), x)
```

sympy [C] time = 0.77, size = 34, normalized size = 0.14

$$\frac{2^{\frac{3}{4}}x^5e^{-\frac{i\pi}{4}}\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{3x^2e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x**5*exp(-I*pi/4)*hyper((1/4, 5/2), (7/2,), 3*x**2*exp_polar(I*pi)
/2)/10
```

3.900 $\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=224

$$\frac{4\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{15\sqrt{3}x}-\frac{2}{15}\left(-3x^2-2\right)^{3/4}x-\frac{8\sqrt[4]{-3x^2-}}{15\left(\sqrt{-3x^2-2}\right.}$$

```
[Out] -2/15*x*(-3*x^2-2)^(3/4)-8/15*x*(-3*x^2-2)^(1/4)/(2^(1/2)+(-3*x^2-2)^(1/2))
-8/45*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*a
rctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(-3*x^2-2)^
(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*
x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)+4/45*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^
(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*Ellipt
icF(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x
^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 230, 305, 220, 1196}

$$-\frac{2}{15}\left(-3x^2-2\right)^{3/4}x-\frac{8\sqrt[4]{-3x^2-2}\,x}{15\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}+\frac{4\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right.}{15\sqrt{3}x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (-2*x*(-2 - 3*x^2)^(3/4))/15 - (8*x*(-2 - 3*x^2)^(1/4))/(15*(Sqrt[2] + Sqrt
[-2 - 3*x^2])) - (8*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sq
rt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1
/2])/(15*Sqrt[3]*x) + (4*2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)
]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4
)], 1/2])/(15*Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx &= -\frac{2}{15}x\left(-2-3x^2\right)^{3/4}-\frac{4}{15}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx\\ &= -\frac{2}{15}x\left(-2-3x^2\right)^{3/4}+\frac{\left(4\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{15x}\\ &= -\frac{2}{15}x\left(-2-3x^2\right)^{3/4}+\frac{\left(8\sqrt{-x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{15\sqrt{3}\,x}-\frac{\left(8\sqrt{-x^2}\right)\mathrm{Subst}\left(\int\right.}{}\\ &= -\frac{2}{15}x\left(-2-3x^2\right)^{3/4}-\frac{8x\sqrt[4]{-2-3x^2}}{15\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}-\frac{8\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right.}{15\sqrt{3}\,x}\end{aligned}$$

Mathematica [C] time = 0.01, size = 58, normalized size = 0.26

$$\frac{2x\left(-2^{3/4}\sqrt[4]{3x^2+2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)+3x^2+2\right)}{15\sqrt[4]{-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-2 - 3*x^2)^(1/4),x]
```

```
[Out] (2*x*(2 + 3*x^2 - 2^(3/4)*(2 + 3*x^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2
, (-3*x^2)/2]))/(15*(-2 - 3*x^2)^(1/4))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\frac{45\,x\mathrm{integral}\left(\frac{16\left(-3\,x^2-2\right)^{\frac{3}{4}}}{45\left(3\,x^4+2\,x^2\right)},x\right)-2\left(3\,x^2-4\right)\left(-3\,x^2-2\right)^{\frac{3}{4}}}{45\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/45*(45*x*integral(16/45*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2*(3*x^2
 - 4)*(-3*x^2 - 2)^(3/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(1/4),x, algorithm="giac")
[Out] integrate(x^2/(-3*x^2 - 2)^(1/4), x)
```

maple [C] time = 0.27, size = 41, normalized size = 0.18

$$\frac{2\,(-1)^{\frac{3}{4}}\,2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{15}+\frac{2\left(3x^2+2\right)x}{15\left(-3x^2-2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2-2)^(1/4),x)
[Out] 2/15*x*(3*x^2+2)/(-3*x^2-2)^(1/4)+2/15*(-1)^(3/4)*2^(3/4)*x*hypergeom([1/4,
1/2],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(1/4),x, algorithm="maxima")
[Out] integrate(x^2/(-3*x^2 - 2)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(-3\,x^2-2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(- 3*x^2 - 2)^(1/4),x)
[Out] int(x^2/(- 3*x^2 - 2)^(1/4), x)
```

sympy [C] time = 0.72, size = 34, normalized size = 0.15

$$\frac{2^{\frac{3}{4}}x^3e^{-\frac{i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\right|\frac{3x^2e^{i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2-2)**(1/4),x)
[Out] 2**(3/4)*x**3*exp(-I*pi/4)*hyper((1/4, 3/2), (5/2,), 3*x**2*exp_polar(I*pi)
/2)/6
```

3.901 $\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=202

$$-\frac{\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{\sqrt{3}x}+\frac{2\sqrt[4]{-3x^2-2}\,x}{\sqrt{-3x^2-2}+\sqrt{2}}+\frac{2\sqrt[4]{2}\sqrt{-\frac{}{(}}}{}$$

```
[Out] 2*x*(-3*x^2-2)^(1/4)/(2^(1/2)+(-3*x^2-2)^(1/2))+2/3*2^(1/4)*(cos(2*arctan(1
/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^
(3/4)))*EllipticE(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*
(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1
/2)-1/3*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2
*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-2
)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-
3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {230, 305, 220, 1196}

$$\frac{2\sqrt[4]{-3x^2-2}\,x}{\sqrt{-3x^2-2}+\sqrt{2}}-\frac{\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt{3}x}+\frac{2\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - 3*x^2)^(-1/4),x]
```

```
[Out] (2*x*(-2 - 3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 - 3*x^2]) + (2*2^(1/4)*Sqrt[-(x
^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[
2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x) - (2^(1/4)*Sqrt[-(x
^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[
2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
```

```
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx = -\frac{\left(\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{x}$$

$$= -\frac{\left(2\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{\sqrt{3}\,x} + \frac{\left(2\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1-\frac{x^2}{\sqrt{2}}}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x\right.}$$

$$= \frac{2x\sqrt[4]{-2-3x^2}}{\sqrt{2}+\sqrt{-2-3x^2}} + \frac{2\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt{3}\,x} -$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.21

$$\frac{x\sqrt[4]{\frac{3x^2}{2}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{3x^2}{2}\right)}{\sqrt[4]{-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - 3*x^2)^(-1/4),x]
```

```
[Out] (x*(1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2)/2])/(-2
- 3*x^2)^(1/4)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\frac{3\,x\text{integral}\left(-\frac{4\left(-3\,x^2-2\right)^{\frac{3}{4}}}{3\left(3\,x^4+2\,x^2\right)}, x\right) - 2\left(-3\,x^2-2\right)^{\frac{3}{4}}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*integral(-4/3*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2*(-3*x^2 -
2)^(3/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-3*x^2 - 2)^(-1/4), x)
```

maple [C] time = 0.27, size = 21, normalized size = 0.10

$$-\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], -\frac{3x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2)^(1/4),x)
```

```
[Out] -1/2*(-1)^(3/4)*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-3*x^2 - 2)^(-1/4), x)
```

mupad [B] time = 4.88, size = 34, normalized size = 0.17

$$\frac{2^{3/4}\,x\,\left(3\,x^2+2\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{3\,x^2}{2}\right)}{2\,\left(-3\,x^2-2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 3*x^2 - 2)^(1/4),x)
```

```
[Out] (2^(3/4)*x*(3*x^2 + 2)^(1/4)*hypergeom([1/4, 1/2], 3/2, -(3*x^2)/2))/(2*(-
3*x^2 - 2)^(1/4))
```

sympy [C] time = 0.69, size = 32, normalized size = 0.16

$$\frac{2^{\frac{3}{4}} x e^{-\frac{i\pi}{4}} {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{3x^{2} e^{i\pi}}{2}} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*x*exp(-I*pi/4)*hyper((1/4, 1/2), (3/2,), 3*x**2*exp_polar(I*pi)/2)
/2
```

3.902 $\int \frac{1}{x^2 \sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=223

$$-\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2\,2^{3/4}x}+\frac{3\sqrt[4]{-3x^2-2}\,x}{2\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}+\frac{\left(-3x^2-2\right.}{2x}$$

```
[Out] 1/2*(-3*x^2-2)^(3/4)/x+3/2*x*(-3*x^2-2)^(1/4)/(2^(1/2)+(-3*x^2-2)^(1/2))+1/
2*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arcta
n(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(-3*x^2-2)^(1/4
)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-
2)^(1/2))^2)^(1/2)/x*3^(1/2)-1/4*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)
*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(s
in(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)
^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 230, 305, 220, 1196}

$$\frac{3\sqrt[4]{-3x^2-2}\,x}{2\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}+\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{2\,2^{3/4}x}+\frac{\sqrt{3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] (-2 - 3*x^2)^(3/4)/(2*x) + (3*x*(-2 - 3*x^2)^(1/4))/(2*(Sqrt[2] + Sqrt[-2 -
 3*x^2])) + (Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] +
 Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2
^(3/4)*x) - (Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] +
 Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2
*2^(3/4)*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx = \frac{\left(-2-3x^2\right)^{3/4}}{2x} + \frac{3}{4}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{2x} - \frac{\left(\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{2x}$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{2x} - \frac{\left(\sqrt{3}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{2x} + \frac{\left(\sqrt{3}\sqrt{-x^2}\right)\text{Subst}\left(\right.}{}$$

$$= \frac{\left(-2-3x^2\right)^{3/4}}{2x} + \frac{3x\sqrt[4]{-2-3x^2}}{2\left(\sqrt{2}+\sqrt{-2-3x^2}\right)} + \frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)E\left(\right.}{2^{3/4}x}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.21

$$-\frac{\sqrt[4]{\frac{3x^2}{2}+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-\frac{3x^2}{2}\right)}{x\sqrt[4]{-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] -(((1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (-3*x^2)/2])/(x*
(-2 - 3*x^2)^(1/4)))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\frac{2\,x\text{integral}\left(-\frac{3\left(-3\,x^2-2\right)^{\frac{3}{4}}}{4\left(3\,x^2+2\right)}, x\right) + \left(-3\,x^2-2\right)^{\frac{3}{4}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x*integral(-3/4*(-3*x^2 - 2)^(3/4)/(3*x^2 + 2), x) + (-3*x^2 - 2)^(3
/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^{2}-2\right)^{\frac{1}{4}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^2), x)
```

maple [C] time = 0.28, size = 43, normalized size = 0.19

$$-\frac{3\left(-1\right)^{\frac{3}{4}}2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^{2}}{2}\right)}{8}-\frac{3x^{2}+2}{2\left(-3x^{2}-2\right)^{\frac{1}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2-2)^(1/4),x)
```

```
[Out] -1/2*(3*x^2+2)/x/(-3*x^2-2)^(1/4)-3/8*(-1)^(3/4)*2^(3/4)*x*hypergeom([1/4,1
/2],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^{2}-2\right)^{\frac{1}{4}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^2), x)
```

mupad [B] time = 5.04, size = 36, normalized size = 0.16

$$-\frac{2\,3^{3/4}\left(\frac{2}{x^{2}}+3\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{2}{3\,x^{2}}\right)}{9\,x\left(-3\,x^{2}-2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(- 3*x^2 - 2)^(1/4)),x)
```

```
[Out] -(2*3^(3/4)*(2/x^2 + 3)^(1/4)*hypergeom([1/4, 3/4], 7/4, -2/(3*x^2)))/(9*x*
(- 3*x^2 - 2)^(1/4))
```

sympy [C] time = 0.76, size = 36, normalized size = 0.16

$$\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\,\frac{3x^{2}e^{i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)
/(2*x)
```

3.903 $\int \frac{1}{x^4 \sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=244

$$\frac{3\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{8\,2^{3/4}x}-\frac{9\sqrt[4]{-3x^2-2}\,x}{8\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}-\frac{3\left(-3x^2\right.}{8}$$

```
[Out] 1/6*(-3*x^2-2)^(3/4)/x^3-3/8*(-3*x^2-2)^(3/4)/x-9/8*x*(-3*x^2-2)^(1/4)/(2^(
1/2)+(-3*x^2-2)^(1/2))-3/8*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/
4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*a
rctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2)
)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)+3/16*2^(1/4)*(cos(2*a
rctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(
1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^
(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)
/x*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 230, 305, 220, 1196}

$$-\frac{9\sqrt[4]{-3x^2-2}\,x}{8\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}-\frac{3\left(-3x^2-2\right)^{3/4}}{8x}+\frac{\left(-3x^2-2\right)^{3/4}}{6x^3}+\frac{3\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan\right.}{8\,2^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] (-2 - 3*x^2)^(3/4)/(6*x^3) - (3*(-2 - 3*x^2)^(3/4))/(8*x) - (9*x*(-2 - 3*x^
2)^(1/4))/(8*(Sqrt[2] + Sqrt[-2 - 3*x^2])) - (3*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2]
 + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-
2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(4*2^(3/4)*x) + (3*Sqrt[3]*Sqrt[-(x^2/(Sqr
t[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTa
n[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(8*2^(3/4)*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt[4]{-2-3x^2}}\,dx &= \frac{\left(-2-3x^2\right)^{3/4}}{6x^3}-\frac{3}{4}\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx \\
&= \frac{\left(-2-3x^2\right)^{3/4}}{6x^3}-\frac{3\left(-2-3x^2\right)^{3/4}}{8x}-\frac{9}{16}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx \\
&= \frac{\left(-2-3x^2\right)^{3/4}}{6x^3}-\frac{3\left(-2-3x^2\right)^{3/4}}{8x}+\frac{\left(3\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{8x} \\
&= \frac{\left(-2-3x^2\right)^{3/4}}{6x^3}-\frac{3\left(-2-3x^2\right)^{3/4}}{8x}+\frac{\left(3\sqrt{3}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{8x}-\frac{(3\sqrt{}}{} \\
&= \frac{\left(-2-3x^2\right)^{3/4}}{6x^3}-\frac{3\left(-2-3x^2\right)^{3/4}}{8x}-\frac{9x\sqrt[4]{-2-3x^2}}{8\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}-\frac{3\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\right.}{}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.20

$$
-\frac{\sqrt[4]{\frac{3x^2}{2}+1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};-\frac{3x^2}{2}\right)}{3x^3\sqrt[4]{-3x^2-2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] -1/3*((1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (-3*x^2)/2])
/(x^3*(-2 - 3*x^2)^(1/4))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$
\frac{24\,x^3\text{integral}\left(\frac{9\left(-3\,x^2-2\right)^{\frac{3}{4}}}{16\left(3\,x^2+2\right)},x\right)-\left(9\,x^2-4\right)\left(-3\,x^2-2\right)^{\frac{3}{4}}}{24\,x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/24*(24*x^3*integral(9/16*(-3*x^2 - 2)^(3/4)/(3*x^2 + 2), x) - (9*x^2 - 4)
*(-3*x^2 - 2)^(3/4))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^4), x)
```

maple [C] time = 0.28, size = 48, normalized size = 0.20

$$\frac{9\,(-1)^{\frac{3}{4}}\,2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{32}+\frac{27x^4+6x^2-8}{24\left(-3x^2-2\right)^{\frac{1}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2-2)^(1/4),x)
```

```
[Out] 1/24*(27*x^4+6*x^2-8)/x^3/(-3*x^2-2)^(1/4)+9/32*(-1)^(3/4)*2^(3/4)*x*hyperg
eom([1/4,1/2],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\left(-3\,x^2-2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(- 3*x^2 - 2)^(1/4)),x)
```

```
[Out] int(1/(x^4*(- 3*x^2 - 2)^(1/4)), x)
```

sympy [C] time = 0.86, size = 39, normalized size = 0.16

$$\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2}\end{matrix}\right|\frac{3x^2e^{i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2
)/(6*x**3)
```

3.904 $\int \frac{1}{x^6 \sqrt[4]{-2-3x^2}}\,dx$

Optimal. Leaf size=262

$$-\frac{63\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{160\, 2^{3/4}x}+\frac{189\sqrt[4]{-3x^2-2}\,x}{160\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}+\frac{63\,(-$$

```
[Out] 1/10*(-3*x^2-2)^(3/4)/x^5-7/40*(-3*x^2-2)^(3/4)/x^3+63/160*(-3*x^2-2)^(3/4)
/x+189/160*x*(-3*x^2-2)^(1/4)/(2^(1/2)+(-3*x^2-2)^(1/2))+63/160*2^(1/4)*(co
s(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2
-2)^(1/4)*2^(3/4)))*EllipticE(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1
/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^
(1/2)/x*3^(1/2)-63/320*2^(1/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))
^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arcta
n(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-
x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {325, 230, 305, 220, 1196}

$$\frac{189\sqrt[4]{-3x^2-2}\,x}{160\left(\sqrt{-3x^2-2}+\sqrt{2}\right)}+\frac{63\left(-3x^2-2\right)^{3/4}}{160x}-\frac{7\left(-3x^2-2\right)^{3/4}}{40x^3}+\frac{\left(-3x^2-2\right)^{3/4}}{10x^5}-\frac{63\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}\right.}{160\,2^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] (-2 - 3*x^2)^(3/4)/(10*x^5) - (7*(-2 - 3*x^2)^(3/4))/(40*x^3) + (63*(-2 - 3
*x^2)^(3/4))/(160*x) + (189*x*(-2 - 3*x^2)^(1/4))/(160*(Sqrt[2] + Sqrt[-2 -
 3*x^2])) + (63*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2
] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])
/(80*2^(3/4)*x) - (63*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(
Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)],
 1/2])/(160*2^(3/4)*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\sqrt[4]{-2-3x^2}}\,dx &= \frac{\left(-2-3x^2\right)^{3/4}}{10x^5}-\frac{21}{20}\int \frac{1}{x^4\sqrt[4]{-2-3x^2}}\,dx \\
&= \frac{\left(-2-3x^2\right)^{3/4}}{10x^5}-\frac{7\left(-2-3x^2\right)^{3/4}}{40x^3}+\frac{63}{80}\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx \\
&= \frac{\left(-2-3x^2\right)^{3/4}}{10x^5}-\frac{7\left(-2-3x^2\right)^{3/4}}{40x^3}+\frac{63\left(-2-3x^2\right)^{3/4}}{160x}+\frac{189}{320}\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx \\
&= \frac{\left(-2-3x^2\right)^{3/4}}{10x^5}-\frac{7\left(-2-3x^2\right)^{3/4}}{40x^3}+\frac{63\left(-2-3x^2\right)^{3/4}}{160x}-\frac{\left(63\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{2}}}\,d\right.}{160x} \\
&= \frac{\left(-2-3x^2\right)^{3/4}}{10x^5}-\frac{7\left(-2-3x^2\right)^{3/4}}{40x^3}+\frac{63\left(-2-3x^2\right)^{3/4}}{160x}-\frac{\left(63\sqrt{3}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,d\right.}{160x} \\
&= \frac{\left(-2-3x^2\right)^{3/4}}{10x^5}-\frac{7\left(-2-3x^2\right)^{3/4}}{40x^3}+\frac{63\left(-2-3x^2\right)^{3/4}}{160x}+\frac{189x\sqrt[4]{-2-3x^2}}{160\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}+\frac{63\sqrt{3}}{}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.18

$$-\frac{\sqrt[4]{\frac{3x^2}{2}+1}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};-\frac{3x^2}{2}\right)}{5x^5\sqrt[4]{-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-2 - 3*x^2)^(1/4)),x]
```

```
[Out] -1/5*((1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, (-3*x^2)/2])
/(x^5*(-2 - 3*x^2)^(1/4))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\frac{160\,x^5\text{integral}\left(-\frac{189\left(-3\,x^2-2\right)^{\frac{3}{4}}}{320\left(3\,x^2+2\right)},x\right)+\left(63\,x^4-28\,x^2+16\right)\left(-3\,x^2-2\right)^{\frac{3}{4}}}{160\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/160*(160*x^5*integral(-189/320*(-3*x^2 - 2)^(3/4)/(3*x^2 + 2), x) + (63*x
^4 - 28*x^2 + 16)*(-3*x^2 - 2)^(3/4))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^{2}-2\right)^{\frac{1}{4}}x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^6), x)
```

maple [C] time = 0.30, size = 53, normalized size = 0.20

$$-\frac{189\,(-1)^{\frac{3}{4}}\,2^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^{2}}{2}\right)}{640}-\frac{189x^{6}+42x^{4}-8x^{2}+32}{160\left(-3x^{2}-2\right)^{\frac{1}{4}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-3*x^2-2)^(1/4),x)
```

```
[Out] -1/160*(189*x^6+42*x^4-8*x^2+32)/x^5/(-3*x^2-2)^(1/4)-189/640*(-1)^(3/4)*2^
(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^{2}-2\right)^{\frac{1}{4}}x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(1/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{6}\,{\left(-3\,x^{2}-2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(- 3*x^2 - 2)^(1/4)),x)
```

```
[Out] int(1/(x^6*(- 3*x^2 - 2)^(1/4)), x)
```

sympy [C] time = 0.97, size = 39, normalized size = 0.15

$$\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{5}{2},\frac{1}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{3x^{2}e^{i\pi}}{2}\right)}{10x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-3*x**2-2)**(1/4),x)
```

```
[Out] 2**(3/4)*exp(3*I*pi/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2
)/(10*x**5)
```

3.905 $\int \frac{x^6}{\left(-2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=138

$$\frac{160\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2079\sqrt{3}\,x}+\frac{160\sqrt[4]{3x^2-2}\,x}{2079}+\frac{2}{33}\sqrt[4]{3x^2-2}\,x^5+\frac{40}{69}$$

```
[Out] 160/2079*x*(3*x^2-2)^(1/4)+40/693*x^3*(3*x^2-2)^(1/4)+2/33*x^5*(3*x^2-2)^(1
/4)+160/6237*2^(3/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/c
os(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(3*x^2
-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3
*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 234, 220}

$$\frac{2}{33}\sqrt[4]{3x^2-2}\,x^5+\frac{40}{693}\sqrt[4]{3x^2-2}\,x^3+\frac{160\sqrt[4]{3x^2-2}\,x}{2079}+\frac{160\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right.\right.}{2079\sqrt{3}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (160*x*(-2 + 3*x^2)^(1/4))/2079 + (40*x^3*(-2 + 3*x^2)^(1/4))/693 + (2*x^5*
(-2 + 3*x^2)^(1/4))/33 + (160*2^(3/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])
^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1
/4)], 1/2])/(2079*Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\left(-2+3x^2\right)^{3/4}}\,dx = \frac{2}{33}x^5\sqrt[4]{-2+3x^2} + \frac{20}{33}\int \frac{x^4}{\left(-2+3x^2\right)^{3/4}}\,dx$$
$$= \frac{40}{693}x^3\sqrt[4]{-2+3x^2} + \frac{2}{33}x^5\sqrt[4]{-2+3x^2} + \frac{80}{231}\int \frac{x^2}{\left(-2+3x^2\right)^{3/4}}\,dx$$
$$= \frac{160x\sqrt[4]{-2+3x^2}}{2079} + \frac{40}{693}x^3\sqrt[4]{-2+3x^2} + \frac{2}{33}x^5\sqrt[4]{-2+3x^2} + \frac{320\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx}{2079}$$
$$= \frac{160x\sqrt[4]{-2+3x^2}}{2079} + \frac{40}{693}x^3\sqrt[4]{-2+3x^2} + \frac{2}{33}x^5\sqrt[4]{-2+3x^2} + \frac{\left(320\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{2079}$$
$$= \frac{160x\sqrt[4]{-2+3x^2}}{2079} + \frac{40}{693}x^3\sqrt[4]{-2+3x^2} + \frac{2}{33}x^5\sqrt[4]{-2+3x^2} + \frac{160\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\,\left(\right.}{}$$

Mathematica [C] time = 0.02, size = 68, normalized size = 0.49

$$\frac{2x\left(80\sqrt[4]{2}\left(2-3x^2\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)+189x^6+54x^4+120x^2-160\right)}{2079\left(3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-2 + 3*x^2)^(3/4),x]
[Out] (2*x*(-160 + 120*x^2 + 54*x^4 + 189*x^6 + 80*2^(1/4)*(2 - 3*x^2)^(3/4)*Hype
rgeometric2F1[1/2, 3/4, 3/2, (3*x^2)/2]))/(2079*(-2 + 3*x^2)^(3/4))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\left(3\,x^2-2\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(3/4),x, algorithm="fricas")
[Out] integral(x^6/(3*x^2 - 2)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(3/4),x, algorithm="giac")
[Out] integrate(x^6/(3*x^2 - 2)^(3/4), x)
```

maple [C] time = 0.30, size = 65, normalized size = 0.47

$$\frac{160\,2^{\frac{1}{4}}\left(-\mathrm{signum}\left(\frac{3x^2}{2}-1\right)\right)^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{2079\mathrm{signum}\left(\frac{3x^2}{2}-1\right)^{\frac{3}{4}}}+\frac{2\left(63x^4+60x^2+80\right)\left(3x^2-2\right)^{\frac{1}{4}}x}{2079}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2-2)^(3/4),x)
```

```
[Out] 2/2079*x*(63*x^4+60*x^2+80)*(3*x^2-2)^(1/4)+160/2079*2^(1/4)/signum(3/2*x^2
-1)^(3/4)*(-signum(3/2*x^2-1))^(3/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(3*x^2 - 2)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{{\left(3\,x^2-2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2 - 2)^(3/4),x)
```

```
[Out] int(x^6/(3*x^2 - 2)^(3/4), x)
```

sympy [C] time = 0.86, size = 31, normalized size = 0.22

$$\frac{\sqrt[4]{2}\,x^7 e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\right|\frac{3x^2}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**7*exp(-3*I*pi/4)*hyper((3/4, 7/2), (9/2,), 3*x**2/2)/14
```

3.906 $\int \frac{x^4}{\left(-2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=120

$$\frac{8\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{63\sqrt{3}\,x}+\frac{8}{63}\sqrt[4]{3x^2-2}\,x+\frac{2}{21}\sqrt[4]{3x^2-2}\,x^3$$

```
[Out] 8/63*x*(3*x^2-2)^(1/4)+2/21*x^3*(3*x^2-2)^(1/4)+8/189*2^(3/4)*(cos(2*arctan
(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^
(3/4)))*EllipticF(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(
2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 234, 220}

$$\frac{2}{21}\sqrt[4]{3x^2-2}\,x^3+\frac{8}{63}\sqrt[4]{3x^2-2}\,x+\frac{8\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{63\sqrt{3}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (8*x*(-2 + 3*x^2)^(1/4))/63 + (2*x^3*(-2 + 3*x^2)^(1/4))/21 + (8*2^(3/4)*Sq
rt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*Ellipti
cF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(63*Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\left(-2+3x^2\right)^{3/4}}\,dx = \frac{2}{21}x^3\sqrt[4]{-2+3x^2} + \frac{4}{7}\int \frac{x^2}{\left(-2+3x^2\right)^{3/4}}\,dx$$
$$= \frac{8}{63}x\sqrt[4]{-2+3x^2} + \frac{2}{21}x^3\sqrt[4]{-2+3x^2} + \frac{16}{63}\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx$$
$$= \frac{8}{63}x\sqrt[4]{-2+3x^2} + \frac{2}{21}x^3\sqrt[4]{-2+3x^2} + \frac{\left(16\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{63x}$$
$$= \frac{8}{63}x\sqrt[4]{-2+3x^2} + \frac{2}{21}x^3\sqrt[4]{-2+3x^2} + \frac{8\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\right.}{63\sqrt{3}\,x}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 0.52

$$\frac{2x\left(4\sqrt[4]{2}\left(2-3x^2\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)+9x^4+6x^2-8\right)}{63\left(3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(-8 + 6*x^2 + 9*x^4 + 4*2^(1/4)*(2 - 3*x^2)^(3/4)*Hypergeometric2F1[1/
2, 3/4, 3/2, (3*x^2)/2]))/(63*(-2 + 3*x^2)^(3/4))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\left(3x^2-2\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(3*x^2 - 2)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(3x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(3*x^2 - 2)^(3/4), x)
```

maple [C] time = 0.32, size = 60, normalized size = 0.50

$$\frac{8\,2^{\frac{1}{4}}\left(-\text{signum}\left(\frac{3x^2}{2}-1\right)\right)^{\frac{3}{4}}x\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{63\text{signum}\left(\frac{3x^2}{2}-1\right)^{\frac{3}{4}}}+\frac{2\left(3x^2+4\right)\left(3x^2-2\right)^{\frac{1}{4}}x}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2-2)^(3/4),x)
```

```
[Out] 2/63*x*(3*x^2+4)*(3*x^2-2)^(1/4)+8/63*2^(1/4)/signum(3/2*x^2-1)^(3/4)*(-sig
num(3/2*x^2-1))^(3/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(3*x^2 - 2)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(3\,x^2-2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^2 - 2)^(3/4),x)
```

```
[Out] int(x^4/(3*x^2 - 2)^(3/4), x)
```

sympy [C] time = 0.77, size = 31, normalized size = 0.26

$$\frac{\sqrt[4]{2}\,x^5 e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{3x^2}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**5*exp(-3*I*pi/4)*hyper((3/4, 5/2), (7/2,), 3*x**2/2)/10
```

3.907 $\int \frac{x^2}{\left(-2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=102

$$\frac{2\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{9\sqrt{3}\,x}+\frac{2}{9}\sqrt[4]{3x^2-2}\,x$$

```
[Out] 2/9*x*(3*x^2-2)^(1/4)+2/27*2^(3/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4
)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arc
tan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x
^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 234, 220}

$$\frac{2}{9}\sqrt[4]{3x^2-2}\,x+\frac{2\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{9\sqrt{3}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-2 + 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(-2 + 3*x^2)^(1/4))/9 + (2*2^(3/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2
])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^
(1/4)], 1/2])/(9*Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(-2+3x^2\right)^{3/4}}\,dx=\frac{2}{9}x\sqrt[4]{-2+3x^2}+\frac{4}{9}\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx$$
$$=\frac{2}{9}x\sqrt[4]{-2+3x^2}+\frac{\left(4\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{9x}$$
$$=\frac{2}{9}x\sqrt[4]{-2+3x^2}+\frac{2\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{9\sqrt{3}\,x}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.56

$$\frac{2x\left(\sqrt[4]{2}\left(2-3x^2\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)+3x^2-2\right)}{9\left(3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-2 + 3*x^2)^(3/4),x]
[Out] (2*x*(-2 + 3*x^2 + 2^(1/4)*(2 - 3*x^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/
2, (3*x^2)/2]))/(9*(-2 + 3*x^2)^(3/4))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\left(3\,x^2-2\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(3/4),x, algorithm="fricas")
[Out] integral(x^2/(3*x^2 - 2)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)^(3/4),x, algorithm="giac")
[Out] integrate(x^2/(3*x^2 - 2)^(3/4), x)
```

maple [C] time = 0.29, size = 53, normalized size = 0.52

$$\frac{2\,2^{\frac{1}{4}}\left(-\text{signum}\left(\frac{3x^2}{2}-1\right)\right)^{\frac{3}{4}}x\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{9\text{signum}\left(\frac{3x^2}{2}-1\right)^{\frac{3}{4}}}+\frac{2\left(3x^2-2\right)^{\frac{1}{4}}x}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(3*x^2-2)^(3/4),x)

[Out] 2/9*x*(3*x^2-2)^(1/4)+2/9*2^(1/4)/signum(3/2*x^2-1)^(3/4)*(-signum(3/2*x^2-1))^(3/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(3x^2-2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(3*x^2-2)^(3/4),x, algorithm="maxima")

[Out] integrate(x^2/(3*x^2 - 2)^(3/4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(3x^2-2)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(3*x^2 - 2)^(3/4),x)

[Out] int(x^2/(3*x^2 - 2)^(3/4), x)

sympy [C] time = 0.74, size = 31, normalized size = 0.30

$$\frac{\sqrt[4]{2}\,x^3 e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{3x^2}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(3*x**2-2)**(3/4),x)

[Out] 2**(1/4)*x**3*exp(-3*I*pi/4)*hyper((3/4, 3/2), (5/2,), 3*x**2/2)/6

3.908 $\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}\,x}$$

```
[Out] 1/6*2^(3/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arct
an(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(3*x^2-2)^(1/4)
*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(
1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {234, 220}

$$\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x^2)^(-3/4),x]
```

```
[Out] (Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*Elli
pticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(1/4)*Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx &= \frac{\left(\sqrt{\frac{2}{3}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{x}\\ &= \frac{\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}\,x}\end{aligned}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.52

$$\frac{x\left(1-\frac{3x^2}{2}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{3x^2}{2}\right)}{\left(3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x^2)^(-3/4),x]
```

```
[Out] (x*(1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (3*x^2)/2])/(-2 +
 3*x^2)^(3/4)
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(-3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 - 2)^(-3/4), x)
```

maple [C] time = 0.29, size = 40, normalized size = 0.49

$$\frac{2^{\frac{1}{4}}\left(-\mathrm{signum}\left(\frac{3x^2}{2}-1\right)\right)^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{2\mathrm{signum}\left(\frac{3x^2}{2}-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)^(3/4),x)
```

```
[Out] 1/2*2^(1/4)/signum(3/2*x^2-1)^(3/4)*(-signum(3/2*x^2-1))^(3/4)*x*hypergeom(
[1/2,3/4],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 - 2)^(-3/4), x)
```

mupad [B] time = 4.87, size = 34, normalized size = 0.41

$$\frac{2^{1/4}\,x\,{\left(2-3\,x^2\right)}^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ \frac{3\,x^2}{2}\right)}{2\,{\left(3\,x^2-2\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 2)^(3/4),x)
```

```
[Out] (2^(1/4)*x*(2 - 3*x^2)^(3/4)*hypergeom([1/2, 3/4], 3/2, (3*x^2)/2))/(2*(3*x
^2 - 2)^(3/4))
```

sympy [C] time = 0.71, size = 29, normalized size = 0.35

$$\frac{\sqrt[4]{2}\, x e^{-\frac{3i\pi}{4}} {}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\right| \frac{3x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x*exp(-3*I*pi/4)*hyper((1/2, 3/4), (3/2,), 3*x**2/2)/2
```

3.909 $\int \frac{1}{x^2\left(-2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=104

$$\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{4\sqrt[4]{2}\,x}+\frac{\sqrt[4]{3x^2-2}}{2x}$$

```
[Out] 1/2*(3*x^2-2)^(1/4)/x+1/8*2^(3/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)
))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arct
an(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^
2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 234, 220}

$$\frac{\sqrt[4]{3x^2-2}}{2x}+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] (-2 + 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2
]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4
)], 1/2])/(4*2^(1/4)*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(-2+3x^2\right)^{3/4}}\,dx &= \frac{\sqrt[4]{-2+3x^2}}{2x}+\frac{3}{4}\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx\\ &= \frac{\sqrt[4]{-2+3x^2}}{2x}+\frac{\left(\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2+3x^2}\right)}{2x}\\ &= \frac{\sqrt[4]{-2+3x^2}}{2x}+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{2}\,x}\end{aligned}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.44

$$-\frac{\left(1-\frac{3x^2}{2}\right)^{3/4}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};\frac{3x^2}{2}\right)}{x\left(3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-2 + 3*x^2)^(3/4)),x]
[Out] -(((1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (3*x^2)/2])/(x*(
-2 + 3*x^2)^(3/4)))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,x^2-2\right)^{\frac{1}{4}}}{3\,x^4-2\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(3/4),x, algorithm="fricas")
[Out] integral((3*x^2 - 2)^(1/4)/(3*x^4 - 2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^2), x)
```

maple [C] time = 0.29, size = 55, normalized size = 0.53

$$\frac{3\,2^{\frac{1}{4}}\left(-\mathrm{signum}\left(\frac{3x^2}{2}-1\right)\right)^{\frac{3}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{8\mathrm{signum}\left(\frac{3x^2}{2}-1\right)^{\frac{3}{4}}}+\frac{\left(3x^2-2\right)^{\frac{1}{4}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^2-2)^(3/4),x)
```

```
[Out] 1/2*(3*x^2-2)^(1/4)/x+3/8*2^(1/4)/signum(3/2*x^2-1)^(3/4)*(-signum(3/2*x^2-
1))^(3/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^2), x)
```

mupad [B] time = 5.07, size = 23, normalized size = 0.22

$$-\frac{2\,3^{1/4}\left(\frac{1}{x^2}\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ \frac{2}{3\,x^2}\right)}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^2 - 2)^(3/4)),x)
```

```
[Out] -(2*3^(1/4)*(1/x^2)^(3/4)*hypergeom([3/4, 5/4], 9/4, 2/(3*x^2)))/(15*x)
```

sympy [C] time = 0.84, size = 29, normalized size = 0.28

$$\frac{\sqrt[4]{2}\,e^{\frac{i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{1}{2}\end{matrix}\right|\frac{3x^2}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2/2)/(2*x)
```

3.910 $\int \frac{1}{x^4(-2+3x^2)^{3/4}} \, dx$

Optimal. Leaf size=122

$$\frac{5\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{16\sqrt[4]{2}\,x}+\frac{5\sqrt[4]{3x^2-2}}{8x}+\frac{\sqrt[4]{3x^2-2}}{6x^3}$$

```
[Out] 1/6*(3*x^2-2)^(1/4)/x^3+5/8*(3*x^2-2)^(1/4)/x+5/32*2^(3/4)*(cos(2*arctan(1/
2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/
4)))*EllipticF(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(
1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 234, 220}

$$\frac{5\sqrt[4]{3x^2-2}}{8x}+\frac{\sqrt[4]{3x^2-2}}{6x^3}+\frac{5\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{16\sqrt[4]{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] (-2 + 3*x^2)^(1/4)/(6*x^3) + (5*(-2 + 3*x^2)^(1/4))/(8*x) + (5*Sqrt[3]*Sqrt
[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF
[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(16*2^(1/4)*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(-2+3x^2\right)^{3/4}}\,dx &= \frac{\sqrt[4]{-2+3x^2}}{6x^3}+\frac{5}{4}\int \frac{1}{x^2\left(-2+3x^2\right)^{3/4}}\,dx\\ &= \frac{\sqrt[4]{-2+3x^2}}{6x^3}+\frac{5\sqrt[4]{-2+3x^2}}{8x}+\frac{15}{16}\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx\\ &= \frac{\sqrt[4]{-2+3x^2}}{6x^3}+\frac{5\sqrt[4]{-2+3x^2}}{8x}+\frac{\left(5\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2+3x^2}\right)}{8x}\\ &= \frac{\sqrt[4]{-2+3x^2}}{6x^3}+\frac{5\sqrt[4]{-2+3x^2}}{8x}+\frac{5\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{}}{}\right.\right.}{16\sqrt[4]{2}\,x}\end{aligned}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.39

$$-\frac{\left(1-\frac{3x^2}{2}\right)^{3/4}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};\frac{3x^2}{2}\right)}{3x^3\left(3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-2 + 3*x^2)^(3/4)),x]
[Out] -1/3*((1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (3*x^2)/2])/
(x^3*(-2 + 3*x^2)^(3/4))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,x^2-2\right)^{\frac{1}{4}}}{3\,x^6-2\,x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(3/4),x, algorithm="fricas")
[Out] integral((3*x^2 - 2)^(1/4)/(3*x^6 - 2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^4), x)
```

maple [C] time = 0.29, size = 67, normalized size = 0.55

$$\frac{15\,2^{\frac{1}{4}}\left(-\text{signum}\left(\frac{3x^2}{2}-1\right)\right)^{\frac{3}{4}}x\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{32\text{signum}\left(\frac{3x^2}{2}-1\right)^{\frac{3}{4}}}+\frac{45x^4-18x^2-8}{24\left(3x^2-2\right)^{\frac{3}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(3*x^2-2)^(3/4),x)
```

```
[Out] 1/24*(45*x^4-18*x^2-8)/x^3/(3*x^2-2)^(3/4)+15/32*2^(1/4)/signum(3/2*x^2-1)^
(3/4)*(-signum(3/2*x^2-1))^(3/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\left(3\,x^2-2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(3*x^2 - 2)^(3/4)),x)
```

```
[Out] int(1/(x^4*(3*x^2 - 2)^(3/4)), x)
```

sympy [C] time = 0.96, size = 32, normalized size = 0.26

$$\frac{\sqrt[4]{2}\,e^{\frac{i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2/2)/(6*x**3)
```

3.911 $\int \frac{1}{x^6\left(-2+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=140

$$\frac{27\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{64\sqrt[4]{2}\,x}+\frac{27\sqrt[4]{3x^2-2}}{32x}+\frac{\sqrt[4]{3x^2-2}}{10x^5}+\frac{9\sqrt[4]{3x^2-2}}{40x^3}$$

```
[Out] 1/10*(3*x^2-2)^(1/4)/x^5+9/40*(3*x^2-2)^(1/4)/x^3+27/32*(3*x^2-2)^(1/4)/x+2
7/128*2^(3/4)*(cos(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*ar
ctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(3*x^2-2)^(1/
4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(3*x^2-2)^(1/2))*(x^2/(2^(1/2)+(3*x^2-2)
^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 234, 220}

$$\frac{27\sqrt[4]{3x^2-2}}{32x}+\frac{9\sqrt[4]{3x^2-2}}{40x^3}+\frac{\sqrt[4]{3x^2-2}}{10x^5}+\frac{27\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{64\sqrt[4]{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] (-2 + 3*x^2)^(1/4)/(10*x^5) + (9*(-2 + 3*x^2)^(1/4))/(40*x^3) + (27*(-2 + 3
*x^2)^(1/4))/(32*x) + (27*Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*
(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)]
, 1/2])/(64*2^(1/4)*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\left(-2+3x^2\right)^{3/4}}\,dx = \frac{\sqrt[4]{-2+3x^2}}{10x^5} + \frac{27}{20}\int \frac{1}{x^4\left(-2+3x^2\right)^{3/4}}\,dx$$

$$= \frac{\sqrt[4]{-2+3x^2}}{10x^5} + \frac{9\sqrt[4]{-2+3x^2}}{40x^3} + \frac{27}{16}\int \frac{1}{x^2\left(-2+3x^2\right)^{3/4}}\,dx$$

$$= \frac{\sqrt[4]{-2+3x^2}}{10x^5} + \frac{9\sqrt[4]{-2+3x^2}}{40x^3} + \frac{27\sqrt[4]{-2+3x^2}}{32x} + \frac{81}{64}\int \frac{1}{\left(-2+3x^2\right)^{3/4}}\,dx$$

$$= \frac{\sqrt[4]{-2+3x^2}}{10x^5} + \frac{9\sqrt[4]{-2+3x^2}}{40x^3} + \frac{27\sqrt[4]{-2+3x^2}}{32x} + \frac{\left(27\sqrt{\frac{3}{2}}\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x,\right.}{32x}$$

$$= \frac{\sqrt[4]{-2+3x^2}}{10x^5} + \frac{9\sqrt[4]{-2+3x^2}}{40x^3} + \frac{27\sqrt[4]{-2+3x^2}}{32x} + \frac{27\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2\right.}{64\sqrt[4]{2}}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.34

$$-\frac{\left(1-\frac{3x^2}{2}\right)^{3/4}\,{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};\frac{3x^2}{2}\right)}{5x^5\left(3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-2 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/5*((1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, (3*x^2)/2])/
(x^5*(-2 + 3*x^2)^(3/4))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(3\,x^2-2\right)^{\frac{1}{4}}}{3\,x^8-2\,x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 - 2)^(1/4)/(3*x^8 - 2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^6), x)
```

maple [C] time = 0.29, size = 72, normalized size = 0.51

$$\frac{81\,2^{\frac{1}{4}}\left(-\mathrm{signum}\left(\frac{3x^2}{2}-1\right)\right)^{\frac{3}{4}} x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{128\mathrm{signum}\left(\frac{3x^2}{2}-1\right)^{\frac{3}{4}}}+\frac{405x^6-162x^4-24x^2-32}{160\left(3x^2-2\right)^{\frac{3}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(3*x^2-2)^(3/4),x)
```

```
[Out] 1/160*(405*x^6-162*x^4-24*x^2-32)/x^5/(3*x^2-2)^(3/4)+81/128*2^(1/4)/signum
(3/2*x^2-1)^(3/4)*(-signum(3/2*x^2-1))^(3/4)*x*hypergeom([1/2,3/4],[3/2],3/
2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(3\,x^2-2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(3*x^2 - 2)^(3/4)),x)
```

```
[Out] int(1/(x^6*(3*x^2 - 2)^(3/4)), x)
```

sympy [C] time = 1.11, size = 32, normalized size = 0.23

$$\frac{\sqrt[4]{2}\,e^{\frac{i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{5}{2},\frac{3}{4}\\-\frac{3}{2}\end{matrix}\middle|\,\frac{3x^2}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2/2)/(10*x**5)
```

3.912 $\int \frac{x^6}{\left(-2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=139

$$\frac{160\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2079\sqrt{3}\,x}-\frac{160\sqrt[4]{-3x^2-2}\,x}{2079}-\frac{2}{33}\sqrt[4]{-3x^2}$$

```
[Out] -160/2079*x*(-3*x^2-2)^(1/4)+40/693*x^3*(-3*x^2-2)^(1/4)-2/33*x^5*(-3*x^2-2
)^(1/4)+160/6237*2^(3/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1
/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*
(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2
^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 234, 220}

$$-\frac{2}{33}\sqrt[4]{-3x^2-2}\,x^5+\frac{40}{693}\sqrt[4]{-3x^2-2}\,x^3-\frac{160\sqrt[4]{-3x^2-2}\,x}{2079}+\frac{160\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\text{t}\right.}{2079\sqrt{3}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (-160*x*(-2 - 3*x^2)^(1/4))/2079 + (40*x^3*(-2 - 3*x^2)^(1/4))/693 - (2*x^5
*(-2 - 3*x^2)^(1/4))/33 + (160*2^(3/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^
2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/
2^(1/4)], 1/2])/(2079*Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\left(-2-3x^2\right)^{3/4}}\,dx = -\frac{2}{33}x^5\sqrt[4]{-2-3x^2} - \frac{20}{33}\int \frac{x^4}{\left(-2-3x^2\right)^{3/4}}\,dx$$
$$= \frac{40}{693}x^3\sqrt[4]{-2-3x^2} - \frac{2}{33}x^5\sqrt[4]{-2-3x^2} + \frac{80}{231}\int \frac{x^2}{\left(-2-3x^2\right)^{3/4}}\,dx$$
$$= -\frac{160x\sqrt[4]{-2-3x^2}}{2079} + \frac{40}{693}x^3\sqrt[4]{-2-3x^2} - \frac{2}{33}x^5\sqrt[4]{-2-3x^2} - \frac{320\int \frac{1}{\left(-2-3x^2\right)^{3/4}}\,dx}{2079}$$
$$= -\frac{160x\sqrt[4]{-2-3x^2}}{2079} + \frac{40}{693}x^3\sqrt[4]{-2-3x^2} - \frac{2}{33}x^5\sqrt[4]{-2-3x^2} + \frac{\left(320\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{2079x}$$
$$= -\frac{160x\sqrt[4]{-2-3x^2}}{2079} + \frac{40}{693}x^3\sqrt[4]{-2-3x^2} - \frac{2}{33}x^5\sqrt[4]{-2-3x^2} + \frac{160\ 2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{}\right.}{}$$

Mathematica [C] time = 0.02, size = 68, normalized size = 0.49

$$\frac{2x\left(-80\sqrt[4]{2}\left(3x^2+2\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)+189x^6-54x^4+120x^2+160\right)}{2079\left(-3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(160 + 120*x^2 - 54*x^4 + 189*x^6 - 80*2^(1/4)*(2 + 3*x^2)^(3/4)*Hyper
geometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2]))/(2079*(-2 - 3*x^2)^(3/4))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$-\frac{2}{2079}\left(63\,x^5-60\,x^3+80\,x\right)\left(-3\,x^2-2\right)^{\frac{1}{4}}+\text{integral}\left(\frac{320\left(-3\,x^2-2\right)^{\frac{1}{4}}}{2079\left(3\,x^2+2\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/2079*(63*x^5 - 60*x^3 + 80*x)*(-3*x^2 - 2)^(1/4) + integral(320/2079*(-3
*x^2 - 2)^(1/4)/(3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-3*x^2 - 2)^(3/4), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-3x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-3*x^2-2)^(3/4),x)
[Out] int(x^6/(-3*x^2-2)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2-2)^(3/4),x, algorithm="maxima")
[Out] integrate(x^6/(-3*x^2 - 2)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(-3\,x^2-2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(- 3*x^2 - 2)^(3/4),x)
[Out] int(x^6/(- 3*x^2 - 2)^(3/4), x)
```

sympy [C] time = 0.84, size = 36, normalized size = 0.26

$$\frac{\sqrt[4]{2}\,x^7 e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\,\frac{3x^2e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-3*x**2-2)**(3/4),x)
[Out] 2**(1/4)*x**7*exp(-3*I*pi/4)*hyper((3/4, 7/2), (9/2,), 3*x**2*exp_polar(I*p
i)/2)/14
```

3.913 $\int \frac{x^4}{\left(-2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=121

$$-\frac{8\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{63\sqrt{3}\,x}+\frac{8}{63}\sqrt[4]{-3x^2-2}\,x-\frac{2}{21}\sqrt[4]{-3x^2-2}\,x$$

```
[Out] 8/63*x*(-3*x^2-2)^(1/4)-2/21*x^3*(-3*x^2-2)^(1/4)-8/189*2^(3/4)*(cos(2*arct
an(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4
)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/
2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*
3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 234, 220}

$$-\frac{2}{21}\sqrt[4]{-3x^2-2}\,x^3+\frac{8}{63}\sqrt[4]{-3x^2-2}\,x-\frac{8\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{63\sqrt{3}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (8*x*(-2 - 3*x^2)^(1/4))/63 - (2*x^3*(-2 - 3*x^2)^(1/4))/21 - (8*2^(3/4)*Sq
rt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*Elli
pticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(63*Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(-2-3x^2\right)^{3/4}}\,dx &= -\frac{2}{21}x^3\sqrt[4]{-2-3x^2}-\frac{4}{7}\int \frac{x^2}{\left(-2-3x^2\right)^{3/4}}\,dx\\ &= \frac{8}{63}x\sqrt[4]{-2-3x^2}-\frac{2}{21}x^3\sqrt[4]{-2-3x^2}+\frac{16}{63}\int \frac{1}{\left(-2-3x^2\right)^{3/4}}\,dx\\ &= \frac{8}{63}x\sqrt[4]{-2-3x^2}-\frac{2}{21}x^3\sqrt[4]{-2-3x^2}-\frac{\left(16\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}dx,x,\sqrt[4]{-2-3x^2}\right)}{63x}\\ &= \frac{8}{63}x\sqrt[4]{-2-3x^2}-\frac{2}{21}x^3\sqrt[4]{-2-3x^2}-\frac{8\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\,\mathrm{t}\right.}{63\sqrt{3}\,x}\end{aligned}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 0.52

$$\frac{2x\left(4\sqrt[4]{2}\left(3x^2+2\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)+9x^4-6x^2-8\right)}{63\left(-3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (2*x*(-8 - 6*x^2 + 9*x^4 + 4*2^(1/4)*(2 + 3*x^2)^(3/4)*Hypergeometric2F1[1/
2, 3/4, 3/2, (-3*x^2)/2]))/(63*(-2 - 3*x^2)^(3/4))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$-\frac{2}{63}\left(3\,x^3-4\,x\right)\left(-3\,x^2-2\right)^{\frac{1}{4}}+\text{integral}\left(-\frac{16\left(-3\,x^2-2\right)^{\frac{1}{4}}}{63\left(3\,x^2+2\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/63*(3*x^3 - 4*x)*(-3*x^2 - 2)^(1/4) + integral(-16/63*(-3*x^2 - 2)^(1/4)
/(3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-3*x^2 - 2)^(3/4), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-3x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-3*x^2-2)^(3/4),x)
```

```
[Out] int(x^4/(-3*x^2-2)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-3*x^2 - 2)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(-3\,x^2-2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(- 3*x^2 - 2)^(3/4),x)
```

```
[Out] int(x^4/(- 3*x^2 - 2)^(3/4), x)
```

sympy [C] time = 0.76, size = 36, normalized size = 0.30

$$\frac{\sqrt[4]{2}\,x^5 e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**5*exp(-3*I*pi/4)*hyper((3/4, 5/2), (7/2,), 3*x**2*exp_polar(I*p
i)/2)/10
```

3.914 $\int \frac{x^2}{\left(-2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=103

$$\frac{2\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{9\sqrt{3}\,x}-\frac{2}{9}x\sqrt[4]{-3x^2-2}$$

```
[Out] -2/9*x*(-3*x^2-2)^(1/4)+2/27*2^(3/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(
3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2
*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/
2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 234, 220}

$$\frac{2\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{9\sqrt{3}\,x}-\frac{2}{9}x\sqrt[4]{-3x^2-2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-2 - 3*x^2)^(3/4),x]
```

```
[Out] (-2*x*(-2 - 3*x^2)^(1/4))/9 + (2*2^(3/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*
x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4
)/2^(1/4)], 1/2])/(9*Sqrt[3]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(-2-3x^2\right)^{3/4}}\,dx &= -\frac{2}{9}x\sqrt[4]{-2-3x^2} - \frac{4}{9}\int \frac{1}{\left(-2-3x^2\right)^{3/4}}\,dx \\
&= -\frac{2}{9}x\sqrt[4]{-2-3x^2} + \frac{\left(4\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{9x} \\
&= -\frac{2}{9}x\sqrt[4]{-2-3x^2} + \frac{2\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{9\sqrt{3}\,x}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 58, normalized size = 0.56

$$
\frac{2x\left(-\sqrt[4]{2}\left(3x^2+2\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)+3x^2+2\right)}{9\left(-3x^2-2\right)^{3/4}}
$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(-2 - 3*x^2)^(3/4),x]

[Out] (2*x*(2 + 3*x^2 - 2^(1/4)*(2 + 3*x^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2]))/(9*(-2 - 3*x^2)^(3/4))

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$
-\frac{2}{9}\left(-3\,x^2-2\right)^{\frac{1}{4}}x+\text{integral}\left(\frac{4\left(-3\,x^2-2\right)^{\frac{1}{4}}}{9\left(3\,x^2+2\right)},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-3*x^2-2)^(3/4),x, algorithm="fricas")

[Out] -2/9*(-3*x^2 - 2)^(1/4)*x + integral(4/9*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^2}{\left(-3\,x^2-2\right)^{\frac{3}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-3*x^2-2)^(3/4),x, algorithm="giac")

[Out] integrate(x^2/(-3*x^2 - 2)^(3/4), x)

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$
\int \frac{x^2}{\left(-3x^2-2\right)^{\frac{3}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-3*x^2-2)^(3/4),x)

```
[Out] int(x^2/(-3*x^2-2)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-3*x^2 - 2)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(-3\,x^2-2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(- 3*x^2 - 2)^(3/4),x)
```

```
[Out] int(x^2/(- 3*x^2 - 2)^(3/4), x)
```

sympy [C] time = 0.72, size = 36, normalized size = 0.35

$$\frac{\sqrt[4]{2}\,x^3 e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{3x^2e^{i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**3*exp(-3*I*pi/4)*hyper((3/4, 3/2), (5/2,), 3*x**2*exp_polar(I*p
i)/2)/6
```

3.915 $\int \frac{1}{\left(-2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}\,x}$$

```
[Out] -1/6*2^(3/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*ar
ctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-2)^(
1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x
^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {234, 220}

$$-\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - 3*x^2)^(-3/4),x]
```

```
[Out] -((Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])
*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(1/4)*Sqrt[3]*x))
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-2-3x^2\right)^{3/4}}\,dx &= -\frac{\left(\sqrt{\frac{2}{3}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{x}\\ &= -\frac{\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}\,x}\end{aligned}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.51

$$\frac{x\left(\frac{3x^2}{2}+1\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{3x^2}{2}\right)}{\left(-3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - 3*x^2)^(-3/4),x]
```

```
[Out] (x*(1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2])/(-2
- 3*x^2)^(3/4)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-3\,x^2-2\right)^{\frac{1}{4}}}{3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((-3*x^2 - 2)^(-3/4), x)
```

maple [C] time = 0.27, size = 21, normalized size = 0.25

$$-\frac{(-1)^{\frac{1}{4}}\,2^{\frac{1}{4}}x\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-2)^(3/4),x)
```

```
[Out] -1/2*(-1)^(1/4)*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-3/2*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-3*x^2 - 2)^(-3/4), x)
```

mupad [B] time = 4.61, size = 34, normalized size = 0.40

$$\frac{2^{1/4}\,x\,\left(3\,x^2+2\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ -\frac{3\,x^2}{2}\right)}{2\,\left(-3\,x^2-2\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 3*x^2 - 2)^(3/4),x)
```

[Out] (2^(1/4)*x*(3*x^2 + 2)^(3/4)*hypergeom([1/2, 3/4], 3/2, -(3*x^2)/2))/(2*(-3*x^2 - 2)^(3/4))

sympy [C] time = 0.69, size = 34, normalized size = 0.40

$$\frac{\sqrt[4]{2}\, x e^{-\frac{3i\pi}{4}} {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{3x^2 e^{i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x*exp(-3*I*pi/4)*hyper((1/2, 3/4), (3/2,), 3*x**2*exp_polar(I*pi)/2)/2
```

3.916 $\int \frac{1}{x^2\left(-2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\operatorname{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{4\sqrt[4]{2}\,x}+\frac{\sqrt[4]{-3x^2-2}}{2x}$$

```
[Out] 1/2*(-3*x^2-2)^(1/4)/x+1/8*2^(3/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/
4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*a
rctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2)
)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 234, 220}

$$\frac{\sqrt[4]{-3x^2-2}}{2x}+\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-2 - 3*x^2)^(3/4)),x]
```

```
[Out] (-2 - 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])
^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(
1/4)], 1/2])/(4*2^(1/4)*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(-2-3x^2\right)^{3/4}}\,dx = \frac{\sqrt[4]{-2-3x^2}}{2x} - \frac{3}{4}\int \frac{1}{\left(-2-3x^2\right)^{3/4}}\,dx$$
$$= \frac{\sqrt[4]{-2-3x^2}}{2x} + \frac{\left(\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{-2-3x^2}\right)}{2x}$$
$$= \frac{\sqrt[4]{-2-3x^2}}{2x} + \frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{2}\,x}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.44

$$-\frac{\left(\frac{3x^2}{2}+1\right)^{3/4}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};-\frac{3x^2}{2}\right)}{x\left(-3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-2 - 3*x^2)^(3/4)),x]
[Out] -(((1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (-3*x^2)/2])/(x*
(-2 - 3*x^2)^(3/4)))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\frac{2\,x\text{integral}\left(\frac{3\left(-3\,x^2-2\right)^{\frac{1}{4}}}{4\left(3\,x^2+2\right)}, x\right) + \left(-3\,x^2-2\right)^{\frac{1}{4}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(3/4),x, algorithm="fricas")
[Out] 1/2*(2*x*integral(3/4*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x) + (-3*x^2 - 2)^(1/
4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^2-2\right)^{\frac{3}{4}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-3*x^2-2)^(3/4),x)
```

```
[Out] int(1/x^2/(-3*x^2-2)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^{2}-2\right)^{\frac{3}{4}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^2), x)
```

mupad [B] time = 4.73, size = 36, normalized size = 0.34

$$-\frac{2\,3^{1/4}\,\left(\frac{2}{x^2}+3\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ -\frac{2}{3\,x^2}\right)}{15\,x\,\left(-3\,x^2-2\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(- 3*x^2 - 2)^(3/4)),x)
```

```
[Out] -(2*3^(1/4)*(2/x^2 + 3)^(3/4)*hypergeom([3/4, 5/4], 9/4, -2/(3*x^2)))/(15*x
*(- 3*x^2 - 2)^(3/4))
```

sympy [C] time = 0.84, size = 34, normalized size = 0.32

$$\frac{\sqrt[4]{2}\,e^{\frac{i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2}\end{matrix}\right|\frac{3x^2e^{i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)/(
2*x)
```

3.917 $\int \frac{1}{x^4\left(-2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=123

$$-\frac{5\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{16\sqrt[4]{2}\,x}-\frac{5\sqrt[4]{-3x^2-2}}{8x}+\frac{\sqrt[4]{-3x^2-2}}{6x^3}$$

```
[Out] 1/6*(-3*x^2-2)^(1/4)/x^3-5/8*(-3*x^2-2)^(1/4)/x-5/32*2^(3/4)*(cos(2*arctan(
1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2
^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))
*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(
1/2)
```

Rubi [A] time = 0.05, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 234, 220}

$$-\frac{5\sqrt[4]{-3x^2-2}}{8x}+\frac{\sqrt[4]{-3x^2-2}}{6x^3}-\frac{5\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{16\sqrt[4]{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-2 - 3*x^2)^(3/4)),x]
```

```
[Out] (-2 - 3*x^2)^(1/4)/(6*x^3) - (5*(-2 - 3*x^2)^(1/4))/(8*x) - (5*Sqrt[3]*Sqrt
[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*Ellipt
icF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(16*2^(1/4)*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(-2-3x^2\right)^{3/4}}\,dx &= \frac{\sqrt[4]{-2-3x^2}}{6x^3}-\frac{5}{4}\int \frac{1}{x^2\left(-2-3x^2\right)^{3/4}}\,dx\\ &= \frac{\sqrt[4]{-2-3x^2}}{6x^3}-\frac{5\sqrt[4]{-2-3x^2}}{8x}+\frac{15}{16}\int \frac{1}{\left(-2-3x^2\right)^{3/4}}\,dx\\ &= \frac{\sqrt[4]{-2-3x^2}}{6x^3}-\frac{5\sqrt[4]{-2-3x^2}}{8x}-\frac{\left(5\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx,x,\sqrt[4]{-2-3x^2}\right)}{8x}\\ &= \frac{\sqrt[4]{-2-3x^2}}{6x^3}-\frac{5\sqrt[4]{-2-3x^2}}{8x}-\frac{5\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)F\left(2\tan^{-1}\right.}{16\sqrt[4]{2}\,x}\end{aligned}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.39

$$-\frac{\left(\frac{3x^2}{2}+1\right)^{3/4}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};-\frac{3x^2}{2}\right)}{3x^3\left(-3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-2 - 3*x^2)^(3/4)),x]
[Out] -1/3*((1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (-3*x^2)/2])
/(x^3*(-2 - 3*x^2)^(3/4))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\frac{24\,x^3\text{integral}\left(-\frac{15\left(-3\,x^2-2\right)^{\frac{1}{4}}}{16\left(3\,x^2+2\right)},x\right)-\left(15\,x^2-4\right)\left(-3\,x^2-2\right)^{\frac{1}{4}}}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(3/4),x, algorithm="fricas")
[Out] 1/24*(24*x^3*integral(-15/16*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x) - (15*x^2 -
 4)*(-3*x^2 - 2)^(1/4))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^2-2\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-3*x^2-2)^(3/4),x)
```

```
[Out] int(1/x^4/(-3*x^2-2)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\left(-3\,x^2-2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(- 3*x^2 - 2)^(3/4)),x)
```

```
[Out] int(1/(x^4*(- 3*x^2 - 2)^(3/4)), x)
```

sympy [C] time = 0.94, size = 37, normalized size = 0.30

$$\frac{\sqrt[4]{2}\,e^{\frac{i\pi}{4}}\,{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{3}{2},\frac{3}{4}\\-\frac{1}{2}\end{matrix}\right|\frac{3x^2e^{i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2)/
(6*x**3)
```

3.918 $\int \frac{1}{x^6\left(-2-3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=141

$$\frac{27\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{64\sqrt[4]{2}\,x}+\frac{27\sqrt[4]{-3x^2-2}}{32x}+\frac{\sqrt[4]{-3x^2-2}}{10x^5}-$$

```
[Out] 1/10*(-3*x^2-2)^(1/4)/x^5-9/40*(-3*x^2-2)^(1/4)/x^3+27/32*(-3*x^2-2)^(1/4)/
x+27/128*2^(3/4)*(cos(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))^2)^(1/2)/cos(
2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)))*EllipticF(sin(2*arctan(1/2*(-3*x^2-
2)^(1/4)*2^(3/4))),1/2*2^(1/2))*(2^(1/2)+(-3*x^2-2)^(1/2))*(-x^2/(2^(1/2)+(
-3*x^2-2)^(1/2))^2)^(1/2)/x*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 234, 220}

$$\frac{27\sqrt[4]{-3x^2-2}}{32x}-\frac{9\sqrt[4]{-3x^2-2}}{40x^3}+\frac{\sqrt[4]{-3x^2-2}}{10x^5}+\frac{27\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right.\right.}{64\sqrt[4]{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(-2 - 3*x^2)^(3/4)),x]
```

```
[Out] (-2 - 3*x^2)^(1/4)/(10*x^5) - (9*(-2 - 3*x^2)^(1/4))/(40*x^3) + (27*(-2 - 3
*x^2)^(1/4))/(32*x) + (27*Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2
)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/
4)], 1/2])/(64*2^(1/4)*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6(-2-3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{27}{20}\int \frac{1}{x^4(-2-3x^2)^{3/4}}\,dx$$
$$= \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{9\sqrt[4]{-2-3x^2}}{40x^3} + \frac{27}{16}\int \frac{1}{x^2(-2-3x^2)^{3/4}}\,dx$$
$$= \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{9\sqrt[4]{-2-3x^2}}{40x^3} + \frac{27\sqrt[4]{-2-3x^2}}{32x} - \frac{81}{64}\int \frac{1}{(-2-3x^2)^{3/4}}\,dx$$
$$= \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{9\sqrt[4]{-2-3x^2}}{40x^3} + \frac{27\sqrt[4]{-2-3x^2}}{32x} + \frac{\left(27\sqrt{\frac{3}{2}}\sqrt{-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{2}}}\,dx, x, \sqrt[4]{\right.}{32x}$$
$$= \frac{\sqrt[4]{-2-3x^2}}{10x^5} - \frac{9\sqrt[4]{-2-3x^2}}{40x^3} + \frac{27\sqrt[4]{-2-3x^2}}{32x} + \frac{27\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-\right.}{64\sqrt[4]{2}\,x}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.34

$$-\frac{\left(\frac{3x^2}{2}+1\right)^{3/4}\,{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};-\frac{3x^2}{2}\right)}{5x^5\left(-3x^2-2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-2 - 3*x^2)^(3/4)),x]
```

```
[Out] -1/5*((1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, (-3*x^2)/2])
/(x^5*(-2 - 3*x^2)^(3/4))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\frac{160\,x^5\text{integral}\left(\frac{81\left(-3\,x^2-2\right)^{\frac{1}{4}}}{64\left(3\,x^2+2\right)}, x\right) + \left(135\,x^4-36\,x^2+16\right)\left(-3\,x^2-2\right)^{\frac{1}{4}}}{160\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/160*(160*x^5*integral(81/64*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x) + (135*x^4
 - 36*x^2 + 16)*(-3*x^2 - 2)^(1/4))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^2-2\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^6), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^{2}-2\right)^{\frac{3}{4}} x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-3*x^2-2)^(3/4),x)
```

```
[Out] int(1/x^6/(-3*x^2-2)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3\,x^{2}-2\right)^{\frac{3}{4}} x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{6}\left(-3\,x^{2}-2\right)^{3/4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(- 3*x^2 - 2)^(3/4)),x)
```

```
[Out] int(1/(x^6*(- 3*x^2 - 2)^(3/4)), x)
```

sympy [C] time = 1.12, size = 37, normalized size = 0.26

$$\frac{\sqrt[4]{2}\, e^{\frac{i\pi}{4}}\, {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{3x^{2}e^{i\pi}}{2} \right)}{10x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-3*x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*exp(I*pi/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2)/
(10*x**5)
```

3.919 $\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=152

$$-\frac{a^{5/2}c^2(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\mathrm{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{12b^{3/2}\left(a+bx^2\right)^{3/4}}-\frac{a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}{12b^2}+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}+\frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b}$$

```
[Out] 1/30*a*c*(c*x)^(5/2)*(b*x^2+a)^(1/4)/b+1/5*(c*x)^(9/2)*(b*x^2+a)^(1/4)/c-1/
12*a^(5/2)*c^2*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1
/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(
x*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^2+a)^(3/4)-1/12*a^2*c^3*(b*x^2+a)
^(1/4)*(c*x)^(1/2)/b^2
```

Rubi [A] time = 0.11, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {279, 321, 329, 237, 335, 275, 231}

$$-\frac{a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}{12b^2}-\frac{a^{5/2}c^2(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{12b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}+\frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] -(a^2*c^3*Sqrt[c*x]*(a + b*x^2)^(1/4))/(12*b^2) + (a*c*(c*x)^(5/2)*(a + b*x
^2)^(1/4))/(30*b) + ((c*x)^(9/2)*(a + b*x^2)^(1/4))/(5*c) - (a^(5/2)*c^2*(1
 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2]
)/(12*b^(3/2)*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx = \frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}+\frac{1}{10}a\int\frac{(cx)^{7/2}}{\left(a+bx^2\right)^{3/4}}\,dx$$

$$= \frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b}+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}-\frac{\left(a^2c^2\right)\int\frac{(cx)^{3/2}}{\left(a+bx^2\right)^{3/4}}\,dx}{12b}$$

$$= -\frac{a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}{12b^2}+\frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b}+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}+\frac{\left(a^3c^4\right)\int\frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}}{24b^2}$$

$$= -\frac{a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}{12b^2}+\frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b}+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}+\frac{\left(a^3c^3\right)\operatorname{Subst}\left(\int\frac{}{\left(a+\frac{b}{c}\right.}\right.}{12b^2}$$

$$= -\frac{a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}{12b^2}+\frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b}+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}+\frac{\left(a^3c^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx\right.}{}$$

$$= -\frac{a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}{12b^2}+\frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b}+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}-\frac{\left(a^3c^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx\right.}{1}$$

$$= -\frac{a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}{12b^2}+\frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b}+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}-\frac{\left(a^3c^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx\right.}{24}$$

$$= -\frac{a^2c^3\sqrt{cx}\,\sqrt[4]{a+bx^2}}{12b^2}+\frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b}+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}-\frac{a^{5/2}c^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx}{12b^{3/2}\left(\right.}$$

Mathematica [C] time = 0.07, size = 102, normalized size = 0.67

$$\frac{c^3\sqrt{cx}\sqrt[4]{a+bx^2}\left(\sqrt[4]{\frac{bx^2}{a}+1}\left(-5a^2+abx^2+6b^2x^4\right)+5a^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{30b^2\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (c^3*Sqrt[c*x]*(a + b*x^2)^(1/4)*((1 + (b*x^2)/a)^(1/4)*(-5*a^2 + a*b*x^2 +
 6*b^2*x^4) + 5*a^2*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x^2)/a)]))/(30*b
^2*(1 + (b*x^2)/a)^(1/4))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}\,c^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)*c^3*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(7/2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int(cx)^{\frac{7}{2}}\left(b\,x^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(7/2)*(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int(c\,x)^{7/2}\left(b\,x^2+a\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(a + b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(7/2)*(a + b*x^2)^(1/4), x)
```

sympy [C] time = 25.91, size = 46, normalized size = 0.30

$$\frac{\sqrt[4]{a}\, c^{\frac{7}{2}} x^{\frac{9}{2}} \Gamma\left(\frac{9}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{b x^{2} e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*c**(7/2)*x**(9/2)*gamma(9/4)*hyper((-1/4, 9/4), (13/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(13/4))
```

3.920 $\int (cx)^{3/2} \sqrt[4]{a+bx^2}\, dx$

Optimal. Leaf size=118

$$\frac{a^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{6\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}$$

```
[Out] 1/3*(c*x)^(5/2)*(b*x^2+a)^(1/4)/c+1/6*a^(3/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)
*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1
/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^2+a)^(3/4)
/b^(1/2)+1/6*a*c*(b*x^2+a)^(1/4)*(c*x)^(1/2)/b
```

Rubi [A] time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {279, 321, 329, 237, 335, 275, 231}

$$\frac{a^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (a*c*Sqrt[c*x]*(a + b*x^2)^(1/4))/(6*b) + ((c*x)^(5/2)*(a + b*x^2)^(1/4))/(
3*c) + (a^(3/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]
*x)/Sqrt[a]]/2, 2])/(6*Sqrt[b]*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx &= \frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{1}{6}a\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{3/4}}\,dx\\
&= \frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}-\frac{\left(a^2c^2\right)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{12b}\\
&= \frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}-\frac{\left(a^2c\right)\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{6b}\\
&= \frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}-\frac{\left(a^2c\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,d\right.}{6b\left(a+bx^2\right)^{3/4}}\\
&= \frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{\left(a^2c\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,d\right.}{6b\left(a+bx^2\right)^{3/4}}\\
&= \frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{\left(a^2c\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,d\right.}{12b\left(a+bx^2\right)^{3/4}}\\
&= \frac{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{a^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{b}\left(a+bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 85, normalized size = 0.72

$$
\frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}\left(\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{3b\sqrt[4]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a + b*x^2)^(1/4)*((a + b*x^2)*(1 + (b*x^2)/a)^(1/4) - a*Hyper
geometric2F1[-1/4, 1/4, 5/4, -((b*x^2)/a)]))/(3*b*(1 + (b*x^2)/a)^(1/4))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}\,cx, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)*c*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{4}} (cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{3}{2}} \left(b\,x^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)*(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{4}} (cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {(c\,x)}^{3/2}\,{\left(b\,x^2+a\right)}^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(a + b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)*(a + b*x^2)^(1/4), x)
```

sympy [C]   time = 2.71, size = 46, normalized size = 0.39

$$\frac{\sqrt[4]{a}\, c^{\frac{3}{2}} x^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/4, 5/4), (9/4,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(9/4))
```

3.921 $\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}-\frac{\sqrt{a}\,\sqrt{b}\,(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{c^2\left(a+bx^2\right)^{3/4}}$$

```
[Out] -(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)
/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1
/2))),2^(1/2))*a^(1/2)*b^(1/2)/c^2/(b*x^2+a)^(3/4)+(b*x^2+a)^(1/4)*(c*x)^(1
/2)/c
```

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {279, 329, 237, 335, 275, 231}

$$\frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}-\frac{\sqrt{a}\,\sqrt{b}\,(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{c^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/Sqrt[c*x],x]
```

```
[Out] (Sqrt[c*x]*(a + b*x^2)^(1/4))/c - (Sqrt[a]*Sqrt[b]*(1 + a/(b*x^2))^(3/4)*(c
*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(c^2*(a + b*x^2)^(3/
4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx &= \frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}+\frac{1}{2}a\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx \\
&= \frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{c} \\
&= \frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}+\frac{\left(a\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{c\left(a+bx^2\right)^{3/4}} \\
&= \frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}-\frac{\left(a\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{c\left(a+bx^2\right)^{3/4}} \\
&= \frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}-\frac{\left(a\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{2c\left(a+bx^2\right)^{3/4}} \\
&= \frac{\sqrt{cx}\,\sqrt[4]{a+bx^2}}{c}-\frac{\sqrt{a}\sqrt{b}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{c^2\left(a+bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.61

$$
\frac{2x\sqrt[4]{a+bx^2}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt[4]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/Sqrt[c*x],x]
```

```
[Out] (2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x^2)/a)])/(Sq
rt[c*x]*(1 + (b*x^2)/a)^(1/4))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{cx},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{4}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(1/2),x)
[Out] int((b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{1/4}}{\sqrt{c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(1/2),x)
[Out] int((a + b*x^2)^(1/4)/(c*x)^(1/2), x)
```

sympy [C] time = 1.31, size = 46, normalized size = 0.52

$$\frac{\sqrt[4]{a}\,\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(1/2),x)
[Out] a**(1/4)*sqrt(x)*gamma(1/4)*hyper((-1/4, 1/4), (5/4,), b*x**2*exp_polar(I*p
i)/a)/(2*sqrt(c)*gamma(5/4))
```

3.922 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}} \, dx$

Optimal. Leaf size=94

$$-\frac{2b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\,c^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}$$

```
[Out] -2/3*(b*x^2+a)^(1/4)/c/(c*x)^(3/2)-2/3*b^(3/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/2
)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(
1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/c^4/(b*x^2+a)^
(3/4)/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {277, 329, 237, 335, 275, 231}

$$-\frac{2b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,c^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(3*c*(c*x)^(3/2)) - (2*b^(3/2)*(1 + a/(b*x^2))^(3/4)
*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*c^4*(a
 + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}+\frac{b\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{3c^2}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}+\frac{(2b)\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{3c^3}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}+\frac{\left(2b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{3c^3\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}-\frac{\left(2b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{3c^3\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}-\frac{\left(b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{3c^3\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}-\frac{2b^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,c^4\left(a+bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.60

$$
-\frac{2x\sqrt[4]{a+bx^2}\;{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt[4]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(5/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-3/4, -1/4, 1/4, -((b*x^2)/a)])/(
3*(c*x)^(5/2)*(1 + (b*x^2)/a)^(1/4))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{c^3x^3},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(5/2),x)
[Out] int((b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{1/4}}{(c\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(5/2),x)
[Out] int((a + b*x^2)^(1/4)/(c*x)^(5/2), x)
```

sympy [C] time = 3.71, size = 32, normalized size = 0.34

$$-\frac{\sqrt[4]{b}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(5/2),x)
```

```
[Out] -b**(1/4)*hyper((-1/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(c**(5/2)*
x)
```

3.923 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}\,dx$

Optimal. Leaf size=123

$$\frac{4b^{5/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{21a^{3/2}c^6\left(a+bx^2\right)^{3/4}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}$$

```
[Out] -2/7*(b*x^2+a)^(1/4)/c/(c*x)^(7/2)-2/21*b*(b*x^2+a)^(1/4)/a/c^3/(c*x)^(3/2)
+4/21*b^(5/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/
2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x
*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/c^6/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {277, 325, 329, 237, 335, 275, 231}

$$\frac{4b^{5/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}c^6\left(a+bx^2\right)^{3/4}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(9/2),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(7*c*(c*x)^(7/2)) - (2*b*(a + b*x^2)^(1/4))/(21*a*c^
3*(c*x)^(3/2)) + (4*b^(5/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[Arc
Cot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)*c^6*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}+\frac{b\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx}{7c^2}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}-\frac{\left(2b^2\right)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{21ac^4}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}-\frac{\left(4b^2\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{21ac^5}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}-\frac{\left(4b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{21ac^5\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}+\frac{\left(4b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{21ac^5\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}+\frac{\left(2b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{21ac^5\left(a+bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}}+\frac{4b^{5/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}c^6\left(a+bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.46

$$
-\frac{2x\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{4};-\frac{3}{4};-\frac{bx^2}{a}\right)}{7(cx)^{9/2}\sqrt[4]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(9/2),x]
[Out] (-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-7/4, -1/4, -3/4, -((b*x^2)/a)])/
(7*(c*x)^(9/2)*(1 + (b*x^2)/a)^(1/4))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{c^5x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c^5*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(9/2),x)
[Out] int((b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{1/4}}{(c\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(9/2),x)
```

```
[Out] int((a + b*x^2)^(1/4)/(c*x)^(9/2), x)
```

sympy [C] time = 27.11, size = 36, normalized size = 0.29

$$-\frac{\sqrt[4]{b}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^{2}}\right)}{3c^{\frac{9}{2}}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(9/2),x)
```

```
[Out] -b**(1/4)*hyper((-1/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**2))/(3*c**(9/2
)*x**3)
```

3.924 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{8b^{7/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{77a^{5/2}c^8\left(a+bx^2\right)^{3/4}}+\frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}}-\frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}}-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

```
[Out] -2/11*(b*x^2+a)^(1/4)/c/(c*x)^(11/2)-2/77*b*(b*x^2+a)^(1/4)/a/c^3/(c*x)^(7/
2)+4/77*b^2*(b*x^2+a)^(1/4)/a^2/c^5/(c*x)^(3/2)-8/77*b^(7/2)*(1+a/b/x^2)^(3
/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot
(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/
a^(5/2)/c^8/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {277, 325, 329, 237, 335, 275, 231}

$$\frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}}-\frac{8b^{7/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}c^8\left(a+bx^2\right)^{3/4}}-\frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}}-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(13/2),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(11*c*(c*x)^(11/2)) - (2*b*(a + b*x^2)^(1/4))/(77*a*
c^3*(c*x)^(7/2)) + (4*b^2*(a + b*x^2)^(1/4))/(77*a^2*c^5*(c*x)^(3/2)) - (8*
b^(7/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt
[a]]/2, 2])/(77*a^(5/2)*c^8*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}+\frac{b\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx}{11c^2}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}-\frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}}-\frac{\left(6b^2\right)\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx}{77ac^4}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}-\frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}}+\frac{\left(4b^3\right)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{77a^2c^6}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}-\frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}}+\frac{\left(8b^3\right)\operatorname{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{77a^2c^7}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}-\frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}}+\frac{\left(8b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,d\right.}{77a^2c^7\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}-\frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}}-\frac{\left(8b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\operatorname{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,d\right.}{77a^2c^7\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}-\frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}}-\frac{\left(4b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,d\right.}{77a^2c^7\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}-\frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}}-\frac{8b^{7/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}c^8\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.36

$$-\frac{2x\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{11}{4},-\frac{1}{4};-\frac{7}{4};-\frac{bx^2}{a}\right)}{11(cx)^{13/2}\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(13/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, -((b*x^2)/a)])
/(11*(c*x)^(13/2)*(1 + (b*x^2)/a)^(1/4))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{c^7x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c^7*x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

```
[Out] int((b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{1/4}}{{\left(c\,x\right)}^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(13/2),x)
```

```
[Out] int((a + b*x^2)^(1/4)/(c*x)^(13/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(13/2),x)
```

```
[Out] Timed out
```

3.925 $\int (cx)^{5/2}\sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=147

$$\frac{3a^2c^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}-\frac{3a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}+\frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}$$

```
[Out] 1/16*a*c*(c*x)^(3/2)*(b*x^2+a)^(1/4)/b+1/4*(c*x)^(7/2)*(b*x^2+a)^(1/4)/c+3/
32*a^2*c^(5/2)*arctan(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(7/4)-
3/32*a^2*c^(5/2)*arctanh(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(7/
4)
```

Rubi [A] time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {279, 321, 329, 331, 298, 205, 208}

$$\frac{3a^2c^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}-\frac{3a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}+\frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (a*c*(c*x)^(3/2)*(a + b*x^2)^(1/4))/(16*b) + ((c*x)^(7/2)*(a + b*x^2)^(1/4)
)/(4*c) + (3*a^2*c^(5/2)*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1
/4))])/(32*b^(7/4)) - (3*a^2*c^(5/2)*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a + b*x^2)^(1/4))])/(32*b^(7/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int (cx)^{5/2}\sqrt[4]{a+bx^2}\,dx &= \frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}+\frac{1}{8}a\int\frac{(cx)^{5/2}}{\left(a+bx^2\right)^{3/4}}\,dx\\
&= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}-\frac{\left(3a^2c^2\right)\int\frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx}{32b}\\
&= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}-\frac{\left(3a^2c\right)\mathrm{Subst}\left(\int\frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{16b}\\
&= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}-\frac{\left(3a^2c\right)\mathrm{Subst}\left(\int\frac{x^2}{1-\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{16b}\\
&= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}-\frac{\left(3a^2c^3\right)\mathrm{Subst}\left(\int\frac{1}{c-\sqrt{b}x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{32b^{3/2}}+\frac{\left(3a^2c\right.}{}\\
&= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}+\frac{3a^2c^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}-\frac{3a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{}}{\sqrt{c}}\right.}{32b^{7/4}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 85, normalized size = 0.58

$$\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}\left(\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{4b\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(a + b*x^2)^(1/4)*((a + b*x^2)*(1 + (b*x^2)/a)^(1/4) - a*Hyp
ergeometric2F1[-1/4, 3/4, 7/4, -((b*x^2)/a)]))/(4*b*(1 + (b*x^2)/a)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{1}{4}} (cx)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{5}{2}} \left(b\,x^2 + a\right)^{\frac{1}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)*(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{1}{4}} (cx)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{5/2}\,\left(b\,x^2 + a\right)^{1/4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(a + b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)*(a + b*x^2)^(1/4), x)
```

sympy [C] time = 9.47, size = 46, normalized size = 0.31

$$\frac{\sqrt[4]{a}\,c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)\,{}_{2}F_{1}\left(\left.\begin{matrix} -\frac{1}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(11/4))
```

3.926 $\int \sqrt{cx}\sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=116

$$-\frac{a\sqrt{c}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

```
[Out] 1/2*(c*x)^(3/2)*(b*x^2+a)^(1/4)/c-1/4*a*arctan(b^(1/4)*(c*x)^(1/2)/(b*x^2+a
)^(1/4)/c^(1/2))*c^(1/2)/b^(3/4)+1/4*a*arctanh(b^(1/4)*(c*x)^(1/2)/(b*x^2+a
)^(1/4)/c^(1/2))*c^(1/2)/b^(3/4)
```

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {279, 329, 331, 298, 205, 208}

$$-\frac{a\sqrt{c}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a + b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(3/2)*(a + b*x^2)^(1/4))/(2*c) - (a*Sqrt[c]*ArcTan[(b^(1/4)*Sqrt[c*x
])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(3/4)) + (a*Sqrt[c]*ArcTanh[(b^(1/4)*
Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{cx}\,\sqrt[4]{a+bx^2}\,dx &= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c} + \frac{1}{4}a\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx \\
&= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c} + \frac{a\,\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{2c} \\
&= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c} + \frac{a\,\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2c} \\
&= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c} + \frac{(ac)\,\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4\sqrt{b}} - \frac{(ac)\,\text{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx, x, \right.}{4\sqrt{b}} \\
&= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c} - \frac{a\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}} + \frac{a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.48

$$
\frac{2x\sqrt{cx}\,\sqrt[4]{a+bx^2}\;{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt[4]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a + b*x^2)^(1/4),x]
```

```
[Out] (2*x*Sqrt[c*x]*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, -((b*x^2
)/a)])/(3*(1 + (b*x^2)/a)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \sqrt{cx}\,\left(b\,x^{2}+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)*(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x}\,\left(b\,x^2+a\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a + b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)*(a + b*x^2)^(1/4), x)
```

sympy [C] time = 1.81, size = 46, normalized size = 0.40

$$\frac{\sqrt[4]{a}\,\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x**2*exp_
polar(I*pi)/a)/(2*gamma(7/4))
```

3.927 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

```
[Out] -b^(1/4)*arctan(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/c^(3/2)+b^(1/4
)*arctanh(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/c^(3/2)-2*(b*x^2+a)^
(1/4)/c/(c*x)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {277, 329, 331, 298, 205, 208}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(c*Sqrt[c*x]) - (b^(1/4)*ArcTan[(b^(1/4)*Sqrt[c*x])/
(Sqrt[c]*(a + b*x^2)^(1/4))])/c^(3/2) + (b^(1/4)*ArcTanh[(b^(1/4)*Sqrt[c*x]
)/(Sqrt[c]*(a + b*x^2)^(1/4))])/c^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{b\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx}{c^2}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{c^3}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c^3}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}-\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}-\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}+\frac{\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.50

$$-\frac{2x\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(3/2),x]
[Out] (-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, -((b*x^2)/a)])/(
(c*x)^(3/2)*(1 + (b*x^2)/a)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

```
[Out] int((b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{1/4}}{\left(c\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(3/2),x)
```

```
[Out] int((a + b*x^2)^(1/4)/(c*x)^(3/2), x)
```

sympy [C] time = 2.31, size = 49, normalized size = 0.46

$$\frac{\sqrt[4]{a}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(3/2),x)
```

```
[Out] a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

3.928 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

```
[Out] -2/5*(b*x^2+a)^(5/4)/a/c/(c*x)^(5/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(7/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx=-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{2x\left(a+bx^2\right)^{5/4}}{5a(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(7/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(5/4))/(5*a*(c*x)^(7/2))
```

fricas [A] time = 1.14, size = 25, normalized size = 0.89

$$-\frac{2\left(bx^2+a\right)^{\frac{5}{4}}\sqrt{cx}}{5\,ac^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(b*x^2 + a)^(5/4)*sqrt(c*x)/(a*c^4*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.75

$$-\frac{2\left(b\,x^2+a\right)^{\frac{5}{4}}x}{5\left(cx\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(7/2),x)
[Out] -2/5*x*(b*x^2+a)^(5/4)/a/(c*x)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

mupad [B] time = 4.92, size = 37, normalized size = 1.32

$$-\frac{\left(b\,x^2+a\right)^{1/4}\left(\frac{2}{5\,c^3}+\frac{2\,b\,x^2}{5\,a\,c^3}\right)}{x^2\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(7/2),x)
[Out] -((a + b*x^2)^(1/4)*(2/(5*c^3) + (2*b*x^2)/(5*a*c^3)))/(x^2*(c*x)^(1/2))
```

sympy [B] time = 9.13, size = 78, normalized size = 2.79

$$\frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^2\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(7/2),x)
[Out] b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*c**(7/2)*x**2*gamma(-1/4))
+ b**(5/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*a*c**(7/2)*gamma(-1/4))
```

3.929 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx$

Optimal. Leaf size=57

$$\frac{8\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{9/2}}$$

```
[Out] -2/5*(b*x^2+a)^(5/4)/a/c/(c*x)^(9/2)+8/45*(b*x^2+a)^(9/4)/a^2/c/(c*x)^(9/2)
```

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{8\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a + b*x^2)^(9/4))/(45*a^2*
c*(c*x)^(9/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx &= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{9/2}}-\frac{4\int \frac{\left(a+bx^2\right)^{5/4}}{(cx)^{11/2}}\,dx}{5a}\\ &= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{9/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.72

$$\frac{2\sqrt{cx}\left(a+bx^2\right)^{5/4}\left(4bx^2-5a\right)}{45a^2c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (2*Sqrt[c*x]*(a + b*x^2)^(5/4)*(-5*a + 4*b*x^2))/(45*a^2*c^6*x^5)
```

fricas [A] time = 1.10, size = 46, normalized size = 0.81

$$\frac{2\left(4\,b^2x^4-abx^2-5\,a^2\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{45\,a^2c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/45*(4*b^2*x^4 - a*b*x^2 - 5*a^2)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^6*x^5
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

maple [A] time = 0.01, size = 31, normalized size = 0.54

$$-\frac{2\left(b\,x^2+a\right)^{\frac{5}{4}}\left(-4b\,x^2+5a\right)x}{45\left(cx\right)^{\frac{11}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(11/2),x)
```

```
[Out] -2/45*x*(b*x^2+a)^(5/4)*(-4*b*x^2+5*a)/a^2/(c*x)^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

mupad [B] time = 4.98, size = 51, normalized size = 0.89

$$-\frac{\left(b\,x^2+a\right)^{1/4}\,\left(\frac{2}{9\,c^5}+\frac{2\,b\,x^2}{45\,a\,c^5}-\frac{8\,b^2\,x^4}{45\,a^2\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(11/2),x)
```

```
[Out] -((a + b*x^2)^(1/4)*(2/(9*c^5) + (2*b*x^2)/(45*a*c^5) - (8*b^2*x^4)/(45*a^2
*c^5)))/(x^4*(c*x)^(1/2))
```

sympy [B] time = 69.58, size = 124, normalized size = 2.18

$$-\frac{5\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{8c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)}-\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{8ac^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{2a^2c^{\frac{11}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(11/2),x)
```

```
[Out] -5*b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(8*c**(11/2)*x**4*gamma(-1/
4)) - b**(5/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(8*a*c**(11/2)*x**2*gamm
a(-1/4)) + b**(9/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(2*a**2*c**(11/2)*g
amma(-1/4))
```

3.930 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{64\left(a+bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}+\frac{16\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

```
[Out] -2/5*(b*x^2+a)^(5/4)/a/c/(c*x)^(13/2)+16/45*(b*x^2+a)^(9/4)/a^2/c/(c*x)^(13
/2)-64/585*(b*x^2+a)^(13/4)/a^3/c/(c*x)^(13/2)
```

Rubi [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{64\left(a+bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}+\frac{16\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) + (16*(a + b*x^2)^(9/4))/(45*a^
2*c*(c*x)^(13/2)) - (64*(a + b*x^2)^(13/4))/(585*a^3*c*(c*x)^(13/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx &= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}-\frac{8\int \frac{\left(a+bx^2\right)^{5/4}}{(cx)^{15/2}}\,dx}{5a}\\
&= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}+\frac{16\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}+\frac{32\int \frac{\left(a+bx^2\right)^{9/4}}{(cx)^{15/2}}\,dx}{45a^2}\\
&= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}+\frac{16\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{64\left(a+bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.61

$$-\frac{2\sqrt{cx}\left(a+bx^2\right)^{5/4}\left(45a^2-40abx^2+32b^2x^4\right)}{585a^3c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*Sqrt[c*x]*(a + b*x^2)^(5/4)*(45*a^2 - 40*a*b*x^2 + 32*b^2*x^4))/(585*a^
3*c^8*x^7)
```

fricas [A] time = 1.04, size = 57, normalized size = 0.67

$$-\frac{2\left(32\,b^3x^6 - 8\,ab^2x^4 + 5\,a^2bx^2 + 45\,a^3\right)\left(bx^2 + a\right)^{\frac{1}{4}}\sqrt{cx}}{585\,a^3c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="fricas")
```

```
[Out] -2/585*(32*b^3*x^6 - 8*a*b^2*x^4 + 5*a^2*b*x^2 + 45*a^3)*(b*x^2 + a)^(1/4)*
sqrt(c*x)/(a^3*c^8*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{1}{4}}}{(cx)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

maple [A] time = 0.01, size = 42, normalized size = 0.49

$$-\frac{2\left(b\,x^2 + a\right)^{\frac{5}{4}}\left(32b^2x^4 - 40ab\,x^2 + 45a^2\right)x}{585\,(cx)^{\frac{15}{2}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(15/2),x)
```

```
[Out] -2/585*x*(b*x^2+a)^(5/4)*(32*b^2*x^4-40*a*b*x^2+45*a^2)/a^3/(c*x)^(15/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{1}{4}}}{(cx)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

mupad [B] time = 5.00, size = 65, normalized size = 0.76

$$-\frac{\left(b\,x^2 + a\right)^{1/4}\left(\frac{2}{13\,c^7} + \frac{2\,b\,x^2}{117\,a\,c^7} - \frac{16\,b^2\,x^4}{585\,a^2\,c^7} + \frac{64\,b^3\,x^6}{585\,a^3\,c^7}\right)}{x^6\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(15/2),x)
```

```
[Out] -((a + b*x^2)^(1/4)*(2/(13*c^7) + (2*b*x^2)/(117*a*c^7) - (16*b^2*x^4)/(585
*a^2*c^7) + (64*b^3*x^6)/(585*a^3*c^7)))/(x^6*(c*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(15/2),x)
```

```
[Out] Timed out
```

3.931 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx$

Optimal. Leaf size=113

$$\frac{256\left(a+bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}-\frac{64\left(a+bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

```
[Out] -2/5*(b*x^2+a)^(5/4)/a/c/(c*x)^(17/2)+8/15*(b*x^2+a)^(9/4)/a^2/c/(c*x)^(17/
2)-64/195*(b*x^2+a)^(13/4)/a^3/c/(c*x)^(17/2)+256/3315*(b*x^2+a)^(17/4)/a^4
/c/(c*x)^(17/2)
```

Rubi [A] time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{256\left(a+bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}-\frac{64\left(a+bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(17/2)) + (8*(a + b*x^2)^(9/4))/(15*a^2
*c*(c*x)^(17/2)) - (64*(a + b*x^2)^(13/4))/(195*a^3*c*(c*x)^(17/2)) + (256*
(a + b*x^2)^(17/4))/(3315*a^4*c*(c*x)^(17/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx &= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}-\frac{12\int \frac{\left(a+bx^2\right)^{5/4}}{(cx)^{19/2}}\,dx}{5a}\\
&= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}+\frac{32\int \frac{\left(a+bx^2\right)^{9/4}}{(cx)^{19/2}}\,dx}{15a^2}\\
&= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{64\left(a+bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}-\frac{128\int \frac{\left(a+bx^2\right)^{13/4}}{(cx)^{19/2}}\,dx}{195a^3}\\
&= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{64\left(a+bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{256\left(a+bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.56

$$\frac{2\left(a+bx^2\right)^{5/4}\left(-195a^3+180a^2bx^2-160ab^2x^4+128b^3x^6\right)}{3315a^4c^9x^8\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (2*(a + b*x^2)^(5/4)*(-195*a^3 + 180*a^2*b*x^2 - 160*a*b^2*x^4 + 128*b^3*x^
6))/(3315*a^4*c^9*x^8*Sqrt[c*x])
```

fricas [A] time = 1.16, size = 68, normalized size = 0.60

$$\frac{2\left(128\,b^4x^8-32\,ab^3x^6+20\,a^2b^2x^4-15\,a^3bx^2-195\,a^4\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{3315\,a^4c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(128*b^4*x^8 - 32*a*b^3*x^6 + 20*a^2*b^2*x^4 - 15*a^3*b*x^2 - 195*a^
4)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^4*c^10*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{19}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

maple [A] time = 0.01, size = 53, normalized size = 0.47

$$-\frac{2\left(b\,x^2+a\right)^{\frac{5}{4}}\left(-128b^3x^6+160a\,b^2x^4-180a^2b\,x^2+195a^3\right)x}{3315\,(cx)^{\frac{19}{2}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(19/2),x)
```

```
[Out] -2/3315*x*(b*x^2+a)^(5/4)*(-128*b^3*x^6+160*a*b^2*x^4-180*a^2*b*x^2+195*a^3
)/a^4/(c*x)^(19/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{19}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

mupad [B] time = 5.02, size = 79, normalized size = 0.70

$$-\frac{\left(b\,x^2+a\right)^{1/4}\,\left(\frac{2}{17\,c^9}+\frac{2\,b\,x^2}{221\,a\,c^9}-\frac{8\,b^2\,x^4}{663\,a^2\,c^9}+\frac{64\,b^3\,x^6}{3315\,a^3\,c^9}-\frac{256\,b^4\,x^8}{3315\,a^4\,c^9}\right)}{x^8\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(19/2),x)
```

```
[Out] -((a + b*x^2)^(1/4)*(2/(17*c^9) + (2*b*x^2)/(221*a*c^9) - (8*b^2*x^4)/(663*
a^2*c^9) + (64*b^3*x^6)/(3315*a^3*c^9) - (256*b^4*x^8)/(3315*a^4*c^9)))/(x^
8*(c*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(19/2),x)
```

```
[Out] Timed out
```

3.932 $\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=122

$$-\frac{a^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{6\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}-\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b}$$

```
[Out] 1/3*(c*x)^(5/2)*(-b*x^2+a)^(1/4)/c-1/6*a^(3/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3/2
)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^(
1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^2+a)^(3/
4)/b^(1/2)-1/6*a*c*(-b*x^2+a)^(1/4)*(c*x)^(1/2)/b
```

Rubi [A] time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {279, 321, 329, 237, 335, 275, 232}

$$-\frac{a^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}-\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)*(a - b*x^2)^(1/4),x]
```

```
[Out] -(a*c*Sqrt[c*x]*(a - b*x^2)^(1/4))/(6*b) + ((c*x)^(5/2)*(a - b*x^2)^(1/4))/
(3*c) - (a^(3/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b
]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[b]*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx &= \frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}+\frac{1}{6}a\int \frac{(cx)^{3/2}}{\left(a-bx^2\right)^{3/4}}\,dx \\
&= -\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}+\frac{\left(a^2c^2\right)\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{12b} \\
&= -\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}+\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{6b} \\
&= -\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}+\frac{\left(a^2c\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,\right.}{6b\left(a-bx^2\right)^{3/4}} \\
&= -\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}-\frac{\left(a^2c\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,\right.}{6b\left(a-bx^2\right)^{3/4}} \\
&= -\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}-\frac{\left(a^2c\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,\right.}{12b\left(a-bx^2\right)^{3/4}} \\
&= -\frac{ac\sqrt{cx}\,\sqrt[4]{a-bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}-\frac{a^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{b}\left(a-bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 88, normalized size = 0.72

$$\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}\left(a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{bx^2}{a}\right)+\sqrt[4]{1-\frac{bx^2}{a}}\left(bx^2-a\right)\right)}{3b\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)*(a - b*x^2)^(1/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a - b*x^2)^(1/4)*((-a + b*x^2)*(1 - (b*x^2)/a)^(1/4) + a*Hype
rgeometric2F1[-1/4, 1/4, 5/4, (b*x^2)/a]))/(3*b*(1 - (b*x^2)/a)^(1/4))
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}\,cx, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)*c*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int(cx)^{\frac{3}{2}}\left(-b\,x^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)*(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int(c\,x)^{3/2}\,\left(a-b\,x^2\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)*(a - b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)*(a - b*x^2)^(1/4), x)
```

sympy [C] time = 2.64, size = 48, normalized size = 0.39

$$\frac{\sqrt[4]{a}\, c^{\frac{3}{2}} x^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{b x^{2} e^{2 i \pi}}{a}\right)}{2 \Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)*(-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/4, 5/4), (9/4,), b*x**2*exp
_polar(2*I*pi)/a)/(2*gamma(9/4))
```

3.933 $\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{cx}\,\sqrt[4]{a-bx^2}}{c}-\frac{\sqrt{a}\,\sqrt{b}\,(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{c^2\left(a-bx^2\right)^{3/4}}$$

```
[Out] -(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)
/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^(1
/2))),2^(1/2))*a^(1/2)*b^(1/2)/c^2/(-b*x^2+a)^(3/4)+(-b*x^2+a)^(1/4)*(c*x)^
(1/2)/c
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {279, 329, 237, 335, 275, 232}

$$\frac{\sqrt{cx}\,\sqrt[4]{a-bx^2}}{c}-\frac{\sqrt{a}\,\sqrt{b}\,(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{c^2\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/Sqrt[c*x],x]
```

```
[Out] (Sqrt[c*x]*(a - b*x^2)^(1/4))/c - (Sqrt[a]*Sqrt[b]*(1 - a/(b*x^2))^(3/4)*(c
*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(c^2*(a - b*x^2)^(3/
4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx &= \frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c}+\frac{1}{2}a\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx\\
&= \frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{c}\\
&= \frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c}+\frac{\left(a\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{c\left(a-bx^2\right)^{3/4}}\\
&= \frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c}-\frac{\left(a\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{c\left(a-bx^2\right)^{3/4}}\\
&= \frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c}-\frac{\left(a\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{2c\left(a-bx^2\right)^{3/4}}\\
&= \frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c}-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{c^2\left(a-bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.60

$$\frac{2x\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/Sqrt[c*x],x]
```

```
[Out] (2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, (b*x^2)/a])/(Sqrt[
c*x]*(1 - (b*x^2)/a)^(1/4))
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^2+a\right)^{\frac{1}{4}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(1/2),x)
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^2\right)^{1/4}}{\sqrt{c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(1/2),x)
[Out] int((a - b*x^2)^(1/4)/(c*x)^(1/2), x)
```

sympy [C] time = 1.34, size = 39, normalized size = 0.42

$$-\frac{i\sqrt[4]{b}\,xe^{\frac{3i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(1/2),x)
[Out] -I*b**(1/4)*x*exp(3*I*pi/4)*hyper((-1/2, -1/4), (1/2,), a/(b*x**2))/sqrt(c)
```

3.934 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{2b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\,c^4\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}$$

```
[Out] -2/3*(-b*x^2+a)^(1/4)/c/(c*x)^(3/2)+2/3*b^(3/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3/
2)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^
(1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^(1/2))),2^(1/2))/c^4/(-b*x^2+a
)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {277, 329, 237, 335, 275, 232}

$$\frac{2b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,c^4\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(5/2),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(3*c*(c*x)^(3/2)) + (2*b^(3/2)*(1 - a/(b*x^2))^(3/4)
*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*c^4*(a
 - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}-\frac{b\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{3c^2}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}-\frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{3c^3}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}-\frac{\left(2b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{3c^3\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}+\frac{\left(2b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{3c^3\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}+\frac{\left(b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{3c^3\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}+\frac{2b^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,c^4\left(a-bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.59

$$
-\frac{2x\sqrt[4]{a-bx^2}\;{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{1}{4};\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(5/2),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-3/4, -1/4, 1/4, (b*x^2)/a])/(3*(
c*x)^(5/2)*(1 - (b*x^2)/a)^(1/4))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{c^3x^3},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(5/2),x)
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-b\,x^2\right)}^{1/4}}{{\left(c\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(5/2),x)
[Out] int((a - b*x^2)^(1/4)/(c*x)^(5/2), x)
```

sympy [C] time = 3.71, size = 36, normalized size = 0.37

$$-\frac{i\sqrt[4]{b}e^{-\frac{i\pi}{4}}{}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{a}{bx^2}\right)}{c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(5/2),x)
```

```
[Out] -I*b**(1/4)*exp(-I*pi/4)*hyper((-1/4, 1/2), (3/2,), a/(b*x**2))/(c**(5/2)*x
)
```

3.935 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx$

Optimal. Leaf size=127

$$\frac{4b^{5/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{21a^{3/2}c^6\left(a-bx^2\right)^{3/4}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}-\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}$$

```
[Out] -2/7*(-b*x^2+a)^(1/4)/c/(c*x)^(7/2)+2/21*b*(-b*x^2+a)^(1/4)/a/c^3/(c*x)^(3/
2)+4/21*b^(5/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccsc(x*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc
(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/c^6/(-b*x^2+a)^(3/4)
```

Rubi [A] time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {277, 325, 329, 237, 335, 275, 232}

$$\frac{4b^{5/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}c^6\left(a-bx^2\right)^{3/4}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}-\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(9/2),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(7*c*(c*x)^(7/2)) + (2*b*(a - b*x^2)^(1/4))/(21*a*c^
3*(c*x)^(3/2)) + (4*b^(5/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[Arc
Csc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)*c^6*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}-\frac{b\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx}{7c^2}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}-\frac{\left(2b^2\right)\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{21ac^4}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}-\frac{\left(4b^2\right)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{21ac^5}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}-\frac{\left(4b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{21ac^5\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}+\frac{\left(4b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{21ac^5\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}+\frac{\left(2b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{21ac^5\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}+\frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}}+\frac{4b^{5/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}c^6\left(a-bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.45

$$
-\frac{2x\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{4};-\frac{3}{4};\frac{bx^2}{a}\right)}{7(cx)^{9/2}\sqrt[4]{1-\frac{bx^2}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(9/2),x]
[Out] (-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-7/4, -1/4, -3/4, (b*x^2)/a])/(7*
(c*x)^(9/2)*(1 - (b*x^2)/a)^(1/4))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{c^5x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c^5*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(9/2),x)
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-b\,x^2\right)}^{1/4}}{{\left(c\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(9/2),x)
```

```
[Out] int((a - b*x^2)^(1/4)/(c*x)^(9/2), x)
```

sympy [C] time = 26.58, size = 39, normalized size = 0.31

$$\frac{i\sqrt[4]{b}e^{\frac{3i\pi}{4}}{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{3c^{\frac{9}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(9/2),x)
```

```
[Out] I*b**(1/4)*exp(3*I*pi/4)*hyper((-1/4, 3/2), (5/2,), a/(b*x**2))/(3*c**(9/2)
*x**3)
```

3.936 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx$

Optimal. Leaf size=159

$$\frac{8b^{7/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{77a^{5/2}c^8\left(a-bx^2\right)^{3/4}}+\frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}}+\frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}}-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}$$

```
[Out] -2/11*(-b*x^2+a)^(1/4)/c/(c*x)^(11/2)+2/77*b*(-b*x^2+a)^(1/4)/a/c^3/(c*x)^(
7/2)+4/77*b^2*(-b*x^2+a)^(1/4)/a^2/c^5/(c*x)^(3/2)+8/77*b^(7/2)*(1-a/b/x^2)
^(3/4)*(c*x)^(3/2)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arc
csc(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^(1/2))),2^(1/2
))/a^(5/2)/c^8/(-b*x^2+a)^(3/4)
```

Rubi [A] time = 0.11, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {277, 325, 329, 237, 335, 275, 232}

$$\frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}}+\frac{8b^{7/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}c^8\left(a-bx^2\right)^{3/4}}+\frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}}-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(13/2),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(11*c*(c*x)^(11/2)) + (2*b*(a - b*x^2)^(1/4))/(77*a*
c^3*(c*x)^(7/2)) + (4*b^2*(a - b*x^2)^(1/4))/(77*a^2*c^5*(c*x)^(3/2)) + (8*
b^(7/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt
[a]]/2, 2])/(77*a^(5/2)*c^8*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}-\frac{b\int \frac{1}{(cx)^{9/2}\left(a-bx^2\right)^{3/4}}\,dx}{11c^2}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}+\frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}}-\frac{\left(6b^2\right)\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx}{77ac^4}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}+\frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}}-\frac{\left(4b^3\right)\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{77a^2c^6}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}+\frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}}-\frac{\left(8b^3\right)\mathrm{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{77a^2c^7}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}+\frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}}-\frac{\left(8b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x
}{77a^2c^7\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}+\frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}}+\frac{\left(8b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/
}{77a^2c^7\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}+\frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}}+\frac{\left(4b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/
}{77a^2c^7\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}+\frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}}+\frac{8b^{7/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\,2\right)
}{77a^{5/2}c^8\left(a-bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.36

$$-\frac{2x\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{11}{4},-\frac{1}{4};-\frac{7}{4};\frac{bx^2}{a}\right)}{11(cx)^{13/2}\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(13/2),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, (b*x^2)/a])/(1
1*(c*x)^(13/2)*(1 - (b*x^2)/a)^(1/4))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{c^7x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c^7*x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{\left(-b\,x^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

```
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^2\right)^{1/4}}{{\left(c\,x\right)}^{13/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(13/2),x)
```

```
[Out] int((a - b*x^2)^(1/4)/(c*x)^(13/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(13/2),x)
```

```
[Out] Timed out
```

3.937 $\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=343

$$\frac{3a^2c^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{64\sqrt{2}\,b^{7/4}}-\frac{3a^2c^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{64\sqrt{2}\,b^{7/4}}-\frac{3a^2c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}\,b^{7/4}}+\cdots$$

```
[Out] -1/16*a*c*(c*x)^(3/2)*(-b*x^2+a)^(1/4)/b+1/4*(c*x)^(7/2)*(-b*x^2+a)^(1/4)/c
+3/64*a^2*c^(5/2)*arctan(-1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^
(1/2))/b^(7/4)*2^(1/2)+3/64*a^2*c^(5/2)*arctan(1+b^(1/4)*2^(1/2)*(c*x)^(1/2
)/(-b*x^2+a)^(1/4)/c^(1/2))/b^(7/4)*2^(1/2)+3/128*a^2*c^(5/2)*ln(c^(1/2)-b^
(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/
2))/b^(7/4)*2^(1/2)-3/128*a^2*c^(5/2)*ln(c^(1/2)+b^(1/4)*2^(1/2)*(c*x)^(1/2
)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))/b^(7/4)*2^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {279, 321, 329, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{3a^2c^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{64\sqrt{2}\,b^{7/4}}-\frac{3a^2c^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{64\sqrt{2}\,b^{7/4}}-\frac{3a^2c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}\,b^{7/4}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*(a - b*x^2)^(1/4),x]
```

```
[Out] -(a*c*(c*x)^(3/2)*(a - b*x^2)^(1/4))/(16*b) + ((c*x)^(7/2)*(a - b*x^2)^(1/4
))/(4*c) - (3*a^2*c^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a - b*x^2)^(1/4))])/(32*Sqrt[2]*b^(7/4)) + (3*a^2*c^(5/2)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(32*Sqrt[2]*b^(7/4)) +
(3*a^2*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqrt[2]
*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(64*Sqrt[2]*b^(7/4)) - (3*a^2*c^(5/
2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqr
t[c*x])/(a - b*x^2)^(1/4)])/(64*Sqrt[2]*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx = \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}+\frac{1}{8}a\int \frac{(cx)^{5/2}}{\left(a-bx^2\right)^{3/4}}\,dx$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}+\frac{\left(3a^2c^2\right)\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx}{32b}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}+\frac{\left(3a^2c\right)\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{16b}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}+\frac{\left(3a^2c\right)\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{16b}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}-\frac{\left(3a^2c\right)\text{Subst}\left(\int \frac{c-\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{32b^{3/2}}+\frac{(3a^2c}{}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}+\frac{\left(3a^2c^{5/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt[4]{b}}-x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right.}{64\sqrt{2}b^{7/4}}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}+\frac{3a^2c^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{64\sqrt{2}b^{7/4}}-\frac{3a^2c}{}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}-\frac{3a^2c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}b^{7/4}}+\frac{3a^2c^{5/2}\tan^{-1}\left(\right.}{32\sqrt{}}$$

Mathematica [C] time = 0.05, size = 88, normalized size = 0.26

$$\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}\left(a\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^2}{a}\right)+\sqrt[4]{1-\frac{bx^2}{a}}\left(bx^2-a\right)\right)}{4b\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*(a - b*x^2)^(1/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(a - b*x^2)^(1/4)*((-a + b*x^2)*(1 - (b*x^2)/a)^(1/4) + a*Hy
pergeometric2F1[-1/4, 3/4, 7/4, (b*x^2)/a]))/(4*b*(1 - (b*x^2)/a)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{5}{2}} \left(-b\,x^{2}+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)*(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^{2}+a\right)^{\frac{1}{4}} (cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,x)^{5/2}\,{\left(a-b\,x^2\right)}^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(a - b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)*(a - b*x^2)^(1/4), x)
```

sympy [C] time = 9.29, size = 48, normalized size = 0.14

$$\frac{\sqrt[4]{a}\, c^{\frac{5}{2}} x^{\frac{7}{2}} \Gamma\left(\frac{7}{4}\right) {}_{2}F_{1}\left(\left.\begin{matrix} -\frac{1}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\right| \frac{b x^{2} e^{2 i \pi}}{a} \right)}{2 \Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x**2*ex
p_polar(2*I*pi)/a)/(2*gamma(11/4))
```

3.938 $\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=307

$$\frac{a\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{3/4}}-\frac{a\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{3/4}}-\frac{a\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{3/4}}+\frac{a\sqrt{c}\tan}{}$$

```
[Out] 1/2*(c*x)^(3/2)*(-b*x^2+a)^(1/4)/c+1/8*a*arctan(-1+b^(1/4)*2^(1/2)*(c*x)^(1
/2)/(-b*x^2+a)^(1/4)/c^(1/2))*c^(1/2)/b^(3/4)*2^(1/2)+1/8*a*arctan(1+b^(1/4
)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^(1/2))*c^(1/2)/b^(3/4)*2^(1/2)+1/1
6*a*ln(c^(1/2)-b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/
2)/(-b*x^2+a)^(1/2))*c^(1/2)/b^(3/4)*2^(1/2)-1/16*a*ln(c^(1/2)+b^(1/4)*2^(1
/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))*c^(1/2
)/b^(3/4)*2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {279, 329, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{a\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{3/4}}-\frac{a\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{3/4}}-\frac{a\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{3/4}}+\frac{a\sqrt{c}\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a - b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(3/2)*(a - b*x^2)^(1/4))/(2*c) - (a*Sqrt[c]*ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*Sqrt[2]*b^(3/4)) + (a*Sqrt[
c]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*
Sqrt[2]*b^(3/4)) + (a*Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*
x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(3/4))
- (a*Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b
^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx = \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{1}{4}a\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{a\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{2c}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{a\,\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2c}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} - \frac{a\,\text{Subst}\left(\int \frac{c-\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4\sqrt{b}c} + \frac{a\,\text{Subst}\left(\int \frac{c+\sqrt{b}x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4\sqrt{b}c}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{\left(a\sqrt{c}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}b^{3/4}} + \frac{\left(a\sqrt{c}\right)\text{Subst}\left(\int \frac{\sqrt{2}\ldots}{-\frac{c}{\sqrt{b}}+\ldots}\right.}{8\sqrt{2}\ldots}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{a\sqrt{c}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}b^{3/4}} - \frac{a\sqrt{c}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\ldots}{\sqrt[4]{a-\ldots}}\right.}{8\sqrt{2}b^{3/4}}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} - \frac{a\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{3/4}} + \frac{a\sqrt{c}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{3/4}} + \frac{a\sqrt{c}\log\left(\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.19

$$\frac{2x\sqrt{cx}\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^2}{a}\right)}{3\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a - b*x^2)^(1/4),x]
```

```
[Out] (2*x*Sqrt[c*x]*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, (b*x^2)/
a])/(3*(1 - (b*x^2)/a)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \sqrt{cx}\, \left(-b\,x^{2}+a\right)^{\frac{1}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)*(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^{2}+a\right)^{\frac{1}{4}} \sqrt{cx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{c\,x}\, \left(a-b\,x^2\right)^{1/4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a - b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)*(a - b*x^2)^(1/4), x)
```

sympy [C] time = 1.78, size = 48, normalized size = 0.16

$$\frac{\sqrt[4]{a}\,\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| {\frac{bx^{2}e^{2i\pi}}{a}} \right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x**2*exp_
polar(2*I*pi)/a)/(2*gamma(7/4))
```

3.939 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=296

$$-\frac{\sqrt[4]{b}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,c^{3/2}}+\frac{\sqrt[4]{b}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,c^{3/2}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,c^{3/2}}-\frac{\sqrt[4]{b}\tan^{-1}}{}$$

```
[Out] -1/2*b^(1/4)*arctan(-1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^(1/2)
)/c^(3/2)*2^(1/2)-1/2*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+
a)^(1/4)/c^(1/2))/c^(3/2)*2^(1/2)-1/4*b^(1/4)*ln(c^(1/2)-b^(1/4)*2^(1/2)*(c
*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))/c^(3/2)*2^(1
/2)+1/4*b^(1/4)*ln(c^(1/2)+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b
^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))/c^(3/2)*2^(1/2)-2*(-b*x^2+a)^(1/4)/c/(c*x)
^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {277, 329, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,c^{3/2}}+\frac{\sqrt[4]{b}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,c^{3/2}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,c^{3/2}}-\frac{\sqrt[4]{b}\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(c*Sqrt[c*x]) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(Sqrt[2]*c^(3/2)) - (b^(1/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(Sqrt[2]*c
^(3/2)) - (b^(1/4)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqr
t[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(2*Sqrt[2]*c^(3/2)) + (b^(1/4)*
Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[c
*x])/(a - b*x^2)^(1/4)])/(2*Sqrt[2]*c^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{b\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx}{c^2}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^3} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,c^{3/2}} - \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}-2x}{-\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x\right.}{2\sqrt{2}\,c^{3/2}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{\sqrt[4]{b}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,c^{3/2}} + \frac{\sqrt[4]{b}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,c^{3/2}} - \cdots$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} + \frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,c^{3/2}} - \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,c^{3/2}} - \frac{\sqrt[4]{b}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}\right.}{2\sqrt{2}\,c^{3/2}}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.19

$$-\frac{2x\sqrt[4]{a-bx^2}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, (b*x^2)/a])/((c*
x)^(3/2)*(1 - (b*x^2)/a)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(3/2),x)
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-b\,x^2\right)}^{1/4}}{{\left(c\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(3/2),x)
[Out] int((a - b*x^2)^(1/4)/(c*x)^(3/2), x)
```

sympy [C] time = 2.32, size = 51, normalized size = 0.17

$$\frac{\sqrt[4]{a}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(3/2),x)
[Out] a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x**2*exp_polar(2*I*pi)/a
)/(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

3.940 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

[Out] -2/5*(-b*x^2+a)^(5/4)/a/c/(c*x)^(5/2)

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {264}

$$-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a - b*x^2)^(1/4)/(c*x)^(7/2),x]

[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx=-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.93

$$-\frac{2x\left(a-bx^2\right)^{5/4}}{5a(cx)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(7/2),x]

[Out] (-2*x*(a - b*x^2)^(5/4))/(5*a*(c*x)^(7/2))

fricas [A] time = 1.27, size = 35, normalized size = 1.21

$$\frac{2\left(bx^2-a\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{5\,ac^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="fricas")

[Out] 2/5*(b*x^2 - a)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^4*x^3)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$-\frac{2\left(-b\,x^2+a\right)^{\frac{5}{4}}x}{5\,(cx)^{\frac{7}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(7/2),x)
[Out] -2/5*x*(-b*x^2+a)^(5/4)/a/(c*x)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

mupad [B] time = 4.85, size = 38, normalized size = 1.31

$$-\frac{\left(a-b\,x^2\right)^{1/4}\left(\frac{2}{5\,c^3}-\frac{2\,b\,x^2}{5\,a\,c^3}\right)}{x^2\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(7/2),x)
[Out] -((a - b*x^2)^(1/4)*(2/(5*c^3) - (2*b*x^2)/(5*a*c^3)))/(x^2*(c*x)^(1/2))
```

sympy [B] time = 8.76, size = 178, normalized size = 6.14

$$\begin{cases} \frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^2\Gamma\left(-\frac{1}{4}\right)} - \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)} & \text{for}\ \left|\frac{a}{bx^2}\right|>1 \\ \frac{\sqrt[4]{b}\,\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^2\Gamma\left(-\frac{1}{4}\right)} - \frac{b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(7/2),x)
```

```
[Out] Piecewise((b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(2*c**(7/2)*x**2*ga
mma(-1/4)) - b**(5/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(2*a*c**(7/2)*gam
ma(-1/4)), Abs(a/(b*x**2)) > 1), (b**(1/4)*(-a/(b*x**2) + 1)**(1/4)*exp(I*p
i/4)*gamma(-5/4)/(2*c**(7/2)*x**2*gamma(-1/4)) - b**(5/4)*(-a/(b*x**2) + 1)
**(1/4)*exp(I*pi/4)*gamma(-5/4)/(2*a*c**(7/2)*gamma(-1/4)), True))
```

3.941 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx$

Optimal. Leaf size=59

$$\frac{8\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}$$

```
[Out] -2/5*(-b*x^2+a)^(5/4)/a/c/(c*x)^(9/2)+8/45*(-b*x^2+a)^(9/4)/a^2/c/(c*x)^(9/
2)
```

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {273, 264}

$$\frac{8\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a - b*x^2)^(9/4))/(45*a^2*
c*(c*x)^(9/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}-\frac{4\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{11/2}}\,dx}{5a}\\ &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.71

$$-\frac{2\sqrt{cx}\left(a-bx^2\right)^{5/4}\left(5a+4bx^2\right)}{45a^2c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(5/4)*(5*a + 4*b*x^2))/(45*a^2*c^6*x^5)
```

fricas [A] time = 0.86, size = 46, normalized size = 0.78

$$\frac{2\left(4\,b^2x^4+abx^2-5\,a^2\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{45\,a^2c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/45*(4*b^2*x^4 + a*b*x^2 - 5*a^2)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^6*x^
5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

maple [A] time = 0.00, size = 32, normalized size = 0.54

$$-\frac{2\left(-b\,x^2+a\right)^{\frac{5}{4}}\left(4b\,x^2+5a\right)x}{45\,(cx)^{\frac{11}{2}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(11/2),x)
```

```
[Out] -2/45*x*(-b*x^2+a)^(5/4)*(4*b*x^2+5*a)/a^2/(c*x)^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

mupad [B] time = 4.89, size = 51, normalized size = 0.86

$$\frac{{\left(a-b\,x^2\right)}^{1/4}\,\left(\frac{2\,b\,x^2}{45\,a\,c^5}-\frac{2}{9\,c^5}+\frac{8\,b^2\,x^4}{45\,a^2\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(11/2),x)
```

```
[Out] ((a - b*x^2)^(1/4)*((2*b*x^2)/(45*a*c^5) - 2/(9*c^5) + (8*b^2*x^4)/(45*a^2*
c^5)))/(x^4*(c*x)^(1/2))
```

sympy [B] time = 68.25, size = 462, normalized size = 7.83

$$\begin{cases} -\frac{5\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{9}{4}\right)}{8c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{9}{4}\right)}{8ac^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{9}{4}\right)}{2a^2c^{\frac{11}{2}}\Gamma\left(-\frac{1}{4}\right)} \\ \frac{5a^3b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{x^2\left(-8a^3bc^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)+8a^2b^2c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)\right)}-\frac{6a^2b^{\frac{9}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{-8a^3bc^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)+8a^2b^2c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)}-\frac{3ab^{\frac{13}{4}}x^2\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{-8a^3bc^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)+8a^2b^2c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)}+\frac{4b^{\frac{17}{4}}x}{-8a^3bc^{\frac{11}{2}}x} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(11/2),x)
```

```
[Out] Piecewise((-5*b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-9/4)/(8*c**(11/2)*x**
4*gamma(-1/4)) + b**(5/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-9/4)/(8*a*c**(11/2
)*x**2*gamma(-1/4)) + b**(9/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-9/4)/(2*a**2*
c**(11/2)*gamma(-1/4)), Abs(a/(b*x**2)) > 1), (5*a**3*b**(5/4)*(-a/(b*x**2)
 + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(x**2*(-8*a**3*b*c**(11/2)*x**2*gamma(
-1/4) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4))) - 6*a**2*b**(9/4)*(-a/(b*x
**2) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-8*a**3*b*c**(11/2)*x**2*gamma(-1
/4) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4)) - 3*a*b**(13/4)*x**2*(-a/(b*x
**2) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-8*a**3*b*c**(11/2)*x**2*gamma(-1
/4) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4)) + 4*b**(17/4)*x**4*(-a/(b*x**
2) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-8*a**3*b*c**(11/2)*x**2*gamma(-1/4
) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4)), True))
```

3.942 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{64\left(a-bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}+\frac{16\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

```
[Out] -2/5*(-b*x^2+a)^(5/4)/a/c/(c*x)^(13/2)+16/45*(-b*x^2+a)^(9/4)/a^2/c/(c*x)^(
13/2)-64/585*(-b*x^2+a)^(13/4)/a^3/c/(c*x)^(13/2)
```

Rubi [A] time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {273, 264}

$$-\frac{64\left(a-bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}+\frac{16\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) + (16*(a - b*x^2)^(9/4))/(45*a^
2*c*(c*x)^(13/2)) - (64*(a - b*x^2)^(13/4))/(585*a^3*c*(c*x)^(13/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}-\frac{8\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{15/2}}\,dx}{5a}\\
&= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}+\frac{16\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}+\frac{32\int \frac{\left(a-bx^2\right)^{9/4}}{(cx)^{15/2}}\,dx}{45a^2}\\
&= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}+\frac{16\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{64\left(a-bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.60

$$-\frac{2\sqrt{cx}\left(a-bx^2\right)^{5/4}\left(45a^2+40abx^2+32b^2x^4\right)}{585a^3c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(5/4)*(45*a^2 + 40*a*b*x^2 + 32*b^2*x^4))/(585*a^
3*c^8*x^7)
```

fricas [A] time = 1.08, size = 58, normalized size = 0.66

$$\frac{2\left(32\,b^3x^6+8\,ab^2x^4+5\,a^2bx^2-45\,a^3\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{585\,a^3c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="fricas")
```

```
[Out] 2/585*(32*b^3*x^6 + 8*a*b^2*x^4 + 5*a^2*b*x^2 - 45*a^3)*(-b*x^2 + a)^(1/4)*
sqrt(c*x)/(a^3*c^8*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

maple [A] time = 0.00, size = 43, normalized size = 0.49

$$-\frac{2\left(-b\,x^2+a\right)^{\frac{5}{4}}\left(32b^2x^4+40ab\,x^2+45a^2\right)x}{585\,(cx)^{\frac{15}{2}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(15/2),x)
```

```
[Out] -2/585*x*(-b*x^2+a)^(5/4)*(32*b^2*x^4+40*a*b*x^2+45*a^2)/a^3/(c*x)^(15/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

mupad [B] time = 4.91, size = 65, normalized size = 0.74

$$\frac{\left(a-b\,x^2\right)^{1/4}\,\left(\frac{2\,b\,x^2}{117\,a\,c^7}-\frac{2}{13\,c^7}+\frac{16\,b^2\,x^4}{585\,a^2\,c^7}+\frac{64\,b^3\,x^6}{585\,a^3\,c^7}\right)}{x^6\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(15/2),x)
```

```
[Out] ((a - b*x^2)^(1/4)*((2*b*x^2)/(117*a*c^7) - 2/(13*c^7) + (16*b^2*x^4)/(585*
a^2*c^7) + (64*b^3*x^6)/(585*a^3*c^7)))/(x^6*(c*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(15/2),x)
```

```
[Out] Timed out
```

3.943 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx$

Optimal. Leaf size=117

$$\frac{256\left(a-bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}-\frac{64\left(a-bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

```
[Out] -2/5*(-b*x^2+a)^(5/4)/a/c/(c*x)^(17/2)+8/15*(-b*x^2+a)^(9/4)/a^2/c/(c*x)^(1
7/2)-64/195*(-b*x^2+a)^(13/4)/a^3/c/(c*x)^(17/2)+256/3315*(-b*x^2+a)^(17/4)
/a^4/c/(c*x)^(17/2)
```

Rubi [A] time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {273, 264}

$$\frac{256\left(a-bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}-\frac{64\left(a-bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(17/2)) + (8*(a - b*x^2)^(9/4))/(15*a^2
*c*(c*x)^(17/2)) - (64*(a - b*x^2)^(13/4))/(195*a^3*c*(c*x)^(17/2)) + (256*
(a - b*x^2)^(17/4))/(3315*a^4*c*(c*x)^(17/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}-\frac{12\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{19/2}}\,dx}{5a}\\
&= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}+\frac{32\int \frac{\left(a-bx^2\right)^{9/4}}{(cx)^{19/2}}\,dx}{15a^2}\\
&= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{64\left(a-bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}-\frac{128\int \frac{\left(a-bx^2\right)^{13/4}}{(cx)^{19/2}}\,dx}{195a^3}\\
&= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{64\left(a-bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{256\left(a-bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.55

$$-\frac{2\left(a-bx^2\right)^{5/4}\left(195a^3+180a^2bx^2+160ab^2x^4+128b^3x^6\right)}{3315a^4c^9x^8\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4)*(195*a^3 + 180*a^2*b*x^2 + 160*a*b^2*x^4 + 128*b^3*x^
6))/(3315*a^4*c^9*x^8*Sqrt[c*x])
```

fricas [A] time = 1.01, size = 69, normalized size = 0.59

$$\frac{2\left(128\,b^4x^8+32\,ab^3x^6+20\,a^2b^2x^4+15\,a^3bx^2-195\,a^4\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{3315\,a^4c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(128*b^4*x^8 + 32*a*b^3*x^6 + 20*a^2*b^2*x^4 + 15*a^3*b*x^2 - 195*a^
4)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^4*c^10*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{19}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

maple [A] time = 0.01, size = 54, normalized size = 0.46

$$-\frac{2\left(-b\,x^2+a\right)^{\frac{5}{4}}\left(128b^3x^6+160a\,b^2x^4+180a^2b\,x^2+195a^3\right)x}{3315\left(cx\right)^{\frac{19}{2}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(19/2),x)
```

```
[Out] -2/3315*x*(-b*x^2+a)^(5/4)*(128*b^3*x^6+160*a*b^2*x^4+180*a^2*b*x^2+195*a^3
)/a^4/(c*x)^(19/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{19}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

mupad [B] time = 4.93, size = 79, normalized size = 0.68

$$\frac{\left(a-b\,x^2\right)^{1/4}\,\left(\frac{2\,b\,x^2}{221\,a\,c^9}-\frac{2}{17\,c^9}+\frac{8\,b^2\,x^4}{663\,a^2\,c^9}+\frac{64\,b^3\,x^6}{3315\,a^3\,c^9}+\frac{256\,b^4\,x^8}{3315\,a^4\,c^9}\right)}{x^8\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(19/2),x)
```

```
[Out] ((a - b*x^2)^(1/4)*((2*b*x^2)/(221*a*c^9) - 2/(17*c^9) + (8*b^2*x^4)/(663*a
^2*c^9) + (64*b^3*x^6)/(3315*a^3*c^9) + (256*b^4*x^8)/(3315*a^4*c^9)))/(x^8
*(c*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(19/2),x)
```

```
[Out] Timed out
```

3.944 $\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=117

$$-\frac{ac^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}-\frac{ac^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}+\frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b}$$

```
[Out] -1/4*a*c^(3/2)*arctan(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(5/4)-
1/4*a*c^(3/2)*arctanh(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(5/4)+
1/2*c*(b*x^2+a)^(3/4)*(c*x)^(1/2)/b
```

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {321, 329, 240, 212, 208, 205}

$$-\frac{ac^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}-\frac{ac^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}+\frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a + b*x^2)^(3/4))/(2*b) - (a*c^(3/2)*ArcTan[(b^(1/4)*Sqrt[c*x
])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(5/4)) - (a*c^(3/2)*ArcTanh[(b^(1/4)*
Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(5/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx &= \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{\left(ac^2\right)\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx}{4b}\\
&= \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{(ac)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2b}\\
&= \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{(ac)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b}\\
&= \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{\left(ac^2\right)\mathrm{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b} - \frac{\left(ac^2\right)\mathrm{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b}\\
&= \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{ac^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}} - \frac{ac^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 0.83

$$\frac{(cx)^{3/2}\left(2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^{3/4} - a\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right) - a\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(3/2)*(2*b^(1/4)*Sqrt[x]*(a + b*x^2)^(3/4) - a*ArcTan[(b^(1/4)*Sqrt[
x])/(a + b*x^2)^(1/4)] - a*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(
4*b^(5/4)*x^(3/2))
```

fricas [B] time = 1.12, size = 314, normalized size = 2.68

$$\frac{4\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,c + 4\left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}} b\arctan\left(-\frac{\left(\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,ab^4c - \left(b^5x^2+ab^4\right)\left(\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}\sqrt{\frac{\sqrt{bx^2+a}\,a^2c^3x+\sqrt{\frac{a^4c^6}{b^5}}\left(b^3x^2+ab^2\right)}{bx^2+a}}}{a^4bc^6x^2+a^5c^6}\right) - \left(\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*x^2 + a)^(3/4)*sqrt(c*x)*c + 4*(a^4*c^6/b^5)^(1/4)*b*arctan(-((a^
4*c^6/b^5)^(3/4)*(b*x^2 + a)^(3/4)*sqrt(c*x)*a*b^4*c - (b^5*x^2 + a*b^4)*(a
^4*c^6/b^5)^(3/4)*sqrt((sqrt(b*x^2 + a)*a^2*c^3*x + sqrt(a^4*c^6/b^5)*(b^3*
```

```
x^2 + a*b^2))/(b*x^2 + a)))/(a^4*b*c^6*x^2 + a^5*c^6)) - (a^4*c^6/b^5)^(1/4
)*b*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*a*c + (a^4*c^6/b^5)^(1/4)*(b^2*x^2 + a
*b))/(b*x^2 + a)) + (a^4*c^6/b^5)^(1/4)*b*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*
a*c - (a^4*c^6/b^5)^(1/4)*(b^2*x^2 + a*b))/(b*x^2 + a)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{\left(b\,x^2+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a + b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)/(a + b*x^2)^(1/4), x)
```

sympy [C] time = 2.18, size = 44, normalized size = 0.38

$$\frac{c^{\frac{3}{2}} x^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\left.\begin{matrix} \frac{1}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(1/4)*gamma(9/4))
```

3.945 $\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=83

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\,\sqrt{c}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\,\sqrt{c}}$$

```
[Out] arctan(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(1/4)/c^(1/2)+arctanh
(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(1/4)/c^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {329, 240, 212, 208, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\,\sqrt{c}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(b^(1/4)*Sqrt[c]) +
 ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(b^(1/4)*Sqrt[c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a+bx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}$$
$$= \text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right) + \text{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\,\sqrt{c}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\,\sqrt{c}}$$

Mathematica [A] time = 0.01, size = 65, normalized size = 0.78

$$\frac{\sqrt{x}\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{\sqrt[4]{b}\,\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(1/4)),x]
```

```
[Out] (Sqrt[x]*(ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + ArcTanh[(b^(1/4)*Sq
rt[x])/(a + b*x^2)^(1/4)]))/(b^(1/4)*Sqrt[c*x])
```

fricas [B] time = 1.34, size = 241, normalized size = 2.90

$$-2\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,bc\left(\frac{1}{bc^2}\right)^{\frac{3}{4}}-\left(b^2cx^2+abc\right)\sqrt{\frac{\sqrt{bx^2+a}\,cx+\left(bc^2x^2+ac^2\right)\sqrt{\frac{1}{bc^2}}}{bx^2+a}}\left(\frac{1}{bc^2}\right)^{\frac{3}{4}}}{bx^2+a}\right)+\frac{1}{2}\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2*(1/(b*c^2))^(1/4)*arctan(-((b*x^2 + a)^(3/4)*sqrt(c*x)*b*c*(1/(b*c^2))^(
3/4) - (b^2*c*x^2 + a*b*c)*sqrt((sqrt(b*x^2 + a)*c*x + (b*c^2*x^2 + a*c^2)*
sqrt(1/(b*c^2)))/(b*x^2 + a))*(1/(b*c^2))^(3/4))/(b*x^2 + a)) + 1/2*(1/(b*c
^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x) + (b*c*x^2 + a*c)*(1/(b*c^2))^(
1/4))/(b*x^2 + a)) - 1/2*(1/(b*c^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x)
 - (b*c*x^2 + a*c)*(1/(b*c^2))^(1/4))/(b*x^2 + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

[Out] integrate(1/((b*x^2 + a)^(1/4)*sqrt(c*x)), x)

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\left(b\,x^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x)

[Out] int(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")

[Out] integrate(1/((b*x^2 + a)^(1/4)*sqrt(c*x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x}\,{\left(b\,x^2+a\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((c*x)^(1/2)*(a + b*x^2)^(1/4)),x)

[Out] int(1/((c*x)^(1/2)*(a + b*x^2)^(1/4)), x)

sympy [C] time = 1.51, size = 44, normalized size = 0.53

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(1/4),x)

[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(1/4)*sqrt(c)*gamma(5/4))

3.946 $\int \frac{1}{(cx)^{5/2} \sqrt[4]{a+bx^2}} \, dx$

Optimal. Leaf size=28

$$-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

```
[Out] -2/3*(b*x^2+a)^(3/4)/a/c/(c*x)^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {264}

$$-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x]
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2} \sqrt[4]{a+bx^2}} \, dx = -\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{2x\left(a+bx^2\right)^{3/4}}{3a(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x]
[Out] (-2*x*(a + b*x^2)^(3/4))/(3*a*(c*x)^(5/2))
```

fricas [A] time = 1.34, size = 25, normalized size = 0.89

$$-\frac{2\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{3\,ac^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] -2/3*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a*c^3*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.75

$$-\frac{2\left(b\,x^2+a\right)^{\frac{3}{4}}x}{3\,(cx)^{\frac{5}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x)
[Out] -2/3*x*(b*x^2+a)^(3/4)/a/(c*x)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

mupad [B] time = 4.95, size = 25, normalized size = 0.89

$$-\frac{2\left(b\,x^2+a\right)^{3/4}}{3\,a\,c^2\,x\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x)
[Out] -(2*(a + b*x^2)^(3/4))/(3*a*c^2*x*(c*x)^(1/2))
```

sympy [A] time = 3.58, size = 36, normalized size = 1.29

$$\frac{b^{\frac{3}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(1/4),x)
[Out] b**(3/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma(1/4))
```

3.947 $\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=57

$$\frac{8\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{7/2}}$$

```
[Out] -2/3*(b*x^2+a)^(3/4)/a/c/(c*x)^(7/2)+8/21*(b*x^2+a)^(7/4)/a^2/c/(c*x)^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{8\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(7/2)) + (8*(a + b*x^2)^(7/4))/(21*a^2*
c*(c*x)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx&=-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{7/2}}-\frac{4\int \frac{\left(a+bx^2\right)^{3/4}}{(cx)^{9/2}}\,dx}{3a}\\&=-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{7/2}}+\frac{8\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.72

$$\frac{2\sqrt{cx}\left(a+bx^2\right)^{3/4}\left(4bx^2-3a\right)}{21a^2c^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (2*Sqrt[c*x]*(a + b*x^2)^(3/4)*(-3*a + 4*b*x^2))/(21*a^2*c^5*x^4)
```

fricas [A] time = 0.71, size = 35, normalized size = 0.61

$$\frac{2\left(4\,bx^2-3\,a\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{21\,a^2c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/21*(4*b*x^2 - 3*a)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*c^5*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

maple [A] time = 0.01, size = 31, normalized size = 0.54

$$-\frac{2\left(b\,x^2+a\right)^{\frac{3}{4}}\left(-4b\,x^2+3a\right)x}{21\,(cx)^{\frac{9}{2}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] -2/21*x*(b*x^2+a)^(3/4)*(-4*b*x^2+3*a)/a^2/(c*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

mupad [B] time = 5.00, size = 40, normalized size = 0.70

$$-\frac{\left(b\,x^2+a\right)^{3/4}\left(\frac{2}{7\,a\,c^4}-\frac{8\,b\,x^2}{21\,a^2\,c^4}\right)}{x^3\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*(2/(7*a*c^4) - (8*b*x^2)/(21*a^2*c^4)))/(x^3*(c*x)^(1/2
))
```

sympy [A] time = 34.72, size = 80, normalized size = 1.40

$$-\frac{3b^{\frac{3}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{8ac^{\frac{9}{2}}x^2\Gamma\left(\frac{1}{4}\right)}+\frac{b^{\frac{7}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{2a^2c^{\frac{9}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] -3*b**(3/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(8*a*c**(9/2)*x**2*gamma(1/
4)) + b**(7/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(2*a**2*c**(9/2)*gamma(1
/4))
```

3.948 $\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{64\left(a+bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}+\frac{16\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

```
[Out] -2/3*(b*x^2+a)^(3/4)/a/c/(c*x)^(11/2)+16/21*(b*x^2+a)^(7/4)/a^2/c/(c*x)^(11
/2)-64/231*(b*x^2+a)^(11/4)/a^3/c/(c*x)^(11/2)
```

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{64\left(a+bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}+\frac{16\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) + (16*(a + b*x^2)^(7/4))/(21*a^
2*c*(c*x)^(11/2)) - (64*(a + b*x^2)^(11/4))/(231*a^3*c*(c*x)^(11/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx &= -\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}-\frac{8\int \frac{\left(a+bx^2\right)^{3/4}}{(cx)^{13/2}}\,dx}{3a}\\ &= -\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}+\frac{16\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}+\frac{32\int \frac{\left(a+bx^2\right)^{7/4}}{(cx)^{13/2}}\,dx}{21a^2}\\ &= -\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}+\frac{16\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{64\left(a+bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.61

$$-\frac{2\sqrt{cx}\left(a+bx^2\right)^{3/4}\left(21a^2-24abx^2+32b^2x^4\right)}{231a^3c^7x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*Sqrt[c*x]*(a + b*x^2)^(3/4)*(21*a^2 - 24*a*b*x^2 + 32*b^2*x^4))/(231*a^
3*c^7*x^6)
```

fricas [A] time = 0.77, size = 46, normalized size = 0.54

$$-\frac{2\left(32\,b^2x^4-24\,abx^2+21\,a^2\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{231\,a^3c^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2/231*(32*b^2*x^4 - 24*a*b*x^2 + 21*a^2)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^3*
c^7*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}\left(cx\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

maple [A] time = 0.01, size = 42, normalized size = 0.49

$$-\frac{2\left(b\,x^2+a\right)^{\frac{3}{4}}\left(32b^2x^4-24ab\,x^2+21a^2\right)x}{231\left(cx\right)^{\frac{13}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] -2/231*x*(b*x^2+a)^(3/4)*(32*b^2*x^4-24*a*b*x^2+21*a^2)/a^3/(c*x)^(13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}\left(cx\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

mupad [B] time = 5.00, size = 54, normalized size = 0.64

$$-\frac{\left(b\,x^2+a\right)^{3/4}\,\left(\frac{2}{11\,a\,c^6}-\frac{16\,b\,x^2}{77\,a^2\,c^6}+\frac{64\,b^2\,x^4}{231\,a^3\,c^6}\right)}{x^5\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*(2/(11*a*c^6) - (16*b*x^2)/(77*a^2*c^6) + (64*b^2*x^4)/
(231*a^3*c^6)))/(x^5*(c*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] Timed out
```

3.949 $\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}} \, dx$

Optimal. Leaf size=156

$$\frac{7a^{5/2}c^4\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20b^{5/2}\sqrt[4]{a+bx^2}}+\frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}}-\frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2}+\frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b}$$

```
[Out] -7/30*a*c^3*(c*x)^(3/2)*(b*x^2+a)^(3/4)/b^2+1/5*c*(c*x)^(7/2)*(b*x^2+a)^(3/
4)/b+7/20*a^2*c^4*x*(c*x)^(1/2)/b^2/(b*x^2+a)^(1/4)+7/20*a^(5/2)*c^4*(1+a/b
/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b
^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*x
)^(1/2)/b^(5/2)/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {321, 314, 284, 335, 196}

$$\frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}}+\frac{7a^{5/2}c^4\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{20b^{5/2}\sqrt[4]{a+bx^2}}-\frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2}+\frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(9/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] (7*a^2*c^4*x*Sqrt[c*x])/(20*b^2*(a + b*x^2)^(1/4)) - (7*a*c^3*(c*x)^(3/2)*(
a + b*x^2)^(3/4))/(30*b^2) + (c*(c*x)^(7/2)*(a + b*x^2)^(3/4))/(5*b) + (7*a
^(5/2)*c^4*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqr
t[a]]/2, 2])/(20*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 314

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[(x*Sqrt[
c*x])/(a + b*x^2)^(1/4), x] - Dist[a/2, Int[Sqrt[c*x]/(a + b*x^2)^(5/4), x]
, x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx = \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} - \frac{\left(7ac^2\right)\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx}{10b}$$

$$= -\frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} + \frac{\left(7a^2c^4\right)\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx}{20b^2}$$

$$= \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} - \frac{\left(7a^3c^4\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{40b^2}$$

$$= \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} - \frac{\left(7a^3c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{}{(1\text{-}}}{40b^3\sqrt[4]{a+bx^2}}$$

$$= \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} + \frac{\left(7a^3c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subs}}{40b^3\sqrt[4]{a\ -}}$$

$$= \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} + \frac{7a^{5/2}c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\right.}{20b^{5/2}\sqrt[4]{a+b}}$$

Mathematica [C] time = 0.03, size = 87, normalized size = 0.56

$$\frac{c^3(cx)^{3/2}\left(7a^2\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-7a^2-abx^2+6b^2x^4\right)}{30b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(9/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] (c^3*(c*x)^(3/2)*(-7*a^2 - a*b*x^2 + 6*b^2*x^4 + 7*a^2*(1 + (b*x^2)/a)^(1/4
)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*x^2)/a)]))/(30*b^2*(a + b*x^2)^(1/4
))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx}\,c^4x^4}{\left(bx^2+a\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)*c^4*x^4/(b*x^2 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{9}{2}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(9/2)/(b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{9}{2}}}{\left(b\,x^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(9/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(9/2)/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{9}{2}}}{\left(bx^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(9/2)/(b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(c\,x)}^{9/2}}{{\left(b\,x^2+a\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(9/2)/(a + b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(9/2)/(a + b*x^2)^(1/4), x)
```

sympy [C] time = 51.81, size = 44, normalized size = 0.28

$$\frac{c^{\frac{9}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(9/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] c**(9/2)*x**(11/2)*gamma(11/4)*hyper((1/4, 11/4), (15/4,), b*x**2*exp_polar
(I*pi)/a)/(2*a**(1/4)*gamma(15/4))
```

3.950 $\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=125

$$-\frac{a^{3/2}c^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^2}}-\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b}$$

```
[Out] 1/3*c*(c*x)^(3/2)*(b*x^2+a)^(3/4)/b-1/2*a*c^2*x*(c*x)^(1/2)/b/(b*x^2+a)^(1/
4)-1/2*a^(3/2)*c^2*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)
^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2
)/a^(1/2))),2^(1/2))*(c*x)^(1/2)/b^(3/2)/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {321, 314, 284, 335, 196}

$$-\frac{a^{3/2}c^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^2}}-\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] -(a*c^2*x*Sqrt[c*x])/(2*b*(a + b*x^2)^(1/4)) + (c*(c*x)^(3/2)*(a + b*x^2)^(
3/4))/(3*b) - (a^(3/2)*c^2*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(3/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 314

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[(x*Sqrt[
c*x])/(a + b*x^2)^(1/4), x] - Dist[a/2, Int[Sqrt[c*x]/(a + b*x^2)^(5/4), x]
, x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx = \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} - \frac{\left(ac^2\right)\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx}{2b}$$
$$= -\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} + \frac{\left(a^2c^2\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{4b}$$
$$= -\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} + \frac{\left(a^2c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{4b^2\sqrt[4]{a+bx^2}}$$
$$= -\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} - \frac{\left(a^2c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{4b^2\sqrt[4]{a+bx^2}}$$
$$= -\frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} - \frac{a^{3/2}c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.03, size = 69, normalized size = 0.55

$$\frac{c(cx)^{3/2}\left(-a\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+a+bx^2\right)}{3b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4,
3/4, 7/4, -((b*x^2)/a)]))/(3*b*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cx}\,c^2x^2}{\left(bx^2+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)*c^2*x^2/(b*x^2 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(b\,x^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{5/2}}{\left(b\,x^2+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a + b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)/(a + b*x^2)^(1/4), x)
```

sympy [C] time = 6.74, size = 44, normalized size = 0.35

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(1/4)*gamma(11/4))
```

3.951 $\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=83

$$\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}+\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}$$

```
[Out] x*(c*x)^(1/2)/(b*x^2+a)^(1/4)+(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcc
ot(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {314, 284, 335, 196}

$$\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}+\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(1/4),x]
```

```
[Out] (x*Sqrt[c*x])/(a + b*x^2)^(1/4) + (Sqrt[a]*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*
EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 314

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[(x*Sqrt[
c*x])/(a + b*x^2)^(1/4), x] - Dist[a/2, Int[Sqrt[c*x]/(a + b*x^2)^(5/4), x]
, x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx &= \frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}} - \frac{1}{2}a\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx \\
&= \frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}} - \frac{\left(a\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{2b\sqrt[4]{a+bx^2}} \\
&= \frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}} + \frac{\left(a\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2b\sqrt[4]{a+bx^2}} \\
&= \frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}} + \frac{\sqrt{a}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.67

$$
\frac{2x\sqrt{cx}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt[4]{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(1/4),x]
```

```
[Out] (2*x*Sqrt[c*x]*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*
x^2)/a)])/(3*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{1}{4}}}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)/(b*x^2 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$
\int \frac{\sqrt{cx}}{\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x}}{\left(b\,x^2+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a + b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)/(a + b*x^2)^(1/4), x)
```

sympy [C] time = 1.01, size = 44, normalized size = 0.53

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(1/4)*gamma(7/4))
```

3.952 $\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{2\sqrt{b}\,\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\sqrt[4]{a+bx^2}}-\frac{2}{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}$$

```
[Out] -2/c/(b*x^2+a)^(1/4)/(c*x)^(1/2)+2*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1
/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2
*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)*(c*x)^(1/2)/c^2/(b*x^2+a)^(1/4
)/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {316, 284, 335, 196}

$$\frac{2\sqrt{b}\,\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\sqrt[4]{a+bx^2}}-\frac{2}{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] -2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (2*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sqrt[
c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*c^2*(a + b*x^2)^
(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 316

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[-
2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)), x] - Dist[b/c^2, Int[Sqrt[c*x]/(a + b*x^
2)^(5/4), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2}{c\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \frac{b\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{c^2}$$
$$= -\frac{2}{c\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \frac{\left(\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{c^2\sqrt[4]{a+bx^2}}$$
$$= -\frac{2}{c\sqrt{cx}\,\sqrt[4]{a+bx^2}} + \frac{\left(\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{c^2\sqrt[4]{a+bx^2}}$$
$$= -\frac{2}{c\sqrt{cx}\,\sqrt[4]{a+bx^2}} + \frac{2\sqrt{b}\,\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.60

$$-\frac{2x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(1/4)),x]
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, -((b*x^2)/a)]
)/((c*x)^(3/2)*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{bc^2x^4+ac^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^2*x^4 + a*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{2}}\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{3/2}\,\left(b\,x^2+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a + b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(3/2)*(a + b*x^2)^(1/4)), x)
```

sympy [C] time = 2.04, size = 31, normalized size = 0.34

$$-\frac{{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{\sqrt[4]{b}\,c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((1/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(1/4)*c**(3/2)*x
)
```

3.953 $\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=126

$$-\frac{4b^{3/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a+bx^2}}+\frac{4b}{5ac^3\sqrt{cx}\sqrt[4]{a+bx^2}}-\frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

```
[Out] -2/5*(b*x^2+a)^(3/4)/a/c/(c*x)^(5/2)+4/5*b/a/c^3/(b*x^2+a)^(1/4)/(c*x)^(1/2
)-4/5*b^(3/2)*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2
)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(
1/2))),2^(1/2))*(c*x)^(1/2)/a^(3/2)/c^4/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {325, 316, 284, 335, 196}

$$-\frac{4b^{3/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a+bx^2}}+\frac{4b}{5ac^3\sqrt{cx}\sqrt[4]{a+bx^2}}-\frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (4*b)/(5*a*c^3*Sqrt[c*x]*(a + b*x^2)^(1/4)) - (2*(a + b*x^2)^(3/4))/(5*a*c*
(c*x)^(5/2)) - (4*b^(3/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[
(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*c^4*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 316

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[-
2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)), x] - Dist[b/c^2, Int[Sqrt[c*x]/(a + b*x^
2)^(5/4), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx &= -\frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}} - \frac{(2b)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx}{5ac^2}\\
&= \frac{4b}{5ac^3\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}} + \frac{\left(2b^2\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{5ac^4}\\
&= \frac{4b}{5ac^3\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}} + \frac{\left(2b\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{5ac^4\sqrt[4]{a+bx^2}}\\
&= \frac{4b}{5ac^3\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}} - \frac{\left(2b\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{5ac^4\sqrt[4]{a+bx^2}}\\
&= \frac{4b}{5ac^3\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}} - \frac{4b^{3/2}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a+bx^2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.44

$$-\frac{2x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, -((b*x^2)/a)
])/(5*(c*x)^(7/2)*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{bc^4x^6+ac^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^4*x^6 + a*c^4*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx\right)^{\frac{7}{2}} \left(b\,x^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(bx^{2}+a\right)}^{\frac{1}{4}} \left(cx\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{7/2}\,{\left(b\,x^2+a\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a + b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(7/2)*(a + b*x^2)^(1/4)), x)
```

sympy [C] time = 12.23, size = 34, normalized size = 0.27

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{a e^{i \pi}}{b x^{2}}} \right)}{3 \sqrt[4]{b} c^{\frac{7}{2}} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((1/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**2))/(3*b**(1/4)*c**(7/2)
*x**3)
```

3.954 $\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=157

$$\frac{8b^{5/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a+bx^2}}-\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}}+\frac{4b\left(a+bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}}$$

```
[Out] -2/9*(b*x^2+a)^(3/4)/a/c/(c*x)^(9/2)+4/15*b*(b*x^2+a)^(3/4)/a^2/c^3/(c*x)^(
5/2)-8/15*b^2/a^2/c^5/(b*x^2+a)^(1/4)/(c*x)^(1/2)+8/15*b^(5/2)*(1+a/b/x^2)^
(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)
/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*x)^(1/2
)/a^(5/2)/c^6/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {325, 316, 284, 335, 196}

$$-\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}}+\frac{8b^{5/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a+bx^2}}+\frac{4b\left(a+bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-8*b^2)/(15*a^2*c^5*Sqrt[c*x]*(a + b*x^2)^(1/4)) - (2*(a + b*x^2)^(3/4))/(
9*a*c*(c*x)^(9/2)) + (4*b*(a + b*x^2)^(3/4))/(15*a^2*c^3*(c*x)^(5/2)) + (8*
b^(5/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a
]]/2, 2])/(15*a^(5/2)*c^6*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 316

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[-
2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)), x] - Dist[b/c^2, Int[Sqrt[c*x]/(a + b*x^
2)^(5/4), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}} - \frac{(2b)\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx}{3ac^2}$$

$$= -\frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b\left(a+bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}} + \frac{\left(4b^2\right)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx}{15a^2c^4}$$

$$= -\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b\left(a+bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}} - \frac{\left(4b^3\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{15a^2c^6}$$

$$= -\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b\left(a+bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}} - \frac{\left(4b^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \cdots}{15a^2c^6\sqrt[4]{a+b\cdots}}$$

$$= -\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b\left(a+bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}} + \frac{\left(4b^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Su}\cdots}{15a^2c^6\cdots}$$

$$= -\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b\left(a+bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}} + \frac{8b^{5/2}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\cdots\right.}{15a^{5/2}c^6\sqrt[4]{a\cdots}}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.36

$$-\frac{2x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{9}{4},\frac{1}{4};-\frac{5}{4};-\frac{bx^2}{a}\right)}{9(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, -((b*x^2)/a)
])/(9*(c*x)^(11/2)*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{bc^6x^8+ac^6x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^6*x^8 + a*c^6*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{11}{2}} \left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}} (cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{11/2}\,{\left(b\,x^2+a\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(a + b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(11/2)*(a + b*x^2)^(1/4)), x)
```

sympy [C] time = 89.35, size = 34, normalized size = 0.22

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{5\sqrt[4]{b}\,c^{\frac{11}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] -hyper((1/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*b**(1/4)*c**(11/2
)*x**5)
```

3.955 $\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=308

$$-\frac{ac^{3/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{5/4}}+\frac{ac^{3/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{5/4}}-\frac{ac^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{5/4}}+\cdots$$

```
[Out] 1/8*a*c^(3/2)*arctan(-1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^(1/2
))/b^(5/4)*2^(1/2)+1/8*a*c^(3/2)*arctan(1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x
^2+a)^(1/4)/c^(1/2))/b^(5/4)*2^(1/2)-1/16*a*c^(3/2)*ln(c^(1/2)-b^(1/4)*2^(1
/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))/b^(5/4
)*2^(1/2)+1/16*a*c^(3/2)*ln(c^(1/2)+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^
(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))/b^(5/4)*2^(1/2)-1/2*c*(-b*x^2+a)^
(3/4)*(c*x)^(1/2)/b
```

Rubi [A] time = 0.27, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {321, 329, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{ac^{3/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{5/4}}+\frac{ac^{3/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{5/4}}-\frac{ac^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{5/4}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a - b*x^2)^(1/4),x]
```

```
[Out] -(c*Sqrt[c*x]*(a - b*x^2)^(3/4))/(2*b) - (a*c^(3/2)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*Sqrt[2]*b^(5/4)) + (a*c^(3
/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4
*Sqrt[2]*b^(5/4)) - (a*c^(3/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b
*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(5/4))
 + (a*c^(3/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*
b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx = -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b} + \frac{\left(ac^2\right)\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx}{4b}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b} + \frac{(ac)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a-\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2b}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b} + \frac{(ac)\,\text{Subst}\left(\int \frac{1}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2b}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b} + \frac{a\,\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4b} + \frac{a\,\text{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4b}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b} - \frac{\left(ac^{3/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{5/4}} - \frac{\left(ac^{3/2}\right)\text{Subst}\left(\int \frac{}{-\frac{c}{\sqrt{b}}}\right)}{8}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b} - \frac{ac^{3/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{5/4}} + \frac{ac^{3/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}}{\sqrt[4]{}}\right)}{8\sqrt{2}\,b^{5/4}}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b} - \frac{ac^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{5/4}} + \frac{ac^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{5/4}} - ac^{3/2}\log$$

Mathematica [A] time = 0.15, size = 241, normalized size = 0.78

$$-\frac{(cx)^{3/2}\left(8\sqrt[4]{b}\sqrt{x}\left(a-bx^2\right)^{3/4}+\sqrt{2}\,a\log\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)-\sqrt{2}\,a\log\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)+2\sqrt{2}\right.}{16b^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a - b*x^2)^(1/4),x]
```

```
[Out] -1/16*((c*x)^(3/2)*(8*b^(1/4)*Sqrt[x]*(a - b*x^2)^(3/4) + 2*Sqrt[2]*a*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] - 2*Sqrt[2]*a*ArcTan[1 +
 (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + Sqrt[2]*a*Log[1 + (Sqrt[b]*
x)/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] - Sqrt[2]
*a*Log[1 + (Sqrt[b]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x
^2)^(1/4)]))/(b^(5/4)*x^(3/2))
```

fricas [A] time = 0.62, size = 340, normalized size = 1.10

$$-\frac{4\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,c+4\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(-\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,ab^4c-\left(b^5x^2-ab^4\right)\left(-\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}\sqrt{-\frac{\sqrt{-bx^2+a}\,a^2c^3x-\sqrt{-\frac{a^4c^6}{b^5}}\left(b^3\right.}{bx^2-a}}}{a^4bc^6x^2-a^5c^6}\right.}{8\,l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(-b*x^2 + a)^(3/4)*sqrt(c*x)*c + 4*(-a^4*c^6/b^5)^(1/4)*b*arctan(-(
(-a^4*c^6/b^5)^(3/4)*(-b*x^2 + a)^(3/4)*sqrt(c*x)*a*b^4*c - (b^5*x^2 - a*b^
4)*(-a^4*c^6/b^5)^(3/4)*sqrt(-(sqrt(-b*x^2 + a)*a^2*c^3*x - sqrt(-a^4*c^6/b
^5)*(b^3*x^2 - a*b^2))/(b*x^2 - a)))/(a^4*b*c^6*x^2 - a^5*c^6)) + (-a^4*c^6
/b^5)^(1/4)*b*log(((-b*x^2 + a)^(3/4)*sqrt(c*x)*a*c + (-a^4*c^6/b^5)^(1/4)*
(b^2*x^2 - a*b))/(b*x^2 - a)) - (-a^4*c^6/b^5)^(1/4)*b*log(((-b*x^2 + a)^(3
/4)*sqrt(c*x)*a*c - (-a^4*c^6/b^5)^(1/4)*(b^2*x^2 - a*b))/(b*x^2 - a)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x)^{3/2}}{\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a - b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(3/2)/(a - b*x^2)^(1/4), x)
```

sympy [C] time = 2.20, size = 46, normalized size = 0.15

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**2*exp_polar(2*I
*pi)/a)/(2*a**(1/4)*gamma(9/4))
```

3.956 $\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=272

$$-\frac{\log\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}+\frac{\log\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}$$

```
[Out] 1/2*arctan(-1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^(1/2))/b^(1/4)
*2^(1/2)/c^(1/2)+1/2*arctan(1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/
c^(1/2))/b^(1/4)*2^(1/2)/c^(1/2)-1/4*ln(c^(1/2)-b^(1/4)*2^(1/2)*(c*x)^(1/2)
/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))/b^(1/4)*2^(1/2)/c^(1/
2)+1/4*ln(c^(1/2)+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^
(1/2)/(-b*x^2+a)^(1/2))/b^(1/4)*2^(1/2)/c^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {329, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}+\frac{\log\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a - b*x^2)^(1/4)),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]/(Sqrt
[2]*b^(1/4)*Sqrt[c])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a
- b*x^2)^(1/4))]/(Sqrt[2]*b^(1/4)*Sqrt[c]) - Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]
*x)/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(2*Sqr
t[2]*b^(1/4)*Sqrt[c]) + Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] +
 (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(2*Sqrt[2]*b^(1/4)*Sqrt[c])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a-bx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a-\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c}$$

$$= \frac{\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^2} + \frac{\text{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{b}} - \frac{\text{Subs}\cdots}{\cdots}$$

$$= -\frac{\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \frac{\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\cdots\right)}{\sqrt{2}\cdots}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}} - \frac{\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \cdots$$

Mathematica [A] time = 0.04, size = 197, normalized size = 0.72

$$\frac{\sqrt{x}\left(-\log\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)+\log\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}\right.}{2\sqrt{2}\sqrt[4]{b}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a - b*x^2)^(1/4)),x]
```

```
[Out] (Sqrt[x]*(-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + 2*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] - Log[1 + (Sqrt[b]*x)
/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + Log[1 + (
Sqrt[b]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)]))
/(2*Sqrt[2]*b^(1/4)*Sqrt[c*x])
```

fricas [A] time = 0.66, size = 267, normalized size = 0.98

$$-2\left(-\frac{1}{bc^2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,bc\left(-\frac{1}{bc^2}\right)^{\frac{3}{4}}-\left(b^2cx^2-abc\right)\sqrt{-\frac{\sqrt{-bx^2+a}\,cx-\left(bc^2x^2-ac^2\right)\sqrt{-\frac{1}{bc^2}}}{bx^2-a}}\left(-\frac{1}{bc^2}\right)^{\frac{3}{4}}}{bx^2-a}\right)-\frac{1}{2}\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2*(-1/(b*c^2))^(1/4)*arctan(-((-b*x^2 + a)^(3/4)*sqrt(c*x)*b*c*(-1/(b*c^2)
)^(3/4) - (b^2*c*x^2 - a*b*c)*sqrt(-(sqrt(-b*x^2 + a)*c*x - (b*c^2*x^2 - a*
c^2)*sqrt(-1/(b*c^2)))/(b*x^2 - a))*(-1/(b*c^2))^(3/4))/(b*x^2 - a)) - 1/2*
(-1/(b*c^2))^(1/4)*log(((-b*x^2 + a)^(3/4)*sqrt(c*x) + (b*c*x^2 - a*c)*(-1/
(b*c^2))^(1/4))/(b*x^2 - a)) + 1/2*(-1/(b*c^2))^(1/4)*log(((-b*x^2 + a)^(3/
4)*sqrt(c*x) - (b*c*x^2 - a*c)*(-1/(b*c^2))^(1/4))/(b*x^2 - a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x}\,{\left(a-b\,x^2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a - b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(a - b*x^2)^(1/4)), x)
```

sympy [C] time = 1.52, size = 46, normalized size = 0.17

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\,\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**2*exp_polar(2*I*pi)/a)/(2
*a**(1/4)*sqrt(c)*gamma(5/4))
```

3.957 $\int \frac{1}{(cx)^{5/2} \sqrt[4]{a-bx^2}} \, dx$

Optimal. Leaf size=29

$$-\frac{2 \left(a-b x^2\right)^{3/4}}{3 a c (c x)^{3/2}}$$

```
[Out] -2/3*(-b*x^2+a)^(3/4)/a/c/(c*x)^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {264}

$$-\frac{2 \left(a-b x^2\right)^{3/4}}{3 a c (c x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c x)^{5/2} \sqrt[4]{a-b x^2}} \, dx = -\frac{2 \left(a-b x^2\right)^{3/4}}{3 a c (c x)^{3/2}}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.93

$$-\frac{2 x \left(a-b x^2\right)^{3/4}}{3 a (c x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(a - b*x^2)^(3/4))/(3*a*(c*x)^(5/2))
```

fricas [A] time = 0.70, size = 26, normalized size = 0.90

$$-\frac{2 \left(-b x^2+a\right)^{\frac{3}{4}} \sqrt{c x}}{3 \, a c^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2/3*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a*c^3*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$-\frac{2\left(-b\,x^2+a\right)^{\frac{3}{4}} x}{3\,(cx)^{\frac{5}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x)
[Out] -2/3*x*(-b*x^2+a)^(3/4)/a/(c*x)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

mupad [B] time = 5.12, size = 26, normalized size = 0.90

$$-\frac{2\left(a-b\,x^2\right)^{3/4}}{3\,a\,c^2\,x\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x)
[Out] -(2*(a - b*x^2)^(3/4))/(3*a*c^2*x*(c*x)^(1/2))
```

sympy [A] time = 3.62, size = 88, normalized size = 3.03

$$\begin{cases} \frac{b^{\frac{3}{4}}\left(\frac{a}{bx^2}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)} & \text{for}\ \left|\frac{a}{bx^2}\right|>1 \\ -\frac{b^{\frac{3}{4}}\left(-\frac{a}{bx^2}+1\right)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(-b*x**2+a)**(1/4),x)
[Out] Piecewise((b**(3/4)*(a/(b*x**2) - 1)**(3/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma
(1/4)), Abs(a/(b*x**2)) > 1), (-b**(3/4)*(-a/(b*x**2) + 1)**(3/4)*exp(-I*pi
/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma(1/4)), True))
```

3.958 $\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=59

$$\frac{8\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{7/2}}$$

```
[Out] -2/3*(-b*x^2+a)^(3/4)/a/c/(c*x)^(7/2)+8/21*(-b*x^2+a)^(7/4)/a^2/c/(c*x)^(7/
2)
```

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {273, 264}

$$\frac{8\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(7/2)) + (8*(a - b*x^2)^(7/4))/(21*a^2*
c*(c*x)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx &= -\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{7/2}}-\frac{4\int \frac{\left(a-bx^2\right)^{3/4}}{(cx)^{9/2}}\,dx}{3a}\\ &= -\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{7/2}}+\frac{8\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.71

$$-\frac{2\sqrt{cx}\left(a-bx^2\right)^{3/4}\left(3a+4bx^2\right)}{21a^2c^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(3/4)*(3*a + 4*b*x^2))/(21*a^2*c^5*x^4)
```

fricas [A] time = 0.58, size = 36, normalized size = 0.61

$$-\frac{2\left(4\,bx^{2}+3\,a\right)\left(-bx^{2}+a\right)^{\frac{3}{4}}\sqrt{cx}}{21\,a^{2}c^{5}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] -2/21*(4*b*x^2 + 3*a)*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*c^5*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^{2}+a\right)^{\frac{1}{4}}\left(cx\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

maple [A] time = 0.00, size = 32, normalized size = 0.54

$$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{3}{4}}\left(4b\,x^{2}+3a\right)x}{21\left(cx\right)^{\frac{9}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x)
[Out] -2/21*x*(-b*x^2+a)^(3/4)*(4*b*x^2+3*a)/a^2/(c*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^{2}+a\right)^{\frac{1}{4}}\left(cx\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

mupad [B] time = 5.14, size = 41, normalized size = 0.69

$$-\frac{{\left(a-b\,x^2\right)}^{3/4}\,\left(\frac{2}{7\,a\,c^4}+\frac{8\,b\,x^2}{21\,a^2\,c^4}\right)}{x^3\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x)
[Out] -((a - b*x^2)^(3/4)*(2/(7*a*c^4) + (8*b*x^2)/(21*a^2*c^4)))/(x^3*(c*x)^(1/2
))
```

sympy [A] time = 34.34, size = 343, normalized size = 5.81

$$\begin{cases} -\frac{3b^{\frac{3}{4}}\left(\frac{a}{bx^2}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{8ac^{\frac{9}{2}}x^2\Gamma\left(\frac{1}{4}\right)} - \frac{b^{\frac{7}{4}}\left(\frac{a}{bx^2}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{2a^2c^{\frac{9}{2}}\Gamma\left(\frac{1}{4}\right)} & \text{for } \left|\frac{a}{bx^2}\right| \\ -\frac{3a^2b^{\frac{7}{4}}\left(-\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-8a^3bc^{\frac{9}{2}}x^2e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+8a^2b^2c^{\frac{9}{2}}x^4e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)} - \frac{ab^{\frac{11}{4}}x^2\left(-\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-8a^3bc^{\frac{9}{2}}x^2e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+8a^2b^2c^{\frac{9}{2}}x^4e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)} + \frac{4b^{\frac{15}{4}}x^4\left(-\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-8a^3bc^{\frac{9}{2}}x^2e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+8a^2b^2c^{\frac{9}{2}}x^4e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)} & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] Piecewise((-3*b**(3/4)*(a/(b*x**2) - 1)**(3/4)*gamma(-7/4)/(8*a*c**(9/2)*x*
*2*gamma(1/4)) - b**(7/4)*(a/(b*x**2) - 1)**(3/4)*gamma(-7/4)/(2*a**2*c**(9
/2)*gamma(1/4)), Abs(a/(b*x**2)) > 1), (-3*a**2*b**(7/4)*(-a/(b*x**2) + 1)*
*(3/4)*gamma(-7/4)/(-8*a**3*b*c**(9/2)*x**2*exp(I*pi/4)*gamma(1/4) + 8*a**2
*b**2*c**(9/2)*x**4*exp(I*pi/4)*gamma(1/4)) - a*b**(11/4)*x**2*(-a/(b*x**2)
 + 1)**(3/4)*gamma(-7/4)/(-8*a**3*b*c**(9/2)*x**2*exp(I*pi/4)*gamma(1/4) +
8*a**2*b**2*c**(9/2)*x**4*exp(I*pi/4)*gamma(1/4)) + 4*b**(15/4)*x**4*(-a/(b
*x**2) + 1)**(3/4)*gamma(-7/4)/(-8*a**3*b*c**(9/2)*x**2*exp(I*pi/4)*gamma(1
/4) + 8*a**2*b**2*c**(9/2)*x**4*exp(I*pi/4)*gamma(1/4)), True))
```

3.959 $\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=88

$$-\frac{64\left(a-bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}+\frac{16\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

```
[Out] -2/3*(-b*x^2+a)^(3/4)/a/c/(c*x)^(11/2)+16/21*(-b*x^2+a)^(7/4)/a^2/c/(c*x)^(
11/2)-64/231*(-b*x^2+a)^(11/4)/a^3/c/(c*x)^(11/2)
```

Rubi [A] time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {273, 264}

$$-\frac{64\left(a-bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}+\frac{16\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) + (16*(a - b*x^2)^(7/4))/(21*a^
2*c*(c*x)^(11/2)) - (64*(a - b*x^2)^(11/4))/(231*a^3*c*(c*x)^(11/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx &= -\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{11/2}}-\frac{8\int \frac{\left(a-bx^2\right)^{3/4}}{(cx)^{13/2}}\,dx}{3a}\\ &= -\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{11/2}}+\frac{16\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}+\frac{32\int \frac{\left(a-bx^2\right)^{7/4}}{(cx)^{13/2}}\,dx}{21a^2}\\ &= -\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{11/2}}+\frac{16\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{64\left(a-bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.60

$$-\frac{2\sqrt{cx}\left(a-bx^2\right)^{3/4}\left(21a^2+24abx^2+32b^2x^4\right)}{231a^3c^7x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(3/4)*(21*a^2 + 24*a*b*x^2 + 32*b^2*x^4))/(231*a^
3*c^7*x^6)
```

fricas [A] time = 0.54, size = 47, normalized size = 0.53

$$-\frac{2\left(32\,b^2x^4 + 24\,abx^2 + 21\,a^2\right)\left(-bx^2 + a\right)^{\frac{3}{4}}\sqrt{cx}}{231\,a^3c^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2/231*(32*b^2*x^4 + 24*a*b*x^2 + 21*a^2)*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a^3
*c^7*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2 + a\right)^{\frac{1}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

maple [A] time = 0.01, size = 43, normalized size = 0.49

$$-\frac{2\left(-b\,x^2 + a\right)^{\frac{3}{4}}\left(32b^2x^4 + 24ab\,x^2 + 21a^2\right)x}{231\,(cx)^{\frac{13}{2}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] -2/231*x*(-b*x^2+a)^(3/4)*(32*b^2*x^4+24*a*b*x^2+21*a^2)/a^3/(c*x)^(13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2 + a\right)^{\frac{1}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

mupad [B] time = 5.25, size = 55, normalized size = 0.62

$$-\frac{\left(a - b\,x^2\right)^{3/4}\left(\frac{2}{11\,a\,c^6} + \frac{16\,b\,x^2}{77\,a^2\,c^6} + \frac{64\,b^2\,x^4}{231\,a^3\,c^6}\right)}{x^5\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x)
```

```
[Out] -((a - b*x^2)^(3/4)*(2/(11*a*c^6) + (16*b*x^2)/(77*a^2*c^6) + (64*b^2*x^4)/
(231*a^3*c^6)))/(x^5*(c*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] Timed out
```

3.960 $\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=128

$$\frac{a^{3/2}c^2\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a-bx^2}}-\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}$$

```
[Out] -1/3*c*(c*x)^(3/2)*(-b*x^2+a)^(3/4)/b-1/2*a*c^3*(-b*x^2+a)^(3/4)/b^2/(c*x)^
(1/2)+1/2*a^(3/2)*c^2*(1-a/b/x^2)^(1/4)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccsc(x*b^(
1/2)/a^(1/2))),2^(1/2))*(c*x)^(1/2)/b^(3/2)/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{321, 315, 317, 335, 228\}$

$$\frac{a^{3/2}c^2\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a-bx^2}}-\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a - b*x^2)^(1/4),x]
```

```
[Out] -(a*c^3*(a - b*x^2)^(3/4))/(2*b^2*Sqrt[c*x]) - (c*(c*x)^(3/2)*(a - b*x^2)^(
3/4))/(3*b) + (a^(3/2)*c^2*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCsc
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(3/2)*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 315

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[(c*(a +
b*x^2)^(3/4))/(b*Sqrt[c*x]), x] + Dist[(a*c^2)/(2*b), Int[1/((c*x)^(3/2)*(a
 + b*x^2)^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 317

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[(
Sqrt[c*x]*(1 + a/(b*x^2))^(1/4))/(c^2*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx &= -\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}+\frac{\left(ac^2\right)\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx}{2b}\\
&= -\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}-\frac{\left(a^2c^4\right)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx}{4b^2}\\
&= -\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}-\frac{\left(a^2c^2\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}\,x^2}\,dx}{4b^2\sqrt[4]{a-bx^2}}\\
&= -\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}+\frac{\left(a^2c^2\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx,x,\frac{1}{x}\right)}{4b^2\sqrt[4]{a-bx^2}}\\
&= -\frac{ac^3\left(a-bx^2\right)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}+\frac{a^{3/2}c^2\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a-bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.55

$$
\frac{c(cx)^{3/2}\left(a\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^2}{a}\right)-a+bx^2\right)}{3b\sqrt[4]{a-bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a - b*x^2)^(1/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4,
 3/4, 7/4, (b*x^2)/a]))/(3*b*(a - b*x^2)^(1/4))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,c^2x^2}{bx^2-a},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)*c^2*x^2/(b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(cx)^{\frac{5}{2}}}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(1/4), x)

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(5/2)/(-b*x^2+a)^(1/4),x)

[Out] int((c*x)^(5/2)/(-b*x^2+a)^(1/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(-bx^{2}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")

[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(1/4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{5/2}}{\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(5/2)/(a - b*x^2)^(1/4),x)

[Out] int((c*x)^(5/2)/(a - b*x^2)^(1/4), x)

sympy [C] time = 6.78, size = 46, normalized size = 0.36

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(5/2)/(-b*x**2+a)**(1/4),x)

[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**2*exp_polar(2*I*pi)/a)/(2*a**(1/4)*gamma(11/4))

3.961 $\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}-\frac{c\left(a-bx^2\right)^{3/4}}{b\sqrt{cx}}$$

```
[Out] -c*(-b*x^2+a)^(3/4)/b/(c*x)^(1/2)+(1-a/b/x^2)^(1/4)*(cos(1/2*arccsc(x*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*
arccsc(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/b^
(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {315, 317, 335, 228}

$$\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}-\frac{c\left(a-bx^2\right)^{3/4}}{b\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a - b*x^2)^(1/4),x]
```

```
[Out] -((c*(a - b*x^2)^(3/4))/(b*Sqrt[c*x])) + (Sqrt[a]*(1 - a/(b*x^2))^(1/4)*Sqr
t[c*x]*EllipticE[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a - b*x^2)^(1
/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 315

```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[(c*(a +
b*x^2)^(3/4))/(b*Sqrt[c*x]), x] + Dist[(a*c^2)/(2*b), Int[1/((c*x)^(3/2)*(a
 + b*x^2)^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 317

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[(
Sqrt[c*x]*(1 + a/(b*x^2))^(1/4))/(c^2*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx = -\frac{c\left(a-bx^2\right)^{3/4}}{b\sqrt{cx}} - \frac{\left(ac^2\right)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx}{2b}$$

$$= -\frac{c\left(a-bx^2\right)^{3/4}}{b\sqrt{cx}} - \frac{\left(a\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}\,x^2}\,dx}{2b\sqrt[4]{a-bx^2}}$$

$$= -\frac{c\left(a-bx^2\right)^{3/4}}{b\sqrt{cx}} + \frac{\left(a\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x}\right)}{2b\sqrt[4]{a-bx^2}}$$

$$= -\frac{c\left(a-bx^2\right)^{3/4}}{b\sqrt{cx}} + \frac{\sqrt{a}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.63

$$\frac{2x\sqrt{cx}\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^2}{a}\right)}{3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a - b*x^2)^(1/4),x]
```

```
[Out] (2*x*Sqrt[c*x]*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, (b*x^
2)/a])/(3*(a - b*x^2)^(1/4))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{bx^2-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x}}{\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a - b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)/(a - b*x^2)^(1/4), x)
```

sympy [C] time = 1.04, size = 46, normalized size = 0.51

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x**2*exp_polar(2*I*
pi)/a)/(2*a**(1/4)*gamma(7/4))
```

3.962 $\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2\sqrt{b}\,\sqrt{cx}\,\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\sqrt[4]{a-bx^2}}$$

```
[Out] -2*(1-a/b/x^2)^(1/4)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*a
rccsc(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccsc(x*b^(1/2)/a^(1/2))),2^(1
/2))*b^(1/2)*(c*x)^(1/2)/c^2/(-b*x^2+a)^(1/4)/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {317, 335, 228}

$$-\frac{2\sqrt{b}\,\sqrt{cx}\,\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*Sqrt[b]*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCsc[(Sqrt[b]*x)/Sq
rt[a]]/2, 2])/(Sqrt[a]*c^2*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 317

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[(
Sqrt[c*x]*(1 + a/(b*x^2))^(1/4))/(c^2*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx = \frac{\left(\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}\,x^2}\,dx}{c^2\sqrt[4]{a-bx^2}}$$
$$= -\frac{\left(\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x}\right)}{c^2\sqrt[4]{a-bx^2}}$$
$$= -\frac{2\sqrt{b}\,\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.81

$$-\frac{2x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a - b*x^2)^(1/4)),x]
[Out] (-2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (b*x^2)/a])/(
(c*x)^(3/2)*(a - b*x^2)^(1/4))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{bc^2x^4-ac^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^2*x^4 - a*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{2}}\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

[Out] int(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")

[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{3/2}\,\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((c*x)^(3/2)*(a - b*x^2)^(1/4)),x)

[Out] int(1/((c*x)^(3/2)*(a - b*x^2)^(1/4)), x)

sympy [C] time = 2.06, size = 32, normalized size = 0.47

$$\frac{ie^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{\sqrt[4]{b}\,c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)**(3/2)/(-b*x**2+a)**(1/4),x)

[Out] I*exp(I*pi/4)*hyper((1/4, 1/2), (3/2,), a/(b*x**2))/(b**(1/4)*c**(3/2)*x)

3.963 $\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=100

$$-\frac{4b^{3/2}\sqrt{cx}\,\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a-bx^2}}-\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

```
[Out] -2/5*(-b*x^2+a)^(3/4)/a/c/(c*x)^(5/2)-4/5*b^(3/2)*(1-a/b/x^2)^(1/4)*(cos(1/
2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*El
lipticE(sin(1/2*arccsc(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*x)^(1/2)/a^(3/2)/c^4
/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 317, 335, 228}

$$-\frac{4b^{3/2}\sqrt{cx}\,\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a-bx^2}}-\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(5*a*c*(c*x)^(5/2)) - (4*b^(3/2)*(1 - a/(b*x^2))^(1/
4)*Sqrt[c*x]*EllipticE[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*c^4*(a
 - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 317

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[(
Sqrt[c*x]*(1 + a/(b*x^2))^(1/4))/(c^2*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}} + \frac{(2b)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx}{5ac^2}$$

$$= -\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}} + \frac{\left(2b\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}\,x^2}\,dx}{5ac^4\sqrt[4]{a-bx^2}}$$

$$= -\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}} - \frac{\left(2b\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x}\right)}{5ac^4\sqrt[4]{a-bx^2}}$$

$$= -\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}} - \frac{4b^{3/2}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.57

$$-\frac{2x\sqrt[4]{1-\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a - b*x^2)^(1/4)),x]
[Out] (-2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, (b*x^2)/a])/
(5*(c*x)^(7/2)*(a - b*x^2)^(1/4))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{bc^4x^6-ac^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^4*x^6 - a*c^4*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{7}{2}}\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{7/2}\,\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a - b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(7/2)*(a - b*x^2)^(1/4)), x)
```

sympy [C] time = 12.20, size = 39, normalized size = 0.39

$$-\frac{ie^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{a}{bx^2}\right)}{3\sqrt[4]{b}\,c^{\frac{7}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] -I*exp(-3*I*pi/4)*hyper((1/4, 3/2), (5/2,), a/(b*x**2))/(3*b**(1/4)*c**(7/2
)*x**3)
```

3.964 $\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=130

$$-\frac{8b^{5/2}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a-bx^2}}-\frac{4b\left(a-bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}}$$

```
[Out] -2/9*(-b*x^2+a)^(3/4)/a/c/(c*x)^(9/2)-4/15*b*(-b*x^2+a)^(3/4)/a^2/c^3/(c*x)
^(5/2)-8/15*b^(5/2)*(1-a/b/x^2)^(1/4)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2
)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccsc(x*b^(1/
2)/a^(1/2))),2^(1/2))*(c*x)^(1/2)/a^(5/2)/c^6/(-b*x^2+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {325, 317, 335, 228}

$$-\frac{8b^{5/2}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a-bx^2}}-\frac{4b\left(a-bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(9*a*c*(c*x)^(9/2)) - (4*b*(a - b*x^2)^(3/4))/(15*a^
2*c^3*(c*x)^(5/2)) - (8*b^(5/2)*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[A
rcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*a^(5/2)*c^6*(a - b*x^2)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 317

```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[(
Sqrt[c*x]*(1 + a/(b*x^2))^(1/4))/(c^2*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a
/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}} + \frac{(2b)\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx}{3ac^2}$$
$$= -\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}} - \frac{4b\left(a-bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}} + \frac{\left(4b^2\right)\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx}{15a^2c^4}$$
$$= -\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}} - \frac{4b\left(a-bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}} + \frac{\left(4b^2\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}\,x^2}\,dx}{15a^2c^6\sqrt[4]{a-bx^2}}$$
$$= -\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}} - \frac{4b\left(a-bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}} - \frac{\left(4b^2\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x}\right)}{15a^2c^6\sqrt[4]{a-bx^2}}$$
$$= -\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}} - \frac{4b\left(a-bx^2\right)^{3/4}}{15a^2c^3(cx)^{5/2}} - \frac{8b^{5/2}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a-bx^2}}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.44

$$-\frac{2x\sqrt[4]{1-\frac{bx^2}{a}}\ {}_2F_1\left(-\frac{9}{4},\frac{1}{4};-\frac{5}{4};\frac{bx^2}{a}\right)}{9(cx)^{11/2}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a - b*x^2)^(1/4)),x]
[Out] (-2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, (b*x^2)/a])/
(9*(c*x)^(11/2)*(a - b*x^2)^(1/4))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{bc^6x^8-ac^6x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^6*x^8 - a*c^6*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{11}{2}}\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{11/2}\,\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(a - b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(11/2)*(a - b*x^2)^(1/4)), x)
```

sympy [C] time = 87.78, size = 36, normalized size = 0.28

$$\frac{ie^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{5\sqrt[4]{b}\,c^{\frac{11}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] I*exp(I*pi/4)*hyper((1/4, 5/2), (7/2,), a/(b*x**2))/(5*b**(1/4)*c**(11/2)*x
**5)
```

3.965 $\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{a}\,(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\,\left(a+bx^2\right)^{3/4}}+\frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b}$$

```
[Out] (1+a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/
cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/
2))),2^(1/2))*a^(1/2)/(b*x^2+a)^(3/4)/b^(1/2)+c*(b*x^2+a)^(1/4)*(c*x)^(1/2)
/b
```

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {321, 329, 237, 335, 275, 231}

$$\frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b}+\frac{\sqrt{a}\,(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a + b*x^2)^(1/4))/b + (Sqrt[a]*(1 + a/(b*x^2))^(3/4)*(c*x)^(3
/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(3/4)
)
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{3/4}}\,dx &= \frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b}-\frac{\left(ac^2\right)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{2b}\\
&= \frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b}-\frac{(ac)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{b}\\
&= \frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b}-\frac{\left(ac\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{b\left(a+bx^2\right)^{3/4}}\\
&= \frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b}+\frac{\left(ac\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{b\left(a+bx^2\right)^{3/4}}\\
&= \frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b}+\frac{\left(ac\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{2b\left(a+bx^2\right)^{3/4}}\\
&= \frac{c\sqrt{cx}\,\sqrt[4]{a+bx^2}}{b}+\frac{\sqrt{a}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 66, normalized size = 0.77

$$
\frac{c\sqrt{cx}\left(-a\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)+a+bx^2\right)}{b\left(a+bx^2\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a + b*x^2 - a*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/
4, 5/4, -((b*x^2)/a)]))/(b*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx}\,cx}{\left(bx^2+a\right)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral(sqrt(c*x)*c*x/(b*x^2 + a)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(3/4),x)
[Out] int((c*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{\left(b\,x^2+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a + b*x^2)^(3/4),x)
[Out] int((c*x)^(3/2)/(a + b*x^2)^(3/4), x)
```

sympy [C] time = 2.03, size = 44, normalized size = 0.51

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4}, \frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(3/4)*gamma(9/4))
```

3.966 $\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=66

$$-\frac{2\sqrt{b}\,(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,c^2\left(a+bx^2\right)^{3/4}}$$

```
[Out] -2*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/
2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^
(1/2))),2^(1/2))*b^(1/2)/c^2/(b*x^2+a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {329, 237, 335, 275, 231}

$$-\frac{2\sqrt{b}\,(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[b]*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/
Sqrt[a]]/2, 2])/(Sqrt[a]*c^2*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx=\frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{c}$$

$$=\frac{\left(2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{c\left(a+bx^2\right)^{3/4}}$$

$$=-\frac{\left(2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{c\left(a+bx^2\right)^{3/4}}$$

$$=-\frac{\left(\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{c\left(a+bx^2\right)^{3/4}}$$

$$=-\frac{2\sqrt{b}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.82

$$\frac{2x\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(3/4)),x]
```

```
[Out] (2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)])/
(Sqrt[c*x]*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{bcx^3+acx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c*x^3 + a*c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\left(b\,x^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x}\,\left(b\,x^{2}+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(a + b*x^2)^(3/4)), x)
```

sympy [C] time = 1.71, size = 31, normalized size = 0.47

$$-\frac{{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{b^{\frac{3}{4}}\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((1/2, 3/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(3/4)*sqrt(c)*x)
```

3.967 $\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=97

$$\frac{4b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{3a^{3/2}c^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}$$

```
[Out] -2/3*(b*x^2+a)^(1/4)/a/c/(c*x)^(3/2)+4/3*b^(3/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3
/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a
^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/c^4/
(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {325, 329, 237, 335, 275, 231}

$$\frac{4b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}c^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(3*a*c*(c*x)^(3/2)) + (4*b^(3/2)*(1 + a/(b*x^2))^(3/
4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*a^(3/2)*c^4*
(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}} - \frac{(2b)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{3ac^2}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}} - \frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{3ac^3}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}} - \frac{\left(4b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{3ac^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}} + \frac{\left(4b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{3ac^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}} + \frac{\left(2b\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{3ac^3\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}} + \frac{4b^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}c^4\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.58

$$-\frac{2x\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, -((b*x^2)/a)]
)/(3*(c*x)^(5/2)*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{bc^3x^5+ac^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^3*x^5 + a*c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{2}}\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x)
[Out] int(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{5/2}\left(b\,x^2+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a + b*x^2)^(3/4)),x)
[Out] int(1/((c*x)^(5/2)*(a + b*x^2)^(3/4)), x)
```

sympy [C] time = 7.83, size = 48, normalized size = 0.49

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 3/4), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4
)*c**(5/2)*x**(3/2)*gamma(1/4))
```

3.968 $\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=126

$$-\frac{8b^{5/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{7a^{5/2}c^6\left(a+bx^2\right)^{3/4}}+\frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}$$

```
[Out] -2/7*(b*x^2+a)^(1/4)/a/c/(c*x)^(7/2)+4/7*b*(b*x^2+a)^(1/4)/a^2/c^3/(c*x)^(3
/2)-8/7*b^(5/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot
(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/c^6/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {325, 329, 237, 335, 275, 231}

$$-\frac{8b^{5/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}c^6\left(a+bx^2\right)^{3/4}}+\frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(7*a*c*(c*x)^(7/2)) + (4*b*(a + b*x^2)^(1/4))/(7*a^2
*c^3*(c*x)^(3/2)) - (8*b^(5/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[
ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*a^(5/2)*c^6*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} - \frac{(6b)\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx}{7ac^2}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} + \frac{\left(4b^2\right)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{7a^2c^4}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} + \frac{\left(8b^2\right)\mathrm{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{7a^2c^5}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} + \frac{\left(8b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x,\right.}{7a^2c^5\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{\left(8b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1+\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x,\right.}{7a^2c^5\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{\left(4b^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x,\right.}{7a^2c^5\left(a+bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{8b^{5/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}c^6\left(a+bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.44

$$-\frac{2x\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};-\frac{bx^2}{a}\right)}{7(cx)^{9/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, -((b*x^2)/a)
])/(7*(c*x)^(9/2)*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{bc^5x^7+ac^5x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^5*x^7 + a*c^5*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}\left(cx\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx\right)^{\frac{9}{2}}\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}\left(cx\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{9/2}\,{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(9/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] int(1/((c*x)^(9/2)*(a + b*x^2)^(3/4)), x)
```

sympy [C] time = 66.19, size = 34, normalized size = 0.27

$$-\frac{{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{5b^{\frac{3}{4}}c^{\frac{9}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -hyper((3/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*b**(3/4)*c**(9/2)
*x**5)
```

3.969 $\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=157

$$\frac{80b^{7/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{77a^{7/2}c^8\left(a+bx^2\right)^{3/4}}-\frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}}+\frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

```
[Out] -2/11*(b*x^2+a)^(1/4)/a/c/(c*x)^(11/2)+20/77*b*(b*x^2+a)^(1/4)/a^2/c^3/(c*x
)^(7/2)-40/77*b^2*(b*x^2+a)^(1/4)/a^3/c^5/(c*x)^(3/2)+80/77*b^(7/2)*(1+a/b/
x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2
*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^
(1/2))/a^(7/2)/c^8/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {325, 329, 237, 335, 275, 231}

$$-\frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}}+\frac{80b^{7/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}c^8\left(a+bx^2\right)^{3/4}}+\frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(11*a*c*(c*x)^(11/2)) + (20*b*(a + b*x^2)^(1/4))/(77
*a^2*c^3*(c*x)^(7/2)) - (40*b^2*(a + b*x^2)^(1/4))/(77*a^3*c^5*(c*x)^(3/2))
 + (80*b^(7/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCot[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/(77*a^(7/2)*c^8*(a + b*x^2)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} - \frac{(10b)\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx}{11ac^2}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} + \frac{\left(60b^2\right)\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx}{77a^2c^4}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{\left(40b^3\right)\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{3/4}}\,dx}{77a^3c^6}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{\left(80b^3\right)\text{Subst}\left(\int \frac{1}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x\right.}{77a^3c^7}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{\left(80b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Sub}}{77a^3c^7\left(a\right.}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}} + \frac{\left(80b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Sub}}{77a^3c^7\left(a+\right.}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}} + \frac{\left(40b^3\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Sub}}{77a^3c^7\left(a+\right.}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}} + \frac{80b^{7/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\right.}{77a^{7/2}c^8\left(a+bx^2\right)}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.36

$$-\frac{2x\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(-\frac{11}{4},\frac{3}{4};-\frac{7}{4};-\frac{bx^2}{a}\right)}{11(cx)^{13/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-11/4, 3/4, -7/4, -((b*x^2)/a
)])/(11*(c*x)^(13/2)*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{bc^7x^9+ac^7x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^7*x^9 + a*c^7*x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{13}{2}}\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{(c\,x)}^{13/2}\,{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(13/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] int(1/((c*x)^(13/2)*(a + b*x^2)^(3/4)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

3.970 $\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=117

$$\frac{3ac^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}-\frac{3ac^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}$$

```
[Out] 1/2*c*(c*x)^(3/2)*(b*x^2+a)^(1/4)/b+3/4*a*c^(5/2)*arctan(b^(1/4)*(c*x)^(1/2
)/(b*x^2+a)^(1/4)/c^(1/2))/b^(7/4)-3/4*a*c^(5/2)*arctanh(b^(1/4)*(c*x)^(1/2
)/(b*x^2+a)^(1/4)/c^(1/2))/b^(7/4)
```

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {321, 329, 331, 298, 205, 208}

$$\frac{3ac^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}-\frac{3ac^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b) + (3*a*c^(5/2)*ArcTan[(b^(1/4)*Sqrt
[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(7/4)) - (3*a*c^(5/2)*ArcTanh[(b^
(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(7/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}\,dx &= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{(3ac^2)\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx}{4b} \\
&= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{(3ac)\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{2b} \\
&= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{(3ac)\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b} \\
&= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{(3ac^3)\,\mathrm{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b^{3/2}} + \frac{(3ac^3)\,\mathrm{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx,\right.}{4b^{3/2}} \\
&= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} + \frac{3ac^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}} - \frac{3ac^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 0.83

$$\frac{(cx)^{5/2}\left(2b^{3/4}x^{3/2}\sqrt[4]{a+bx^2}+3a\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)-3a\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{7/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] ((c*x)^(5/2)*(2*b^(3/4)*x^(3/2)*(a + b*x^2)^(1/4) + 3*a*ArcTan[(b^(1/4)*Sqr
t[x])/(a + b*x^2)^(1/4)] - 3*a*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]
))/(4*b^(7/4)*x^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(b\,x^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{5/2}}{\left(b\,x^2+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a + b*x^2)^(3/4),x)
```

```
[Out] int((c*x)^(5/2)/(a + b*x^2)^(3/4), x)
```

sympy [C] time = 7.78, size = 44, normalized size = 0.38

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(3/4)*gamma(11/4))
```

3.971 $\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}$$

```
[Out] -arctan(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))*c^(1/2)/b^(3/4)+arctan
h(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))*c^(1/2)/b^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = $\{329, 331, 298, 205, 208\}$

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(3/4),x]
```

```
[Out] -((Sqrt[c]*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(3/4)
) + (Sqrt[c]*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(3
/4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}$$
$$= \frac{c\,\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{b}} - \frac{c\,\text{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{b}}$$
$$= -\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}} + \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}$$

Mathematica [A] time = 0.01, size = 67, normalized size = 0.80

$$\frac{\sqrt{cx}\left(\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)-\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{b^{3/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(3/4),x]
[Out] (Sqrt[c*x]*(-ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + ArcTanh[(b^(1/4)
*Sqrt[x])/(a + b*x^2)^(1/4)]))/(b^(3/4)*Sqrt[x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x}}{{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a + b*x^2)^(3/4),x)
```

```
[Out] int((c*x)^(1/2)/(a + b*x^2)^(3/4), x)
```

sympy [C] time = 1.50, size = 44, normalized size = 0.52

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(3/4)*gamma(7/4))
```

3.972 $\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=26

$$-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

[Out] -2*(b*x^2+a)^(1/4)/a/c/(c*x)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x]

[Out] (-2*(a + b*x^2)^(1/4))/(a*c*Sqrt[c*x])

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.92

$$-\frac{2x\sqrt[4]{a+bx^2}}{a(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x]

[Out] (-2*x*(a + b*x^2)^(1/4))/(a*(c*x)^(3/2))

fricas [A] time = 0.97, size = 25, normalized size = 0.96

$$-\frac{2\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")

[Out] -2*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^2*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.81

$$-\frac{2\left(b\,x^2+a\right)^{\frac{1}{4}}x}{\left(cx\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x)
[Out] -2*x*(b*x^2+a)^(1/4)/a/(c*x)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

mupad [B] time = 4.90, size = 22, normalized size = 0.85

$$-\frac{2\left(b\,x^2+a\right)^{1/4}}{a\,c\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x)
[Out] -(2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(1/2))
```

sympy [A] time = 2.92, size = 36, normalized size = 1.38

$$\frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(3/4),x)
[Out] b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma(3/4))
```

3.973 $\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=55

$$\frac{8\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}$$

```
[Out] -2*(b*x^2+a)^(1/4)/a/c/(c*x)^(5/2)+8/5*(b*x^2+a)^(5/4)/a^2/c/(c*x)^(5/2)
```

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{8\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(5/2)) + (8*(a + b*x^2)^(5/4))/(5*a^2*c*(
c*x)^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}-\frac{4\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx}{a}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}+\frac{8\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.62

$$-\frac{2x\left(a-4bx^2\right)\sqrt[4]{a+bx^2}}{5a^2(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(a - 4*b*x^2)*(a + b*x^2)^(1/4))/(5*a^2*(c*x)^(7/2))
```

fricas [A] time = 0.93, size = 35, normalized size = 0.64

$$\frac{2\left(4\,bx^{2}-a\right)\left(bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}}{5\,a^{2}c^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/5*(4*b*x^2 - a)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^4*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{3}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

maple [A] time = 0.01, size = 29, normalized size = 0.53

$$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(-4b\,x^{2}+a\right)x}{5\,(cx)^{\frac{7}{2}}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/5*x*(b*x^2+a)^(1/4)*(-4*b*x^2+a)/a^2/(c*x)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{3}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

mupad [B] time = 4.98, size = 40, normalized size = 0.73

$$-\frac{\left(b\,x^{2}+a\right)^{1/4}\left(\frac{2}{5\,a\,c^{3}}-\frac{8\,b\,x^{2}}{5\,a^{2}\,c^{3}}\right)}{x^{2}\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] -((a + b*x^2)^(1/4)*(2/(5*a*c^3) - (8*b*x^2)/(5*a^2*c^3)))/(x^2*(c*x)^(1/2)
)
```

sympy [A] time = 24.76, size = 78, normalized size = 1.42

$$-\frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{5}{4}\right)}{8ac^{\frac{7}{2}}x^{2}\Gamma\left(\frac{3}{4}\right)}+\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{5}{4}\right)}{2a^{2}c^{\frac{7}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(8*a*c**(7/2)*x**2*gamma(3/4)
) + b**(5/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*a**2*c**(7/2)*gamma(3/4
))
```

3.974 $\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{3/4}} dx$

Optimal. Leaf size=83

$$-\frac{64\left(a+bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}+\frac{16\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}$$

```
[Out] -2*(b*x^2+a)^(1/4)/a/c/(c*x)^(9/2)+16/5*(b*x^2+a)^(5/4)/a^2/c/(c*x)^(9/2)-6
4/45*(b*x^2+a)^(9/4)/a^3/c/(c*x)^(9/2)
```

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{64\left(a+bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}+\frac{16\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(9/2)) + (16*(a + b*x^2)^(5/4))/(5*a^2*c*
(c*x)^(9/2)) - (64*(a + b*x^2)^(9/4))/(45*a^3*c*(c*x)^(9/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{3/4}} dx &= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}-\frac{8\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}} dx}{a}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}+\frac{16\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}+\frac{32\int \frac{\left(a+bx^2\right)^{5/4}}{(cx)^{11/2}} dx}{5a^2}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}+\frac{16\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{64\left(a+bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.63

$$-\frac{2\sqrt{cx}\sqrt[4]{a+bx^2}\left(5a^2-8abx^2+32b^2x^4\right)}{45a^3c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[c*x]*(a + b*x^2)^(1/4)*(5*a^2 - 8*a*b*x^2 + 32*b^2*x^4))/(45*a^3*c
^6*x^5)
```

fricas [A] time = 0.61, size = 46, normalized size = 0.55

$$-\frac{2\left(32\,b^2x^4-8\,abx^2+5\,a^2\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{45\,a^3c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/45*(32*b^2*x^4 - 8*a*b*x^2 + 5*a^2)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^3*c^6
*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

maple [A] time = 0.00, size = 42, normalized size = 0.51

$$-\frac{2\left(b\,x^2+a\right)^{\frac{1}{4}}\left(32b^2x^4-8ab\,x^2+5a^2\right)x}{45\,(cx)^{\frac{11}{2}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/45*x*(b*x^2+a)^(1/4)*(32*b^2*x^4-8*a*b*x^2+5*a^2)/a^3/(c*x)^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

mupad [B] time = 5.17, size = 54, normalized size = 0.65

$$-\frac{\left(b\,x^2+a\right)^{1/4}\left(\frac{2}{9\,a\,c^5}-\frac{16\,b\,x^2}{45\,a^2\,c^5}+\frac{64\,b^2\,x^4}{45\,a^3\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] -((a + b*x^2)^(1/4)*(2/(9*a*c^5) - (16*b*x^2)/(45*a^2*c^5) + (64*b^2*x^4)/(
45*a^3*c^5)))/(x^4*(c*x)^(1/2))
```

sympy [B] time = 172.06, size = 483, normalized size = 5.82

$$\frac{5a^4b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^5b^4c^{\frac{11}{2}}x^4\Gamma\left(\frac{3}{4}\right)+64a^4b^5c^{\frac{11}{2}}x^6\Gamma\left(\frac{3}{4}\right)+32a^3b^6c^{\frac{11}{2}}x^8\Gamma\left(\frac{3}{4}\right)}+\frac{2a^3b^{\frac{21}{4}}x^2\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^5b^4c^{\frac{11}{2}}x^4\Gamma\left(\frac{3}{4}\right)+64a^4b^5c^{\frac{11}{2}}x^6\Gamma\left(\frac{3}{4}\right)+32a^3b^6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] 5*a**4*b**(17/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2
)*x**4*gamma(3/4) + 64*a**4*b**5*c**(11/2)*x**6*gamma(3/4) + 32*a**3*b**6*c
**(11/2)*x**8*gamma(3/4)) + 2*a**3*b**(21/4)*x**2*(a/(b*x**2) + 1)**(1/4)*g
amma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*gamma(3/4) + 64*a**4*b**5*c**(11/2)
*x**6*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*gamma(3/4)) + 21*a**2*b**(25
/4)*x**4*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*g
amma(3/4) + 64*a**4*b**5*c**(11/2)*x**6*gamma(3/4) + 32*a**3*b**6*c**(11/2)
*x**8*gamma(3/4)) + 56*a*b**(29/4)*x**6*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)
/(32*a**5*b**4*c**(11/2)*x**4*gamma(3/4) + 64*a**4*b**5*c**(11/2)*x**6*gamm
a(3/4) + 32*a**3*b**6*c**(11/2)*x**8*gamma(3/4)) + 32*b**(33/4)*x**8*(a/(b*
x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*gamma(3/4) + 64*
a**4*b**5*c**(11/2)*x**6*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*gamma(3/4
))
```

3.975 $\int \frac{(cx)^{3/2}}{\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{a}\,(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\,\left(a-bx^2\right)^{3/4}}-\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b}$$

```
[Out] -(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)
/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^(1
/2))),2^(1/2))*a^(1/2)/(-b*x^2+a)^(3/4)/b^(1/2)-c*(-b*x^2+a)^(1/4)*(c*x)^(1
/2)/b
```

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {321, 329, 237, 335, 275, 232}

$$-\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b}-\frac{\sqrt{a}\,(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a - b*x^2)^(3/4),x]
```

```
[Out] -((c*Sqrt[c*x]*(a - b*x^2)^(1/4))/b) - (Sqrt[a]*(1 - a/(b*x^2))^(3/4)*(c*x)
^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a - b*x^2)^(3
/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{3/2}}{\left(a-bx^2\right)^{3/4}}\,dx &= -\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b}+\frac{\left(ac^2\right)\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{2b}\\
&= -\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b}+\frac{(ac)\,\mathrm{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{b}\\
&= -\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b}+\frac{\left(ac\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{b\left(a-bx^2\right)^{3/4}}\\
&= -\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b}-\frac{\left(ac\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{b\left(a-bx^2\right)^{3/4}}\\
&= -\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b}-\frac{\left(ac\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{2b\left(a-bx^2\right)^{3/4}}\\
&= -\frac{c\sqrt{cx}\,\sqrt[4]{a-bx^2}}{b}-\frac{\sqrt{a}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a-bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.75

$$\frac{c\sqrt{cx}\left(a\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{bx^2}{a}\right)-a+bx^2\right)}{b\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a - b*x^2)^(3/4),x]
```

```
[Out] (c*Sqrt[c*x]*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3
/4, 5/4, (b*x^2)/a]))/(b*(a - b*x^2)^(3/4))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}\,cx}{bx^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)*c*x/(b*x^2 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(-b*x^2+a)^(3/4),x)
[Out] int((c*x)^(3/2)/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{\left(a-b\,x^2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a - b*x^2)^(3/4),x)
[Out] int((c*x)^(3/2)/(a - b*x^2)^(3/4), x)
```

sympy [C] time = 2.20, size = 46, normalized size = 0.51

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*exp_polar(2*I
*pi)/a)/(2*a**(3/4)*gamma(9/4))
```

3.976 $\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=68

$$-\frac{2\sqrt{b}\,(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,c^2\left(a-bx^2\right)^{3/4}}$$

```
[Out] -2*(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/
2)/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^
(1/2))),2^(1/2))*b^(1/2)/c^2/(-b*x^2+a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {329, 237, 335, 275, 232}

$$-\frac{2\sqrt{b}\,(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[b]*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/
Sqrt[a]]/2, 2])/(Sqrt[a]*c^2*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\,\left(a-bx^2\right)^{3/4}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{\left(2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx, x, \sqrt{cx}\right)}{c\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{\left(2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{c\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{\left(\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{cx}\right)}{c\left(a-bx^2\right)^{3/4}}$$

$$= -\frac{2\sqrt{b}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,c^2\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.81

$$\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{bx^2}{a}\right)}{\sqrt{cx}\,\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a - b*x^2)^(3/4)),x]
[Out] (2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (b*x^2)/a])/(Sq
rt[c*x]*(a - b*x^2)^(3/4))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{bcx^3-acx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c*x^3 - a*c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
[Out] int(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^{2}+a\right)^{\frac{3}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x}\,\left(a-b\,x^{2}\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a - b*x^2)^(3/4)),x)
[Out] int(1/((c*x)^(1/2)*(a - b*x^2)^(3/4)), x)
```

sympy [C] time = 1.89, size = 32, normalized size = 0.47

$$\frac{ie^{-\frac{i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{a}{bx^{2}}\right)}{b^{\frac{3}{4}}\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(-b*x**2+a)**(3/4),x)
[Out] I*exp(-I*pi/4)*hyper((1/2, 3/4), (3/2,), a/(b*x**2))/(b**(3/4)*sqrt(c)*x)
```

3.977 $\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{4b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{3a^{3/2}c^4\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}$$

```
[Out] -2/3*(-b*x^2+a)^(1/4)/a/c/(c*x)^(3/2)-4/3*b^(3/2)*(1-a/b/x^2)^(3/4)*(c*x)^(
3/2)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/
a^(1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/c^4
/(-b*x^2+a)^(3/4)
```

Rubi [A] time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {325, 329, 237, 335, 275, 232}

$$-\frac{4b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}c^4\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(3*a*c*(c*x)^(3/2)) - (4*b^(3/2)*(1 - a/(b*x^2))^(3/
4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*a^(3/2)*c^4*
(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}+\frac{(2b)\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{3ac^2}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}+\frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{3ac^3}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}+\frac{\left(4b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{cx}\right)}{3ac^3\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}-\frac{\left(4b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right)}{3ac^3\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}-\frac{\left(2b\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{3ac^3\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}-\frac{4b^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}c^4\left(a-bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.57

$$
-\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};\frac{bx^2}{a}\right)}{3(cx)^{5/2}\left(a-bx^2\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, (b*x^2)/a])/(
3*(c*x)^(5/2)*(a - b*x^2)^(3/4))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{bc^3x^5-ac^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^3*x^5 - a*c^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{1}{(cx)^{\frac{5}{2}}\left(-b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{(c\,x)^{5/2}\left(a-b\,x^2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a - b*x^2)^(3/4)),x)
```

```
[Out] int(1/((c*x)^(5/2)*(a - b*x^2)^(3/4)), x)
```

sympy [C] time = 8.31, size = 39, normalized size = 0.39

$$-\frac{ie^{\frac{3i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\right|\frac{a}{bx^2}\right)}{3b^{\frac{3}{4}}c^{\frac{5}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] -I*exp(3*I*pi/4)*hyper((3/4, 3/2), (5/2,), a/(b*x**2))/(3*b**(3/4)*c**(5/2)
*x**3)
```

3.978 $\int \frac{1}{(cx)^{9/2}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=130

$$-\frac{8b^{5/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{7a^{5/2}c^6\left(a-bx^2\right)^{3/4}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}$$

```
[Out] -2/7*(-b*x^2+a)^(1/4)/a/c/(c*x)^(7/2)-4/7*b*(-b*x^2+a)^(1/4)/a^2/c^3/(c*x)^
(3/2)-8/7*b^(5/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccsc(x*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcc
sc(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/c^6/(-b*x^2+a)^(3/4)
```

Rubi [A] time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {325, 329, 237, 335, 275, 232}

$$-\frac{8b^{5/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}c^6\left(a-bx^2\right)^{3/4}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(7*a*c*(c*x)^(7/2)) - (4*b*(a - b*x^2)^(1/4))/(7*a^2
*c^3*(c*x)^(3/2)) - (8*b^(5/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[
ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*a^(5/2)*c^6*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{9/2}\left(a-bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}+\frac{(6b)\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx}{7ac^2}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}+\frac{\left(4b^2\right)\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{7a^2c^4}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}+\frac{\left(8b^2\right)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{7a^2c^5}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}+\frac{\left(8b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2}{bx^4}\right)^{3/4}x^3}\,dx,x,\sqrt{c}\right.}{7a^2c^5\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{\left(8b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ac^2x^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{\sqrt{cx}}\right.}{7a^2c^5\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{\left(4b^2\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ac^2x^2}{b}\right)^{3/4}}\,dx,x,\frac{1}{cx}\right)}{7a^2c^5\left(a-bx^2\right)^{3/4}}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}-\frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}}-\frac{8b^{5/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}c^6\left(a-bx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.44

$$
-\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};\frac{bx^2}{a}\right)}{7(cx)^{9/2}\left(a-bx^2\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, (b*x^2)/a])/
(7*(c*x)^(9/2)*(a - b*x^2)^(3/4))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{bc^5x^7-ac^5x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^5*x^7 - a*c^5*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{9}{2}}\left(-b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x)
[Out] int(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{9/2}\left(a-b\,x^2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(9/2)*(a - b*x^2)^(3/4)),x)
[Out] int(1/((c*x)^(9/2)*(a - b*x^2)^(3/4)), x)
```

sympy [C] time = 68.93, size = 36, normalized size = 0.28

$$\frac{ie^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{3}{4}, \frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{5b^{\frac{3}{4}}c^{\frac{9}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] I*exp(-I*pi/4)*hyper((3/4, 5/2), (7/2,), a/(b*x**2))/(5*b**(3/4)*c**(9/2)*x
**5)
```

3.979 $\int \frac{1}{(cx)^{13/2}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=162

$$-\frac{80b^{7/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right),2\right)}{77a^{7/2}c^8\left(a-bx^2\right)^{3/4}}-\frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}}-\frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

```
[Out] -2/11*(-b*x^2+a)^(1/4)/a/c/(c*x)^(11/2)-20/77*b*(-b*x^2+a)^(1/4)/a^2/c^3/(c
*x)^(7/2)-40/77*b^2*(-b*x^2+a)^(1/4)/a^3/c^5/(c*x)^(3/2)-80/77*b^(7/2)*(1-a
/b/x^2)^(3/4)*(c*x)^(3/2)*(cos(1/2*arccsc(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(
1/2*arccsc(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x*b^(1/2)/a^(1/2)))
,2^(1/2))/a^(7/2)/c^8/(-b*x^2+a)^(3/4)
```

Rubi [A] time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {325, 329, 237, 335, 275, 232}

$$-\frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}}-\frac{80b^{7/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}c^8\left(a-bx^2\right)^{3/4}}-\frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(11*a*c*(c*x)^(11/2)) - (20*b*(a - b*x^2)^(1/4))/(77
*a^2*c^3*(c*x)^(7/2)) - (40*b^2*(a - b*x^2)^(1/4))/(77*a^3*c^5*(c*x)^(3/2))
 - (80*b^(7/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcCsc[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/(77*a^(7/2)*c^8*(a - b*x^2)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{13/2}\left(a-bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} + \frac{(10b)\int \frac{1}{(cx)^{9/2}\left(a-bx^2\right)^{3/4}}\,dx}{11ac^2}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} + \frac{\left(60b^2\right)\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx}{77a^2c^4}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} + \frac{\left(40b^3\right)\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}\,dx}{77a^3c^6}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} + \frac{\left(80b^3\right)\text{Subst}\left(\int \frac{1}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{\right.}{77a^3c^7}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} + \frac{\left(80b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\right.}{77a^3c^7\left(a-bx\right.}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{\left(80b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\right.}{77a^3c^7\left(a-bx^2\right.}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{\left(40b^3\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\right)\text{Subst}\left(\right.}{77a^3c^7\left(a-bx^2\right)}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{80b^{7/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}F\left(\frac{1}{2}\csc^{-}\right.}{77a^{7/2}c^8\left(a-bx^2\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.35

$$-\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(-\frac{11}{4},\frac{3}{4};-\frac{7}{4};\frac{bx^2}{a}\right)}{11(cx)^{13/2}\left(a-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-11/4, 3/4, -7/4, (b*x^2)/a])
/(11*(c*x)^(13/2)*(a - b*x^2)^(3/4))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{bc^7x^9-ac^7x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^7*x^9 - a*c^7*x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{13}{2}}\left(-b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{13/2}\left(a-b\,x^2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(13/2)*(a - b*x^2)^(3/4)),x)
```

```
[Out] int(1/((c*x)^(13/2)*(a - b*x^2)^(3/4)), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

3.980 $\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=308

$$\frac{3ac^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{7/4}}-\frac{3ac^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{7/4}}-\frac{3ac^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{7/4}}+\cdots$$

```
[Out] -1/2*c*(c*x)^(3/2)*(-b*x^2+a)^(1/4)/b+3/8*a*c^(5/2)*arctan(-1+b^(1/4)*2^(1/
2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^(1/2))/b^(7/4)*2^(1/2)+3/8*a*c^(5/2)*arct
an(1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^(1/2))/b^(7/4)*2^(1/2)+
3/16*a*c^(5/2)*ln(c^(1/2)-b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^
(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))/b^(7/4)*2^(1/2)-3/16*a*c^(5/2)*ln(c^(1/2)+b
^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1
/2))/b^(7/4)*2^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {321, 329, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{3ac^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{7/4}}-\frac{3ac^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{7/4}}-\frac{3ac^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{7/4}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a - b*x^2)^(3/4),x]
```

```
[Out] -(c*(c*x)^(3/2)*(a - b*x^2)^(1/4))/(2*b) - (3*a*c^(5/2)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*Sqrt[2]*b^(7/4)) + (3*
a*c^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4)
)])/(4*Sqrt[2]*b^(7/4)) + (3*a*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sq
rt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*
b^(7/4)) - (3*a*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] +
 (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{\left(a-bx^2\right)^{3/4}}\,dx = -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}+\frac{\left(3ac^2\right)\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx}{4b}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}+\frac{(3ac)\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{2b}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}+\frac{(3ac)\,\mathrm{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2b}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}-\frac{(3ac)\,\mathrm{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4b^{3/2}}+\frac{(3ac)\,\mathrm{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx,x,\right.}{4b^{3/2}}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}+\frac{\left(3ac^{5/2}\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{7/4}}+\frac{\left(3ac^{5/2}\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}+\frac{3ac^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{7/4}}-\frac{3ac^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\right.}{8\sqrt{2}\,b^{7/4}}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}-\frac{3ac^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{3ac^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{3ac^{5/}}{}$$

Mathematica [A] time = 0.06, size = 112, normalized size = 0.36

$$-\frac{(cx)^{5/2}\left(-3a\sqrt[4]{-b}\,\tan^{-1}\left(\frac{\sqrt[4]{-b}\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)+3a\sqrt[4]{-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{-b}\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)+2bx^{3/2}\sqrt[4]{a-bx^2}\right)}{4b^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a - b*x^2)^(3/4),x]
```

```
[Out] -1/4*((c*x)^(5/2)*(2*b*x^(3/2)*(a - b*x^2)^(1/4) - 3*a*(-b)^(1/4)*ArcTan[((
-b)^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + 3*a*(-b)^(1/4)*ArcTanh[((-b)^(1/4)*
Sqrt[x])/(a - b*x^2)^(1/4)]))/(b^2*x^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(-bx^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x)^{5/2}}{\left(a-b\,x^2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a - b*x^2)^(3/4),x)
```

```
[Out] int((c*x)^(5/2)/(a - b*x^2)^(3/4), x)
```

sympy [C] time = 7.82, size = 46, normalized size = 0.15

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar(2*
I*pi)/a)/(2*a**(3/4)*gamma(11/4))
```

3.981 $\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=272

$$\frac{\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}b^{3/4}}-\frac{\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}b^{3/4}}-\frac{\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}b^{3/4}}+\frac{\sqrt{c}\tan^{-}}{}$$

```
[Out] 1/2*arctan(-1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/c^(1/2))*c^(1/2)
/b^(3/4)*2^(1/2)+1/2*arctan(1+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/
c^(1/2))*c^(1/2)/b^(3/4)*2^(1/2)+1/4*ln(c^(1/2)-b^(1/4)*2^(1/2)*(c*x)^(1/2)
/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^(1/2)/(-b*x^2+a)^(1/2))*c^(1/2)/b^(3/4)*2^(1/
2)-1/4*ln(c^(1/2)+b^(1/4)*2^(1/2)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)+x*b^(1/2)*c^
(1/2)/(-b*x^2+a)^(1/2))*c^(1/2)/b^(3/4)*2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {329, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}b^{3/4}}-\frac{\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}b^{3/4}}-\frac{\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}b^{3/4}}+\frac{\sqrt{c}\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a - b*x^2)^(3/4),x]
```

```
[Out] -((Sqrt[c]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4
))]])/(Sqrt[2]*b^(3/4))) + (Sqrt[c]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(
Sqrt[c]*(a - b*x^2)^(1/4))]])/(Sqrt[2]*b^(3/4)) + (Sqrt[c]*Log[Sqrt[c] + (Sq
rt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^
(1/4)]])/(2*Sqrt[2]*b^(3/4)) - (Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sq
rt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)]])/(2*Sqrt[2]*
b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx=\frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{c}$$
$$=\frac{2\,\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c}$$
$$=-\frac{\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{\sqrt{b}\,c}+\frac{\text{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{\sqrt{b}\,c}$$
$$=\frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,b^{3/4}}+\frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt{c}}{\sqrt[4]{b}}-2x}{-\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,b^{3/4}}-$$
$$=\frac{\sqrt{c}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,b^{3/4}}-\frac{\sqrt{c}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,b^{3/4}}+\frac{\sqrt{c}\,\text{Subst}\left(\int\right.}{}$$
$$=-\frac{\sqrt{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,b^{3/4}}+\frac{\sqrt{c}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,b^{3/4}}+\frac{\sqrt{c}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}}{\sqrt[4]{a-b}}\right.}{2\sqrt{2}\,b^{3/4}}$$

Mathematica [A] time = 0.02, size = 75, normalized size = 0.28

$$\frac{\sqrt{cx}\left(\tanh^{-1}\left(\frac{\sqrt[4]{-b}\,\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)-\tan^{-1}\left(\frac{\sqrt[4]{-b}\,\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)\right)}{(-b)^{3/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a - b*x^2)^(3/4),x]
```

```
[Out] (Sqrt[c*x]*(-ArcTan[((-b)^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + ArcTanh[((-b)
^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)]))/((-b)^(3/4)*Sqrt[x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(3/4), x)

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(1/2)/(-b*x^2+a)^(3/4),x)

[Out] int((c*x)^(1/2)/(-b*x^2+a)^(3/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(-bx^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")

[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(3/4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x}}{{\left(a-b\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(1/2)/(a - b*x^2)^(3/4),x)

[Out] int((c*x)^(1/2)/(a - b*x^2)^(3/4), x)

sympy [C] time = 1.56, size = 46, normalized size = 0.17

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(1/2)/(-b*x**2+a)**(3/4),x)

[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(2*I*pi)/a)/(2*a**(3/4)*gamma(7/4))

3.982 $\int \frac{1}{(cx)^{3/2}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=27

$$-\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

```
[Out] -2*(-b*x^2+a)^(1/4)/a/c/(c*x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {264}

$$-\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(a*c*Sqrt[c*x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\left(a-bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.93

$$-\frac{2x\sqrt[4]{a-bx^2}}{a(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4))/(a*(c*x)^(3/2))
```

fricas [A] time = 0.97, size = 26, normalized size = 0.96

$$-\frac{2\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}} (cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

maple [A] time = 0.00, size = 22, normalized size = 0.81

$$-\frac{2\left(-b\,x^2+a\right)^{\frac{1}{4}} x}{(cx)^{\frac{3}{2}}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x)
[Out] -2*x*(-b*x^2+a)^(1/4)/a/(c*x)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}} (cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

mupad [B] time = 5.08, size = 23, normalized size = 0.85

$$-\frac{2\left(a-b\,x^2\right)^{1/4}}{a\,c\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x)
[Out] -(2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(1/2))
```

sympy [A] time = 3.00, size = 90, normalized size = 3.33

$$\begin{cases} \frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} & \text{for}\ \left|\frac{a}{bx^2}\right|>1 \\ -\frac{\sqrt[4]{b}\,\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{-\frac{3i\pi}{4}}\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(-b*x**2+a)**(3/4),x)
[Out] Piecewise((b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma
(3/4)), Abs(a/(b*x**2)) > 1), (-b**(1/4)*(-a/(b*x**2) + 1)**(1/4)*exp(-3*I*
pi/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma(3/4)), True))
```

3.983 $\int \frac{1}{(cx)^{7/2}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=57

$$\frac{8\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}$$

```
[Out] -2*(-b*x^2+a)^(1/4)/a/c/(c*x)^(5/2)+8/5*(-b*x^2+a)^(5/4)/a^2/c/(c*x)^(5/2)
```

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {273, 264}

$$\frac{8\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(5/2)) + (8*(a - b*x^2)^(5/4))/(5*a^2*c*(
c*x)^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{7/2}\left(a-bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}-\frac{4\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx}{a}\\ &= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}+\frac{8\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.61

$$-\frac{2x\sqrt[4]{a-bx^2}\left(a+4bx^2\right)}{5a^2(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*(a + 4*b*x^2))/(5*a^2*(c*x)^(7/2))
```

fricas [A] time = 0.89, size = 34, normalized size = 0.60

$$-\frac{2 \left(4 \, b x^2 + a\right) \left(-b x^2 + a\right)^{\frac{1}{4}} \sqrt{c x}}{5 \, a^2 c^4 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/5*(4*b*x^2 + a)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^4*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b x^2 + a\right)^{\frac{3}{4}} (c x)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

maple [A] time = 0.01, size = 30, normalized size = 0.53

$$-\frac{2 \left(-b \, x^2 + a\right)^{\frac{1}{4}} \left(4 b \, x^2 + a\right) x}{5 \, (c x)^{\frac{7}{2}} \, a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] -2/5*x*(-b*x^2+a)^(1/4)*(4*b*x^2+a)/a^2/(c*x)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b x^2 + a\right)^{\frac{3}{4}} (c x)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

mupad [B] time = 5.10, size = 41, normalized size = 0.72

$$-\frac{\left(a - b \, x^2\right)^{1/4} \left(\frac{2}{5 \, a \, c^3} + \frac{8 \, b \, x^2}{5 \, a^2 \, c^3}\right)}{x^2 \, \sqrt{c \, x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x)
```

```
[Out] -((a - b*x^2)^(1/4)*(2/(5*a*c^3) + (8*b*x^2)/(5*a^2*c^3)))/(x^2*(c*x)^(1/2)
)
```

sympy [A] time = 25.45, size = 352, normalized size = 6.18

$$\begin{cases} -\frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{5}{4}\right)}{8ac^{\frac{7}{2}}x^2\Gamma\left(\frac{3}{4}\right)} - \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{5}{4}\right)}{2a^2c^{\frac{7}{2}}\Gamma\left(\frac{3}{4}\right)} \\ -\frac{a^2b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{-8a^3bc^{\frac{7}{2}}x^2e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)+8a^2b^2c^{\frac{7}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)} - \frac{3ab^{\frac{9}{4}}x^2\sqrt[4]{-\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{-8a^3bc^{\frac{7}{2}}x^2e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)+8a^2b^2c^{\frac{7}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)} + \frac{4b^{\frac{13}{4}}x^4\sqrt[4]{-\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{-8a^3bc^{\frac{7}{2}}x^2e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)+8a^2b^2c^{\frac{7}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] Piecewise((-b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(8*a*c**(7/2)*x**2
*gamma(3/4)) - b**(5/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(2*a**2*c**(7/2
)*gamma(3/4)), Abs(a/(b*x**2)) > 1), (-a**2*b**(5/4)*(-a/(b*x**2) + 1)**(1/
4)*gamma(-5/4)/(-8*a**3*b*c**(7/2)*x**2*exp(3*I*pi/4)*gamma(3/4) + 8*a**2*b
**2*c**(7/2)*x**4*exp(3*I*pi/4)*gamma(3/4)) - 3*a*b**(9/4)*x**2*(-a/(b*x**2
) + 1)**(1/4)*gamma(-5/4)/(-8*a**3*b*c**(7/2)*x**2*exp(3*I*pi/4)*gamma(3/4)
 + 8*a**2*b**2*c**(7/2)*x**4*exp(3*I*pi/4)*gamma(3/4)) + 4*b**(13/4)*x**4*(
-a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(-8*a**3*b*c**(7/2)*x**2*exp(3*I*pi/4)*
gamma(3/4) + 8*a**2*b**2*c**(7/2)*x**4*exp(3*I*pi/4)*gamma(3/4)), True))
```

3.984 $\int \frac{1}{(cx)^{11/2}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=86

$$-\frac{64\left(a-bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}+\frac{16\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}$$

```
[Out] -2*(-b*x^2+a)^(1/4)/a/c/(c*x)^(9/2)+16/5*(-b*x^2+a)^(5/4)/a^2/c/(c*x)^(9/2)
-64/45*(-b*x^2+a)^(9/4)/a^3/c/(c*x)^(9/2)
```

Rubi [A] time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {273, 264}

$$-\frac{64\left(a-bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}+\frac{16\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(9/2)) + (16*(a - b*x^2)^(5/4))/(5*a^2*c*
(c*x)^(9/2)) - (64*(a - b*x^2)^(9/4))/(45*a^3*c*(c*x)^(9/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{11/2}\left(a-bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}-\frac{8\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx}{a}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}+\frac{16\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}+\frac{32\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{11/2}}\,dx}{5a^2}\\
&= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}+\frac{16\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{64\left(a-bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.62

$$-\frac{2\sqrt{cx}\,\sqrt[4]{a-bx^2}\left(5a^2+8abx^2+32b^2x^4\right)}{45a^3c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(1/4)*(5*a^2 + 8*a*b*x^2 + 32*b^2*x^4))/(45*a^3*c
^6*x^5)
```

fricas [A] time = 1.05, size = 47, normalized size = 0.55

$$-\frac{2\left(32\,b^{2}x^{4}+8\,abx^{2}+5\,a^{2}\right)\left(-bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}}{45\,a^{3}c^{6}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/45*(32*b^2*x^4 + 8*a*b*x^2 + 5*a^2)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^3*c^
6*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^{2}+a\right)^{\frac{3}{4}}\left(cx\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

maple [A] time = 0.01, size = 43, normalized size = 0.50

$$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{1}{4}}\left(32b^{2}x^{4}+8ab\,x^{2}+5a^{2}\right)x}{45\left(cx\right)^{\frac{11}{2}}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] -2/45*x*(-b*x^2+a)^(1/4)*(32*b^2*x^4+8*a*b*x^2+5*a^2)/a^3/(c*x)^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^{2}+a\right)^{\frac{3}{4}}\left(cx\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

mupad [B] time = 5.15, size = 55, normalized size = 0.64

$$-\frac{\left(a-b\,x^{2}\right)^{1/4}\left(\frac{2}{9\,a\,c^{5}}+\frac{16\,b\,x^{2}}{45\,a^{2}\,c^{5}}+\frac{64\,b^{2}\,x^{4}}{45\,a^{3}\,c^{5}}\right)}{x^{4}\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x)
```

```
[Out] -((a - b*x^2)^(1/4)*(2/(9*a*c^5) + (16*b*x^2)/(45*a^2*c^5) + (64*b^2*x^4)/(
45*a^3*c^5)))/(x^4*(c*x)^(1/2))
```

sympy [C] time = 171.23, size = 1263, normalized size = 14.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] Piecewise((-5*a**4*b**(17/4)*(a/(b*x**2) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/
4)/(32*a**5*b**4*c**(11/2)*x***4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**
(11/2)*x***6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x***8*exp(3*I*
pi/4)*gamma(3/4)) + 2*a**3*b**(21/4)*x***2*(a/(b*x**2) - 1)**(1/4)*exp(-I*pi
/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x***4*exp(3*I*pi/4)*gamma(3/4) - 64*
a**4*b**5*c**(11/2)*x***6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*
x***8*exp(3*I*pi/4)*gamma(3/4)) - 21*a**2*b**(25/4)*x***4*(a/(b*x**2) - 1)**(
1/4)*exp(-I*pi/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x***4*exp(3*I*pi/4)*ga
mma(3/4) - 64*a**4*b**5*c**(11/2)*x***6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b
**6*c**(11/2)*x***8*exp(3*I*pi/4)*gamma(3/4)) + 56*a*b**(29/4)*x***6*(a/(b*x*
*2) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x***4*exp(3
*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x***6*exp(3*I*pi/4)*gamma(3/4)
+ 32*a**3*b**6*c**(11/2)*x***8*exp(3*I*pi/4)*gamma(3/4)) - 32*b**(33/4)*x***8
*(a/(b*x**2) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x
**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x***6*exp(3*I*pi/4)*ga
mma(3/4) + 32*a**3*b**6*c**(11/2)*x***8*exp(3*I*pi/4)*gamma(3/4)), Abs(a/(b*
x**2)) > 1), (-5*a**4*b**(17/4)*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a*
*5*b**4*c**(11/2)*x***4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x*
*6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x***8*exp(3*I*pi/4)*gam
ma(3/4)) + 2*a**3*b**(21/4)*x***2*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a
**5*b**4*c**(11/2)*x***4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x
**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x***8*exp(3*I*pi/4)*ga
mma(3/4)) - 21*a**2*b**(25/4)*x***4*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32
*a**5*b**4*c**(11/2)*x***4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)
*x***6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x***8*exp(3*I*pi/4)*
gamma(3/4)) + 56*a*b**(29/4)*x***6*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*
a**5*b**4*c**(11/2)*x***4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*
x***6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x***8*exp(3*I*pi/4)*g
amma(3/4)) - 32*b**(33/4)*x***8*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**
5*b**4*c**(11/2)*x***4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x***
6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x***8*exp(3*I*pi/4)*gamm
a(3/4)), True))
```

3.985 $\int \frac{(cx)^{7/2}}{\left(a+bx^2\right)^{5/4}} \, dx$

Optimal. Leaf size=146

$$-\frac{5ac^{7/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{5ac^{7/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}$$

```
[Out] 1/2*c*(c*x)^(5/2)/b/(b*x^2+a)^(1/4)-5/4*a*c^(7/2)*arctan(b^(1/4)*(c*x)^(1/2
)/(b*x^2+a)^(1/4)/c^(1/2))/b^(9/4)-5/4*a*c^(7/2)*arctanh(b^(1/4)*(c*x)^(1/2
)/(b*x^2+a)^(1/4)/c^(1/2))/b^(9/4)+5/2*a*c^3*(c*x)^(1/2)/b^2/(b*x^2+a)^(1/4
)
```

Rubi [A] time = 0.08, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {285, 288, 329, 240, 212, 208, 205}

$$\frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}}-\frac{5ac^{7/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{5ac^{7/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (5*a*c^3*Sqrt[c*x])/(2*b^2*(a + b*x^2)^(1/4)) + (c*(c*x)^(5/2))/(2*b*(a + b
*x^2)^(1/4)) - (5*a*c^(7/2)*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)
^(1/4))])/(4*b^(9/4)) - (5*a*c^(7/2)*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a + b*x^2)^(1/4))])/(4*b^(9/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
```

```
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^{7/2}}{\left(a+bx^2\right)^{5/4}}\,dx &= \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^2\right)\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{4b} \\
&= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^4\right)\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx}{4b^2} \\
&= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^3\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2b^2} \\
&= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^3\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b^2} \\
&= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^4\right)\operatorname{Subst}\left(\int \frac{1}{c-\sqrt{b}x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b^2} - \frac{\left(5ac^4\right)\operatorname{Subst}\left(\int \frac{}{c+}\right.}{4b} \\
&= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{5ac^{7/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}} - \frac{5ac^{7/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 63, normalized size = 0.43

$$
\frac{c(cx)^{5/2}\left(1-\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{bx^2}{a}\right)\right)}{2b\sqrt[4]{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c*(c*x)^(5/2)*(1 - (1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4,
-((b*x^2)/a)]))/(2*b*(a + b*x^2)^(1/4))
```

fricas [B] time = 0.87, size = 389, normalized size = 2.66

$$4\left(bc^3x^2+5\,ac^3\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}+20\left(\frac{a^4c^{14}}{b^9}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\arctan\left(-\frac{\left(\frac{a^4c^{14}}{b^9}\right)^{\frac{3}{4}}\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,ab^7c^3-\left(b^8x^2+ab^7\right)\left(\frac{a^4c^{14}}{b^9}\right)^{\frac{3}{4}}\sqrt{}}{a^4bc^{14}x^2+a^5c^{14}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*c^3*x^2 + 5*a*c^3)*(b*x^2 + a)^(3/4)*sqrt(c*x) + 20*(a^4*c^14/b^9
)^(1/4)*(b^3*x^2 + a*b^2)*arctan(-((a^4*c^14/b^9)^(3/4)*(b*x^2 + a)^(3/4)*s
qrt(c*x)*a*b^7*c^3 - (b^8*x^2 + a*b^7)*(a^4*c^14/b^9)^(3/4)*sqrt((sqrt(b*x^
2 + a)*a^2*c^7*x + sqrt(a^4*c^14/b^9)*(b^5*x^2 + a*b^4))/(b*x^2 + a)))/(a^4
*b*c^14*x^2 + a^5*c^14)) - 5*(a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*(
(b*x^2 + a)^(3/4)*sqrt(c*x)*a*c^3 + (a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2))
/(b*x^2 + a)) + 5*(a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*((b*x^2 + a)
^(3/4)*sqrt(c*x)*a*c^3 - (a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2))/(b*x^2 + a
)))/(b^3*x^2 + a*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x\right)}^{7/2}}{{\left(b\,x^2+a\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(a + b*x^2)^(5/4),x)
```

```
[Out] int((c*x)^(7/2)/(a + b*x^2)^(5/4), x)
```

sympy [C] time = 24.49, size = 44, normalized size = 0.30

$$\frac{c^{\frac{7}{2}} x^{\frac{9}{2}} \Gamma\left(\frac{9}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{5}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{b x^{2} e^{i\pi}}{a}\right)}{2 a^{\frac{5}{4}} \Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c**(7/2)*x**(9/2)*gamma(9/4)*hyper((5/4, 9/4), (13/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/4)*gamma(13/4))
```

3.986 $\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=107

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

```
[Out] c^(3/2)*arctan(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(5/4)+c^(3/2)
*arctanh(b^(1/4)*(c*x)^(1/2)/(b*x^2+a)^(1/4)/c^(1/2))/b^(5/4)-2*c*(c*x)^(1/
2)/b/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {288, 329, 240, 212, 208, 205}

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (-2*c*Sqrt[c*x])/(b*(a + b*x^2)^(1/4)) + (c^(3/2)*ArcTan[(b^(1/4)*Sqrt[c*x]
)/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(5/4) + (c^(3/2)*ArcTanh[(b^(1/4)*Sqrt[c*
x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(5/4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{5/4}}\,dx &= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{c^2\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a+bx^2}}\,dx}{b}\\
&= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{(2c)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{b}\\
&= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{(2c)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{b}\\
&= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{c^2\,\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{b}+\frac{c^2\,\text{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{b}\\
&= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.55

$$\frac{2x(cx)^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{bx^2}{a}\right)}{5a\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*x*(c*x)^(3/2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, -((
b*x^2)/a)])/(5*a*(a + b*x^2)^(1/4))
```

fricas [B] time = 0.95, size = 319, normalized size = 2.98

$$-\frac{4\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,c+4\left(b^2x^2+ab\right)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,b^4c\left(\frac{c^6}{b^5}\right)^{\frac{3}{4}}-\left(b^5x^2+ab^4\right)\left(\frac{c^6}{b^5}\right)^{\frac{3}{4}}\sqrt{\frac{\sqrt{bx^2+a}\,c^3x+\left(b^3x^2+ab^2\right)\sqrt{\frac{c^6}{b^5}}}{bx^2+a}}}{bc^6x^2+ac^6}\right)-\Big(}{2\left(b^2x^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(b*x^2 + a)^(3/4)*sqrt(c*x)*c + 4*(b^2*x^2 + a*b)*(c^6/b^5)^(1/4)*a
rctan(-((b*x^2 + a)^(3/4)*sqrt(c*x)*b^4*c*(c^6/b^5)^(3/4) - (b^5*x^2 + a*b^
4)*(c^6/b^5)^(3/4)*sqrt((sqrt(b*x^2 + a)*c^3*x + (b^3*x^2 + a*b^2)*sqrt(c^6
/b^5))/(b*x^2 + a)))/(b*c^6*x^2 + a*c^6)) - (b^2*x^2 + a*b)*(c^6/b^5)^(1/4)
*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*c + (b^2*x^2 + a*b)*(c^6/b^5)^(1/4))/(b*x
^2 + a)) + (b^2*x^2 + a*b)*(c^6/b^5)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x)
*c - (b^2*x^2 + a*b)*(c^6/b^5)^(1/4))/(b*x^2 + a)))/(b^2*x^2 + a*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2 + a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(b\,x^2 + a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(5/4),x)
[Out] int((c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2 + a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{\left(b\,x^2 + a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a + b*x^2)^(5/4),x)
[Out] int((c*x)^(3/2)/(a + b*x^2)^(5/4), x)
```

sympy [C] time = 4.26, size = 44, normalized size = 0.41

$$\frac{c^{\frac{3}{2}} x^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{5}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{2 a^{\frac{5}{4}} \Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(5/4)*gamma(9/4))
```

3.987 $\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

```
[Out] 2*(c*x)^(1/2)/a/c/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$\frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*Sqrt[c*x])/(a*c*(a + b*x^2)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{5/4}}\,dx=\frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.92

$$\frac{2x}{a\sqrt{cx}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*x)/(a*Sqrt[c*x]*(a + b*x^2)^(1/4))
```

fricas [A] time = 0.81, size = 31, normalized size = 1.19

$$\frac{2\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{abcx^2+a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 2*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a*b*c*x^2 + a^2*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/4)*sqrt(c*x)), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.81

$$\frac{2x}{\left(b\,x^2+a\right)^{\frac{1}{4}}\sqrt{cx}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x)
[Out] 2*x/(b*x^2+a)^(1/4)/a/(c*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/4)*sqrt(c*x)), x)
```

mupad [B] time = 4.97, size = 29, normalized size = 1.12

$$\frac{2\,x\,\left(b\,x^2+a\right)^{3/4}}{\left(a^2+b\,a\,x^2\right)\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a + b*x^2)^(5/4)),x)
[Out] (2*x*(a + b*x^2)^(3/4))/((a^2 + a*b*x^2)*(c*x)^(1/2))
```

sympy [A] time = 3.14, size = 34, normalized size = 1.31

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2a\sqrt[4]{b}\,\sqrt{c}\,\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(5/4),x)
[Out] gamma(1/4)/(2*a*b**(1/4)*sqrt(c)*(a/(b*x**2) + 1)**(1/4)*gamma(5/4))
```

3.988 $\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=55

$$\frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{3/2}}$$

```
[Out] 2/a/c/(c*x)^(3/2)/(b*x^2+a)^(1/4)-8/3*(b*x^2+a)^(3/4)/a^2/c/(c*x)^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*c*(c*x)^(3/2)*(a + b*x^2)^(1/4)) - (8*(a + b*x^2)^(3/4))/(3*a^2*c*(c*x
)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx &= \frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}+\frac{4\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx}{a}\\ &= \frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.62

$$-\frac{2x\left(a+4bx^2\right)}{3a^2(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(a + 4*b*x^2))/(3*a^2*(c*x)^(5/2)*(a + b*x^2)^(1/4))
```

fricas [A] time = 0.94, size = 48, normalized size = 0.87

$$-\frac{2\left(4\,bx^2+a\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{3\left(a^2bc^3x^4+a^3c^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] -2/3*(4*b*x^2 + a)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*b*c^3*x^4 + a^3*c^3*x^2
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}\left(cx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)), x)
```

maple [A] time = 0.01, size = 29, normalized size = 0.53

$$-\frac{2\left(4b\,x^2+a\right)x}{3\left(b\,x^2+a\right)^{\frac{1}{4}}\left(cx\right)^{\frac{5}{2}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x)
[Out] -2/3*x*(4*b*x^2+a)/(b*x^2+a)^(1/4)/a^2/(c*x)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}\left(cx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)), x)
```

mupad [B] time = 5.10, size = 57, normalized size = 1.04

$$-\frac{\left(b\,x^2+a\right)^{3/4}\,\left(\frac{2}{3\,a\,b\,c^2}+\frac{8\,x^2}{3\,a^2\,c^2}\right)}{x^3\,\sqrt{c\,x}+\frac{a\,x\,\sqrt{c\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x)
[Out] -((a + b*x^2)^(3/4)*(2/(3*a*b*c^2) + (8*x^2)/(3*a^2*c^2)))/(x^3*(c*x)^(1/2)
 + (a*x*(c*x)^(1/2))/b)
```

sympy [A] time = 14.99, size = 78, normalized size = 1.42

$$\frac{\Gamma\left(-\frac{3}{4}\right)}{8a\sqrt[4]{b}\,c^{\frac{5}{2}}x^2\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{5}{4}\right)}+\frac{b^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2a^2c^{\frac{5}{2}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] gamma(-3/4)/(8*a*b**(1/4)*c**(5/2)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(5/4))
 + b**(3/4)*gamma(-3/4)/(2*a**2*c**(5/2)*(a/(b*x**2) + 1)**(1/4)*gamma(5/4)
)
```

3.989 $\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=83

$$\frac{64\left(a+bx^2\right)^{7/4}}{21a^3c(cx)^{7/2}}-\frac{16\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{7/2}}+\frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 2/a/c/(c*x)^(7/2)/(b*x^2+a)^(1/4)-16/3*(b*x^2+a)^(3/4)/a^2/c/(c*x)^(7/2)+64
/21*(b*x^2+a)^(7/4)/a^3/c/(c*x)^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{64\left(a+bx^2\right)^{7/4}}{21a^3c(cx)^{7/2}}-\frac{16\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{7/2}}+\frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*c*(c*x)^(7/2)*(a + b*x^2)^(1/4)) - (16*(a + b*x^2)^(3/4))/(3*a^2*c*(c*
x)^(7/2)) + (64*(a + b*x^2)^(7/4))/(21*a^3*c*(c*x)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx &= \frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}+\frac{8\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx}{a}\\
&= \frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{7/2}}-\frac{32\int \frac{\left(a+bx^2\right)^{3/4}}{(cx)^{9/2}}\,dx}{3a^2}\\
&= \frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{7/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{21a^3c(cx)^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 0.57

$$-\frac{2x\left(3a^2-8abx^2-32b^2x^4\right)}{21a^3(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(3*a^2 - 8*a*b*x^2 - 32*b^2*x^4))/(21*a^3*(c*x)^(9/2)*(a + b*x^2)^(1/
4))
```

fricas [A] time = 0.55, size = 61, normalized size = 0.73

$$\frac{2\left(32\,b^{2}x^{4}+8\,abx^{2}-3\,a^{2}\right)\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{cx}}{21\left(a^{3}bc^{5}x^{6}+a^{4}c^{5}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 2/21*(32*b^2*x^4 + 8*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^3*b*c^
5*x^6 + a^4*c^5*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{5}{4}}\left(cx\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)), x)
```

maple [A] time = 0.01, size = 42, normalized size = 0.51

$$-\frac{2\left(-32b^{2}x^{4}-8ab\,x^{2}+3a^{2}\right)x}{21\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(cx\right)^{\frac{9}{2}}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/21*x*(-32*b^2*x^4-8*a*b*x^2+3*a^2)/(b*x^2+a)^(1/4)/a^3/(c*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{5}{4}}\left(cx\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)), x)
```

mupad [B] time = 5.15, size = 70, normalized size = 0.84

$$\frac{\left(b\,x^2+a\right)^{3/4}\,\left(\frac{16\,x^2}{21\,a^2\,c^4}-\frac{2}{7\,a\,b\,c^4}+\frac{64\,b\,x^4}{21\,a^3\,c^4}\right)}{x^5\,\sqrt{c\,x}+\frac{a\,x^3\,\sqrt{c\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] ((a + b*x^2)^(3/4)*((16*x^2)/(21*a^2*c^4) - 2/(7*a*b*c^4) + (64*b*x^4)/(21*
a^3*c^4)))/(x^5*(c*x)^(1/2) + (a*x^3*(c*x)^(1/2))/b)
```

sympy [B] time = 109.04, size = 384, normalized size = 4.63

$$-\frac{3a^3b^{\frac{19}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{32a^5b^4c^{\frac{9}{2}}x^2\Gamma\left(\frac{5}{4}\right)+64a^4b^5c^{\frac{9}{2}}x^4\Gamma\left(\frac{5}{4}\right)+32a^3b^6c^{\frac{9}{2}}x^6\Gamma\left(\frac{5}{4}\right)}+\frac{5a^2b^{\frac{23}{4}}x^2\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{32a^5b^4c^{\frac{9}{2}}x^2\Gamma\left(\frac{5}{4}\right)+64a^4b^5c^{\frac{9}{2}}x^4\Gamma\left(\frac{5}{4}\right)+32a^3b^6c^{\frac{9}{2}}x^6\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] -3*a**3*b**(19/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(32*a**5*b**4*c**(9/2
)*x**2*gamma(5/4) + 64*a**4*b**5*c**(9/2)*x**4*gamma(5/4) + 32*a**3*b**6*c*
*(9/2)*x**6*gamma(5/4)) + 5*a**2*b**(23/4)*x**2*(a/(b*x**2) + 1)**(3/4)*gam
ma(-7/4)/(32*a**5*b**4*c**(9/2)*x**2*gamma(5/4) + 64*a**4*b**5*c**(9/2)*x**
4*gamma(5/4) + 32*a**3*b**6*c**(9/2)*x**6*gamma(5/4)) + 40*a*b**(27/4)*x**4
*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(32*a**5*b**4*c**(9/2)*x**2*gamma(5/4)
 + 64*a**4*b**5*c**(9/2)*x**4*gamma(5/4) + 32*a**3*b**6*c**(9/2)*x**6*gamma
(5/4)) + 32*b**(31/4)*x**6*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(32*a**5*b**
4*c**(9/2)*x**2*gamma(5/4) + 64*a**4*b**5*c**(9/2)*x**4*gamma(5/4) + 32*a**
3*b**6*c**(9/2)*x**6*gamma(5/4))
```

3.990 $\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=109

$$-\frac{256\left(a+bx^2\right)^{11/4}}{77a^4c(cx)^{11/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{7a^3c(cx)^{11/2}}-\frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}}+\frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

```
[Out] 2/a/c/(c*x)^(11/2)/(b*x^2+a)^(1/4)-8*(b*x^2+a)^(3/4)/a^2/c/(c*x)^(11/2)+64/
7*(b*x^2+a)^(7/4)/a^3/c/(c*x)^(11/2)-256/77*(b*x^2+a)^(11/4)/a^4/c/(c*x)^(1
1/2)
```

Rubi [A] time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{256\left(a+bx^2\right)^{11/4}}{77a^4c(cx)^{11/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{7a^3c(cx)^{11/2}}-\frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}}+\frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*c*(c*x)^(11/2)*(a + b*x^2)^(1/4)) - (8*(a + b*x^2)^(3/4))/(a^2*c*(c*x)
^(11/2)) + (64*(a + b*x^2)^(7/4))/(7*a^3*c*(c*x)^(11/2)) - (256*(a + b*x^2)
^(11/4))/(77*a^4*c*(c*x)^(11/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx &= \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}+\frac{12\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx}{a}\\
&= \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}}-\frac{32\int \frac{\left(a+bx^2\right)^{3/4}}{(cx)^{13/2}}\,dx}{a^2}\\
&= \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{7a^3c(cx)^{11/2}}+\frac{128\int \frac{\left(a+bx^2\right)^{7/4}}{(cx)^{13/2}}\,dx}{7a^3}\\
&= \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{7a^3c(cx)^{11/2}}-\frac{256\left(a+bx^2\right)^{11/4}}{77a^4c(cx)^{11/2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 58, normalized size = 0.53

$$-\frac{2x\left(7a^3-12a^2bx^2+32ab^2x^4+128b^3x^6\right)}{77a^4(cx)^{13/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(7*a^3 - 12*a^2*b*x^2 + 32*a*b^2*x^4 + 128*b^3*x^6))/(77*a^4*(c*x)^(1
3/2)*(a + b*x^2)^(1/4))
```

fricas [A] time = 0.53, size = 72, normalized size = 0.66

$$-\frac{2\left(128\,b^3x^6+32\,ab^2x^4-12\,a^2bx^2+7\,a^3\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{77\left(a^4bc^7x^8+a^5c^7x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -2/77*(128*b^3*x^6 + 32*a*b^2*x^4 - 12*a^2*b*x^2 + 7*a^3)*(b*x^2 + a)^(3/4)
*sqrt(c*x)/(a^4*b*c^7*x^8 + a^5*c^7*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)), x)
```

maple [A] time = 0.01, size = 53, normalized size = 0.49

$$-\frac{2\left(128b^3x^6+32a\,b^2x^4-12a^2b\,x^2+7a^3\right)x}{77\left(b\,x^2+a\right)^{\frac{1}{4}}(cx)^{\frac{13}{2}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/77*x*(128*b^3*x^6+32*a*b^2*x^4-12*a^2*b*x^2+7*a^3)/(b*x^2+a)^(1/4)/a^4/(
c*x)^(13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)), x)
```

mupad [B] time = 5.20, size = 85, normalized size = 0.78

$$-\frac{\left(b\,x^2+a\right)^{3/4}\,\left(\frac{2}{11\,a\,b\,c^6}-\frac{24\,x^2}{77\,a^2\,c^6}+\frac{64\,b\,x^4}{77\,a^3\,c^6}+\frac{256\,b^2\,x^6}{77\,a^4\,c^6}\right)}{x^7\,\sqrt{c\,x}+\frac{a\,x^5\,\sqrt{c\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*(2/(11*a*b*c^6) - (24*x^2)/(77*a^2*c^6) + (64*b*x^4)/(7
7*a^3*c^6) + (256*b^2*x^6)/(77*a^4*c^6)))/(x^7*(c*x)^(1/2) + (a*x^5*(c*x)^(
1/2))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

3.991 $\int \frac{(cx)^{13/2}}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=155

$$\frac{77a^{5/2}c^6\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}}+\frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}}-\frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}}$$

```
[Out] 77/60*a^2*c^5*(c*x)^(3/2)/b^3/(b*x^2+a)^(1/4)-11/30*a*c^3*(c*x)^(7/2)/b^2/(
b*x^2+a)^(1/4)+1/5*c*(c*x)^(11/2)/b/(b*x^2+a)^(1/4)+77/20*a^(5/2)*c^6*(1+a/
b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*
b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*
x)^(1/2)/b^(7/2)/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {285, 284, 335, 196}

$$\frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}}+\frac{77a^{5/2}c^6\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}}-\frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(13/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (77*a^2*c^5*(c*x)^(3/2))/(60*b^3*(a + b*x^2)^(1/4)) - (11*a*c^3*(c*x)^(7/2)
)/(30*b^2*(a + b*x^2)^(1/4)) + (c*(c*x)^(11/2))/(5*b*(a + b*x^2)^(1/4)) + (
77*a^(5/2)*c^6*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)
/Sqrt[a]]/2, 2])/(20*b^(7/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{(cx)^{13/2}}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} - \frac{\left(11ac^2\right)\int \frac{(cx)^{9/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{10b}$$

$$= -\frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} + \frac{\left(77a^2c^4\right)\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{60b^2}$$

$$= \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} - \frac{\left(77a^3c^6\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{40b^3}$$

$$= \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} - \frac{\left(77a^3c^6\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,d}{40b^4\sqrt[4]{a+bx^2}}$$

$$= \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} + \frac{\left(77a^3c^6\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ax}{b}\right.}\right.}{40b^4\sqrt[4]{a+bx^2}}$$

$$= \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} + \frac{77a^{5/2}c^6\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{20b^{7/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.04, size = 87, normalized size = 0.56

$$\frac{c^5(cx)^{3/2}\left(-77a^2\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+77a^2-22abx^2+12b^2x^4\right)}{60b^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c^5*(c*x)^(3/2)*(77*a^2 - 22*a*b*x^2 + 12*b^2*x^4 - 77*a^2*(1 + (b*x^2)/a)
^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*x^2)/a)]))/(60*b^3*(a + b*x^2)
^(1/4))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,c^6x^6}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)*c^6*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{13}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(13/2)/(b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{13}{2}}}{\left(b\,x^{2}+a\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((c*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{13}{2}}}{\left(bx^{2}+a\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(13/2)/(b*x^2 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{13/2}}{\left(b\,x^2+a\right)^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/2)/(a + b*x^2)^(5/4),x)
```

```
[Out] int((c*x)^(13/2)/(a + b*x^2)^(5/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

3.992 $\int \frac{(cx)^{9/2}}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=124

$$-\frac{7a^{3/2}c^4\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}-\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}$$

```
[Out] -7/6*a*c^3*(c*x)^(3/2)/b^2/(b*x^2+a)^(1/4)+1/3*c*(c*x)^(7/2)/b/(b*x^2+a)^(1
/4)-7/2*a^(3/2)*c^4*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2
)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/
2)/a^(1/2))),2^(1/2))*(c*x)^(1/2)/b^(5/2)/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {285, 284, 335, 196}

$$-\frac{7a^{3/2}c^4\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}-\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(9/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (-7*a*c^3*(c*x)^(3/2))/(6*b^2*(a + b*x^2)^(1/4)) + (c*(c*x)^(7/2))/(3*b*(a
+ b*x^2)^(1/4)) - (7*a^(3/2)*c^4*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[
ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*b^(5/2)*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{(cx)^{9/2}}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} - \frac{\left(7ac^2\right)\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{6b}$$

$$= -\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} + \frac{\left(7a^2c^4\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{4b^2}$$

$$= -\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} + \frac{\left(7a^2c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{4b^3\sqrt[4]{a+bx^2}}$$

$$= -\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} - \frac{\left(7a^2c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{4b^3\sqrt[4]{a+bx^2}}$$

$$= -\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} - \frac{7a^{3/2}c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.03, size = 74, normalized size = 0.60

$$\frac{c^3(cx)^{3/2}\left(7a\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)-7a+2bx^2\right)}{6b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(9/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c^3*(c*x)^(3/2)*(-7*a + 2*b*x^2 + 7*a*(1 + (b*x^2)/a)^(1/4)*Hypergeometric
2F1[3/4, 5/4, 7/4, -((b*x^2)/a)]))/(6*b^2*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,c^4x^4}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)*c^4*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{9}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{9}{2}}}{\left(b\,x^{2}+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(9/2)/(b*x^2+a)^(5/4),x)
[Out] int((c*x)^(9/2)/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{9}{2}}}{\left(bx^{2}+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate((c*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{9/2}}{\left(b\,x^{2}+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(9/2)/(a + b*x^2)^(5/4),x)
[Out] int((c*x)^(9/2)/(a + b*x^2)^(5/4), x)
```

sympy [C] time = 64.02, size = 44, normalized size = 0.35

$$\frac{c^{\frac{9}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{5}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(9/2)/(b*x**2+a)**(5/4),x)
[Out] c**(9/2)*x**(11/2)*gamma(11/4)*hyper((5/4, 11/4), (15/4,), b*x**2*exp_polar
(I*pi)/a)/(2*a**(5/4)*gamma(15/4))
```

3.993 $\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=90

$$\frac{3\sqrt{a}\,c^2\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}$$

```
[Out] c*(c*x)^(3/2)/b/(b*x^2+a)^(1/4)+3*c^2*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(
1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*(c*x)^(1/2)/b^(3/2)/(b*x^2+
a)^(1/4)
```

Rubi [A] time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {285, 284, 335, 196}

$$\frac{3\sqrt{a}\,c^2\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c*(c*x)^(3/2))/(b*(a + b*x^2)^(1/4)) + (3*Sqrt[a]*c^2*(1 + a/(b*x^2))^(1/4
)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^
2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{5/4}}\,dx &= \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}} - \frac{\left(3ac^2\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{2b} \\
&= \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}} - \frac{\left(3ac^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{2b^2\sqrt[4]{a+bx^2}} \\
&= \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}} + \frac{\left(3ac^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2b^2\sqrt[4]{a+bx^2}} \\
&= \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}} + \frac{3\sqrt{a}\,c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 60, normalized size = 0.67

$$
\frac{c(cx)^{3/2}\left(1-\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{b\sqrt[4]{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(1 - (1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4,
-((b*x^2)/a)]))/(b*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,c^2x^2}{b^2x^4+2\,abx^2+a^2}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)*c^2*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$
\int \frac{(cx)^{\frac{5}{2}}}{\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((c*x)^(5/2)/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{5/2}}{\left(b\,x^2+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a + b*x^2)^(5/4),x)
```

```
[Out] int((c*x)^(5/2)/(a + b*x^2)^(5/4), x)
```

sympy [C] time = 7.78, size = 44, normalized size = 0.49

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((5/4, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/4)*gamma(11/4))
```

3.994 $\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=63

$$-\frac{2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

```
[Out] -2*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*a
rccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1
/2))*(c*x)^(1/2)/(b*x^2+a)^(1/4)/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {284, 335, 196}

$$-\frac{2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(5/4),x]
```

```
[Out] (-2*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2
, 2])/(Sqrt[a]*Sqrt[b]*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule Rubi steps

$$\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{\left(\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{b\sqrt[4]{a+bx^2}}$$

$$= -\frac{\left(\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{b\sqrt[4]{a+bx^2}}$$

$$= -\frac{2\sqrt[4]{1+\frac{a}{bx^2}}\,\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.94

$$\frac{2x\sqrt{cx}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3a\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*x*Sqrt[c*x]*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*
x^2)/a)])/(3*a*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((c*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x}}{{\left(b\,x^2+a\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a + b*x^2)^(5/4),x)
```

```
[Out] int((c*x)^(1/2)/(a + b*x^2)^(5/4), x)
```

sympy [C] time = 2.10, size = 44, normalized size = 0.70

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(5/4)*gamma(7/4))
```

3.995 $\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/4}}\,dx$

Optimal. Leaf size=93

$$\frac{4\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}$$

```
[Out] -2/a/c/(b*x^2+a)^(1/4)/(c*x)^(1/2)+4*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^
(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1
/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)*(c*x)^(1/2)/a^(3/2)/c^2/(b*x
^2+a)^(1/4)
```

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {286, 284, 335, 196}

$$\frac{4\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] -2/(a*c*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (4*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sqr
t[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*c^2*(a + b*x^2
)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2}{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \frac{(2b)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{ac^2}$$

$$= -\frac{2}{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}} - \frac{\left(2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{ac^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{2}{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}} + \frac{\left(2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{ac^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{2}{ac\sqrt{cx}\,\sqrt[4]{a+bx^2}} + \frac{4\sqrt{b}\,\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.61

$$-\frac{2x\sqrt[4]{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{a(cx)^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(5/4)),x]
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, -((b*x^2)/a)]
)/(a*(c*x)^(3/2)*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{b^2c^2x^6+2\,abc^2x^4+a^2c^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*c^2*x^6 + 2*a*b*c^2*x^4 + a^2*c^2
*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(3/2)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{2}}\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{3/2}\,\left(b\,x^2+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] int(1/((c*x)^(3/2)*(a + b*x^2)^(5/4)), x)
```

sympy [C] time = 6.01, size = 48, normalized size = 0.52

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 5/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4
)*c**(3/2)*sqrt(x)*gamma(3/4))
```

3.996 $\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=126

$$-\frac{24b^{3/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}c^4\sqrt[4]{a+bx^2}}+\frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -2/5/a/c/(c*x)^(5/2)/(b*x^2+a)^(1/4)+12/5*b/a^2/c^3/(b*x^2+a)^(1/4)/(c*x)^(
1/2)-24/5*b^(3/2)*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^
(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)
/a^(1/2))),2^(1/2))*(c*x)^(1/2)/a^(5/2)/c^4/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {286, 284, 335, 196}

$$-\frac{24b^{3/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}c^4\sqrt[4]{a+bx^2}}+\frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] -2/(5*a*c*(c*x)^(5/2)*(a + b*x^2)^(1/4)) + (12*b)/(5*a^2*c^3*Sqrt[c*x]*(a +
 b*x^2)^(1/4)) - (24*b^(3/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcC
ot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(5/2)*c^4*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} - \frac{(6b)\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx}{5ac^2}$$

$$= -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}} + \frac{\left(12b^2\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{5/4}}\,dx}{5a^2c^4}$$

$$= -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}} + \frac{\left(12b\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\int \frac{1}{\left(1+\frac{a}{bx^2}\right)^{5/4}x^2}\,dx}{5a^2c^4\sqrt[4]{a+bx^2}}$$

$$= -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{\left(12b\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5}}\right.}{5a^2c^4\sqrt[4]{a+bx^2}}$$

$$= -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{24b^{3/2}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right.}{5a^{5/2}c^4\sqrt[4]{a+bx^2}}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.47

$$-\frac{2x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, -((b*x^2)/a)
])/(5*a*(c*x)^(7/2)*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{b^2c^4x^8+2\,abc^4x^6+a^2c^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*c^4*x^8 + 2*a*b*c^4*x^6 + a^2*c^4
*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(7/2)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{7}{2}} \left(b\,x^{2}+a\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x)
[Out] int(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{5}{4}} (cx)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{7/2}\,{\left(b\,x^2+a\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a + b*x^2)^(5/4)),x)
[Out] int(1/((c*x)^(7/2)*(a + b*x^2)^(5/4)), x)
```

sympy [C] time = 41.49, size = 34, normalized size = 0.27

$$-\frac{{}_{2}F_{1}\left(\left.\begin{matrix}\frac{5}{4}, \frac{5}{2}\\ \frac{7}{2}\end{matrix}\right| \frac{ae^{i\pi}}{bx^{2}}\right)}{5b^{\frac{5}{4}}c^{\frac{7}{2}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(5/4),x)
[Out] -hyper((5/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*b**(5/4)*c**(7/2)
*x**5)
```

3.997 $\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=157

$$\frac{16b^{5/2}\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{7/2}c^6\sqrt[4]{a+bx^2}}-\frac{8b^2}{3a^3c^5\sqrt{cx}\,\sqrt[4]{a+bx^2}}+\frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

```
[Out] -2/9/a/c/(c*x)^(9/2)/(b*x^2+a)^(1/4)+4/9*b/a^2/c^3/(c*x)^(5/2)/(b*x^2+a)^(1
/4)-8/3*b^2/a^3/c^5/(b*x^2+a)^(1/4)/(c*x)^(1/2)+16/3*b^(5/2)*(1+a/b/x^2)^(1
/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a
^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*x)^(1/2)/
a^(7/2)/c^6/(b*x^2+a)^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {286, 284, 335, 196}

$$-\frac{8b^2}{3a^3c^5\sqrt{cx}\,\sqrt[4]{a+bx^2}}+\frac{16b^{5/2}\sqrt{cx}\,\sqrt[4]{\frac{a}{bx^2}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{3a^{7/2}c^6\sqrt[4]{a+bx^2}}+\frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}}-\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] -2/(9*a*c*(c*x)^(9/2)*(a + b*x^2)^(1/4)) + (4*b)/(9*a^2*c^3*(c*x)^(5/2)*(a
+ b*x^2)^(1/4)) - (8*b^2)/(3*a^3*c^5*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (16*b^(
5/2)*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/
2, 2])/(3*a^(7/2)*c^6*(a + b*x^2)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 284

```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Dist[(Sqrt[c
*x]*(1 + a/(b*x^2))^(1/4))/(b*(a + b*x^2)^(1/4)), Int[1/(x^2*(1 + a/(b*x^2)
)^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}} - \frac{(10b)\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx}{9ac^2}$$

$$= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}} + \frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{\left(4b^2\right)\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx}{3a^2c^4}$$

$$= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}} + \frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}} - \frac{8b^2}{3a^3c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \cdots$$

$$= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}} + \frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}} - \frac{8b^2}{3a^3c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \cdots$$

$$= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}} + \frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}} - \frac{8b^2}{3a^3c^5\sqrt{cx}\sqrt[4]{a+bx^2}} + \cdots$$

$$= -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}} + \frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}} - \frac{8b^2}{3a^3c^5\sqrt{cx}\sqrt[4]{a+bx^2}} + \cdots$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.38

$$-\frac{2x\sqrt[4]{\frac{bx^2}{a}+1}\ {}_2F_1\left(-\frac{9}{4},\frac{5}{4};-\frac{5}{4};-\frac{bx^2}{a}\right)}{9a(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-9/4, 5/4, -5/4, -((b*x^2)/a)
])/(9*a*(c*x)^(11/2)*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{b^2c^6x^{10}+2\,abc^6x^8+a^2c^6x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*c^6*x^10 + 2*a*b*c^6*x^8 + a^2*c^
6*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(11/2)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{11}{2}} \left(b\,x^{2}+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x)
[Out] int(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{5}{4}} (cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(11/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x\right)}^{11/2}\,{\left(b\,x^2+a\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(a + b*x^2)^(5/4)),x)
[Out] int(1/((c*x)^(11/2)*(a + b*x^2)^(5/4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(b*x**2+a)**(5/4),x)
[Out] Timed out
```

3.998 $\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{4(cx)^{9/4}\sqrt[4]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{9}{8};\frac{17}{8};-\frac{bx^2}{a}\right)}{9c\sqrt[4]{a+bx^2}}$$

```
[Out] 4/9*(c*x)^(9/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/4, 9/8],[17/8],-b*x^2/a)/c/(
b*x^2+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{9/4}\sqrt[4]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{9}{8};\frac{17}{8};-\frac{bx^2}{a}\right)}{9c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/4)/(a + b*x^2)^(1/4),x]
```

```
[Out] (4*(c*x)^(9/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 9/8, 17/8, -((b
*x^2)/a)])/(9*c*(a + b*x^2)^(1/4))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx &= \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{(cx)^{5/4}}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}\\ &= \frac{4(cx)^{9/4}\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{9}{8};\frac{17}{8};-\frac{bx^2}{a}\right)}{9c\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 0.97

$$\frac{4x(cx)^{5/4}\sqrt[4]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{9}{8};\frac{17}{8};-\frac{bx^2}{a}\right)}{9\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/4)/(a + b*x^2)^(1/4),x]
```

```
[Out] (4*x*(c*x)^(5/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 9/8, 17/8, -(
(b*x^2)/a)])/(9*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^{\frac{1}{4}}cx}{\left(bx^2+a\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(1/4)*c*x/(b*x^2 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{4}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/4)/(b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{4}}}{\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{4}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/4)/(b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{5/4}}{\left(b\,x^2+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/4)/(a + b*x^2)^(1/4),x)
```

[Out] int((c*x)^(5/4)/(a + b*x^2)^(1/4), x)

sympy [C] time = 12.12, size = 44, normalized size = 0.76

$$\frac{c^{\frac{5}{4}}x^{\frac{9}{4}}\Gamma\left(\frac{9}{8}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{9}{8}\\\frac{17}{8}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{17}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(5/4)/(b*x**2+a)**(1/4),x)

[Out] c**(5/4)*x**(9/4)*gamma(9/8)*hyper((1/4, 9/8), (17/8,), b*x**2*exp_polar(I*pi)/a)/(2*a**(1/4)*gamma(17/8))

3.999 $\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{4(cx)^{7/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{7}{8};\frac{15}{8};-\frac{bx^2}{a}\right)}{7c\sqrt[4]{a+bx^2}}$$

```
[Out] 4/7*(c*x)^(7/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/4, 7/8],[15/8],-b*x^2/a)/c/(
b*x^2+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{7/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{7}{8};\frac{15}{8};-\frac{bx^2}{a}\right)}{7c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/4)/(a + b*x^2)^(1/4),x]
```

```
[Out] (4*(c*x)^(7/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 7/8, 15/8, -((b
*x^2)/a)])/(7*c*(a + b*x^2)^(1/4))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx &= \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{(cx)^{3/4}}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}\\ &= \frac{4(cx)^{7/4}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{7}{8};\frac{15}{8};-\frac{bx^2}{a}\right)}{7c\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 0.97

$$\frac{4x(cx)^{3/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{7}{8};\frac{15}{8};-\frac{bx^2}{a}\right)}{7\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x)^(3/4)/(a + b*x^2)^(1/4),x]

[Out] (4*x*(c*x)^(3/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 7/8, 15/8, -((b*x^2)/a)])/(7*(a + b*x^2)^(1/4))

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^{\frac{3}{4}}}{\left(bx^2+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")

[Out] integral((c*x)^(3/4)/(b*x^2 + a)^(1/4), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{4}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="giac")

[Out] integrate((c*x)^(3/4)/(b*x^2 + a)^(1/4), x)

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{4}}}{\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(3/4)/(b*x^2+a)^(1/4),x)

[Out] int((c*x)^(3/4)/(b*x^2+a)^(1/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{4}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")

[Out] integrate((c*x)^(3/4)/(b*x^2 + a)^(1/4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{3/4}}{\left(b\,x^2+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(3/4)/(a + b*x^2)^(1/4),x)

```
[Out] int((c*x)^(3/4)/(a + b*x^2)^(1/4), x)
```

sympy [C] time = 3.69, size = 44, normalized size = 0.76

$$\frac{c^{\frac{3}{4}}x^{\frac{7}{4}}\Gamma\left(\frac{7}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{8}\\ \frac{15}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{15}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/4)/(b*x**2+a)**(1/4),x)
```

```
[Out] c**(3/4)*x**(7/4)*gamma(7/8)*hyper((1/4, 7/8), (15/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(1/4)*gamma(15/8))
```

3.1000 $\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{4(cx)^{5/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{5}{8};\frac{13}{8};-\frac{bx^2}{a}\right)}{5c\sqrt[4]{a+bx^2}}$$

```
[Out] 4/5*(c*x)^(5/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/4, 5/8],[13/8],-b*x^2/a)/c/(
b*x^2+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{5/4}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{5}{8};\frac{13}{8};-\frac{bx^2}{a}\right)}{5c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/4)/(a + b*x^2)^(1/4),x]
```

```
[Out] (4*(c*x)^(5/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 5/8, 13/8, -((b
*x^2)/a)])/(5*c*(a + b*x^2)^(1/4))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx &= \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{\sqrt[4]{cx}}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}\\ &= \frac{4(cx)^{5/4}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{5}{8};\frac{13}{8};-\frac{bx^2}{a}\right)}{5c\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 0.97

$$\frac{4x\sqrt[4]{cx}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{5}{8};\frac{13}{8};-\frac{bx^2}{a}\right)}{5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/4)/(a + b*x^2)^(1/4),x]
```

```
[Out] (4*x*(c*x)^(1/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 5/8, 13/8, -(
(b*x^2)/a)])/(5*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^{\frac{1}{4}}}{\left(bx^2+a\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(1/4)/(b*x^2 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{4}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/4)/(b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{4}}}{\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{4}}}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(1/4)/(b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{1/4}}{\left(b\,x^2+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/4)/(a + b*x^2)^(1/4),x)
```

[Out] int((c*x)^(1/4)/(a + b*x^2)^(1/4), x)

sympy [C] time = 1.23, size = 44, normalized size = 0.76

$$\frac{\sqrt[4]{c}\,x^{\frac{5}{4}}\Gamma\left(\frac{5}{8}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{5}{8}\\\frac{13}{8}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{13}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(1/4)/(b*x**2+a)**(1/4),x)

[Out] c**(1/4)*x**(5/4)*gamma(5/8)*hyper((1/4, 5/8), (13/8,), b*x**2*exp_polar(I*pi)/a)/(2*a**(1/4)*gamma(13/8))

3.1001 $\int \frac{1}{\sqrt[4]{cx}\,\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{4(cx)^{3/4}\sqrt[4]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-\frac{bx^2}{a}\right)}{3c\sqrt[4]{a+bx^2}}$$

```
[Out] 4/3*(c*x)^(3/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/4, 3/8],[11/8],-b*x^2/a)/c/(
b*x^2+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{3/4}\sqrt[4]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-\frac{bx^2}{a}\right)}{3c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(1/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (4*(c*x)^(3/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/8, 11/8, -((b
*x^2)/a)])/(3*c*(a + b*x^2)^(1/4))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{cx}\,\sqrt[4]{a+bx^2}}\,dx &= \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{cx}\,\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}\\ &= \frac{4(cx)^{3/4}\sqrt[4]{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-\frac{bx^2}{a}\right)}{3c\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 0.97

$$\frac{4x\sqrt[4]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-\frac{bx^2}{a}\right)}{3\sqrt[4]{cx}\,\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(1/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (4*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/8, 11/8, -((b*x^2)/a)])
/(3*(c*x)^(1/4)*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{3}{4}}}{bcx^3+acx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(c*x)^(3/4)/(b*c*x^3 + a*c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(1/4)), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{1}{4}}\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{(c\,x)}^{1/4}\,{\left(b\,x^2+a\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/4)*(a + b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(1/4)*(a + b*x^2)^(1/4)), x)
```

sympy [C] time = 1.30, size = 44, normalized size = 0.76

$$\frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{8}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{3}{8}\\ \frac{11}{8}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\,\Gamma\left(\frac{11}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/4)/(b*x**2+a)**(1/4),x)
```

```
[Out] x**(3/4)*gamma(3/8)*hyper((1/4, 3/8), (11/8,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(1/4)*c**(1/4)*gamma(11/8))
```

3.1002 $\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=56

$$\frac{4\sqrt[4]{cx}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{a+bx^2}}$$

```
[Out] 4*(c*x)^(1/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/8, 1/4],[9/8],-b*x^2/a)/c/(b*x
^2+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4\sqrt[4]{cx}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (4*(c*x)^(1/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -((b*
x^2)/a)])/(c*(a + b*x^2)^(1/4))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx &= \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{(cx)^{3/4}\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}\\
&= \frac{4\sqrt[4]{cx}\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.96

$$\frac{4x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{(cx)^{3/4}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (4*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -((b*x^2)/a)])/
((c*x)^(3/4)*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{1}{4}}}{bcx^3+acx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(c*x)^(1/4)/(b*c*x^3 + a*c*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/4)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{4}}\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{(c\,x)}^{3/4}\,{\left(b\,x^2+a\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/4)*(a + b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(3/4)*(a + b*x^2)^(1/4)), x)
```

sympy [C] time = 2.09, size = 44, normalized size = 0.79

$$\frac{\sqrt[4]{x}\,\Gamma\left(\frac{1}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{8},\frac{1}{4}\\ \frac{9}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,c^{\frac{3}{4}}\Gamma\left(\frac{9}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/4)/(b*x**2+a)**(1/4),x)
```

```
[Out] x**(1/4)*gamma(1/8)*hyper((1/8, 1/4), (9/8,), b*x**2*exp_polar(I*pi)/a)/(2*
a**(1/4)*c**(3/4)*gamma(9/8))
```

3.1003 $\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=56

$$-\frac{4\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{cx}\,\sqrt[4]{a+bx^2}}$$

```
[Out] -4*(1+b*x^2/a)^(1/4)*hypergeom([-1/8, 1/4],[7/8],-b*x^2/a)/c/(c*x)^(1/4)/(b
*x^2+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$-\frac{4\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{cx}\,\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-4*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/8, 1/4, 7/8, -((b*x^2)/a)])/
(c*(c*x)^(1/4)*(a + b*x^2)^(1/4))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx &= \frac{\sqrt[4]{1+\frac{bx^2}{a}}\int \frac{1}{(cx)^{5/4}\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a+bx^2}}\\
&= -\frac{4\sqrt[4]{1+\frac{bx^2}{a}}\,{}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{c\sqrt[4]{cx}\,\sqrt[4]{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.96

$$-\frac{4x\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{(cx)^{5/4}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/4)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-4*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/8, 1/4, 7/8, -((b*x^2)/a)]
)/((c*x)^(5/4)*(a + b*x^2)^(1/4))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{3}{4}}}{bc^2x^4+ac^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/4)*(c*x)^(3/4)/(b*c^2*x^4 + a*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/4)), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{4}}\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(c\,x\right)}^{5/4}\,{\left(b\,x^2+a\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/4)*(a + b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(5/4)*(a + b*x^2)^(1/4)), x)
```

sympy [C] time = 3.76, size = 48, normalized size = 0.86

$$\frac{\Gamma\left(-\frac{1}{8}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{8},\frac{1}{4}\\\frac{7}{8}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,c^{\frac{5}{4}}\sqrt[4]{x}\,\Gamma\left(\frac{7}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/4)/(b*x**2+a)**(1/4),x)
```

```
[Out] gamma(-1/8)*hyper((-1/8, 1/4), (7/8,), b*x**2*exp_polar(I*pi)/a)/(2*a**(1/4
)*c**(5/4)*x**(1/4)*gamma(7/8))
```

3.1004 $\int \frac{(cx)^{5/4}}{\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=61

$$\frac{4(cx)^{9/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{9}{8},\frac{7}{4};\frac{17}{8};-\frac{bx^2}{a}\right)}{9ac\left(a+bx^2\right)^{3/4}}$$

```
[Out] 4/9*(c*x)^(9/4)*(1+b*x^2/a)^(3/4)*hypergeom([9/8, 7/4],[17/8],-b*x^2/a)/a/c
/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{9/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{9}{8},\frac{7}{4};\frac{17}{8};-\frac{bx^2}{a}\right)}{9ac\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*(c*x)^(9/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[9/8, 7/4, 17/8, -((b
*x^2)/a)])/(9*a*c*(a + b*x^2)^(3/4))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^{5/4}}{\left(a+bx^2\right)^{7/4}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{(cx)^{5/4}}{\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\left(a+bx^2\right)^{3/4}}\\ &= \frac{4(cx)^{9/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{9}{8},\frac{7}{4};\frac{17}{8};-\frac{bx^2}{a}\right)}{9ac\left(a+bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 0.97

$$\frac{4x(cx)^{5/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{9}{8},\frac{7}{4};\frac{17}{8};-\frac{bx^2}{a}\right)}{9a\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*x*(c*x)^(5/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[9/8, 7/4, 17/8, -(
(b*x^2)/a)])/(9*a*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{1}{4}}cx}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(1/4)*c*x/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{4}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/4)/(b*x^2 + a)^(7/4), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{4}}}{\left(b\,x^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{4}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/4)/(b*x^2 + a)^(7/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x\right)}^{5/4}}{{\left(b\,x^2+a\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/4)/(a + b*x^2)^(7/4),x)
```

```
[Out] int((c*x)^(5/4)/(a + b*x^2)^(7/4), x)
```

sympy [C] time = 23.70, size = 44, normalized size = 0.72

$$\frac{c^{\frac{5}{4}}x^{\frac{9}{4}}\Gamma\left(\frac{9}{8}\right){}_2F_1\left(\begin{matrix}\frac{9}{8},\frac{7}{4}\\\frac{17}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{17}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] c**(5/4)*x**(9/4)*gamma(9/8)*hyper((9/8, 7/4), (17/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(7/4)*gamma(17/8))
```

3.1005 $\int \frac{(cx)^{3/4}}{\left(a+bx^2\right)^{7/4}} \, dx$

Optimal. Leaf size=61

$$\frac{4(cx)^{7/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{7}{8},\frac{7}{4};\frac{15}{8};-\frac{bx^2}{a}\right)}{7ac\left(a+bx^2\right)^{3/4}}$$

```
[Out] 4/7*(c*x)^(7/4)*(1+b*x^2/a)^(3/4)*hypergeom([7/8, 7/4],[15/8],-b*x^2/a)/a/c
/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{7/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{7}{8},\frac{7}{4};\frac{15}{8};-\frac{bx^2}{a}\right)}{7ac\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*(c*x)^(7/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[7/8, 7/4, 15/8, -((b
*x^2)/a)])/(7*a*c*(a + b*x^2)^(3/4))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{3/4}}{\left(a+bx^2\right)^{7/4}} \, dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/4} \int \frac{(cx)^{3/4}}{\left(1+\frac{bx^2}{a}\right)^{7/4}} \, dx}{a\left(a+bx^2\right)^{3/4}}\\
&= \frac{4(cx)^{7/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{7}{8},\frac{7}{4};\frac{15}{8};-\frac{bx^2}{a}\right)}{7ac\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 0.97

$$\frac{4x(cx)^{3/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{7}{8},\frac{7}{4};\frac{15}{8};-\frac{bx^2}{a}\right)}{7a\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*x*(c*x)^(3/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[7/8, 7/4, 15/8, -(
(b*x^2)/a)])/(7*a*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{4}}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(3/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{4}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/4)/(b*x^2 + a)^(7/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{4}}}{\left(b\,x^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{4}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/4)/(b*x^2 + a)^(7/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{3/4}}{\left(b\,x^2+a\right)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/4)/(a + b*x^2)^(7/4),x)
```

```
[Out] int((c*x)^(3/4)/(a + b*x^2)^(7/4), x)
```

sympy [C] time = 13.71, size = 44, normalized size = 0.72

$$\frac{c^{\frac{3}{4}}x^{\frac{7}{4}}\Gamma\left(\frac{7}{8}\right){}_2F_1\left(\begin{matrix}\frac{7}{8},\frac{7}{4}\\\frac{15}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{15}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] c**(3/4)*x**(7/4)*gamma(7/8)*hyper((7/8, 7/4), (15/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(7/4)*gamma(15/8))
```

3.1006 $\int \frac{\sqrt[4]{cx}}{\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=61

$$\frac{4(cx)^{5/4}\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(\frac{5}{8},\frac{7}{4};\frac{13}{8};-\frac{bx^2}{a}\right)}{5ac\left(a+bx^2\right)^{3/4}}$$

```
[Out] 4/5*(c*x)^(5/4)*(1+b*x^2/a)^(3/4)*hypergeom([5/8, 7/4],[13/8],-b*x^2/a)/a/c
/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{5/4}\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(\frac{5}{8},\frac{7}{4};\frac{13}{8};-\frac{bx^2}{a}\right)}{5ac\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/4)/(a + b*x^2)^(7/4),x]
```

```
[Out] (4*(c*x)^(5/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[5/8, 7/4, 13/8, -((b
*x^2)/a)])/(5*a*c*(a + b*x^2)^(3/4))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{cx}}{\left(a+bx^2\right)^{7/4}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{\sqrt[4]{cx}}{\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\left(a+bx^2\right)^{3/4}}\\
&= \frac{4(cx)^{5/4}\left(1+\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{5}{8},\frac{7}{4};\frac{13}{8};-\frac{bx^2}{a}\right)}{5ac\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 0.97

$$\frac{4x\sqrt[4]{cx}\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(\frac{5}{8},\frac{7}{4};\frac{13}{8};-\frac{bx^2}{a}\right)}{5a\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/4)/(a + b*x^2)^(7/4),x]
[Out] (4*x*(c*x)^(1/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[5/8, 7/4, 13/8, -(
(b*x^2)/a)])/(5*a*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{1}{4}}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(1/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{4}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
[Out] integrate((c*x)^(1/4)/(b*x^2 + a)^(7/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{4}}}{\left(b\,x^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/4)/(b*x^2+a)^(7/4),x)
[Out] int((c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{4}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
[Out] integrate((c*x)^(1/4)/(b*x^2 + a)^(7/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{(c\,x)}^{1/4}}{{\left(b\,x^2+a\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/4)/(a + b*x^2)^(7/4),x)
```

```
[Out] int((c*x)^(1/4)/(a + b*x^2)^(7/4), x)
```

sympy [C] time = 7.48, size = 44, normalized size = 0.72

$$\frac{\sqrt[4]{c}\,x^{\frac{5}{4}}\Gamma\left(\frac{5}{8}\right){}_2F_1\left(\begin{matrix}\frac{5}{8},\frac{7}{4}\\\frac{13}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{13}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] c**(1/4)*x**(5/4)*gamma(5/8)*hyper((5/8, 7/4), (13/8,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(7/4)*gamma(13/8))
```

3.1007 $\int \frac{1}{\sqrt[4]{cx}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=61

$$\frac{4(cx)^{3/4}\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{3}{8},\frac{7}{4};\frac{11}{8};-\frac{bx^2}{a}\right)}{3ac\left(a+bx^2\right)^{3/4}}$$

```
[Out] 4/3*(c*x)^(3/4)*(1+b*x^2/a)^(3/4)*hypergeom([3/8, 7/4],[11/8],-b*x^2/a)/a/c
/(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4(cx)^{3/4}\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{3}{8},\frac{7}{4};\frac{11}{8};-\frac{bx^2}{a}\right)}{3ac\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(1/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (4*(c*x)^(3/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[3/8, 7/4, 11/8, -((b
*x^2)/a)])/(3*a*c*(a + b*x^2)^(3/4))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[4]{cx}\left(a+bx^2\right)^{7/4}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\sqrt[4]{cx}\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\left(a+bx^2\right)^{3/4}}\\
&= \frac{4(cx)^{3/4}\left(1+\frac{bx^2}{a}\right)^{3/4}{}_2F_1\left(\frac{3}{8},\frac{7}{4};\frac{11}{8};-\frac{bx^2}{a}\right)}{3ac\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 0.97

$$\frac{4x\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{3}{8},\frac{7}{4};\frac{11}{8};-\frac{bx^2}{a}\right)}{3a\sqrt[4]{cx}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(1/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (4*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[3/8, 7/4, 11/8, -((b*x^2)/a)])
/(3*a*(c*x)^(1/4)*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{4}}}{b^2cx^5+2\,abcx^3+a^2cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(3/4)/(b^2*c*x^5 + 2*a*b*c*x^3 + a^2*c*x),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}(cx)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(1/4)), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{1}{4}}\left(b\,x^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

```
[Out] int(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}(cx)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(c\,x)^{1/4}\,\left(b\,x^2+a\right)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/4)*(a + b*x^2)^(7/4)),x)
```

```
[Out] int(1/((c*x)^(1/4)*(a + b*x^2)^(7/4)), x)
```

sympy [C]   time = 8.14, size = 44, normalized size = 0.72

$$\frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{8}\right){}_2F_1\left(\begin{matrix}\frac{3}{8},\frac{7}{4}\\ \frac{11}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\sqrt[4]{c}\,\Gamma\left(\frac{11}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] x**(3/4)*gamma(3/8)*hyper((3/8, 7/4), (11/8,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(7/4)*c**(1/4)*gamma(11/8))
```

3.1008 $\int \frac{1}{(cx)^{3/4}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=59

$$\frac{4\sqrt[4]{cx}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{8},\frac{7}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{ac\left(a+bx^2\right)^{3/4}}$$

```
[Out] 4*(c*x)^(1/4)*(1+b*x^2/a)^(3/4)*hypergeom([1/8, 7/4],[9/8],-b*x^2/a)/a/c/(b
*x^2+a)^(3/4)
```

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$\frac{4\sqrt[4]{cx}\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{8},\frac{7}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{ac\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (4*(c*x)^(1/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/8, 7/4, 9/8, -((b*
x^2)/a)])/(a*c*(a + b*x^2)^(3/4))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{3/4}\left(a+bx^2\right)^{7/4}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{(cx)^{3/4}\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\left(a+bx^2\right)^{3/4}}\\
&= \frac{4\sqrt[4]{cx}\left(1+\frac{bx^2}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{8},\frac{7}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{ac\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 0.97

$$\frac{4x\left(\frac{bx^2}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{8},\frac{7}{4};\frac{9}{8};-\frac{bx^2}{a}\right)}{a(cx)^{3/4}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (4*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/8, 7/4, 9/8, -((b*x^2)/a)])/
(a*(c*x)^(3/4)*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{1}{4}}}{b^2cx^5+2\,abcx^3+a^2cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(1/4)/(b^2*c*x^5 + 2*a*b*c*x^3 + a^2*c*x),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}(cx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(3/4)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{4}}\left(b\,x^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

```
[Out] int(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}(cx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(c\,x)^{3/4}\,\left(b\,x^2+a\right)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/4)*(a + b*x^2)^(7/4)),x)
```

[Out] int(1/((c*x)^(3/4)*(a + b*x^2)^(7/4)), x)

sympy [C] time = 14.68, size = 44, normalized size = 0.75

$$\frac{\sqrt[4]{x}\,\Gamma\left(\frac{1}{8}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{8},\frac{7}{4}\\ \frac{9}{8}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}c^{\frac{3}{4}}\Gamma\left(\frac{9}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)**(3/4)/(b*x**2+a)**(7/4),x)

[Out] x**(1/4)*gamma(1/8)*hyper((1/8, 7/4), (9/8,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4)*c**(3/4)*gamma(9/8))

3.1009 $\int \frac{1}{(cx)^{5/4}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=59

$$-\frac{4\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(-\frac{1}{8},\frac{7}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{ac\sqrt[4]{cx}\left(a+bx^2\right)^{3/4}}$$

```
[Out] -4*(1+b*x^2/a)^(3/4)*hypergeom([-1/8, 7/4],[7/8],-b*x^2/a)/a/c/(c*x)^(1/4)/
(b*x^2+a)^(3/4)
```

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {365, 364}

$$-\frac{4\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(-\frac{1}{8},\frac{7}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{ac\sqrt[4]{cx}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/4)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-4*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/8, 7/4, 7/8, -((b*x^2)/a)])/
(a*c*(c*x)^(1/4)*(a + b*x^2)^(3/4))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{5/4}\left(a+bx^2\right)^{7/4}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{(cx)^{5/4}\left(1+\frac{bx^2}{a}\right)^{7/4}}\,dx}{a\left(a+bx^2\right)^{3/4}}\\ &= -\frac{4\left(1+\frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(-\frac{1}{8},\frac{7}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{ac\sqrt[4]{cx}\left(a+bx^2\right)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 0.97

$$-\frac{4x\left(\frac{bx^2}{a}+1\right)^{3/4} {}_2F_1\left(-\frac{1}{8},\frac{7}{4};\frac{7}{8};-\frac{bx^2}{a}\right)}{a(cx)^{5/4}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/4)*(a + b*x^2)^(7/4)),x]
[Out] (-4*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/8, 7/4, 7/8, -((b*x^2)/a)]
)/(a*(c*x)^(5/4)*(a + b*x^2)^(3/4))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{4}}}{b^2c^2x^6+2\,abc^2x^4+a^2c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/4)*(c*x)^(3/4)/(b^2*c^2*x^6 + 2*a*b*c^2*x^4 + a^2*c
^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}(cx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(5/4)), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{4}}\left(b\,x^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x)
[Out] int(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}(cx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(5/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(c\,x)^{5/4}\left(b\,x^2+a\right)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/4)*(a + b*x^2)^(7/4)),x)
```

```
[Out] int(1/((c*x)^(5/4)*(a + b*x^2)^(7/4)), x)
```

sympy [C] time = 25.45, size = 48, normalized size = 0.81

$$\frac{\Gamma\left(-\frac{1}{8}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{8}, \frac{7}{4} \\ \frac{7}{8} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}c^{\frac{5}{4}}\sqrt[4]{x}\,\Gamma\left(\frac{7}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/4)/(b*x**2+a)**(7/4),x)
```

```
[Out] gamma(-1/8)*hyper((-1/8, 7/4), (7/8,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4
)*c**(5/4)*x**(1/4)*gamma(7/8))
```

3.1010 $\int x^6\sqrt[6]{a+bx^2}\,dx$

Optimal. Leaf size=345

$$-\frac{81\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^4\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{2816b^4x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] 81/2816*a^3*x*(b*x^2+a)^(1/6)/b^3-9/704*a^2*x^3*(b*x^2+a)^(1/6)/b^2+3/352*a
*x^5*(b*x^2+a)^(1/6)/b+3/22*x^7*(b*x^2+a)^(1/6)-81/2816*3^(3/4)*a^4*(b*x^2+
a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/
(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((
1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^
2)^(1/2)/b^4/x/(a/(b*x^2+a))^(1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a
))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {279, 321, 241, 236, 219}

$$-\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2}+\frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3}-\frac{81\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^4\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\right.}{2816b^4x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^(1/6),x]
```

```
[Out] (81*a^3*x*(a + b*x^2)^(1/6))/(2816*b^3) - (9*a^2*x^3*(a + b*x^2)^(1/6))/(70
4*b^2) + (3*a*x^5*(a + b*x^2)^(1/6))/(352*b) + (3*x^7*(a + b*x^2)^(1/6))/22
 - (81*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^4*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))
^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[
3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^
2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2816*b
^4*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
- (a/(a + b*x^2))^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^6\sqrt[6]{a+bx^2}\,dx = \frac{3}{22}x^7\sqrt[6]{a+bx^2}+\frac{1}{22}a\int\frac{x^6}{\left(a+bx^2\right)^{5/6}}\,dx$$

$$=\frac{3ax^5\sqrt[6]{a+bx^2}}{352b}+\frac{3}{22}x^7\sqrt[6]{a+bx^2}-\frac{\left(15a^2\right)\int\frac{x^4}{\left(a+bx^2\right)^{5/6}}\,dx}{352b}$$

$$=-\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2}+\frac{3ax^5\sqrt[6]{a+bx^2}}{352b}+\frac{3}{22}x^7\sqrt[6]{a+bx^2}+\frac{\left(27a^3\right)\int\frac{x^2}{\left(a+bx^2\right)^{5/6}}\,dx}{704b^2}$$

$$=\frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3}-\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2}+\frac{3ax^5\sqrt[6]{a+bx^2}}{352b}+\frac{3}{22}x^7\sqrt[6]{a+bx^2}-\frac{\left(81a^4\right)\int\frac{1}{\left(a+bx^2\right)^{5/6}}\,d}{2816b^3}$$

$$=\frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3}-\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2}+\frac{3ax^5\sqrt[6]{a+bx^2}}{352b}+\frac{3}{22}x^7\sqrt[6]{a+bx^2}-\frac{\left(81a^4\right)\text{Subst}\Big(\int\frac{}{(1-}}{2816b^3\sqrt[3]{\frac{}{a+}}}$$

$$=\frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3}-\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2}+\frac{3ax^5\sqrt[6]{a+bx^2}}{352b}+\frac{3}{22}x^7\sqrt[6]{a+bx^2}+\frac{\Big(243a^4\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{}}{}$$

$$=\frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3}-\frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2}+\frac{3ax^5\sqrt[6]{a+bx^2}}{352b}+\frac{3}{22}x^7\sqrt[6]{a+bx^2}-\frac{81\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^4}{}$$

Mathematica [C] time = 0.06, size = 105, normalized size = 0.30

$$\frac{3x\sqrt[6]{a+bx^2}\left(\sqrt[6]{\frac{bx^2}{a}+1}\left(27a^3-3a^2bx^2+2ab^2x^4+32b^3x^6\right)-27a^3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{704b^3\sqrt[6]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6)*((1 + (b*x^2)/a)^(1/6)*(27*a^3 - 3*a^2*b*x^2 + 2*a*b
^2*x^4 + 32*b^3*x^6) - 27*a^3*Hypergeometric2F1[-1/6, 1/2, 3/2, -((b*x^2)/a
)]))/(704*b^3*(1 + (b*x^2)/a)^(1/6))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{6}}x^6,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)*x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{1}{6}}x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)*x^6, x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int\left(b\,x^2+a\right)^{\frac{1}{6}}x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^(1/6),x)
```

```
[Out] int(x^6*(b*x^2+a)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{1}{6}}x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6)*x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\,\left(b\,x^2+a\right)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^(1/6),x)
```

```
[Out] int(x^6*(a + b*x^2)^(1/6), x)
```

sympy [A] time = 1.19, size = 29, normalized size = 0.08

$$\frac{\sqrt[6]{a}\,x^7\,{}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**(1/6),x)
```

```
[Out] a**(1/6)*x**7*hyper((-1/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

3.1011 $\int x^4 \sqrt[6]{a+bx^2}\,dx$

Optimal. Leaf size=321

$$\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{640b^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] -27/640*a^2*x*(b*x^2+a)^(1/6)/b^2+3/160*a*x^3*(b*x^2+a)^(1/6)/b+3/16*x^5*(b
*x^2+a)^(1/6)+27/640*3^(3/4)*a^3*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*El
lipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I
-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))
^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b^3/x/(a/(b*x^2+a))^(1/3)/
((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {279, 321, 241, 236, 219}

$$-\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\right|}{640b^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(1/6),x]
```

```
[Out] (-27*a^2*x*(a + b*x^2)^(1/6))/(640*b^2) + (3*a*x^3*(a + b*x^2)^(1/6))/(160*
b) + (3*x^5*(a + b*x^2)^(1/6))/16 + (27*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(a +
b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) +
(a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[A
rcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^
(1/3))], -7 + 4*Sqrt[3]])/(640*b^3*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(
a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
```

```
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\sqrt[6]{a+bx^2}\,dx = \frac{3}{16}x^5\sqrt[6]{a+bx^2}+\frac{1}{16}a\int\frac{x^4}{\left(a+bx^2\right)^{5/6}}\,dx$$

$$=\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}-\frac{\left(9a^2\right)\int\frac{x^2}{\left(a+bx^2\right)^{5/6}}\,dx}{160b}$$

$$=-\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}+\frac{\left(27a^3\right)\int\frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{640b^2}$$

$$=-\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}+\frac{\left(27a^3\right)\text{Subst}\left(\int\frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+b}}\right.}{640b^2\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$=-\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}-\frac{\left(81a^3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int\right.}{1280b^3x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$=-\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}+\frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+b}}{640b}$$

Mathematica [C] time = 0.05, size = 93, normalized size = 0.29

$$\frac{3x\sqrt[6]{a+bx^2}\left(\sqrt[6]{\frac{bx^2}{a}+1}\left(-9a^2+abx^2+10b^2x^4\right)+9a^2\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{160b^2\sqrt[6]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6)*((1 + (b*x^2)/a)^(1/6)*(-9*a^2 + a*b*x^2 + 10*b^2*x^
4) + 9*a^2*Hypergeometric2F1[-1/6, 1/2, 3/2, -((b*x^2)/a)]))/(160*b^2*(1 +
(b*x^2)/a)^(1/6))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{6}}x^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{6}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)*x^4, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{1}{6}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/6),x)
```

```
[Out] int(x^4*(b*x^2+a)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{6}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(b\,x^2+a\right)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(1/6),x)
```

```
[Out] int(x^4*(a + b*x^2)^(1/6), x)
```

sympy [A] time = 1.06, size = 29, normalized size = 0.09

$$\frac{\sqrt[6]{a}\,x^5{}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/6),x)
```

```
[Out] a**(1/6)*x**5*hyper((-1/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

3.1012 $\int x^2\sqrt[6]{a+bx^2}\,dx$

Optimal. Leaf size=297

$$-\frac{3\ 3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{40b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] 3/40*a*x*(b*x^2+a)^(1/6)/b+3/10*x^3*(b*x^2+a)^(1/6)-3/40*3^(3/4)*a^2*(b*x^2
+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))
/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*(
(1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))
^2)^(1/2)/b^2/x/(a/(b*x^2+a))^(1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+
a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {279, 321, 241, 236, 219}

$$-\frac{3\ 3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\ F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{40b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{3}{10}x$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(1/6),x]
```

```
[Out] (3*a*x*(a + b*x^2)^(1/6))/(40*b) + (3*x^3*(a + b*x^2)^(1/6))/10 - (3*3^(3/4
)*Sqrt[2 - Sqrt[3]]*a^2*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[
(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a +
b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1
 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(40*b^2*x*(a/(a + b*
x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2
))^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
```

```
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2\sqrt[6]{a+bx^2}\,dx = \frac{3}{10}x^3\sqrt[6]{a+bx^2} + \frac{1}{10}a\int\frac{x^2}{\left(a+bx^2\right)^{5/6}}\,dx$$

$$= \frac{3ax\sqrt[6]{a+bx^2}}{40b} + \frac{3}{10}x^3\sqrt[6]{a+bx^2} - \frac{\left(3a^2\right)\int\frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{40b}$$

$$= \frac{3ax\sqrt[6]{a+bx^2}}{40b} + \frac{3}{10}x^3\sqrt[6]{a+bx^2} - \frac{\left(3a^2\right)\text{Subst}\left(\int\frac{1}{\left(1-bx^2\right)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{40b\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= \frac{3ax\sqrt[6]{a+bx^2}}{40b} + \frac{3}{10}x^3\sqrt[6]{a+bx^2} + \frac{\left(9a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{80b^2x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= \frac{3ax\sqrt[6]{a+bx^2}}{40b} + \frac{3}{10}x^3\sqrt[6]{a+bx^2} - \frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\cdots}}{\left(1\cdots\right.}}}{40b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\cdots}}{\left(1-\sqrt{3}-\sqrt[3]{\cdots}\right.}}}$$

Mathematica [C] time = 0.05, size = 62, normalized size = 0.21

$$\frac{3x\sqrt[6]{a+bx^2}\left(-\frac{a\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[6]{\frac{bx^2}{a}+1}}+a+bx^2\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6)*(a + b*x^2 - (a*Hypergeometric2F1[-1/6, 1/2, 3/2, -(
(b*x^2)/a)])/(1 + (b*x^2)/a)^(1/6)))/(10*b)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^2+a\right)^{\frac{1}{6}}x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/6),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/6)*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{6}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/6),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/6)*x^2, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(b\,x^2+a\right)^{\frac{1}{6}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/6),x)
[Out] int(x^2*(b*x^2+a)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{6}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/6)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(b\,x^2+a\right)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(1/6),x)
[Out] int(x^2*(a + b*x^2)^(1/6), x)
```

sympy [A] time = 0.98, size = 29, normalized size = 0.10

$$\frac{\sqrt[6]{a}\,x^3{}_2F_1\left(\left.\begin{matrix}-\frac{1}{6},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/6),x)
[Out] a**(1/6)*x**3*hyper((-1/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

3.1013 $\int \sqrt[6]{a+bx^2}\,dx$

Optimal. Leaf size=273

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{4bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{3}{4}x\sqrt[6]{a}$$

```
[Out] 3/4*x*(b*x^2+a)^(1/6)+1/4*3^(3/4)*a*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))
*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),
2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+
a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b/x/(a/(b*x^2+a))^(1/3)
/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {195, 241, 236, 219}

$$\frac{3}{4}x\sqrt[6]{a+bx^2}+\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6))/4 + (3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a + b*x^2)^(1/6)*
(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2
))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sq
rt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 +
 4*Sqrt[3]])/(4*b*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3)
)/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
```

```
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt[6]{a+bx^2}\,dx &= \frac{3}{4}x\sqrt[6]{a+bx^2}+\frac{1}{4}a\int\frac{1}{\left(a+bx^2\right)^{5/6}}\,dx\\
&= \frac{3}{4}x\sqrt[6]{a+bx^2}+\frac{a\,\text{Subst}\left(\int\frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{4\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\\
&= \frac{3}{4}x\sqrt[6]{a+bx^2}-\frac{\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{8bx\sqrt[3]{\frac{a}{a+bx^2}}}\\
&= \frac{3}{4}x\sqrt[6]{a+bx^2}+\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,F\left(\sin\right.}{4bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\sqrt{-1+\frac{a}{a+bx^2}}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.17

$$\frac{x\sqrt[6]{a+bx^2}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[6]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6),x]
```

```
[Out] (x*(a + b*x^2)^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(1/6)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{1}{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \left(b\,x^{2}+a\right)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/6),x)
```

```
[Out] int((b*x^2+a)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2}+a\right)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6), x)
```

mupad [B] time = 4.88, size = 37, normalized size = 0.14

$$\frac{x\,\left(b\,x^2+a\right)^{1/6}\,{}_2\mathrm{F}_1\left(-\frac{1}{6},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{1/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/6),x)
```

```
[Out] (x*(a + b*x^2)^(1/6)*hypergeom([-1/6, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(1/6)
```

sympy [A] time = 0.89, size = 26, normalized size = 0.10

$$\sqrt[6]{a}\,x\,{}_{2}F_{1}\left(\left.\begin{matrix} -\frac{1}{6}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\right| \frac{bx^{2}e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/6),x)
```

```
[Out] a**(1/6)*x*hyper((-1/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

3.1014 $\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=266

$$\frac{\sqrt{2-\sqrt{3}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}\,x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{\sqrt[6]{a+bx^2}}{x}$$

```
[Out] -(b*x^2+a)^(1/6)/x+1/3*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1
-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2)
)*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1
-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/x/(a/(b*x^2+a))^(1/3)/((-1+(
a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {277, 241, 236, 219}

$$\frac{\sqrt{2-\sqrt{3}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\ F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{\sqrt[6]{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6)/x^2,x]
```

```
[Out] -((a + b*x^2)^(1/6)/x) + (Sqrt[2 - Sqrt[3]]*(a + b*x^2)^(1/6)*(1 - (a/(a +
b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1
- Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a
 + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/
(3^(1/4)*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sq
rt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
```

```
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx &= -\frac{\sqrt[6]{a+bx^2}}{x}+\frac{1}{3}b\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx\\
&= -\frac{\sqrt[6]{a+bx^2}}{x}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{3\sqrt[3]{\frac{a}{a+bx^2}}\,\sqrt[3]{a+bx^2}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{x}-\frac{\left(\sqrt{-\frac{bx^2}{a+bx^2}}\,\sqrt[6]{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{2x\sqrt[3]{\frac{a}{a+bx^2}}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{x}+\frac{\sqrt{2-\sqrt{3}}\,\sqrt{-\frac{bx^2}{a+bx^2}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}\cdots}{1-\sqrt{3}\cdots}\right.\right.}{\sqrt[4]{3}\,x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,\sqrt{-1+\frac{a}{a+bx^2}}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.18

$$
-\frac{\sqrt[6]{a+bx^2}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[6]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6)/x^2,x]
```

```
[Out] -(((a + b*x^2)^(1/6)*Hypergeometric2F1[-1/2, -1/6, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(1/6)))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{6}}}{x^2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2+a)^{\frac{1}{6}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^2, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^{\frac{1}{6}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/6)/x^2,x)
```

```
[Out] int((b*x^2+a)^(1/6)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2+a)^{\frac{1}{6}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^2, x)
```

mupad [B] time = 5.08, size = 40, normalized size = 0.15

$$-\frac{3\,\left(b\,x^2+a\right)^{1/6}\,{}_2\mathrm{F}_1\left(-\frac{1}{6},\frac{1}{3};\ \frac{4}{3};\ -\frac{a}{b\,x^2}\right)}{2\,x\,\left(\frac{a}{b\,x^2}+1\right)^{1/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/6)/x^2,x)
```

```
[Out] -(3*(a + b*x^2)^(1/6)*hypergeom([-1/6, 1/3], 4/3, -a/(b*x^2)))/(2*x*(a/(b*x
^2) + 1)^(1/6))
```

sympy [A] time = 0.98, size = 29, normalized size = 0.11

$$-\frac{\sqrt[6]{a}\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{6} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/6)/x**2,x)
```

```
[Out] -a**(1/6)*hyper((-1/2, -1/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

3.1015 $\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx$

Optimal. Leaf size=297

$$-\frac{2\sqrt{2-\sqrt{3}}\,b\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}\,ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{b\sqrt[6]{a+}}{9ax}$$

```
[Out] -1/3*(b*x^2+a)^(1/6)/x^3-1/9*b*(b*x^2+a)^(1/6)/a/x-2/27*b*(b*x^2+a)^(1/6)*(
1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x
^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^
2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3
^(3/4)/a/x/(a/(b*x^2+a))^(1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(
1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 325, 241, 236, 219}

$$-\frac{b\sqrt[6]{a+bx^2}}{9ax}-\frac{\sqrt[6]{a+bx^2}}{3x^3}-\frac{2\sqrt{2-\sqrt{3}}\,b\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle|-\right)}{9\sqrt[4]{3}\,ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6)/x^4,x]
```

```
[Out] -(a + b*x^2)^(1/6)/(3*x^3) - (b*(a + b*x^2)^(1/6))/(9*a*x) - (2*Sqrt[2 - Sq
rt[3]]*b*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*
x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^
2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a
/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a*x*(a/(a + b*x^2))^(1/3
)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^
2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx &= -\frac{\sqrt[6]{a+bx^2}}{3x^3}+\frac{1}{9}b\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx\\
&= -\frac{\sqrt[6]{a+bx^2}}{3x^3}-\frac{b\sqrt[6]{a+bx^2}}{9ax}-\frac{\left(2b^2\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{27a}\\
&= -\frac{\sqrt[6]{a+bx^2}}{3x^3}-\frac{b\sqrt[6]{a+bx^2}}{9ax}-\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{27a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{3x^3}-\frac{b\sqrt[6]{a+bx^2}}{9ax}+\frac{\left(b\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{9ax\sqrt[3]{\frac{a}{a+bx^2}}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{3x^3}-\frac{b\sqrt[6]{a+bx^2}}{9ax}-\frac{2\sqrt{2-\sqrt{3}}\,b\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+b\cdots}\right.}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx\cdots}}\right.}}}{9\sqrt[4]{3}\,ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.17

$$
-\frac{\sqrt[6]{a+bx^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{6};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[6]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6)/x^4,x]
```

```
[Out] -1/3*((a + b*x^2)^(1/6)*Hypergeometric2F1[-3/2, -1/6, -1/2, -((b*x^2)/a)])/
(x^3*(1 + (b*x^2)/a)^(1/6))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2 + a\right)^{\frac{1}{6}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^4,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/6)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{1}{6}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^4,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/6)/x^4, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2 + a\right)^{\frac{1}{6}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/6)/x^4,x)
[Out] int((b*x^2+a)^(1/6)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2 + a\right)^{\frac{1}{6}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^4,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/6)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2 + a\right)^{1/6}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/6)/x^4,x)
[Out] int((a + b*x^2)^(1/6)/x^4, x)
```

sympy [A] time = 1.11, size = 34, normalized size = 0.11

$$-\frac{\sqrt[6]{a}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, -\frac{1}{6} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/6)/x**4,x)
```

```
[Out] -a**(1/6)*hyper((-3/2, -1/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

3.1016 $\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx$

Optimal. Leaf size=323

$$\frac{16\sqrt{2-\sqrt{3}}\,b^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{135\sqrt[4]{3}\,a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{8b^2\sqrt[6]{a}}{135}$$

```
[Out] -1/5*(b*x^2+a)^(1/6)/x^5-1/45*b*(b*x^2+a)^(1/6)/a/x^3+8/135*b^2*(b*x^2+a)^(
1/6)/a^2/x+16/405*b^2*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-
(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))
*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-
(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/a^2/x/(a/(b*x^2+a))^(1/3)/((-
1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 325, 241, 236, 219}

$$\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x}+\frac{16\sqrt{2-\sqrt{3}}\,b^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{135\sqrt[4]{3}\,a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6)/x^6,x]
```

```
[Out] -(a + b*x^2)^(1/6)/(5*x^5) - (b*(a + b*x^2)^(1/6))/(45*a*x^3) + (8*b^2*(a +
 b*x^2)^(1/6))/(135*a^2*x) + (16*Sqrt[2 - Sqrt[3]]*b^2*(a + b*x^2)^(1/6)*(1
 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))
^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt
[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4
*Sqrt[3]])/(135*3^(1/4)*a^2*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x
^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx &= -\frac{\sqrt[6]{a+bx^2}}{5x^5}+\frac{1}{15}b\int \frac{1}{x^4\left(a+bx^2\right)^{5/6}}\,dx\\
&= -\frac{\sqrt[6]{a+bx^2}}{5x^5}-\frac{b\sqrt[6]{a+bx^2}}{45ax^3}-\frac{\left(8b^2\right)\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx}{135a}\\
&= -\frac{\sqrt[6]{a+bx^2}}{5x^5}-\frac{b\sqrt[6]{a+bx^2}}{45ax^3}+\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x}+\frac{\left(16b^3\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{405a^2}\\
&= -\frac{\sqrt[6]{a+bx^2}}{5x^5}-\frac{b\sqrt[6]{a+bx^2}}{45ax^3}+\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x}+\frac{\left(16b^3\right)\text{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{405a^2\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{5x^5}-\frac{b\sqrt[6]{a+bx^2}}{45ax^3}+\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x}-\frac{\left(8b^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx,\right.}{135a^2x\sqrt[3]{\frac{a}{a+bx^2}}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{5x^5}-\frac{b\sqrt[6]{a+bx^2}}{45ax^3}+\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x}+\frac{16\sqrt{2-\sqrt{3}}\,b^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right.}{135\sqrt[4]{3}\,a^2x\sqrt[3]{\frac{a}{a+bx^2}}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.16

$$
-\frac{\sqrt[6]{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{6};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt[6]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6)/x^6,x]
```

```
[Out] -1/5*((a + b*x^2)^(1/6)*Hypergeometric2F1[-5/2, -1/6, -3/2, -((b*x^2)/a)])/
(x^5*(1 + (b*x^2)/a)^(1/6))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{6}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{6}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^6, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{6}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/6)/x^6,x)
```

```
[Out] int((b*x^2+a)^(1/6)/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{6}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{1/6}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/6)/x^6,x)
```

```
[Out] int((a + b*x^2)^(1/6)/x^6, x)
```

sympy [A] time = 1.29, size = 34, normalized size = 0.11

$$-\frac{\sqrt[6]{a}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, -\frac{1}{6} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{5x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/6)/x**6,x)
```

```
[Out] -a**(1/6)*hyper((-5/2, -1/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

3.1017 $\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx$

Optimal. Leaf size=347

$$-\frac{32\sqrt{2-\sqrt{3}}\,b^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{405\sqrt[4]{3}\,a^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{16b^3}{4\ldots}$$

```
[Out] -1/7*(b*x^2+a)^(1/6)/x^7-1/105*b*(b*x^2+a)^(1/6)/a/x^5+2/135*b^2*(b*x^2+a)^
(1/6)/a^2/x^3-16/405*b^3*(b*x^2+a)^(1/6)/a^3/x-32/1215*b^3*(b*x^2+a)^(1/6)*
(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*
x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x
^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*
3^(3/4)/a^3/x/(a/(b*x^2+a))^(1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a)
)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 325, 241, 236, 219}

$$-\frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x}+\frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3}-\frac{32\sqrt{2-\sqrt{3}}\,b^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\ldots}{-\sqrt[3]{\frac{a}{bx^2+a}}-\ldots}\right.\right.}{405\sqrt[4]{3}\,a^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/6)/x^8,x]
```

```
[Out] -(a + b*x^2)^(1/6)/(7*x^7) - (b*(a + b*x^2)^(1/6))/(105*a*x^5) + (2*b^2*(a
+ b*x^2)^(1/6))/(135*a^2*x^3) - (16*b^3*(a + b*x^2)^(1/6))/(405*a^3*x) - (3
2*Sqrt[2 - Sqrt[3]]*b^3*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[
(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a +
b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1
 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(405*3^(1/4)*a^3*x*(
a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(
a + b*x^2))^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx &= -\frac{\sqrt[6]{a+bx^2}}{7x^7}+\frac{1}{21}b\int \frac{1}{x^6\left(a+bx^2\right)^{5/6}}\,dx\\
&= -\frac{\sqrt[6]{a+bx^2}}{7x^7}-\frac{b\sqrt[6]{a+bx^2}}{105ax^5}-\frac{\left(2b^2\right)\int \frac{1}{x^4\left(a+bx^2\right)^{5/6}}\,dx}{45a}\\
&= -\frac{\sqrt[6]{a+bx^2}}{7x^7}-\frac{b\sqrt[6]{a+bx^2}}{105ax^5}+\frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3}+\frac{\left(16b^3\right)\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx}{405a^2}\\
&= -\frac{\sqrt[6]{a+bx^2}}{7x^7}-\frac{b\sqrt[6]{a+bx^2}}{105ax^5}+\frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3}-\frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x}-\frac{\left(32b^4\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{1215a^3}\\
&= -\frac{\sqrt[6]{a+bx^2}}{7x^7}-\frac{b\sqrt[6]{a+bx^2}}{105ax^5}+\frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3}-\frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x}-\frac{\left(32b^4\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx \cdots\right.}{1215a^3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+\cdots}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{7x^7}-\frac{b\sqrt[6]{a+bx^2}}{105ax^5}+\frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3}-\frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x}+\frac{\left(16b^3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\mathrm{Su}\cdots}{405a^3\cdots}\\
&= -\frac{\sqrt[6]{a+bx^2}}{7x^7}-\frac{b\sqrt[6]{a+bx^2}}{105ax^5}+\frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3}-\frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x}-\frac{32\sqrt{2-\sqrt{3}}\,b^3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a\cdots}}{4\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.15

$$
-\frac{\sqrt[6]{a+bx^2}\,{}_2F_1\left(-\frac{7}{2},-\frac{1}{6};-\frac{5}{2};-\frac{bx^2}{a}\right)}{7x^7\sqrt[6]{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/6)/x^8,x]
```

```
[Out] -1/7*((a + b*x^2)^(1/6)*Hypergeometric2F1[-7/2, -1/6, -5/2, -((b*x^2)/a)])/
(x^7*(1 + (b*x^2)/a)^(1/6))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{6}}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^8,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{6}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^8, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{6}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/6)/x^8,x)
```

```
[Out] int((b*x^2+a)^(1/6)/x^8,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{6}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/6)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/6)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^{1/6}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/6)/x^8,x)
```

```
[Out] int((a + b*x^2)^(1/6)/x^8, x)
```

sympy [A] time = 1.46, size = 34, normalized size = 0.10

$$-\frac{\sqrt[6]{a}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{7}{2}, -\frac{1}{6} \\ -\frac{5}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{7x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/6)/x**8,x)
[Out] -a**(1/6)*hyper((-7/2, -1/6), (-5/2,), b*x**2*exp_polar(I*pi)/a)/(7*x**7)
```

3.1018 $\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=659

$$\frac{81\,3^{3/4}a^4\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{448\sqrt{2}\,b^4x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{243\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^4\left(\right.}{}$$

```
[Out] -243/896*a^3*x/b^3/(b*x^2+a)^(1/6)+81/448*a^2*x*(b*x^2+a)^(5/6)/b^3-9/56*a*
x^3*(b*x^2+a)^(5/6)/b^2+3/20*x^5*(b*x^2+a)^(5/6)/b-243/896*a^4*x/b^3/(a/(b*
x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))+81/896*3^(3/4
)*a^4*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-
(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x
^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b^4/x/(a/(b*x^2+a))^
(2/3)/(b*x^2+a)^(1/6)*2^(1/2)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1
/3)-3^(1/2))^2)^(1/2)-243/1792*3^(1/4)*a^4*(1-(a/(b*x^2+a))^(1/3))*Elliptic
E((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(
1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3
^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^4/x/(a/(b*x^2+a))^(2/3)/(b*x^2
+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2
)
```

Rubi [A] time = 0.69, antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {321, 238, 198, 235, 304, 219, 1879}

$$\frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3}-\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}}-\frac{243a^4x}{896b^3\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{81\,3^{3/4}a^4\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right.}{448\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(1/6),x]
```

```
[Out] (-243*a^3*x)/(896*b^3*(a + b*x^2)^(1/6)) + (81*a^2*x*(a + b*x^2)^(5/6))/(44
8*b^3) - (9*a*x^3*(a + b*x^2)^(5/6))/(56*b^2) + (3*x^5*(a + b*x^2)^(5/6))/(
20*b) - (243*a^4*x)/(896*b^3*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - S
qrt[3] - (a/(a + b*x^2))^(1/3))) - (243*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^4*(1 -
(a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2
/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3]
 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sq
rt[3]])/(1792*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/
(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (81*3^(3/4
)*a^4*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a +
 b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(
1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]
, -7 + 4*Sqrt[3]])/(448*Sqrt[2]*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/
6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
^2)])
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
```

```
2]], x] /; FreeQ[{a, b}, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx = \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} - \frac{(3a)\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx}{4b}$$

$$= -\frac{9ax^3\left(a+bx^2\right)^{5/6}}{56b^2} + \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} + \frac{\left(27a^2\right)\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx}{56b^2}$$

$$= \frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3} - \frac{9ax^3\left(a+bx^2\right)^{5/6}}{56b^2} + \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} - \frac{\left(81a^3\right)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{448b^3}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3} - \frac{9ax^3\left(a+bx^2\right)^{5/6}}{56b^2} + \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} + \frac{\left(81a^4\right)\int \frac{}{(a+b}}{896b^3}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3} - \frac{9ax^3\left(a+bx^2\right)^{5/6}}{56b^2} + \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} + \frac{\left(81a^4\right)\text{Subst}}{896b^3\,(}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3} - \frac{9ax^3\left(a+bx^2\right)^{5/6}}{56b^2} + \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} - \frac{\left(243a^4\sqrt{-\frac{b}{a+}}\right.}{17}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3} - \frac{9ax^3\left(a+bx^2\right)^{5/6}}{56b^2} + \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} + \frac{\left(243a^4\sqrt{-\frac{b}{a+}}\right.}{17}$$

$$= -\frac{243a^3x}{896b^3\sqrt[6]{a+bx^2}} + \frac{81a^2x\left(a+bx^2\right)^{5/6}}{448b^3} - \frac{9ax^3\left(a+bx^2\right)^{5/6}}{56b^2} + \frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} + \frac{243}{896b^4x\left(\frac{a}{a+bx}\right.}$$

Mathematica [C] time = 0.03, size = 90, normalized size = 0.14

$$\frac{3\left(-135a^3x\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+135a^3x+15a^2bx^3-8ab^2x^5+112b^3x^7\right)}{2240b^3\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(1/6),x]
```

```
[Out] (3*(135*a^3*x + 15*a^2*b*x^3 - 8*a*b^2*x^5 + 112*b^3*x^7 - 135*a^3*x*(1 + (
b*x^2)/a)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, -((b*x^2)/a)]))/(2240*b^3*
(a + b*x^2)^(1/6))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\left(bx^2+a\right)^{\frac{1}{6}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(x^6/(b*x^2 + a)^(1/6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/6),x, algorithm="giac")
[Out] integrate(x^6/(b*x^2 + a)^(1/6), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(b\,x^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(1/6),x)
[Out] int(x^6/(b*x^2+a)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(1/6),x, algorithm="maxima")
[Out] integrate(x^6/(b*x^2 + a)^(1/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(b\,x^2+a\right)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(1/6),x)
[Out] int(x^6/(a + b*x^2)^(1/6), x)
```

sympy [A] time = 1.11, size = 27, normalized size = 0.04

$$\frac{x^7\,{}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7\sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(1/6),x)
[Out] x**7*hyper((1/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(1/6))
```

3.1019 $\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=635

$$-\frac{27\,3^{3/4}a^3\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{112\sqrt{2}\,b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^3\left(\right.}{}$$

```
[Out] 81/224*a^2*x/b^2/(b*x^2+a)^(1/6)-27/112*a*x*(b*x^2+a)^(5/6)/b^2+3/14*x^3*(b
*x^2+a)^(5/6)/b+81/224*a^3*x/b^2/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/
(b*x^2+a))^(1/3)-3^(1/2))-27/224*3^(3/4)*a^3*(1-(a/(b*x^2+a))^(1/3))*Ellipt
icF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3
^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)
-3^(1/2))^2)^(1/2)/b^3/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)*2^(1/2)/((-1+(
a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)+81/448*3^(1/4)
*a^3*(1-(a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(
a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^
2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1
/2))/b^3/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1
-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {321, 238, 198, 235, 304, 219, 1879}

$$\frac{81a^3x}{224b^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}}-\frac{27\,3^{3/4}a^3\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}{112\sqrt{2}\,b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(1/6),x]
```

```
[Out] (81*a^2*x)/(224*b^2*(a + b*x^2)^(1/6)) - (27*a*x*(a + b*x^2)^(5/6))/(112*b^
2) + (3*x^3*(a + b*x^2)^(5/6))/(14*b) + (81*a^3*x)/(224*b^2*(a/(a + b*x^2))
^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (81*3^(1/
4)*Sqrt[2 + Sqrt[3]]*a^3*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^
2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]
*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(
a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(448*b^3*x*(a/(a + b*x^2))^(2/3)*(a +
 b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x
^2))^(1/3))^2)]) - (27*3^(3/4)*a^3*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a
/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))
^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt
[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(112*Sqrt[2]*b^3*x*(a/(a +
b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
```

```
2]], x] /; FreeQ[{a, b}, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx = \frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} - \frac{(9a)\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx}{14b}$$

$$= -\frac{27ax\left(a+bx^2\right)^{5/6}}{112b^2} + \frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} + \frac{\left(27a^2\right)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{112b^2}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax\left(a+bx^2\right)^{5/6}}{112b^2} + \frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} - \frac{\left(27a^3\right)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{224b^2}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax\left(a+bx^2\right)^{5/6}}{112b^2} + \frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} - \frac{\left(27a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx}}\right.}{224b^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax\left(a+bx^2\right)^{5/6}}{112b^2} + \frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} + \frac{\left(81a^3\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,d\right.}{448b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+b}}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax\left(a+bx^2\right)^{5/6}}{112b^2} + \frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} - \frac{\left(81a^3\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,d\right.}{448b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+b}}$$

$$= \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax\left(a+bx^2\right)^{5/6}}{112b^2} + \frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} - \frac{81a^3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx}}}{224b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}\right.}$$

Mathematica [C] time = 0.02, size = 79, normalized size = 0.12

$$\frac{3\left(9a^2x\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right) - 9a^2x - abx^3 + 8b^2x^5\right)}{112b^2\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(1/6),x]
```

```
[Out] (3*(-9*a^2*x - a*b*x^3 + 8*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(1/6)*Hypergeo
metric2F1[1/6, 1/2, 3/2, -((b*x^2)/a)]))/(112*b^2*(a + b*x^2)^(1/6))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\left(bx^2+a\right)^{\frac{1}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(x^4/(b*x^2 + a)^(1/6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/6),x, algorithm="giac")
[Out] integrate(x^4/(b*x^2 + a)^(1/6), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/6),x)
[Out] int(x^4/(b*x^2+a)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/6),x, algorithm="maxima")
[Out] integrate(x^4/(b*x^2 + a)^(1/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(1/6),x)
[Out] int(x^4/(a + b*x^2)^(1/6), x)
```

sympy [A] time = 0.96, size = 27, normalized size = 0.04

$$\frac{x^5 {}_2F_1\left(\begin{matrix} \frac{1}{6}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/6),x)
[Out] x**5*hyper((1/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/6))
```

3.1020 $\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=611

$$\frac{3\,3^{3/4}a^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{8\sqrt{2}\,b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^2\left(1-\right.}{3}$$

```
[Out] -9/16*a*x/b/(b*x^2+a)^(1/6)+3/8*x*(b*x^2+a)^(5/6)/b-9/16*a^2*x/b/(a/(b*x^2+
a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))+3/16*3^(3/4)*a^2*
(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*
x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))
^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b^2/x/(a/(b*x^2+a))^(2/3)/
(b*x^2+a)^(1/6)*2^(1/2)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^
(1/2))^2)^(1/2)-9/32*3^(1/4)*a^2*(1-(a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(b
*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+
(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)
^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^2/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/
((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 611, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {321, 238, 198, 235, 304, 219, 1879}

$$\frac{3\,3^{3/4}a^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\ F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{8\sqrt{2}\,b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right.}{32b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(1/6),x]
```

```
[Out] (-9*a*x)/(16*b*(a + b*x^2)^(1/6)) + (3*x*(a + b*x^2)^(5/6))/(8*b) - (9*a^2*
x)/(16*b*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x
^2))^(1/3))) - (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^2*(1 - (a/(a + b*x^2))^(1/3))
*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a
/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/
3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(32*b^2*x*(a/(
a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 -
 Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (3*3^(3/4)*a^2*(1 - (a/(a + b*x^2)
)^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt
[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x
^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(8*Sqr
t[2]*b^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^
2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx = \frac{3x\left(a+bx^2\right)^{5/6}}{8b} - \frac{(3a)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{8b}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x\left(a+bx^2\right)^{5/6}}{8b} + \frac{\left(3a^2\right)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{16b}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x\left(a+bx^2\right)^{5/6}}{8b} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{16b\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x\left(a+bx^2\right)^{5/6}}{8b} - \frac{\left(9a^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{32b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x\left(a+bx^2\right)^{5/6}}{8b} + \frac{\left(9a^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{32b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} - \frac{\left(9\sqrt{\frac{1}{2}\left(\right.\right.}{}$$

$$= -\frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x\left(a+bx^2\right)^{5/6}}{8b} + \frac{9a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{16b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)} - \frac{9\sqrt[4]{3}\sqrt{2}}{}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.10

$$\frac{3x\left(-a\sqrt[6]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{8b\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(1/6),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2,
-((b*x^2)/a)]))/(8*b*(a + b*x^2)^(1/6))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\left(bx^2+a\right)^{\frac{1}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*x^2 + a)^(1/6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/6),x, algorithm="giac")
[Out] integrate(x^2/(b*x^2 + a)^(1/6), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(b\,x^{2}+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/6),x)
[Out] int(x^2/(b*x^2+a)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(bx^{2}+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/6),x, algorithm="maxima")
[Out] integrate(x^2/(b*x^2 + a)^(1/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(b\,x^2+a\right)}^{1/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(1/6),x)
[Out] int(x^2/(a + b*x^2)^(1/6), x)
```

sympy [A] time = 0.84, size = 27, normalized size = 0.04

$$\frac{x^{3} {{}_{2}F_{1}\left(\left.\begin{matrix} \frac{1}{6}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\right| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{3 \sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/6),x)
[Out] x**3*hyper((1/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/6))
```

3.1021 $\int \frac{1}{\sqrt[6]{a+bx^2}}\, dx$

Optimal. Leaf size=577

$$-\frac{3^{3/4}a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt{2}\,bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{3x}{2\sqrt[6]{a+bx^2}}+\frac{}{2\left(\frac{a}{a+bx^2}\right)^{2/3}}$$

```
[Out] 3/2*x/(b*x^2+a)^(1/6)+3/2*a*x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*
x^2+a))^(1/3)-3^(1/2))-1/2*3^(3/4)*a*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(
a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*
((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)
)^2)^(1/2)/b/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)*2^(1/2)/((-1+(a/(b*x^2+a
))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)+3/4*3^(1/4)*a*(1-(a/(b*x
^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1
/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1
-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(a/(b*
x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1
/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {238, 198, 235, 304, 219, 1879}

$$\frac{3x}{2\sqrt[6]{a+bx^2}}+\frac{3ax}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{3^{3/4}a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\right.\right.}{\sqrt{2}\,bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-}{\left(-\sqrt[3]{}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1/6),x]
```

```
[Out] (3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*x)/(2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^
(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]
*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*
x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 +
 Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -
7 + 4*Sqrt[3]])/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 -
(a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (3^(3/4
)*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b
*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1
+ Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))],
-7 + 4*Sqrt[3]])/(Sqrt[2]*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[
-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \frac{3x}{2\sqrt[6]{a+bx^2}} - \frac{1}{2}a\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx$$

$$= \frac{3x}{2\sqrt[6]{a+bx^2}} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$

$$= \frac{3x}{2\sqrt[6]{a+bx^2}} + \frac{\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{3x}{2\sqrt[6]{a+bx^2}} - \frac{\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} + \frac{\left(3\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\,a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{S}}{2bx\left(\frac{a}{a+bx^2}\right)}$$

$$= \frac{3x}{2\sqrt[6]{a+bx^2}} - \frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)} + \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a\sqrt{-\frac{bx^2}{a+bx^2}}\left(1-\right.}{4bx\left(\frac{a}{a+bx^2}\right)}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.08

$$\frac{x\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1/6),x]
[Out] (x*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(1/6)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/6),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(-1/6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/6),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-1/6), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/6),x)
```

```
[Out] int(1/(b*x^2+a)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-1/6), x)
```

mupad [B] time = 4.88, size = 37, normalized size = 0.06

$$\frac{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{1/6}\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(b\,x^2+a\right)}^{1/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/6),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(1/6)*hypergeom([1/6, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(1/6)
```

sympy [A] time = 0.85, size = 24, normalized size = 0.04

$$\frac{x{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{6},\frac{1}{2}\\\frac{3}{2}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{\sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/6),x)
```

```
[Out] x*hyper((1/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/6)
```

3.1022 $\int \frac{1}{x^2 \sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=586

$$-\frac{\sqrt{2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}\,x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{bx}{a\sqrt[6]{a+bx^2}}+\frac{}{\left(\frac{a}{a+bx^2}\right)^{2/3}(a}$$

```
[Out] b*x/a/(b*x^2+a)^(1/6)-(b*x^2+a)^(5/6)/a/x+b*x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)
^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))-1/3*(1-(a/(b*x^2+a))^(1/3))*Elliptic
F((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(
1/2))*2^(1/2)*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))
^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1
+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)+1/2*3^(1/4)*
(1-(a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*
x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))
^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/
x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^
2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 586, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {325, 238, 198, 235, 304, 219, 1879}

$$\frac{bx}{a\sqrt[6]{a+bx^2}}+\frac{bx}{\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{\left(a+bx^2\right)^{5/6}}{ax}-\frac{\sqrt{2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}{\sqrt[4]{3}\,x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(1/6)),x]
```

```
[Out] (b*x)/(a*(a + b*x^2)^(1/6)) - (a + b*x^2)^(5/6)/(a*x) + (b*x)/((a/(a + b*x^
2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (3^(1/
4)*Sqrt[2 + Sqrt[3]]*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^
(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*Ell
ipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a +
b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1
/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3)
)^2)]) - (Sqrt[2]*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/
3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*Ellipt
icF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x
^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)
^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1
/3))^2)])
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx = -\frac{\left(a+bx^2\right)^{5/6}}{ax} + \frac{(2b)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{3a}$$
$$= \frac{bx}{a\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{ax} - \frac{1}{3}b\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx$$
$$= \frac{bx}{a\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{ax} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{3\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$
$$= \frac{bx}{a\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{ax} + \frac{\sqrt{-\frac{bx^2}{a+bx^2}}\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$
$$= \frac{bx}{a\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{ax} - \frac{\sqrt{-\frac{bx^2}{a+bx^2}}\,\mathrm{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} + \frac{\left(\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\,\sqrt{\right.}{}$$
$$= \frac{bx}{a\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{ax} - \frac{\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{-}}{}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.08

$$-\frac{\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(1/6)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-1/2, 1/6, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(1/6)))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}}{bx^4+ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b*x^4 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^2), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{6}} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/6),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{1}{6}} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^2), x)
```

mupad [B] time = 5.09, size = 40, normalized size = 0.07

$$-\frac{3\left(\frac{a}{b\,x^2}+1\right)^{1/6}\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{2}{3};\ \frac{5}{3};\ -\frac{a}{b\,x^2}\right)}{4\,x\,\left(b\,x^2+a\right)^{1/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(1/6)),x)
```

```
[Out] -(3*(a/(b*x^2) + 1)^(1/6)*hypergeom([1/6, 2/3], 5/3, -a/(b*x^2)))/(4*x*(a +
 b*x^2)^(1/6))
```

sympy [A] time = 0.97, size = 27, normalized size = 0.05

$$-\frac{{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{\sqrt[6]{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/6),x)
```

```
[Out] -hyper((-1/2, 1/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(1/6)*x)
```

3.1023 $\int \frac{1}{x^4 \sqrt[6]{a+bx^2}}\,dx$

Optimal. Leaf size=633

$$\frac{4\sqrt{2}\,b\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}\,ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}}+\frac{4b\left(a+b\right.}{9a^2x}$$

```
[Out] -4/9*b^2*x/a^2/(b*x^2+a)^(1/6)-1/3*(b*x^2+a)^(5/6)/a/x^3+4/9*b*(b*x^2+a)^(5
/6)/a^2/x-4/9*b^2*x/a/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^
(1/3)-3^(1/2))+4/27*b*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1
/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(a/
(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1
/2)*3^(3/4)/a/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3
))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)-2/9*b*(1-(a/(b*x^2+a))^(1/3))*E
llipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*
I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^
(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a/x/(a/(b*x^2+a))
^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(
1/2))^2)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 633, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {325, 238, 198, 235, 304, 219, 1879}

$$-\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}}+\frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x}-\frac{4b^2x}{9a\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}+\frac{4\sqrt{2}\,b\left(1-\sqrt[3]{\frac{a}{a+b}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(1/6)),x]
```

```
[Out] (-4*b^2*x)/(9*a^2*(a + b*x^2)^(1/6)) - (a + b*x^2)^(5/6)/(3*a*x^3) + (4*b*(
a + b*x^2)^(5/6))/(9*a^2*x) - (4*b^2*x)/(9*a*(a/(a + b*x^2))^(2/3)*(a + b*x
^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) - (2*Sqrt[2 + Sqrt[3]]*b*(
1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2)
)^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqr
t[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 +
4*Sqrt[3]])/(3*3^(3/4)*a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((
1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (4*
Sqrt[2]*b*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/
(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcS
in[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/
3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/
6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
^2)])
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
```

```
2]], x] /; FreeQ[{a, b}, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx &= -\frac{\left(a+bx^2\right)^{5/6}}{3ax^3} - \frac{(4b)\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx}{9a} \\
&= -\frac{\left(a+bx^2\right)^{5/6}}{3ax^3} + \frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x} - \frac{\left(8b^2\right)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{27a^2} \\
&= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{3ax^3} + \frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x} + \frac{\left(4b^2\right)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{27a} \\
&= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{3ax^3} + \frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x} + \frac{\left(4b^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{27a\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}} \\
&= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{3ax^3} + \frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x} - \frac{\left(2b\sqrt{-\frac{bx^2}{a+bx^2}}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\cdots}\right.}{9ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} \\
&= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{3ax^3} + \frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x} + \frac{\left(2b\sqrt{-\frac{bx^2}{a+bx^2}}\right)\mathrm{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\cdots}\right.}{9ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} \\
&= -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{3ax^3} + \frac{4b\left(a+bx^2\right)^{5/6}}{9a^2x} + \frac{4b\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}\cdots}{9ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+\cdots}}\right.\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.08

$$
-\frac{\sqrt[6]{\frac{bx^2}{a}+1}\ {}_2F_1\left(-\frac{3}{2},\frac{1}{6};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt[6]{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(1/6)),x]
```

```
[Out] -1/3*((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-3/2, 1/6, -1/2, -((b*x^2)/a)
])/(x^3*(a + b*x^2)^(1/6))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}}{bx^6+ax^4}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b*x^6 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}x^4}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/6),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^4), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{6}} x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/6),x)
[Out] int(1/x^4/(b*x^2+a)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{1}{6}} x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/6),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{4}\left(b\,x^{2}+a\right)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(1/6)),x)
[Out] int(1/(x^4*(a + b*x^2)^(1/6)), x)
```

sympy [A] time = 1.12, size = 32, normalized size = 0.05

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{1}{6} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a} \right)}{3\sqrt[6]{a}\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/6),x)
[Out] -hyper((-3/2, 1/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/6)*x**3)
```

3.1024 $\int \frac{1}{x^6 \sqrt[6]{a+bx^2}} \, dx$

Optimal. Leaf size=661

$$-\frac{8\sqrt{2}\, b^2 \left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\, \text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right), 4\sqrt{3}-7\right)}{27\sqrt[4]{3}\, a^2 x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{8b^3 x}{27a^3 \sqrt[6]{a+bx^2}}-\frac{8b^2 (}{2}$$

[Out] 8/27*b^3*x/a^3/(b*x^2+a)^(1/6)-1/5*(b*x^2+a)^(5/6)/a/x^5+2/9*b*(b*x^2+a)^(5/6)/a^2/x^3-8/27*b^2*(b*x^2+a)^(5/6)/a^3/x+8/27*b^3*x/a^2/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))-8/81*b^2*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/a^2/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)+4/27*b^2*(1-(a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^2/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)

Rubi [A] time = 0.67, antiderivative size = 661, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {325, 238, 198, 235, 304, 219, 1879}

$$\frac{8b^3 x}{27a^3 \sqrt[6]{a+bx^2}}+\frac{8b^3 x}{27a^2 \left(\frac{a}{a+bx^2}\right)^{2/3} \left(a+bx^2\right)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{8b^2 \left(a+bx^2\right)^{5/6}}{27a^3 x}-\frac{8\sqrt{2}\, b^2 \left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+b}\right.}{\left(-\right.}}}{27\sqrt[4]{3}\, a^2 x \left(\frac{}{a}\right.}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(a + b*x^2)^(1/6)),x]

[Out] (8*b^3*x)/(27*a^3*(a + b*x^2)^(1/6)) - (a + b*x^2)^(5/6)/(5*a*x^5) + (2*b*(a + b*x^2)^(5/6))/(9*a^2*x^3) - (8*b^2*(a + b*x^2)^(5/6))/(27*a^3*x) + (8*b^3*x)/(27*a^2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (4*Sqrt[2 + Sqrt[3]]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(3/4)*a^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (8*Sqrt[2]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(27*3^(1/4)*a^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
```

```
2]], x] /; FreeQ[{a, b}, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[6]{a+bx^2}}\,dx = -\frac{\left(a+bx^2\right)^{5/6}}{5ax^5} - \frac{(2b)\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx}{3a}$$

$$= -\frac{\left(a+bx^2\right)^{5/6}}{5ax^5} + \frac{2b\left(a+bx^2\right)^{5/6}}{9a^2x^3} + \frac{\left(8b^2\right)\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx}{27a^2}$$

$$= -\frac{\left(a+bx^2\right)^{5/6}}{5ax^5} + \frac{2b\left(a+bx^2\right)^{5/6}}{9a^2x^3} - \frac{8b^2\left(a+bx^2\right)^{5/6}}{27a^3x} + \frac{\left(16b^3\right)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{81a^3}$$

$$= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{5ax^5} + \frac{2b\left(a+bx^2\right)^{5/6}}{9a^2x^3} - \frac{8b^2\left(a+bx^2\right)^{5/6}}{27a^3x} - \frac{\left(8b^3\right)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{81a^2}$$

$$= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{5ax^5} + \frac{2b\left(a+bx^2\right)^{5/6}}{9a^2x^3} - \frac{8b^2\left(a+bx^2\right)^{5/6}}{27a^3x} - \frac{\left(8b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\right.}{81a^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a\right.}$$

$$= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{5ax^5} + \frac{2b\left(a+bx^2\right)^{5/6}}{9a^2x^3} - \frac{8b^2\left(a+bx^2\right)^{5/6}}{27a^3x} + \frac{\left(4b^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}}{27a^2x\left(\frac{a}{a+bx\right.}}$$

$$= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{5ax^5} + \frac{2b\left(a+bx^2\right)^{5/6}}{9a^2x^3} - \frac{8b^2\left(a+bx^2\right)^{5/6}}{27a^3x} - \frac{\left(4b^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}}{27a^2x\left(\frac{a}{a+bx\right.}}$$

$$= \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{\left(a+bx^2\right)^{5/6}}{5ax^5} + \frac{2b\left(a+bx^2\right)^{5/6}}{9a^2x^3} - \frac{8b^2\left(a+bx^2\right)^{5/6}}{27a^3x} - \frac{8b^2\sqrt{-\frac{bx^2}{a+bx^2}}}{27a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.08

$$-\frac{\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{6};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(1/6)),x]
```

```
[Out] -1/5*((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-5/2, 1/6, -3/2, -((b*x^2)/a)
])/(x^5*(a + b*x^2)^(1/6))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}}{bx^8+ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b*x^8 + a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/6),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^6), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{6}}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(1/6),x)
[Out] int(1/x^6/(b*x^2+a)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{6}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(1/6),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/6)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\,\left(b\,x^2+a\right)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^(1/6)),x)
[Out] int(1/(x^6*(a + b*x^2)^(1/6)), x)
```

sympy [A] time = 1.30, size = 32, normalized size = 0.05

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{5}{2}, \frac{1}{6}\\ -\frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[6]{a}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(1/6),x)
[Out] -hyper((-5/2, 1/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/6)*x**5)
```

3.1025 $\int \frac{x^6}{\left(a+bx^2\right)^{5/6}}\,dx$

Optimal. Leaf size=324

$$-\frac{81\, 3^{3/4}\sqrt{2-\sqrt{3}}\, a^3 \sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{128 b^4 x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+$$

```
[Out] 81/128*a^2*x*(b*x^2+a)^(1/6)/b^3-9/32*a*x^3*(b*x^2+a)^(1/6)/b^2+3/16*x^5*(b
*x^2+a)^(1/6)/b-81/128*3^(3/4)*a^3*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*
EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2
*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a
))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b^4/x/(a/(b*x^2+a))^(1/3
)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {321, 241, 236, 219}

$$\frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3}-\frac{81\, 3^{3/4}\sqrt{2-\sqrt{3}}\, a^3 \sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle|-7\right.}{128 b^4 x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(5/6),x]
```

```
[Out] (81*a^2*x*(a + b*x^2)^(1/6))/(128*b^3) - (9*a*x^3*(a + b*x^2)^(1/6))/(32*b^
2) + (3*x^5*(a + b*x^2)^(1/6))/(16*b) - (81*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^3*(
a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3
) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*Ellipti
cF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^
2))^(1/3))], -7 + 4*Sqrt[3]])/(128*b^4*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 -
(a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\left(a+bx^2\right)^{5/6}}\,dx &= \frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{(15a)\int \frac{x^4}{\left(a+bx^2\right)^{5/6}}\,dx}{16b}\\
&= -\frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2}+\frac{3x^5\sqrt[6]{a+bx^2}}{16b}+\frac{\left(27a^2\right)\int \frac{x^2}{\left(a+bx^2\right)^{5/6}}\,dx}{32b^2}\\
&= \frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3}-\frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2}+\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{\left(81a^3\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{128b^3}\\
&= \frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3}-\frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2}+\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{\left(81a^3\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+b}}\right.}{128b^3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\\
&= \frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3}-\frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2}+\frac{3x^5\sqrt[6]{a+bx^2}}{16b}+\frac{\left(243a^3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\mathrm{Subst}\left(\right.}{256b^4x\sqrt[3]{\frac{a}{a+bx}}}\\
&= \frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3}-\frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2}+\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{81\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+b}}{128}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 89, normalized size = 0.27

$$\frac{3x\left(-27a^3\left(\frac{bx^2}{a}+1\right)^{5/6}{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};-\frac{bx^2}{a}\right)+27a^3+15a^2bx^2-4ab^2x^4+8b^3x^6\right)}{128b^3\left(a+bx^2\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(5/6),x]
```

```
[Out] (3*x*(27*a^3 + 15*a^2*b*x^2 - 4*a*b^2*x^4 + 8*b^3*x^6 - 27*a^3*(1 + (b*x^2)
/a)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, -((b*x^2)/a)]))/(128*b^3*(a + b*
x^2)^(5/6))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\left(bx^2+a\right)^{\frac{5}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/6),x, algorithm="fricas")
[Out] integral(x^6/(b*x^2 + a)^(5/6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/6),x, algorithm="giac")
[Out] integrate(x^6/(b*x^2 + a)^(5/6), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(b\,x^2+a\right)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(5/6),x)
[Out] int(x^6/(b*x^2+a)^(5/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/6),x, algorithm="maxima")
[Out] integrate(x^6/(b*x^2 + a)^(5/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(b\,x^2+a\right)^{5/6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(5/6),x)
[Out] int(x^6/(a + b*x^2)^(5/6), x)
```

sympy [A] time = 0.99, size = 27, normalized size = 0.08

$$\frac{x^7 {}_2F_1\left(\begin{matrix} \frac{5}{6}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{7a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(5/6),x)
```

```
[Out] x**7*hyper((5/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(5/6))
```

3.1026 $\int \frac{x^4}{(a+bx^2)^{5/6}} dx$

Optimal. Leaf size=300

$$\frac{27\, 3^{3/4}\sqrt{2-\sqrt{3}}\, a^2 \sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{40b^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{27}{}$$

```
[Out] -27/40*a*x*(b*x^2+a)^(1/6)/b^2+3/10*x^3*(b*x^2+a)^(1/6)/b+27/40*3^(3/4)*a^2
*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3
^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(
1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3
^(1/2))^2)^(1/2)/b^3/x/(a/(b*x^2+a))^(1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/
(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {321, 241, 236, 219}

$$\frac{27\, 3^{3/4}\sqrt{2-\sqrt{3}}\, a^2 \sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\ F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{40b^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{27ax\sqrt[6]{}}{40}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(5/6),x]
```

```
[Out] (-27*a*x*(a + b*x^2)^(1/6))/(40*b^2) + (3*x^3*(a + b*x^2)^(1/6))/(10*b) + (
27*3^(3/4)*Sqrt[2 - Sqrt[3]]*a^2*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/
3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] -
 (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^
(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(40*b^3*x*(
a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(
a + b*x^2))^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{5/6}}\,dx = \frac{3x^3\sqrt[6]{a+bx^2}}{10b} - \frac{(9a)\int \frac{x^2}{\left(a+bx^2\right)^{5/6}}\,dx}{10b}$$

$$= -\frac{27ax\sqrt[6]{a+bx^2}}{40b^2} + \frac{3x^3\sqrt[6]{a+bx^2}}{10b} + \frac{\left(27a^2\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{40b^2}$$

$$= -\frac{27ax\sqrt[6]{a+bx^2}}{40b^2} + \frac{3x^3\sqrt[6]{a+bx^2}}{10b} + \frac{\left(27a^2\right)\text{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx, x, \frac{x}{\sqrt[6]{a+bx^2}}\right)}{40b^2\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= -\frac{27ax\sqrt[6]{a+bx^2}}{40b^2} + \frac{3x^3\sqrt[6]{a+bx^2}}{10b} - \frac{\left(81a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a\ldots}}\right.}{80b^3x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= -\frac{27ax\sqrt[6]{a+bx^2}}{40b^2} + \frac{3x^3\sqrt[6]{a+bx^2}}{10b} + \frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\ldots}{40b^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{\ldots}{(1-\ldots}}}$$

Mathematica [C] time = 0.02, size = 79, normalized size = 0.26

$$\frac{3\left(9a^2x\left(\frac{bx^2}{a}+1\right)^{5/6}{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};-\frac{bx^2}{a}\right)-9a^2x-5abx^3+4b^2x^5\right)}{40b^2\left(a+bx^2\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(5/6),x]
```

```
[Out] (3*(-9*a^2*x - 5*a*b*x^3 + 4*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(5/6)*Hyperg
eometric2F1[1/2, 5/6, 3/2, -((b*x^2)/a)]))/(40*b^2*(a + b*x^2)^(5/6))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\left(bx^2+a\right)^{\frac{5}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(x^4/(b*x^2 + a)^(5/6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(5/6), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(5/6),x)
```

```
[Out] int(x^4/(b*x^2+a)^(5/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(5/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(b\,x^2+a\right)}^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(5/6),x)
```

```
[Out] int(x^4/(a + b*x^2)^(5/6), x)
```

sympy [A] time = 0.92, size = 27, normalized size = 0.09

$$\frac{x^5\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{6},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(5/6),x)
```

```
[Out] x**5*hyper((5/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/6))
```

3.1027 $\int \frac{x^2}{\left(a+bx^2\right)^{5/6}}\,dx$

Optimal. Leaf size=276

$$\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right)\right.}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] 3/4*x*(b*x^2+a)^(1/6)/b-3/4*3^(3/4)*a*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3
))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)
),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^
2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b^2/x/(a/(b*x^2+a))^(
1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {321, 241, 236, 219}

$$\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle|-7+\right.}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(5/6),x]
```

```
[Out] (3*x*(a + b*x^2)^(1/6))/(4*b) - (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a + b*x^2)^
(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a +
 b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(
1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]
, -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2
))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
```

```
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{5/6}}\,dx = \frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{(3a)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{4b}$$

$$=\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{4b\sqrt[3]{\frac{a}{a+bx^2}}\,\sqrt[3]{a+bx^2}}$$

$$=\frac{3x\sqrt[6]{a+bx^2}}{4b}+\frac{\left(9a\sqrt{-\frac{bx^2}{a+bx^2}}\,\sqrt[6]{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{8b^2x\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$=\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt{-\frac{bx^2}{a+bx^2}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,F\left(\mathrm{si}\right.}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,\sqrt{-1+\frac{a}{a+bx^2}}}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.22

$$\frac{3x\left(-a\left(\frac{bx^2}{a}+1\right)^{5/6}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{4b\left(a+bx^2\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(5/6),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2,
-((b*x^2)/a)]))/(4*b*(a + b*x^2)^(5/6))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^2}{\left(bx^2+a\right)^{\frac{5}{6}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*x^2 + a)^(5/6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^2 + a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/6),x, algorithm="giac")
[Out] integrate(x^2/(b*x^2 + a)^(5/6), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^2 + a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(5/6),x)
[Out] int(x^2/(b*x^2+a)^(5/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^2 + a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/6),x, algorithm="maxima")
[Out] integrate(x^2/(b*x^2 + a)^(5/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(b\,x^2 + a)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(5/6),x)
[Out] int(x^2/(a + b*x^2)^(5/6), x)
```

sympy [A] time = 0.94, size = 27, normalized size = 0.10

$$\frac{x^3\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{6}, \frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(5/6),x)
[Out] x**3*hyper((5/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/6))
```

3.1028 $\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx$

Optimal. Leaf size=252

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] 3^(3/4)*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^
(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-
1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^
(1/3)-3^(1/2))^2)^(1/2)/b/x/(a/(b*x^2+a))^(1/3)/((-1+(a/(b*x^2+a))^(1/3))/(
1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {241, 236, 219}

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-5/6),x]
```

```
[Out] (3^(3/4)*Sqrt[2 - Sqrt[3]]*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sq
rt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a
 + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a/(a + b*x^
2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))
^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
```

```
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$= -\frac{\left(3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{2bx\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$= \frac{3^{3/4}\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}}{1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}}\right)\right.}{bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\sqrt{-1+\frac{a}{a+bx^2}}}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.18

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{5/6} {}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(a+bx^2\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-5/6),x]
[Out] (x*(1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, -((b*x^2)/a)])/(a
 + b*x^2)^(5/6)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/6),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(-5/6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/6),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-5/6), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/6),x)
```

```
[Out] int(1/(b*x^2+a)^(5/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(-5/6), x)
```

mupad [B] time = 4.91, size = 37, normalized size = 0.15

$$\frac{x\left(\frac{b\,x^2}{a}+1\right)^{5/6}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{6};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{5/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(5/6),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(5/6)*hypergeom([1/2, 5/6], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(5/6)
```

sympy [A] time = 0.87, size = 24, normalized size = 0.10

$$\frac{x{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{6}\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/6),x)
```

```
[Out] x*hyper((1/2, 5/6), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/6)
```

3.1029 $\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx$

Optimal. Leaf size=273

$$-\frac{2\sqrt{2-\sqrt{3}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}\,ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{\sqrt[6]{a+}}{a}$$

```
[Out] -(b*x^2+a)^(1/6)/a/x-2/3*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF(
(1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/
2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/
(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/a/x/(a/(b*x^2+a))^(1/3)/((
-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 241, 236, 219}

$$-\frac{\sqrt[6]{a+bx^2}}{ax}-\frac{2\sqrt{2-\sqrt{3}}\,\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(5/6)),x]
```

```
[Out] -((a + b*x^2)^(1/6)/(a*x)) - (2*Sqrt[2 - Sqrt[3]]*(a + b*x^2)^(1/6)*(1 - (a
/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3
))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
 (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt
[3]])/(3^(1/4)*a*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
```

```
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx &= -\frac{\sqrt[6]{a+bx^2}}{ax}-\frac{(2b)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{3a}\\
&= -\frac{\sqrt[6]{a+bx^2}}{ax}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{3a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{ax}+\frac{\left(\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right)}{ax\sqrt[3]{\frac{a}{a+bx^2}}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{ax}-\frac{2\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\frac{a}{a+bx^2}}+\left(\frac{a}{a+bx^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\,F\left(\sin^{-1}\right.}{\sqrt[4]{3}\,ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)^2}}\sqrt{-1+\frac{a}{a+bx^2}}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.18

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{5/6}\,{}_2F_1\left(-\frac{1}{2},\frac{5}{6};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(a+bx^2\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(5/6)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[-1/2, 5/6, 1/2, -((b*x^2)/a)])/(
x*(a + b*x^2)^(5/6)))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{6}}}{bx^4+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(1/6)/(b*x^4 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^2), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{5}{6}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(5/6),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(5/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^2), x)
```

mupad [B] time = 5.10, size = 40, normalized size = 0.15

$$-\frac{3\,{\left(\frac{a}{b\,x^2}+1\right)}^{5/6}\,{}_2\mathrm{F}_1\left(\frac{5}{6},\frac{4}{3};\ \frac{7}{3};\ -\frac{a}{b\,x^2}\right)}{8\,x\,{\left(b\,x^2+a\right)}^{5/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(5/6)),x)
```

```
[Out] -(3*(a/(b*x^2) + 1)^(5/6)*hypergeom([5/6, 4/3], 7/3, -a/(b*x^2)))/(8*x*(a +
 b*x^2)^(5/6))
```

sympy [A] time = 1.15, size = 27, normalized size = 0.10

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{6} \\ \frac{1}{2} \end{matrix}\middle| {\frac{bx^2e^{i\pi}}{a}}\right)}{a^{\frac{5}{6}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(5/6),x)
```

```
[Out] -hyper((-1/2, 5/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(5/6)*x)
```

3.1030 $\int \frac{1}{x^4(a+bx^2)^{5/6}}\,dx$

Optimal. Leaf size=300

$$\frac{16\sqrt{2-\sqrt{3}}\,b\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}\,a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{8b\sqrt[6]{a+}}{9a^2}$$

```
[Out] -1/3*(b*x^2+a)^(1/6)/a/x^3+8/9*b*(b*x^2+a)^(1/6)/a^2/x+16/27*b*(b*x^2+a)^(1
/6)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a
/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/
(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1
/2)*3^(3/4)/a^2/x/(a/(b*x^2+a))^(1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^
2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 241, 236, 219}

$$\frac{8b\sqrt[6]{a+bx^2}}{9a^2x}+\frac{16\sqrt{2-\sqrt{3}}\,b\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(5/6)),x]
```

```
[Out] -(a + b*x^2)^(1/6)/(3*a*x^3) + (8*b*(a + b*x^2)^(1/6))/(9*a^2*x) + (16*Sqrt
[2 - Sqrt[3]]*b*(a + b*x^2)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/
(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^
(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[
3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*a^2*x*(a/(a + b*x
^2))^(1/3)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2)
)^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^2\right)^{5/6}}\,dx &= -\frac{\sqrt[6]{a+bx^2}}{3ax^3}-\frac{(8b)\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx}{9a}\\
&= -\frac{\sqrt[6]{a+bx^2}}{3ax^3}+\frac{8b\sqrt[6]{a+bx^2}}{9a^2x}+\frac{\left(16b^2\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{27a^2}\\
&= -\frac{\sqrt[6]{a+bx^2}}{3ax^3}+\frac{8b\sqrt[6]{a+bx^2}}{9a^2x}+\frac{\left(16b^2\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{27a^2\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{3ax^3}+\frac{8b\sqrt[6]{a+bx^2}}{9a^2x}-\frac{\left(8b\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{\frac{a}{a+bx^2}}\right.}{9a^2x\sqrt[3]{\frac{a}{a+bx^2}}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{3ax^3}+\frac{8b\sqrt[6]{a+bx^2}}{9a^2x}+\frac{16\sqrt{2-\sqrt{3}}\,b\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\cdots}}{\left(1-\sqrt{\cdots}\right.}}}{9\sqrt[4]{3}\,a^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\cdots}}{\left(1-\sqrt{3}-\cdots\right.}}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.17

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{5/6}{}_2F_1\left(-\frac{3}{2},\frac{5}{6};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(a+bx^2\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(5/6)),x]
```

```
[Out] -1/3*((1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[-3/2, 5/6, -1/2, -((b*x^2)/a)
])/(x^3*(a + b*x^2)^(5/6))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{6}}}{bx^6+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/6),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/6)/(b*x^6 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/6),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^4), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{5}{6}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(5/6),x)
[Out] int(1/x^4/(b*x^2+a)^(5/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/6),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(5/6)),x)
[Out] int(1/(x^4*(a + b*x^2)^(5/6)), x)
```

sympy [A] time = 1.46, size = 32, normalized size = 0.11

$$-\frac{{}_{2}F_{1}\left(\left.\begin{matrix} -\frac{3}{2}, \frac{5}{6} \\ -\frac{1}{2} \end{matrix}\right| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{6}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(5/6),x)
[Out] -hyper((-3/2, 5/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/6)*x**3)
```

3.1031 $\int \frac{1}{x^6\left(a+bx^2\right)^{5/6}}\,dx$

Optimal. Leaf size=326

$$-\frac{224\sqrt{2-\sqrt{3}}\,b^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{135\sqrt[4]{3}\,a^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-$$

```
[Out] -1/5*(b*x^2+a)^(1/6)/a/x^5+14/45*b*(b*x^2+a)^(1/6)/a^2/x^3-112/135*b^2*(b*x
^2+a)^(1/6)/a^3/x-224/405*b^2*(b*x^2+a)^(1/6)*(1-(a/(b*x^2+a))^(1/3))*Ellip
ticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*
3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2
/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/a^3/x/(a/(b*x^2+a))^(
1/3)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 241, 236, 219}

$$-\frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x}-\frac{224\sqrt{2-\sqrt{3}}\,b^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle|-7\right.}{135\sqrt[4]{3}\,a^3x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(5/6)),x]
```

```
[Out] -(a + b*x^2)^(1/6)/(5*a*x^5) + (14*b*(a + b*x^2)^(1/6))/(45*a^2*x^3) - (112
*b^2*(a + b*x^2)^(1/6))/(135*a^3*x) - (224*Sqrt[2 - Sqrt[3]]*b^2*(a + b*x^2
)^(1/6)*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a
 + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3)
)], -7 + 4*Sqrt[3]])/(135*3^(1/4)*a^3*x*(a/(a + b*x^2))^(1/3)*Sqrt[-((1 - (
a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/
(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] &
& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(a+bx^2\right)^{5/6}}\,dx &= -\frac{\sqrt[6]{a+bx^2}}{5ax^5}-\frac{(14b)\int \frac{1}{x^4\left(a+bx^2\right)^{5/6}}\,dx}{15a}\\
&= -\frac{\sqrt[6]{a+bx^2}}{5ax^5}+\frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3}+\frac{\left(112b^2\right)\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx}{135a^2}\\
&= -\frac{\sqrt[6]{a+bx^2}}{5ax^5}+\frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3}-\frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x}-\frac{\left(224b^3\right)\int \frac{1}{\left(a+bx^2\right)^{5/6}}\,dx}{405a^3}\\
&= -\frac{\sqrt[6]{a+bx^2}}{5ax^5}+\frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3}-\frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x}-\frac{\left(224b^3\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-bx^2\right)^{2/3}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right.}{405a^3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{5ax^5}+\frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3}-\frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x}+\frac{\left(112b^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{}{\sqrt{}}\right.}{135a^3x\sqrt[3]{\frac{a}{a+bx^2}}}\\
&= -\frac{\sqrt[6]{a+bx^2}}{5ax^5}+\frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3}-\frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x}-\frac{224\sqrt{2-\sqrt{3}}\,b^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\right.}{135\sqrt[4]{3}\,a^3x}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.16

$$
-\frac{\left(\frac{bx^2}{a}+1\right)^{5/6}{}_2F_1\left(-\frac{5}{2},\frac{5}{6};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\left(a+bx^2\right)^{5/6}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(5/6)),x]
```

```
[Out] -1/5*((1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[-5/2, 5/6, -3/2, -((b*x^2)/a)
])/(x^5*(a + b*x^2)^(5/6))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{1}{6}}}{bx^8+ax^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/6),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(1/6)/(b*x^8 + a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/6),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^6), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{5}{6}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(5/6),x)
[Out] int(1/x^6/(b*x^2+a)^(5/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{6}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(5/6),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/6)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\,{\left(b\,x^2+a\right)}^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^(5/6)),x)
[Out] int(1/(x^6*(a + b*x^2)^(5/6)), x)
```

sympy [A] time = 1.49, size = 32, normalized size = 0.10

$$-\frac{{}_{2}F_{1}\left(\begin{matrix}-\frac{5}{2},\frac{5}{6}\\-\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{6}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(5/6),x)
```

```
[Out] -hyper((-5/2, 5/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/6)*x**5)
```

3.1032 $\int \frac{x^6}{\left(a+bx^2\right)^{7/6}}\,dx$

Optimal. Leaf size=654

$$-\frac{405\, 3^{3/4} a^3 \left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{112\sqrt{2}\, b^4 x \left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{1215\sqrt[4]{3}\sqrt{2+\cdots}}{\cdots}$$

```
[Out] 1215/224*a^2*x/b^3/(b*x^2+a)^(1/6)-3*x^5/b/(b*x^2+a)^(1/6)-405/112*a*x*(b*x
^2+a)^(5/6)/b^3+45/14*x^3*(b*x^2+a)^(5/6)/b^2+1215/224*a^3*x/b^3/(a/(b*x^2+
a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))-405/224*3^(3/4)*a
^3*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/
(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+
a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b^4/x/(a/(b*x^2+a))^(2/
3)/(b*x^2+a)^(1/6)*2^(1/2)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)
-3^(1/2))^2)^(1/2)+1215/448*3^(1/4)*a^3*(1-(a/(b*x^2+a))^(1/3))*EllipticE((
1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2
))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1
/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^4/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)
^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 654, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {288, 321, 238, 198, 235, 304, 219, 1879}

$$\frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}}+\frac{1215a^3x}{224b^3\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{405\, 3^{3/4} a^3\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\cdots}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{\cdots}\right)}}\cdots}{112\sqrt{2}\, b^4 x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(7/6),x]
```

```
[Out] (1215*a^2*x)/(224*b^3*(a + b*x^2)^(1/6)) - (3*x^5)/(b*(a + b*x^2)^(1/6)) -
(405*a*x*(a + b*x^2)^(5/6))/(112*b^3) + (45*x^3*(a + b*x^2)^(5/6))/(14*b^2)
 + (1215*a^3*x)/(224*b^3*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[
3] - (a/(a + b*x^2))^(1/3))) + (1215*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^3*(1 - (a/
(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3)
)/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
(a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[
3]])/(448*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (405*3^(3/4)*a
^3*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*
x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 +
 Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -
7 + 4*Sqrt[3]])/(112*Sqrt[2]*b^4*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*
Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)
])
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
```

```
2]], x] /; FreeQ[{a, b}, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^{7/6}}\,dx = -\frac{3x^5}{b\sqrt[6]{a+bx^2}} + \frac{15\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx}{b}$$

$$= -\frac{3x^5}{b\sqrt[6]{a+bx^2}} + \frac{45x^3\left(a+bx^2\right)^{5/6}}{14b^2} - \frac{(135a)\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx}{14b^2}$$

$$= -\frac{3x^5}{b\sqrt[6]{a+bx^2}} - \frac{405ax\left(a+bx^2\right)^{5/6}}{112b^3} + \frac{45x^3\left(a+bx^2\right)^{5/6}}{14b^2} + \frac{\left(405a^2\right)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{112b^3}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}} - \frac{3x^5}{b\sqrt[6]{a+bx^2}} - \frac{405ax\left(a+bx^2\right)^{5/6}}{112b^3} + \frac{45x^3\left(a+bx^2\right)^{5/6}}{14b^2} - \frac{\left(405a^3\right)\int \cdots}{224b^3}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}} - \frac{3x^5}{b\sqrt[6]{a+bx^2}} - \frac{405ax\left(a+bx^2\right)^{5/6}}{112b^3} + \frac{45x^3\left(a+bx^2\right)^{5/6}}{14b^2} - \frac{\left(405a^3\right)\text{Subs}\cdots}{224b^3\left(\cdots\right.}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}} - \frac{3x^5}{b\sqrt[6]{a+bx^2}} - \frac{405ax\left(a+bx^2\right)^{5/6}}{112b^3} + \frac{45x^3\left(a+bx^2\right)^{5/6}}{14b^2} + \frac{\left(1215a^3\sqrt{-\cdots}\right.}{44\cdots}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}} - \frac{3x^5}{b\sqrt[6]{a+bx^2}} - \frac{405ax\left(a+bx^2\right)^{5/6}}{112b^3} + \frac{45x^3\left(a+bx^2\right)^{5/6}}{14b^2} - \frac{\left(1215a^3\sqrt{-\cdots}\right.}{44\cdots}$$

$$= \frac{1215a^2x}{224b^3\sqrt[6]{a+bx^2}} - \frac{3x^5}{b\sqrt[6]{a+bx^2}} - \frac{405ax\left(a+bx^2\right)^{5/6}}{112b^3} + \frac{45x^3\left(a+bx^2\right)^{5/6}}{14b^2} - \frac{1215\cdots}{224b^4x\left(\frac{a}{a+bx^2}\cdots\right.}$$

Mathematica [C] time = 0.02, size = 79, normalized size = 0.12

$$\frac{-405a^2x\sqrt[6]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};-\frac{bx^2}{a}\right)+405a^2x-90abx^3+48b^2x^5}{224b^3\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(7/6),x]
```

```
[Out] (405*a^2*x - 90*a*b*x^3 + 48*b^2*x^5 - 405*a^2*x*(1 + (b*x^2)/a)^(1/6)*Hype
rgeometric2F1[1/2, 7/6, 3/2, -((b*x^2)/a)])/(224*b^3*(a + b*x^2)^(1/6))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}x^6}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(7/6),x, algorithm="giac")
[Out] integrate(x^6/(b*x^2 + a)^(7/6), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(b\,x^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(7/6),x)
[Out] int(x^6/(b*x^2+a)^(7/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(7/6),x, algorithm="maxima")
[Out] integrate(x^6/(b*x^2 + a)^(7/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(b\,x^2+a\right)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(7/6),x)
[Out] int(x^6/(a + b*x^2)^(7/6), x)
```

sympy [A] time = 1.05, size = 27, normalized size = 0.04

$$\frac{x^7\,{}_2F_1\left(\begin{matrix} \frac{7}{6}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(7/6),x)
[Out] x**7*hyper((7/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(7/6))
```

3.1033 $\int \frac{x^4}{(a+bx^2)^{7/6}}\,dx$

Optimal. Leaf size=630

$$\frac{27\,3^{3/4}a^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{8\sqrt{2}\,b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^2}{}$$

```
[Out] -81/16*a*x/b^2/(b*x^2+a)^(1/6)-3*x^3/b/(b*x^2+a)^(1/6)+27/8*x*(b*x^2+a)^(5/
6)/b^2-81/16*a^2*x/b^2/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))
^(1/3)-3^(1/2))+27/16*3^(3/4)*a^2*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(
b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1
+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2
)^(1/2)/b^3/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)*2^(1/2)/((-1+(a/(b*x^2+a)
)^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)-81/32*3^(1/4)*a^2*(1-(a/(
b*x^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))
^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))
/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^3/x/(
a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a
))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {288, 321, 238, 198, 235, 304, 219, 1879}

$$-\frac{81a^2x}{16b^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{27\,3^{3/4}a^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{}}{-\sqrt[3]{}}\right.\right.}{8\sqrt{2}\,b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(7/6),x]
```

```
[Out] (-81*a*x)/(16*b^2*(a + b*x^2)^(1/6)) - (3*x^3)/(b*(a + b*x^2)^(1/6)) + (27*
x*(a + b*x^2)^(5/6))/(8*b^2) - (81*a^2*x)/(16*b^2*(a/(a + b*x^2))^(2/3)*(a
+ b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) - (81*3^(1/4)*Sqrt[2
+ Sqrt[3]]*a^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3)
+ (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE
[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2)
)^(1/3))], -7 + 4*Sqrt[3]])/(32*b^3*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/
6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
^2)]) + (27*3^(3/4)*a^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2
))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a
 + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(8*Sqrt[2]*b^3*x*(a/(a + b*x^2))^(2/3)
*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a
+ b*x^2))^(1/3))^2)])
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
```

```
2]], x] /; FreeQ[{a, b}, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{7/6}}\,dx = -\frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{9\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx}{b}$$

$$= -\frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{27x\left(a+bx^2\right)^{5/6}}{8b^2} - \frac{(27a)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{8b^2}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}} - \frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{27x\left(a+bx^2\right)^{5/6}}{8b^2} + \frac{\left(27a^2\right)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{16b^2}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}} - \frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{27x\left(a+bx^2\right)^{5/6}}{8b^2} + \frac{\left(27a^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right.}{16b^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}} - \frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{27x\left(a+bx^2\right)^{5/6}}{8b^2} - \frac{\left(81a^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx\right.}{32b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}} - \frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{27x\left(a+bx^2\right)^{5/6}}{8b^2} + \frac{\left(81a^2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\mathrm{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx\right.}{32b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= -\frac{81ax}{16b^2\sqrt[6]{a+bx^2}} - \frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{27x\left(a+bx^2\right)^{5/6}}{8b^2} + \frac{81a^2\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{16b^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}\right.}$$

Mathematica [C] time = 0.02, size = 65, normalized size = 0.10

$$\frac{3x\left(9a\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};-\frac{bx^2}{a}\right)-9a+2bx^2\right)}{16b^2\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(7/6),x]
```

```
[Out] (3*x*(-9*a + 2*b*x^2 + 9*a*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/2, 7/6
, 3/2, -((b*x^2)/a)]))/(16*b^2*(a + b*x^2)^(1/6))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}x^4}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(7/6),x, algorithm="giac")
[Out] integrate(x^4/(b*x^2 + a)^(7/6), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(7/6),x)
[Out] int(x^4/(b*x^2+a)^(7/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(7/6),x, algorithm="maxima")
[Out] integrate(x^4/(b*x^2 + a)^(7/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(7/6),x)
[Out] int(x^4/(a + b*x^2)^(7/6), x)
```

sympy [A] time = 1.04, size = 27, normalized size = 0.04

$$\frac{x^5 {}_2F_1\left(\left.\begin{matrix}\frac{7}{6}, \frac{5}{2}\\ \frac{7}{2}\end{matrix}\right| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(7/6),x)
[Out] x**5*hyper((7/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/6))
```

3.1034 $\int \frac{x^2}{\left(a+bx^2\right)^{7/6}}\,dx$

Optimal. Leaf size=583

$$-\frac{3\,3^{3/4}a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt{2}\,b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a\left(1\right.}{}$$

```
[Out] 3/2*x/b/(b*x^2+a)^(1/6)+9/2*a*x/b/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a
/(b*x^2+a))^(1/3)-3^(1/2))-3/2*3^(3/4)*a*(1-(a/(b*x^2+a))^(1/3))*EllipticF(
(1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/
2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(
1/2))^2)^(1/2)/b^2/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)*2^(1/2)/((-1+(a/(b
*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)+9/4*3^(1/4)*a*(1-(
a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+
a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/
3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^2/
x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^
2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 583, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {288, 238, 198, 235, 304, 219, 1879}

$$-\frac{3\,3^{3/4}a\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}\,b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a\left(1-\sqrt[3]{\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(7/6),x]
```

```
[Out] (3*x)/(2*b*(a + b*x^2)^(1/6)) + (9*a*x)/(2*b*(a/(a + b*x^2))^(2/3)*(a + b*x
^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt
[3]]*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a
+ b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[
(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))
], -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-
((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (
3*3^(3/4)*a*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (
a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[Ar
cSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(
1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(
1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3
))^2)])
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{7/6}}\,dx = -\frac{3x}{b\sqrt[6]{a+bx^2}} + \frac{3\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{b}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} - \frac{(3a)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{2b}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} + \frac{\left(9a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{4b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} - \frac{\left(9a\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{4b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} + \frac{\left(9\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\,a\sqrt{-\frac{b}{a+}}\right.}{2b^2x\,(}$$

$$= \frac{3x}{2b\sqrt[6]{a+bx^2}} - \frac{9a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{2b^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)} + \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a\sqrt{-\frac{bx^2}{a+bx^2}}}{4b^2x}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.10

$$\frac{3x - 3x\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};-\frac{bx^2}{a}\right)}{2b\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(7/6),x]
```

```
[Out] (3*x - 3*x*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/2, 7/6, 3/2, -((b*x^2)
/a)])/(2*b*(a + b*x^2)^(1/6))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}x^2}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(7/6),x, algorithm="giac")
[Out] integrate(x^2/(b*x^2 + a)^(7/6), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(b\,x^{2}+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(7/6),x)
[Out] int(x^2/(b*x^2+a)^(7/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(bx^{2}+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(7/6),x, algorithm="maxima")
[Out] integrate(x^2/(b*x^2 + a)^(7/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(b\,x^2+a\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(7/6),x)
[Out] int(x^2/(a + b*x^2)^(7/6), x)
```

sympy [A] time = 1.03, size = 27, normalized size = 0.05

$$\frac{x^{3}{}_{2}F_{1}\left(\begin{matrix}\frac{7}{6},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|{\frac{bx^{2}e^{i\pi}}{a}}\right)}{3a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(7/6),x)
[Out] x**3*hyper((7/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(7/6))
```

3.1035 $\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx$

Optimal. Leaf size=555

$$\frac{\sqrt{2}\,3^{3/4}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{}{\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7}}$$

```
[Out] -3*x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))+3^
(3/4)*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-
(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(a/(b*x^2+a))^(1/3)
+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)/b/x/(a/(b*x^
2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3
)-3^(1/2))^2)^(1/2)-3/2*3^(1/4)*(1-(a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(b*
x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(
a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^
(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-
1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {198, 235, 304, 219, 1879}

$$-\frac{3x}{\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{\sqrt{2}\,3^{3/4}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+}{-\sqrt[3]{\frac{a}{bx^2+a}}-}\right.\right.}{bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-7/6),x]
```

```
[Out] (-3*x)/((a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^
2))^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (a/(a + b*x^2))^(1/3))*Sqrt
[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a +
 b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(
1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*(a/(a + b*x^
2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
 - (a/(a + b*x^2))^(1/3))^2)]) + (Sqrt[2]*3^(3/4)*(1 - (a/(a + b*x^2))^(1/3
))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] -
(a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(
1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a/(a +
 b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sq
rt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}} \\
&= -\frac{\left(3\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} \\
&= \frac{\left(3\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\frac{a}{a+bx^2}}\right)}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} - \frac{\left(3\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\sqrt{-\frac{bx^2}{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}} \cdots\right.}{bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}} \\
&= \frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{a}{a+bx^2}}\right)} - \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{1+\sqrt[3]{\cdots}}{\left(1-\cdots\right.}} \cdots}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2} \cdots}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.09

$$
\frac{x\sqrt[6]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};-\frac{bx^2}{a}\right)}{a\sqrt[6]{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-7/6),x]
[Out] (x*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/2, 7/6, 3/2, -((b*x^2)/a)])/(a
*(a + b*x^2)^(1/6))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/6),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^(5/6)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/6),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-7/6), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(7/6),x)
[Out] int(1/(b*x^2+a)^(7/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(7/6),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(-7/6), x)
```

mupad [B] time = 4.94, size = 37, normalized size = 0.07

$$\frac{x\left(\frac{b\,x^2}{a}+1\right)^{7/6}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{6};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{7/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(7/6),x)
```

```
[Out] (x*((b*x^2)/a + 1)^(7/6)*hypergeom([1/2, 7/6], 3/2, -(b*x^2)/a))/(a + b*x^2
)^(7/6)
```

sympy [A] time = 0.98, size = 24, normalized size = 0.04

$$\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{6}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(7/6),x)
```

```
[Out] x*hyper((1/2, 7/6), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/6)
```

3.1036 $\int \frac{1}{x^2\left(a+bx^2\right)^{7/6}}\,dx$

Optimal. Leaf size=614

$$-\frac{4\sqrt{2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}\,ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{4bx}{a^2\sqrt[6]{a+bx^2}}-\frac{4\left(a+\right.}{a^2}$$

```
[Out] 3/a/x/(b*x^2+a)^(1/6)+4*b*x/a^2/(b*x^2+a)^(1/6)-4*(b*x^2+a)^(5/6)/a^2/x+4*b
*x/a/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))-4/
3*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(
b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(a/(b*x^2+a))^(1/3)+(a/
(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/a/x/(a/(
b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^
(1/3)-3^(1/2))^2)^(1/2)+2*3^(1/4)*(1-(a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(
b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1
+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2
)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/a/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/(
(-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 325, 238, 198, 235, 304, 219, 1879}

$$\frac{4bx}{a^2\sqrt[6]{a+bx^2}}-\frac{4\left(a+bx^2\right)^{5/6}}{a^2x}+\frac{4bx}{a\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{3}{ax\sqrt[6]{a+bx^2}}-\frac{4\sqrt{2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right.}{\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(7/6)),x]
```

```
[Out] 3/(a*x*(a + b*x^2)^(1/6)) + (4*b*x)/(a^2*(a + b*x^2)^(1/6)) - (4*(a + b*x^2
)^(5/6))/(a^2*x) + (4*b*x)/(a*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 -
Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (2*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (a/(a
 + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/
(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a
/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]
])/(a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))
^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) - (4*Sqrt[2]*(1 - (a/(a
+ b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(
1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/
(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]]
)/(3^(1/4)*a*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a +
b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^{7/6}}\,dx = \frac{3}{ax\sqrt[6]{a+bx^2}} + \frac{4\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx}{a}$$

$$= \frac{3}{ax\sqrt[6]{a+bx^2}} - \frac{4\left(a+bx^2\right)^{5/6}}{a^2x} + \frac{(8b)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{3a^2}$$

$$= \frac{3}{ax\sqrt[6]{a+bx^2}} + \frac{4bx}{a^2\sqrt[6]{a+bx^2}} - \frac{4\left(a+bx^2\right)^{5/6}}{a^2x} - \frac{(4b)\int \frac{1}{\left(a+bx^2\right)^{7/6}}\,dx}{3a}$$

$$= \frac{3}{ax\sqrt[6]{a+bx^2}} + \frac{4bx}{a^2\sqrt[6]{a+bx^2}} - \frac{4\left(a+bx^2\right)^{5/6}}{a^2x} - \frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-bx^2}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{3a\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}$$

$$= \frac{3}{ax\sqrt[6]{a+bx^2}} + \frac{4bx}{a^2\sqrt[6]{a+bx^2}} - \frac{4\left(a+bx^2\right)^{5/6}}{a^2x} + \frac{\left(2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\vphantom{x}}\right.}{ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{3}{ax\sqrt[6]{a+bx^2}} + \frac{4bx}{a^2\sqrt[6]{a+bx^2}} - \frac{4\left(a+bx^2\right)^{5/6}}{a^2x} - \frac{\left(2\sqrt{-\frac{bx^2}{a+bx^2}}\right)\mathrm{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{\vphantom{x}}\right.}{ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$= \frac{3}{ax\sqrt[6]{a+bx^2}} + \frac{4bx}{a^2\sqrt[6]{a+bx^2}} - \frac{4\left(a+bx^2\right)^{5/6}}{a^2x} - \frac{4\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt{-1+\frac{a}{a+bx^2}}}{ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\left(1-\sqrt{3}-\sqrt[3]{\frac{}{a+}}\right.}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.08

$$-\frac{\sqrt[6]{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{1}{2},\frac{7}{6};\frac{1}{2};-\frac{bx^2}{a}\right)}{ax\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(7/6)),x]
```

```
[Out] -(((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-1/2, 7/6, 1/2, -((b*x^2)/a)])/(
a*x*(a + b*x^2)^(1/6)))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}}{b^2x^6+2\,abx^4+a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{6}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^2), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{7}{6}} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(7/6),x)
```

```
[Out] int(1/x^2/(b*x^2+a)^(7/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{7}{6}} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^2), x)
```

mupad [B] time = 5.12, size = 40, normalized size = 0.07

$$-\frac{3\left(\frac{a}{b\,x^{2}}+1\right)^{7/6}\,{}_2\mathrm{F}_1\left(\frac{7}{6},\frac{5}{3};\ \frac{8}{3};\ -\frac{a}{b\,x^{2}}\right)}{10\,x\left(b\,x^{2}+a\right)^{7/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(7/6)),x)
```

```
[Out] -(3*(a/(b*x^2) + 1)^(7/6)*hypergeom([7/6, 5/3], 8/3, -a/(b*x^2)))/(10*x*(a
+ b*x^2)^(7/6))
```

sympy [A] time = 1.24, size = 27, normalized size = 0.04

$$-\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{7}{6} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{a^{\frac{7}{6}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(7/6),x)
```

```
[Out] -hyper((-1/2, 7/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(7/6)*x)
```

3.1037 $\int \frac{1}{x^4(a+bx^2)^{7/6}}\,dx$

Optimal. Leaf size=652

$$\frac{40\sqrt{2}\,b\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{9\sqrt[4]{3}\,a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}-\frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}}+\frac{40b\,(}{}$$

```
[Out] 3/a/x^3/(b*x^2+a)^(1/6)-40/9*b^2*x/a^3/(b*x^2+a)^(1/6)-10/3*(b*x^2+a)^(5/6)
/a^2/x^3+40/9*b*(b*x^2+a)^(5/6)/a^3/x-40/9*b^2*x/a^2/(a/(b*x^2+a))^(2/3)/(b
*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))+40/27*b*(1-(a/(b*x^2+a))^(1/3
))*EllipticF((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)
),2*I-I*3^(1/2))*2^(1/2)*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a
/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/a^2/x/(a/(b*x^2+a))^(2/3)/(b*x^
2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/
2)-20/9*b*(1-(a/(b*x^2+a))^(1/3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))
/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/
(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2
*2^(1/2))*3^(1/4)/a^2/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+
a))^(1/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 652, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 325, 238, 198, 235, 304, 219, 1879}

$$-\frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}}-\frac{40b^2x}{9a^2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}+\frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x}-\frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3}+\frac{40\sqrt{2}\,b\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(7/6)),x]
```

```
[Out] 3/(a*x^3*(a + b*x^2)^(1/6)) - (40*b^2*x)/(9*a^3*(a + b*x^2)^(1/6)) - (10*(a
 + b*x^2)^(5/6))/(3*a^2*x^3) + (40*b*(a + b*x^2)^(5/6))/(9*a^3*x) - (40*b^2
*x)/(9*a^2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b
*x^2))^(1/3))) - (20*Sqrt[2 + Sqrt[3]]*b*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(
1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b
*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1
- Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(3/4)*a^2*x*(a/(
a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 -
 Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)]) + (40*Sqrt[2]*b*(1 - (a/(a + b*x^2))
^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[
3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^
2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1
/4)*a^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2
))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
```

```
2]], x] /; FreeQ[{a, b}, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{7/6}}\,dx = \frac{3}{ax^3\sqrt[6]{a+bx^2}} + \frac{10\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx}{a}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3} - \frac{(40b)\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx}{9a^2}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3} + \frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x} - \frac{\left(80b^2\right)\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx}{27a^3}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3} + \frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x} + \frac{\left(40b^2\right)\int \frac{1}{\left(a+bx^2\right)^{7}}}{27a^2}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3} + \frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x} + \frac{\left(40b^2\right)\mathrm{Subst}\left(\int\right.}{27a^2\left(\frac{a}{a+bx^2}\right.}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3} + \frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x} - \frac{\left(20b\sqrt{-\frac{bx^2}{a+bx^2}}\right)}{9a^2x}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3} + \frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x} + \frac{\left(20b\sqrt{-\frac{bx^2}{a+bx^2}}\right)}{9a^2x}$$

$$= \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3} + \frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x} + \frac{40b\sqrt{-}}{9a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.08

$$-\frac{\sqrt[6]{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{3}{2},\frac{7}{6};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3ax^3\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(7/6)),x]
```

```
[Out] -1/3*((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-3/2, 7/6, -1/2, -((b*x^2)/a)
])/(a*x^3*(a + b*x^2)^(1/6))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}}{b^2x^8+2\,abx^6+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{6}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(7/6),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^4), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{7}{6}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(7/6),x)
[Out] int(1/x^4/(b*x^2+a)^(7/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{6}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(7/6),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\left(b\,x^2+a\right)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(7/6)),x)
[Out] int(1/(x^4*(a + b*x^2)^(7/6)), x)
```

sympy [A] time = 1.45, size = 32, normalized size = 0.05

$$-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{7}{6} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{7}{6}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(7/6),x)
[Out] -hyper((-3/2, 7/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(7/6)*x**3)
```

3.1038 $\int \frac{1}{x^6(a+bx^2)^{7/6}}\,dx$

Optimal. Leaf size=680

$$-\frac{128\sqrt{2}\,b^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right),4\sqrt{3}-7\right)}{27\sqrt[4]{3}\,a^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}+\frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}}-$$

```
[Out] 3/a/x^5/(b*x^2+a)^(1/6)+128/27*b^3*x/a^4/(b*x^2+a)^(1/6)-16/5*(b*x^2+a)^(5/
6)/a^2/x^5+32/9*b*(b*x^2+a)^(5/6)/a^3/x^3-128/27*b^2*(b*x^2+a)^(5/6)/a^4/x+
128/27*b^3*x/a^3/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(7/6)/(1-(a/(b*x^2+a))^(1/3)
-3^(1/2))-128/81*b^2*(1-(a/(b*x^2+a))^(1/3))*EllipticF((1-(a/(b*x^2+a))^(1/
3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(a/(
b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/
2)*3^(3/4)/a^3/x/(a/(b*x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/
3))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2))^2)^(1/2)+64/27*b^2*(1-(a/(b*x^2+a))^(1/
3))*EllipticE((1-(a/(b*x^2+a))^(1/3)+3^(1/2))/(1-(a/(b*x^2+a))^(1/3)-3^(1/2
)),2*I-I*3^(1/2))*((1+(a/(b*x^2+a))^(1/3)+(a/(b*x^2+a))^(2/3))/(1-(a/(b*x^2
+a))^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^3/x/(a/(b*
x^2+a))^(2/3)/(b*x^2+a)^(1/6)/((-1+(a/(b*x^2+a))^(1/3))/(1-(a/(b*x^2+a))^(1
/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 325, 238, 198, 235, 304, 219, 1879}

$$\frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}}+\frac{128b^3x}{27a^3\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)}-\frac{128b^2\left(a+bx^2\right)^{5/6}}{27a^4x}-\frac{128\sqrt{2}\,b^2\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right.}{27\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^(7/6)),x]
```

```
[Out] 3/(a*x^5*(a + b*x^2)^(1/6)) + (128*b^3*x)/(27*a^4*(a + b*x^2)^(1/6)) - (16*
(a + b*x^2)^(5/6))/(5*a^2*x^5) + (32*b*(a + b*x^2)^(5/6))/(9*a^3*x^3) - (12
8*b^2*(a + b*x^2)^(5/6))/(27*a^4*x) + (128*b^3*x)/(27*a^3*(a/(a + b*x^2))^(
2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (64*Sqrt[2
+ Sqrt[3]]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3)
+ (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE
[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2)
)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(3/4)*a^3*x*(a/(a + b*x^2))^(2/3)*(a + b*x
^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))
^(1/3))^2)]) - (128*Sqrt[2]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a
 + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1
/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
 - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(27*3^(1/4)*a^3*x*(a/(a + b*x^
2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
 - (a/(a + b*x^2))^(1/3))^2)])
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Dist[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3)), Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x^
2]], x] /; FreeQ[{a, b}, x]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[(3*x)/(2*(a + b*x^2)^(1
/6)), x] - Dist[a/2, Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^2\right)^{7/6}}\,dx = \frac{3}{ax^5\sqrt[6]{a+bx^2}} + \frac{16\int \frac{1}{x^6\sqrt[6]{a+bx^2}}\,dx}{a}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} - \frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5} - \frac{(32b)\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx}{3a^2}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} - \frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5} + \frac{32b\left(a+bx^2\right)^{5/6}}{9a^3x^3} + \frac{\left(128b^2\right)\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx}{27a^3}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} - \frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5} + \frac{32b\left(a+bx^2\right)^{5/6}}{9a^3x^3} - \frac{128b^2\left(a+bx^2\right)^{5/6}}{27a^4x} + \frac{\left(256b^3\right)\int}{81a}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} + \frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}} - \frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5} + \frac{32b\left(a+bx^2\right)^{5/6}}{9a^3x^3} - \frac{128b^2\left(a+bx^2\right)}{27a^4x}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} + \frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}} - \frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5} + \frac{32b\left(a+bx^2\right)^{5/6}}{9a^3x^3} - \frac{128b^2\left(a+bx^2\right)}{27a^4x}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} + \frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}} - \frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5} + \frac{32b\left(a+bx^2\right)^{5/6}}{9a^3x^3} - \frac{128b^2\left(a+bx^2\right)}{27a^4x}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} + \frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}} - \frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5} + \frac{32b\left(a+bx^2\right)^{5/6}}{9a^3x^3} - \frac{128b^2\left(a+bx^2\right)}{27a^4x}$$

$$= \frac{3}{ax^5\sqrt[6]{a+bx^2}} + \frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}} - \frac{16\left(a+bx^2\right)^{5/6}}{5a^2x^5} + \frac{32b\left(a+bx^2\right)^{5/6}}{9a^3x^3} - \frac{128b^2\left(a+bx^2\right)}{27a^4x}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.08

$$-\frac{\sqrt[6]{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{5}{2},\frac{7}{6};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5ax^5\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^(7/6)),x]
```

```
[Out] -1/5*((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-5/2, 7/6, -3/2, -((b*x^2)/a)
])/(a*x^5*(a + b*x^2)^(1/6))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{5}{6}}}{b^2x^{10}+2\,abx^8+a^2x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(5/6)/(b^2*x^10 + 2*a*b*x^8 + a^2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{6}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^6), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{7}{6}}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^(7/6),x)
```

```
[Out] int(1/x^6/(b*x^2+a)^(7/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{6}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/6)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\,\left(b\,x^2+a\right)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^(7/6)),x)
```

```
[Out] int(1/(x^6*(a + b*x^2)^(7/6)), x)
```

sympy [A] time = 1.70, size = 32, normalized size = 0.05

$$-\frac{{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{5}{2},\frac{7}{6}\\-\frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{7}{6}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**(7/6),x)
```

```
[Out] -hyper((-5/2, 7/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/6)*x**5)
```

3.1039 $\int x^7 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=100

$$-\frac{a^3 \left(a+bx^2\right)^{p+1}}{2b^4(p+1)}+\frac{3a^2 \left(a+bx^2\right)^{p+2}}{2b^4(p+2)}-\frac{3a \left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}$$

```
[Out] -1/2*a^3*(b*x^2+a)^(1+p)/b^4/(1+p)+3/2*a^2*(b*x^2+a)^(2+p)/b^4/(2+p)-3/2*a*
(b*x^2+a)^(3+p)/b^4/(3+p)+1/2*(b*x^2+a)^(4+p)/b^4/(4+p)
```

Rubi [A] time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^3 \left(a+bx^2\right)^{p+1}}{2b^4(p+1)}+\frac{3a^2 \left(a+bx^2\right)^{p+2}}{2b^4(p+2)}-\frac{3a \left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^p,x]
```

```
[Out] -(a^3*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (3*a^2*(a + b*x^2)^(2 + p))/(2
*b^4*(2 + p)) - (3*a*(a + b*x^2)^(3 + p))/(2*b^4*(3 + p)) + (a + b*x^2)^(4
+ p)/(2*b^4*(4 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a+bx^2\right)^p \, dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^p \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3(a+bx)^p}{b^3}+\frac{3a^2(a+bx)^{1+p}}{b^3}-\frac{3a(a+bx)^{2+p}}{b^3}+\frac{(a+bx)^{3+p}}{b^3}\right) dx, x, x^2\right)\\
&= -\frac{a^3 \left(a+bx^2\right)^{1+p}}{2b^4(1+p)}+\frac{3a^2 \left(a+bx^2\right)^{2+p}}{2b^4(2+p)}-\frac{3a \left(a+bx^2\right)^{3+p}}{2b^4(3+p)}+\frac{\left(a+bx^2\right)^{4+p}}{2b^4(4+p)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 95, normalized size = 0.95

$$\frac{1}{2}\left(-\frac{a^3 \left(a+bx^2\right)^{p+1}}{b^4(p+1)}+\frac{3a^2 \left(a+bx^2\right)^{p+2}}{b^4(p+2)}-\frac{3a \left(a+bx^2\right)^{p+3}}{b^4(p+3)}+\frac{\left(a+bx^2\right)^{p+4}}{b^4(p+4)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^p,x]
```

```
[Out] (-((a^3*(a + b*x^2)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*x^2)^(2 + p))/(
b^4*(2 + p)) - (3*a*(a + b*x^2)^(3 + p))/(b^4*(3 + p)) + (a + b*x^2)^(4 + p
)/(b^4*(4 + p)))/2
```

fricas [A] time = 0.48, size = 148, normalized size = 1.48

$$\frac{\left(\left(b^4p^3+6\,b^4p^2+11\,b^4p+6\,b^4\right)x^8+6\,a^3bpx^2+\left(ab^3p^3+3\,ab^3p^2+2\,ab^3p\right)x^6-3\,\left(a^2b^2p^2+a^2b^2p\right)x^4-6\,a^4\right)\left(bx^2\right.}{2\,\left(b^4p^4+10\,b^4p^3+35\,b^4p^2+50\,b^4p+24\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*((b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^8 + 6*a^3*b*p*x^2 + (a*b^3*
p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*x^6 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^4 - 6*a^
4)*(b*x^2 + a)^p/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4)
```

giac [B] time = 0.59, size = 410, normalized size = 4.10

$$\frac{\left(bx^2+a\right)^4\left(bx^2+a\right)^pp^3-3\left(bx^2+a\right)^3\left(bx^2+a\right)^pap^3+3\left(bx^2+a\right)^2\left(bx^2+a\right)^pa^2p^3-\left(bx^2+a\right)\left(bx^2+a\right)^pa^3p^3+6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)^4*(b*x^2 + a)^p*p^3 - 3*(b*x^2 + a)^3*(b*x^2 + a)^p*a*p^3
+ 3*(b*x^2 + a)^2*(b*x^2 + a)^p*a^2*p^3 - (b*x^2 + a)*(b*x^2 + a)^p*a^3*p^3
 + 6*(b*x^2 + a)^4*(b*x^2 + a)^p*p^2 - 21*(b*x^2 + a)^3*(b*x^2 + a)^p*a*p^2
 + 24*(b*x^2 + a)^2*(b*x^2 + a)^p*a^2*p^2 - 9*(b*x^2 + a)*(b*x^2 + a)^p*a^3
*p^2 + 11*(b*x^2 + a)^4*(b*x^2 + a)^p*p - 42*(b*x^2 + a)^3*(b*x^2 + a)^p*a*
p + 57*(b*x^2 + a)^2*(b*x^2 + a)^p*a^2*p - 26*(b*x^2 + a)*(b*x^2 + a)^p*a^3
*p + 6*(b*x^2 + a)^4*(b*x^2 + a)^p - 24*(b*x^2 + a)^3*(b*x^2 + a)^p*a + 36*
(b*x^2 + a)^2*(b*x^2 + a)^p*a^2 - 24*(b*x^2 + a)*(b*x^2 + a)^p*a^3)/((b^3*p
^4 + 10*b^3*p^3 + 35*b^3*p^2 + 50*b^3*p + 24*b^3)*b)
```

maple [A] time = 0.01, size = 132, normalized size = 1.32

$$-\frac{\left(-b^3p^3x^6-6b^3p^2x^6-11b^3p\,x^6+3a\,b^2p^2x^4-6b^3x^6+9a\,b^2p\,x^4+6a\,b^2x^4-6a^2bp\,x^2-6a^2b\,x^2+6a^3\right)\left(b\,x^2+a\right.}{2\left(p^4+10p^3+35p^2+50p+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^p,x)
```

```
[Out] -1/2*(b*x^2+a)^(p+1)*(-b^3*p^3*x^6-6*b^3*p^2*x^6-11*b^3*p*x^6+3*a*b^2*p^2*x
^4-6*b^3*x^6+9*a*b^2*p*x^4+6*a*b^2*x^4-6*a^2*b*p*x^2-6*a^2*b*x^2+6*a^3)/b^4
/(p^4+10*p^3+35*p^2+50*p+24)
```

maxima [A] time = 1.45, size = 106, normalized size = 1.06

$$\frac{\left(\left(p^3+6\,p^2+11\,p+6\right)b^4x^8+\left(p^3+3\,p^2+2\,p\right)ab^3x^6-3\,\left(p^2+p\right)a^2b^2x^4+6\,a^3bpx^2-6\,a^4\right)\left(bx^2+a\right)^p}{2\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^3 + 6*p^2 + 11*p + 6)*b^4*x^8 + (p^3 + 3*p^2 + 2*p)*a*b^3*x^6 - 3*(
p^2 + p)*a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 6*a^4)*(b*x^2 + a)^p/((p^4 + 10*p^3
+ 35*p^2 + 50*p + 24)*b^4)
```

mupad [B] time = 4.97, size = 183, normalized size = 1.83

$$\left(b\,x^2+a\right)^p\left(\frac{x^8\,\left(p^3+6\,p^2+11\,p+6\right)}{2\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}-\frac{3\,a^4}{b^4\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}+\frac{3\,a^3\,p\,x^2}{b^3\,\left(p^4+10\,p^3+35\,p\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^p,x)
```

```
[Out] (a + b*x^2)^p*((x^8*(11*p + 6*p^2 + p^3 + 6))/(2*(50*p + 35*p^2 + 10*p^3 +
p^4 + 24)) - (3*a^4)/(b^4*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (3*a^3*p*x
^2)/(b^3*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (a*p*x^6*(3*p + p^2 + 2))/(
2*b*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (3*a^2*p*x^4*(p + 1))/(2*b^2*(50
*p + 35*p^2 + 10*p^3 + p^4 + 24)))
```

sympy [A] time = 11.06, size = 2025, normalized size = 20.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((a**p*x**8/8, Eq(b, 0)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(1
2*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a**3*l
og(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x
**4 + 12*b**7*x**6) + 11*a**3/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6
*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(12*a*
*3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x*
*2*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b*
*6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4 + 36*a**2*b**5*x**2
+ 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b)
+ x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 1
8*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**
2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(12*a**3*b**4 + 36*a**2
*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(-I*sqrt(a)*sq
rt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x
**6) + 6*b**3*x**6*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**
5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -4)), (-6*a**3*log(-I*sqrt(a
)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a**3*log(I
*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 9*a**
3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2*log(-I*sqrt(
a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x
**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4
) - 12*a**2*b*x**2/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a*b**2*x
**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**
4) - 6*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**
2 + 4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4),
 Eq(p, -3)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2)
 + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**3/(4
*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*b
**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 +
 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4 + 4*b**5*x**2) + b**3*x**6/(4*a*b**
4 + 4*b**5*x**2), Eq(p, -2)), (-a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**4)
 - a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**4) + a**2*x**2/(2*b**3) - a*x**4
/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**2)**p/(2*b**4*p**4 +
 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*a**3*b*p*x**2*(a +
```

```
 b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b*
*4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 7
0*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x**4*(a + b*x**2)**p/(2
*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + a*b**3*p
**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b
**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**
4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**3*p*x**6*(a + b*x**2
)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + b
**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 +
100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*
b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*b**4*p*x**8*(a + b*x*
*2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) +
 6*b**4*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 1
00*b**4*p + 48*b**4), True))
```

3.1040 $\int x^5 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=72

$$\frac{a^2 \left(a+bx^2\right)^{p+1}}{2b^3(p+1)}-\frac{a \left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}$$

```
[Out] 1/2*a^2*(b*x^2+a)^(1+p)/b^3/(1+p)-a*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*(b*x^2+a)
^(3+p)/b^3/(3+p)
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2 \left(a+bx^2\right)^{p+1}}{2b^3(p+1)}-\frac{a \left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^p,x]
```

```
[Out] (a^2*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*(a + b*x^2)^(2 + p))/(b^3*(2
 + p)) + (a + b*x^2)^(3 + p)/(2*b^3*(3 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^2\right)^p \, dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^p \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{1+p}}{b^2}+\frac{(a+bx)^{2+p}}{b^2}\right) dx, x, x^2\right)\\
&= \frac{a^2 \left(a+bx^2\right)^{1+p}}{2b^3(1+p)}-\frac{a \left(a+bx^2\right)^{2+p}}{b^3(2+p)}+\frac{\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.89

$$\frac{\left(a+bx^2\right)^{p+1} \left(2a^2-2ab(p+1)x^2+b^2 \left(p^2+3p+2\right) x^4\right)}{2b^3(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^2)*x^4))
/(2*b^3*(1 + p)*(2 + p)*(3 + p))
```

fricas [A] time = 0.68, size = 98, normalized size = 1.36

$$\frac{\left(\left(b^3p^2 + 3\,b^3p + 2\,b^3\right)x^6 - 2\,a^2bpx^2 + \left(ab^2p^2 + ab^2p\right)x^4 + 2\,a^3\right)\left(bx^2 + a\right)^p}{2\left(b^3p^3 + 6\,b^3p^2 + 11\,b^3p + 6\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*((b^3*p^2 + 3*b^3*p + 2*b^3)*x^6 - 2*a^2*b*p*x^2 + (a*b^2*p^2 + a*b^2*p
)*x^4 + 2*a^3)*(b*x^2 + a)^p/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

giac [B] time = 0.58, size = 231, normalized size = 3.21

$$\frac{\left(bx^2+a\right)^3\left(bx^2+a\right)^p p^2 - 2\left(bx^2+a\right)^2\left(bx^2+a\right)^p ap^2 + \left(bx^2+a\right)\left(bx^2+a\right)^p a^2p^2 + 3\left(bx^2+a\right)^3\left(bx^2+a\right)^p p - 8\,(bx}{2\left(b^2p^3+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)^3*(b*x^2 + a)^p*p^2 - 2*(b*x^2 + a)^2*(b*x^2 + a)^p*a*p^2
+ (b*x^2 + a)*(b*x^2 + a)^p*a^2*p^2 + 3*(b*x^2 + a)^3*(b*x^2 + a)^p*p - 8*(
b*x^2 + a)^2*(b*x^2 + a)^p*a*p + 5*(b*x^2 + a)*(b*x^2 + a)^p*a^2*p + 2*(b*x
^2 + a)^3*(b*x^2 + a)^p - 6*(b*x^2 + a)^2*(b*x^2 + a)^p*a + 6*(b*x^2 + a)*(
b*x^2 + a)^p*a^2)/((b^2*p^3 + 6*b^2*p^2 + 11*b^2*p + 6*b^2)*b)
```

maple [A] time = 0.01, size = 80, normalized size = 1.11

$$\frac{\left(b^2p^2x^4 + 3b^2p\,x^4 + 2b^2x^4 - 2abp\,x^2 - 2ab\,x^2 + 2a^2\right)\left(b\,x^2 + a\right)^{p+1}}{2\left(p^3 + 6p^2 + 11p + 6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^p,x)
```

```
[Out] 1/2*(b*x^2+a)^(p+1)*(b^2*p^2*x^4+3*b^2*p*x^4+2*b^2*x^4-2*a*b*p*x^2-2*a*b*x^
2+2*a^2)/b^3/(p^3+6*p^2+11*p+6)
```

maxima [A] time = 1.41, size = 73, normalized size = 1.01

$$\frac{\left(\left(p^2 + 3\,p + 2\right)b^3x^6 + \left(p^2 + p\right)ab^2x^4 - 2\,a^2bpx^2 + 2\,a^3\right)\left(bx^2 + a\right)^p}{2\left(p^3 + 6\,p^2 + 11\,p + 6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)
*(b*x^2 + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

mupad [B] time = 4.90, size = 117, normalized size = 1.62

$$\left(b\,x^2+a\right)^p\left(\frac{a^3}{b^3\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{x^6\,\left(p^2+3\,p+2\right)}{2\,\left(p^3+6\,p^2+11\,p+6\right)}-\frac{a^2\,p\,x^2}{b^2\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{a\,p\,x^4\,\left(p+\right.}{2\,b\,\left(p^3+6\,p^2+1\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^p,x)
```

```
[Out] (a + b*x^2)^p*(a^3/(b^3*(11*p + 6*p^2 + p^3 + 6)) + (x^6*(3*p + p^2 + 2))/(
2*(11*p + 6*p^2 + p^3 + 6)) - (a^2*p*x^2)/(b^2*(11*p + 6*p^2 + p^3 + 6)) +
(a*p*x^4*(p + 1))/(2*b*(11*p + 6*p^2 + p^3 + 6)))
```

sympy [A] time = 5.05, size = 981, normalized size = 13.62

$$\begin{cases} \frac{a^p x^6}{6} \\ \frac{2a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \ldots \\ -\frac{2a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2} - \frac{2a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2} - \frac{2a^2}{2ab^3+2b^4x^2} - \frac{2abx^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2} - \frac{2abx^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^3+2b^4x^2} + \frac{b^2x^4}{2ab^3+2b^4x^2} \\ \frac{a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^3} + \frac{a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b} \\ \frac{2a^3(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} - \frac{2a^2bpx^2(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2p^2x^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2px^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{b^3p^2x^6(a \ldots}{2b^3p^3+12b^3p^2 \ldots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4
*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b)
+ x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*
a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*
*2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b)
 + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3
 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)
/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*sqrt(a)*sq
rt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a
**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*s
qrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*
x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) -
 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x*
*4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x
)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2) +
 x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**
2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 1
2*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b*
*3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)*
*p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a +
 b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x
**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*
b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3
), True))
```

3.1041 $\int x^3 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=48

$$\frac{\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}-\frac{a\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}$$

```
[Out] -1/2*a*(b*x^2+a)^(1+p)/b^2/(1+p)+1/2*(b*x^2+a)^(2+p)/b^2/(2+p)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}-\frac{a\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^p,x]
```

```
[Out] -(a*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (a + b*x^2)^(2 + p)/(2*b^2*(2 +
p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^2\right)^p \, dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^p \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right) dx, x, x^2\right)\\
&= -\frac{a\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.83

$$\frac{\left(a+bx^2\right)^{p+1}\left(b(p+1)x^2-a\right)}{2b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(-a + b*(1 + p)*x^2))/(2*b^2*(1 + p)*(2 + p))
```

fricas [A] time = 0.73, size = 58, normalized size = 1.21

$$\frac{\left(abpx^2 + \left(b^2p + b^2\right)x^4 - a^2\right)\left(bx^2 + a\right)^p}{2\left(b^2p^2 + 3\,b^2p + 2\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*p*x^2 + (b^2*p + b^2)*x^4 - a^2)*(b*x^2 + a)^p/(b^2*p^2 + 3*b^2*p
+ 2*b^2)
```

giac [B] time = 0.58, size = 94, normalized size = 1.96

$$\frac{\left(bx^2 + a\right)^2\left(bx^2 + a\right)^p p - \left(bx^2 + a\right)\left(bx^2 + a\right)^p ap + \left(bx^2 + a\right)^2\left(bx^2 + a\right)^p - 2\left(bx^2 + a\right)\left(bx^2 + a\right)^p a}{2\left(p^2 + 3\,p + 2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)^2*(b*x^2 + a)^p*p - (b*x^2 + a)*(b*x^2 + a)^p*a*p + (b*x^2
 + a)^2*(b*x^2 + a)^p - 2*(b*x^2 + a)*(b*x^2 + a)^p*a)/((p^2 + 3*p + 2)*b^2
)
```

maple [A] time = 0.00, size = 42, normalized size = 0.88

$$-\frac{\left(-x^2pb - b\,x^2 + a\right)\left(b\,x^2 + a\right)^{p+1}}{2\left(p^2 + 3p + 2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p,x)
```

```
[Out] -1/2*(b*x^2+a)^(p+1)*(-b*p*x^2-b*x^2+a)/b^2/(p^2+3*p+2)
```

maxima [A] time = 1.29, size = 47, normalized size = 0.98

$$\frac{\left(b^2\left(p + 1\right)x^4 + abpx^2 - a^2\right)\left(bx^2 + a\right)^p}{2\left(p^2 + 3\,p + 2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p/((p^2 + 3*p + 2)*b^2)
```

mupad [B] time = 4.89, size = 68, normalized size = 1.42

$$\left(b\,x^2 + a\right)^p\left(\frac{x^4\,\left(p + 1\right)}{2\,\left(p^2 + 3\,p + 2\right)} - \frac{a^2}{2\,b^2\,\left(p^2 + 3\,p + 2\right)} + \frac{a\,p\,x^2}{2\,b\,\left(p^2 + 3\,p + 2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^p,x)
```

```
[Out] (a + b*x^2)^p*((x^4*(p + 1))/(2*(3*p + p^2 + 2)) - a^2/(2*b^2*(3*p + p^2 +
2)) + (a*p*x^2)/(2*b*(3*p + p^2 + 2)))
```

sympy [A] time = 2.00, size = 364, normalized size = 7.58

$$\begin{cases} \frac{a^p x^4}{4} & \text{for } b = 0 \\ \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} + \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2ab^2+2b^3x^2} & \text{for } p = -2 \\ -\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2} - \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b^2} + \frac{x^2}{2b} & \text{for } p = -1 \\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b*
*2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2)
 + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b
**2 + 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3
*x**2), Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sq
rt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)
**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*
p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b*
*2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2
), True))
```

3.1042 $\int x\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=23

$$\frac{\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

```
[Out] 1/2*(b*x^2+a)^(1+p)/b/(1+p)
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^p,x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*(1 + p))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^p\,dx=\frac{\left(a+bx^2\right)^{1+p}}{2b(1+p)}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{\left(a+bx^2\right)^{p+1}}{2bp+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^p,x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b + 2*b*p)
```

fricas [A] time = 0.67, size = 25, normalized size = 1.09

$$\frac{\left(bx^2+a\right)\left(bx^2+a\right)^p}{2\left(bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^p/(b*p + b)
```

giac [A] time = 0.57, size = 21, normalized size = 0.91

$$\frac{\left(bx^2+a\right)^{p+1}}{2\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p,x, algorithm="giac")
[Out] 1/2*(b*x^2 + a)^(p + 1)/(b*(p + 1))
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{\left(b\,x^2+a\right)^{p+1}}{2\left(p+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p,x)
[Out] 1/2*(b*x^2+a)^(p+1)/b/(p+1)
```

maxima [A] time = 1.38, size = 21, normalized size = 0.91

$$\frac{\left(bx^2+a\right)^{p+1}}{2\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p,x, algorithm="maxima")
[Out] 1/2*(b*x^2 + a)^(p + 1)/(b*(p + 1))
```

mupad [B] time = 4.89, size = 21, normalized size = 0.91

$$\frac{\left(b\,x^2+a\right)^{p+1}}{2\,b\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^p,x)
[Out] (a + b*x^2)^(p + 1)/(2*b*(p + 1))
```

sympy [A] time = 0.73, size = 97, normalized size = 4.22

$$\begin{cases} \frac{x^2}{2a} & \text{for}\ b=0 \wedge p=-1 \\ \frac{a^p x^2}{2} & \text{for}\ b=0 \\ \frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b}+\frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b} & \text{for}\ p=-1 \\ \frac{a\left(a+bx^2\right)^p}{2bp+2b}+\frac{bx^2\left(a+bx^2\right)^p}{2bp+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((x**2/(2*a), Eq(b, 0) & Eq(p, -1)), (a**p*x**2/2, Eq(b, 0)), (log
(-I*sqrt(a)*sqrt(1/b) + x)/(2*b) + log(I*sqrt(a)*sqrt(1/b) + x)/(2*b), Eq(p
, -1)), (a*(a + b*x**2)**p/(2*b*p + 2*b) + b*x**2*(a + b*x**2)**p/(2*b*p +
2*b), True))
```

3.1043 $\int \frac{\left(a+bx^2\right)^p}{x}\,dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

[Out] -1/2*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2/a)/a/(1+p)

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 65}

$$-\frac{\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^p/x,x]

[Out] -((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^p}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)\\ &= -\frac{\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$-\frac{\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^p/x,x]

```
[Out] -1/2*((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a]
)/(a*(1 + p))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x,x)
```

```
[Out] int((b*x^2+a)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x,x)
```

```
[Out] int((a + b*x^2)^p/x, x)
```

sympy [C] time = 2.49, size = 39, normalized size = 0.95

$$-\frac{b^p x^{2p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x,x)
```

```
[Out] -b**p*x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**
2))/(2*gamma(1 - p))
```

3.1044 $\int \frac{(a+bx^2)^p}{x^3}\,dx$

Optimal. Leaf size=42

$$\frac{b\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}$$

```
[Out] 1/2*b*(b*x^2+a)^(1+p)*hypergeom([2, 1+p],[2+p],1+b*x^2/a)/a^2/(1+p)
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 65}

$$\frac{b\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^3,x]
```

```
[Out] (b*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a])/(
2*a^2*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^p}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^p}{x^2}\,dx,x,x^2\right)\\ &= \frac{b\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(2,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a^2(1+p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{b\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^3,x]
```

[Out] (b*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a])/(
2*a^2*(1 + p))

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^p/x^3,x, algorithm="fricas")

[Out] integral((b*x^2 + a)^p/x^3, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^p/x^3,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^p/x^3, x)

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^p/x^3,x)

[Out] int((b*x^2+a)^p/x^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^p/x^3,x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^p/x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^p/x^3,x)

[Out] int((a + b*x^2)^p/x^3, x)

sympy [C] time = 5.13, size = 42, normalized size = 1.00

$$-\frac{b^p x^{2p}\Gamma\left(1-p\right){}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^2\Gamma\left(2-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**3,x)
```

```
[Out] -b**p*x**(2*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/
(b*x**2))/(2*x**2*gamma(2 - p))
```

3.1045 $\int x^6 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=40

$$\frac{x^7 \left(a+bx^2\right)^{p+1} {}_2F_1\left(1,p+\frac{9}{2};\frac{9}{2};-\frac{bx^2}{a}\right)}{7a}$$

```
[Out] 1/7*x^7*(b*x^2+a)^(1+p)*hypergeom([1, 9/2+p],[9/2],-b*x^2/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {365, 364}

$$\frac{1}{7}x^7 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^p,x]
```

```
[Out] (x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(1 + (
b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int x^6 \left(a+bx^2\right)^p \, dx &= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int x^6 \left(1+\frac{bx^2}{a}\right)^p \, dx \\
&= \frac{1}{7}x^7 \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.22

$$\frac{1}{7}x^7 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^p,x]
```

```
[Out] (x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(1 + (
b*x^2)/a)^p)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^6, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int x^6 \left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^p,x)
[Out] int(x^6*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^6 \left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^p,x)
[Out] int(x^6*(a + b*x^2)^p, x)
```

sympy [C] time = 11.85, size = 26, normalized size = 0.65

$$\frac{a^p x^7 {}_2F_1\left(\left.\begin{matrix}\frac{7}{2}, -p\\ \frac{9}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**p,x)
[Out] a**p*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

3.1046 $\int x^4 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=40

$$\frac{x^5 \left(a+bx^2\right)^{p+1} {}_2F_1\left(1,p+\frac{7}{2};\frac{7}{2};-\frac{bx^2}{a}\right)}{5a}$$

```
[Out] 1/5*x^5*(b*x^2+a)^(1+p)*hypergeom([1, 7/2+p],[7/2],-b*x^2/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {365, 364}

$$\frac{1}{5}x^5 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^p,x]
```

```
[Out] (x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (
b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int x^4 \left(a+bx^2\right)^p \, dx &= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int x^4 \left(1+\frac{bx^2}{a}\right)^p \, dx \\
&= \frac{1}{5}x^5 \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.22

$$\frac{1}{5}x^5 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^p,x]
```

```
[Out] (x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (
b*x^2)/a)^p)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^4, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int x^4\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^p,x)
[Out] int(x^4*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^4\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^p,x)
[Out] int(x^4*(a + b*x^2)^p, x)
```

sympy [C] time = 6.61, size = 26, normalized size = 0.65

$$\frac{a^p x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**p,x)
[Out] a**p*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

3.1047 $\int x^2 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=40

$$\frac{x^3 \left(a+bx^2\right)^{p+1} {}_2F_1\left(1,p+\frac{5}{2};\frac{5}{2};-\frac{bx^2}{a}\right)}{3a}$$

```
[Out] 1/3*x^3*(b*x^2+a)^(1+p)*hypergeom([1, 5/2+p],[5/2],-b*x^2/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {365, 364}

$$\frac{1}{3}x^3 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^p,x]
```

```
[Out] (x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(3*(1 + (
b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^2\right)^p \, dx &= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int x^2 \left(1+\frac{bx^2}{a}\right)^p \, dx\\ &= \frac{1}{3}x^3 \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.22

$$\frac{1}{3}x^3 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^p,x]
```

```
[Out] (x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(3*(1 + (
b*x^2)/a)^p)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^2, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int x^2\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^p,x)
[Out] int(x^2*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^p,x)
[Out] int(x^2*(a + b*x^2)^p, x)
```

sympy [C] time = 3.81, size = 26, normalized size = 0.65

$$\frac{a^p x^3 {}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, -p\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**p,x)
[Out] a**p*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

3.1048 $\int \left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=35

$$\frac{x\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+\frac{3}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{a}$$

[Out] x*(b*x^2+a)^(1+p)*hypergeom([1, 3/2+p],[3/2],-b*x^2/a)/a

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {246, 245}

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^p,x]

[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 44, normalized size = 1.26

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^p,x]

[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \left(b\,x^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p,x)
[Out] int((b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p, x)
```

mupad [B] time = 5.41, size = 41, normalized size = 1.17

$$\frac{x\left(b\,x^2 + a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a} + 1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p,x)
[Out] (x*(a + b*x^2)^p*hypergeom([1/2, -p], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^p
```

sympy [C] time = 2.32, size = 22, normalized size = 0.63

$$a^p x {}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p,x)
[Out] a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

3.1049 $\int \frac{(a+bx^2)^p}{x^2}\,dx$

Optimal. Leaf size=38

$$-\frac{(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{ax}$$

```
[Out] -(b*x^2+a)^(1+p)*hypergeom([1, 1/2+p],[1/2],-b*x^2/a)/a/x
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {365, 364}

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^2,x]
```

```
[Out] -(((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b
*x^2)/a)^p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{x^2}\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx\\ &= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.24

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^2,x]
```

```
[Out] -(((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b
*x^2)/a)^p))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^2, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^2,x)
```

```
[Out] int((b*x^2+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/x^2, x)
```

mupad [B] time = 5.03, size = 58, normalized size = 1.53

$$\frac{\left(b\,x^2+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2}-p,-p;\ \frac{3}{2}-p;\ -\frac{a}{b\,x^2}\right)}{x\,\left(2\,p-1\right)\,\left(\frac{a}{b\,x^2}+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^2,x)
```

```
[Out] ((a + b*x^2)^p*hypergeom([1/2 - p, -p], 3/2 - p, -a/(b*x^2)))/(x*(2*p - 1)*
(a/(b*x^2) + 1)^p)
```

sympy [C] time = 3.75, size = 26, normalized size = 0.68

$$-\frac{a^{p}{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**2,x)
```

```
[Out] -a**p*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

3.1050 $\int x^{7/2}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=42

$$\frac{2x^{9/2}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,p+\frac{13}{4};\frac{13}{4};-\frac{bx^2}{a}\right)}{9a}$$

```
[Out] 2/9*x^(9/2)*(b*x^2+a)^(1+p)*hypergeom([1, 13/4+p],[13/4],-b*x^2/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{2}{9}x^{9/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{9}{4},-p;\frac{13}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(9/2)*(a + b*x^2)^p*Hypergeometric2F1[9/4, -p, 13/4, -((b*x^2)/a)])/(9
*(1 + (b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{7/2}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{2}{9}x^{9/2}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{9}{4},-p;\frac{13}{4};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.21

$$\frac{2}{9}x^{9/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{9}{4},-p;\frac{13}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(9/2)*(a + b*x^2)^p*Hypergeometric2F1[9/4, -p, 13/4, -((b*x^2)/a)])/(9
*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^(7/2), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int x^{\frac{7}{2}}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^p,x)
[Out] int(x^(7/2)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{7/2}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2)^p,x)
[Out] int(x^(7/2)*(a + b*x^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**p,x)
[Out] Timed out
```

3.1051 $\int x^{5/2} (a + bx^2)^p \, dx$

Optimal. Leaf size=42

$$\frac{2x^{7/2} (a + bx^2)^{p+1} \, {}_2F_1\left(1, p + \frac{11}{4}; \frac{11}{4}; -\frac{bx^2}{a}\right)}{7a}$$

```
[Out] 2/7*x^(7/2)*(b*x^2+a)^(1+p)*hypergeom([1, 11/4+p],[11/4],-b*x^2/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{2}{7} x^{7/2} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{7}{4}, -p; \frac{11}{4}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(7/2)*(a + b*x^2)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^2)/a)])/(7
*(1 + (b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int x^{5/2} (a + bx^2)^p \, dx &= \left((a + bx^2)^p \left(1 + \frac{bx^2}{a}\right)^{-p}\right) \int x^{5/2} \left(1 + \frac{bx^2}{a}\right)^p \, dx \\ &= \frac{2}{7} x^{7/2} (a + bx^2)^p \left(1 + \frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{7}{4}, -p; \frac{11}{4}; -\frac{bx^2}{a}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.21

$$\frac{2}{7} x^{7/2} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{7}{4}, -p; \frac{11}{4}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(7/2)*(a + b*x^2)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^2)/a)])/(7
*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^(5/2), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}} \left(b\,x^2+a\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^p,x)
[Out] int(x^(5/2)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{5/2} \left(b\,x^2+a\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2)^p,x)
[Out] int(x^(5/2)*(a + b*x^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**p,x)
[Out] Timed out
```

3.1052 $\int x^{3/2}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=42

$$\frac{2x^{5/2}\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+\frac{9}{4};\frac{9}{4};-\frac{bx^2}{a}\right)}{5a}$$

```
[Out] 2/5*x^(5/2)*(b*x^2+a)^(1+p)*hypergeom([1, 9/4+p],[9/4],-b*x^2/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{2}{5}x^{5/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(5/2)*(a + b*x^2)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^2)/a)])/(5*
(1 + (b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{3/2}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{2}{5}x^{5/2}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.21

$$\frac{2}{5}x^{5/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(5/2)*(a + b*x^2)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^2)/a)])/(5*
(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^(3/2), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^p,x)
[Out] int(x^(3/2)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{3/2}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2)^p,x)
[Out] int(x^(3/2)*(a + b*x^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**p,x)
[Out] Timed out
```

3.1053 $\int \sqrt{x} \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=42

$$\frac{2x^{3/2}\left(a+bx^2\right)^{p+1} {}_2F_1\left(1,p+\frac{7}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3a}$$

```
[Out] 2/3*x^(3/2)*(b*x^2+a)^(1+p)*hypergeom([1, 7/4+p],[7/4],-b*x^2/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{2}{3}x^{3/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(3/2)*(a + b*x^2)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^2)/a)])/(3*
(1 + (b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^p \, dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \sqrt{x}\left(1+\frac{bx^2}{a}\right)^p \, dx\\ &= \frac{2}{3}x^{3/2}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.21

$$\frac{2}{3}x^{3/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^p,x]
```

```
[Out] (2*x^(3/2)*(a + b*x^2)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^2)/a)])/(3*
(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*sqrt(x), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2+a)^p,x)
[Out] int(x^(1/2)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x}\,\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2)^p,x)
[Out] int(x^(1/2)*(a + b*x^2)^p, x)
```

sympy [C] time = 57.09, size = 37, normalized size = 0.88

$$\frac{a^p x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x**2+a)**p,x)
[Out] a**p*x**(3/2)*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**2*exp_polar(I*pi)/a)
/(2*gamma(7/4))
```

3.1054 $\int \frac{(a+bx^2)^p}{\sqrt{x}}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{x}\,(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+\frac{5}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{a}$$

```
[Out] 2*(b*x^2+a)^(1+p)*hypergeom([1, 5/4+p],[5/4],-b*x^2/a)*x^(1/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$2\sqrt{x}\,(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(a + b*x^2)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^p
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{\sqrt{x}}\,dx &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{\sqrt{x}}\,dx\\ &= 2\sqrt{x}\,(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.22

$$2\sqrt{x}\,(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(a + b*x^2)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^p
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{\sqrt{x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^p}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/sqrt(x), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^2+a\right)^p}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^(1/2),x)
[Out] int((b*x^2+a)^p/x^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^p}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(b\,x^2+a\right)^p}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(1/2),x)
[Out] int((a + b*x^2)^p/x^(1/2), x)
```

sympy [C] time = 39.00, size = 37, normalized size = 0.92

$$\frac{a^p\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**(1/2),x)
```

```
[Out] a**p*sqrt(x)*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**2*exp_polar(I*pi)/a)/
(2*gamma(5/4))
```

3.1055 $\int \frac{(a+bx^2)^p}{x^{3/2}} \, dx$

Optimal. Leaf size=40

$$-\frac{2\left(a+bx^2\right)^{p+1} {}_2F_1\left(1,p+\frac{3}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{a\sqrt{x}}$$

```
[Out] -2*(b*x^2+a)^(1+p)*hypergeom([1, 3/4+p],[3/4],-b*x^2/a)/a/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$-\frac{2\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-\frac{1}{4},-p;\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^(3/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-1/4, -p, 3/4, -((b*x^2)/a)])/(Sqrt[x]*
(1 + (b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\left(a+bx^2\right)^p}{x^{3/2}} \, dx &= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^{3/2}} \, dx \\ &= -\frac{2\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{4},-p;\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{x}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.22

$$-\frac{2\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-\frac{1}{4},-p;\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^(3/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-1/4, -p, 3/4, -((b*x^2)/a)])/(Sqrt[x]*
(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{x^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/x^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^(3/2), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^p}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^(3/2),x)
```

```
[Out] int((b*x^2+a)^p/x^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^p}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(3/2),x)
```

```
[Out] int((a + b*x^2)^p/x^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**(3/2),x)
```

```
[Out] Timed out
```

3.1056 $\int \frac{\left(a+bx^2\right)^p}{x^{5/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+\frac{1}{4};\frac{1}{4};-\frac{bx^2}{a}\right)}{3ax^{3/2}}$$

```
[Out] -2/3*(b*x^2+a)^(1+p)*hypergeom([1, 1/4+p],[1/4],-b*x^2/a)/a/x^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$-\frac{2\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{3}{4},-p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^(5/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-3/4, -p, 1/4, -((b*x^2)/a)])/(3*x^(3/2
)*(1 + (b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^p}{x^{5/2}}\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^{5/2}}\,dx\\ &= -\frac{2\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{3}{4},-p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.21

$$-\frac{2\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{3}{4},-p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^(5/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-3/4, -p, 1/4, -((b*x^2)/a)])/(3*x^(3/2
)*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^2+a)^p}{x^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/x^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2+a)^p}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^(5/2), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^(5/2),x)
```

```
[Out] int((b*x^2+a)^p/x^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2+a)^p}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/x^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x^2+a)^p}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(5/2),x)
```

```
[Out] int((a + b*x^2)^p/x^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**(5/2),x)
```

```
[Out] Timed out
```

3.1057 $\int \frac{(a+bx^2)^p}{x^{7/2}}\, dx$

Optimal. Leaf size=42

$$-\frac{2\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p-\frac{1}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5ax^{5/2}}$$

```
[Out] -2/5*(b*x^2+a)^(1+p)*hypergeom([1, -1/4+p],[-1/4],-b*x^2/a)/a/x^(5/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$-\frac{2\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{5}{4},-p;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^(7/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-5/4, -p, -1/4, -((b*x^2)/a)])/(5*x^(5/
2)*(1 + (b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^p}{x^{7/2}}\, dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^{7/2}}\, dx\\ &= -\frac{2\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{5}{4},-p;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.21

$$-\frac{2\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{5}{4},-p;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^(7/2),x]
```

```
[Out] (-2*(a + b*x^2)^p*Hypergeometric2F1[-5/4, -p, -1/4, -((b*x^2)/a)])/(5*x^(5/
2)*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{x^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/x^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{x^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^(7/2), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^p}{x^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^(7/2),x)
```

```
[Out] int((b*x^2+a)^p/x^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{x^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/x^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^p}{x^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(7/2),x)
```

```
[Out] int((a + b*x^2)^p/x^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**(7/2),x)
```

```
[Out] Timed out
```

3.1058 $\int x^m \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=53

$$\frac{x^{m+1}\left(a+bx^2\right)^{p+1}\, {}_2F_1\left(1,\frac{1}{2}(m+2p+3);\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*(b*x^2+a)^(1+p)*hypergeom([1, 3/2+1/2*m+p],[3/2+1/2*m],-b*x^2/a)/a/
(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {365, 364}

$$\frac{x^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\, {}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{bx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^p,x]
```

```
[Out] (x^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -((b*x
^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^m\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{x^{1+m}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\, {}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{bx^2}{a}\right)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.01, size = 63, normalized size = 1.19

$$\frac{x^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\, {}_2F_1\left(\frac{m+1}{2},-p;\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^p,x]
```

```
[Out] (x^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2, -(
(b*x^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^m, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int x^m \left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^p,x)
```

```
[Out] int(x^m*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^p,x)
```

```
[Out] int(x^m*(a + b*x^2)^p, x)
```

sympy [C] time = 21.28, size = 51, normalized size = 0.96

$$\frac{a^p x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\left.\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**p,x)
```

```
[Out] a**p*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

3.1059 $\int (cx)^m \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=66

$$\frac{(cx)^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{bx^2}{a}\right)}{c(m+1)}$$

```
[Out] (c*x)^(1+m)*(b*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/c/(
1+m)/((1+b*x^2/a)^p)
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{(cx)^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{bx^2}{a}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x^2)^p,x]
```

```
[Out] ((c*x)^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -(
(b*x^2)/a)])/(c*(1 + m)*(1 + (b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(a+bx^2\right)^p \, dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int (cx)^m\left(1+\frac{bx^2}{a}\right)^p \, dx\\ &= \frac{(cx)^{1+m}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{bx^2}{a}\right)}{c(1+m)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 64, normalized size = 0.97

$$\frac{x(cx)^m\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x^2)^p,x]
```

```
[Out] (x*(c*x)^m*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2, -(
(b*x^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p (cx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(c*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*(c*x)^m, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (cx)^m \left(b\,x^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2+a)^p,x)
[Out] int((c*x)^m*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*(c*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^m \left(b\,x^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x^2)^p,x)
[Out] int((c*x)^m*(a + b*x^2)^p, x)
```

sympy [C] time = 21.46, size = 54, normalized size = 0.82

$$\frac{a^p c^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\left.\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**2+a)**p,x)
```

```
[Out] a**p*c**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), b*x*
*2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

3.1060 $\int x^{-8-2p} \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=53

$$-\frac{x^{-2p-7} \left(a+bx^2\right)^{p+1} {}_2F_1\left(-\frac{5}{2},1;\frac{1}{2}(-2p-5);-\frac{bx^2}{a}\right)}{a(2p+7)}$$

```
[Out] -x^(-7-2*p)*(b*x^2+a)^(1+p)*hypergeom([-5/2, 1],[-5/2-p],-b*x^2/a)/a/(7+2*p
)
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$-\frac{x^{-2p-7} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{1}{2}(-2p-7),-p;\frac{1}{2}(-2p-5);-\frac{bx^2}{a}\right)}{2p+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-8 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-7 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-7 - 2*p)/2, -p, (-5 - 2*p
)/2, -((b*x^2)/a)])/((7 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int x^{-8-2p} \left(a+bx^2\right)^p \, dx &= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int x^{-8-2p} \left(1+\frac{bx^2}{a}\right)^p \, dx \\
&= -\frac{x^{-7-2p} \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2}(-7-2p),-p;\frac{1}{2}(-5-2p);-\frac{bx^2}{a}\right)}{7+2p}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 1.25

$$-\frac{x^{-2p-7} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-p-\frac{7}{2},-p;-p-\frac{5}{2};-\frac{bx^2}{a}\right)}{2p+7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-8 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-7 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-7/2 - p, -p, -5/2 - p, -((
b*x^2)/a)])/((7 + 2*p)*(1 + (b*x^2)/a)^p))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^{-2p-8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-8-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^(-2*p - 8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p-8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-8-2*p)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 8), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int x^{-2p-8}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-8-2*p)*(b*x^2+a)^p,x)
[Out] int(x^(-8-2*p)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p-8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-8-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^p}{x^{2\,p+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 8),x)
[Out] int((a + b*x^2)^p/x^(2*p + 8), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-8-2*p)*(b*x**2+a)**p,x)
[Out] Timed out
```

3.1061 $\int x^{-7-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=105

$$-\frac{b^2x^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{a^3(p+1)(p+2)(p+3)}+\frac{bx^{-2(p+2)}\left(a+bx^2\right)^{p+1}}{a^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(a+bx^2\right)^{p+1}}{2a(p+3)}$$

```
[Out] -b^2*(b*x^2+a)^(1+p)/a^3/(2+p)/(p^2+4*p+3)/(x^(2+2*p))+b*(b*x^2+a)^(1+p)/a^
2/(2+p)/(3+p)/(x^(4+2*p))-1/2*(b*x^2+a)^(1+p)/a/(3+p)/(x^(6+2*p))
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {271, 264}

$$-\frac{b^2x^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{a^3(p+1)(p+2)(p+3)}+\frac{bx^{-2(p+2)}\left(a+bx^2\right)^{p+1}}{a^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(a+bx^2\right)^{p+1}}{2a(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-7 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((b^2*(a + b*x^2)^(1 + p))/(a^3*(1 + p)*(2 + p)*(3 + p)*x^(2*(1 + p)))) +
(b*(a + b*x^2)^(1 + p))/(a^2*(2 + p)*(3 + p)*x^(2*(2 + p))) - (a + b*x^2)^(
1 + p)/(2*a*(3 + p)*x^(2*(3 + p)))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^{-7-2p}\left(a+bx^2\right)^p\,dx &= -\frac{x^{-2(3+p)}\left(a+bx^2\right)^{1+p}}{2a(3+p)}-\frac{(2b)\int x^{-5-2p}\left(a+bx^2\right)^p\,dx}{a(3+p)}\\
&= \frac{bx^{-2(2+p)}\left(a+bx^2\right)^{1+p}}{a^2(2+p)(3+p)}-\frac{x^{-2(3+p)}\left(a+bx^2\right)^{1+p}}{2a(3+p)}+\frac{\left(2b^2\right)\int x^{-3-2p}\left(a+bx^2\right)^p\,dx}{a^2(2+p)(3+p)}\\
&= -\frac{b^2x^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{a^3(1+p)(2+p)(3+p)}+\frac{bx^{-2(2+p)}\left(a+bx^2\right)^{1+p}}{a^2(2+p)(3+p)}-\frac{x^{-2(3+p)}\left(a+bx^2\right)^{1+p}}{2a(3+p)}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 62, normalized size = 0.59

$$-\frac{x^{-2(p+3)}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-p-3,-p;-p-2;-\frac{bx^2}{a}\right)}{2(p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-7 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -1/2*((a + b*x^2)^p*Hypergeometric2F1[-3 - p, -p, -2 - p, -((b*x^2)/a)])/((
3 + p)*x^(2*(3 + p))*(1 + (b*x^2)/a)^p)
```

fricas [A] time = 0.71, size = 106, normalized size = 1.01

$$-\frac{\left(2\,b^{3}x^{7}-2\,ab^{2}px^{5}+\left(a^{2}bp^{2}+a^{2}bp\right)x^{3}+\left(a^{3}p^{2}+3\,a^{3}p+2\,a^{3}\right)x\right)\left(bx^{2}+a\right)^{p}x^{-2\,p-7}}{2\,\left(a^{3}p^{3}+6\,a^{3}p^{2}+11\,a^{3}p+6\,a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*x^7 - 2*a*b^2*p*x^5 + (a^2*b*p^2 + a^2*b*p)*x^3 + (a^3*p^2 + 3*
a^3*p + 2*a^3)*x)*(b*x^2 + a)^p*x^(-2*p - 7)/(a^3*p^3 + 6*a^3*p^2 + 11*a^3*
p + 6*a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^{2}+a\right)^{p}x^{-2\,p-7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 7), x)
```

maple [A] time = 0.01, size = 81, normalized size = 0.77

$$-\frac{\left(2b^{2}x^{4}-2abp\,x^{2}+a^{2}p^{2}-2ab\,x^{2}+3a^{2}p+2a^{2}\right)x^{-2p-6}\left(b\,x^{2}+a\right)^{p+1}}{2\left(p+3\right)\left(p+2\right)\left(p+1\right)a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-7-2*p)*(b*x^2+a)^p,x)
```

```
[Out] -1/2*(b*x^2+a)^(p+1)*x^(-6-2*p)*(2*b^2*x^4-2*a*b*p*x^2+a^2*p^2-2*a*b*x^2+3*
a^2*p+2*a^2)/(3+p)/(p+2)/(p+1)/a^3
```

maxima [A] time = 1.46, size = 84, normalized size = 0.80

$$-\frac{\left(2\,b^{3}x^{6}-2\,ab^{2}px^{4}+\left(p^{2}+p\right)a^{2}bx^{2}+\left(p^{2}+3\,p+2\right)a^{3}\right)e^{\left(p\log\left(bx^{2}+a\right)-2\,p\log\left(x\right)\right)}}{2\,\left(p^{3}+6\,p^{2}+11\,p+6\right)a^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3*x^6 - 2*a*b^2*p*x^4 + (p^2 + p)*a^2*b*x^2 + (p^2 + 3*p + 2)*a^3
)*e^(p*log(b*x^2 + a) - 2*p*log(x))/((p^3 + 6*p^2 + 11*p + 6)*a^3*x^6)
```

mupad [B] time = 5.09, size = 154, normalized size = 1.47

$$-\left(b\,x^{2}+a\right)^{p}\,\left(\frac{x\,\left(p^{2}+3\,p+2\right)}{2\,x^{2\,p+7}\,\left(p^{3}+6\,p^{2}+11\,p+6\right)}+\frac{b^{3}\,x^{7}}{a^{3}\,x^{2\,p+7}\,\left(p^{3}+6\,p^{2}+11\,p+6\right)}-\frac{b^{2}\,p\,x^{5}}{a^{2}\,x^{2\,p+7}\,\left(p^{3}+6\,p^{2}+11\,p+6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 7),x)
```

```
[Out] -(a + b*x^2)^p*((x*(3*p + p^2 + 2))/(2*x^(2*p + 7)*(11*p + 6*p^2 + p^3 + 6)
) + (b^3*x^7)/(a^3*x^(2*p + 7)*(11*p + 6*p^2 + p^3 + 6)) - (b^2*p*x^5)/(a^2
*x^(2*p + 7)*(11*p + 6*p^2 + p^3 + 6)) + (b*p*x^3*(p + 1))/(2*a*x^(2*p + 7)
*(11*p + 6*p^2 + p^3 + 6)))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-7-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

3.1062 $\int x^{-6-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=53

$$-\frac{x^{-2p-5}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(-\frac{3}{2},1;\frac{1}{2}(-2p-3);-\frac{bx^2}{a}\right)}{a(2p+5)}$$

```
[Out] -x^(-5-2*p)*(b*x^2+a)^(1+p)*hypergeom([-3/2, 1],[-3/2-p],-b*x^2/a)/a/(5+2*p
)
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$-\frac{x^{-2p-5}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-2p-5),-p;\frac{1}{2}(-2p-3);-\frac{bx^2}{a}\right)}{2p+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-6 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-5 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-5 - 2*p)/2, -p, (-3 - 2*p
)/2, -((b*x^2)/a)])/((5 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-6-2p}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{-6-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= -\frac{x^{-5-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-5-2p),-p;\frac{1}{2}(-3-2p);-\frac{bx^2}{a}\right)}{5+2p}\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 1.25

$$-\frac{x^{-2p-5}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-p-\frac{5}{2},-p;-p-\frac{3}{2};-\frac{bx^2}{a}\right)}{2p+5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-6 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-5 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-5/2 - p, -p, -3/2 - p, -((
b*x^2)/a)])/((5 + 2*p)*(1 + (b*x^2)/a)^p))
```

fricas **[F]** time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^{-2p-6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(-2*p - 6), x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p-6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 6), x)
```

maple **[F]** time = 0.28, size = 0, normalized size = 0.00

$$\int x^{-2p-6} \left(b\,x^2+a\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2*p-6)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(-2*p-6)*(b*x^2+a)^p,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p-6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-6-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 6), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^p}{x^{2\,p+6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 6),x)
```

```
[Out] int((a + b*x^2)^p/x^(2*p + 6), x)
```

sympy **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-6-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1063 $\int x^{-5-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=67

$$\frac{bx^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{2a^2(p+1)(p+2)}-\frac{x^{-2(p+2)}\left(a+bx^2\right)^{p+1}}{2a(p+2)}$$

```
[Out] 1/2*b*(b*x^2+a)^(1+p)/a^2/(1+p)/(2+p)/(x^(2+2*p))-1/2*(b*x^2+a)^(1+p)/a/(2+
p)/(x^(4+2*p))
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {271, 264}

$$\frac{bx^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{2a^2(p+1)(p+2)}-\frac{x^{-2(p+2)}\left(a+bx^2\right)^{p+1}}{2a(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (b*(a + b*x^2)^(1 + p))/(2*a^2*(1 + p)*(2 + p)*x^(2*(1 + p))) - (a + b*x^2)
^(1 + p)/(2*a*(2 + p)*x^(2*(2 + p)))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^{-5-2p}\left(a+bx^2\right)^p\,dx&=-\frac{x^{-2(2+p)}\left(a+bx^2\right)^{1+p}}{2a(2+p)}-\frac{b\int x^{-3-2p}\left(a+bx^2\right)^p\,dx}{a(2+p)}\\&=\frac{bx^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{2a^2(1+p)(2+p)}-\frac{x^{-2(2+p)}\left(a+bx^2\right)^{1+p}}{2a(2+p)}\end{aligned}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.93

$$-\frac{x^{-2(p+2)}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-p-2,-p;-p-1;-\frac{bx^2}{a}\right)}{2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -1/2*((a + b*x^2)^p*Hypergeometric2F1[-2 - p, -p, -1 - p, -((b*x^2)/a)])/((
2 + p)*x^(2*(2 + p))*(1 + (b*x^2)/a)^p)
```

fricas [A] time = 0.70, size = 67, normalized size = 1.00

$$\frac{\left(b^2x^5 - abpx^3 - \left(a^2p + a^2\right)x\right)\left(bx^2 + a\right)^p x^{-2\,p-5}}{2\left(a^2p^2 + 3\,a^2p + 2\,a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^5 - a*b*p*x^3 - (a^2*p + a^2)*x)*(b*x^2 + a)^p*x^(-2*p - 5)/(a^2
*p^2 + 3*a^2*p + 2*a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^p x^{-2\,p-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 5), x)
```

maple [A] time = 0.00, size = 45, normalized size = 0.67

$$-\frac{\left(-b\,x^2 + ap + a\right)x^{-2p-4}\left(b\,x^2 + a\right)^{p+1}}{2\left(p+2\right)\left(p+1\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-5-2*p)*(b*x^2+a)^p,x)
```

```
[Out] -1/2*(b*x^2+a)^(p+1)*x^(-4-2*p)*(-b*x^2+a*p+a)/(p+2)/(p+1)/a^2
```

maxima [A] time = 1.41, size = 59, normalized size = 0.88

$$\frac{\left(b^2x^4 - abpx^2 - a^2\left(p+1\right)\right)e^{\left(p\log\left(bx^2+a\right) - 2\,p\log\left(x\right)\right)}}{2\left(p^2 + 3\,p + 2\right)a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*x^4 - a*b*p*x^2 - a^2*(p + 1))*e^(p*log(b*x^2 + a) - 2*p*log(x))/(
(p^2 + 3*p + 2)*a^2*x^4)
```

mupad [B] time = 5.02, size = 96, normalized size = 1.43

$$-\left(b\,x^2 + a\right)^p\left(\frac{x\left(p+1\right)}{2\,x^{2\,p+5}\left(p^2 + 3\,p + 2\right)} - \frac{b^2\,x^5}{2\,a^2\,x^{2\,p+5}\left(p^2 + 3\,p + 2\right)} + \frac{b\,p\,x^3}{2\,a\,x^{2\,p+5}\left(p^2 + 3\,p + 2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 5),x)
```

```
[Out] -(a + b*x^2)^p*((x*(p + 1))/(2*x^(2*p + 5)*(3*p + p^2 + 2)) - (b^2*x^5)/(2*
a^2*x^(2*p + 5)*(3*p + p^2 + 2)) + (b*p*x^3)/(2*a*x^(2*p + 5)*(3*p + p^2 +
2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-5-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

3.1064 $\int x^{-4-2p} \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=53

$$-\frac{x^{-2p-3} \left(a+bx^2\right)^{p+1} {}_2F_1\left(-\frac{1}{2},1;\frac{1}{2}(-2p-1);-\frac{bx^2}{a}\right)}{a(2p+3)}$$

```
[Out] -x^(-3-2*p)*(b*x^2+a)^(1+p)*hypergeom([-1/2, 1],[-1/2-p],-b*x^2/a)/a/(3+2*p
)
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$-\frac{x^{-2p-3} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{1}{2}(-2p-3),-p;\frac{1}{2}(-2p-1);-\frac{bx^2}{a}\right)}{2p+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-3 - 2*p)/2, -p, (-1 - 2*p
)/2, -((b*x^2)/a)])/((3 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-4-2p} \left(a+bx^2\right)^p \, dx &= \left(\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p}\right) \int x^{-4-2p} \left(1+\frac{bx^2}{a}\right)^p \, dx\\ &= -\frac{x^{-3-2p} \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2}(-3-2p),-p;\frac{1}{2}(-1-2p);-\frac{bx^2}{a}\right)}{3+2p}\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 1.25

$$-\frac{x^{-2p-3} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-p-\frac{3}{2},-p;-p-\frac{1}{2};-\frac{bx^2}{a}\right)}{2p+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-3/2 - p, -p, -1/2 - p, -((
b*x^2)/a)])/((3 + 2*p)*(1 + (b*x^2)/a)^p))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^{-2p-4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^(-2*p - 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 4), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int x^{-2p-4}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2*p-4)*(b*x^2+a)^p,x)
[Out] int(x^(-2*p-4)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^p}{x^{2\,p+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 4),x)
[Out] int((a + b*x^2)^p/x^(2*p + 4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4-2*p)*(b*x**2+a)**p,x)
[Out] Timed out
```

3.1065 $\int x^{-3-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=30

$$-\frac{x^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{2a(p+1)}$$

```
[Out] -1/2*(b*x^2+a)^(1+p)/a/(1+p)/(x^(2+2*p))
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {264}

$$-\frac{x^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -(a + b*x^2)^(1 + p)/(2*a*(1 + p)*x^(2*(1 + p)))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-3-2p}\left(a+bx^2\right)^p\,dx=-\frac{x^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{2a(1+p)}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.97

$$\frac{x^{-2p-2}\left(a+bx^2\right)^{p+1}}{a(-2p-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(-2 - 2*p)*(a + b*x^2)^(1 + p))/(a*(-2 - 2*p))
```

fricas [A] time = 0.67, size = 34, normalized size = 1.13

$$-\frac{\left(bx^3+ax\right)\left(bx^2+a\right)^p x^{-2p-3}}{2\left(ap+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^3 + a*x)*(b*x^2 + a)^p*x^(-2*p - 3)/(a*p + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^p x^{-2p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 3), x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.97

$$-\frac{x^{-2p-2}\left(b\,x^{2}+a\right)^{p+1}}{2\left(p+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-2*p)*(b*x^2+a)^p,x)
```

```
[Out] -1/2*x^(-2-2*p)*(b*x^2+a)^(p+1)/a/(p+1)
```

maxima [A] time = 1.48, size = 37, normalized size = 1.23

$$-\frac{\left(bx^{2}+a\right)e^{\left(p\log\left(bx^{2}+a\right)-2\,p\log\left(x\right)\right)}}{2\,a\left(p+1\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)*e^(p*log(b*x^2 + a) - 2*p*log(x))/(a*(p + 1)*x^2)
```

mupad [B] time = 5.05, size = 52, normalized size = 1.73

$$-\left(b\,x^{2}+a\right)^{p}\left(\frac{x}{2\,x^{2\,p+3}\,\left(p+1\right)}+\frac{b\,x^{3}}{2\,a\,x^{2\,p+3}\,\left(p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 3),x)
```

```
[Out] -(a + b*x^2)^p*(x/(2*x^(2*p + 3)*(p + 1)) + (b*x^3)/(2*a*x^(2*p + 3)*(p + 1
)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

3.1066 $\int x^{-2-2p} \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=53

$$-\frac{x^{-2p-1}\left(a+bx^2\right)^{p+1} {}_2F_1\left(\frac{1}{2},1;\frac{1}{2}(1-2p);-\frac{bx^2}{a}\right)}{a(2p+1)}$$

```
[Out] -x^(-1-2*p)*(b*x^2+a)^(1+p)*hypergeom([1/2, 1],[1/2-p],-b*x^2/a)/a/(1+2*p)
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$-\frac{x^{-2p-1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(\frac{1}{2}(-2p-1),-p;\frac{1}{2}(1-2p);-\frac{bx^2}{a}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-1 - 2*p)/2, -p, (1 - 2*p)
/2, -((b*x^2)/a)])/((1 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-2-2p}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{-2-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= -\frac{x^{-1-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1}{2}(-1-2p),-p;\frac{1}{2}(1-2p);-\frac{bx^2}{a}\right)}{1+2p}\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 1.25

$$-\frac{x^{-2p-1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-p-\frac{1}{2},-p;\frac{1}{2}-p;-\frac{bx^2}{a}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((x^(-1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-1/2 - p, -p, 1/2 - p, -((b
*x^2)/a)])/((1 + 2*p)*(1 + (b*x^2)/a)^p))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^{-2p-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^(-2*p - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 2), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int x^{-2p-2}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2*p-2)*(b*x^2+a)^p,x)
```

```
[Out] int(x^(-2*p-2)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^p}{x^{2\,p+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 2),x)
```

```
[Out] int((a + b*x^2)^p/x^(2*p + 2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

3.1067 $\int x^{-1-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=43

$$-\frac{x^{-2p}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,1;1-p;-\frac{bx^2}{a}\right)}{2ap}$$

```
[Out] -1/2*(b*x^2+a)^(1+p)*hypergeom([1, 1],[1-p],-b*x^2/a)/a/p/(x^(2*p))
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$-\frac{x^{-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-p,-p;1-p;-\frac{bx^2}{a}\right)}{2p}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((a + b*x^2)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^2)/a)])/(2*p*x^(2*p
)*(1 + (b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-1-2p}\left(a+bx^2\right)^p\,dx&=\left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{-1-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\&=-\frac{x^{-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-p,-p;1-p;-\frac{bx^2}{a}\right)}{2p}\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.30

$$-\frac{x^{-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-p,-p;1-p;-\frac{bx^2}{a}\right)}{2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -1/2*((a + b*x^2)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^2)/a)])/(p*x^(2
*p)*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^{-2p-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^(-2*p - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*p)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 1), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int x^{-2p-1}\left(b\,x^2+a\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*p)*(b*x^2+a)^p,x)
[Out] int(x^(-1-2*p)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^p}{x^{2\,p+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 1),x)
[Out] int((a + b*x^2)^p/x^(2*p + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*p)*(b*x**2+a)**p,x)
[Out] Timed out
```

3.1068 $\int x^{-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=52

$$\frac{x^{1-2p}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,\frac{3}{2};\frac{1}{2}(3-2p);-\frac{bx^2}{a}\right)}{a(1-2p)}$$

```
[Out] x^(1-2*p)*(b*x^2+a)^(1+p)*hypergeom([1, 3/2],[3/2-p],-b*x^2/a)/a/(1-2*p)
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {365, 364}

$$\frac{x^{1-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(1-2p),-p;\frac{1}{2}(3-2p);-\frac{bx^2}{a}\right)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/x^(2*p),x]
```

```
[Out] (x^(1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(1 - 2*p)/2, -p, (3 - 2*p)/2, -((b*x^2)/a)])/((1 - 2*p)*(1 + (b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-2p}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{x^{1-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(1-2p),-p;\frac{1}{2}(3-2p);-\frac{bx^2}{a}\right)}{1-2p}\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.25

$$\frac{x^{1-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}-p,-p;\frac{3}{2}-p;-\frac{bx^2}{a}\right)}{1-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/x^(2*p),x]
```

```
[Out] (x^(1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[1/2 - p, -p, 3/2 - p, -((b*x^2
)/a)])/((1 - 2*p)*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{x^{2p}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(x^(2*p)),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/x^(2*p), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{x^{2p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(x^(2*p)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/x^(2*p), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int x^{-2p}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(x^(2*p)),x)
```

```
[Out] int((b*x^2+a)^p/(x^(2*p)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^p}{x^{2p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(x^(2*p)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/x^(2*p), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^p}{x^{2\,p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p),x)
```

```
[Out] int((a + b*x^2)^p/x^(2*p), x)
```

sympy [C] time = 15.08, size = 24, normalized size = 0.46

$$b^p x\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\right|\frac{ae^{i\pi}}{bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(x**(2*p)),x)
```

```
[Out] b**p*x*hyper((-1/2, -p), (1/2,), a*exp_polar(I*pi)/(b*x**2))
```

3.1069 $\int x^{1-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=49

$$\frac{x^{2-2p}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,2;2-p;-\frac{bx^2}{a}\right)}{2a(1-p)}$$

```
[Out] 1/2*x^(2-2*p)*(b*x^2+a)^(1+p)*hypergeom([1, 2],[2-p],-b*x^2/a)/a/(1-p)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{x^{2-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^2}{a}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(2 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x^2)/a)
])/(2*(1 - p)*(1 + (b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{1-2p}\left(a+bx^2\right)^p\,dx&=\left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{1-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\&=\frac{x^{2-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^2}{a}\right)}{2(1-p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 1.24

$$\frac{x^{2-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^2}{a}\right)}{2-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(2 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x^2)/a)
])/((2 - 2*p)*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^{-2p+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^(-2*p + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-2*p)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 1), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int x^{-2p+1}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1-2*p)*(b*x^2+a)^p,x)
[Out] int(x^(1-2*p)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{1-2p}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1 - 2*p)*(a + b*x^2)^p,x)
[Out] int(x^(1 - 2*p)*(a + b*x^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1-2*p)*(b*x**2+a)**p,x)
[Out] Timed out
```

3.1070 $\int x^{2-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=52

$$\frac{x^{3-2p}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,\frac{5}{2};\frac{1}{2}(5-2p);-\frac{bx^2}{a}\right)}{a(3-2p)}$$

```
[Out] x^(3-2*p)*(b*x^2+a)^(1+p)*hypergeom([1, 5/2],[5/2-p],-b*x^2/a)/a/(3-2*p)
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{x^{3-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(3-2p),-p;\frac{1}{2}(5-2p);-\frac{bx^2}{a}\right)}{3-2p}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(3 - 2*p)/2, -p, (5 - 2*p)/2, -((b*x^2)/a)])/((3 - 2*p)*(1 + (b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2-2p}\left(a+bx^2\right)^p\,dx&=\left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{2-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\&=\frac{x^{3-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(3-2p),-p;\frac{1}{2}(5-2p);-\frac{bx^2}{a}\right)}{3-2p}\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.25

$$\frac{x^{3-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{3}{2}-p,-p;\frac{5}{2}-p;-\frac{bx^2}{a}\right)}{3-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[3/2 - p, -p, 5/2 - p, -((b*x^2
)/a)])/((3 - 2*p)*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^{-2p+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^(-2*p + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2-2*p)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 2), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int x^{-2p+2}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2*p+2)*(b*x^2+a)^p,x)
[Out] int(x^(-2*p+2)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{2-2p}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2 - 2*p)*(a + b*x^2)^p,x)
[Out] int(x^(2 - 2*p)*(a + b*x^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2-2*p)*(b*x**2+a)**p,x)
[Out] Timed out
```

3.1071 $\int x^{3-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=49

$$\frac{x^{4-2p}\left(a+bx^2\right)^{p+1}\,{}_2F_1\left(1,3;3-p;-\frac{bx^2}{a}\right)}{2a(2-p)}$$

```
[Out] 1/2*x^(4-2*p)*(b*x^2+a)^(1+p)*hypergeom([1, 3],[3-p],-b*x^2/a)/a/(2-p)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{x^{4-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(2-p,-p;3-p;-\frac{bx^2}{a}\right)}{2(2-p)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(4 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x^2)/a)
])/(2*(2 - p)*(1 + (b*x^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3-2p}\left(a+bx^2\right)^p\,dx &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^{3-2p}\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{x^{4-2p}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(2-p,-p;3-p;-\frac{bx^2}{a}\right)}{2(2-p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 1.24

$$\frac{x^{4-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(2-p,-p;3-p;-\frac{bx^2}{a}\right)}{4-2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(4 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x^2)/a)
])/((4 - 2*p)*(1 + (b*x^2)/a)^p)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2+a\right)^p x^{-2p+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^(-2*p + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-2*p)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 3), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int x^{-2p+3}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2*p+3)*(b*x^2+a)^p,x)
[Out] int(x^(-2*p+3)*(b*x^2+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^(-2*p + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{3-2p}\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3 - 2*p)*(a + b*x^2)^p,x)
[Out] int(x^(3 - 2*p)*(a + b*x^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3-2*p)*(b*x**2+a)**p,x)
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```